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TWO POINT EXPONENTIAL APPROXIMATION METHOD FOR STRUCTURAL OPTIMIZATION OF PROBLEMS WITH FREQUENCY CONSTRAINTS

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TWO POINT EXPONENTIAL APPROXIMATION METHOD FOR STRUCTURAL OPTIMIZATION OF PROBLEMS WITH FREQUENCY CONSTRAINTS

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Abstract The Two Point Exponential Approximation Method was introduced by Fadel et al. (Fadel, 1990), and tested on structural optimization problems with stress and displacement constraints. The results reported in earlier papers were promising, and the method, which consists in correcting Taylor series approximations using previous design history, is tested in the present paper on optimization problems with frequency constraints. The aim of the research is to verify the robustness and speed of convergence of the Two Point Exponential Approximation method when highly non-linear constraints are used.

Introduction

In the practice of optimization, especially when complex structural, thermal, aerodynamic or other analyses are needed, the computer time required to perform the analyses is critical. Most large optimization problems have been formulated such that the number of full scale analyses are minimal. This is generally accomplished by reducing the original problem to an approximate, simpler model which can be optimized within certain constraints. The original problem is then solved with the optimized approximate design variables, and iterations are performed until overall convergence is attained. The critical aspect of the procedure is the quality of the approximation. For a very highly non-linear problem, linear approximations are valid only in a very small domain around the original design point, whereas in better behaved problems, larger moves can be accomplished. The trade-off between the quality of approximation and number of real analyses is what dictates the overall time needed for reaching the optimum (if at all reachable).

Derivation of the Two Point Exponential Approximation

Several traditional approximation methods were summarized in the paper by Fadel et al (Fadel, 1990) ranging from the simple Taylor series in the form:
\[ g(X) = g(X_0) + \sum_i (x_i - x_{oi}) \frac{\partial g(X_0)}{\partial x_i} \]

to the reciprocal, hybrid, and higher order approximations. The authors then introduced the Two Point Exponential approximation which is an extension of the simpler Taylor series, adjusted by matching the derivatives at the previous design point. This correction term is incorporated into an exponent which is computed after each real analysis for each constraint, and with respect to each design variable. The exponent acts as a measure of goodness of fit: If the linear approximation is valid for a certain constraint, the exponent is close to or equal to 1, if the reciprocal approximation is more appropriate, the exponent approaches or is equal to -1. In other cases, the exponent varies between -1 and 1, correcting the approximation and improving the fit of the data.

The Two Point Exponential Approximation is derived as mentioned earlier by matching the slopes at previous design points. Initially, one substitutes \( x^{pi} \) for \( x \) in the Taylor series:

\[ g(X) = g(X_0) + \sum_i (x^{pi}_i - x_{oi}) \frac{\partial g(X_0)}{\partial x^{pi}_i} \]

and after resubstitution, one can write:

\[ g(X) = g(X_0) + \sum_i \left( \frac{x_i}{x_{oi}} \right)^{p_i} \frac{x_{oi}}{p_i} \frac{\partial g(X_0)}{\partial x_i} \]

with the exponent evaluated according to:

\[ p_i = \frac{\log \left( \frac{\partial g(x_i)}{\partial x_i} \right)}{\log \left( \frac{x_{o1}}{x_{oi}} \right)} \]

The point \( X_1 \) refers to the design point at the previous iteration and \( X_0 \) refers to the current design point from where the approximation is carried out. Note that at the first iteration, since no previous design history exists, a linear or reciprocal step is carried out, depending on the preference of the user.
The results reported in the earlier paper compared the linear, reciprocal and Two Point Exponential approximations on structural problems with stress and displacement constraints. Three problems of different sizes were used, namely the standard three bar truss problem, a 25 bar truss transmission tower, and a 52 bar truss tower. The results showed that the Two Point Exponential approximation generally displayed a much smoother behavior than the other two methods. It contributed to reducing the oscillations between successive iterations, and required less iterations to reach the optimum in most cases. The overhead involved in computing the exponents proved to be insignificant. The exponents have to be computed after each real analysis and used during the optimization of the approximate problem. Care has to be taken in the code development to avoid divisions by zero, and to avoid having to compute the logarithm of a negative number. In such cases, the algorithm should be written in a way that it would revert to a linear or reciprocal step.

