PREDICTING THUNDERSTORM EVOLUTION USING
GROUND-BASED LIGHTNING DETECTION NETWORKS

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Lightning measurements acquired principally by a ground-based network of magnetic direction finders are used to diagnose and predict the existence, temporal evolution, and decay of thunderstorms over a wide range of space and time scales extending over four orders of magnitude. The non-linear growth and decay of thunderstorms and their accompanying cloud-to-ground lightning activity is described by the three parameter logistic growth model. The growth rate is shown to be a function of the storm size and duration, and the limiting value of the total lightning activity is related to the available energy in the environment. A new technique is described for removing systematic bearing errors from direction finder data where radar echoes are used to constrain site error correction and optimization (best point estimate) algorithms. A nearest neighbor pattern recognition algorithm is employed to cluster the discrete lightning discharges into storm cells and the advantages and limitations of different clustering strategies for storm identification and tracking are examined.
ACKNOWLEDGMENTS

I wish to thank the many individuals who allowed me to have engaging discussions with them. Dr. Robert Brown of the University of Alabama in Huntsville introduced me to the time series analysis and forecasting literature, and helped me down a path of enlightenment to discover a broad range of approaches to modeling the behavior of non-linear dynamical systems. Drs. Hugh Christian of Marshall Space Flight Center, Marx Brook of the New Mexico Institute of Mining and Technology and Bernard Vonnegut of the State University of New York at Albany helped me to appreciate the complexity of natural phenomena and gain a greater understanding of thunderstorm electrification and cloud physics. Dr. Richard Johnson of SouthWest Research Institute not only taught me a great deal about radio direction finding and provided assistance in obtaining the FFIX eigenvector optimization source code and documentation, but also supplied me with the AZRN subroutine for computing the distance between two points on a sphere.

Mr. Dennis Buechler and Mr. Patrick Wright assisted in the daily operation of the lightning location network and also provided some of the radar and precipitation data sets. Mr. Steven Williams assisted in the installation of the magnetic direction finder sites. This research was supported by the Physical Climate and Hydrologic Systems research program (formerly the Mesoscale Processes research program) at NASA Headquarters.
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<tr>
<td>a</td>
<td>semimajor axis of confidence ellipse</td>
<td>145</td>
</tr>
<tr>
<td>b</td>
<td>semiminor axis of confidence ellipse</td>
<td>145</td>
</tr>
<tr>
<td>d_L</td>
<td>lower bound of Durbin-Watson test</td>
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</tr>
<tr>
<td>d_U</td>
<td>upper bound of Durbin-Watson test</td>
<td>121</td>
</tr>
<tr>
<td>D</td>
<td>Durbin-Watson test statistic</td>
<td>121</td>
</tr>
<tr>
<td>D</td>
<td>lightning data matrix used in cluster analysis</td>
<td>73</td>
</tr>
<tr>
<td>D_i</td>
<td>distance from station S_i to the best point estimate</td>
<td>141</td>
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<td>e_t</td>
<td>residual error at time t</td>
<td>121</td>
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<td>e_t-1</td>
<td>residual error at lag 1</td>
<td>121</td>
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<tr>
<td>e_2</td>
<td>eigenvector corresponding to the eigenvalue ( \lambda_2 )</td>
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<td>H_o</td>
<td>null hypothesis</td>
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<tr>
<td>H_1</td>
<td>alternative hypothesis</td>
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<tr>
<td>k</td>
<td>rate constant of the logistic growth curve</td>
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</tr>
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<td>M</td>
<td>liquid water content</td>
<td>19</td>
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<tr>
<td>N_i</td>
<td>bearing plane normal vector from the ith station</td>
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<td>P</td>
<td>probability of discharge occurring within ellipse having semimajor axis ( a ) and semiminor axis ( b )</td>
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<td>P</td>
<td>matrix of rotation</td>
<td>142</td>
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<tr>
<td>r</td>
<td>correlation coefficient</td>
<td>20</td>
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<tr>
<td>R</td>
<td>rainfall rate</td>
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$s_k$ sum of squared bearing errors for a set of $k$ bearings 

$S$ lightning seed matrix for cluster analysis 

$S_i$ direction finder station $i$ 

$t$ time 

$t_i$ time of inflection of the logistic growth curve 

$T$ environmental temperature 

$T_v$ virtual temperature of air parcel 

$T$ target vector 

$w$ vertical velocity 

$w_i$ weighting factor for the $i$th bearing 

$W_i$ matrix of weights for $i$th station 

$Z$ radar reflectivity 

$Z_e$ equivalent radar reflectivity 

$Z_{DR}$ differential reflectivity 

$Z_H$ radar reflectivity at horizontal polarization 

$\alpha$ limiting value parameter of the logistic growth curve 

$\alpha_i$ angle defining the dot product of the target and $i$th normal vector 

$\alpha_o$ significance level in hypothesis test 

$\beta$ time scale parameter of the logistic growth curve 

$\epsilon_i$ bearing error at the $i$th station 

$\lambda_i$ eigenvalues of the matrix of rotation $P$ 

$\Lambda$ diagonal ($\lambda_1, \lambda_2, \lambda_3$) 

$\Theta$ bearing angle 

$\rho$ autocorrelation parameter 

$\rho_i$ angle defining the dot product of the target and $i$th station vector 

$\sigma_i$ standard deviation of the bearing errors at the $i$th station 

$\sigma_{i*}$ range weighted bearing standard deviation at the $i$th station
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<td>$\Delta t$</td>
<td>time increment</td>
<td>13</td>
</tr>
<tr>
<td>$\chi^2$</td>
<td>chi-square test statistic</td>
<td>36</td>
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CHAPTER I.
INTRODUCTION

Weather radars and imaging sensors on geostationary weather satellites are currently the most widely used remote sensing tools for the short-term forecasting or nowcasting of warm season convective storms and for warning of severe thunderstorm hazards. Zipser (1983) defines nowcasting as "the description of the state of the current weather and forecasts within the valid extrapolation range for each phenomenon which are based on intensive observations". The valid extrapolation range is further defined "as the period within which weather forecasts based upon observations and extrapolation are useful". The valid extrapolation period as well as the amount of lightning activity depend on the phenomena being described, geography, season of occurrence, instability of the atmosphere, and structure of the storm environment (Table 1). Dynamical forecast models with explicit physics are presently more applicable to greater time and space scales (Figure 1).

In a discussion of the stages of nowcasting, Wilson and Carbone (1984) state "the first element of forecasting is simple extrapolation of event position and intensity. Prediction of completely new development or onset of dissipation of the existing event is a distinctly more ambitious nowcast objective". Forecasts of the future location and intensity of clouds, precipitation, lightning, or storm severity can be assessed by asking yes/no or how much. Did it rain at all? Was there any lightning with that storm? Was there severe weather (flooding, hail, tornadoes, microbursts)? How much rain was forecast? How much lightning? Extrapolation forecasting is akin to conditional expectation. What is the probability of rain at point P in the next hour or two, given that it is
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<th>Nonlinear Predictive Capability (Beyond Nowcast)</th>
<th>Accompanying Lightning Activity</th>
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<td>Downburst/Microburst</td>
<td>~1 to a Few Minutes</td>
<td>Very Limited</td>
<td>Often Many Intracloud Flashes, Few Ground Flashes</td>
</tr>
<tr>
<td>Tornado</td>
<td>~1 to a Few Minutes</td>
<td>Very Limited</td>
<td>Often Many Intracloud Flashes, Ground Flash Rates Variable</td>
</tr>
<tr>
<td>Thunderstorm, Individual</td>
<td>5–20 Minutes</td>
<td>Very Limited</td>
<td>Variable Ratio of Intracloud to Ground Flashes</td>
</tr>
<tr>
<td>Severe Thunderstorm</td>
<td>10 Minutes to 1 Hour</td>
<td>Very Limited</td>
<td>Typically Many Intracloud Flashes, Ground Flashes Variable</td>
</tr>
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<td>~1–2 Hours</td>
<td>Some</td>
<td>Typically Many Intracloud and Ground Flashes</td>
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<td>Flash-Flood Rainfall</td>
<td>~1 to a Few Hours</td>
<td>Very Limited</td>
<td>Varies From Many Ground Flashes to None</td>
</tr>
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<td>High Wind, Orographic</td>
<td>~1 to a Few Hours</td>
<td>Some</td>
<td>—</td>
</tr>
<tr>
<td>Lake-Effect Snowstorms</td>
<td>A Few Hours</td>
<td>Very Limited</td>
<td>Some</td>
</tr>
<tr>
<td>Heavy Snow/Winter Storm/Blizzard</td>
<td>A Few Hours</td>
<td>Some</td>
<td>Not Usually</td>
</tr>
<tr>
<td>Frost/Freeze</td>
<td>Hours</td>
<td>Some</td>
<td>—</td>
</tr>
<tr>
<td>Low Visibility</td>
<td>A Few Hours</td>
<td>Some</td>
<td>—</td>
</tr>
<tr>
<td>Air-Pollution Episode</td>
<td>Hours</td>
<td>Some</td>
<td>—</td>
</tr>
<tr>
<td>Wind</td>
<td>Hours</td>
<td>Some</td>
<td>—</td>
</tr>
<tr>
<td>Precipitation</td>
<td>Hours</td>
<td>Some</td>
<td>Variable</td>
</tr>
<tr>
<td>Hurricane</td>
<td>Many Hours</td>
<td>Fair</td>
<td>Variable</td>
</tr>
<tr>
<td>Frontal Passage</td>
<td>Many Hours</td>
<td>Fair to Good</td>
<td>Variable</td>
</tr>
</tbody>
</table>

*Adapted From Zipser (1983); Doswell (1986)
Figure 1. Efficacy of different approaches to short-term forecasting over a range of time and space scales (after Doswell, 1986).
raining at point \( P_0 \) now? Any ability to determine the presence, increase, or decrease of lightning activity as a storm approaches a given location or facility will provide valuable information to the user of these data.

With the recent advent of lightning detection and location technology, it is now possible to directly measure the existence and frequency of lightning activity in storms over large areas. The deployment of regional and national networks (Figure 2) using magnetic direction finding (Krider et al., 1976) and time-of-arrival (Bent and Lyons, 1984) techniques to detect and locate cloud-to-ground lightning offers a new and decidedly different nowcasting data source for real-time multisensor data fusion. In the future total lightning rates (intracloud and cloud-to-ground) will also be observed in real-time by a lightning sensor in geostationary orbit (Christian et al., 1989).

The observed lightning activity may be used in determining the existence, initiation, movement, dissipation, configuration, areal extent, intensity, and redevelopment of convective storms (Goodman et al., 1988a; Lewis, 1989). A recent evaluation of the operational use of lightning data by forecasters at the National Severe Storms Forecast Center (NSSFC) demonstrated great value in monitoring lightning activity for assessing the threat of existing storms and in issuing weather advisories; most frequently when storms were classified as strong (5-min update interval) and less frequently when storms were weak (15-min update interval). Furthermore, when forecasters were asked if lightning activity added knowledge about the general convective activity that could not be obtained from either satellite or radar data, a positive response was acknowledged for 78% of 153 storm episodes considered strong and for 64% of 301 cases of storms considered weak (as subjectively characterized by the forecasters).

These preliminary results suggest that lightning activity and its association with a storm or complex of storms should be quantified and used as a source of nowcasting information in knowledge-based and expert system/artificial intelligence/neural network algorithms being developed and tested (Browning and Collier, 1989; Roberts et al., 1990).
Figure 2. Demonstration national lightning network direction finder (DF) sites. Three sites are part of the MSFC network at Tullahoma, TN (DF 2 = T1), Centerville, TN (DF 3 = Ce), and Barton, AL (DF 4 = MS). DF 1 (at MSFC) is not part of the national network.
The quantitative assessment of lightning activity offers a wide range of opportunities to develop algorithms to evaluate the synergism of these data as an adjunct to the weather radar, satellite, and conventional meteorological data (Watson et al., 1987; Goodman et al., 1988). The Advanced Weather Interactive Processing System (AWIPS) currently under development by the National Weather Service will be the first opportunity for all local forecast offices to integrate, process, and transmit high volume radar, satellite, upper air, and surface data.

The Need for Real-Time Lightning Observations

A recent survey of federal agency requirements provides a clear impetus for developing real-time techniques to monitor lightning hazards (MSI Services, 1986). Examples include: 1) data requirements by the National Weather Service (NWS) during active thunderstorm periods to issue severe weather warnings; 2) timely information needed by the Federal Aviation Administration for flight safety, dispatch, and air traffic control/operations; 3) reliable tracking of lightning to support the safe and efficient operation of a variety of naval activities including operational and training flights, weapons and munitions handling, and aircraft and in-port ship refueling; real-time display of the location and direction of movement of cloud-to-ground lightning strikes within 300 km to support Air Force strategic and defensive activities including refueling operations, munitions handling, radar operations, computer operations, safety, and field exercises.

Facility applications of lightning data include various test range activities such as those conducted by the Air Force and National Aeronautics and Space Administration (NASA) in support of the Space Shuttle and unmanned space vehicles, and by the Department of Energy in support of the Nevada test site where underground nuclear tests are conducted. Presently, NASA operates lightning detection systems at Marshall
Space Flight Center (MSFC), AL; Wallops Island, VA; and Kennedy Space Center, FL. Lightning data from a Navy operated network are also distributed to the Stennis Space Center, MS. The operational requirements for lightning data at test facilities are driven primarily by the concern over personal safety. However, improved lightning warnings at Kennedy Space Center also permit safe fueling operations during stormy weather periods at an estimated annual savings of one million dollars. Real-time users of the MSFC lightning network are shown in Table 2.

Utilities presently use lightning data to design protection for power lines and distribution systems, and to deploy repair crews during severe storms (Fischer and Krider, 1982). In general, lightning warnings tend to be very conservative, with many operational and training opportunities cancelled unnecessarily resulting in lost productivity. This brief summary of applications strongly suggests that any lightning sensitive tasks concerned with optimizing the safety and use of material and human resources can benefit from the currently available lightning detection and location technology.

**Review of Nowcasting and Extrapolation Forecasting Algorithms**

A nowcasting system consists of two main parts: 1) some type of characterization of the present weather situation and 2) a means (i.e., a model) to project the situation forward in time and space. Forecast methods using weather radar to extrapolate rainfall (an appropriate analog for lightning patterns), storm position, and intensity typically use some reflectivity (intensity) threshold to define the convective storms as either cells (clusters) or rain areas, correlate two or more successive observations to get a storm motion vector, and extrapolate the intensity pattern some time into the future.

The existing operational radar nowcasting systems extrapolate the characteristics and full intensity of the precipitation pattern without consideration for growth/decay of the rain intensity or its spatial distribution. The primary source of forecast error in a
Table 2. Real-Time Users of the MSFC Lightning Network

<table>
<thead>
<tr>
<th>U.S. Air Force, Arnold Engineering and Development Center, TN</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. Army, Redstone Arsenal, AL Test and Engineering Directorate</td>
</tr>
<tr>
<td>NASA, Marshall Space Flight Center, AL</td>
</tr>
<tr>
<td>Earth Science and Applications Division</td>
</tr>
<tr>
<td>Rocket Motor Test Areas</td>
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<tr>
<td>Information Systems Office</td>
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<td>Safety, Reliability, Maintainability, and Quality Assurance Office</td>
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<tr>
<td>Neutral Buoyancy Simulation Facility</td>
</tr>
<tr>
<td>*Redstone Airfield Flight Operations</td>
</tr>
<tr>
<td>WAFF 48, Huntsville, AL Television Station</td>
</tr>
<tr>
<td>*National Weather Service Office, Huntsville, AL</td>
</tr>
<tr>
<td>†State University of New York at Albany, National Lightning Network</td>
</tr>
</tbody>
</table>

*Future Users
†Raw Bearing Information Provided by Three MSFC Antennas

8
study of rain patterns associated with weather fronts in Great Britain was attributed to
the development or decay of rain areas in 16 of 29 (55%) events (Browning et al., 1982).
Storm growth/decay, mergers, splitting, and fragmentation also compromise the perfor-
mance of peak reflectivity trackers (Crane, 1979; Rosenfeld, 1987), centroid trackers
(Barclay and Wilk, 1970; Duda and Blackmer, 1972; Zittel, 1976; Bjerkaas and Forsyth,
1980), and (pattern) correlation trackers (Austin and Bellon, 1974; Rinehart and Garvey,
1978; Browning et al., 1982).

Peak reflectivity trackers isolate and track local maxima in the reflectivity or
precipitation field. Such techniques tend to overestimate the number of physically
realistic storm cells, but do not miss the small, potentially severe storms that may not be
identified by the other methods which depend on intensity thresholds to delineate
storms. The centroid trackers use the 3-dimensional reflectivity weighted centroids
(storm mass) to delineate, characterize, and follow storm movement. Correlation track-
ers compare a field of reflectivity values at two successive times to get a motion vector
for the entire system (more applicable to widespread light rain situations) or in localized
sub-areas (applicable to individual thunderstorms).

The Bjerkaas and Forsyth (1980) mass weighted centroid tracker has been imple-
mented as a Next Generation Weather Radar (NEXRAD) system algorithm (NEXRAD,
1985). NEXRAD is the Doppler radar system being jointly deployed across the United
States and overseas by the National Weather Service, Federal Aviation Administration,
and Air Weather Service to replace the aging WSR-57 and WSR-74 network radars now
in service (Leone et al., 1989). These new radars offer numerous advantages over
present weather radars in severe thunderstorm warning, rainfall estimation, and the
detection of wind shears.

In a study comparing the performance of different types of storm trackers, El-
vander (1976) found the cross-correlation tracker performed best when presented only
base-scan (lowest elevation level) reflectivity data and the centroid tracker superior for
volume-scan data (the data acquisition mode for NEXRAD). A comparison between the Crane (1979) peak reflectivity tracker and centroid trackers showed general agreement of the storm motion vectors. Brasunas (1984) recommended using the correlation tracker on slowly moving widespread rain areas and the centroid tracker on the more convective storms. For large areas with multiple storm motion vectors, Browning and Collier (1989) suggested applying the correlation tracker to subareas within the confines of the larger system.

Other sources of forecast error can be attributed to incorrect specification and delineation of the initial pattern (measurement errors), errors in estimating the initial pattern trajectory, and errors due to changes in the trajectory during the forecast period. Examples of this latter effect are storms that upon becoming severe tend to move to the right of the mean lower tropospheric wind (Newton and Fankhauser, 1964; 1975), and rainbands associated with tropical hurricanes and extra-tropical cyclones where the ambient wind field imparts both a translational and rotational component to produce a curvilinear trajectory.

Quantifying the Valid Extrapolation Range

Linear extrapolation of the present trend is perhaps the easiest and most widely used method for nowcasting. However, the motion of the atmosphere and growth/decay of convective phenomena are examples of complex non-linear dynamical systems. The limitations of linear extrapolation with increasing time scale are readily apparent as the forecast error becomes large and then exceeds the threshold value for an acceptable or "useful" forecast (Figure 3). As noted earlier, the valid extrapolation period will depend on the process under study, which itself will have some mean lifetime. A non-linear
Figure 3. Possible extrapolation model fits to a non-linear process based on two prior observations (OB1, OB2) and the current observation: a, linear extrapolation model; b, valid extrapolation range defined by acceptable error bounds; c, non-linear model fit that is worse than linear fit; d, good non-linear model fit (after Doswell, 1986).
model may be inferior to a linear one if the nonlinear model is applied beyond the valid extrapolation range. In general, a non-linear model should provide the best forecast of a non-linear process such as the growth/decay of a thunderstorm.

For large, approximately steady state weather systems containing widespread light to moderate rain showers, extrapolation forecasts can be very accurate for up to 6 h (Browning et al., 1982). Projecting the motion of intense thunderstorms that persist for several hours (supercells) is not too difficult, but predicting if, when, and where it might spawn a tornado is beyond present capabilities. The tornadic storm that struck Huntsville, AL on 15 November 1989 is one example of an isolated supercell storm that could be identified on radar in Mississippi 5 h before it reached the city and produced an F4 intensity tornado that killed 22 people. Yet, the tornado struck with almost no warning. Forecasts of convective weather phenomena at smaller space and time scales can be much more difficult since they change size, shape, and intensity more readily.

Study Objectives

The objectives of this study are twofold. First, the applicability and limitations of extrapolation techniques are explored relative to the problem of forecasting lightning (i.e., thunderstorm) activity at a future location and time using data acquired primarily from the NASA/MSFC ground strike lightning network installed, operated, and maintained by the author since 1985. Radar echoes and the spatial distribution of the lightning itself will be used to analyze, characterize, track, and examine extrapolative forecasts of storm position and the accompanying lightning activity. Second, the usefulness of physically-based non-linear models will also be examined for their applicability to the lightning forecast problem and to determine the valid extrapolation range for different weather scenarios.
In Chapter 2 the lightning and precipitation time series for storm systems over a wide range of space and time scales are examined. These results are used to develop a conceptual understanding of the thunderstorm life-cycle. The three parameter logistic model is offered as a candidate model of the thunderstorm life-cycle. Its properties and their relevance to the extrapolation nowcasting problem are examined. The success of an extrapolation forecast at time \( t + \Delta t \) is influenced by the quality of the data source (analyzed in Chapter 3), an accurate description of the present situation at time \( t \) (examined in Chapter 4), and correctly accounting for changes during the forecast period (investigated in Chapter 5).

Chapter 3 discusses the process of finding the most accurate locations of the cloud-to-ground lightning discharges. This process involves removing systematic errors from the data and the application of an optimization technique to locate the most probable position of each lightning discharge. A novel approach using isolated radar echoes to constrain the error correction and optimization procedures to remove systematic errors is described.

Chapter 4 presents a pattern recognition scheme that is used to generate initial seeds or "first guess" fields for clustering the discrete lightning discharges into storm cells. The clustering process is critically dependent on the prior accuracy of the lightning location estimates (Chapter 3) and the generation of subsequent storm life-cycle time series (Chapter 5) relies on the cluster analysis procedure assigning the correct number of lightning discharges (objects) to the proper storms (groups). The advantages and limitations of different clustering strategies for storm identification and tracking are examined. Storm identification with lightning data alone is compared to storm identification with radar alone, and some synergies for sensor fusion are explored.
In Chapter 5 the logistic growth model is utilized to examine the storm life-cycle over a wide range of space and time scales and address the potential of non-linear regression models to improve upon short-term extrapolation forecasts. A physical interpretation of the logistic model parameters and the resulting implications for determining the valid extrapolation range is considered.

Chapter 6 summarizes the chief results of this research and discusses how these results move the state of knowledge forward. Chapter 7 concludes the discussion and offers future areas for additional research.

