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**PROBABILISTIC FATIGUE METHODOLOGY  
FOR SIX NINES RELIABILITY**

**R. A. Everett, Jr., F. D. Bartlett, Jr., and W. Elber**

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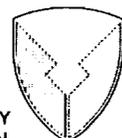
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**US ARMY  
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## SUMMARY

Fleet readiness and flight safety strongly depend on the degree of reliability and maintainability that can be designed into rotorcraft flight critical components. The current U.S. Army fatigue life specification for new rotorcraft is the so-called "six nines" reliability, or a probability of failure of one in a million. This report reviews the progress of a round robin which was established by the American Helicopter Society (AHS) Subcommittee for Fatigue and Damage Tolerance to investigate reliability-based fatigue methodology. The participants in this cooperative effort are the U.S. Army Aviation Systems Command (AVSCOM) and the rotorcraft industry. One phase of the joint activity examined fatigue reliability under uniquely defined conditions for which only one answer was correct. The other phases were set up to learn how the different industry methods in defining fatigue strength affected the mean fatigue life and reliability calculations. Hence, constant amplitude and spectrum fatigue test data were provided so that each participant could perform their standard fatigue life analysis. As a result of this round robin, the probabilistic logic which includes both fatigue strength and spectrum loading variability in developing a consistent reliability analysis was established. In this first study, the reliability analysis was limited to the linear cumulative damage approach. However, it is expected that superior fatigue life prediction methods will ultimately be developed through this open AHS forum. To that end, these preliminary results were useful in identifying some topics for additional study.

## SYMBOLS

a	crack length, in.
f(z)	probability density function
F	boundary correction factor
K <sub>C</sub>	fracture toughness , ksi·in <sup>½</sup>
K <sub>T</sub>	net section elastic stress concentration factor
ΔK	stress intensity factor range, ksi·in <sup>½</sup>
n	occurrences of stress level in loading spectrum, cycles
N	fatigue life, cycles
ΔP <sub>i</sub>	interval probability on fatigue strength or applied load
ΔP <sub>ij</sub>	interval joint probability
R	ratio of minimum to maximum stress
s <sub>m</sub>	mean of applied stress, ksi
s <sub>r</sub>	range of applied stress, ksi
s <sub>r</sub> '	range of α-scaled and Goodman-corrected stress, ksi
S	range of fatigue strength at specified unreliability, ksi
S <sub>e</sub>	fatigue limit strength, ksi
S <sub>max</sub>	maximum stress in a cycle, ksi
S <sub>o</sub>	crack-opening stress, ksi
S <sub>r</sub>	range of stress in one cycle, ksi
S <sub>u</sub>	ultimate strength, ksi
z	independent variable in probability density function
α	scaling parameter for applied load (stress)
φ	difference between applied and fatigue limit stresses, ksi
σ	standard deviation

## INTRODUCTION

The U.S. Army operates a fleet of almost 8000 helicopters. Each of these rotorcraft has on the order of 100 flight critical components. In general, the airframes have been kept in service much longer than originally anticipated, while many flight critical parts are replaced at predetermined intervals. Still, at any one time, almost one million of these components are in service

and they must serve their function safely. For that reason, the rotorcraft industry aims to design and operate these components with a risk of failure of roughly one in a million, or a reliability of six nines.

While this might seem to be a protection against a very rare event, other industries must work at reliabilities which are one or two orders of magnitude greater. Examples of these are front-end wheel spindles in automobiles and individual transistors in computers, where reliabilities of seven and eight nines are being demonstrated. And, just like the rotorcraft structural design requirements, that reliability must be achieved under severe weight and size constraints.

Traditionally, rotorcraft fatigue design has combined constant amplitude tests of full-scale parts with flight loads and usage data in a conservative manner to provide "safe life" component replacement times with a high, but unquantified, reliability. The conservatism in fatigue strength is based on a one in a thousand probability of failure, but the conservatisms in loads and usage have not been quantified. One of the reasons for conservative design has been a lack of accurate and detailed structural analyses. Over the years advances in computer speed and memory have resulted in more efficient methods for loads analysis. However, these methods have been implemented without a better definition of the level of conservatism. As a consequence, the U.S. Army is requiring that the next rotorcraft to be developed have an expected value of six nines reliability for flight critical components.

The exploration and adoption of new approaches in design and fleet management may be necessary to achieve a reliability of six nines with minimum impact on structural weight. Actual fleet loads monitoring may be required to reduce the uncertainty in usage. Fracture mechanics fatigue life approaches may be required to provide more accurate estimates of damage progression. Also,

flight-by-flight spectrum testing of full-scale parts may be required to reduce the uncertainty of spectrum fatigue life predictions and possibly lower the coefficients of variation. Reference 1 is one of the first attempts to define the effects of six nines reliability for rotorcraft fatigue life design. Furthermore, this six nines reliability requirement and the associated concerns about implementing such a philosophy have resulted in establishing a round robin involving the U.S. Army Aviation Systems Command (AVSCOM) and the rotorcraft industry. This study was undertaken by the American Helicopter Society (AHS) Subcommittee for Fatigue and Damage Tolerance. It is a thought-provoking exercise which requires the exploration of the computational methods necessary to determine fatigue life as a function of the reliability criterion. In this first study, the reliability analysis is limited to the linear cumulative damage approach. But through this open AHS forum it is expected that industry-wide approaches for a superior fatigue life methodology will ultimately be developed. The probabilistic fatigue life approach and the results of this first round-robin activity are summarized in this report.

#### ROUND ROBIN DESCRIPTION

As discussed in the Introduction, the development of a methodology for the probabilistic fatigue analysis of U.S. Army helicopters was set up as an AHS round robin. The objectives of this round robin were to develop a consistent reliability analysis, to evaluate the different U.S. rotorcraft industry methods in defining fatigue strength, and to examine the issue of fleet versus individual aircraft component replacement. A further objective of the exercise was to contend with the probabilistic complexities associated with defining loads variability to demonstrate the benefits of loads monitoring for achieving six nines reliability. To that end, a normal distribution for fatigue strength under precisely defined cyclic loads and a normal distribution for a simple loading spectrum were prescribed so that fatigue lives could be

calculated to six nines reliability. The round robin was conducted in three phases. In Phase I, the problem was defined such that only one numerical answer was correct. Phases II and III were set up so that the experience of individual contributors could show how differences in assumptions affected the results. A description of the three phases in this AHS round robin is presented in this section of the report. Table 4 lists the round-robin participants and the appropriate identifier codes.

Phase I - Identical Methods and Inputs. In Phase I of the exercise, all fatigue-related variables were strictly controlled to insure that the participants developed consistent solutions for fatigue lives at the prescribed reliability. For computational purposes, a mathematically defined S-N curve was provided in terms of stress range,  $S_r$ , and fatigue limit (fatigue strength at very large loading cycles),  $S_e$ , for a stress ratio  $R=0$ . The loading spectrum used in this round robin was based on the Felix/28 sequence which is explained in a later section. To illustrate trends due to overall spectrum severity, a baseline spectrum level was established and other spectrum severities were created by multiplying all loads in the spectrum by a scaling parameter called  $\alpha$ . To include variability of loads in the reliability analysis, the severity of the spectrum was assumed to have a normal distribution about a mean severity (or mean  $\alpha$ ) level. It is important to recognize that this is a theoretical exercise and that  $\alpha$  is a mathematical artifice used to simulate changes in the baseline spectrum which account for differences in usage, pilot technique, weather, vehicle configuration, etc. Thus, for the purposes of this exercise,  $\alpha$  is an operational variable that combines both usage and loads variability to discriminate in mission intensity.

Based upon these fatigue strength and spectrum loading definitions, the following three problems were proposed and solved by the AHS round-robin participants. For all three problems, it was assumed that fatigue strength was normally distributed about the

fatigue limit with a 7% coefficient of variation (COV). Also, for the purposes of this exercise, it was assumed that: (1) Palmgren-Miner's linear damage rule was valid; and (2) Goodman corrections needed to convert the applied stresses to equivalent-damage stresses at  $R=0$  were valid.

Problem 1. Assume that the loading spectrum does not vary. Calculate fatigue lives for the mean, mean- $3\sigma$ , and mean- $5\sigma$  fatigue strength curves using the Felix/28 loading spectrum factored by deterministic mean values of the fleet severity parameter,  $\alpha$ , between 0.3 and 0.9.

Problem 2. Assume that the loading spectrum severity parameter,  $\alpha$ , is normally distributed with a 7% COV. Calculate the fleet fatigue lives at six nines reliability for mean values of  $\alpha$  between 0.3 and 0.9.

Problem 3. Assume that the actual  $\alpha$  for a subset of helicopters (or individual aircraft) has a normal distribution and can be measured to within a 3% coefficient of variation. Calculate the six nines reliability fatigue lives for the subset of aircraft over a range of  $\alpha$ 's. Then, assuming that the mean  $\alpha$ 's of the subsets in the fleet are normally distributed with a 7% COV, calculate the mean fatigue life at six nines reliability for a fleet mean  $\alpha$  of 0.6 (0.85 for Phase II).

Phase II - Independent Methods and Inputs. In Phase I, the same S-N curve and fatigue limit COV were used by each participant. In Phase II, instead of using a prescribed mathematical expression for the S-N curve, the round-robin participants were provided six constant amplitude test points which were mutually agreed to be typical of the six data points obtained in full-scale helicopter fatigue substantiation testing. Each participant then used this data set to develop an independent S-N curve formulation, fatigue limit, and fatigue limit COV. The three problems in Phase I were

solved again to compare the effects of each participant's choice of S-N curve.

Phase III - Spectrum Fatigue Tests. In addition to the constant amplitude tests, AVSCOM's Aerostructures Directorate (ASTD) conducted spectrum fatigue tests using the Felix/28 loading spectrum over a range of  $\alpha$  values. This measured  $\alpha$  versus life curve was used to assess the accuracy of a cumulative damage model and a fracture mechanics model for predicting the measured spectrum fatigue lives.

### RELIABILITY ANALYSIS

This section explains the solution methods used to answer the three questions postulated for this round-robin exercise. Problem 1 does not address reliability but was included to assure that all participants properly accounted for loads (alpha) scaling and the Goodman correction when calculating the fatigue life. Problems 2 and 3 were intended to address the probability of failure (POF) for a given distribution of fatigue strength and applied loading. The results are presented in terms of reliability or the probability of no failure, (1-POF). Thus, the six nines reliability requirement is equivalent to one failure in a million for each major component in the life of the helicopter fleet. An underlying objective of this analysis was to demonstrate the advantages of load monitoring for achieving six nines reliability. The solutions to these problems are presented and discussed in a later section of this report.

Problem 1. As mentioned earlier, the S-N curve shape, the ultimate strength (180 ksi), and the fatigue limit stress range (40 ksi) were prescribed in Phase I. Figure 1 shows the S-N curve formulation as plotted on a log-log scale. Based upon this straight line definition, the fatigue life equation can be written as

$$N = 500000 \cdot (S_R - S_e)^{-1.51785}, \quad S_R > S_e \quad (1)$$

and

$$N = 10^{15}, \quad S_R \leq S_e \quad (2)$$

Recall that the coefficient of variation on the fatigue limit was assumed to be 7%. It is further assumed that the standard deviation at the fatigue limit also applies to the fatigue strength distributions at all points on the S-N curve. In other words, the fatigue strength standard deviation is constant. The  $\alpha$  versus fatigue life curves for the mean, mean-3 $\sigma$ , and mean-5 $\sigma$  fatigue limits were computed using the Palmgren-Miner linear cumulative damage model which is described in a later section. The solution process consists of four basic steps. First, the Felix/28 loading spectrum stresses (mean and amplitude) are multiplied by the  $\alpha$  scaling parameter. Second, the scaled stresses are adjusted using the Goodman correction. The intent of the Goodman correction is to convert a given stress mean and range into an equivalent stress range which produces equivalent fatigue damage at R=0. Figure 2 describes the Goodman correction for the  $\alpha$ -scaled stress mean and range. The equivalent stress range,  $s_R'$ , is defined as

$$s_R' = \alpha \cdot S_u \cdot s_R / (S_u - \alpha \cdot s_m + \alpha \cdot s_R/2) \quad (3)$$

at the R=0 fatigue life curve for an arbitrary value of  $\alpha$ . Equation (1) is used with  $S_R$  replaced by  $s_R'$  to calculate the fatigue life. Third, for the mean and the reduced-strength definitions of the S-N curves, Palmgren-Miner's Rule is used to calculate the fatigue life for the Felix/28 spectrum. Finally, in the fourth step,  $\alpha$  is plotted versus the number of sequences through the loading spectrum.