Two Point Exponential Approximation and Frequency Constraints

After ascertaining the merit of the approximation in the case of stress and displacement constraints, it was suggested to test the method on frequency type constraints. The frequency constraints are generally highly non-linear, and further testing of the method was warranted to confirm its value for general structural optimization problems. For this purpose, two test problems of different complexity and size were selected. The approximation method is tested on both the problems, and results and conclusions are reported. Both problems were taken from the literature to ensure correctness.

Test problem: Cantilever Beam

The first test problem is taken from Pritchard and Adelman (Pritchard, 1990). The 193 inch long hollow cantilever beam with square cross section (Figure 1) has four design variables: the height and width of the beam cross-section, and the two wall thicknesses (sides, top and bottom). The beam is divided into ten elements. The first element near the base has a slightly different modulus of elasticity, but all other characteristics are uniform over the length of the beam. The dimensions and physical characteristics of the standard beam $X_0$ are:

\[
\begin{align*}
H &= 5.00 \text{ in} \\
B &= 3.75 \text{ in} \\
t &= 0.80 \text{ in} \\
d &= 0.10 \text{ in}
\end{align*}
\]
and moduli of elasticity: (Element 1 is at the wall)

\[
\begin{align*}
E_{2-10} &= 5.85 \times 10^6 \\
E_1 &= 4.90 \times 10^6
\end{align*}
\]

The problem was analyzed using the ANSYS (Swanson, 1990) finite element package, and optimizations were carried out with the program CONMIN (Vanderplaats, 1973). The first test consisted in evaluating the approximations to the first bending frequency when one variable was modified. The height of the beam cross section was selected as the design variable, and the results are tabulated in Table 1.

<table>
<thead>
<tr>
<th>H</th>
<th>linear</th>
<th>reciprocal</th>
<th>exact</th>
<th>2 pt exp.</th>
<th>H</th>
<th>rel err lin [%]</th>
<th>rel err 2pt [%]</th>
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<td>-3.64094</td>
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<td>5.31106</td>
<td>5.31106</td>
<td>5.31106</td>
<td>5</td>
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<td>6.305727</td>
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<td>1.002004366</td>
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\[
\text{dfdh0} = (H=5) \quad 1.1936
\]

\[
\text{dfdh1} = (H=6) \quad 1.17833
\]

Table 1. Cantilever Beam analysis. Evaluation of approximations based on Beam Height H.
These results are illustrated in Figure 2. The errors resulting from both the linear and Two Point Exponential approximations are plotted as a function of the beam height H. The reference point $X_0$ is the point $H=5$, and the Two point Exponential approximation uses the previous analysis point at $X_1$ as the point where $H=6$. The graphs show the superior performance of the new approximation in the case of changes in one design variable.

![Figure 2. Approximation error as a function of beam height H](image)

When one considers variations in multiple variables simultaneously, the advantages of one approximation versus another are less easily demonstrated. In this study, we first considered changing all four variables of the cantilever beam problem simultaneously by progressive percentages. Figure 3 illustrates the relative errors of the approximations of the first bending frequency as a function of the relative change in all four design variables. These changes are certainly not indicative of performance within an optimization exercise, but they do provide some measure of goodness easily displayable. Note that the starting point for the approximation is the point with abscissa 0. The linear approximation is carried out from this point forward (increasing all four design variables by $x\%$), and then backward (decreasing all four variables by $x\%$). For the Two Point Exponential approximation, the starting point is the same $X_0$, and the "previous" design point $X_1$ is at abscissa $x=.2$. In this case, increasing $x$ means backtracking, whereas decreasing $x$ means progressing in the direction established by the
two successive design points. The figure shows that the Two Point Exponential approximation seems to better fit the real function below the design point, and is slightly worse than the linear approximation above the design point. It is hoped that during an optimization, the design variables would either increase or decrease monotonically, and the Two Point Exponential approximation would perform better than the linear. Note that the results of both approximations were very sensitive to the derivatives obtained through finite differences in the analysis program (ANSYS). The true test of an approximation however, is to perform the optimization exercise. This is the subject of the next section.

\[ \text{Figure 3. Approximation error as a function of all four variables} \]

\text{Cantilever Beam Problem}

**Optimization of the Cantilever Beam Problem.**

Since a true test of the approximations can only be obtained in an optimization problem, the Cantilever Beam example discussed above was reformulated as an optimization exercise. The initial design variables are the ones given above as vector \( X_0 \), and the object of the problem is to find the minimal weight subject to frequency constraints. The first frequency constraint is the first bending frequency of the beam which has to be below a certain minimum value and the second frequency above another value. This would ensure a separation of natural frequencies, and could be used as a design problem. The first attempt to solve the problem considered two
design variables, namely the height and width of the beam, leaving the thicknesses constant. The constraints (first and second frequencies) are limited to 5 Hz and 30 Hz respectively (F1 < 5Hz, F2 > 30Hz). The allowable error is 0.1 and the move limits are 50% in all three cases. The results are tabulated below:

<table>
<thead>
<tr>
<th></th>
<th>Linear</th>
<th>Reciprocal</th>
<th>2 Pt. Exp.</th>
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</thead>
<tbody>
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<td>6.68</td>
<td>6.68</td>
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<td>3.60146</td>
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<td>1.54845</td>
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<tr>
<td>7</td>
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</table>

Table 2. Variation of Cross sectional area as function of iteration number.

Because of the similarity of results, a graph of the variation of objective (cross sectional area) with respect to iteration number would not provide any additional information. From the table above, one can only deduce that in this particular case, the three approximations perform relatively similarly. All three reach the optimum in roughly the same amount of steps. The linear approximation seems to reach a smaller optimum, but this result is because this particular approximation in this problem causes one of the constraints to be slightly violated, and at the final result, the second frequency constraint is active, but very close to be violated, whereas in the two other methods, the second frequency constraint is active, and satisfied. Table 3 lists some of the results for the above problem. In all three cases, the beam width is driven to the minimum (0.5 in), and the second frequency constraint becomes active.

<table>
<thead>
<tr>
<th></th>
<th>Linear B</th>
<th>Linear F2</th>
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</table>

Table 3 Cantilever Beam. Active constraints as function of iteration number. Beam width B driven to >= 0.5 in, second natural frequency driven to >= 30Hz.

When one considers all four parameters: height, width and thicknesses, as design variables, the problem should be more complicated and the approximations less well behaved.
Figure 4. illustrates the variation of the objective with respect to iteration number in the case of four design variables. In this case again, all three approximations behave relatively similarly, with the linear and Two Point Exponential approximations reaching the minimum in 5 steps whereas the reciprocal this time, takes one additional step. The interesting observation however, is the path to the minimum taken by all three approximations. In order to show the differences, the value axis (area) was magnified with a maximum at 2 inches. The first two iterations are therefore not visible, but one can see that the Two Point Exponential approximation is the smoothest behaved function.

Figure 5. illustrates in the same problem (four design variables, two constraints), the variation of the second natural frequency. The problem consisted in minimizing the area subject to the second frequency remaining above 30Hz. The figure shows that the Two Point Exponential method shows similar oscillative behavior as the other methods, but with a smaller amplitude.

The two results described sofar show that for two relatively simple problems with frequency constraints, the Two Point Exponential approximation behaves at least as good, if not better than the best of the linear or reciprocal approximations.
Conclusion

The Two point Exponential Approximation was tested on problems with frequency constraints. The results obtained sofar show that the method is at least as performing as the best of the traditional methods like the linear or reciprocal approximation. It does also perform as a more controlled method which should be used when the problem to be solved does not have uniformly linearly behaved or uniformly reciprocally behaved constraints and objectives.

Acknowledgements

The author wishes to thank Dr. Jean Francois Barthelemy and Dr. Jacek Sobieski from NASA Langley for their comments and suggestions. This work was supported in part by NASA under contract NAG-1-1144.

References


### APPENDIX A
Numerical Results of Optimization runs

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Cantilever Beam results 2 variables 2 constraints
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Cantilever Beam results. 4 variables 2 constraints
APPENDIX B
Ansly input file and Program listing

H=5.
B=3.75
T=0.8
D=0.1
/TITLE, Beam model for approximation testing
/3D model
FINISH
/PREP7
KAN,2
KAY,1,-1
KAY,2,3
KAY,7,3
C*** compute area and IZZ
IYY1=(T**3)*B
IYY2=IYY1/12
PAR1=H-(T*2)
PAR3=B-(D*2)
PAR2=(H-T)/2
IYY3=(T*B)*(PAR2**2)
IYY4=(IYY2+IYY3)**2
IYY5=D*(PAR1**3)
IYY6=IYY5/6
IYY=IYY6+IYY4
AREA=((T*B)+(PAR1*D))*2
C*** end of calculations
ET,1,3
R,1,AREA,IYY,H
MP,EX,1,4.9e6
MP,DENS,1,0.00018
MP,EX,2,5.85e6
MP,DENS,2,0.00018
N,1,0
N,11,193
FILL
/PNUM,NODE,1
N PLOT
MAT,1
E,1,2
MAT,2
E,2,3
EGEN,9,1,2
EPLOT
D,1,ALL
M,2,UY,11,UX,ROTZ
SAVE
ITER,1,1
SWRITE
FINISH
/SOLVE
FINISH