Appendix A provides the mathematical formulation for the optimization algorithm (not currently available in the open literature) employed in Chapter 3 to find the most probable flash locations. Appendix B contains the FORTRAN-77 source code used to convert the raw data into geophysical quantities, correct the systematic errors in the data, and compute the optimal flash location. Lastly, Appendix C contains the FORTRAN-77 source code for the cluster analysis.
CHAPTER II.
A CONCEPTUAL MODEL OF THE THUNDERSTORM LIFE-CYCLE

A conceptual understanding or model of the thunderstorm life-cycle is needed for addressing the utility and limitations of extrapolation forecasts of non-steady-state weather phenomena such as thunderstorms and their accompanying lightning activity. The following discussion examines the lightning and precipitation time series for storm systems ranging in size from an isolated airmass storm to a large mesoscale storm complex encompassing an area of nearly 12,000 km².

Air Mass Thunderstorms

The relationships between the early electrical development of a thunderstorm cell and the vertical development of the radar echo, precipitation, and cloud top are depicted in Figure 4. A significant fraction of the total lightning (50-95%) occurs between regions of opposite charge within the cloud (intracloud lightning) without ever reaching the earth (cloud-to-ground lightning). The initial discharge will almost always be intracloud. Typically this discharge occurs 5-10 min after initial electrification, which itself begins 5-10 min after the detection of a 35-40 dBZ radar echo aloft. Based on lightning and radar data collected in three different climatic environments (New Mexico, Alabama, and Florida), Buechler and Goodman (1990) find that the time lag from the reflectivity exceeding 40 dBZ at the -10°C level (the height of the main negative charge region in the thunderstorm central dipole) to the first intracloud discharge ranges from 4-33 min. The time lag is related to the rate of vertical development of the cloud. On
Figure 4. Composite chart of thunderstorm development observed by radar, radiosonde and electrical sensors (after Workman and Reynolds 1949).
average, the first cloud-to-ground discharge will be of negative polarity (it lowers negative charge to ground) and will occur 15-20 min after the initial radar echo is observed in a vertically growing cloud. The stage is now set for the active lightning phase of the thunderstorm life-cycle.

As the cell continues to develop, the active electrical phase may have a duration lasting from a few minutes to many hours. The total amount of lightning and peak flash rates of a storm are a non-linear function of its height, size, mass, duration, and environment (Shackford, 1960; Livingston and Krider, 1978; Williams, 1985; Cherna and Stansbury, 1986; Goodman and MacGorman, 1986; Goodman et al., 1988b).

Figure 5 shows the relationship between lightning occurrence and precipitation in a small airmass thunderstorm 26 km² in area (>18 dBZ) observed near Huntsville, AL by the NCAR CP2 radar on 20 July 1986. The storm produced a strong microburst with a velocity differential of 30 m s⁻¹. Total lightning (intracloud and cloud-to-ground) activity was measured by an instrumented mobile laboratory operated by the National Severe Storms Laboratory (Rust, 1989). The mobile laboratory was also used for ground truthing the lightning strike network, discussed in greater detail in Chapter 3.

The 20 July case represents a Byers and Braham (1949) type airmass thunderstorm. The mobile laboratory was situated under the storm throughout its 45 min life-cycle and recorded 110 intracloud flashes and 6 cloud-to-ground flashes, all 6 of which were detected by the ground strike network. The first intracloud discharge was observed about 4 min after hail was initially indicated by radar, during a period of rapid vertical development as the cloud top neared its maximum height of 14 km. The first ground discharge occurred 5 min later when the maximum reflectivity core descended to 5.5 km and a weak outflow was detected by the radar.

Storm rain rates are computed from empirical Z-R relations developed by Marshall and Palmer (1948), Jones (1956), and Seliga et al. (1986). The storm rainflux (kg s⁻¹), mass (kg), and vertically integrated liquid water content or VIL (kg m⁻²) (Greene
Figure 5. Lightning and precipitation history of an airmass thunderstorm that produced a microburst on 20 July 1966 at Huntsville, AL: above left, total flash rate time series where \( N_{ic} \) and \( N_{cg} \) represent the number of intracloud and cloud-to-ground flashes produced by the parent storm; right, time-height cross-section of cloud top infrared temperature (°C), 0 dBZ and 30 dBZ echo contours and maximum reflectivity; lower left, vertically integrated liquid water content (VIL), storm mass, rain flux and echo volume; right, peak reflectivity (\( Z \)) and storm average rain rates (\( R \)) as indicated (after Goodman et al., 1988).
and Clark, 1972) are calculated using a 30 dBZ threshold and the Jones (1956) relations 
\[ Z = 486R^{1.87} \quad \text{and} \quad M = 0.052R^{0.97}, \]
where \( Z \) (mm\(^6\) m\(^{-3}\)), or \( Z_H \), is the CP2 reflectivity at horizontal polarization, \( R \) (mm h\(^{-1}\)) is the rain rate and \( M \) (g m\(^{-3}\)) is the liquid water content.

The peak total flash rate of 23 min\(^{-1}\) was reached another 3–4 min after the initial lightning, 6 min prior to the maximum microburst outflows, and in conjunction with the peak in vertically integrated liquid water content (5.3 \( \times \) 10\(^3\) kg m\(^{-2}\)), echo volume (1.9 \( \times \) 10\(^{11}\) m\(^3\)), and storm mass (3.3 \( \times \) 10\(^8\) km). The rainflux (1.5 \( \times \) 10\(^5\) kg s\(^{-1}\)) and storm averaged rain rates (18.2–28.2 mm h\(^{-1}\)) reached their maximum values in association with a visual confirmation of pea-sized hail mixed with heavy rain about 2 min after the peak flash rate.

Rapid-scan (5-min interval) satellite imagery from the GOES-E geostationary weather satellite were collected during the storm life-cycle. The infrared temperature of the cloud continued to show cooling (which could be misinterpreted as continued vertical development) even as the radar echo top and lightning rates decrease (indicating storm collapse). The misleading satellite signature is due to the small size of the storm and the (effectively 4 km x 8 km) field of view of the infrared radiometer (which also "sees" the earth's surface). The radiometer field of view is underfilled at 1900 UTC. The field of view becomes more fully filled as the anvil expands, thereby sensing a decreasing cloud top temperature. However, the abrupt decrease in the total flash rate indicates storm collapse and thus serves as a microburst precursor. Yet, no such signature exists in the cloud-to-ground lightning evolution due to the small number of events (samples).
A Multi-cellular Thunderstorm

Figure 6 shows the cloud-to-ground lightning and convective rainflux calculated every 10 min from the WSR-57 radar at Nashville, TN for a multi-cellular storm observed on 25 July 1986 over a period of 90 min. The convective rain area is simply defined here as the precipitating area within the 30 dBZ reflectivity contour. The lightning and rainflux are in-phase and are fairly well correlated ($r=0.77$). However, as the storm decays the lighter rainfall area contributes more to the total precipitation such that there is more rainfall per flash during storm decay than during storm growth.

Convective Storm Complexes

Figure 7 shows cloud-to-ground lightning and rainflux during a 7 h period of observation on 13 July 1986 of a mesoscale convective system in the Tennessee Valley that develops an extensive trailing stratiform rain region in the latter part of its lifecycle. The lightning data recording was briefly interrupted for a tape change at 2240 UTC and continued at 2248 UTC, but the latter period is not shown here. Not long after 2300 UTC the entire storm system could not be sampled adequately from the Nashville radar as the storm moved out of range to the south. This case again shows excellent agreement ($r=0.96$) between the lightning and convective rainflux time histories.

Figure 8 presents a summary of the cloud-to-ground lightning and rainfall time histories of mesoscale convective complexes (MCCs) in the Central United States studied by Goodman and MacGorman (1986) and McAnelly and Cotton (1986). Such storm systems are readily identified by their persistence and extensive cold cloud shields in infrared satellite imagery (Figure 9). The typical precipitating lifetimes of MCCs are on the order of 12 h with spatial extents of a few hundred kilometers. Much of the warm season rainfall in the Northern Hemisphere (up to 70% in the major crop growing
Figure 6. Lightning-rainfall relationships for a small multi-cellular storm on 25 July 1986.
Figure 7. Lightning-rainfall relationships for a mesoscale convective system on 13 July 1986.
Figure 8. Hourly cloud-to-ground lightning rates and rainfall as a function of Mesoscale Convective Complex (MCC) life-cycle (adapted from Goodman and MacGorman, 1986; McAnelly and Cotton, 1986).
Figure 9. NSSL lightning network shown in satellite projection with the infrared cloud top image of a MCC during its mature phase (maximum cloud shield extent). The 4-DF deployment during 1983 and the 350 km range ring are denoted by crosses (after Goodman and MacGorman, 1986).
regions of the U.S.), extensive flooding, and severe weather is a result of these organized mesoscale circulations (Fritsch et al., 1986). Up to 25% of the entire annual lightning strikes at a given site can be accounted for by the passage of just one MCC (Goodman and MacGorman, 1986). Due to their meteorological and economic significance, any nowcast/forecast skill that can be demonstrated for mesoscale storm systems is a worthwhile endeavor.

The cloud-to-ground lightning activity increases and decreases exponentially over the life-cycle of MCCs. Based on earlier studies and the preceding analysis, this relationship appears to be generally valid for isolated storms, multi-cellular storms, and the ensemble convection embedded in organized mesoscale convective weather systems. The high correlations (>0.9) between lightning and rainflux extends over three orders of magnitude from 10^1-10^4 km^2. Earlier scaling studies indicate that precipitating cloud dimensions are self-similar over 5 orders of magnitude (Lovejoy, 1982). Clearly, the non-linear physical interactions that produce the microphysical and dynamical properties of clouds are also relevant to their electrification.

**Positive Polarity Cloud-to-Ground Discharges**

Positive polarity cloud-to-ground discharges are often observed during the dissipation phase of the storm (Krehbiel, 1986). In addition, positive polarity flashes frequently occur 1) from thunderstorm anvils and storms which become severe and produce mesocyclones, large hail, or tornadoes (Rust, 1986); 2) in association with long-lived wet microbursts in low shear environments (Buechler et al., 1988); 3) in the trailing stratiform rain region of mesoscale weather systems (Rutledge and MacGorman, 1988); and 4) in the northern section of mesoscale systems, aligned with the geostrophic wind and downwind from the most vigorous convection, which is dominated by negative
polarity ground discharges (Orville et al., 1988; Engholm et al., 1990). The mesoscale system that led to the Huntsville tornado produced positive polarity discharges in each category listed above during some portion of its life-cycle.

A Conceptual Model of Cloud Electrical Development and Lightning Activity

Figure 10 summarizes these lightning observations into a conceptual model of the growth and decay of a typical thunderstorm and its associated total lightning activity. The temporal evolution of the lightning activity is in-phase with the development of the storm updraft and is strongly coupled to the life-cycle of the thunderstorm described above and in earlier studies by Byers and Braham (1949), Workman and Reynolds (1949), and others. These results show the electrical development of the cloud is intimately connected to its dynamical and microphysical development. Laboratory measurements by Jayaratne et al. (1983) suggest that the charge transferred per collision is a complicated function of particle size and type, cloud liquid water content, temperature, and even chemical composition. A possible inference from these observations is that the greater the production rate of precipitation and ice particles in a cloud, the greater the charging rate of the storm. This is partially supported by the growing success of numerical models in simulating the initial electrification of small thunderstorms (e.g., Ziegler et al., 1986; Helsdon and Farley, 1987). Multi-cellular storms will exhibit impulsive updraft, downdraft, precipitation, and flash rate growth and decay. Thus, the time rate-of-change of flash rates also provides a signature of the growth and decay of the thunderstorm.
Figure 10. Conceptual model of the temporal evolution of cloud electrical, kinematic, and microphysical development.
The Logistic Growth Model

Hald (1952) states that "the function used to represent the relationship between variables should as far as possible be chosen on the basis of professional knowledge about the problem under discussion and the reasons advanced for this choice are of fundamental importance as regards confidence in extrapolations". The storm system size and precipitation particle population have been shown to be important factors in maintaining the charging/discharging process. One can attempt to characterize this process by simple first order differential equations which have been applied to population dynamics to describe the phenomena of growth and decay (Hald, 1952; Bard, 1974; Boyce and DiPrima, 1977; Haberman, 1977).

The type of model needed depends on the type of growth that occurs. These types of models are mechanistic in nature, rather than empirical. Mechanistic models are derived from assumptions on the type of growth, and these assumptions can be represented by differential or difference equations (Draper and Smith, 1981). Empirical models are chosen to approximate the unknown mechanistic models. One likely candidate mechanistic growth equation is the "logistic" or sigmoid curve (Figure 11). The logistic curve has frequently been used to describe the growth rates of populations (cells, human and animal populations, chemical kinetics, telephone subscribers, and business transactions).

Let t denote time or the magnitude of a growth factor which influences the size y of the phenomenon observed, then dy/dt denotes the rate of growth per unit time. Let the process be characterized by the general equation

\[
\frac{dy}{dt} = f(t,y) \tag{2.1}
\]

where the growth rate depends on both time and the size of the population.
Figure 11. Logistic model with limiting value \( \alpha \) plotted as a function of time \( t \).
Consider the following three special cases where

\[
\frac{dy}{dt} = f(y)g(t). \quad (2.2)
\]

Letting \(f(y)=1\), \(y\), and \(y(\alpha-y)\) gives

\[
\frac{dy}{dt} = g(t) \quad (2.3)
\]

\[
\frac{dy}{dt} = yg(t) \quad (2.4)
\]

\[
\frac{dy}{dt} = y(\alpha-y)g(t), \quad (0<y<\alpha). \quad (2.5)
\]

In (2.3) the growth rate \(y\) depends on time, but not on the size reached. In (2.4) the growth rate is proportional to the size reached and to a function of time. In (2.5) the growth rate is proportional to both the size reached and the remaining size, as well as a function of time. The latter case is the one of most interest. Now, write (2.2) as

\[
\frac{dy}{f(y)} = g(t)dt. \quad (2.6)
\]

Introducing the "logarithmic differential coefficient" (Hald, 1952, p.659)

\[
\frac{d\ln(y)}{dt} = \frac{1}{y} \frac{dy}{dt} \quad (2.7)
\]

and letting \(f(y) = y(\alpha-y)\) and \(g(t)=\beta\) gives a relation where the growth rate is proportional to the size of the population and remaining size (where \(\alpha\) denotes the growth is limited to some maximum amount, i.e., the value that \(y\) approaches as \(t\) increases) as well as a function of time. The growth rate relative to its present size, \(1/y \ (dy/dt)\), decreases linearly as \(y\) increases. The resulting solution of the differential equation is the logistic function

\[
y = \frac{\alpha}{(1+\beta e^{-kt})}. \quad (2.8)
\]
This model is but one of many possible exponential growth models having similar forms (Hald, 1952; Williams, 1959; Richards, 1959; Draper and Smith, 1981; Bard, 1974; Haberman, 1977; and Myers, 1986). At \( t=0 \), the starting growth rate is \( \alpha/(1+\beta) \). Also, as \( t \to \infty \), \( y \to \alpha \).

The slope of the logistic curve is positive and the second derivative (Draper and Smith, 1981; Prof. Don Ryan, personal communication)

\[
\frac{d^2y}{dt^2} = \left( \frac{k^2}{\alpha^2} \right) y (\alpha-y)(\alpha-2y) \tag{2.9}
\]

has inflection points at \( y=0 \), \( \alpha \), and \( \alpha/2 \). At the point of inflection \( y = \alpha/2 \), substitution in Eq. (2.8) gives the time of inflection as \( t_i=(\ln\beta)/k \). We note that the curve is symmetric about this point, i.e., the system decays or diminishes at the same rate at which it grows. Thus, for nowcasting purposes one might first compute the rate at which the lightning activity increases (i.e., a growth rate) and the time required for a storm to reach its peak discharge rate (the point of inflection). Based on symmetry, one would then predict the storm to decay at this same rate and reach the end of its life-cycle in the same number of time steps needed to produce the initial 50% of the total lightning.

In order to test and evaluate this model, one must develop a methodology for associating the discrete lightning events with their parent thunderstorms. This process is addressed in Chapter 4. Next, generate a time series at uniform sampling intervals. During each successive sampling period the parent storm must be tracked with time and correlated with its past position. The extrapolation forecast results are presented in Chapter 5. However, the pattern recognition process is strongly dependent on the quality and limitations of the lightning strike data which is described next in Chapter 3.
CHAPTER III.

OPTIMIZATION METHODS FOR LOCATING LIGHTNING FLASHES USING MAGNETIC DIRECTION FINDING NETWORKS

Introduction

Magnetic direction finding (DF) networks for locating lightning strikes to ground require that two or more receivers detect the characteristic radio signal produced by return strokes (Krider et al., 1976). Once the signal is detected, an estimate of the most probable flash position, sometimes referred to as the best point estimate (BPE), and a confidence region can be constructed. The spatial distribution (or clustering) of the lightning flashes, however, is a function of the dimension and vigor of the storm, the orientation of the lightning channel and hence its radiation field, and the errors (both random and systematic) associated with the technique.

The systematic errors due to DF site effects are a major source of network degradation (Ross and Horner, 1952; Horner, 1954; Gething, 1978). When one or more of the network DFs do not detect the flash, the location estimate must be determined from a less favorable geometry (e.g., a flash along the baseline of two DFs, or a flash more distant from one site than another site in a more optimal geometry). The reliability of a fix (i.e., position estimate) can generally be maximized by using only the two closest stations to the target (Stansfield, 1947). However, near the baseline of the two DFs it is better to use a more distant receiver in a more favorable geometry (which will usually produce a smaller confidence ellipse). This is the basis for the real-time algorithm implemented in the earlier versions of the lightning DF networks manufactured
by Lightning, Location, and Protection (LLP), Inc. (Krider et al., 1980). This algorithm chooses the DF pair having the greatest signal strengths (presumably the two closest DFs to the flash). If the flash is near the baseline of the DF pair, a solution can be computed with the DF having the next strongest signal strength. An algorithm called "multiple correlation optimization" now replaces the simple 2 DF technique described above when three or more DFs detect a flash (LLP, Inc., 1988). This algorithm basically performs a least squares minimization between the most probable flash position and the sum of the bearing errors.

This latter algorithm, first introduced by Hiscox et al. (1984) and a more recent eigen-vector algorithm introduced by Orville (1987) are very similar to a technique first proposed more than ten years earlier by Wangsness (1973). All three algorithms attempt to minimize the same objective function although different methods are used to reject or flag "wild" bearings. Hiscox et al. (1984) also proposed the use of properly normalized signal amplitudes as an additional weighting factor (or constraint) to determine the optimal location. The improvement in solution accuracy by this latter method is a function of network geometry and the lightning location relative to that geometry. More recently, stochastic optimization techniques known as simulated annealing (Kirkpatrick et al., 1983; Szu and Harley, 1987a; 1987b) have been successfully applied to the general multiple DF/multiple bearing problem.

This chapter describes the application of an eigen-vector algorithm (called FFIX) which is based on the technique described by Wangsness (1973). FFIX was developed on or before January, 1973 (but apparently never published) for finding radio transmitter locations anywhere on earth from multiple DF bearings. The technique characterizes the measured bearings by bearing planes that pass through the center of the earth and by their unit normal vectors. The BPE is determined from the vector from the center of
the earth that minimizes the weighted sum of squares of its inner products with the normal vectors. A BPE and confidence ellipse are computed from the bearing data in terms of the eigenvalues and eigenvectors of a 3 x 3 matrix.

This paper offers the first ever adaptation of the FFIX algorithm to the lightning location problem. The author is indebted to Dr. R. Johnson of the Southwest Research Institute and his sponsors at the Department of Defense for providing some documentation and the source code in 1985. The mathematical formulation for FFIX is provided in Appendix A. Appendix B contains the FORTRAN-77 source code for the FFIX subroutine.

Overview of the FFIX Algorithm

Individual DF bearings are corrected for systematic errors and correlated in time for each lightning discharge before being submitted to FFIX to determine the most probable lightning ground strike point (Figure 12). The input data consist of n station bearings and their respective standard deviations (random plus any remaining systematic errors). The root mean square (RMS) bearing error for each LLP DF is about 1°. Previous attempts to iteratively remove the systematic error have not wholly eliminated them (Mach et al., 1986; Schutte et al., 1987). These techniques reduce the total bearing error (i.e., the standard deviation) to a value approaching 2° at best. It will be shown below that the standard deviation impacts both the BPE calculation and the confidence ellipse.

Sines of the bearing errors are assumed to be independent normally distributed random variables with zero means, but some bearings may be "wild". "Wild" bearings are rejected by a process where all combinations of bearings are exhaustively evaluated until
Figure 12. FFIX algorithm flowchart.
A consistent subset of bearings is obtained. A set of \( n \) bearings is "consistent" if the associated sum of squared bearing errors from the best point estimate is less than the 80% value of \( \chi^2_{n-2} \).

The BPE and confidence ellipse are computed from the largest "consistent" subset of bearings. The algorithm employs two approaches for rejecting the "wild" bearings. The first approach, called the "exhaustive method", is invoked when ten or fewer bearings are submitted for a BPE. If the submitted bearings lack sufficient consistency to form a BPE, then all subsets of \( n \) bearings are taken \((n-j)\) at a time, where \( j = 1, 2, 3, \ldots \), until an acceptable solution is identified or a lower limit on the number of bearings is reached, in which case there is no solution. The lower limit is the greater of \( n/2 \) or 3. Thus, the largest subset of consistent bearings forms the BPE. A "sequential method" is employed when more than ten bearings are submitted for a BPE. If all \( n \) bearings fail the consistency test, then the bearing that is more "inconsistent" with the bearing set is rejected (i.e., the wild bearing). The remaining set of \((n-j)\) bearings is examined as before. In this way the most "wild" bearings are rejected sequentially.

In the event that the chi-square consistency test fails, the "best solution" is chosen as the intersection of the bearing pair having the minimum semimajor axis in its confidence ellipse. This argument assumes the best network geometry also gives the best solution, all other factors being equal. This approach is supported by the earlier work of Stansfield (1947). This final iteration is necessary in a small direction finder network such as that run by MSFC (4 DFs) because nearly 50% of all flashes are seen by only 2 DFs. The probability of detection for the network as a whole would be significantly poorer without this last iteration process.
Proof of Concept

BPE Calculation for a 4-DF Network

The Marshall Space Flight Center (MSFC) 4-DF network has been in operation in the Tennessee Valley (southern TN and northern AL) since 1984. Figure 13 shows the deployment of the DF stations and other ground-based remote sensing systems such as radars and rawinsondes. The additional systems were operational during June and July 1986 in support of the Cooperative Huntsville Meteorological Experiment (COHMEX) multi-agency field program (Dodge, et al., 1986). The radar and rawinsonde data are used in this study to determine the location and characteristics of thunderstorms as well as the structure (vertical profiles of temperature, humidity, and winds) of the storm environment.