Problem 2. The purpose of Problem 2 was to include the variability of fatigue strength and applied load in the fatigue life calculations for six nines reliability. For this problem, both strength and load are normally distributed with a 7% coefficient of variation. The probability of failure for particular distribu-

tions of  $\alpha$  and strength is a joint probability of occurrence problem. Joint probability distributions associated with two random variables are discussed in reference 2. The normal distribution curve or probability density function, as depicted in figure 3, is used to calculate the probability that a value of the parameter (strength or load) exists in the interval between  $f(z_i)$  and  $f(z_{i+1})$ . The product of the fractional or interval probabilities for load,  $\Delta P_i(\alpha)$ , and strength,  $\Delta P_j(S)$ , defines the joint probability that both occur simultaneously. Thus, the joint probability is given by

$$\Delta P_{ij}(\alpha, S) = \Delta P_i(\alpha) \cdot \Delta P_j(S) \quad (4)$$

The current problem is solved numerically by creating a joint probability matrix for normal distributions of  $\alpha$  and strength, as illustrated in figure 4. These normal distribution curves for  $\alpha$  and strength were divided into 50 uniform increments from  $-5\sigma$  to  $+5\sigma$  about the mean. A sensitivity study verified that 50 increments gave results to within 2.5% of the converged solution which required 200 increments.

The procedures which were discussed in Problem 1 were used to compute the fatigue life,  $N_{ij}(\alpha, S)$ , for each combination of  $\alpha$  and strength in the joint probability matrix. Thus, there is an associated fatigue life for each element in the joint probability matrix. The reliability is calculated by reordering the  $[\Delta P_{ij}(\alpha, S), N_{ij}(\alpha, S)]$  pair into a one-dimensional array from the smallest life to the largest life. The reliability at any specified fatigue life is the sum of all values of  $\Delta P_{ij}(\alpha, S)$  in the array above that value of life. Figure 5 presents an example of this cumulative joint probability versus fatigue life. As indicated in figure 5, the fatigue life for six nines reliability (or a POF = 0.000001) can be easily calculated by interpolation. This process is repeated for each mean  $\alpha$  from 0.3 to 0.9. It turns out that only the shaded region of the joint probability matrix in figure 4 contributes to the calculation of six nines reliability. Thus, the

number of computations needed to obtain six nines reliability is only about one-eighth of the 2500 load and strength combinations.

There is another method which gives essentially the same answers as the numerical joint probability/life matrix approach. This alternative method is a closed-form solution which uses the following ideas and procedures. Each  $\alpha$ -scaled and Goodman-corrected applied stress range ( $s_{r'}$ ) in the loading spectrum of Table 2 is assumed to be normally distributed with a mean value given by equation (3) and a COV equal to the  $\alpha$  distribution COV. Furthermore, because  $s_{r'}$  and  $S_e$  are assumed to be normally distributed, the S-N curve in equation (2) can be defined in terms of a new variable,  $\phi = (s_{r'} - S_e)$ , which is also normally distributed with a mean,  $\phi_m$ , and a standard deviation,  $\sigma_\phi$ . The S-N equation becomes

$$N = 500000 \cdot (\phi)^{-1.51785} \quad , \quad \phi > 0 \quad (5)$$

In the linear damage fraction ( $n/N$ ),  $n$  is defined for each stress range in the loading spectrum. The damage that will be exceeded with a POF equal to  $10^{-6}$  (or six nines reliability) at each stress range in the spectrum is determined by using equation (5) to calculate  $N$  when

$$\phi = (\phi_m + 4.75 \cdot \sigma_\phi) \quad (6)$$

where,  $\phi_m = (s_{r'_m} - S_{e_m})$  and  $\sigma_\phi = (\sigma_{s_{r'}}^2 + \sigma_{S_e}^2)^{1/2}$ . Finally, the fatigue life at six nines reliability is computed as the reciprocal of the total damage in the loading spectrum. A more detailed description of the linear cumulative damage model is presented in a later section.

Although this alternative method has produced answers which are close to those calculated by the "matrix" method, the two methods have not been shown to be theoretically equivalent. One of the differences between the "matrix" method and the closed-form

method can be attributed to differences between the normal population statistics for  $\alpha$  and  $s_{\mathbf{r}'}$ . While the  $\alpha$ -scaled stresses have the same COV as  $\alpha$ , the  $\alpha$ -scaled and Goodman-corrected stresses ( $s_{\mathbf{r}'}$ ) do not. There was no attempt in the current study to evaluate the limitations of assuming the same COV for  $\alpha$  and  $s_{\mathbf{r}'}$  on the accuracy of the results. However, the closed-form method does require much less computation than the "matrix" method and both approaches were used in the round robin.

Problem 3. Problem 3 was posed to examine the difference between defining a six nines reliability fatigue life for a fleet of helicopters versus defining a six nines reliability fatigue life for an individual aircraft based on loads monitoring. Thus, this problem is an application of the theorem of total probability where the total set of helicopters (fleet) exhibits a normal distribution of  $\alpha$  with a 7% COV whereas each of the subsets of helicopters (ship) exhibits normal distributions with a 3% COV. The solution process consists of two basic steps. First, the individual aircraft (ship) six nines reliability fatigue lives are calculated versus the applied load (stress) severity parameter,  $\alpha$ . The procedures for these calculations are the same as described in Problem 2, but with the COV changed from the fleet value of 7% to the ship value of 3%. Second, the mean fatigue life at any specified  $\alpha$  of the fleet is computed using the method of conditional expectation, as explained in reference 3. Thus, the mean fatigue life of the fleet,  $N_{\text{mean}}$ , for a specified mean fleet load severity,  $\alpha$ , is calculated as

$$N_{\text{mean}} = \int_{\alpha=-5\sigma}^{+5\sigma} N_{\text{ship}}(\alpha) \cdot P_{\text{fleet}}(\alpha) \quad (7)$$

where  $N_{\text{ship}}(\alpha)$  is computed for a 3% COV on applied loads and  $P_{\text{fleet}}(\alpha)$  is computed for a 7% COV on the mean  $\alpha$ . Figure 6 shows the details of this process.

## FATIGUE TEST PROGRAM

As discussed earlier, Phase I of this joint exercise was set up so that there was only one unique answer. Phases II and III were formulated to allow each participant to solve the problems using their standard fatigue methodology. In that way the effects of different assumptions and approaches could be assessed. To support this part of the exercise, ASTD conducted a test program which included constant amplitude and spectrum fatigue tests. This section describes the test specimen and explains how the fatigue tests were performed.

Material and Specimen Configuration. The material selected for this study was AISI 4340 steel plate supplied in the annealed condition with a thickness of 3/8 inch. The steel plate was heat treated to Rockwell C scale values between 43 and 45 by a one hour soak at 840 degrees Celsius. After heat treatment the steel plates were then tempered in a vacuum at 440 degrees Celsius for two hours followed by furnace cooling in nitrogen gas. All test specimens were machined from the plate with the longitudinal axis of the specimen being aligned in the rolling direction of the plate. The tensile test specimens were machined according to ASTM standards and the resulting tensile strength, which was calculated from an average of five tests, was 212 ksi. The fatigue test specimens were configured as shown in figure 7 and had a 32 rms surface finish. The hole diameter of 0.25 inches was machined using several drill sizes with the last process removing only 0.002 inches maximum to minimize residual stresses. The surface finish of the hole after machine polishing was 8 rms. The net section elastic stress concentration factor,  $K_T$ , as determined from the boundary force method of reference 4, is 2.42. The same value is given by Peterson in reference 5.

Constant Amplitude Tests. The constant amplitude fatigue tests were run in servo-hydraulic, electronically controlled test stands. All tests were run at a stress ratio,  $R$ , of zero with

cyclic frequencies between 10 and 20 Hertz. Command and feedback signals were controlled to within one percent. The fatigue lives reported herein were to specimen failure. Maximum stress values ranged from 50 to 170 ksi on the net section of the test specimen.

Table 1 presents data for all constant amplitude tests. As stated previously, results of six constant amplitude tests were provided for the Phase II portion of the round robin. These test points are shown in bold in Table 1. Figure 8 shows all constant amplitude fatigue tests plotted on a typical stress versus life cycle (S-N) curve. The fatigue limit for these tests was determined to be 55.8 ksi.

Spectrum Tests. The spectrum fatigue tests were also performed using servo-hydraulic, electronically controlled test stands. In these computer controlled tests, if the command versus feedback signals are not within 0.050 volts, the next command signal is delayed until this 0.050 volt difference is satisfied. This insured an error of less than two percent in command versus feedback signals at the higher loads. Errors are most likely to exist when the command load approaches the maximum load capacity of the test stand. The command signal is generated by taking the difference between two successive end points (load levels) and increasing the load in 16 step increments from one load to the next.

The loading spectrum chosen for these tests was a helicopter loading sequence developed in a collaborative effort by three European countries. Two standardized spectra were developed by this effort. One spectrum, called Helix, is a loading sequence representative of hinged or articulated rotors. The other spectrum, called Felix, represents a loading sequence for fixed or semi-rigid rotors (ref. 6). A shortened version of Felix called Felix/28 was chosen for these tests. The full Felix sequence has slightly more than two million loading cycles through one pass of the spectrum while Felix/28 has only 161034 cycles.

As with all fatigue test loading spectra, many modifications are made to the recorded flight loads before the final version of the test loading sequence is established (ref. 7). Loading sequences from a Westland Lynx and an MBB BO-105 were used in developing Felix. The Felix spectrum is scaled in Felix units with the maximum load in the sequence being 100. In arriving at the final version of Felix all alternating loads below 16 Felix units were omitted. The ground load at landing is -28 Felix units. The Felix/28 spectrum was developed by further omitting all alternating loads that were below 28 Felix units. The full Felix version contained 22 unique maneuvers. If any of these maneuvers were eliminated in the Felix/28 spectrum, the maneuver effects were retained by including one loading cycle for the highest load at or below 28 Felix units.

Four types of flights at three different flight lengths make up the 140 flights which represent one pass through the spectrum. The three different flight lengths are 0.75, 2.25, and 3.75 hours which when combined represent 190.5 flight hours. The four types of flights consist of loading sequences that represent training, transport, anti-submarine warfare, and search and rescue missions. Figure 9 shows a typical loading sequence for the transport mission.

Two forms of the Felix/28 spectrum were used during this test program. The actual Felix/28 sequence as well as a rainflow-counted version were run at several  $\alpha$  values. The rainflow-counted version stress ranges were arranged in a low-to-high sequence. The rainflow-counted spectrum was used because this was the order of the loads given to each participant for their fatigue life analysis. This loading sequence is listed in Table 2 and shown graphically in figure 10. Figure 11 presents the spectrum fatigue test results for the actual Felix/28 and the rainflow-counted spectrum loadings. The data are plotted as the maximum stress in the spectrum versus the number of loading cycles to failure.

Table 3 shows these test results in tabular form at the respective  $\alpha$  values. The data show that for this material and hole configuration, a maximum stress in the spectrum of 100 ksi will give fatigue test lives at about one pass through the spectrum while a runout occurs at a maximum stress of about 70 ksi in the spectrum. These tests also indicate that consideration must be given to randomizing the rainflow-counted sequence so that it will produce fatigue lives equivalent to the actual Felix/28 loading sequence. This is particularly true at the higher  $\alpha$  values. However, for the tests where the maximum stress in the spectra approaches the fatigue limit, the data for the two sequences appear to converge.

#### FATIGUE LIFE PREDICTION METHODS

Two major design philosophies, safe life and damage tolerance, are currently used in predicting fatigue lives of aircraft components. In the safe life approach, a conservative fatigue life is generally calculated using the Palmgren-Miner (P-M) linear cumulative damage model. It is normally assumed that this safe life includes fatigue damage which results from both the crack initiation and the crack growth phases of damage accumulation (ref. 8). While a fracture mechanics model could also be used to calculate a safe life, it is not currently being used in design. However, in the damage tolerance approach, a fracture mechanics model based on crack growth alone is used to calculate a safe inspection interval (ref. 9). For Phases I and II, all round-robin participants calculated the total fatigue life (life to catastrophic failure) using the P-M model. In Phase III, both the P-M model and a fracture mechanics crack growth model were used to calculate the total fatigue lives. The details of these two methods are described in this section.

Linear Cumulative Damage Analysis. The Palmgren-Miner (P-M) linear damage accumulation model defines fatigue damage by the cycle ratio  $n/N$ , where the numerator ( $n$ ) represents the number of

cycles of applied loading and the denominator (N) represents the number of cycles to failure at that loading. This method assumes that fatigue failure occurs when the sum of the cycle ratios (n/N) for all loads is equal to one. A counting technique is used on the flight loads to group the loads at discrete loading levels so the numerator (n) of the cycle ratio can be defined. As described in Mil-Hdbk-5 (ref. 10), the number of allowable cycles to failure (N) is determined from full-scale S-N data at a specific stress ratio (R) or steady stress. However, for the purposes of the round robin, the cycles to failure were determined from coupon S-N data. The data were acquired at an R=0 stress ratio and a specified stress concentration factor for the test specimen. Because the Felix/28 loading cycles are at various R ratios, these loads (stresses) must be "corrected" to the R ratio (or steady stress) of the fatigue test. The fatigue life is then calculated by: (1) summing all the cycle ratios for the different stress levels determined from the counting technique and, (2) inverting this sum and multiplying by the number of cycles per pass in the loading spectrum. Equation (8) describes this process.

$$\text{Fatigue Life} = 1/[\Sigma(n/N)] \cdot \text{cycles per pass} \quad (8)$$

Fracture Mechanics Analysis. In Phase III of this round robin, the Aerostructures Directorate used a fracture mechanics approach to calculate the fatigue life. In this approach, herein called the total life analysis (TLA), fatigue life is calculated by integration using a crack growth rate versus stress intensity factor relationship of the form

$$da/dN = C \cdot (\Delta K)^q \quad (9)$$

where da/dN is the crack growth rate,  $\Delta K$  is the stress intensity factor range, and C and q are numerical curve fit parameters. The main difference between the TLA analysis and the more conventional

crack growth analysis is that crack-closure concepts (ref. 11) are used to define an effective stress intensity factor range,  $\Delta K_{\text{eff}}$ . For the purposes of this analysis, the initial crack length used to calculate the fatigue life was about 0.0005 inches. This value was obtained from a small crack study on 4340 steel (ref. 12) in which initial defect sizes were evaluated at 34 crack initiation sites. The median crack length was about 0.0005 inches with the largest and smallest values ranging from about 0.002 inches to 0.00008 inches.