13
/POST1
set,,1
*get,fre1,freq
set,,2
*get,fre2,freq
set,,3
*get,fre3,freq
FINISH
/OPT
FACT=.99999
H1=H*FACT
H2=H/FACT
B1=B*FACT
B2=B/FACT
OPVAR,H,DV,H1,H2
OPVAR,B,DV,B1,B2
OPVAR,AREA,OBJ
OPVAR,PAR1,SV,.1,H
OPVAR,PAR3,SV,.1,B
OPVAR,FRE1,SV,.1,10
OPVAR,FRE2,SV,.1,100.
OPVAR,FRE2,SV,.1,150.
OPCOPY
H=H*1.001
RUN,2
B=B*1.001
H=H/1.001
RUN,3
T=T*1.001
B=B/1.001
RUN,4
D=D*1.001
T=T/1.001
RUN,5
OPLIST,ALL,,1
FINISH
/EOF
program to read an ANSYS file and extract the necessary data for
optimization, call conmin, and use approx to solve approximate prob:

Georges Fadel Sept 1990
Oct 1990
Jan 1991

IMPLICIT DOUBLE PRECISION (A-H,O-Z)

common for CONMIN call
COMMON /CNMN1/ DELFUN,DABFUN,FDCH,FDCHM,CT,CTMIN,CTL,CTLMIN,
1 ALPHAX,ABOBJ1,THETA,OBJ,NDV,NCON,NSIDE,IPRINT,NFDG,NSCAL,LINOBJ,
2 ITMAX,ITRM,ICNDIR,IGOTO,NAC,INFO,INFOG,ITER
COMMON /CNMN2/ X(6),DF(6),G(15),ISC(15),IC(15),A(6,15),AF(7)
COMMON /CNMN4/ VLB(6),VUB(6),SCAL(6)

the next two are for approx subroutine. Second common just to pass
flags to approx
COMMON /INFOIN/ DV(4), FUNC(7), GRAD(4,7)
COMMON /INFOLD/ DV1(4), FUNC1(7), GRAD1(4,7)
COMMON /FLAGS/ IFLAG,ICALL,IDEBUG
DIMENSION P(4,7), RATX(4), RATDER(4,7)
DIMENSION S(6),GI(15),G2(15),B(15,15),C(15),MSI(30)
end of conmin non-executable
DIMENSION CONS(5,5),OOBJ(4),GMAX(6)
CHARACTER*4 START(5),T(5)
CHARACTER*12 FILNM,FILNM1,FILNM2,FILNM3
CHARACTER*80 TT
LOGICAL TOF
DATA START(1),START(2),START(3),START(4),START(5)='/LIST',' OPT','
1 'IMIZ','ATIO','N SE'/
name of file (File=' ') written from batch file into
temp.dat is read into FNAM

some parameters that have to be set for each optimization program:

nlines in output file
number of design variables NDV
Number of constraints NCON
Increment factor used to compute finite differences in
finite element program: FACT = 1. - actual FACT
DF means derivative of objective wrt design variable
A means derivative of constraint wrt design variable
and remember to adjust dimensions to read all needed data
in X(NDV), CONS(NCON,NDV), OOBJ(NDV)
DF(NDV),A(NDV,NCON)

Also, the output data includes a maximum of 6 cases per row. If
NCON is more than 6, then, an additional read statement has to be
written for the next batch of results.

conmin requirements ++++++++++++++++++++++++++++++++++++++
IGOTO  Sets start of optimization loop
IPRINT  Print control: 0 print nothing
1 print initial and final function informat
2 1st debug level print 1 + control paramet
function value and X at each iteration.
3 2nd debug level print 2 + constraints, ac
or violated constraints, move parameters.
approaches 0 as optimum gets closer
4 full debug

NDV Number of decision variables
ITMAX Max number of iterations
NCON Number of constraint functions G(J)
NSIDE Number of side constraints (upper, lower bounds)
G Constraints at initial design point
CONS Constraints at finite differences
ICNDIR Conjugate direction restart parameter
NSCAL Scaling control parameter
NFDG Gradient calculation control parameter 0: calculated by F
1: externally supp
2: obj external, r