At the present time FFIX is used only in post analysis and follows the exhaustive rejection path shown in Figure 12. An example of the information computed for each flash is given in Table 3. The flashes are stored in rows and columns which indicate the hour of occurrence and flash sequence number. The file also contains the Julian day (DAY); hour (UTC) of occurrence (TIME); number of flashes in the selected interval (CMA); flash time in hours, minutes, and seconds (HMS); latitude (LAT) and longitude (LON) of the flash; an estimate of the first stroke peak current in kAmps (KA); the maximum normalized signal strength (NSTR); the number of return strokes (RS); the semimajor axis (SMA) in km; semiminor axis (SMI) in km; orientation (ORI) in degrees; and area (AREA) of the error ellipse in km²; flash polarity (FLAG= '.' for negative and '+' for positive); and time of the flash in hundredths after the last second (MS). The flash polarity is also indicated by the sign of the KA and NSTR fields.
Figure 13. Deployment of the lightning, radar and rawinsonde network for the Cooperative Huntsville Meteorological Experiment (COHMEX) conducted near Huntsville, AL in 1986.
Table 3. Lightning Discharge Data Base

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<tr>
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</thead>
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<tr>
<td>[DAY = 86194 SYD ] [ TIME = 230000 HMS ] [ CMAX = 11 ]</td>
</tr>
<tr>
<td>[HMS = 232021 HMS ] [ LAT = 35.8841 DEG ] [ LON = 86.0345 DEG ]</td>
</tr>
<tr>
<td>[KA = 87.6 ] [ NSTR = -483.1 ] [ RS = 2 ]</td>
</tr>
<tr>
<td>[SMA = 6.3 KM ] [ SMI = 1.1 KM ] [ ORI = 5.9 DEG ]</td>
</tr>
<tr>
<td>[AREA = 23.6 KM2 ] [ FLAG = ] [ MS = 0.00 HMS ]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
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</tr>
</thead>
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</tr>
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</tr>
<tr>
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</tr>
<tr>
<td>[SMA = 9.2 KM ] [ SMI = 2.5 KM ] [ ORI = 79.0 DEG ]</td>
</tr>
<tr>
<td>[AREA = 72.3 KM2 ] [ FLAG = ] [ MS = 0.62 HMS ]</td>
</tr>
</tbody>
</table>

<table>
<thead>
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<tbody>
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<tr>
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</tr>
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</tr>
<tr>
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</tr>
</tbody>
</table>

<table>
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</thead>
<tbody>
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</tr>
<tr>
<td>[HMS = 232301 HMS ] [ LAT = 35.8586 DEG ] [ LON = 86.1433 DEG ]</td>
</tr>
<tr>
<td>[KA = -70.2 ] [ NSTR = -178.6 ] [ RS = 1 ]</td>
</tr>
<tr>
<td>[SMA = 3.5 KM ] [ SMI = 2.1 KM ] [ ORI = 62.7 DEG ]</td>
</tr>
<tr>
<td>[AREA = 24.1 KM2 ] [ FLAG = ] [ MS = 0.75 HMS ]</td>
</tr>
</tbody>
</table>
Figures 14 and 15 show an expanded view of a radar echo (as seen from the CP-2 radar) and the accompanying lightning strikes (shown as dots enclosed by the 50% error ellipse) for a small storm in Tennessee on 13 July 1986. The ellipse is generated from the type of information contained in Table 3. The lightning centroid is less than 5 km from the main echo. Figure 15 is produced using a bearing standard deviation of 2°. Figure 16 shows the corrected locations, but with a 1.5° bearing standard deviation, superimposed on the radar reflectivity. The spatial dispersion of strikes is on the order of 10 km and is well correlated with the radar echo. By reducing the bearing standard deviation from 2° to 1.5°, the semimajor axes of the 50% ellipse for flashes 6 and 7 (Table 3) are reduced (i.e., improved by) 40% and 24%, respectively.

Earlier studies of the natural distribution of lightning strikes produced by isolated storms show that the lightning clusters are approximately 10 km in diameter (Feteris, 1952; Hatakeyama, 1958; Krider, 1988; Goodman et al., 1988). In contrast, however, the spatial distribution of the lightning in the trailing stratiform rain region behind summertime squall lines and in wintertime regimes can be very sparse and widespread (Rutledge and MacGorman, 1988; Engholm et al., 1990).

BPE Calculation for a 6-DF Network

Figure 17 shows an example of a flash detected by the 6 (now 7) DF lightning network operated by the National Severe Storms Laboratory (NSSL) in Norman, OK (Rutledge and MacGorman, 1988). This example shows how the consistency criterion is used to reject "wild" bearings. The BPE labeled FIX #1 (SMA=9.2 km) uses all six DFs as input, but the BPE uses only five bearings (DF 6 is rejected). The BPE labeled FIX #2 (SMA=32.1 km) uses only the four DFs in Oklahoma as input, using all four. FIX #2 using only the Oklahoma DFs is located 28 km northwest of FIX #1. The other
Figure 14. Lightning during the period 2315–2325 UTC on 13 July 1986 superimposed onto the CP-2 radar echo prior to ground truth corrections: • = negative polarity discharges; + = positive polarity discharges. Radar reflectivity contoured at 18 dBZ and 40 dBZ.
Figure 15. The location and 50% error ellipses (2° bearing standard deviation) for each discharge after radar ground truth corrections.
Figure 16. Overlay of corrected lightning location estimates with the radar echo. Same as in Figure 15 but with $1.5^\circ$ bearing standard deviation.
Figure 17. FFIX solutions for cloud-to-ground flash detected in Kansas by the NSSL DF network.
Intersection points where bearing pairs intersect represent false or ghost targets. Figure 18 also shows the corresponding radar echo distribution and the location of FIX #1 (indicated by the large cross in southwestern Kansas).

Table 4 gives the error ellipse characteristics for FIXs #1 and #2, and for each bearing pair. When a BPE cannot be computed for \( \sigma_i = 1.5^\circ \), one can increase \( \sigma_i \) by 0.5° increments until a solution is acquired. In this way, the standard deviation is treated more as a tolerance factor, rather than as an absolute.

The results of this test show that FIX #1 has the smallest error ellipse of all submitted DF combinations. For all paired-bearing combinations the DF (1,7) BPE has the smallest semi-major axis, but the DF (4,7) BPE has the smallest ellipse area. Although a solution is possible near the baseline of DFs (2,3), the error ellipse is quite substantial because of the poor geometry, reflected in the large value of \( \sigma_i \). The three greatest signal strengths are reported at DFs 4, 3 and 6, in that order. An algorithm that uses the bearing pair having the greatest signal strengths would form a solution using the DF (3,4) combination, but the resulting semi-major axis for the ellipse is 201.9 km and the solution is displaced 14 km from the FIX #1 BPE. A solution could not be obtained until \( \sigma_i \) was increased to 5°, again indicating a poor geometry. However, a solution formed with the DF (4,6) combination reduces the semi-major axis to 23.3 km. Yet, any solution with DF 6 must be treated with caution, since it is inconsistent with the other bearings. One should ask whether the DF 6 bearing deviation is due to site errors or if it is just a "wild" bearing. If this particular DF 6 bearing angle is inconsistent with other flash BPEs in the same direction, then the deviation is due to systematic or site errors. However, if it is just a chance occurrence, say 9 of 10 bearings at this angle are consistent with the other DFs, then it was most likely a "wild" bearing. Following the completion of the aforementioned analysis a large systematic error at DF 6 was independently confirmed and the station moved to a new location in northern Kansas.
Figure 18. Distribution of radar echoes and lightning activity corresponding to the time of the FFIX solution in Kansas. Large cross indicates solution #1. Radar contours at 18, 30, 40, 45, 50 and 55 dBZ.
Table 4. Error Ellipse Characteristics for 6-DF Network

<table>
<thead>
<tr>
<th>DFs Submitted</th>
<th>$\sigma$ (deg)</th>
<th>LAT (deg)</th>
<th>LONG (deg)</th>
<th>SMA (km)</th>
<th>SMI (km)</th>
<th>ORI (deg)</th>
<th>AREA ($km^2$)</th>
<th>RADIUS (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,2,3,4,6,7</td>
<td>1.5</td>
<td>37.659</td>
<td>100.058</td>
<td>9.2</td>
<td>4.0</td>
<td>151.3</td>
<td>116.9</td>
<td>7.4</td>
</tr>
<tr>
<td>1,2,3,4</td>
<td>1.5</td>
<td>37.874</td>
<td>100.220</td>
<td>32.1</td>
<td>4.3</td>
<td>149.1</td>
<td>437.4</td>
<td>24.6</td>
</tr>
<tr>
<td>1,2</td>
<td>5.0</td>
<td>37.976</td>
<td>100.230</td>
<td>258.2</td>
<td>35.2</td>
<td>147.7</td>
<td>28,508.9</td>
<td>198.0</td>
</tr>
<tr>
<td>1,3</td>
<td>5.0</td>
<td>38.086</td>
<td>100.347</td>
<td>213.0</td>
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<td>150.3</td>
<td>22,201.9</td>
<td>163.4</td>
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<tr>
<td>1,4</td>
<td>1.5</td>
<td>39.592</td>
<td>101.995</td>
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<td>100.098</td>
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<td>99.573</td>
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<td>100.057</td>
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<td>99.272</td>
<td>23.0</td>
<td>6.9</td>
<td>23.5</td>
<td>500.3</td>
<td>17.9</td>
</tr>
</tbody>
</table>
Systematic Error Corrections

The FFIX algorithm has also been used in this study to correct for the systematic errors arising from site effects (Horner, 1954; Gething, 1978). Site errors cause bearing errors that are themselves a function of direction. The 12° bearing deviation from the BPE at DF 6 in the previous example was due to unresolved site errors. Site errors are one of the chief limitations to achieving the optimal location accuracy, and perhaps the most difficult problem degrading network performance.

Previous attempts to correct for the systematic errors associated with lightning direction finding systems employed some type of optimization procedure that minimized the difference between the observed bearings and the "true" target location. The "true" target location can be determined by visual ground-truth (Mach et al., 1986) or by assuming that one or more of the direction finder bearings is correct (Hiscox, 1984; Orville, 1987). This latter method will give self-consistent solutions (as applied to the DF 6 bearing deviation described above), but spatial bias effects may still be present (e.g., lightning clusters offset from radar echoes). Rocket triggered lightning strikes at Kennedy Space Center, FL have also been used to provide ground-truth, but this is only applicable to a single bearing line from any direction finder station. In practice, it is very difficult to obtain visual ground-truth at a large number of discrete bearings. Schutte et al. (1987) reversed the role of transmitter and receiver by radiating a 1 MHz signal successively through each loop of the direction finder antenna whose amplitude could then be measured by a radio receiver at a number of bearings from the site.

The ultimate litmus test of any of these methods (and the practical usefulness of these data over a large domain) should be determined by the degree of spatial correlation between clusters of lightning strikes and their associated radar echoes. Most lightning strike networks have either partial or full weather radar coverage, thus making this
validation technique practical for all but a few users. A new technique is presented below that uses isolated radar echoes to constrain the error correction and optimization procedures to remove the systematic errors.

Defining the Sample Subset

A sample subset of lightning strikes is constructed for a short time interval corresponding to low-level radar scans of isolated storms distributed about each DF (refer to Figure 14). The interval should account for the propagation of the echo. If the echo moves slowly, then the time increment can be increased to enlarge the lightning sample size.

Computing the Bearing Deviations

Next, the lightning strikes are superimposed on the radar image and the position of the reflectivity core is marked. The bearings from each DF to the storm core are computed and these are referred to as the "true" bearings. The lightning flashes are replotted (Figure 19) and the deviations between the observed and "true" bearings are computed. There are 11 flashes associated with this storm in the 10 min interval between 2315 and 2325 UTC.

Flashes 6 and 7 (refer to Table 3) have the two largest semimajor axes of 6.3 km and 9.2 km, respectively. Table 5 shows that these are two of the three flashes only detected by just two DFs. The low number of flashes seen by DF 2 indicates another source of network performance degradation. Poor detection efficiencies and even "blind spots" due to site effects (e.g., poor ground conductivity) have been noted by other users of such systems. Because one or more sites may not detect a flash, the location must be determined from a less favorable geometry (e.g., a flash along the baseline of two DFs or
Figure 19. Bearing lines from each of the DFs to the reflectivity core of the subject storm.
Table 5. DF Bearings and Site Error Corrections

<table>
<thead>
<tr>
<th>Flash</th>
<th>DF 1</th>
<th>DF 2</th>
<th>DF 3</th>
<th>DF 4</th>
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<td>86.3</td>
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</tr>
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<td>18.2</td>
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<td>4.6</td>
<td>88.4</td>
<td>52.1</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>Median</th>
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<td>87.5</td>
<td>51.5</td>
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<tr>
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<td>350.4</td>
<td>84.8</td>
<td>49.0</td>
</tr>
<tr>
<td>Deviation</td>
<td>-1.6</td>
<td>-14.7</td>
<td>-2.7</td>
<td>-2.5</td>
</tr>
</tbody>
</table>
a flash more distant from one site than another in a more optimal geometry). Over a large area, say 300 km from the center of the network, we find that nearly 50% of all flashes are only detected by two DFs.

Because of the small sample sizes, choose the median bearing as the measure of central tendency. The median is a more robust estimator in that it is less sensitive to outliers or "wild" bearings than is the sample mean. Others have chosen median estimation for dealing with bearing information for just this reason (e.g., Lenth, 1981). In practice, one would try to find a number of storms close to the bearing angles in use here (we try to get samples in 6° azimuth bins) to further increase the sample size. The median observed bearing and "true" bearing to the storm are also presented in Table 5. The nearly 15° deviation at DF 2 is further evidence that the site is poor. Large site errors such as this in certain directions, however, are not that uncommon (e.g., Schutte et al., 1987).

Computing New Solutions With the Corrected Bearings

Figure 20 shows a close-up view of the lightning locations before any site error corrections are used (shown as dots) and after the initial bearing corrections are implemented (again described by the 50% error ellipse). The lightning centroid is displaced 10 km to the southeast of the main echo. Figures 15 and 16 show the lightning locations described by the 50% error ellipse after the bearing corrections from Table 5 are implemented. The lightning centroid is now less than 5 km from the main echo. The impact of uncorrected bearing errors on the clustering process and storm identification is discussed further in Chapter 4.
Figure 20. Location estimates and 50% error ellipses for each flash before radar ground truth corrections.
Site Error Polynomials

Once the data base on site error corrections is generated, one would fit a polynomial to the data having the form

\[ y = a_0 + \sum_{k} (\sin k\Theta + \cos k\Theta), \quad k=1,2,3,\ldots,6 \]  

(3.1)

where \( y \) is the site error correction to the observed bearing \( \Theta \). One of the motivations for choosing a polynomial fit of this form is that the real-time hardware can implement the polynomial (but only to 4th-order) in real-time. Figure 21 shows the site error correction curves for the four DFs. The curves were produced by computing site error corrections in 6° bins using the technique described by Mach et al. (1986) and Orville (1987). A nonlinear regression was performed using the method of Marquardt (1963) to compute the polynomial fit to Eqn. (3.1).

Another way to examine the network performance using the error ellipse is to make plots such as Figure 22 where 30 min of lightning data are shown from 2300-2330 UTC with increasing thresholds of the semimajor axis (SMA). The locations compare reasonably well with the radar data even with a semimajor axis of 20 km.
Figure 21a. Site error polynomial and residual for DF 1.
Figure 21b. Site error polynomial and residual for DF 2.
Figure 21c. Site error polynomial and residual for DF 3.
Figure 21d. Site error polynomial and residual for DF 4.
Figure 22. Lightning plots during the period 2300-2330 UTC with semimajor ellipse axis thresholds: a, infinity; b, 2 km; c, 10 km; d, 20 km.
CHAPTER IV.
CLUSTERING METHODOLOGY

Storm Identification

Before generating a lightning time series, a pattern recognition scheme is needed to identify the collection of flashes belonging to an individual storm or storm complex. For example, Peckham et al. (1984) manually identified storms and storm systems during a 3-h sample period using a 2-station lightning network in Florida (Figure 23). Closer inspection of Figure 23 shows subareas of greater flash density within the closed contours representing the boundaries of the 13 storm cells labeled A–M. Given that the average thunderstorm lifetime is less than 1 h, these subareas are highly suggestive of smaller thunderstorms that existed during a portion of the 3-h observation period.

In a recent investigation examining the fusion of lightning ground strike information with satellite imagery, Goodman et al. (1988a) subdivided the lightning flashes contained within a $4 \times 10^6 \text{ km}^2$ area into grid cells having a dimension of $0.1^\circ$ latitude by $0.1^\circ$ longitude. The selected grid cell dimension of approximately 10 km is much greater than the random position errors, yet also represents the typical diameter of an individual thunderstorm cell. This process was applied during a 1-h period to successive 5-min sample intervals to permit the manual (human judgment) identification of individual and multi-cellular storms using an interactive workstation (Figure 24). The closed contours outline the lightning density maxima and can be used to track the movement, merger, and splitting of clusters. The 5-min sample period is commensurate with the NEXRAD radar and geostationary weather satellite sampling intervals. In the section below this process is taken a step further and investigate the use of a clustering
Figure 23. Map of ground discharges in the Tampa Bay, FL area on 8 August 1979 from 1700-2000 UTC (1300-1600 EDT) by a 2 DF network. Map scale is in thousands of feet (adapted from Peckham et al., 1984).
Figure 24. Cloud-to-ground lightning contour maps (number of discharges per cell per 5 min) of storms embedded within a mesoscale convective system in North Alabama on 11 June 1986. Time in UTC. (after Goodman et al., 1988).
algorithm to assign the individual lightning flashes (the objects) to their respective storms (the groups). New members can be assigned iteratively to a new or existing storm during each sampling interval and tracked with time.

Selecting a Clustering Algorithm

Various clustering methods can be used for such object classifications. Sokal and Sneath (1963), Anderberg (1973), Hartigan (1975), and Romesburg (1984), for example, have written entire books on the subject of cluster analysis. At its most basic level, clustering is the grouping of similar objects. All clustering algorithms are procedures for searching through the set of all possible clusterings to find the one that fits the data reasonably well (Hartigan, 1975).

The clustering procedure begins with the choice of some initial partition of the data which is then modified so as to obtain a better partition. The basic concept of these methods is similar to that of the steepest descent algorithms used for unconstrained optimization in non-linear programming (Anderberg, 1973). These algorithms begin with an initial point and generate a sequence of moves to find an improved value of the objective function until a local optimum is found. The search may involve sorting by variables, switching objects between clusters, joining objects together, splitting objects apart, adding objects to pre-existing clusters, or specialized searching of a subset of clusters (Hartigan, 1975).

Joining algorithms have been used previously to identify storms from radar base-scan reflectivity patterns for the purpose of extrapolating storm motion (Blackmer and Duda, 1976; Browning et al., 1982). The basic method follows:

1. **Step 1.** Identify all gridpoints with local reflectivity maxima above some threshold value.

2. **Step 2.** Compute the spatial separation between each pair of maxima.
Step 3. Combine the nearest pair together to form a new cluster.

Step 4. Repeat steps 1-3 until only one cluster remains or a stopping condition has been reached (e.g., a threshold on the minimum separation distance needed to define a cluster as a storm or storm complex). The result can be visualized as a linkage tree or dendrogram describing the level (distance) where individual clusters join together.

Step 5. Repeat steps 1-4 for successive radar images. Compute motion vectors from the individual cluster displacements between sampling intervals.

The K-Means Algorithm

The joining algorithms produce a large number of clusters which are then reduced to a manageable number more for the sake of convenience rather than for quantitative reasons related to the physical process itself. Chiefly for this reason, and because of computational expense (as many as 8000 discharges per hour have been detected by the MSFC network), a switching (or transfer) algorithm called the K-means or K-nearest neighbor algorithm was selected for grouping the lightning discharges into storm clusters. The algorithm begins with an initial partition of the data (referred to as seed points) and obtains new partitions until no additional switches in the neighborhood of the initial partition improve the classification. Thus, a local rather than a global optimum is sought. The stopping criterion is reached when no movement of an object from one cluster to another will reduce the within-cluster total sum of squares. The IMSL implementation of algorithm AS 136 developed by Hartigan and Wong (1979) was used for this purpose. Appendix C contains the FORTRAN-77 programs developed by the author for generating the seeds and calling the IMSL subroutine KMEANS. The clustering of the lightning data is implemented as follows:
**Step 1.** Define an area of interest and subdivide the lightning flashes occurring during a sample interval into grid cells having a dimension of 10 km x 10 km. Sample intervals of 5-min and 10-min are used with the choice a function of either the corresponding radar sampling interval (when used for intercomparison or validation experiments) or storm lifetime.

**Step 2.** Search the data matrix for isolated lightning density maxima and label these as possible seeds. A default threshold of 0.02 discharges km$^{-2}$ (2 ground discharges per 100 km$^2$) is used almost exclusively (see Step 5 below).

**Step 3.** Use the K-means algorithm to obtain optimal partitioning of the lightning into storm clusters.

**Step 4.** (For storm tracking purposes). Repeat steps 1-3 for successive sample intervals. A storm motion vector can be computed from the successive cluster centroid displacements and a time series can be constructed from the successive cluster memberships.

**Step 5.** (Hybrid Scheme). Step 2 may generate more seeds than desired (or physically realistic) in which case the clusters are not sufficiently coherent to track during successive time intervals in Step 4. Occasionally too few seeds are generated and Step 3 fails to converge to a solution. In the former case with a richness of seeds, the initial clusters are joined until the number of remaining lightning clusters can be correlated (i.e., tracked) over successive sampling intervals. The clusters can be joined further until only a single cluster representing an entire complex of storms remains. This final cluster can be used to generate and analyze a time series for the entire storm system. In the latter case having a deficit of seeds, the seed threshold value is lowered to 0.01 discharges km$^{-2}$ or a 5 km x 5 km grid subarea is searched for additional seeds.

**Step 6.** (Sensor Fusion Scheme). Steps 1-5 address thunderstorm identification with the ground discharges alone. An alternative seeding method employing radar data has been evaluated. In this case, the peak reflectivity tracker or NEXRAD tracker schemes can be used to identify the storm cells. These cell centroids then serve as the seed points for clustering the ground discharges with the K-Means algorithm.

**Demonstration of Method**

Storm systems ranging from individual thunderstorms to large complexes of storms are used to evaluate the performance of the clustering procedures. The basic algorithm is first described for a case of thunderstorm cells embedded within a fast
moving pre-frontal squall line that produced widespread severe weather as it crossed the Tennessee Valley during a 9 h period in the afternoon of 15 November 1989. The widespread precipitation pattern (rain/no rain) and reflectivity peaks (above a threshold of 40 dBZ) are depicted at 2000 UTC (1400 CST) in Figure 25. The most active hour of cloud-to-ground lightning activity takes place from 2000-2100 UTC. The image is a composite constructed from several NWS network (e.g., Nashville, TN; Centerville, AL; Jackson, MS) and local warning (e.g., Huntsville, AL; Tupelo, MS) radars in the region.