From crack-closure considerations,  $\Delta K$  in equation (9) is replaced by  $\Delta K_{\text{eff}}$  which states that only that portion of the cyclic stress range which is between the crack-opening stress and the maximum stress in the loading cycle causes fatigue crack growth. In this analysis,  $\Delta K_{\text{eff}}$  is defined as

$$\Delta K_{\text{eff}} = (S_{\text{max}} - S_0) \cdot (\pi a)^{\frac{1}{2}} \cdot F \quad (10)$$

where  $S_0$  is the crack-opening stress as calculated from the analytical closure model developed by Newman in reference 11 and  $F$  is the boundary correction factor which accounts for the effects of specimen and crack configuration on the stress intensity factors. To calculate the crack growth rate, equation (9) becomes

$$da/dN = C \cdot [(S_{\text{max}} - S_0) \cdot (\pi a)^{\frac{1}{2}} \cdot F]^q \quad (11)$$

Total life is then calculated by integrating equation (11) from the initial crack length to failure and is given by

$$\text{Total Life} = \int_{a_i}^{a_f} \frac{da}{C \cdot [(S_{\text{max}} - S_0) \cdot (\pi a)^{\frac{1}{2}} \cdot F]^q} \quad (12)$$

where  $a_i$  is the initial crack length as determined from the small crack studies and  $a_f$  is the final crack length at failure. Fatigue cycles are summed as the crack grows until  $K_{\text{max}} = K_C$ ,

where  $K_C$  is the fracture toughness. When  $K_{max} = K_C$ , the summation of the loading cycles,  $N$ , becomes the total fatigue life.

In figure 12 the analytical predictions from the P-M analysis and the crack-growth (TLA) analysis are compared with the spectra test results. The P-M predictions follow the trend of the spectrum test data. However, the P-M analysis predicts the same life for both the Felix/28 and the low-to-high rainflow loading spectra. For these test results, the P-M lives are on the low side of the rainflow data and on the high side of the Felix/28 data. Because the TLA analysis can account for acceleration and retardation effects on crack growth that exist in most loading spectra, different fatigue lives are predicted for the Felix/28 and the rainflow spectra. For the Felix/28 test data, the TLA analysis accurately predicts the trend in the fatigue lives. The predicted lives fall along the high-life edge of the scatter in the test lives. For the rainflow spectra, the TLA predicted lives fall between the scatter of the test data. As seen from equation (12), the TLA-calculated fatigue lives can be sensitive to the initial crack length. For the predicted lives shown in figure 12, the initial crack length was the median value obtained from reference 12.

#### ROUND ROBIN RESULTS

The results of all three phases of this joint exercise are presented in this section.

Phase I - Identical Methods and Inputs. In Phase I, all participants used the same S-N curve, spectrum loading sequence, and statistical parameters in solving the three problems. Tables 5 through 7 present the fatigue life results for Problem 1. Tables 8 and 9 present the fatigue life calculations at six nines reliability for Problems 2 and 3, respectively. Fatigue lives in

the tables are given in terms of the number of passes through the loading spectrum. For Problems 2 and 3, the first numbers tabulated were calculated using the joint probability/life matrix approach while the numbers in parentheses were calculated using the closed-form method. The results from Phase I confirmed that all participants could calculate about the same answers for all three problems when the S-N curve formulation, loading spectrum, and statistical parameters were identical.

Recall that Problem 2 was set up to calculate fatigue lives at six nines reliability without any knowledge about loads on individual aircraft. On the other hand, Problem 3 assumed that loads on each aircraft could be measured to within a 3% COV. Comparing the fatigue lives at six nines reliability between Problems 2 and 3 may provide some measure of the benefits of loads monitoring. For Problem 3, the calculated six nines reliability fatigue life at a mean  $\alpha$  of 0.6 was about 4.2 times greater than the six nines fatigue life calculated for Problem 2. Thus, this 4.2 factor could also be used to quantify potential cost savings if retirement lives could be increased through loads monitoring.

Phase II - Independent Methods and Inputs. In Phase II, each participant used the constant-amplitude test points provided by ASTD to develop an independent S-N curve, fatigue limit, and fatigue limit COV. The form of the fatigue life equation is given as

$$N = A \cdot (S_r - S_e)^{-B} \quad (13)$$

where A and B are curve fit parameters. Table 10 presents the parameters from equation (13) and the fatigue limit COV which were used by each participant. Tables 11 through 13 present the Phase II fatigue life results for Problem 1 in terms of the number of passes through the loading spectrum. As expected, there are differences in the fatigue life predictions. The most significant contributor to these differences appears to be the value of

fatigue limit which was determined from the given constant-amplitude fatigue data. Figures 13 and 14 highlight some of these differences for Problem 1 at  $\alpha$ 's of 0.7 and 1.0, respectively. For  $\alpha$  equal to 0.7 (fig. 13) the maximum difference in fatigue life predictions is almost a factor of 30. However, for  $\alpha$  equal to 1.0 (fig. 14) the maximum difference is less than a factor of three. In terms of current engineering practice for fatigue life prediction, a factor of four difference is generally considered to be reasonable. At an  $\alpha$  of 0.7, many of the loads are below the fatigue limit and small differences in the fatigue limit could result in large differences in the predicted fatigue life. As seen from Table 10, the fatigue limits used by the participants differed by almost 20 percent. The results are consistent with this reasoning in that the longest fatigue life predictions (fig. 13) were calculated by using the highest fatigue limits (Table 10). At the higher  $\alpha$ 's, more of the loads are above the fatigue limit and contribute to fatigue damage. Thus, according to Miner's Rule, less scatter in the fatigue life predictions should be expected, as shown in figure 14.

Besides the effects of fatigue limit on mean life predictions, the manner in which fatigue strength reductions are applied will contribute to the scatter in life predictions. Recall that in Phase I the statistical variations on strength were based on a constant standard deviation and not a constant coefficient of variation. In Phase II, some of the participants based the strength reductions on a constant COV and not a constant standard deviation. Assuming all other parameters equal, the predicted fatigue lives would be shorter when using a constant COV to account for strength variability. Another slight difference in the statistical analysis was the use of a log-normal distribution for strength by one of the participants. The six nines reliability results from Problem 2 show larger differences among the participants than do the mean fatigue life results of Problem 1. One cause for these differences appears to be the magnitude of the fatigue limit COV that was used. In addition, the differences at higher  $\alpha$  values

may be attributed to the use of a constant COV rather than a constant standard deviation.

Tables 14 and 15 show the Phase II fatigue life predictions at six nines reliability for Problem 2 and Problem 3, respectively. Again, for Problems 2 and 3 the first numbers represent the joint probability/life matrix approach while the numbers in parentheses represent the closed-form method. Figure 15 shows some of the differences in the six nines fatigue lives for Problem 2 at an  $\alpha$  of 0.6. The maximum difference between the predicted fatigue lives at six nines reliability is almost a factor of 60. This is about twice the scatter that was obtained in Problem 1 for the mean fatigue life predictions. Comparing the ratio of fatigue lives between Problems 2 and 3 at mean  $\alpha$ 's of 0.8 and 0.85, shows increases in fatigue lives at six nines reliability from as low as 1.8 to as high as 40. Again, the fatigue limit value and the method used for fatigue strength reduction (constant standard deviation versus constant COV) may account for these differences.

Phase III - Spectrum Fatigue Tests. In this phase, each industry participant used their standard linear cumulative damage methodology and the mean S-N curves derived from the Phase II constant amplitude fatigue tests to calculate mean spectrum fatigue lives. In addition to this approach, ASTD used a fracture mechanics approach to calculate mean fatigue lives. Table 16 presents these mean fatigue life calculations at the same  $\alpha$ 's used to conduct the spectrum tests. Figure 16 shows a comparison between the spectrum test data and the round-robin predictions. Although all the predictions are greater than the test lives, the predictions are within a factor of 4 difference. The relatively small scatter among predictions is consistent with the Phase II/Problem 2 predictions for  $\alpha > 1$ . Also presented in figure 16 are the round-robin predictions for fatigue lives at six nines reliability from Phase II/Problem 2. One clear observation is the larger variation in six nines reliability predictions for  $\alpha$  values less than one.

## CONCLUDING REMARKS

The purposes of this AHS round robin were twofold. First, it was intended to develop the logic for performing a reliability analysis for fatigue life prediction. Phase I was set up so that only one answer was correct and the results for each participant were compared to assure that a consistent approach was used. Second, in Phases II and III the participants applied the same logic but used their company's standard fatigue methodology to solve for six nines reliability. The intent of these two phases was not to prove that any one answer was "right" or "better" but instead to find out what contributed to the differences.

The two major contributors which affected the results were the S-N curve formulation and the method used for strength reduction. One of the questions which was raised as a result of this round robin was whether the COV or the standard deviation is constant over the S-N curve. In Phase I, all participants used a constant standard deviation to solve the fatigue life and reliability problems. However, in Phase II some participants used a constant standard deviation while others used a constant COV. There is a real need to establish a uniform methodology for developing S-N curves and coefficients of variation.

Another question raised during the round robin was what is the "best" fatigue life analysis method. The U.S. helicopter industry has traditionally used the P-M linear cumulative damage model to predict fatigue lives. As a result, this approach was used by all participants in the round robin. In Phase III, ASTD also used a fracture mechanics model based on crack growth to predict the mean fatigue life. While the P-M fatigue life predictions were reasonable, the P-M rule could not distinguish between the two spectra which were used in the fatigue tests. On the other hand, the fracture mechanics approach can account for load interaction effects and the TLA method did predict the differences in fatigue lives for the two spectra. Even though the P-M fatigue

life predictions for the Felix/28 spectrum tests were reasonable, it is important to continue the search for a better fatigue life methodology.

Some aspects of the probabilistic approach to six nines reliability were not explored in this round robin. There was no attempt to incorporate confidence levels in the current effort. By definition (and intent) the results which are presented in this report are based upon a 50% confidence level. The aim of this exercise was to develop a methodology which included loads variability but restrained the fatigue analyst's freedom to manipulate conservatism in the measured loads. Most of the goals of this first round-robin activity were achieved by only considering the expected value (50% confidence) of reliability. The need to include confidence levels is a topic which may require further evaluation. Another purpose of this exercise was to try and quantify the benefits of individual component replacement versus fleet replacement. The  $\alpha$  parameter was devised to account for loads variability in the reliability analysis. While these preliminary results showed the potential for increasing mean retirement lives through loads monitoring (with commensurate reductions in costs), no attempt was made in this first round robin to separate usage from other sources of variability in the loads. Additional study is needed to examine how usage should be treated to properly account for operational variability.

All of these questions reaffirm that much more work is needed before reliability-based fatigue design becomes standard industry practice. These preliminary round-robin results have demonstrated that consistent reliability-based design cannot be implemented without the cooperation of all the rotorcraft industry. In addition to the study areas already mentioned, follow-on efforts to this round robin are needed to:

- (1) Extend the statistical and reliability analysis complexity to account for both usage and other sources of load variability, and to assess reliability versus confidence levels. If cur-

rent flight data recorders largely monitor usage, what does this imply for load accuracy and the merit of individual part replacement?

(2) Apply the reliability methodology to metals using a damage tolerance or fracture mechanics approach.

(3) Repeat the P-M and TLA approaches on other metallic materials with different ultimate strengths and stress concentration factors to develop confidence in the fatigue reliability approach.

(4) Evaluate the effects of coupon versus full-scale testing on data scatter.

(5) Investigate flight loads survey methodology to better define the variabilities of usage and pilotage (simulated mission flights versus maneuver-by-maneuver flights).

(6) Extend the reliability-based fatigue methodology to composites.

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Table 1. Constant amplitude fatigue test data for R = 0.

$S_{max}$ (ksi)	Cycles-to-failure			
50	5993030			
52.5	3757353,	<b>1000000</b>	(run-out)	
55	2577077,	10000000	(run-out)	
60	<b>206790,</b>	116768,	<b>839331</b>	
60	10000000	(runout)		
65	97278,	81773		
70	308435,	61361,	58233	
80	80827,	49277,	37095,	<b>34059</b>
	<b>28099</b>			
120	<b>7434,</b>	7306,	7056	
175	1531,	1336,	1325	

Note: Numbers in **bold** are original data points given to participants.