FDCH Relative change of decision variable for FD calc.
FDCHM Minimum step for FD
CT Constraint thickness parameter
CTMIN Minimum abs value of CT
CTL Constraint thickness for linear and side constraints
CTLMIN Minimum abs value of CTL
THETA Mean value of push off factor (for highly non-linear probl
NACMX1 Estimate of number of active constraints
DELFUN Minimum change in OBJ to indicate convergence
DABFUN Same as DELFUN, but absolute not relative error
LINOBJ 0 means non-linear, 1 means linear
ITRM (3) number of consecutive iterations for convergence
X(N1) Vector of decision variables
VLB(N1) Lower bound on variables X(I)
VUB(N1) Upper bound on variables X(I)
SCAL(N5) Vector of scaling parameters not used if NSCAL=0
ISC(N2) Linear constraint identification vector

GMAX(NCON) LIMITS OF CONSTRAINTS

IPRINT=2 SUPPLIED IN EXTERNAL FILE
NDV=4 SUPPLIED IN EXTERNAL FILE
SET NUMBER OF CONSTRAINTS TO REQUIRED NUMBER (INITIALLY 1, THEN 6)
NCON=1 SUPPLIED IN EXTERNAL FILE

IGOTO=0
NFDG=0
ITMAX=50

NACMX1=15
NSIDE=8
ICNDIR=0
NSCAL=0
LINOBJ=0
N1=6
N2=15
N3=15
N4=15
N5=30
ITRM=3
FDCH=0.
FDCHM=0.
CT=0.
CTMIN=0.
CTL=0.
CTLMIN=0.
THETA=0.
DELFUN=10.E-8
DABFUN=10.E-8
NAC=0
ALPHAX=0.1
AOBJJ=0.1
ICALL=1
DO 9 I=1,N2
   ISC(I)=0
9 CONTINUE
end of common variables definition

NLINES=20000
FACT=0.001

create a file called OPTIM.DAT in which the filenames of the initial result data file and file to be used to store results are written. One name on each line. Next, enter a number representing the magnitude of the move limits in %

OPEN(UNIT=3,STATUS='OLD',FILE='OPTIM.DAT')
read(3,99)FILNM,FILNM1,FILNM2,FILNM3
read(3,98)GMOVE
read(3,*)NDV,NCON
read(3,*)IDEBUG,IPRINT
read(3,*) (GMAX(I),I=1,NCON)
CLOSE(UNIT=3)

OPEN(UNIT=8,STATUS='OLD',FILE=FILNM)
OPEN(UNIT=9,ACCESS='TRANSPARENT',FORM='UNFORMATTED',FILE=FILNM1)
OPEN(UNIT=10,STATUS='OLD',FILE='HISTORY.DAT')
OPEN(UNIT=11,STATUS='OLD',FILE='FLAGS.DAT')

READ(8,100)T(1),T(2),T(3),T(4),T(5)
DO 1000 L=1,NLINES
   find first line of results
   IF(T(1).NE.START(1).OR.T(5).NE.START(5)) THEN
      READ(8,100)T(1),T(2),T(3),T(4),T(5)
   ELSE
      READ(8,101)
   END IF
   read some blank lines to get to beginning of data
   initially do 10 i=1,ndv THIS SHOULD BE ACCORDING TO FILE
   if(idebug.ge.3) PRINT *,/' DESIGN VARIABLES '
   DO 10 I=1,4
      read the design variables X(I)
      READ(8,102)X(I)
      if(idebug.ge.3) PRINT *,X(I)
      Compute the move limits
      VLB(I)=X(I)*(1.-GMOVE/100.)
      VUB(I)=X(I)*(1.+GMOVE/100.)
      if(idebug.ge.3) PRINT *,VLB(I),VUB(I)
10 CONTINUE

ADD THE FOLLOWING LOWER BOUNDS FOR PROBLEM TO BE REALISTIC
IF(VLB(1).LE.2.) VLB(1)=2.0
IF(VLB(2).LE.0.5) VLB(2)=0.5
IF(VLB(3).LE.0.1) VLB(3)=0.1
IF(VLB(4).LE.0.05) VLB(4)=0.05

and upper limits
IF(VUB(1).GE.15.) VUB(1)=15.0
IF(VUB(2).GE.15.) VUB(2)=15.0
IF(VUB(3).GE.(X(2)/2.)) VUB(3)=X(2)/2.
IF(VUB(4).GE.(X(1)/2.)) VUB(4)=X(1)/2.