An isolated supercell storm ahead of the line (labeled T) was overtaken as the line moved northeastward more rapidly than the cell (Figures 26-28). The average system motion vector was 25 m s\(^{-1}\) from 235° (due north is defined as 0°). The merger and subsequent interaction of the gust front from the squall line with the supercell led to an F4 intensity tornado (estimated wind speeds of 92-116 m s\(^{-1}\)) at 2230 UTC that killed 22 people in Huntsville, AL. The ambient wind in the lower troposphere derived from atmospheric soundings at 1200 UTC (235° at 6.7 m s\(^{-1}\) at Centerville, AL (CKL) and 230° at 8.5 m s\(^{-1}\) at Nashville, TN (BNA)) is much less than the storm motion vector. In addition, the supercell storm had an average storm motion vector (indicated by the large dots) of 243° at 18.3 m s\(^{-1}\), 8° to the right of the mean wind.

Define the Region of Interest

Isolated lightning discharges of both positive and negative polarity occurred both ahead of the line from thunderstorm anvils and from the trailing stratiform rain region behind the main line of storms. Lightning activity during a 1.5 h period prior to the Huntsville, AL tornado is shown in Figure 29. The analysis region of interest is shown in Figure 30. The lightning flashes are converted from earth coordinates (latitude, longitude) to rectangular coordinates (x,y). For convenience, the geographic coordinate (0,0) at the center of the map represents the location of the MIT/Lincoln Laboratory.
Figure 25. Composite precipitation pattern at 2000 UTC (1400 CST) on 15 November 1989 observed by the regional network of NWS radars. The radar reflectivity is contoured at 18, 30, 40, 45, 50, 55 dBZ (VIP levels 1-6).

Small solid circle = Reflectivity cores > 45 DBZ; large solid circle = track of the tornadic supercell storm every 15 min. The motion vector of cells within the squall line indicated by the wind barb is 25 m s\(^{-1}\) from 235°.
Figure 26. Composite precipitation pattern at 2100 UTC on 15 November 1989.
Figure 27. Composite precipitation pattern at 2200 UTC on 15 November 1989.
Figure 28. Composite precipitation pattern at 2230 UTC on 15 November 1989.
Figure 29. Cloud-to-ground lightning activity from 2105-2236 UTC on 15 November 1989. . = negative polarity discharges; + = positive polarity discharges.
Figure 30. County and state outlines in the lightning analysis region. Range rings are every 50 km. The site of the MIT/Lincoln Laboratory FL2 Doppler radar is at the center of the rings.
FL2 Doppler radar (refer also to Figure 24). The radar was situated just north of the Huntsville airport (HSV) during the COHMEX field campaign to study thunderstorm downdrafts, outflows, and gust fronts hazardous to aviation. This transformation is for the convenience of some lightning-radar intercomparisons discussed later. Figure 31 depicts the centroid of all lightning activity in successive 5-min intervals from 2130-2235 UTC. The plotted track of the tornadic supercell storm, T, was computed every 15-min during the interval 2130-2230 UTC from the National Climatic Data Center 16 mm film-archive of the Nashville, TN (BNA) radar scope. The initial damage from the tornado (labeled t) occurred on Redstone Arsenal at 2230 UTC.

The clustering procedure begins by summing the lightning discharges into 10 km grids within an m-row by n-column matrix such as that pictured in Figure 32. The evolution of the lightning activity associated with the main line of storms can be followed from the series of contour maps (> 0.02 flashes km\(^{-2}\) shaded) in Figure 33. A contoured lightning map during the 5-min interval 2215-2220 UTC indicates the locations of greatest lightning density that might serve as candidate seed points (Figure 34). In the following discussion, the gridpoint (20,20) is at the center of the data matrix and corresponds to the aforementioned geographic reference point at (0,0).

Identify the Cluster Seeds

**Step 1.** A copy of the m x n data matrix D is generated and designated as the seed matrix S.

**Step 2.** A 3 x 3 point window searches directionally by rows through S from the upper left-hand corner (1,1) to the lower right-hand corner (m,n) to identify isolated lightning density maxima (Figure 35). Any non-zero grid point S(i,j) at the center of the 3 x 3 window having a neighbor greater or equal to itself is also set equal to zero. Otherwise, the point is left undisturbed in S as a candidate seed. For example, the gridpoint S(15,20)=8 is greater than its neighbors and is designated as a seed. When the center of the window reaches the gridpoint S(16,20)=6, the value of 6 is less than the neighbor above it and is set equal to zero. A single pass through S would produce the seeds indicated in Figure 36. However, note that the points
Figure 31. Centroid of all lightning within the analysis region in 5-min intervals (labeled 1-9-A-D) during the period 2130-2235 UTC. T = track of the tornadic storm echo every 15 min beginning at 2130 UTC; t = location of tornado touchdown and tree damage on Redstone Arsenal. The FL2 radar is at the geographic center of the map at the point (0,0).
Figure 32. Lightning analysis region with 10 km gridpoints.
Figure 33. Evolution of cloud-to-ground lightning activity in 5-min intervals between 2135-2235 UTC. Contours are approximately 0.01 discharges km\(^{-2}\) and 0.02 discharges km\(^{-2}\) (shaded).
Figure 34. Contoured lightning density map during the period 2215–2220 UTC. Contour interval is every 0.01 discharges km\(^{-2}\) beginning with the value 0.02 discharges km\(^{-2}\).
Figure 35. Diagram of the 3 x 3 point search window moving through the m-row x n-column data matrix D. Lightning shown for the period 2215-2220 UTC at each 10 km gridpoint.
S(13,22)=2 and S(17,20)=2 are not isolated maxima in D, but result from the left to right search direction of the 3 x 3 window. Such spurious seeds are removed by step 3.

**Step 3.** The original data matrix D is scanned locally for a neighbor that might be larger than the candidate seed itself, but had been set to zero in S in the prior step. If a neighbor is not a seed, but is still greater than the candidate point in question, then the candidate point is not an isolated maxima and is set to zero. For example, let the window continue moving through S until the center of the window is at S(17,20)=2. S(17,20) is now a candidate because S(16,20)=0 resulted from step 2. However, S(17,20) < D(16,20) so S(17,20) should also be set to zero. Thus, one pass produces the final S matrix of exactly K seeds (Figure 37).

**Step 4.** Define a set of criteria for establishing the number of valid seeds. In this example, there are 17 seed points having at least 0.01 flashes km\(^{-2}\) (or 1 flash within a 10 km x 10 km grid). The possible seeds consist of 7 seeds produced by single lightning discharges of positive polarity, 5 produced by single discharges of negative polarity, and 5 seeds with a density greater than 0.02 flashes km\(^{-2}\). A sensitivity study of possible criteria and their justification are discussed below.

Determine the Cluster Memberships

The initial partition of K seeds is critical since the K-means algorithm must optimally assign the lightning flashes to one of exactly K clusters. A different initial partition (determined by the number and location of the seeds) might produce a different final partition and storm motion vector based on the tracking of the seeds. Furthermore, any change in cluster membership also alters its time series.

Figure 38 shows the the cluster assignment for each flash occurring between 2215-2220 UTC and the location of the 5 seeds (labeled a-e) identified in step 4 above with a density of 0.02 flashes km\(^{-2}\) or greater. Seed densities of 0.01 flashes km\(^{-2}\) are excepted and the location of the tornadic supercell storm (labeled T) as observed by the BNA radar is indicated. The Tennessee-Alabama border is approximately 45 km north of the reference point.
Figure 36. Diagram of the seed matrix $S$ after one pass through the data matrix. The search window is centered at the point $S(i,j) = (17,20)$.

Figure 37. Diagram of the final seed matrix $S$. 
Figure 38. Cluster assignments for each flash during the period 2215-2220 UTC. Circled lower case letters (a-e) = cluster seeds with values greater than 0.02 discharges km$^{-2}$; T = location of tornadic storm echo at 2215 UTC.
Clusters B, C, and D have the smallest displacements between their seed (first guess) and final centroid. Clusters A and E are more spread out with the four northwesternmost flashes assigned to cluster A being of positive polarity. A comparison with the lightning density contour maps (Figures 33; 34) and the regional radar image at 2215 UTC (Figure 27) indicates good agreement between the 5 lightning centroids and the largest thunderstorm echoes. The positive polarity flashes in the northwest quadrant of cluster A are seen to be located in the trailing stratiform rain behind the main line of thunderstorms.

Next, let the 5 single discharges of negative polarity serve as additional seeds (f-j) and consider the impact on the preceding example (Figure 39). The distribution of the 10 seeds splits apart cluster E into E, I, and J. Cluster A is split into clusters A and F, where F is comprised entirely of the scattered positive polarity flashes in the trailing stratiform rain. Cluster B is subdivided into clusters B and G and lastly, cluster H is split off from cluster C. Comparison with the 2215 UTC radar image indicates that cluster H is probably best represented by the solitary echo southeast of the main line of thunderstorms. However, the remaining subdivisions appear less realistic.

The effect of changing cluster memberships is presented in Table 6. The original cluster membership changes (and decreases) when the number of seeds increases from 5 to 10 seeds. Cluster D maintains the same total membership, but exchanges members with clusters C and E. In addition, the within group sum of squares (WSS) decreases as the number of seeds is increased.

Due to the natural spatial variability of the lightning strikes, a single flash within a 100 km² grid is just as likely to have occurred in any of the bordering grids. Indeed, ground-based radar and high altitude airplane lightning measurements of large weather systems such as this have shown lightning flashes occasionally propagating over 100 km horizontally before striking the earth (Ligda, 1956; Goodman et al., 1988c).
Figure 39. Cluster assignments for each flash during the period 2215-2220 UTC. Circled lower case letters (a-e) = cluster seeds with values greater than 0.02 discharges km\(^{-2}\); Circled lower case letters (f-i) = cluster seeds each represented by a single negative polarity discharge.
Table 6. Cluster Characteristics at 2215-2220 UTC

<table>
<thead>
<tr>
<th>Cluster</th>
<th>Seed (x, y) (km)</th>
<th>Seed Density (x 10^{-2} km^{-2})</th>
<th>Cluster (x, y) (km)</th>
<th>Members (5)</th>
<th>Members (10)</th>
<th>WSS (km^2) (5)</th>
<th>WSS (km^2) (10)</th>
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<tr>
<td>A</td>
<td>(40,90)</td>
<td>6</td>
<td>(32.2, 110.3)</td>
<td>17</td>
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<td>23543</td>
<td>4137</td>
</tr>
<tr>
<td>B</td>
<td>(0, 50)</td>
<td>8</td>
<td>(7.4, 57.5)</td>
<td>30</td>
<td>22</td>
<td>5662</td>
<td>1984</td>
</tr>
<tr>
<td>C</td>
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<td>(-6.2, -1.5)</td>
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<td>12</td>
<td>9000</td>
<td>3225</td>
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<td>16</td>
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<td>2226</td>
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<td>(-105.3, -45.1)</td>
<td>11</td>
<td>8</td>
<td>8580</td>
<td>2335</td>
</tr>
<tr>
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<td>-</td>
<td>-</td>
<td>3</td>
<td>-</td>
<td>2811</td>
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<td>G</td>
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<td>-</td>
<td>1</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>I</td>
<td>(-150, -60)</td>
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<td>-</td>
<td>-</td>
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<td>0</td>
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<td>J</td>
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<td>-</td>
<td>3</td>
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<td>0</td>
</tr>
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</table>
Tracking Seeds and Clusters

The eastward propagation of the lightning activity can be followed over the three subsequent 5-min periods 2220-2225 UTC (Figure 40), 2225-2230 UTC (Figure 41), and 2230-2235 UTC (Figure 42). In each case a threshold seed density of 0.02 flashes km\(^{-2}\) is employed to identify the clusters. The seed and cluster letter identifiers are newly assigned each 5-min sample period in the order the seeds are identified. Thus, the assignment follows the search direction from upper left to lower right within the seed matrix S.

Cluster A beginning at 2215 UTC can be tracked from the continuity between 5-min observation periods from 2215-2230 UTC as A-A-A-A. Since no local seed is identified at 2230 UTC, the singular A event near the coordinate (70,120) in Figure 42 is misclassified as a member of the cluster to its south, rather than to the decaying northern storm. Cluster B beginning at 2215 UTC can be tracked as B-(B+C)-B-A. The tornadic storm cluster can be tracked as C-(D+E)-C-C. The two clusters initially identified at 2215 UTC as B and C are each split apart into two distinct groups at 2220 UTC before merging again. The additional lightning clusters at 2220 UTC makes subsequent tracking of all but the largest storms difficult. However, such storms also produce a greater number of discharges and hence pose the greatest threat. Isolated storms, large or small, tend to maintain their identity. Storms that merge and split apart cause identification problems for the radar echo tracking techniques as well.

An approach that might be considered for reducing the number of clusters to only those that are coherent (i.e., trackable) from sample to sample is to make use of a hybrid scheme using multiple methods. One possible hybrid approach has been applied to this problem. The K-means algorithm is run initially as before, but then a variation of the joining algorithm is used to reduce the number of clusters to the same number identified in the prior sampling interval. The final number of clusters desired can be
Figure 40. Cluster assignments for each flash during the period 2220–2225 UTC. Circled lower case letters (a–i) = cluster seeds with values greater than 0.02 discharges km$^{-2}$. 
Figure 41. Cluster assignments for each flash during the period 2225-2230 UTC. Circled lower case letters (a-e) = cluster seeds with values greater than 0.02 discharges km$^{-2}$. 
Figure 42. Cluster assignments for each flash during the period 2230–2235 UTC. Circled lower case letters (a–f) = cluster seeds with values greater than 0.02 discharges km$^{-2}$. 
based on having a manageable number of entities to track, correspondence with radar measurements, or perhaps based on some criteria such as desired storm size (by setting a minimum areal extent threshold). The result is shown in Figure 43 for the period 2220-2225 UTC. The hybrid implementation involves finding the mean \((x,y)\) coordinate of the two nearest clusters to be paired until the total number of seeds is reduced from the original set of 9 seeds (refer to Figure 40) to a revised set of 5 seeds. Table 7 lists the results of changing the number of seeds. When clusters D and E are joined, the seed location of cluster E is used instead of a mean location for the pair because of the greater seed density and greater number of members assigned to E in the first application of the K-means algorithm.

Alternatively, the grid cell dimension could be increased to 20 km x 20 km or greater to allow greater seed densities and fewer seeds, but this approach will reduce the accuracy of the seed locations and the ability to resolve small storms as individual entities. We have found that it is better to generate more seeds and iteratively join the clusters than to generate too few seeds and be unable to resolve new cells that may produce low flash rates or identify decaying storms which produce scattered discharges. The Huntsville, AL tornadic storm, for example, produces a flash density of 1 discharge per 100 km\(^2\) earlier in its life-cycle. Other alternative approaches using only lightning data, considered to be beyond the scope of this study, would be 1) to set a threshold on the seed density or total cluster membership at some level greater than 0.02 flashes km\(^{-2}\) that might be considered to be physically meaningful, or 2) define a maximum search radius, say 20 km, which would be the maximum distance a flash could be from a seed point for consideration as a cluster member.

Figure 44 shows 30-min seed tracks (representing the gridpoint with the largest local flash density) from 2200-2230 UTC for the clusters identified at 2215 UTC as A-E. The lower case letters identify only the 5-min periods when the seed value is greater than the 0.02 flashes km\(^{-2}\) threshold. The upper case letters indicate the location.
Figure 43. Hybrid clustering algorithm assignments during the period 2220-2225 UTC. Circled lower case letters (a-e) = cluster seeds with values greater than 0.02 discharges km$^{-2}$. 
Table 7. Hybrid Clustering Algorithm at 2220-2225 UTC

<table>
<thead>
<tr>
<th>Cluster</th>
<th>Seed (x, y) (km)</th>
<th>Seed Density (x 10⁻² km⁻²)</th>
<th>Cluster (x̄, ȳ) (km)</th>
<th>Members</th>
<th>WSS (km²)</th>
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<td>(6)</td>
<td>(9)</td>
<td>(11.0, 57.2)</td>
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<tr>
<td>C</td>
<td>(10, 40)</td>
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<td>(11.8, 44.8)</td>
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<td>891</td>
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<td>D</td>
<td>(-20, 10)</td>
<td>(0, 0)</td>
<td>(-20.4, 15.8)</td>
<td>5</td>
<td>410</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>(9)</td>
<td>(-0.8, 6.2)</td>
<td>13</td>
<td>(4772)</td>
</tr>
<tr>
<td>E</td>
<td>(0, 0)</td>
<td>(0, 0)</td>
<td>(6.7, 2.5)</td>
<td>13</td>
<td>1073</td>
</tr>
<tr>
<td>F</td>
<td>(-40, -20)</td>
<td>(-30, -25)</td>
<td>(-22.6, -32.8)</td>
<td>7</td>
<td>1924</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>(9)</td>
<td>(-37.1, -27.1)</td>
<td>(22)</td>
<td>(6303)</td>
</tr>
<tr>
<td>G</td>
<td>(-60, -30)</td>
<td>(-80, -35)</td>
<td>(-92.6, -42.0)</td>
<td>10</td>
<td>1066</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>(9)</td>
<td>(-110.2, -51.1)</td>
<td>(16)</td>
<td>(6917)</td>
</tr>
<tr>
<td>H</td>
<td>(-100, -40)</td>
<td>(3)</td>
<td>(42.6, -23.3)</td>
<td>14</td>
<td>1302</td>
</tr>
<tr>
<td></td>
<td>(9)</td>
<td>(5)</td>
<td>(9)</td>
<td>(9)</td>
<td>(9)</td>
</tr>
</tbody>
</table>
Figure 44. Seed tracks during the period 2200-2230 UTC. Lower case letters (a-e) = seed position for each 5-min interval that the seed value exceeds a threshold of 0.02 discharges km$^{-2}$. Upper case letters (A-E) = seed location at 2215 UTC; T = Position of tornadic storm echo at 2200, 2215 and 2230 UTC.
of the seeds at 2215 UTC and the location of the tornadic storm echo at 2200 UTC, 2215 UTC, and 2230 UTC. All clusters are generally moving from west to east, with lightning cluster C within 1-gridpoint of the storm echo. However, only seeds B and D are greater than the threshold value during each 5-min interval. The track variations suggest a moving or weighted average of past tracks be used instead of updating the track with each observation. Such techniques are also employed in the radar echo trackers described earlier.

**Synergism With Radar**

A primary objective of using a clustering algorithm for pattern recognition is to allow the identification and tracking of storms and their changing membership using the lightning data alone. However, the previous intercomparisons between the lightning clusters and the radar echoes suggest that a fusion of the data sources would be useful. For example, the radar echo trackers could provide seeds for clustering the lightning. Consider the tornadic storm at 2135 UTC when the maximum flash density is only 1 flash per gridpoint. Using a seed threshold of 0.01 flashes km$^{-2}$ produces 21 total clusters and permits a separate cluster of two flashes, labeled T, for the tornadic storm (Figure 45). Using a seed threshold of 0.02 flashes km$^{-2}$ causes the two lightning flashes to be assigned to cluster C.

On the other hand, the size and intensity criteria used in radar echo tracking could be given adaptive thresholds to capture small, electrically active storms that do not meet the default threshold criteria. In a study of the effect of radar echo size and asymmetry on the identification of small, electrically active microbursts occurring in Alabama and Florida, Buechler and Goodman (1990) find that the operational NEXRAD storm identification algorithms have a probability of detection less than 0.5. The algorithms are designed to detect large severe storms. Small storms (less than 5 km in
Figure 45. Cluster assignments during the period 2135-2140 UTC. A seed threshold of 0.01 discharges km\(^{-2}\) produces 21 clusters (A-T). \(I\) = tornadic storm discharges.
diameter), changing echo shapes, and mergers sometimes cause the radar echo trackers to lose (fail to identify) a storm from one 5-min scan period to the next. The continued occurrence of lightning could thus be used to alert the echo tracker to adjust threshold.

A comparison of the different lightning clusters produced from radar echo seeds follows. The NEXRAD storm identification and tracking algorithm (Bjerkaas and Forsyth, 1980) is implemented with S-band radar data collected on 13 July 1986 at 2328 UTC by the CP2 radar. The tracking algorithm yields two echo centroids, labeled A and B, for this multi-cellular complex of storms (Figure 46). A smaller length threshold (5 km) and intensity threshold (30 dBZ) could produce more than two distinct storms. A peak reflectivity tracker (e.g., Crane, 1979; Rosenfeld, 1987) would produce at least four echo centroids, indicated by the shaded area representing reflectivity values in excess of 55 dBZ. The 3-dimensional structure of these storms is portrayed in Figure 47. The lightning seeding algorithm (with its 10 km grid) would produce only a single seed located between the points (36,115) and (46,125) depending on the placement of the grid. In this isolated storm example, a joining-type clustering algorithm could be used instead to assign the 28 lightning flashes to possibly four distinct lightning clusters. The lightning locations in Figures 48 and 49 suggest four storm clusters offset ahead of the peak reflectivity maxima. The offset is partially physical in that the lightning strikes generally occur outside of the storm reflectivity core, but there also appears to be a directional bias to the southeast of the echo centroids, likely due to unrecovered systematic errors at one or more of the antennas. This initial examination of the performance and limitations of lightning clustering algorithms demonstrates their usefulness in identifying and tracking thunderstorms. In addition, it appears that more than one method and stopping criteria is best suited to the various space and time scales examined here.
Figure 46. Plan-view of a small multicellular storm complex observed by the CP2 radar on 13 July 1986 at 2328 UTC. Circled upper case letters (A,B) = two storm centroids identified by the NEXRAD storm identification and tracking algorithm; solid contour = reflectivity > 40 dBZ; shaded region = reflectivity > 55 dBZ. Distance units in kilometers from CP2.
Figure 47. Three-dimensional structure of the storm complex in Figure 46.
Figure 48. Cluster assignments using two centroids identified by NEXRAD algorithm. Circled lower case letters (a,b) = radar echo cluster seeds; Upper case letters (A,B) = cluster assignments.
Figure 49. Cluster assignments using four centroids identified from peak reflectivity ($Z > 55$ dBZ) maxima. *Circled lower case letters (a,b,c,e) = radar echo cluster seeds; Upper case letters (A,B,C,E) = cluster assignments.*
CHAPTER V.
THUNDERSTORM LIFE-CYCLES AND EXTRAPOLATION FORECASTING

Testing and Evaluation of the Logistic Growth Model

The storm identification and tracking process using clustering methods sets the stage for assessing the validity of the logistic growth model and predictability of lightning activity during the storm life-cycle. Table 8 gives the model parameters for a selected number of cases. A total of 14 storm systems have been examined ranging in size from small, short-lived air mass storms (< 1 h duration) to large, long-lived mesoscale convective systems. The total (intracoud and cloud-to-ground) lightning life-cycle has been computed for 3 of the short-lived storms. These latter cases are included to demonstrate the applicability of the logistic model to storms for which the total lightning rates portray the growth and decay process, yet the storm may produce too few discrete cloud-to-ground flashes to establish a trend (e.g., the 20 July case). The cloud-to-ground lightning data were summed over 1, 5, 10, 30 and 60 min sampling intervals depending on the availability of data or the duration of the storm life-cycle. The SAS (1985) non-linear regression procedure NLIN using the Gauss-Newton method was used to estimate the parameters $\beta$ and $k$ from each set of observations and limiting values of $\alpha$ corresponding to the total number of lightning flashes produced by each storm event.