Table 2. Rainflow Low-High Load Sequence Derived From Felix28

NOMINAL STRESS RANGE (KSI)	NOMINAL STRESS MEAN (KSI)	NUMBER OF CYCLES
2.80	25.59	354
2.80	32.83	334
6.42	29.21	416
10.04	29.21	609
10.04	36.45	1228
10.04	40.07	810
13.66	36.45	2
17.28	18.35	140
17.28	32.83	78
20.91	32.83	2061
20.91	36.45	90
24.53	-7.00	140
24.53	18.35	140
24.53	36.45	2040
28.15	29.21	833
31.77	25.59	346
35.39	25.59	7904
35.39	29.21	56
35.39	32.83	71072
35.39	43.69	2529
39.01	21.97	3014
39.01	25.59	42825
39.01	29.21	6393
39.01	43.69	252
42.63	25.59	480
42.63	29.21	207
42.63	36.45	1274
46.25	21.97	274
46.25	25.59	6239
46.25	29.21	4274
46.25	40.07	604
49.87	3.86	268
49.87	25.59	956
49.87	29.21	2179
53.49	25.59	2
53.49	29.21	116
57.12	25.59	5
57.12	29.21	185
60.74	29.21	25
64.36	25.59	7
64.36	29.21	8
64.36	32.83	75
67.98	29.21	9
71.60	29.21	16
75.22	25.59	7
78.84	18.35	5
78.84	25.59	1
82.46	21.97	128
82.46	29.21	16
89.70	25.59	8

Table 3. Spectra Fatigue Test Data

a) Felix/28 Spectrum

$S_{max}$ (ksi)	$\alpha$	Cycles-to-failure			
65	0.92	41000000 (run-out)			
70	0.99	11031000			
73.3	1.04	4396500			
80	1.14	3128200,	2898600,	2095100,	1177900
		1032500,	841860,	552600,	404510
85	1.21	121080			
86.7	1.23	176308			
90	1.28	227510,	179180		
93.3	1.32	279190			
100	1.42	187490,	175650,	107580	
120	1.70	52079,	41228		

b) Rainflow Low-High Spectrum (of Felix/28)

70	0.99	42290000 (run-out)			
80	1.14	2116500			
90	1.28	4112834,	3176269,	629160	
100	1.42	577140,	219872,	214420	
120	1.70	206495,	65124,	49055	

Table 4. AHS Round-Robin Participants.

Organization	ID Code
Aerostructures Directorate (AVSCOM)	ASTD
Bell Helicopter Textron	BHT
Boeing Helicopters	BH
Kaman Aerospace Corporation	KAC
McDonnell Douglas Helicopter Company	MDHC
Sikorsky Aircraft Division	SA

Table 5. Fatigue Life Predictions for Mean Strength, Phase I/Problem 1.

Alpha	ASTD	BHT	BH	KAC	MDHC	SA
0.3	6.21e9	--	--	--		--
0.4	6.21e9	--	--	--		--
0.5	14740	--	14900	14700	16236	14895
0.6	171.1	170.5	168.4	171.0	175.0	168.4
0.7	47.80	50.0	46.85	47.8	48.0	46.85
0.8	18.69	19.3	18.42	18.7	19.0	18.42
0.9	3.49	3.5	3.48	3.49	3.7	3.48
1.0	1.03	1.0	1.02	--	1.1	1.02

Note: Fatigue life values are defined in terms of the number of passes through the spectrum (1 pass = 161,034 cycles).

Table 6. Fatigue Life Predictions for Mean Minus 3-Sigma Strength, Phase I/Problem 1.

Alpha	ASTD	BHT	BH	KAC	MDHC	SA
0.3	6.21e9	--	--	6.21e9	--	--
0.4	10850	--	11112	10900	12420	11112
0.5	135.0	131.5	132.2	135.0	136.0	132.2
0.6	36.9	37.2	36.2	36.9	37.0	36.2
0.7	6.72	6.68	6.69	6.72	7.0	6.69
0.8	1.36	1.39	1.36	1.36	1.4	1.36
0.9	.294	.30	.294	.294	--	.29
1.0	.117	.118	.117	.117	--	.12

Note: Fatigue life values are defined in terms of the number of passes through the spectrum (1 pass = 161,034 cycles).

Table 7. Fatigue Life Predictions for Mean Minus 5-Sigma Strength, Phase I/Problem 1.

Alpha	ASTD	BHT	BH	KAC	MDHC	SA
0.3	2.67e6	--	2.65e6	2.67e6	1204.0	--
0.4	214.9	200.7	210.6	215.0	217.0	210.6
0.5	44.2	43.9	43.4	44.2	44.0	43.4
0.6	5.18	5.0	5.15	5.18	5.4	5.15
0.7	.891	.90	.89	.891	--	.89
0.8	.209	.211	.209	.209	--	.21
0.9	.096	.097	.096	.096	--	.10
1.0	.057	.057	.057	.057	--	.06

Note: Fatigue life values are defined in terms of the number of passes through the spectrum (1 pass = 161,034 cycles).

Table 8. Fatigue Life Predictions for Six Nines Reliability, Phase I/Problem 2.

Alpha	ASTD	BHT	BH	KAC	MDHC	SA
0.3	--	--	22697 (23062)	--	--	(22663)
0.4	110.7 (105.0)	111.4	105.2 (105.4)	--	162.6	(105.3)
0.5	21.48 (22.0)	24.05	21.26 (22.1)	(22.3)	23.6	(22.08)
0.6	2.14 (2.19)	2.23	2.01 (2.2)	(2.21)	2.08	(2.20)
0.7	.315 (.35)	.280	.310 (.351)	(.35)	.31	(.35)
0.8	.108 (.114)	.112	.104 (.114)	(.114)	.11	(.11)
0.9	.056 (.058)	.056	.052 (.058)	(.058)	.06	(.06)
1.0	.035 (.036)	--	.033 (.036)	--	.04	(.04)

- Notes: 1. Fatigue life values are defined in terms of the number of passes through the spectrum (1 pass = 161,034 cycles).  
 2. Fatigue life values in parentheses were calculated using the closed-form method.

Table 9. Fatigue Life Predictions for Six Nines Reliability, Phase I/Problem 3.

Alpha	ASTD	BHT	BH	KAC	MDHC	SA
0.6	8.88 (8.60)	9.50	8.67	(8.65)	9.14	(8.90)

- Notes: 1. Fatigue life values are defined in terms of the number of passes through the spectrum (1 pass = 161,034 cycles).  
 2. Fatigue life values in parentheses were calculated using the closed-form method.

Table 10. Phase II Results for S-N Curve Formulation.

S-N Curve Parameter	ASTD	BHT	BH(1)	KAC	MDHC(2)	SA
A	3.500e6	3.828e6	1.40e9	1.148e8	--	3.855e6
B	1.47164	1.37	2.927	2.31	--	1.3699
Se	54.5	49.6	44.75	48.0	--	53.56
COV(Se)	.07	.109	.0785	.0831	--	.10

- Notes: 1. Log Normal distribution assumed.  
 2. Round robin S-N formulation not applicable.

Table 11. Fatigue Life Predictions for Mean Strength, Phase II/Problem 1.

Alpha	ASTD	BHT (2)	BH (2,3)	KAC	MDHC	SA (2)
0.5	--	--	--	6.20e9	--	--
0.6	--	190735.	--	1.59e6	90356	--
0.7	38180.	2316.5	10610.	10900.	1934.0	52354.
0.8	1144.0	649.0	1545.3	1760.0	655.0	1418.8
0.9	376.1	288.0	447.9	580.0	292.0	449.5
1.0	175.8	121.5	175.8	231.0	80.0	205.6
1.1	82.21	--	59.2	69.2	--	87.74
1.2	22.37	--	20.3	22.8	--	22.23

- Notes: 1. Fatigue life values are defined in terms of the number of passes through the spectrum (1 pass = 161,034 cycles).  
 2. Constant COV for strength reduction.  
 3. Modified Goodman Correction.

Table 12. Fatigue Life Predictions for Mean Minus 3-Sigma Strength, Phase II/Problem 1.

Alpha	ASTD	BHT (2)	BH (2,3)	KAC	MDHC	SA (2)
0.4	--	141125.	258469.	--	--	--
0.5	--	1001.8	2997.0	41800.	10373.	74206.
0.6	2723.	277.6	645.6	2850.	1034.	950.0
0.7	545.1	75.0	106.0	739.0	350.0	281.2
0.8	214.2	12.0	30.7	209.0	94.0	110.0
0.9	75.48	2.92	6.32	48.2	17.0	19.24
1.0	15.89	.98	2.62	12.7	5.0	5.88
1.1	5.501	--	1.32	4.06	--	1.51
1.2	1.664	--	.75	1.77	--	.65

Table 13. Fatigue Life Predictions for Mean Minus 5-Sigma Strength, Phase II/Problem 1.

Alpha	ASTD	BHT (2)	BH (2,3)	KAC	MDHC	SA (2)
0.4	--	282.7	2078.5	33400.	294055.	3031.1
0.5	2637.	31.0	282.0	2420.	1543.	338.5
0.6	504.8	4.0	27.4	549.0	411.0	94.55
0.7	186.6	.80	6.3	89.9	104.0	11.35
0.8	32.0	.40	2.64	18.0	16.0	2.36
0.9	8.48	.20	1.34	4.99	4.0	.70
1.0	2.14	--	.774	2.06	1.0	.36
1.1	.896	--	.487	1.07	--	.22
1.2	.512	--	.326	.635	--	.15

- Notes:
1. Fatigue life values are defined in terms of the number of passes through the spectrum (1 pass = 161,034 cycles).
  2. Constant COV for strength reduction.
  3. Modified Goodman Correction.

Table 14. Fatigue Life Predictions for Six Nines Reliability, Phase II/Problem 2.

Alpha	ASTD	BHT	BH	KAC	MDHC	SA
0.3	--	4786.	17360.	--	--	--
0.4	257200. (217500.)	324.	1428.0	--	45914.	(2744.2)
0.5	1166. (1088.)	42.0	121.2	(1340.)	814.0	(326.2)
0.6	283.6 (275.6)	4.8	18.3	(278.0)	180.0	(88.51)
0.7	78.06 (79.56)	.90	4.28	(45.2)	19.0	(10.74)
0.8	12.25 (12.85)	.40	1.73	(8.98)	3.7	(2.20)
0.85	6.82	--	1.19	--	--	(1.09)
0.9	2.88 (3.24)	.25	.878	(2.75)	1.2	(.67)
1.0	.924 (1.01)	--	.492	--	.66	(.35)
1.1	.468 (.508)	--	.679	--	--	(.22)
1.2	.288 (.314)	--	.206	--	--	(.15)

- Notes: 1. Fatigue life values are defined in terms of the number of passes through the spectrum (1 pass = 161,034 cycles).  
 2. Fatigue life values in parentheses were calculated using the closed-form method.

Table 15. Fatigue Life Predictions for Six Nines Reliability, Phase II/Problem 3.

Alpha	ASTD	BHT	BH	KAC	MDHC	SA
0.80	--	--	3.17	(30.5)		
0.85	25.74 (25.49)	13.55	2.13		9.14	(2.7)

- Notes: 1. Fatigue life values are defined in terms of the number of passes through the spectrum (1 pass = 161,034 cycles).  
 2. Fatigue life values in parentheses were calculated using the closed-form method.

Table 16. Fatigue Life Predictions for Mean Strength, Phase III.  
 (Smax = 70.44 for alpha = 1.0)

Smax	Alpha	ASTD	BHT	BH	KAC	MDHC	SA
73.3	1.04	134.7	121.49	109.8	--	38.74	156.1
80.0	1.14	51.72	18.3	37.5	38.5	13.67	53.7
86.7	1.23	15.3	7.90	15.4	17.5	4.41	15.4
90.0	1.28	9.27	4.60	9.32	9.18	2.56	9.5
93.3	1.32	6.69	2.88	5.46	5.78	1.85	6.8
100.	1.42	2.48	1.23	2.42	2.49	1.03	2.2
120.	1.70	.375	.38	.448	.466	--	.38

Note: Fatigue life values are defined in terms of the number of passes through the spectrum (1 pass = 161,034 cycles).