Read some more blank lines
READ(8,103)

and then the Objective function at the design point OBJ
and the objective at finite differences from the origin
READ(8,104)OBJ,(OOBJ(J),J=1,NDV)
if(idebug.ge.1) THEN
   PRINT *,'OBJECTIVE AND RESULTS OF FDs'
   PRINT *,OBJ,(OOBJ(J),J=1,NDV)
ENDIF

convert constraints into <=0 constraints and scale
if(idebug.ge.1)PRINT *,'CONSTRAINTS AND RESULTS OF FDs'
DO 11 J=1,NCON
   READ(8,104) G(J),(CONS(J,K),K=1,NDV)
   if(idebug.ge.1) PRINT *,G(J),(CONS(J,K),K=1,NDV)
   G(J)=G(J)/GMAX(J)-1.
   IF(J.EQ.2) G(J)=-G(J)
   DO 13 KK=1,NDV
      CONS(J,KK)=CONS(J,KK)/GMAX(J)-1
      IF(J.EQ.2) CONS(J,KK)=-CONS(J,KK)
   13 CONTINUE
   if(idebug.ge.1) PRINT *,",CORR",G(J),(CONS(J,K)
   ,K=1,NDV)
11 CONTINUE

Now compute the derivatives:

DO 12 I=1,NDV
   DF(I)=(OOBJ(I)-OBJ)/X(I)/FACT
   if(idebug.ge.1) PRINT *,"OBJ DER",DF(I)
   DO 12 J=1,NCON
      A(I,J)=(CONS(J,I)-G(J))/X(I)/FACT
      if(idebug.ge.1) PRINT *,"DERIV",A(I,J)
   12 CONTINUE

write values to confirm
WRITE(9)NDV,(X(I),I=1,NDV),OBJ,NCON,(G(J),J=1,NCON),
(DF(I),I=1,NDV),((A(K,M),K=1,NDV),M=1,NCON)
if(idebug.ge.2) THEN
   print *,"SUMMARY'
   print *,NDV,(X(I),I=1,NDV)
   print *,OBJ,NCON,(G(J),J=1,NCON)
   print *,(DF(I),I=1,NDV)
   print *,((A(K,M),K=1,NDV),M=1,NCON)
ENDIF

replace values into conmin arrays and form. they will
be passed to approx through common.
FUNC(1)=OBJ
DO 20 I=1,NDV
   DV(I)=X(I)
   GRAD(I,1)=DF(I)
DO 20 JJ=1,NCON
    GRAD(I,JJ+1)=A(I,JJ)

20    CONTINUE
    DO 21 J=1,NCON
        FUNC(J+1)=G(J)
    21    CONTINUE
    GOTO 999
ENDIF
1000    CONTINUE
C
999    CONTINUE
C
** INITIALIZE CONSTRAINT IDENTIFICATION VECTOR, ISC. **
    DO 310 J=1,NCON+1
        ISC(J)=0
310
C** SOLVE OPTIMIZATION. **
C
350    CONTINUE
    IF(idebug.ge.2)print *, 'before conmin',X(1),X(2),X(3),X(4)
    CALL CONMIN(X,VLB,VUB,G,SCAL,DF,A,S,G1,G2,B,C,ISC,IC,MS1,  
        1 N1,N2,N3,N4,N5)
    IF(idebug.ge.2)print *, 'after conmin',X(1),X(2),X(3),X(4)
    IF(IGOTO.EQ.0) THEN
        reached optimum
        IF(idebug.ge.2)then
            print *, 'final results'
            print *, '
            print *, 'OBJECTIVE = ',OBJ
            print *, 'X VECTOR ',(X(I),I=1,NDV)
            print *, 'G VECTOR ',(G(J),J=1,NCON)
        endif
        WRITE(10,*) OBJ,(X(I),I=1,NDV),(G(J),J=1,NCON)
        C
        write info to new file to rerun ansys
        C
        first, we have to read the input file for ansys and then rewrite
        C
        it with new values
        OPEN(UNIT=4,STATUS='OLD',FILE=FILNM2)
        OPEN(UNIT=5,STATUS='UNKNOWN',FILE=FILNM3)
        C
        READ AND WRITE FILE
        WRITE(5,110)(X(I),I=1,NDV)
        DO 363 NN=1,NDV
            READ(4,*) TT
        363    CONTINUE
        DO 361 NN=1,NLINES
            READ(4,111,END=362) TT
        361    CONTINUE
        WRITE(5,111)TT
        362    CONTINUE
        ICALL=1
        REWIND(11)
        WRITE(11,112)ICALL,IFLAG
        CLOSE(UNIT=11)
C