Isolated and Multicellular Storms

Figure 50 shows the evolution of the cloud-to-ground lightning activity associated with 3 distinct storms observed in Southern Tennessee on 17 July 1986. The storms were identified from the cluster analysis during the interval 1605-1845 UTC.
### Table 8. Parameters of the Logistic Growth Model

<table>
<thead>
<tr>
<th>Case</th>
<th>Geographic Location</th>
<th>α</th>
<th>β</th>
<th>k</th>
<th>Observation Periods</th>
<th>Δt (min)</th>
<th>Duration of Lightning Activity (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>17 July 1986 (A)</td>
<td>TN</td>
<td>102</td>
<td>54.9</td>
<td>0.89</td>
<td>8</td>
<td>10</td>
<td>80</td>
</tr>
<tr>
<td>17 July 1986 (C)</td>
<td>TN</td>
<td>453</td>
<td>33.0</td>
<td>0.76</td>
<td>11</td>
<td>10</td>
<td>110</td>
</tr>
<tr>
<td>3 June 1986</td>
<td>AL/TN</td>
<td>3089</td>
<td>136.1</td>
<td>0.32</td>
<td>40</td>
<td>10</td>
<td>400</td>
</tr>
<tr>
<td>25 June 1986</td>
<td>TN</td>
<td>1626</td>
<td>128.7</td>
<td>0.41</td>
<td>21</td>
<td>10</td>
<td>210</td>
</tr>
<tr>
<td>13 July 1986</td>
<td>AL/TN</td>
<td>6094</td>
<td>419.9</td>
<td>0.23</td>
<td>37</td>
<td>10</td>
<td>370</td>
</tr>
<tr>
<td>14 July 1986 (1)</td>
<td>TN</td>
<td>72</td>
<td>106.7</td>
<td>0.87</td>
<td>10</td>
<td>5</td>
<td>50</td>
</tr>
<tr>
<td>14 July 1986 (2)</td>
<td>TN</td>
<td>61</td>
<td>41.5</td>
<td>0.94</td>
<td>9</td>
<td>5</td>
<td>45</td>
</tr>
<tr>
<td>15 November 1989</td>
<td>AL/TN</td>
<td>9442</td>
<td>52.4</td>
<td>0.15</td>
<td>53</td>
<td>10</td>
<td>530</td>
</tr>
<tr>
<td>20 July 1986*</td>
<td>AL</td>
<td>91</td>
<td>156.6</td>
<td>0.86</td>
<td>13</td>
<td>1</td>
<td>13</td>
</tr>
<tr>
<td>8 August 1977*</td>
<td>FL</td>
<td>12.5</td>
<td>36.4</td>
<td>0.52</td>
<td>15</td>
<td>5</td>
<td>75</td>
</tr>
<tr>
<td>11 July 1978*</td>
<td>FL</td>
<td>155</td>
<td>24.7</td>
<td>0.94</td>
<td>8</td>
<td>5</td>
<td>40</td>
</tr>
<tr>
<td>9 June 1985</td>
<td>OK/KS</td>
<td>122</td>
<td>121.3</td>
<td>0.77</td>
<td>12</td>
<td>10</td>
<td>120</td>
</tr>
<tr>
<td>3 June 1985</td>
<td>OK/KS</td>
<td>6070</td>
<td>24.2</td>
<td>0.40</td>
<td>18</td>
<td>30</td>
<td>540</td>
</tr>
<tr>
<td>MCC Composite</td>
<td>OK/KS</td>
<td>1.0</td>
<td>32.7</td>
<td>0.42</td>
<td>18</td>
<td>60</td>
<td>1080</td>
</tr>
</tbody>
</table>

* Total Lightning Activity
Figure 50. Cloud-to-ground lightning clusters during 5-min intervals on 17 July 1986. Distance units in kilometers from FL2 radar. (Time in UTC.)
The maps were produced every 10 min from 1605-1735 UTC, a period which encompasses the entire 90 min life-cycle of cell A. The corresponding radar echo time history is summarized in Figure 51 from 30 min observations made with the Nashville, TN radar. The center coordinate of each radar map is approximately 6 km east and 9 km north of the FL2 radar (used as the center of the lightning maps).

The lightning and rainflux time series for storms A and C are shown in Figure 52. Cell A undergoes a secondary surge in lightning activity after an initial peak. The rainflux time history also shows two peaks, each lagging the lightning maxima by 10 min. Cell C shows a single maxima in both lightning activity and rainflux.

The logistic model regression (P), observed flash rate (A), and residual error (A-P), for storms A and C are plotted in Figures 53 and 54. The model fit is excellent in both cases with correlation coefficient $r > 0.998$. Cell A produced 102 ground discharges (the observed $a$) and Cell C produced 453 ground discharges during its 120 min life-cycle. The steepness of the curves along the time axis are described by the parameter $k$. For storm lifetimes of 1-2 h (sampled at 10 min intervals), the median value of $k$ (Table 8) is approximately 0.8.

**Total Lightning Activity for Airmass Storms**

The value of $k$ increases to 0.9 for storms with lifetimes less than about 1 h. Figure 55 shows the logistic model results for the total lightning produced by the 20 July airmass storm described in Chapter 2. Due to the short-lived 13 min duration of the lightning activity a more frequent sampling period of 1 min was chosen. Again, the residual error is only a few percent. The other two storms occurred in Florida in 1977 and 1978 (Figure 56), and have been studied previously by Piepgrass et al. (1982) and Krehbiel (1986). The 11 July 1978 single peak storm is similar to the 20 July storm, but the secondary peak of the 8 August storm is likened more to Storm A on 17 July 1986.
Figure 51. Base scan radar echoes observed by the Nashville radar: upper left, 1600 UTC; upper right, 1630 UTC; lower left, 1700 UTC; lower right, 1730 UTC. Contour interval every 10 dBZ beginning at 10 dBZ.
Figure 52. Lightning and rainflux time series for storms A and C:
Upper case letters, number of ground discharges in 10 min;
lower case letters, radar estimated rainflux.
17 July 1986 Storm A
(Cloud to Ground Lightning)

- Model
- Observed

\[ \hat{y} = \frac{102}{{(1 + 54.9e^{-0.69t})}} \]

Figure 53. Logistic model regression and residual error for 17 July 1986 Storm A. 

Top: For each 10 min observation period, \( \Delta \) = the observed flash rate; 


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Figure 54. Logistic model regression and residual error for 17 July 1986 Storm C. 
Top: For each 10 min observation period, $A$ = the observed flash rate; $P$ = model prediction. 
Bottom: Residual error = $(A - P)$. 

17 July 1986 Storm C  
(Cloud-to-Ground Lightning)  
$P$-Model  
$A$-Observed  
\[ y = \frac{453}{1 + 33.0e^{-0.76t}} \]
20 July 1986 Air Mass Thunderstorm
(Total Lightning)

\[ P \text{ - Model} \]
\[ A \text{ - Observed} \]
\[ \hat{y} = \frac{91}{1 + 156.65e^{-0.86t}} \]

Figure 55. Logistic model regression and residual error for 20 July 1986 airmass storm. **Top:** For each 10 min observation period, \( A \) = the observed flash rate; \( P \) = model prediction. **Bottom:** Residual error = (A - P).
Figure 56. Total lightning time series (discharges per 5 min) for two thunderstorms observed near Cape Canaveral, Florida.
In the latter case the total system life-cycle is nearly symmetric within the envelope encompassing the storm, but the large individual peaks indicate 2 or 3 embedded cells or resurgent growth and decay. Few ground discharges were observed in all three cases.

Long-lived Storm Complexes

Figure 57 depicts the life-cycle of the 15 November 1989 mesoscale weather system through much of its life-cycle. The time series covers the period 1600 UTC 15 November to 0100 UTC 16 November in 10 min increments. Despite the multiple peaks superimposed upon the curve (due primarily to cell mergers), the envelope of the lightning activity exhibits symmetry about the maxima of 360 flashes which occurs at time period 27 (2030-2040 UTC). There are 19 time steps between 100 flashes and the maximum during the growth phase and 17 time steps from the maximum to 100 flashes during the decay phase. This 2 time step difference is just 20 min over a 6 h period.

The logistic growth curve in Figure 58 again fits the data well, thus reinforcing the concept of symmetric growth and decay at yet larger space and time scales. The long-lived (>2 h) storm systems have k values less than about 0.4, with k inversely proportional to storm lifetime (Table 8). Using the symmetry that characterizes logistic growth, we can estimate that the duration of the decay phase will approximately equal the duration of the growth phase. This estimate also defines the valid extrapolation period for yes/no (presence/absence of lightning activity) forecasts. Forecasts of the duration of these large storms are valuable because their long lifetimes and severe weather production makes them the most disruptive and hazardous weather systems during the Spring and Summer.

Figure 59 shows ten individual lightning ground strike time histories and a composite life-cycle for convective storm complexes observed in the Oklahoma/Kansas region of the Southern Great Plains with the NSSL ground strike network (Goodman and
Figure 57. Cloud-to-ground lightning time series (discharges per 10 min) during the period 1600 UTC 15 November to 0100 UTC 16 November 1989 for a mesoscale convective system observed in the Tennessee Valley.
Figure 58. Logistic model regression and residual error for 15 November 1989 mesoscale weather system. Top: For each 10 min observation period, A = the observed flash rate; P = model prediction. Bottom: Residual error = (A - P).
Figure 59. Cloud-to-ground lightning discharge histograms and composite lighting life-cycle for mesoscale convective complexes (MCCs). The four life-cycle phases are identified as first storms (F), initiation (I), cold cloud shield maximum extent (M), and termination (T). The composite is plotted with respect to the time and magnitude (± one standard deviation) of the average peak flash rate. (After Goodman and MacGorman, 1986).
MacGorman, 1986). The life-cycle is identified from the infrared cloud temperature criteria developed by Maddox (1980). The dimension of the cold cloud top observed by satellite is used to define four phases of the storm system life-cycle. These phases identify the formation of the first storms (F), the initial time at which the size criteria threshold is reached (I), maximum extent of the cloud shield (M), and the final time at which the size threshold is exceeded (T).

In Figure 60 the composite life-cycle has been normalized to the maximum hourly flash rate and to the time at which it occurred. The flash rates increase and decrease exponentially with time. The exponential relations best fitting the data are also shown where \( N \) is the fraction of discharges in a given hour occurring at a time, \( t \), relative to the magnitude and occurrence of the peak. The logistic model fit to the composite is given in Figure 61.

**Interpretation of Results**

In general, the longer a convective storm takes to reach its maximum intensity, the longer it takes to decay. A storm cell that grows rapidly often decays rapidly. The logistic model provides a good fit to the observed data. In each case the model residual error is only a few percent for individual time series of cloud-to-ground lightning and total lightning activity. The rate constant \( k \) is greatest for the shorter-lived storms (refer to Table 8). The limiting value, \( \alpha \), is a function of storm duration and the environmental factors that serve to sustain strong convection (e.g., atmospheric instability, a moisture source, and a triggering mechanism such as an upper level jet streak, front, storm merger, or outflow boundary). The limiting value \( \alpha \) and residual error will now be considered in greater detail.
Figure 60. Average hourly cloud-to-ground discharge rates are normalized to the average peak flash rate of MCCs and are shown relative to the time of occurrence of the peak. \( N \) is the percentage of the peak rate at a given hour and \( R^2 \) is the correlation coefficient for each of the exponential curves. Open circles denote the time and magnitude of the lightning rates for each of the MCC life-cycle phases F, I, M, T (After Goodman and MacGorman, 1986).
Figure 61. Logistic model regression and residual error for MCC composite life-cycle. **Top:** For each hourly observation period, \( A \) = the observed flash rate; \( P \) = model prediction. **Bottom:** Residual error = \( (A - P) \).
Role of the Environment in Limiting $\alpha$

One frequently used index to describe the instability of the storm environment is the lifted index. The lifted index ($^\circ$C) is found by lifting a positively buoyant saturated parcel of air, having the thermodynamic characteristics of the lowest 100 mb layer of the atmosphere, from its level of free convection (where the buoyancy is first positive) to 500 mb. The temperature excess of the parcel, $T_v$, to that of the environment, $T$, at 500 mb is the lifted index in $^\circ$C. Figure 62 shows the relationship between maximum hourly flash rate during June 1986 within the 8 state area shown in Figure 29 and the 1200 UTC lifted index computed from soundings made at Redstone Arsenal, AL. Total flash rates are seen to increase non-linearly with decreasing atmospheric stability.

A related parameter that describes the environment and offers additional physical insight is the convective available potential energy (CAPE). CAPE is proportional to the square of the maximum parcel updraft speed, $w^2$, and thus represents the increase in kinetic energy of a parcel associated with its vertical acceleration (Weisman and Klemp, 1982). CAPE is also commonly referred to as the available buoyant energy, i.e., the kinetic energy due to buoyancy. Figures 63 and 64 show, respectively, the relationship between 1) total cloud-to-ground lightning and CAPE; and 2) maximum rain rate and maximum parcel energy (CAPE as defined by Zawadzki et al., 1981). Both cloud-to-ground lightning ($r=0.62$) and rainfall ($r=0.79$) are positively correlated with the buoyant energy in the environment. It should be noted that the predicted value of $w$ ignores the effects of precipitation (or mass) loading, vertical pressure gradient perturbations, and mixing which would lower $w$ estimates by as much as 50%. Yet, it is suspected that the total lightning rates would produce an even greater correlation. Such measurements will not be possible for large storm systems until total lightning observations are available from space in the late 1990s with NASAs lightning mapping sensors (see Chapter 7).
Figure 62. Regression plot of maximum hourly flash rate ($Y$) as a function of the lifted index ($X$) computed from the Redstone Arsenal 1200 UTC soundings taken during June 1986. The 95 percent confidence limits are indicated by the dotted lines.

$\ln Y = 5.84 - 0.35X$

$r = -0.82$
Figure 63. Regression plot of total cloud-to-ground flashes observed each day (y) as a function of Convective Available Potential Energy (CAPE) (x) computed from the Redstone Arsenal 1200 UTC soundings taken during June and July 1986. The 95 percent confidence limits are indicated by the dotted lines.

\[ y = 985.16 + 8.15x \]

\[ r = 0.62 \]
Figure 64. Maximum rain rate as a function of maximum parcel energy for 67 storm days near Ottawa, Canada during the summer of 1969-1970. (After Zawadzki et al., 1981).
An Examination of the Residual Errors

Although we claim the logistic model describes the thunderstorm life-cycle reasonably well, there may be other non-linear models that describe the life-cycle just as well, if not better. The logistic model offers a high degree of correlation with the observed data and errors are small, but we note that the residuals are consistently above or below the fitted values for short periods. Such patterns suggest positive autocorrelation in the error terms. Such correlation patterns further suggest improved models can be constructed by adding one or more independent variables to the present model. We can evaluate the error terms quantitatively with the Durbin-Watson test statistic (Neter et al., 1983). The test statistic, $D$, is defined as

$$D = \frac{\sum_{t=2}^{n} (e_t - e_{t-1})^2}{\sum_{t=1}^{n} e_t^2}$$

(5.1)

where $n$ is the number of sample periods, $e_t$ is the residual error from the least squares regression, $\sum e_t^2$ is the residual sum of squares, and the term $(e_t - e_{t-1})$ is the difference in the residuals at two successive times. The usual hypothesis test alternatives are

$$H_0 : \rho = 0$$
$$H_1 : \rho > 0,$$

(5.2)

where $\rho$ is the autocorrelation parameter. The decision rule is as follows:

If $D > d_U$, conclude $H_0$,
If $D < d_L$, conclude $H_1$.

(5.3)
where \( d_U \) and \( d_L \) are the upper and lower bounds obtained by Watson and Durbin, such that a value of \( D \) outside these bounds leads to a decision and any \( d_L < D < d_U \) gives an inconclusive result.

Estimating the two parameters \( \beta \) and \( k \) (\( \alpha \) is proscribed), results in \( d_L=1.34 \) and \( d_U=1.42 \) at a level of significance \( \alpha=0.01 \) (Table A-6 in Neter et al., 1983). For example, the 15 November 1989 storm system has \( n=53 \) and a value of \( D=0.06 << d_L \). Thus, one concludes the error terms are positively correlated. For the MCC composite \( n=18 \), \( d_L=0.90 \), and \( d_U=1.12 \), \( D=0.84 << d_L \), and again conclude \( H_1 \). Small values of \( D \) usually lead to the conclusion that \( \rho>0 \) because successive error terms tend to be of the same magnitude. The residual errors tend to be largest when cells merge or cells redevelop/intensify, both factors which lead to a short term increase in the observed flash rates. The 15 November 1989 case shows a number of such peaks associated with cell mergers. Much of this variation is reduced in the MCC composite by averaging the ten cases. These smaller scale interactions can be included in the basic logistic model by the inclusion of additional (e.g., exponential) terms.

Applications to Extrapolation Forecasting

Thunderstorm Duration

The relationship between the environmental instability and energy can be used as a "first guess" of \( \alpha \). The storm system duration will increase and the system growth rate will decrease as \( \alpha \) increases. Thus, \( \alpha \) can serve as an index to provide some insight into the potential life-cycle of the storms to the forecaster. Since the logistic curve is symmetric about its inflection point, knowledge of when a storm or storm complex has begun the decay phase of its life-cycle (as indicated by decreasing lightning discharge rates) can be used to estimate the valid extrapolation range. One might also predict that
lightning activity will decrease exponentially in the same time it took to reach its peak. This type of information can be used in the context of yes/no (lightning/no lightning) forecasts. The existing extrapolation forecasting procedures use an arbitrary forecast period determined by some ad-hoc method. Using an appropriate storm or system motion vector, one now has a physical basis and empirical evidence upon which to make the determination of longer or shorter extrapolations.

Thunderstorm Existence

Consider the yes/no forecast of lightning activity for the 15 November 1989 storm system using the lightning grids. A total of 1269 cloud-to-ground flashes occurred in association with the line of storms that came through the Tennessee Valley during the 1 h interval 2130-2235 UTC within the analysis sub-region shown in Figure 33. The time series (in 5-min intervals) over this 1 h period is shown in Figure 65.

A full intensity extrapolation forecast would simply move the lightning in one map with the storm motion vector $\Delta t$ (5-min) steps forward. If the storm motion vector placed the system and individual embedded storms in the correct location $t+\Delta t$ step ahead, the existence question would be answered correctly.

Since the total cloud-to-ground lightning for the system is changing (growth and decay), the forecast error will depend on the number of steps ahead to forecast. For example, the system motion vector of the lightning from 2130-2135 UTC to the next 5-min interval is 4.8 km from the southwest at $246^\circ$ (due north is $0^\circ$). An extrapolation of the pattern $t+\Delta t$ steps ahead is simply

$$\hat{x} = x_o + n\Delta t(\Delta x) \tag{5.4}$$
$$\hat{y} = y_o + n\Delta t(\Delta y) \tag{5.5}$$
Figure 65. Cloud-to-ground lightning time series (per 5 min) of the 15 November 1989 storm system during the period 2130-2235 UTC.
where \( n \) is the number of sample periods ahead, and \( \Delta x \) and \( \Delta y \) are the \( x \) and \( y \) components of the storm motion vector. The displacement of each lightning discharge is computed first and then a grid is generated at the desired time. Consider the lightning activity at 2130-2135 UTC which is extrapolated in space 11 periods ahead (Figure 66). Compare this forecast grid against the observed grid at 2225-2230 UTC (Figure 67). A qualitative evaluation of the forecast can be made using the evaluation criteria developed by Donaldson et al. (1975) and successfully used by Browning et al. (1982) and many others for evaluating radar echo extrapolations. The scoring criteria for lightning are given in Table 9. The evaluation criteria of interest are the Threat Score or Critical Success Index (CSI), probability of detection (POD), and false alarm rate (FAR) defined as

\[
\begin{align*}
\text{CSI} &= \frac{A}{(A+B+C)} \quad \text{(5.6)} \\
\text{POD} &= \frac{A}{(A+B)} \quad \text{(5.7)} \\
\text{FAR} &= \frac{C}{(A+C)} \quad \text{(5.8)}
\end{align*}
\]

\( A \) is defined as the number of gridpoints predicted to have a flash density \( \geq 0.02 \) \( \text{km}^{-2} \) at the valid forecast time \( (t+\Delta t) \) and an observed density of \( 0.01 \) \( \text{km}^{-2} \) or greater within a local neighborhood of the gridpoint. A 3x3 point verification grid centered at the gridpoint of interest is used for this purpose. \( B \) is defined as the number of gridpoints where there is no flash density \( \geq 0.02 \) \( \text{km}^{-2} \) predicted at the valid forecast time, yet there is a gridpoint observed in its neighborhood with a density \( \geq 0.02 \) \( \text{km}^{-2} \). Lastly, \( C \) is defined as the number of gridpoints predicted to have a flash density \( \geq 0.02 \) \( \text{km}^{-2} \), yet no lightning is observed in the local neighborhood. The validation focuses on hitting
Figure 66. Eleven-period ahead extrapolation (valid 2225-2230 UTC) of lightning activity observed during the period 2130-2135 UTC.

Figure 67. Observed lightning activity during the period 2225-2230 UTC.
Table 9. Objective Forecast Scoring Criteria

<table>
<thead>
<tr>
<th>Observed</th>
<th>Forecast</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lightning</td>
<td>Lightning</td>
</tr>
<tr>
<td></td>
<td>A</td>
</tr>
<tr>
<td>No Lightning</td>
<td>C</td>
</tr>
</tbody>
</table>
or missing the major storm centers and the validation area is similar to that of Browning et al. (1982) who used 20 km x 20 km grids to validate their rain/no rain extrapolation forecasts.