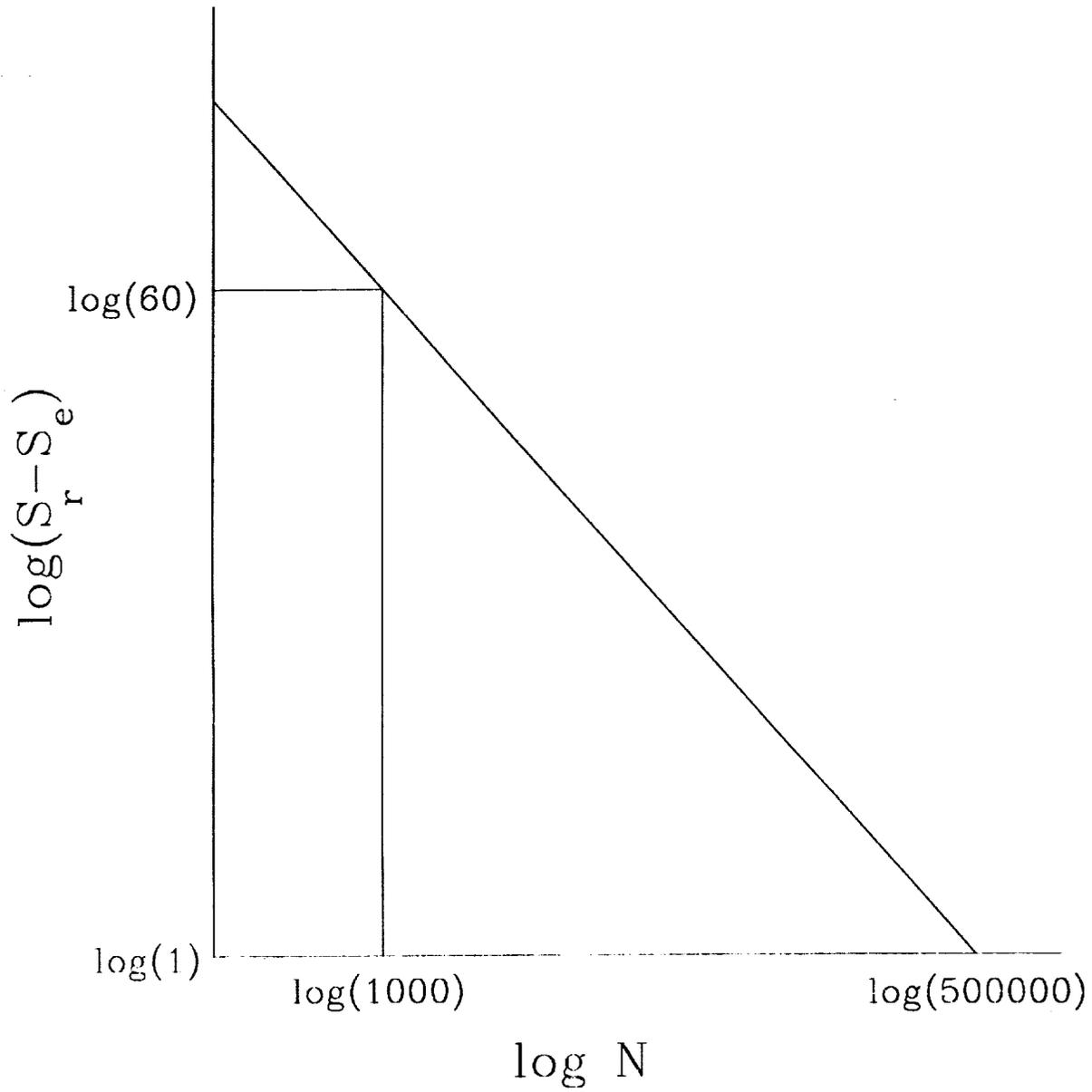


Fig. 1. Theoretical S-N Curve

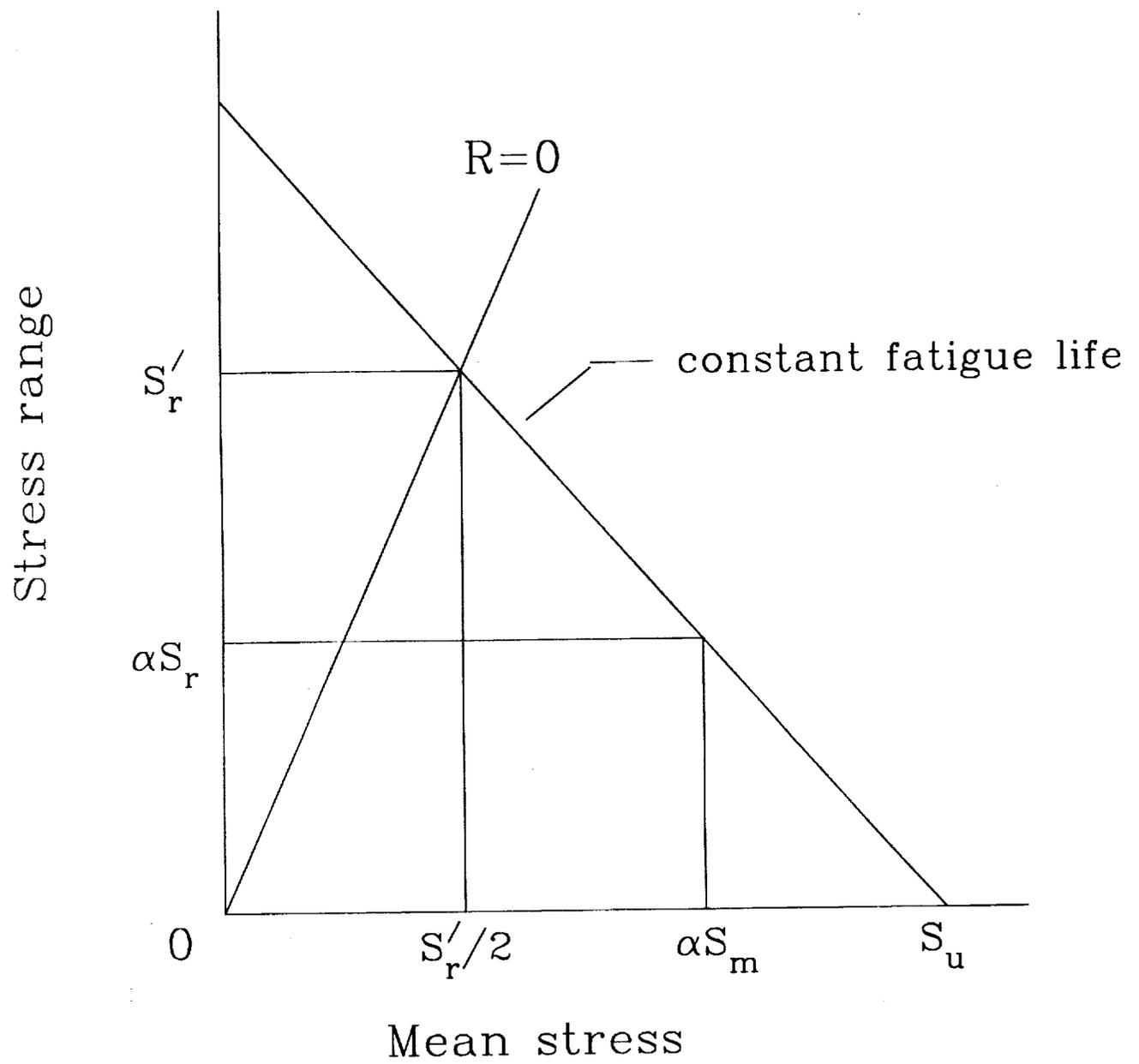


Fig 2. Linear Goodman diagram

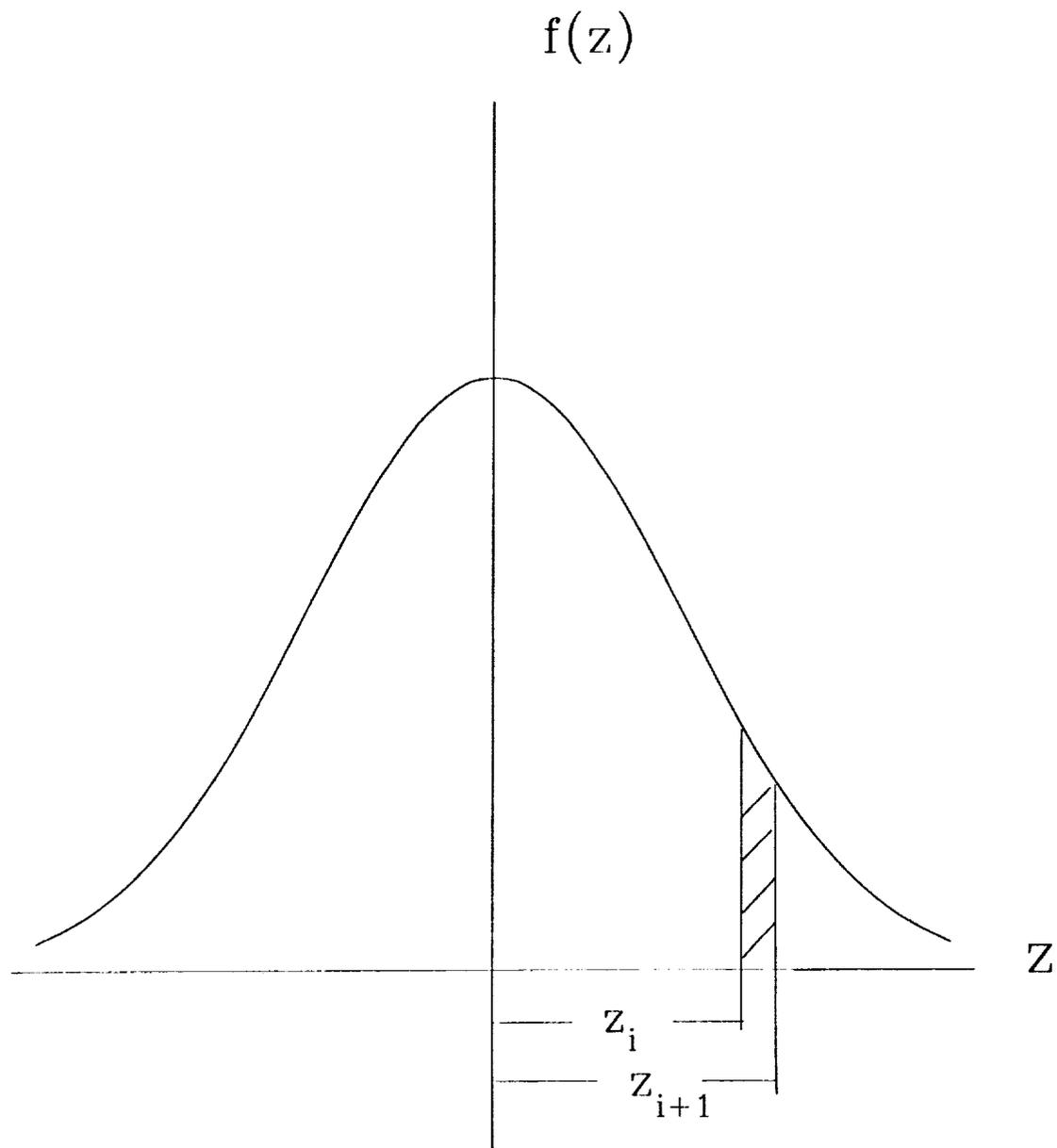


Fig. 3. Probability density function(PDF)