ELSE

C 
no convergence yet ...
rewind(11)
READ(11,112)ICALL,IFLAG
IF(ICALL.EQ.1) THEN
C 
first call to approximation, copy file and compute exponent
C
IFLAG=  
1 LINEAR
C
2 RECIPROCAL
C
3 TWO POINT EXPONENTIAL
C
ICALL=0
REWIND(11)
WRITE(11,112)ICALL,IFLAG
IFLAG=IFLAG

IF(IFLAG.EQ.3) THEN
INQUIRE(FILE='SCNDGRD.DAT',EXIST=TOF)
IF(TOF) THEN
OPEN(UNIT=7,ACCESS='TRANSPARENT',FORM='UNFORMATTED'
 ,STATUS='OLD',FILE='SCNDGRD.DAT')
READ(7)NDV,(DV1(I),I=1,NDV),FUNC1(1),NCON,(FUNC1(J)
,J=2,NCON+1),,(GRAD1(L,1)L=1,NDV),((GRAD1(K,M)
,K=1,NDV),M=2,NCON+1)
if(idb.ge.4) then
print *, 'old point: ',(DV1(I),I=1,NDV)
print *, 'old obj. ', FUNC1(1)
print *, 'old constr ',(FUNC1(J),J=2,NCON+1)
print *, 'old grads',((GRAD1(K,M),K=1,NDV)
 ,M=1,NCON)
endif
REWIND(7)
WRITE(7) NDV, (DV(I),I=1,NDV), FUNC(1), NCON, (FUNC(J)
,J=2,NCON+1),,(GRAD(L,1),L=1,NDV),((GRAD(K,M)
,K=1,NDV),M=2,NCON+1)
if(idb.ge.4) then
print *, 'Xo point: ',(DV(I),I=1,NDV)
print *, 'obj. ', FUNC(1)
print *, 'constr ',(FUNC(J),J=2,NCON+1)
print *, 'grads',((GRAD(K,M),K=1,NDV)
 ,M=1,NCON)
endif
CLOSE(UNIT=7)

ELSE

C 
first call, no data in SCNDGRD.DAT yet. put it in
IFLAG=1
OPEN(UNIT=7,ACCESS='TRANSPARENT',FORM='UNFORMATTED'
 ,STATUS='NEW',FILE='SCNDGRD.DAT')
WRITE(7)NDV,(DV(I),I=1,NDV),FUNC(1),NCON,(FUNC(J)
,J=2,NCON+1),,(GRAD(L,1),L=1,NDV),((GRAD(K,M)
,K=1,NDV),M=2,NCON+1)
CLOSE(UNIT=7)
ENDIF
ENDIF

NFUNCS=NCON+1
COMPUTATION OF EXPONENT BASED ON IFLAG

DO 710 I=1,NDV
  IF(DV(I).EQ.0.) THEN
    RATX(I)=1.E8
  ELSE
    RATX(I)=DV1(I)/DV(I)
  ENDIF
  if(idebug.ge.3) THEN
    PRINT *, 'INITIAL CALCULATIONS IFLAG= ',IFLAG,'X(I) = ',X(I), 'DV1(I)/DV(I) ',RATX(I)
  ENDIF
  IF((IFLAG.EQ.I) .OR. (X(I).EQ.0.) .OR. (RATX(I).EQ.1.)) THEN
    if(idebug.ge.2) PRINT *,' In linear code '
    LINEAR APPROXIMATION
    DO 711 J=I,NFUNCS
      P(I,J)=1.
    CONTINUE
  ELSE
    if(idebug.ge.2) PRINT *,' In Reciprocal code '
    RECIPROCAL APPROXIMATION
    DO 712 J=I,NFUNCS
      P(I,J)=-1.
    CONTINUE
  ELSE
    if(idebug.ge.2) PRINT *,' In 2 point code '
    2 POINT EXPONENTIAL APPROXIMATION
    DO 713 J=I,NFUNCS
      IF(GRAD(I,J).EQ.0.) THEN
        P(I,J)=1.
      ELSE
        RATDER(I,J)=GRAD1(I,J)/GRAD(I,J)
        IF((RATX(I).LE.0.) .OR. (RATDER(I,J).LE.0.)) THEN
          P(I,J)=1.
        ELSE
          P(I,J)=DLOG(RATDER(I,J))/DLOG(RATX(I)) +1
          IF(P(I,J).GE.1.) THEN
            P(I,J)=1.
          ELSE
          ENDIF
        ENDIF
      ENDIF
    CONTINUE
  ENDIF
  if(idebug.ge.2) PRINT *,' ,I, EXPONENT ***',I,P(I,1)
CONTINUE