The 11 period ahead forecast produced a CSI=0.46, POD=0.71, and FAR=0.43. The residual error $e_t=3$ for forecasting the number of seeds or clusters. A total of 8 seeds were identified at 2130-2135 UTC (forecast to still be extant at 2225-2230 UTC since the full system was moved forward in time) and only 5 seeds are observed (refer to Chapter 4). The loss of storms most severely reduces the CSI and FAR since we assume the storm clusters will still be present nearly an hour later. Due to the birth and death process, the CSI and FAR tend to improve with decreasing lag. We note that the lightning cluster identified as I (Figure 45) has long since dissipated (2140 UTC), yet it is the forecast valid at 2225-2230 UTC.

Lastly, consider the 1-period ahead extrapolation of the lightning activity from (2225-2230 UTC) to (2230-2235 UTC) (Figure 68) and compare this grid with the observed lightning grid at (2230-2235 UTC) (Figure 69). We find the CSI=0.67, POD=0.71, the FAR=0.08, and the number of clusters increases from 5 to 6. In this example the FAR is greatly reduced with a lesser improvement in the CSI. Although we generally expect better forecasts at shorter lags, a number of varied weather scenarios need to be studied to understand ways to improve the forecast scores.
Figure 68. One-period ahead extrapolation (valid 2230-2235 UTC) of lightning activity observed during the period 2225-2230 UTC.

Figure 69. Observed lightning activity during the period 2230-2235 UTC.
CHAPTER VI.

SUMMARY AND CONCLUSIONS

The goal of this research was to develop a mechanistic model that can be used for extrapolative forecasting of lightning and thunderstorm activity. The original research reported herein has resulted in the development of a 3-parameter logistic growth model to explain the exponential growth and decay of lightning activity accompanying thunderstorms, at different space and time scales extending over four orders of magnitude. The logistic model depicts a process where the growth rate is proportional to the size reached, the remaining size of the population, and is a function of time. The growth rate constant depends on the size of the storm while the limiting value of the total lightning activity and lifetime is related to the available energy in the environment.

Short-lived storms may not produce sufficient cloud-to-ground lightning discharges to exhibit a continuous growth and decay life-cycle. Instead, the total (intracloud and ground) discharge rate best describes the thunderstorm life-cycle. In two severe weather cases, a short-lived microburst-producing storm and a long-lived tornadic supercell storm, peak total lightning rates were greater than 22 flashes min$^{-1}$ and 40 flashes min$^{-1}$, respectively. Yet cloud-to-ground flashes occurred at rates of only 1-5 min$^{-1}$. The logistic growth model using cloud-to-ground lightning data is more appropriate for describing the evolution of storm complexes, where the longer sampling period is less affected by sudden surges in growth. The short-lived cells produce too few samples (ground discharges) to adequately describe the system and detect significant trends.
The model residual errors, though small, have been shown to exhibit positive serial correlation. This is a result of the merger and reinvigoration of storms giving rise to multiple local maxima in the lightning time series during the growth and decay of the storm system. Suggested techniques for resolving or deconvolving these multiple peaks to gain further insight into their behavior (and implications for nowcasting) might include other non-linear models with additional parameters, stochastic time series models (Box and Jenkins, 1976; Abraham and Ledolter, 1983), and simple first-order difference equation formulations of the logistic model that are used to describe the chaotic behavior of non-linear dynamical systems. The predator-prey relationships in animal and biological populations have been modeled in this way (May, 1974; May 1976).

Many of the physical (non-instrumental) sources of forecast error examined have the same root causes that affect the radar echo extrapolation systems, i.e., incorrect extrapolation vectors, new storms (births), renewed growth (due to cell mergers or a change in the storm environment, storm decay or total dissipation (deaths). The most promising improvements to extrapolation forecasts should come from the use of appropriate non-linear models to understand the birth and death process. It appears appropriate to maintain a history on the life-cycle of the system when two or more storms merge, and not, as current methods do, begin a new description from the time of the merger. This will lead to incomplete life-cycle descriptions and delete the information needed to predict the decay of storm complexes.

A novel constrained optimization approach was developed to remove systematic errors from the cloud-to-ground lightning data base. An optimization algorithm constrained by the observed position of isolated radar echoes produces the best estimate of the lightning locations for subsequent analysis. These lightning locations are next clustered into groups that define single thunderstorm cells and storm complexes. This is accomplished by creating grids of lightning activity in 5–10 min intervals and finding isolated lightning density maxima within the grids. These maxima, typically 2 flashes per
100 km$^2$, are then used to determine the total number of storms and as seed (initial guess) points for a K-Means nearest neighbor clustering algorithm that optimally assigns the flashes to the proper storm. Subsequent groupings of the individual storms permitted the identification of entire storm complexes. Uncorrected location errors will degrade the process of objectively identifying and assigning the individual lightning strikes to their parent thunderstorms. These misclassifications, in turn, could alter the true nature of the lightning life-cycle time series.

Lightning data offers synergistic information to the radar tracking and cell identification algorithms in operational use. The NEXRAD radar storm identification techniques rely on a base scan reflectivity threshold to identify storm cells. It has been demonstrated that lightning strikes can be clustered into discrete cells, thus offering an opportunity for augmenting the decision criteria that are based on the radar algorithms alone.
CHAPTER VII.

RECOMMENDATIONS

During the 1990s, the NEXRAD radars will provide storm coverage over the conterminous United States (Figure 70). Figure 71 shows the radar coverage at a range of 125 km with the track of the 15 November 1989 Huntsville, AL tornado superimposed. At 125 km, for example, the 1° NEXRAD radar beam is about 1 km in diameter and the base-scan height (minimum elevation) is 1 km above the surface of the earth. Note that the distance from the Nashville, TN and Birmingham, AL NEXRAD radars to the tornado is 190 km. At this distance the beam width is 3.4 km. Such a broad sampling volume will degrade the performance of the storm identification and tracking algorithms, and the Doppler wind velocity estimates will be compromised. Yet, the ground-based lightning network will provide overlapping coverage with the radar for improved storm identification.

Finally, a lightning mapping sensor using CCD-focal plane technology will allow total lightning activity to be monitored continuously from space (Christian et al., 1989). This instrument is planned for the next generation of operational weather satellites (called GOES-Next) and will provide coverage from Canada to Brazil (Figure 72). These data will have a spatial resolution of 10 km, sufficient for the identification of individual thunderstorms. The satellite lightning data will not require the sensitive site error corrections required by the cloud-to-ground lightning networks. The clustering and extrapolation forecasting methodologies are equally applicable to the data collected by
Figure 70. NEXRAD network sites.
Figure 71. NEXRAD sites in the Southeastern United States with 125 km range circles. The track of the 15 November 1989 tornado at Huntsville, Alabama is indicated by the cross and solid line.
Figure 72. Proposed 10.5° field of view centered at 2°N latitude and 75° longitude for the lightning mapper sensor on GOES-Next.
the lightning mapper. Sensor fusion with lightning data should provide more capability for diagnosing the present and future weather situation than any one of these sensors can possibly offer by itself.
APPENDIX A: THE FFIX ALGORITHM

Mathematical Formulation of the Best Point Estimate (BPE)

FFIX is based on the result that if there are no bearing errors, then the target vector \( T \) lies in each of the individual bearing planes and, therefore, is normal to the normal vectors, \( N_i \), to each of the bearing planes, i.e., \( T \cdot N_i = 0 \) (Figure 73). The observed bearing planes are defined by their respective bearing and station vectors as measured from the center of the earth. The bearing vector, \( \beta_i \), gives the direction from which the signal from the lightning strike is sensed at station \( S_i \). Due to the random and systematic errors, the observed bearing planes do not generally contain a common target vector, \( T \). The true bearing to the target is represented by \( \rho_i \). Thus, we choose \( T \) such that the dot products, \( T \cdot N_i \), are a minimum. FFIX applies the method of weighted least squares that minimizes the objective function

\[
f(T) = w_i (T \cdot N_i)^2, \quad (A.1)
\]

subject to the constraint \( |T| = 1 \). The choice of the weights, \( w_i \), is based on the principle of maximum likelihood first examined by Stansfield (1947). This is equivalent to minimizing

\[
s_k = \Sigma (\epsilon_i / q_i^*)^2, \quad (A.2)
\]
Figure 73. Spherical triangle relationships used in the FFIX algorithm.
where \( s_k \) is the sum of squared bearing errors for a set of \( k \) bearings, \( \epsilon_i \) is the difference between the observed bearing and the bearing from station \( S_i \) to the BPE and \( \sigma_i^* \) is the range weighted bearing standard deviation for the \( i \)th station. That is,

\[
\sigma_i^* = \sigma_i D_i, \tag{A.3}
\]

where \( \sigma_i \) is the standard deviation of the bearing errors and \( D_i \) is the distance from DF station \( S_i \) to the BPE. For the first iteration when there is no BPE the weight for DF station \( S_i \) is simply

\[
w_i = (1/\sigma_i)^2. \tag{A.4}
\]

Each successive iteration uses the previous BPE for its initial location estimate, which in turn determines each of the \( D_i \)'s. Thus, DF stations having larger bearing errors and at a further distance from the BPE will be given lesser weight. Define

\[ T \text{ as a } 3 \times 1 \text{ column vector,} \]
\[ N \text{ as a } n \times 3 \text{ matrix of normals} \]
\[ W \text{ as a } n \times n \text{ diagonal matrix of weights.} \]

If \( C \) is defined as \( C = N^T W N \), then the objective function (Eqn. A.1) can be expressed as

\[
f(T) = T^T C T. \tag{A.5}
\]

For real data \( C \) will be positive definite, i.e., \( f(T) > 0 \) if \( T \neq 0 \).
Simplifying the Problem

First, $T^TCT = k$ is the equation of an ellipsoid for any $k > 0$. The problem is simplified if we rotate the axes of the ellipsoid to get the equation in standard form (see also Gething, 1978). Note that

$$T = Py$$  \hspace{1cm} (A.6)

is a rotation if

$$P^TP = I,$$  \hspace{1cm} (A.7)

where $P$ is the matrix of rotation and $I$ is the identity matrix. The required rotation gives

$$T^TCT = y^TP^TCy = y^T \Lambda y,$$  \hspace{1cm} (A.8)

where

$$\Lambda = \text{diagonal } (\lambda_1, \lambda_2, \lambda_3).$$  \hspace{1cm} (A.9)

Thus,

$$f(T) = \lambda_1y_1^2 + \lambda_2y_2^2 + \lambda_3y_3^2,$$  \hspace{1cm} (A.10)

$$0 < \lambda_1 \leq \lambda_2 \leq \lambda_3.$$

The minimum value of $f(T)$ is $\lambda_1$ for $y = (1,0,0)$ or

$$T = Py = \begin{pmatrix} P_{11} \\ P_{21} \\ P_{31} \end{pmatrix}.$$  \hspace{1cm} (A.11)

The problem is reduced to determining $\lambda_1$ and $P$. The multipliers $\lambda_i$ must satisfy the equation
\[ |C - \lambda I| = 0. \]  \hspace{1cm} (A.12)

Eqn. (A.12) is the characteristic polynomial of \( C \) and the minimum root \( \lambda_1 \) can be found directly without an iteration procedure. The solution vector \( T \) can now be computed from the matrix equations \( C = \lambda I \) and \( |T| = 1 \).

From the geometry in Figure 73 and the properties of the dot and cross products we get

\[ T \cdot N_i = \cos \alpha_i \]  \hspace{1cm} (A.13)
\[ |T \times S_i| = \sin \rho_i \]  \hspace{1cm} (A.14)

For the spherical triangle defined by the points \( T S_i N_i \) we get the relationship

\[ \cos \alpha_i = \sin \rho_i \sin \epsilon_i \]  \hspace{1cm} (A.15)

Therefore,

\[ (T \cdot N_i)^2 = \cos^2 \alpha_i \]
\[ = |T \times S_i|^2 \sin^2 \epsilon_i \]  \hspace{1cm} (A.16)

From this result it follows that by defining

\[ W_i = (1/\sigma_i^2) |T \times S_i|^2 \]  \hspace{1cm} (A.17)

the function

\[ \Sigma(\sin^2 \epsilon_i/\sigma_i^{-2}) \approx \Sigma(\epsilon_i/\sigma_i^{-1})^2 \]  \hspace{1cm} (A.18)

is minimized. The error introduced by the approximation \( \sin \epsilon_i = \epsilon_i \) is \( < 1\% \) for \( \epsilon_i = 14^\circ \).
The Confidence Region

The confidence ellipse calculation is based on the perturbations (variance) of the bearings to the BPE, i.e., bearing errors and target position errors are treated as differentials (see Stansfield, 1947). Linearization of the functional relationship between the BPE and observed bearings gives an approximate expression for the inverse covariance matrix of T. Therefore, the bearings to the BPE are used, not the observed bearings. Since both T and S_i lie in the bearing plane, the normal vector can be calculated from

\[ N_i = (T \times S_i) / |T \times S_i| . \] (A.19)

As before, \[ C = N^TWN, \] with the exception that now C has the value of zero for an eigenvalue. Since \( T \cdot N_i = 0, \) or in matrix terms, \( NT = 0 \) implying that \( CT = 0, \) the equation

\[ x^T C x = k^2 \] (A.20)

can be rotated by \( x = Py \) to

\[ y^T \Lambda y = \lambda_2 y_2^2 + \lambda_3 y_3^2 = k^2 \] (A.21)

where \( P^TCP = \Lambda = \text{diagonal} \ (0, \lambda_2, \lambda_3). \)

When the site errors have been removed, the random errors are normally distributed and the errors with respect to the lines of position (i.e., the BPE) are independent. Eqn. (A.21) represents an ellipse in the \( y_2y_3 \) plane which is the plane tangent to the earth at T. If \( x \) is the deviation from T, then \( x \) is normally distributed in a plane tangent at T, and \( x^T C x \) has the chi-square distribution with 2 degrees of freedom.

Given the probability \( (x^2 < k^2) = P \) for

\[ k^2 = -2 \ln (1-P), \] (A.22)
the axes of the confidence ellipse for probability level $P$ are given by

$$a = (Rk/\sqrt{\lambda_2}) ; b = (Rk/\sqrt{\lambda_3})$$  \hfill (A.23)

where $a$ and $b$ are the semi-major and semi-minor axes of the ellipse in units of km and $R = 6378$ km is the radius of the earth. The major axis lies along the eigenvector $e_2$ corresponding to $\lambda_2$, and the orthographic projection of $e_2$ onto the plane tangent to the earth at $T$ determines the orientation of the confidence ellipse. If $T = (t_1, t_2, t_3)$ and $e_2 = (a_1, a_2, a_3)$ and $\Theta$ is the bearing of the major axis, then

$$\sin \Theta = (t_1a_2 - t_2a_1)/\sqrt{t_1^2 + t_2^2}.$$  \hfill (A.24)

The ellipse represents a region of minimum area associated with a given probability that the lightning strike occurred on or within the ellipse perimeter. The values of $k^2$ for the 90% ($P=0.9$) and 50% ($P=0.5$) ellipses, respectively, are 4.61 and 1.39. (See Gething, 1978 for more discussion on the confidence ellipse.)
APPENDIX B: LIGHTNING ANALYSIS SOFTWARE

The lightning analysis software consists of programs to decode the hexadecimal data archived on 1600 bpi magnetic tape, convert the data into geophysical quantities, perform data quality control, remove site errors, compute the optimal flash position, and compute error statistics.
SUBROUTINE MAIN0

LLP LIGHTNING DATA DECODER, USES THE DDAH TAPE FORMAT

CONVERTS RAW DATA TO PROPER UNITS, REMOVES SITE ERRORS FOR THE NSFC NETWORK, CALLS NETFIX FOR OPTIMAL LOCATION ESTIMATES WHEN 2 OR MORE DFS DETECT THE FLASH, CALLS NETFIX FOR TWO BEARING CUT SOLN

IMPLICIT DOUBLE PRECISION(A-H,O-Z)
DIMENSION IDATA(128)
CHARACTER GDATE*8
CHARACTER*1 CR, LF, AT
CHARACTER*1 ICHAR1(512), ICHAR2(46), ICHAR3(256)
CHARACTER*2 NFUF(256)
EQUIVALENCE (IBUF, ICHAR1), (ITMF1, ICHAR2), (NFUF, IBUF)
EQUIVALENCE (IBUF, IBUF1)
COMMON/BUFF/BUFFER(128), IBUF2(128)
COMMON/COM1/IBFLAG, NREC, INCH, IN(5), IN2(5)
COMMON/SOLN/SNKA, SNGL, TLAT, TLON, RADIUS, AREA, TSOLN,
*YDAY, YHR, MNR, MNR, NNR, NRR, NEVT, SHA, SNI, ORIG
COMMON/DFSTUF/DFSTUF(128), NS11(128), DFR10(128), NS10(128)

COMMON/SUMRY/KTWOER, NTHRS(4), NBADF, NBDF2, NDUP(4), NOVR(4),
*MDFHT(4), KBIG
COMMON/EFNST/TXX, RSML1, RSML2, RBIG, SBIG, BINSIZ
DATA CR, LF, AT/ZOD, Z25, ZTC/
DATA KLI, KL2, ISTAT, IA4NUM/26, 46, 0, 8000/
CALL DATEG (GDATE)
CALL TIME (ITIME)
IHTIM = INT ( ITIME / 360000 )
AHTIM = FLOAT ( ITIME ) / 360000
ASTIM = ( AHTIM - IHTIM ) * 60.
INTIM = INT ( ASTIM )
STIM = FLOAT ( INTIM + IHTIM ) * 100 + ASTIM
WRITE ( 6,3 ) GDATE, STIM

 THESE VARIABLES ARE USED AS CONSTRAINTS IN ERSTAT FOR SITE ERRORS
IF NOT WANTING SITE ERRORS, SET IDFTST=0, AND ISECOD=2
IDFTST = 0
TMX = 20.
RSML1 = 30.
RSML2 = 50.
RBIG = 200.
SBIG = 5.
BINSIZ = 6.

SITE ERROR CODES: 0 = NONE, 1 = SITERC, 2 = SITER2, 3 = SITERR
ISECOD = 2
IOUNIT = 10

DO 56 I=1,128
DFSTF(I)=0.
NS11(I)=0
DPR10(I)=0.
NS10(I)=0
DPR15(I)=0.
NS15(I)=0.
SQ1(I)=0.
SQ10(I)=0.
SQ15(I)=0.
56 CONTINUE
DO 66 I=1,5
IN2(I)=0
66 IN(I)=0

INITIALIZE COUNTERS
KNT1 = 0
KNT2 = 0
KNT3 = 0
KNT4 = 0
NBADF = 0
NEVT = 0
KBIG = 0
KBIG = 0
KNT5 = 0
DO 68 L=1,4
   QC(L)=0
   NDFP(L)=0
   NTHRS(L)=0
68 CONTINUE
   IBFLAG=0
   LEN=512
   INCHK=0
   IF (ISTAT.EQ.1) CALL ERSUM
   DO 5 IJK=1,MMNUM
      WRITE (6,72)
      72 FORMAT (1X/20X, 'RECORD NO. = ', I6/)
      IF (ISTAT.EQ.1) CALL ERSUM
      CALL GETREC(LEN, ISTAT)
      NSIZ=0
      NEXT=1
      DO 10 I=1,LEN
         IDATA(I)=IBUF(I)
         IPT=2*I
         NEXT=I+1
      10 NSIZ=I-NEXT+I
         IF (ichar1(NEXT).EQ.LF.AND.NSIZ.EQ.0) THEN
            CALL DFDATA (ichar1, IPT, IMES)
            NEXT=Z+1
            IF (IMES.EQ.1) GOTO 10
            IBFLAG=0
            ELSE IF (ichar1(NEXT).EQ.AT.AND.NSIZ.EQ.0) THEN
               CALL PADATA (ichar1, IPT)
               NEXT=I+1
               GO TO 10
         ENDIF
      11 SORTING AND SAVING DATA FROM NEXT BUFFER TO COMPLETE A PARTIAL STRING
         IF (IBFLAG.NE.0) THEN
            IBFLAG=IBFLAG+1
            ISAVPT=IBFLAG
            ICHAR2(IBFLAG)=ICHAR1(I)
         ENDIF
         IF (ichar1(NEXT).EQ.LF.AND.ISIZ.NE.1.AND.ISAVPT.EQ.0) THEN
            CALL DFDATA (ICHAR1, IPT, IMES)
            NEXT=I+1
            IF (IMES.EQ.1) GOTO 10
            IBFLAG=0
            ELSE IF (IBFLAG.EQ.0) THEN
            ENDIF
50 CONTINUE
900 FORMAT ('//1X,' 'NO LINE FEED AT END OF RECORD')
      GOTO 5
   ENDIF
   IPTM(K)=ICHAR1(J)
   IF (ICTM(K).EQ.0) THEN
      WRITE(6,500)
         500 FORMAT (1X, 'NEXT, LEN, IBFLAG, K, IPTM(K), IPTM(K-1)
      IBFLAG=LEN-NEXT+1
         ISAVPT=2*IBFLAG
         ISAVPT=IBFLAG
      ENDIF
      CONTINUE
      SAVE THE LAST OF OR PA PARTIAL DATA BLOCK FROM BUFFER I TO COMPLETE THE CHARACTER STRING WHICH CONTINUES IN THE NEXT BUFFER
         IF (ICHAR1(LEN).NE.LF) THEN
            K=1
            DO 50 J=NEXT,LEN
               IF (ICTM(K).NE.0) THEN
                  WRITE(6,900)
                  900 FORMAT ('//1X,' 'NO LINE FEED AT END OF RECORD')
               GOTO 5
            ENDIF
            IPTM(K)=ICHAR1(J)
            IF (ICTM(K).EQ.0) THEN
               WRITE(6,500)
                  500 FORMAT (1X, 'NEXT, LEN, IBFLAG, K, IPTM(K), IPTM(K-1)
               IBFLAG=LEN-NEXT+1
                  ISAVPT=2*IBFLAG
                  ISAVPT=IBFLAG
               ENDIF
            ELSE IF (K.EQ.1) THEN
            CONTINUE
      ENDIF
      CONTINUE
SUBROUTINE GETREC(LEN, ISTAT)

    IMPLICIT DOUBLE PRECISION(A-H,O-Z)
    EQUIVALENCE (IBOF, IBUF1)
COMMON/BUFS/IBUF1(128), IBUF2(128)
CHARACTER*20 DFFIL
INTEGER BFLN
BFLN=128
NREC=0
IBFLAG=0
READ(9,2,END=400) (IBUFI(IO), IO=1, 128)
2 FORMAT (128A4)
NEXT=1
LAST=LEN
WRITE(6,300) (IBUFI(IO), IO=1, 128)
300 FORMAT (1X,32A4)
GOTO 500
400 ISTAT=1
500 RETURN
END

SUBROUTINE DFDATA(LINE, IPT)