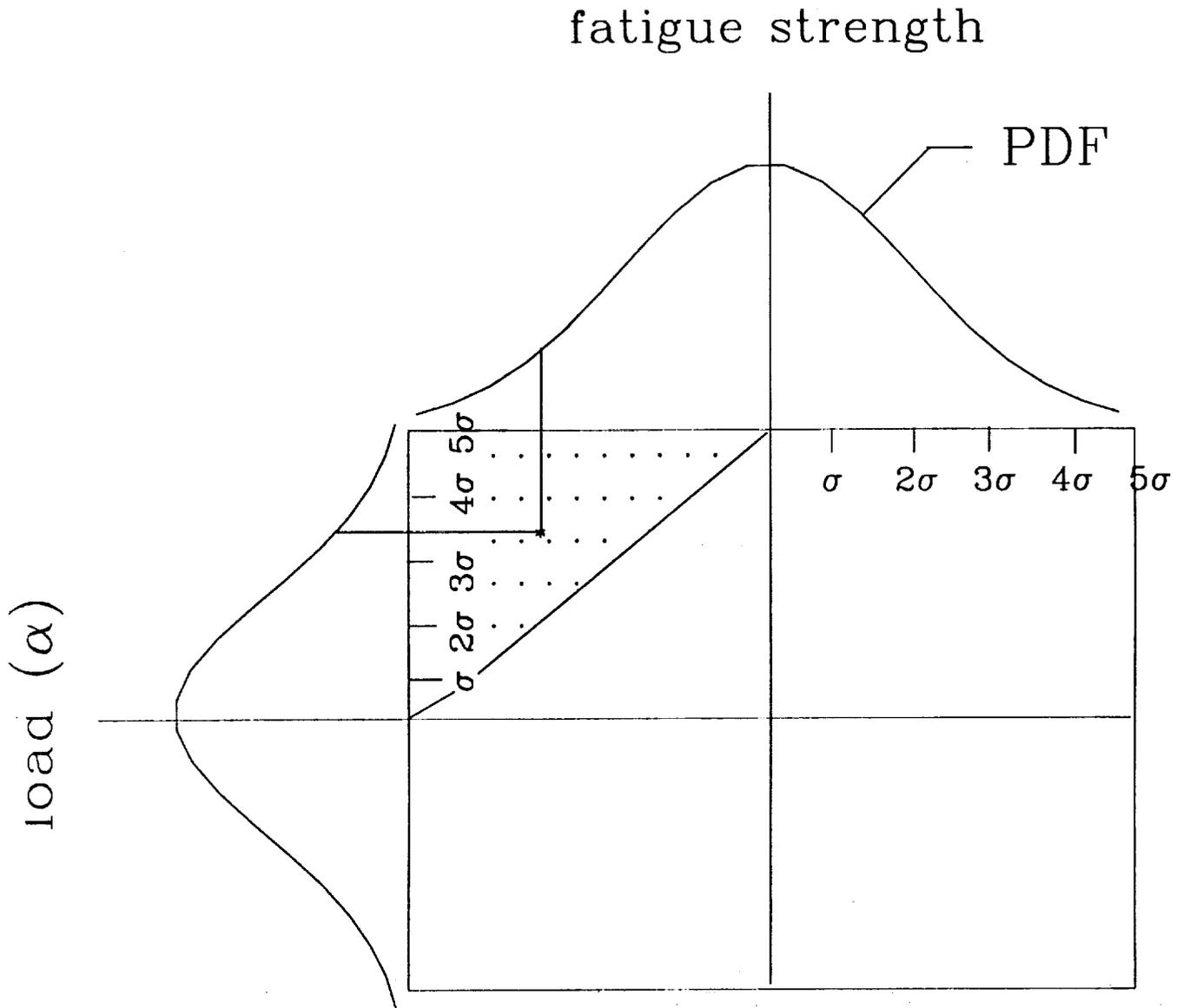


Fig. 4. Joint probability matrix

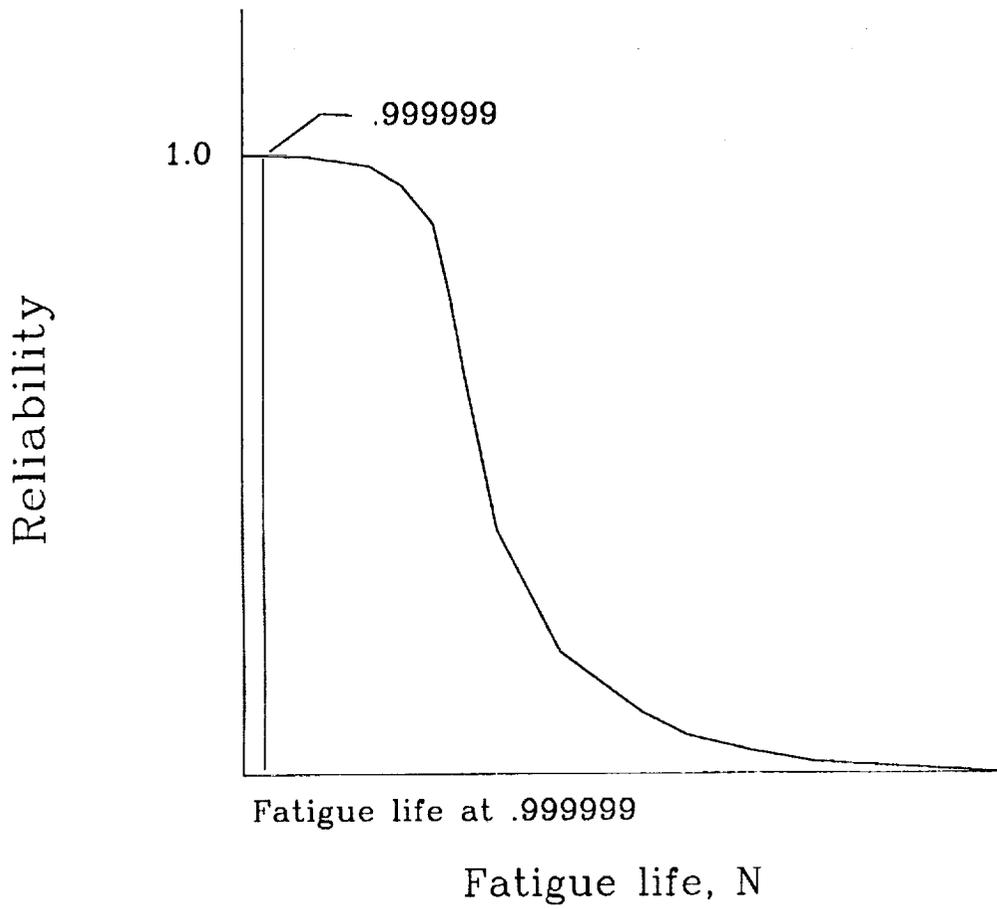


Fig. 5. Reliability versus fatigue life.

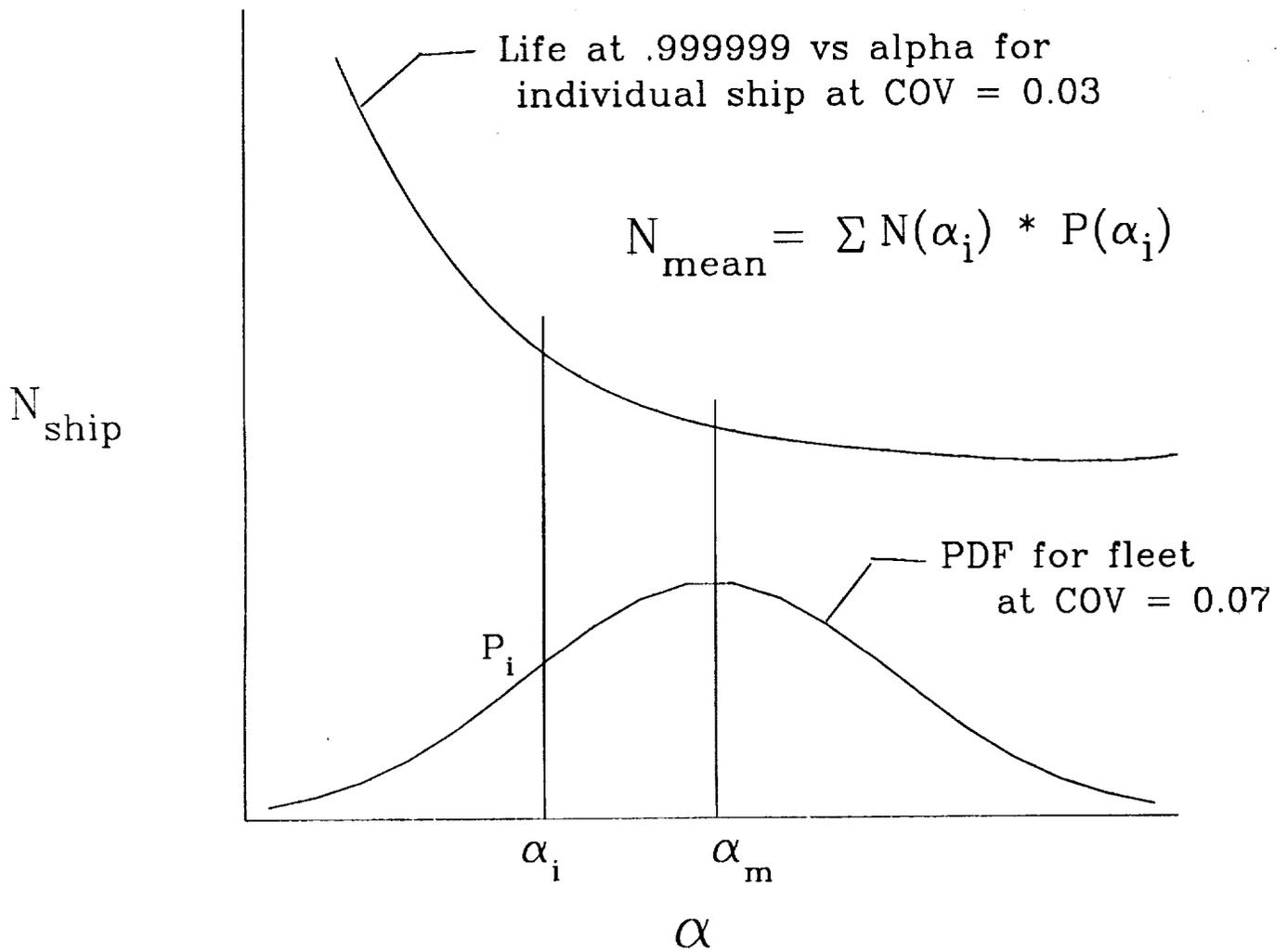
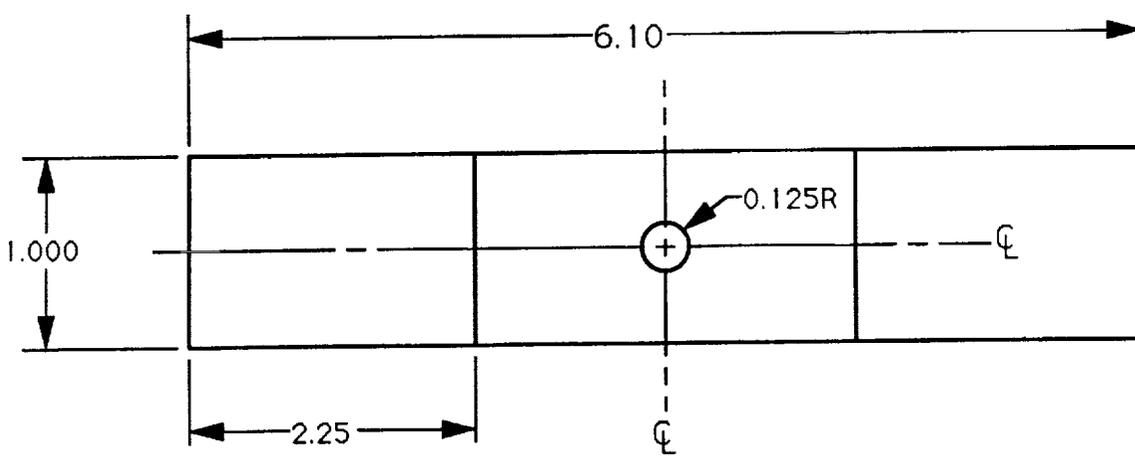


Fig. 6. Total probability process for problem 3.



Note: Dimensions in inches

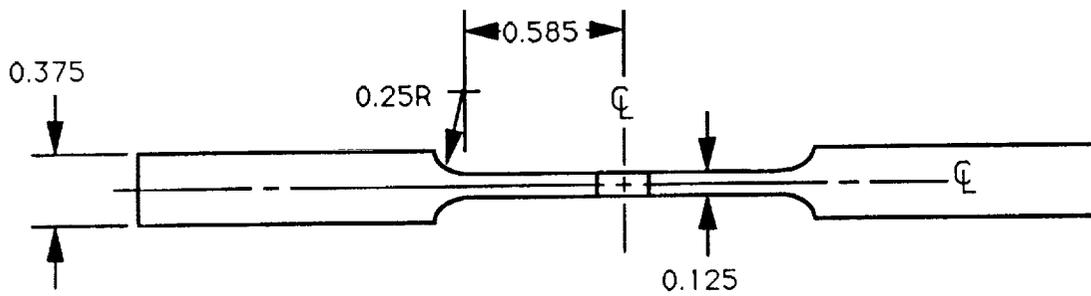


Fig. 7. Fatigue test specimen configuration





Fig. 9. Felix28 long transport flight (3.75 hrs)

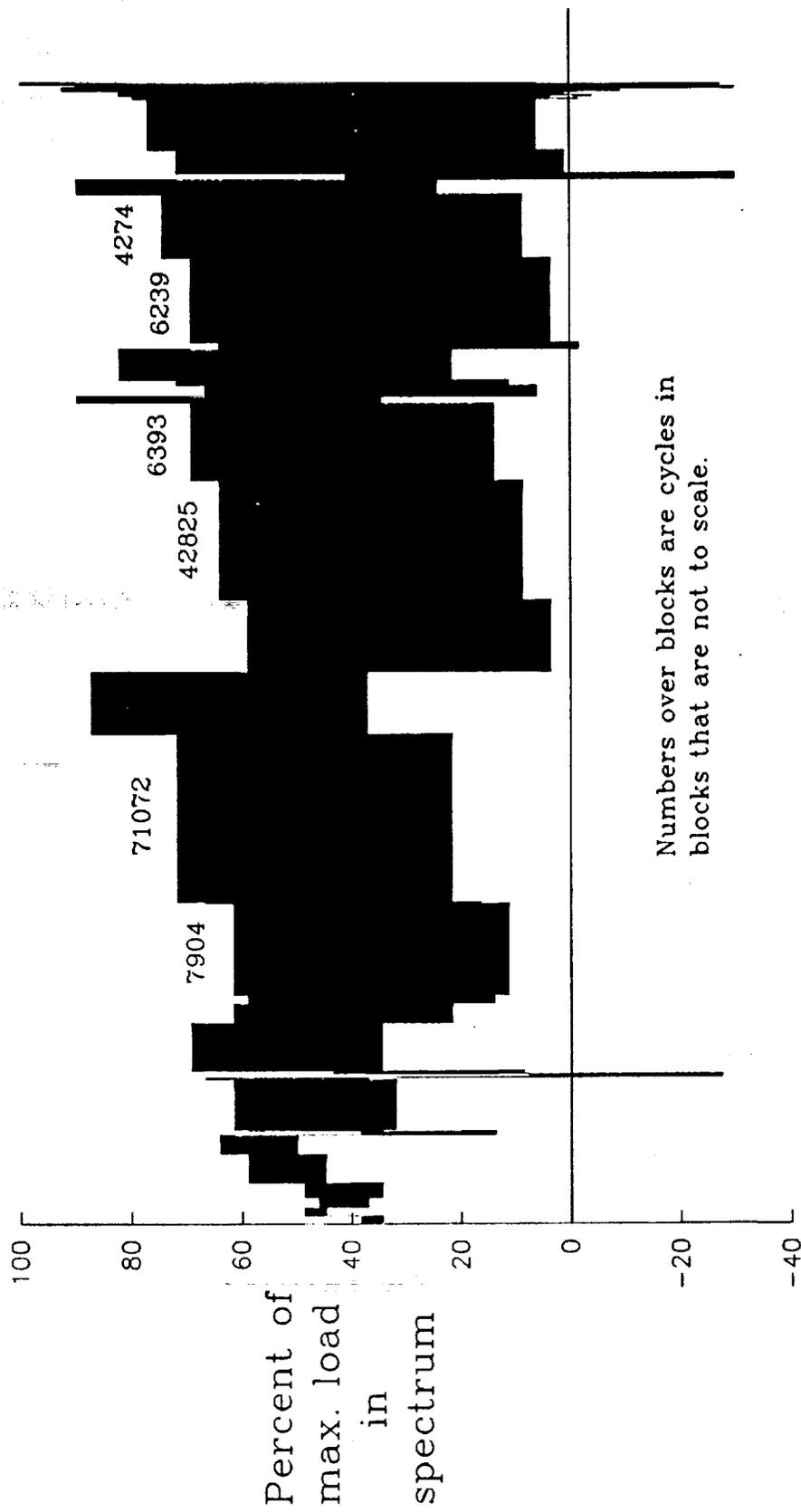


Fig. 10. Rainflow low-high sequence derived from Felix28.