IFLAG=IFLAGT
ENDIF
IF(INFO.EQ.1) THEN
  AF(1)=FUNC(1)
  if(idebug.ge.3) PRINT*, 'OBJ ',obj
  DO 359 J=1,NCON
AF(J+1)=FUNC(J+1)
if(idebug.ge.3)PRINT*,'CONS #',j, G(J)
CONTINUE
if(idebug.ge.2)PRINT *, 'CALL TO APPROXIMATION'
this is the call to the approximation
CALL APPROX(X,AF,P,NDV,NCON)
Resubstituting values in OBJ and CONS
OBJ=AF(1)
if(idebug.ge.3) PRINT*, 'OBJ ',obj
DO 360 J=1,NCON
G(J)=AF(J+1)
if(idebug.ge.3) PRINT *, 'CONS #',j, G(J)
CONTINUE
ELSE
if(idebug.ge.3)PRINT *, '# Info ne 1 ??? ', INFO
ENDIF
ENDIF
GOTO 350

FORMATS
F O R M A T S
98 FORMAT(F3.0)
99 FORMAT(A12/A12/A12/A12)
100 FORMAT(5A4)
101 FORMAT(1X,///)
102 FORMAT(5X,E12.6)
103 FORMAT(1X,///////////)
104 FORMAT(5X,6E13.6)
110 FORMAT('H=',E12.6/'B=',E12.6/'T=',E12.6/'D=',E12.6)
111 FORMAT(A80)
112 FORMAT(2I2)

END

SUBROUTINE APPROX(AV,AF,P,NDV,NCON)

This subroutine is called from optrun to perform various approximations of the functions (objectives and constraints). A flag will select linear, reciprocal or improved approximation. Two sets of data are needed since the improved approximation relies on past analyses to improve the approximation.

Georges Fadel June 1989
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AV is the vector of VARIABLES

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DIMENSION AV(4), AF(7), P(4,7)
COMMON/INFOIN/ DV(4), FUNC(7), GRAD(4,7)
COMMON/FLAGS/ IFLAG,ICALL, IDEBUG
NFUNCS=NCON+1

Now the exponent is known, lets compute the approximating function
if(idebug.ge.1) PRINT*, 'EXponent KNOWN'
DO 30 J=1,NFUNCS
  DO 20 I=1,NDV
    IF((P(I,J).EQ.1.).OR.(ABS(P(I,J)).LE.0.000001))THEN
      AF(J)=AF(J)+(AV(I)-DV(I))*GRAD(I,J)
    ELSE
IF(P(I,J).EQ.-1.) THEN
   AF(J)=AF(J)+(AV(I)-DV(I))*(DV(I)/AV(I))*GRAD(I,J)
ELSE
   AF(J)=AF(J)+((AV(I)/DV(I))**P(I,J)-1.)
1
ENDIF
ENDIF
20 CONTINUE
30 CONTINUE
999 RETURN
END
APPENDIX C
OPTIM.DAT file

BEAM4.OUT
BEAM4.INP
BEAM4.OLD
BEAM4.DAT

50.
4
3
0 4
5.0 30. 80.
APPENDIX D
OPTIMI.BAT batch file to execute optimization

```batch
echo off
cls
echo OPTIMIZATION BATCH FILE TO TEST APPROXIMATIONS WITH ANSYS AND CONMI
echo ....... START OF OPTIMIZATION PROCEDURE ......
echo call to ANSYS with initial design variables set in file
   xxxxxxx.dat in ansys format
ERASE $1.OUT
echo
call ANSYS -I %1.dat -O %1.out
COPY %1.DAT %1.OLD
echo
echo First ANSYS run COMPLETED. Results are written to %1.out
echo :loop1
   call browse %1.out
echo
   echo ++++++++++++ IN LOOP +++++++++++++++++
echo
call optimization program using design variables and derivatives
echo
call optrun2
echo
echo ========= CONVERGED IN APPROXIMATION LOOP =========
copy hist.dat+history.dat hist.dat
echo
ERASE $1.OUT
call ANSYS -I %1.DAT -O %1.out
echo
echo
echo RERUN ANALYSIS (ANSYS)
echo
REM if not converged
goto loop1
echo
REM else
stop
```