    IMPLICIT DOUBLE PRECISION(A-H,O-Z)
LINE CONTAINS RAW DF DATA STRING W/ CRLF
TMP CONTAINS SUBSTRINGS FOR SWAPPING MSB/LSB
CHARACTER*1 LINE (512)
CHARACTER TMP*4
CHARACTER OUTBUF(24)
CHARACTER*1 IAPOL
COMMON/BUFS/IBUF1(128), IBUF2(128)
COMMON/COM1/IBFLAG, NREC, INCHX, IN(5), IN2(5)
COMMON/DFSTUF/DFEI (128), NSI (128), DFR10 (128), NSI0 (128),
*DFR15 (128), NSI5 (128), SQI0 (128), SQI12 (128), SQI15 (128)
COMMON/SUFS/IBUFI (128), IBUF2 (128)
COMMON/COMI/I, BFLAG, NREC, IN, IN(5), IN2(5)
COMMON/SOLE/SMKA, SSSQL, TSLAT, TIJON, AREA, TSOLN,
*IYDSN, IYR, NON, NDAY, NRS, NBR, IRES, ORIER
COMMON/ECONST/TMX, RSML2, RBLG, SBLG, SBSI2
INTEGER IODF(24), DFITST
INTEGER HXSO
INTEGER*4 JD, JDAY, IYEAR
EQUIVALENCE (IBUF1, IBUF2)
DIMENSION IRS(4), IHH(4), IMS(4), IMS(4), ATVMP(4), IYYVDD(4)
DIMENSION POL(4), CBPR2(4), AZM(4), AMP(4), TM(4), CBPR1(4), AZM(4)
DIMENSION IRS(4), TAMP(4), B(4), IA(2), AB(2)
DIMENSION DIAT(4), DLOM1(4)
DATA DIAT/34.649167, 35.399167, 35.83750, 34.71667/
DATA DLOM1/66.669167, 66.076944, 87.443889, 87.881667/
DATA ISM,NMDF/26, 4/
DATA HXSO/280/
DTR=.01745329
EMISS1=-999.99
EBADI=-888.88
ERAD1=-371.
ISTART=IPT-IST+1
IEND=IPT
WRITE(6,200) ISTART, IPT, (LINK(IS), IS=ISTART, IEND)
200 FORMAT (1X,'ISTRART, IPT', 2I4, 2X, 26A4)
J=1
I=1

ORIGINAL PAGE IS OF POOR QUALITY
.. WRITE DF STRING INTO OUTBUF FOR INTERNAL I/O, SWAPPING MSB, LSB

```
OUTBUF(1)=LINE(J)
OUTBUF(2)=LINE(J+1)
OUTBUF(3)=LINE(J+4)
OUTBUF(4)=LINE(J+5)
OUTBUF(5)=LINE(J+2)
OUTBUF(6)=LINE(J+3)
OUTBUF(7)=LINE(J+8)
OUTBUF(8)=LINE(J+9)
OUTBUF(9)=LINE(J+6)
OUTBUF(10)=LINE(J+7)
OUTBUF(11)=LINE(J+12)
OUTBUF(12)=LINE(J+13)
OUTBUF(13)=LINE(J+10)
OUTBUF(14)=LINE(J+11)
OUTBUF(15)=LINE(J+14)
OUTBUF(16)=LINE(J+15)
OUTBUF(17)=LINE(J+18)
OUTBUF(18)=LINE(J+19)
OUTBUF(19)=LINE(J+16)
OUTBUF(20)=LINE(J+17)
OUTBUF(21)=LINE(J+22)
OUTBUF(22)=LINE(J+21)
OUTBUF(23)=LINE(J+20)
OUTBUF(24)=LINE(J+21)
```

.. CHECK FOR ILLEGAL CHARACTERS IN DF DATA STRING

```
IMES=0
DO 30 KC=1,24
  IF (OUTBUF(KC) .LT. 'A' OR. OUTBUF(KC) .GT. '9') THEN
    WRITE (6,455) OUTBUF
    455 FORMAT (5X,'BAD CHARACTER IN DF STRING = ',24AI)
  WRITE(6,230) (IBUFI(IO),IO=1,128)
30 CONTINUE
```

C230 FORMAT (1X, 32A4)

```
INES=1
RETURN
```

C.. GET DF IDENTITY, IDF=IO1+1

```
IDF=IO1+1
IF (IDF.LT.1.OR.IDF.GT.MMODF) THEN
  IQCKEY=1
  WRITE (6,553) OUTBUF
  553 FORMAT (5X,'ILLEGAL DF ID NUMBER IN STRING = ',24A1)
  RETURN
ENDIF
```

C.. COUNT SUM OF EACH DF DETECTIONS

```
IF (IDF.LE.MMODF) NDFHT(IDF)=NDFHT(IDF)+1
```

C.. GET FLASH POLARITY HEX80 ADDED TO BYTE 12 ON TAPE IF POSITIVE

```
POL(IDF)=1.0
IF (IO7.GE.HX80) POL(IDF)=1.0
```

C.. GET AMPLITUDE, 1500 IS OVERANGE VALUE (DF SIGNAL SATURATION)

```
AMP(IDF)=POL(IDF)* (MOD(IO7,HX80)*256. +FLOAT( IO8))/10.
IF (ABS(AMP(IDF)).GE.1500.) THEN
  NOVR(IDF)=NOVR(IDF)+1
```

C.. GET MINUTE OF THE DAY AND CONVERT TO HHMM FORMAT

```
IH(N)H(IDF)=INT(IO2/60)
IMM(IDF)=IO2-60*IHH(IDF)
```

C GET MILLISECONDS OF THE MINUTE

ISS(IDF) = INT(IO3/1000)
IMS(IDF) = IO3-1000*ISS(IDF)

C GET TIME OF DAY HOURS Format

TM(IDF) = FLOAT(((IHH(IDF)*100)+IMM(IDF))*100+ISS(IDF))
** FLOAT(IMS(IDF))/100.000

C GET THE YEAR

JDAY=JDAY+1
NYEAR=JDAY/365
LNPYEAR=NYEAR/4
JD=LPYEAR+365*NYEAR
IF(JDAY-JD) 350,100,400
300 JDAY=365
NYEAR=LPYEAR+4
IF(NYEAR.EQ.0) JDAY=366
NYEAR=NYEAR-1
350 NYEAR=NYEAR-1
LPYEAR=NYEAR/4
JD=LPYEAR+365*NYEAR
JDAY=JDAY-JD
IF(JDAY.EQ.0) GOTO 300
400 JDAY=JDAY-JD
450 JDAY=JDAY-1

C GET YYDDD Format FROM NYEAR AND JDAY

IYYDDD(IDF) = NYEAR + 1000 * JDAY

C GET THE EQUIVALENT MONTH, DAY, AND YEAR

C GET NUMBER RETURN STrokes

IRS(IDF) = IO5

C GET UNCORRECTED DF AZIMUTH TO FLASH

AZN(IDF) = (FLOAT(IO6)/65536.)*360.

C CHECK DF DATA FOR UNACCEPTABLE VALUES

IQCKEY=0
IF(IDF.LT.1.OR.IDF.GT.MMODF) THEN
IQCKEY=-1
WRITE(6,55) IQCKEY = " BAD CHARACTER IN DF STRING = '",24A1)
RETURN
ENDIF
IF(IHH(IDF).LT.0.OR.IHH(IDF).GT.24) IQCKEY=-1
IF(IMM(IDF).LT.0.OR.IMM(IDF).GT.60) IQCKEY=-1
IF(ISS(IDF).LT.0.OR.ISS(IDF).GT.60) IQCKEY=-1
IF(IDYDDD(IDF).LT.83000.OR.IYDDD(IDF).GT.90000) IQCKEY=-1
IF(IYSD(IDF).LT.0.OR.IYSD(IDF).GT.IO) IQCKEY=-1
IF(AZM(IDF).LT.0.OR.AZM(IDF).GT.360.) IQCKEY=-1
C
C IF DATA UNACCEPTABLE DO NOT CONTINUE WITH STRING PROCESSING

C IF(IQCKEY.EQ.-1) THEN
NQC(IDF) = NQC(IDF) + 1
WRITE(6,196) IQCKEY, IDF, IHH(IDF), IMM(IDF), ISS(IDF), IMS(IDF),
* TM(IDF), IYYDDD(IDF), IRS(IDF), AZM(IDF), POL(IDF), AMP(IDF)
196 FORMAT(1X,'$196 DF DATA ',I2,' D= ',I2,2X,3I2,13,4X,F10.3,
* , F6.1,F8.2)
WRITE(6,230) (IBUFI(IO), IO=1,128)
230 FORMAT(1X,32A4)
C
STOP
RETURN
ENDIF

ORIGINAL PAGE IS OF POOR QUALITY
C
GET A CORRECTED BEARING ANGLE FOR THIS DF
C
IF(AZM(IDF).EQ.0..AND.IDFSTT.EQ.IDF) THEN
WRITE(6,188) IDF,AZM(IDF),TM(IDF),AMP(IDF)
188 FORMAT(1X,'ZERO DEGR. ',I2,F12.3)
CBRG1(IDF)=0.0
GOTO 285
ENDIF
C
CBRG1(IDF)=AZM(IDF)+SITERC(IDF,AZM(IDF))
C
CBRG1(IDF)=AZM(IDF)+SITERR(IDF,AZM(IDF))
C
CBRG1(IDF)=AZM(IDF)
C
CHECK THAT CORRECTED ANGLE NON NEGATIVE, ADD 360 FOR PROPER ONE
C
IF(CBRG1(IDF).LT.0.) THEN
WRITE(6,211) CBRG1(IDF),IDF,AZM(IDF)
211 FORMAT(1X,'ANGLE LESS THAN 0. ',F12.3,I4,F12.3/)
CBRG1(IDF)=CBRG1(IDF)+360.
ENDIF
C
IF(TM(IDF).GT.225300..AND.TM(IDF).LT.225500.) THEN
WRITE(6,199) TM(IDF),IDF,AZM(IDF),CBRG1(IDF)
199 FORMAT(1X,' DF, BEARINGS ',F12.3,2X,2F10.4)
C
ENDIF
GOTO 285
271 CONTINUE
299 FORMAT(1X,'NO CORRECTION YET AVAILABLE FOR DF1')
C
CHECK FOR NUMBER OF DFS IN TIME COINCIDENCE
C
CALL TIMECO(IDF,TM(IDF),ICNT1,ICC,ICYYDLD)
C
FIND NUMBER OF DFS IN COINCIDENCE, IF #DFS=2 CALL NETFIX
C
TO GET A GOOD SOLN, THEN GET BEST CUT FROM NETFIX BASED
C
ON MIN. SEMIMAJOR AXIS OF THE ERROR ELLIPSE.
C
IF(IN(5).NE.I) GOTO 275
IERR=0
C
IF ICNT1 IS GREATER THAN 4, ONE OR MORE DFS SAW MORE THAN ONE
C
EVENT WITHIN THE TIME COINCIDENCE WINDOW
C
IF(ICNT1.GT.4) THEN
WRITE(6,707) ICNT1,TM(IDF)
707 FORMAT(1X,'DF SAW MORE THAN ONE FLASH WITHIN TIME WINDOW..',
* I4,F12.3)
ICNT1=4
ENDIF
NCNT=ICNT1
ISUM=0
C
DO 711 IC=1,NCNT
DO 711 IK=1,MNODF
IF(IN(IK).EQ.1) THEN
IN2(IK)=IK
ISUM=ISUM+1
AZM(IK)=AZM(IDF)
CBRG2(IK)=ATMP(IK)
ENDIF
711 CONTINUE
C
CHECK TO SEE IF NUMBER OF DFS IN SOLN EQUALS COUNTER NCNT
C
IF(ISUM.NE.NCNT) THEN
WRITE(6,7277) TM(IDF),ISUM,NCNT,IN,IN2
7277 FORMAT(1X,'TIME CORR ERROR AT ',F12.3,12(I2,1X))
GOTO 901
ENDIF
C
IF(TM(IDF).GT.185500..AND.NDAY.EQ.14) STOP
C
USE THIS FOR COMPUTING DF SITE ERRORS
C
IF(IDFTST.EQ.0) GOTO 75
IF(IN2(IDFTST)).EQ.0) GOTO 901
C
WRITE(6,2112) IN,IN2,CBRG2
2112 FORMAT(1X,'IN,IN2,CBRG2 ',10I2,4F10.4)
DO 717 IK=1,MNODF
IF(IN2(IK).EQ.IDFTST) THEN
IN2(IK)=0
ENDIF
717 CONTINUE
CONTINUE
IF(NCNT.EQ.2.AND.IN2(1).EQ.IDFTST) GOTO 901
IF(NCNT.EQ.2.AND.IN2(2).EQ.IDFTST) GOTO 901
DO 712 KI=1,MNODF-1
  IF(IN2(KI).EQ.0.AND.IN2(KI+1).NE.0) THEN
    IN2(KI)=IN2(KI+1)
    AZM2(KI)=AZM2(KI+1)
    CBRG2(KI)=CBRG2(KI+1)
    IN2(KI+1)=0
  IF(TM(IDF).GT.185200..AND.TM(IDF).LT.185500.) THEN
    WRITE(6,76) KI,NCNT,IN,IN2,AZM2,CBRG2
    FORMAT(IX,'DEBUG KI',12(3,1X),8(F7.3,1X))
    ENDIF
  ENDIF
CONTINUE
C MAKE A SECOND PASS THROUGH ABOVE SORTING LOOP IF DPS NOT
C LISTED CONSECUTIVELY
C IF(IN2(1).EQ.0.OR.IN2(2).EQ.0) GOTO 75
C IF(NEVENT.GE.80) STOP
C IF(IN2(1).EQ.0.OR.IN2(2).EQ.0) GOTO 75
C IF(IN2(1).EQ.0.OR.IN2(2).EQ.0) GOTO 75
C CALL FFIX(IN2,CBRG2,TM)
C IF(IN2(1).EQ.0.OR.IN2(2).EQ.0) GOTO 75
C CALL FFIX(IN2,CBRG2,TM)
C WRITE(6,848) TSOLN,IERR,TLAT,TLON,SMA,SMI,ORIEM, IN2eCBRG2,TM
C IF(IERR.EQ.-1) GOTO 901
C IF(IERR.NE.Z.OR.ICNTZ.EQ.2) THEN
C WRITE(6,841) TM(4),IERR
C NBADF=NBADF+1
C IF(IERR.EQ.0.OR.ICNT1.EQ.2) THEN
C IF(ICNT1.GE.2) THEN
C THIS FORCES BEST 2 DF FIX (BEST = MIN(SMA))
C IBDCHK=0
C TSTOR3=999.
C SMA=999.
C ICNT1=2
C NCNT=ICNT1
C DO 745 I=1,MNODF-1
C IF(I.GE.J) GOTO 745
C IA(1)=IN2(I)
C IA(2)=IN2(J)
C IF(IA(1).EQ.O.OR.IA(2).EQ.0) GOTO 7S4
C ABM1=AB(K)+5.
C ABP1=AB(K)-5.
C RP=RPI*ERADI
C AZP=API/DTR
C IF(AZP.GE.ABP1.OR.AZP.LT.ABM1) THEN
C WRITE(6,7979) TSOLN,TLAT,TLON,RP,
C FORMAT(IX,'DVGA',4X, SFI2.3,2X,212,3F8.2)
C IF(MEVENT.GE.80) STOP
C KDVG=KDVG+1
C GOTO 754
C CONTINUE
]

ORIGINAL PAGE IS OF POOR QUALITY
C
C TSTOR1=TLAT
TSTOR2=TLON
TSTOR3=SMA
TSTOR4=SMI
TSTOR5=ORIEN
TSTOR6=AREA
TSTOR7=RADIUS
IDT1=IA(1)
IDT2=IA(2)
TBR1=AB(1)
TBR2=AB(2)
IBDCHK=1
C
CONTINUE
C
754 CONTINUE
745 CONTINUE
C
TSTOR1=TLAT
TSTOR2=TLON
TSTOR3=SMA
TSTOR4=SMI
TSTOR5=ORIEN
TSTOR6=AREA
RADIUS=TSTOR7
IN2(1)=IDT1
IN2(2)=IDT2
CBRG2(1)=TBR1
CBRG2(2)=TBR2
DO 775 KI=3,4
IN2(KP)=0
CBRG2(KP)=0
775 CONTINUE
C
CONTINUE
C
841 FORMAT(1X,'... NO FIX FROM FFIX AT',F12.3,',' ERROR=','I3)
C
IF(ICNT1.EQ.2.AND. IN(1).EQ.1) GOTO 890
NCNT_2
C
IF(IERR.EQ.0) GOTO 811
IF(IERR.EQ.-1) GOTO 901
IF(IERR.NE.1) THEN
WRITE(6,841) TM(4),IERR
CALL NETFIX(IN2(1),CBRG2(1),IN2(2),CBRG2(2),TLAT,TLON)
TLON = 1. * TLON
C
C WRITE(6,841) TM(4),IERR
C CALL NETFIX(IN2(1),CBRG2(1),IN2(2),CBRG2(2),TLAT,TLON)
TLON = 1. * TLON
C
754 CONTINUE
745 CONTINUE
C
WRITE(6,870) TSOLN, TLAT, TLON, SMA, IN2, CBRG2
FORMAT(1X,'IN 870 CHK SOL',4(1X,12.3,2X,5F12.3,2X,5F8.2))
IF(IBDCHK.NE.1) THEN
NBADF=NBADF+1
C
GOTO 1801
C
GOTO 901
ENDIF
ENDIF
C
WRITE(6,870) TSOLN, TLAT, TLON, SMA, IN2, CBRG2
FORMAT(1X,'... NO FIX FROM FFIX AT',F12.3,',' ERROR=','I3)
C
IF(IN(1).EQ.0) GOTO 901
XTL=TLAT*DTR
XTL0=TLON*DTR
ZF(IDFTST.EQ.0) GOTO 1028
GOTO 1015
C
C
C IF(ICNT1.EQ.2.AND. IN(1).EQ.1) GOTO 890
NCNT_2
C
IF(IERR.EQ.0) GOTO 811
IF(IERR.EQ.-1) GOTO 901
IF(IERR.NE.1) THEN
WRITE(6,1101) NCNT, ICNT1, NBR
FORMAT(1X,'HERE WE ARE..........','I3)
C
CALL FFIX(IN2, CBRG2, TM)
C
IF(IERR.EQ.0) GOTO 811
IF(IERR.EQ.-1) GOTO 901
IF(IERR.NE.1) THEN
WRITE(6,1101) NCNT, ICNT1, NBR
FORMAT(1X,'HERE WE ARE..........','I3)
C
CALL FFIX(IN2, CBRG2, TM)
C
1028 CONTINUE
ENDIF
715 CONTINUE
C
510 FORMAT(1X,'DFDATA..ICNT1,NCNT,IN,IN2,CBRG2,ATMP',12I4/1X,8F12.4)
C
IF(ICNT1.EQ.3.AND.IN(1).EQ.1.OR.ICNT1.EQ.2.AND.IN(1).EQ.0) THEN
IF(ICNT1.EQ.2) THEN
C
WRITE(6,510) ICNT1, NCNT, IN, IN2, CBRG2, ATMP
DO 715 KI=1,4
IF(ATMP(KI).EQ.0) THEN
GOTO 715
ENDIF
IF(IN(KI).EQ.1.AND.IN2(KI).NE.0) THEN
IN2(KI)=KI
CBRG2(KI)=ATMP(KI)
ENDIF
IF(IN(KI).EQ.1.AND.IN2(KI).NE.0) THEN
IN2(KI)=KI
CBRG2(KI)=ATMP(KI)
ENDIF
715 CONTINUE
C
ENDIF
510 FORMAT(1X,'DFDATA..ICNT1,NCNT,IN,IN2,CBRG2,ATMP',12I4/1X,8F12.4)
C
IF(IERR.EQ.0) GOTO 811
IF(IERR.EQ.-1) GOTO 901
IF(IERR.EQ.0) THEN
C
WRITE(6,1101) NCNT, ICNT1, NBR
FORMAT(1X,'HERE WE ARE..........','I3)
C
CALL FFIX(IN2, CBRG2, TM)
C
IF(IERR.EQ.0) GOTO 811
IF(IERR.NE.1) THEN
C
WRITE(6,1101) NCNT, ICNT1, NBR
FORMAT(1X,'HERE WE ARE..........','I3)
C
CALL FFIX(IN2, CBRG2, TM)
C
1101 CONTINUE
C
C WRITE(6,841) TM(4),IERR
C CALL NETFIX(IN2(1),CBRG2(1),IN2(2),CBRG2(2),TLAT,TLON)
TLON = 1. * TLON
C
C WRITE(6,841) TM(4),IERR
C CALL NETFIX(IN2(1),CBRG2(1),IN2(2),CBRG2(2),TLAT,TLON)
TLON = 1. * TLON
C
WRITE(6,2099) TSOLN,IN2(1),CBRG2(1),IN2(2),CBRG2(2)

2099 FORMAT(IX,'2 DF FIX',F12.3,2X,I2,F12.3,2X,I2,3F12.3)

ENDIF
C IF(IN(1).EQ.0) GOTO 901
1801 XTL=TLAT*DTR
XTL0=TLON*DTR
IF(IDFTST.GT.0) GOTO 1015
ENDIF
IF(IDFTST.EQ.0) GOTO 1028
C
CONVERT SOLN TO RADIANS FOR AZRN TO GET DF1 SITE ERRORS
C IF DF1 NOT IN SOLN THEN SKIP THIS
C
XTL=TLAT*DTR
XTL0=TLON*DTR*(-1.0)
1015 XSL=-DLAT(IDFTST)*DTR
XSLO=-DLONG(IDFTST)*DTR
CALL AZRN(XSL, XSLO, XTL, XTLO, AZ1, R1)
C
C WRITE (6,912) AZ1,R1
912 FORMAT(IX,'AZRN SENDS DF1 DATA AZI,RI ',2FI0.6)

C.. CONVERT FROM RADIANS BACK TO KM, EARTH RADIUS=6370 KM
C
C WRITE(6,3111) TA21,TA22,DIFF,DSL,SM
3111 FORMAT(IX,'DIFF BEFORE TAZ CHECK ',F12.3,2X,2F12.6)

C IF (ABS (TA22-TA21).GT.100.) THEN
C IF (TA22.GT.TA21) TA21=TA21+360.
C IF (TA22.LT.TA21) TA22=TA22+360.
ENDIF
DIF=TA22-TA21
C

C WRITE(6,3112) TA21,TA22,DIFF,DSL,SM
3112 FORMAT(IX,'AFTER TAZ CHECK ',7FI0.4)

C IF (ABS (TA22-TA21).GT.100.) THEN
C IF (TA22.GT.TA21) TA21=TA21+360.
C IF (TA22.LT.TA21) TA22=TA22+360.
ENDIF
DIF=TA22-TA21
C
C WRITE(6,680) SM,TA21,TA22,DIFF,DSL
680 FORMAT(IX,'DISTANT OR BAD SOLN..REJECT IT AT ',F12.3,2X,2F12.6)