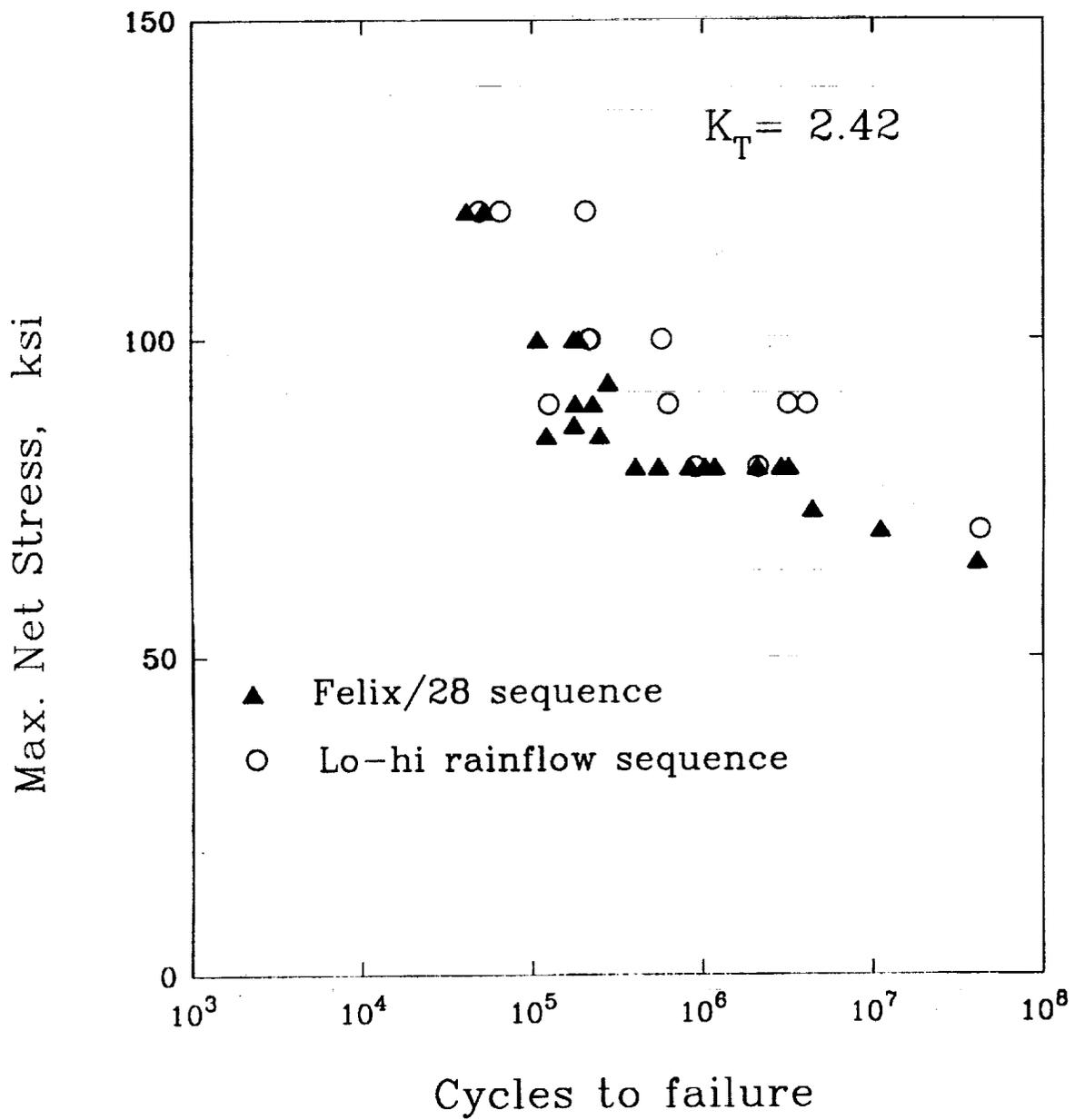


Fig. 11. Spectra test results.

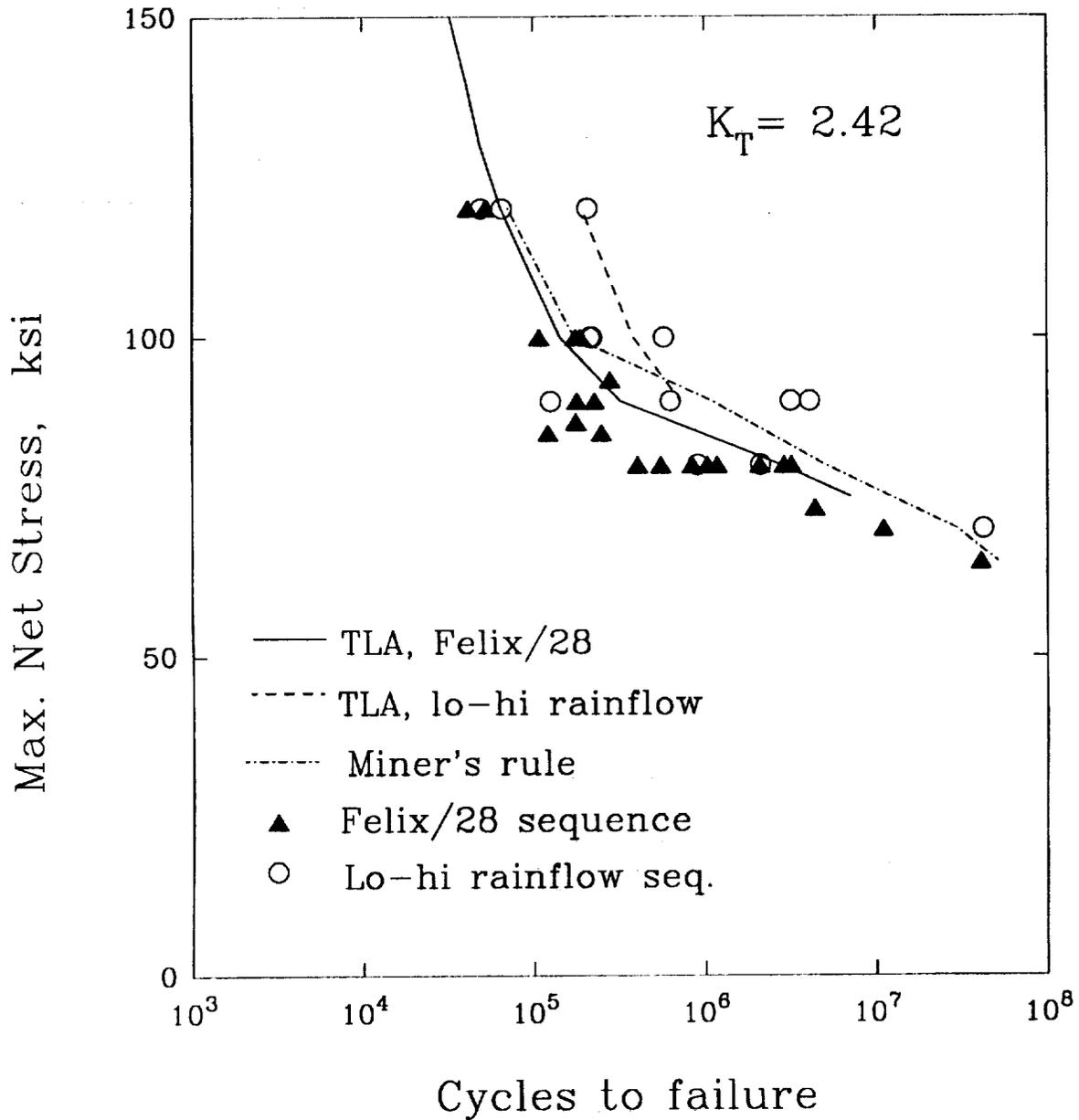


Fig. 12. Analytical predictions of fatigue spectra test results.

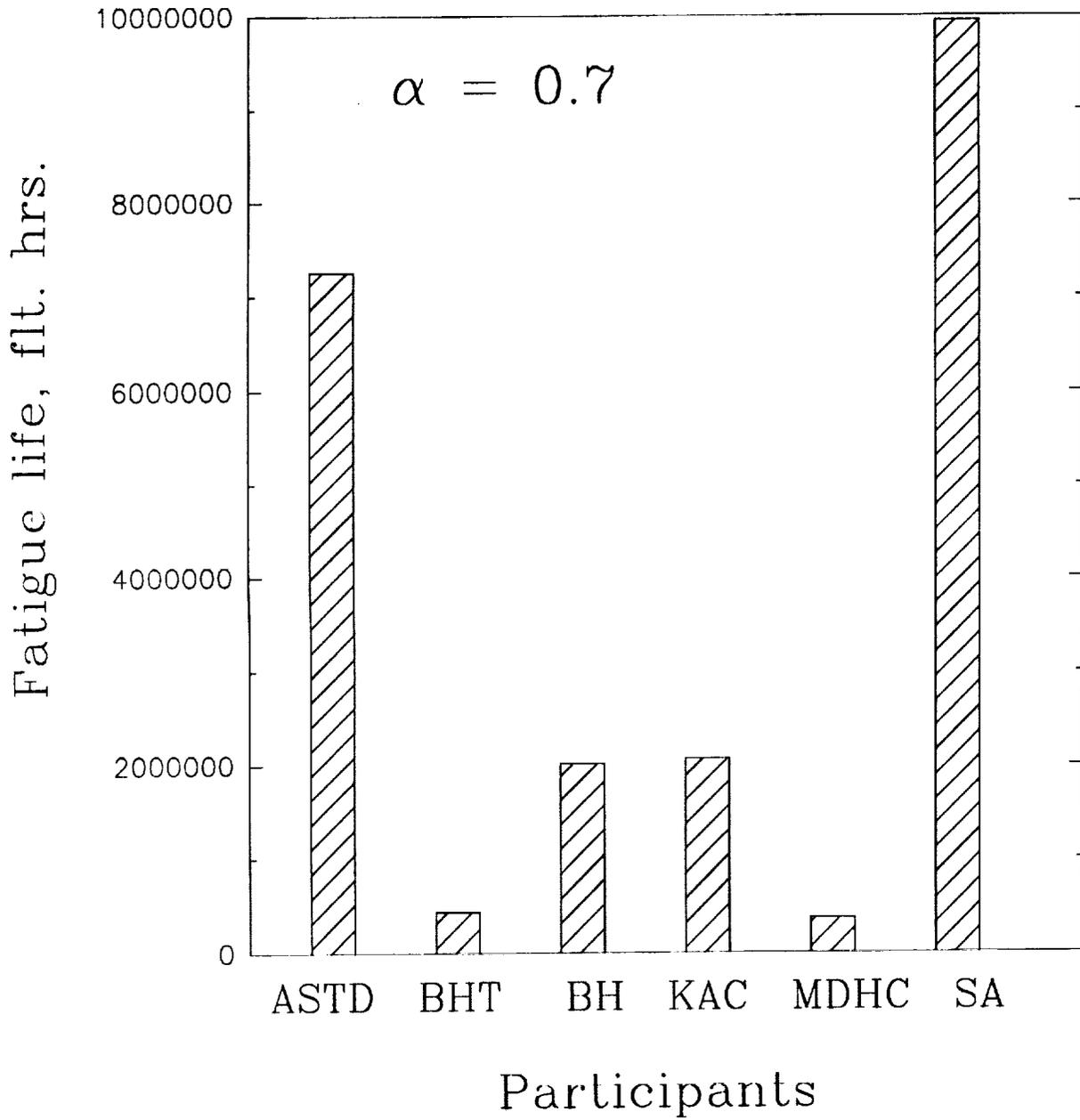


Fig. 13. Phase II/Problem 1 results for mean strength.

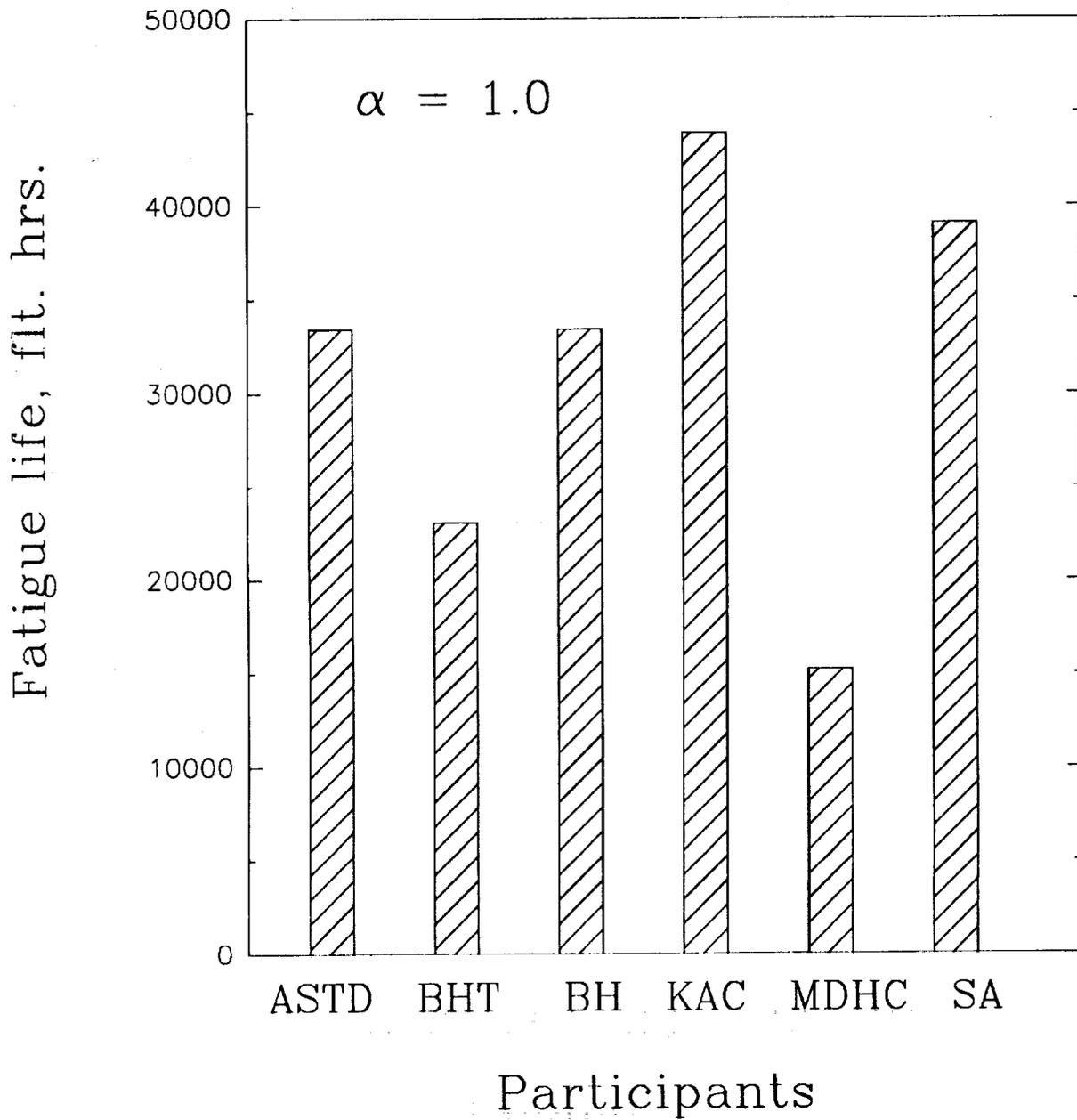


Fig. 14. Phase II/Problem 1 results for mean strength.

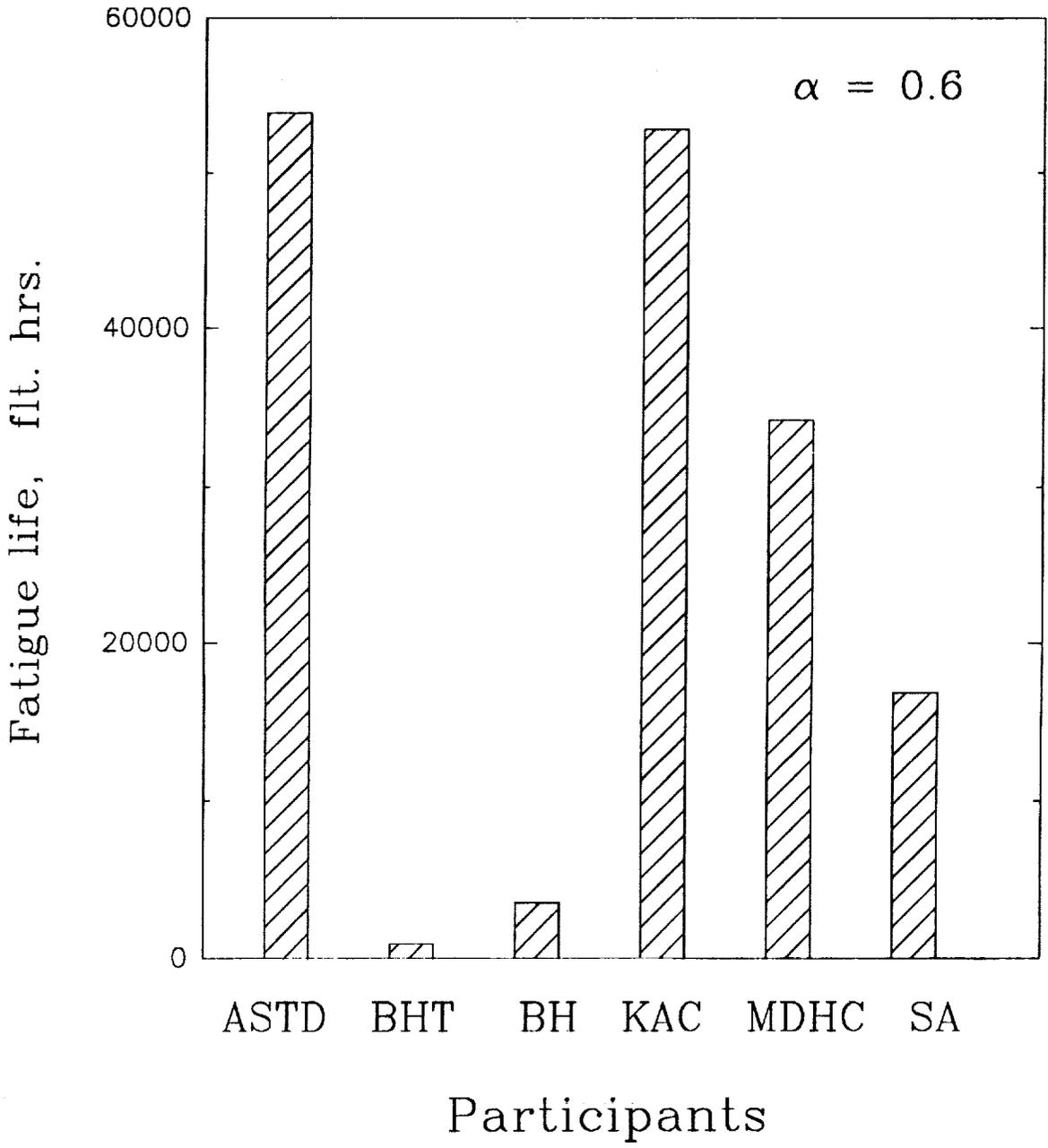


Fig. 15. Phase II/Problem 2 results for six nines reliability.

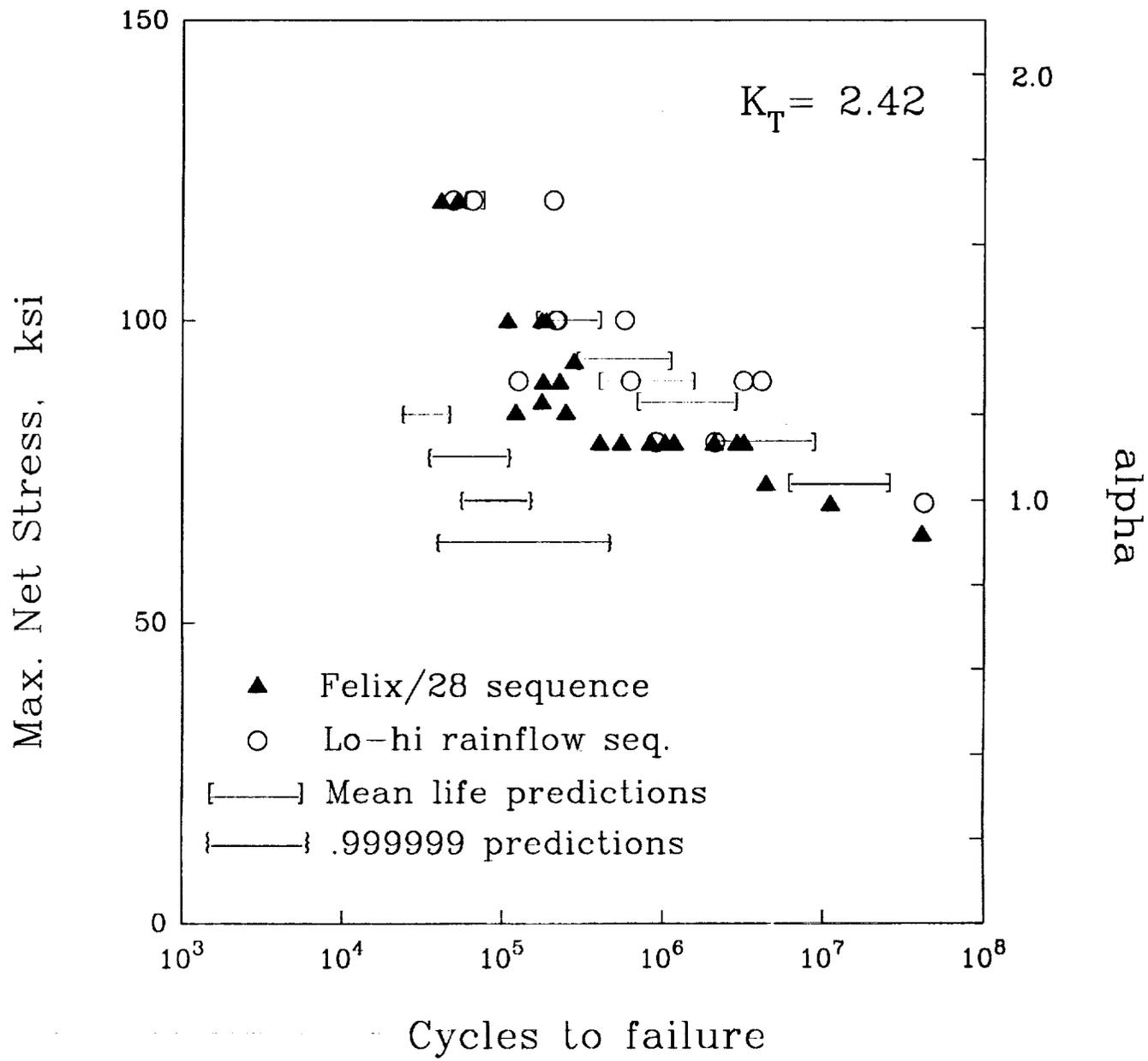


Fig. 16 Round-robin comparisons with spectra tests.



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16. Abstract Fleet readiness and flight safety strongly depend on the degree of reliability that can be designed into rotorcraft flight critical components. The current U.S. Army fatigue life specification for new rotorcraft is the so-called "six nines" reliability, or a probability of failure of one in a million. This report reviews the progress of a round robin which was established by the American Helicopter Society (AHS) Subcommittee for Fatigue and Damage Tolerance to investigate reliability-based fatigue methodology. The participants in this cooperative effort are in the U.S. Army Aviation Systems Command (AVSCOM) and the rotorcraft industry. One phase of the joint activity examined fatigue reliability under uniquely defined conditions for which only one answer was correct. The other phases were set up to learn how the different industry methods in defining fatigue strength affected the mean fatigue life and reliability calculations. Hence, constant amplitude and spectrum fatigue test data were provided so that each participant could perform their standard fatigue life analysis. As a result of this round robin, the probabilistic logic which includes both fatigue strength and spectrum loading variability in developing a consistent reliability analysis was established. In this first study, the reliability analysis was limited to the linear cumulative damage approach. However, it is expected that superior fatigue life prediction methods will ultimately be developed through this open AHS forum. To that end, these preliminary results were useful in identifying some topics for additional study.					
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