C.. CHECK FOR A BAD SOLN USING 2 BEARINGS AND GMNG
C THE SAME SOLN AS PREVIOUS SOLN. THIS IS DONE TO NARROW CUT-ANGLE
C NBDF2 IS NO. OF WRONG SOLNS FROM FFIX, CUT SOLN BETTER
C
635 IF (ABS (DIF).GT.TMX) THEN
NBDF2=NBDF2+1
GOTO 901
ENDIF
C
C.. SORT DF ANGLE ERROR AS A FCN OF ANGLE IN ERSTAT
C
C CALL ERSTAT(ADEG,RKM,DIFDF)
C CALL ERSTAT(TA21,RKM,DIFDF,CRG)
C
C.. ESTIMATE THE PEAK CURRENT FOR THIS FLASH WITH FCN FKA
C
1028 SKA=FKA(TMP)
SKA=SKA
C
C.. FIND THE LARGEST NUMBER OF STROKES IN A FLASH
C
NRS=0
DO 1411 I-I,NCNT
1411 IF (ITRS(IN2(I)) .GT. NRS) NRS=ITRS(IN2(I))
C NCNT=0
C GET THE EQUIVALENT MONTH, DAY, AND YEAR FOR A FLASH
CALL YDMY(YDNR,NDAY,MON,IYR)
C SET NO RADIUS FROM SOLN TO EMISS1=-999.99 IN SOLN SET
IF (RADIUS .EQ. 0. ) THEN
  AREA=EMISS1
  SMA=EMISS1
  SMI=EMISS1
  RADIUS=EMISS1
  ORIEN=EMISS1
ENDIF
IF (RADIUS .GE. 200.0) THEN
  AREA=EMISS1
  SMA=EMISS1
  SMI=EMISS1
  RADIUS=EMISS1
  ORIEN=EMISS1
ENDIF
C COUNT NUMBER OF 2 DF AND 3 DF SOLNS
IF (ICNT1.EQ. 2) ICNT2=ICNT1+1
IF (ICNT1.EQ. 3) ICNT3=ICNT1+1
IF (ICNT1.EQ. 4) ICNT4=ICNT1+1
IF (ICNT1.EQ. 5) ICNT5=ICNT1+1
C CHECK FOR TIME >24 HOURS, DUE TO DAY CHANGE, NEED TO SUBTRACT 24
IF (TSOLN.GE. 240000.) TSOLN=TSOLN-240000.
C CORRECT FOR LEAP YEAR, NDAY=NDAY+1 (ILLF DAY OF CENTURY COUNT IS
C INCORRECTLY COMPUTING THIS.
C IF (IYR.EQ. 88) NDAY=NDAY+366
C WRITE SOLN TO DISK FILE ON EADS (UNIT i0)
NEVT=NEVT+1
WRITE(10,3015) NEVT,TSOLN,IYR,MON,NDAY,TLAT,TLON,SNKA,SSGNL,NRS
3015 FORMAT(I6,2X,F12.3,1X,312,2F12.6,2X,F10.2,I2)
WRITE(10,4015) IN2,CBRG2,SMA,SMI,ORIEN,RADIUS,AREA
4015 FORMAT(512,4(F6.2,1X),5(FI0.2,1X))
IF (NDAY.EQ.14.AND.TSOLN.GT.201818.) THEN
  WRITE(6,915) TSOLN,IYR,MON,NDAY,AZDEG,RK,ATMP(1),DIFDEG,
  Tlat,TLON,NRS,SSGNL,SNKA,DIAM,SMA,AREA
  FORMAT(IX,' SOLN AT',F12.3,1X,312,2X,4F12.6,'- DF 1 ERROR'/
  2F12.6,4F6.2)
  WRITE(6,918) IN2,CBRG2
  C918 FORMAT(1X,'IN2,CBRG2-',512,4(F6.2,1X))
  WRITE(6,2017) LSTHH(IDF)
  WRITE(6,2015) FORMAT(2OX,'EVT',4X,'TIME',6X,'YMMDD',6X,'LAT',10X,'LON',
  8X,'KAMP',5X,'VOLTS',2X,'NS',4X,'RADIUS',4X,'SHAJ',4X,
  'SMIN',3X,'AREA')
  WRITE(6,2015) IN2,CBRG2,SMA,SMI,ORIEN,RADIUS,AREA
2015 FORMAT(/1X,'IST SOLN (TAPE)','=14,2X,F12.3,1X,312,2F12.6,2X,
  2F10.2,12,4F10.2)
ENDIF
IF (NEVT.EQ.1) THEN
  LSTHH=INH(IDF)
  WRITE(6,2017)
2017 FORMAT(20X,'EVT',1X,'TIME',5X,'YMMDD',6X,'LAT',10X,'LON',
  8X,'KAMP',5X,'VOLTS',2X,'NS',4X,'RADIUS',4X,'SHAJ',4X,
  'SMIN',3X,'AREA')
  WRITE(6,2017) NEVT,TSOLN,IYR,MON,NDAY,TLAT,TLON,SNKA,SSGNL,
  NRS,RADIUS,SHAJ,SMI,AREA
ENDIF
C WRITE HOURLY SOLN SUMMARY
IF (NEVT.EQ.1) THEN
  LSTHH=INH(IDF)
  WRITE(6,2017)
2017 FORMAT(20X,'EVT',1X,'TIME',5X,'YMMDD',6X,'LAT',10X,'LON',
  8X,'KAMP',5X,'VOLTS',2X,'NS',4X,'RADIUS',4X,'SHAJ',4X,
  'SMIN',3X,'AREA')
  WRITE(6,2017) NEVT,TSOLN,IYR,MON,NDAY,TLAT,TLON,SNKA,SSGNL,
  NRS,RADIUS,SHAJ,SMI,AREA
ENDIF
C WRITE SOLN TO DISK FILE ON EADS (UNIT i0)
NEVT=NEVT+1
WRITE(10,3015) NEVT,TSOLN,IYR,MON,NDAY,TLAT,TLON,SNKA,SSGNL,NRS
3015 FORMAT(I6,2X,F12.3,1X,312,2F12.6,2F10.2,2X,I2)
WRITE(10,4015) IN2,CBRG2,SMA,SMI,ORIEN,RADIUS,AREA
4015 FORMAT(512,4(F6.2,1X),5(FI0.2,1X))
IF (TSOLN.GE. 201818.) STOP
ENDIF
C WRITE HOURLY SOLN SUMMARY
IF (NEVT.EQ.1) THEN
  LSTHH=INH(IDF)
  WRITE(6,2017)
LSTH=fIH(IDF)
IF (LSTH.LT.0) LSTH=0
WRITE(6,3020) LSTH, INTH, YR, MON, NDY, NEVT
FORMAT(1X, 'HOURLY SUMMARY BEGINNING AT ', I2, ' UT IS ', I6, 4X,
LSTH=fIH(IDF)
ENDIF
C
C C890 WRITE (6,899)
890 CONTINUE
899 FORMAT(1X, 'SOLN REQUIRES DP1 FOR 2 DF FIX')
C
C CLEAR POINTERS
C
901 NCNT=0
ICNT=0
LSTDF-IDF
PREVB=CBRG1(IDF)
PREVA=AMP(IDF)
PREVR=IRS(IDF)
C
IF (LSTDF.NE.1) AIM(1)=0.
AZIM=0.
RKM=0.
DIFIMEG=0.
PLAT=TLAT
TLON=0.
AREA=0.
SMA=0.
SMI=0.
ORIEN=0.
DO 340 INF=1,5
IN2(INF)=0.
340 IN(INF)=0.
XTL=-1.
XLO=-1.
TCo=0
DO 345 INF=1,4
B(INF)=0.
AMP(INF)=0.
IRS(INF)=0.
CBRG2(INF)=0.
345 CBRG1(INF)=0.
CBRG1(IDF)=PREVB
AMP(IDF)=PREVA
IRS(IDF)=PREVR
275 DO 2275 INF=1,4
ATMP(INF)=CBRG1(INF)
AMP(INF)=AMP(INF)
IRS(INF)=IRS(INF)
2275 CONTINUE
RETURN
END
SUBROUTINE TIMECO ( IDN, TMN, ICNT, ICC, IYYDDD)

IMPLICIT DOUBLE PRECISION(A-H, O-2)
COMMON/COM/IT1, YPR, IN, IMH, IN, IN2(5)
COMMON/SOL/SNMA, SSANL, TLAT, TLOI, RADIUS, AREA, TSOLOI
*YPR, YMV, MDY, MBR, MDF2, NDUP
COMMON/SUN/KNTH, KNTH, NTH, TMTST, IDP, ISECOD
DIMENSION IYYDDD(4)
TCO=0.016
TCP=0.008
C
C FIRST TIME THROUGH HERE ONLY
C
ICNT= NUMBER OF DFS IN TIME COINCIDENCE
C
C
ORIGINAL PAGE IS OF POOR QUALITY
IF(INCHK.EQ.1) GOTO 15
IF(IN(IDN).EQ.0)THEN
  ICO=0
  IN(IDN)=1
  INCHK=1
  FTIM=TMN
  LTIM=FTIM
  "TMN=FTIM+TCP"
  CLOCK=FTIM+TCP
  LIDF=IDN
  ICNT=1
  GOTO 20
ENDIF

C
C. SUBSEQUENT PASSES GO THRU HERE
C
C. CHECK FOR CHANGE OF DAY AFTER MIDNIGHT
C
15 IF(IYYDDD(IDN).GT.IYYDDD(LIDF))THEN
  IF((TMN+240000.).GT.CLOCK) ITMN=1
  IF((ITMN.EQ.1).THEN
     TMN=TMN+240000.
     WRITE(6,202) TMN, LTIM
     FORMAT(IX,'DAY-TIME CHANGE CUR TIME=',F12.3,2X,'LAST=',
            F12.3)
  ENDIF
ENDIF

C.
C. SOLN EXISTS WITHIN THE TIME CORRELATION WINDOW
C
IF(TMN.GT.CLOCK)THEN
  IF(ICNT.GE.2)THEN
    IN(5)=1
    ICNT=ICNT
    TSOLN=FTIM
    IYDSN=IYYDDD(LIDF)
  ENDIF
GOTO 30
ENDIF

C.
C. CURRENT TIME STILL WITHIN CORRELATION WINDOW
C
IF(TMN.LE.CLOCK)THEN
  IF(TMN.GT.CTIM)THEN
    IF(ICNT.EQ.1) GOTO 30
  C.
  CHECK FOR DUPLICATE DF IN TIME WINDOW
C
  IF(IN(IDM).EQ.1)THEN
    IN(5)=1
    ICNT=ICNT
    TSOLN=FTIM
    IYDSN=IYYDDD(LIDF)
  ENDIF
GOTO 30
ENDIF
ENDIF

C.
C. DO NOT ALLOW TIME TO DECREASE FROM EVENT I TO EVENT I+1
C
IF(TMN.LT.LTIM)THEN
  DO 40 I=1,4
    IN(I)=0
  CONTINUE
  WRITE(6,300) TMN,IDN,LTIM,LIDF
  C300 FORMAT(5X,'CURRENT TIME ',F12.3,' FOR DF',I2,' LESS THAN',
          '',' LAST TIME OF ',F12.3,' FOR DF',I2)
  MTKNT=MTKNT+1
  GOTO 30
ENDIF

C.
C. LOOK FOR MULTIPLE SOLNS WITHIN TIME WINDOW
C
IF(TMN.LE.CTIM.AND.ICNT.GE.2.AND.IN(IDN).EQ.1) GOTO 35
IF(TMN.LE.CTIM.AND.ICNT.EQ.1.AND.IDN.EQ.LIDF) GOTO 30

C.
C. ICNT=ICNT+1
C
IN(IDN)=1

C.
C. RECOVER THE PREVIOUS DF AND PUT IN SOLN SPACE
C
IF(ITMP.EQ.1)THEN
  ITMP=0
IM(LIDF)=1
ENDIF
LIDF=IDM
GOTO 20
ENDIF
C.
RESET THE POINTERS AND TIME VARIABLES
30
ITMP=1
FTIM=ITM
CTIM=FTIM+TCP
LIDF=IDN
ICNT=1
C
IF (CLOCK.GT.201815. .AND. CLOCK.LT.201818.) THEN
WRITE(6,23) ICNTI, IDN, IN, ICNT, LIDF, LTIM, FTIM, CLOCK,
* CTIM FOR $23",4F12.3)
ENDIF
C
RETURN
20 RETURN
END
SUBROUTINE ERSTAT(OBSAZ,REM,DIFDEG,CBRG2)

IMPLICIT DOUBLE PRECISION(A-H,O-Z)
COMMON/CON1/IBFIJ_G,NREC, INCHK, IN(S), IN2 (5)
COMMON/SOLN/SNKA,SSGNL,TLAT,TLON,
RADIUS,TSOLN,
• IYDSN, IYR, NON, NDAY, NRS, NBR, RES, NEVT, SMA, SMI,
ORIEN
COMMON/DFSTUF/DFEl(128),MS1(128),DFRI0 (128),NSI0 (128),ADFR15 (128),NSI5 (128),SQI (128),SQ10(128),SQ15 (128)
COMMON/SUMRY/KTWO ER,NTHRS (4),NBRADF,NDP2,NDI (4),NOVR (4),
• KDVG
COMMON/KOUNT/KNT1,KNT2,KNT3,KNT4,KNT5,FKT5,IFDSTT,ISECOD
COMMON/ANGLE/ANWAZM(61)
COMMON/DFangs/AFR15(4)
* 75., 78., 81., 84., 87., 90., 93., 96., 99., 102., 105., 108., 111.,
• 333., 336., 339., 342., 345., 348., 351., 354., 357., 360./
DATA ANWAZM/O. ,6., 12., 18., 24., 30., 36., 42., 48., 54., 60.,
• 144., 150., 156., 162., 168., 174., 180., 186., 192., 198., 204., 210.,
• 216., 222., 228., 234., 240., 246., 252., 258., 264., 270., 276., 282.,
* 360./
C
LI2=128
LI2=60
LIM=LI2-1
DO 15 II=1,LI2
IF(OBSAZ.GE.AZM(II).AND.OBSAZ.LT.AZM(II+1))THEN
IF(OBSAZ.GE.ANWAEM(II).AND.OBSAZ.LT.ANWAEM(II+1))THEN
DFRI0(II)=DFRI0(II)+DIFDEG
SQI0(II)=SQI0(II)+DIFDEG**2
NSI0(II)=NSI0(II)+1
ENDIF
IF(SMA.GE.0. .AND.SMA.LE.SBIG.AND.RM.GE.RSML1.AND.
* RM. LT.RBIG) THEN
DFR10(II)=DFR10(II)+DIFDEG
SQ10(II)=SQ10(II)+DIFDEG**2
NS10(II)=NS10(II)+1
ENDIF
IF(SMA.GE.0. .AND.SMA.LE.SBIG.AND.RM.GE.RSML2.AND.
* RM. LT.RBIG) THEN
DFR15(II)=DFR15(II)+DIFDEG
SQ15(II)=SQ15(II)+DIFDEG**2
NS15(II)=NS15(II)+1
ENDIF
ENDIF
C
40
C
44
C
IF(II.EQ.14)THEN
WRITE(6,40) II,OBSAZ,DIFDEG,ANWAZM(II),DFEI(II),NSI(II),SMA,
FORMAT(1X,'ERR STATS',I4,4FS.2,I4,2F8.2,I4,FS.2,I4,3(F8.2,
2X))
WRITE(6,44)
FORMAT(1X,'SOLN ',F12.3,2F10.4,1012,4F10.4)
ENDIF
ENDIF
CONTINUE
RETURN
END
SUBROUTINE ERSUM
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C C
C SUMMARIZES AND PRINTS THE DF ERROR STATISTICS C
C C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
IMPLICIT DOUBLE PRECISION(A-H,O-Z)
CHARACTER GDATE*8
COMMON/SOLN/SNKA,SSGNL,TLAT,TLON,RADIUS,AREA,TSOLN,
*IDYM,YPD,MN,NR,DN,TYR,NSI,RSML,RSML2,RSL,RSML3,RSL2,
COMMON/DFSTUF/DFEI(128),NSI(128),DFR10(128),NSI0(128),
ADFRI5(128),SMA,ADFR15(128),SQI(128),SQI0(128),SQ15(128)
COMMON/GCONST/TMX,RSML1,RSML2,RBIG,SBIG,BINSIZ
EQUIVALENCE (NCNT,NBR)
DIMENSION AZM(129),ANWAZM(61)
DATA AMES,NTI,NTIO,NTIS/-999.99,0,O,0/
DATA AZM/0.,2.,5.,8.,II.,14.,16.,19.,22.,25.,28.,30..33.,
*75.,78.,81.,84.,87.,90.,92.,95.,98.,101.,104.,106.,109.,
*343.,345.,348.,351.,354.,357.,360.,
DATA ANWAZM/0.,6.,12.,18.,24.,30.,36.,42.,48.,54.,60.,
*144.,150.,156.,162.,168.,174.,180.,186.,192.,198.,204.,210.,
*360.,
C LI3=128
C... IF IDFTST=0 THEN NOT WANTING SITE ERRORS, SKIP TO SUMMARY
C
C IF IDFTST=0 THEN NOT WANTING SITE ERRORS, SKIP TO SUMMARY
C IF,IDFTST.EQ.0) GOTO 600
WRITE(6,3000) NEVT,TSOLN,IYR,MON,NDAY,TLAT,TLON,SNKA,SSGNL,
*HRS,RADIUS,AREA
3000 FORMAT(/IX,'LAST SOLN ON TAPE=',I6,2X,F12.3,1X,312,2FI2.6,2X,
*2FI8.2/)
WRITE(6,510) TSOLN,KNTHH,IYR,MON,NDAY
510 FORMAT(//1X,'FINAL HOURLY SUMMARY ON TAPE ENDING AT ',F12.3,
*' IS',I6,4X,'DATE=',312//)
WRITE(6,190) IDFTST,TMX,RSML1,RSML2,RBIG,SBIG,BINSIZ
190 FORMAT(1H1,IX,'ERROR SUMMARY FOR DF',12//4X,' CONSTRAINTS:',
*5X,'MIN. ANGLE DEV.=',F4.1,' RANGES=',3F6.1,' MIN. SMA=',F6.1,
*' BEARING ANGLE BIN=',F4.1/)
C
C IF,IDFTST=0 THEN NOT WANTING SITE ERRORS, SKIP TO SUMMARY
C IF,IDFTST.EQ.0) GOTO 600
WRITE(6,210)
210 FORMAT(//4X,'I',4X,'AZM',4X,'N',5X,
*3X,'VAR',8X,'SD',6X,'W',6X,'AV10',6X,'VAR10',6X,'SD10',6X,
*N',6X,'AV15',5X,'VAR15',6X,'SD15')
DO 15 IK=1,LI3
IF(IK.EQ.II)THEN
X1=DFEI(IK)
X1=AMES
SD1=AMES
GOTO 300
ENDIF
ASI=FLOAT(NS1(IK))
AS1=ASI-1.
X1=DFEI(IX)/AS1
C
C
C  FUNCTION FKA RECEIVES THE AMPLITUDES (AMP) OF THE DF'S AND USES THE GREATEST SIGNAL STRENGTH (BMAX) TO COMPUTE A RANGE NORMALIZED SIGNAL STRENGTH, WHERE DIST = DISTANCE BETWEEN THE DF AND THE SOIL POINT. THIS PRODUCT IS DIVIDED BY 298, A CALIBRATION FACTOR BASED ON THE LLP ANTENNA AND LIN ET AL.'S TRANSMISSION MODEL OF THE RETURN STROKE, RETURNS PEAK CURRENT ESTIMATE FOR DISCHARGE.

C

FUNCTION FKA

IMPLICIT DOUBLE PRECISION (A-H,O-S)

COMMON/COM1/IBF1AG,MR,INCHK,IM(5),IN2(5)
COMMON/SOLA/SNMA,SQML,TLAT,TLOD,RADIUS,AREA,TSOIL,
*YDAM,YIR,YOR,MON,NODAY,WSR,WBR,NNP,SHV,SNMA,SNM,ORIEN
COMMON/DFSTUF/DFR1(128),NSI(128),DFR10(128),NSI10(128),
ADFR15(128),NSI15(128),SOL1(128),SOL10(128),SOL15(128)

IMPLICIT DOUBLE PRECISION (A-H,O-S)

COMMON/COM1/IBF1AG,MR,INCHK,IM(5),IN2(5)
COMMON/SOLA/SNMA,SQML,TLAT,TLOD,RADIUS,AREA,TSOIL,
*YDAM,YIR,YOR,MON,NODAY,WSR,WBR,NNP,SHV,SNMA,SNM,ORIEN
COMMON/DFSTUF/DFR1(128),NSI(128),DFR10(128),NSI10(128),
ADFR15(128),NSI15(128),SOL1(128),SOL10(128),SOL15(128)

FUNCTION FKA

RECEIVES THE AMPLITUDES (AMP) OF THE DF'S AND USES THE GREATEST SIGNAL STRENGTH (BMAX) TO COMPUTE A RANGE NORMALIZED SIGNAL STRENGTH, WHERE DIST = DISTANCE BETWEEN THE DF AND THE SOIL POINT. THIS PRODUCT IS DIVIDED BY 298, A CALIBRATION FACTOR BASED ON THE LLP ANTENNA AND LIN ET AL.'S TRANSMISSION MODEL OF THE RETURN STROKE, RETURNS PEAK CURRENT ESTIMATE FOR DISCHARGE.

IMPLICIT DOUBLE PRECISION (A-H,O-S)

COMMON/COM1/IBF1AG,MR,INCHK,IM(5),IN2(5)
COMMON/SOLA/SNMA,SQML,TLAT,TLOD,RADIUS,AREA,TSOIL,
*YDAM,YIR,YOR,MON,NODAY,WSR,WBR,NNP,SHV,SNMA,SNM,ORIEN
COMMON/DFSTUF/DFR1(128),NSI(128),DFR10(128),NSI10(128),
ADFR15(128),NSI15(128),SOL1(128),SOL10(128),SOL15(128)
COMMON/SUMRY/KTWOER,NTHRS(4),HBADF,NDUP(4),NOVR(4),
*DFNT(4),KDVG
COMMON/KOUNT/RKNT, KNT2, KNT3, KNT4, NTHHR, MTNRT, DFTST, ISECCOD
EQUIVALENCE (NCNT,NBR)
DIMENSION TAMP(4)
DIMENSION ALATI(4),ALONGI(4)
DATA ALATI/34.649167,35.399167,35.83750,34.716667/
DATA ALONGI/86.669167,86.076944,87.443889,87.881667/
DATA DTR, ERAD2, NRT/0.01745329,6371.,4/
ISPOL=0
BMAX=0.

C FIND ID AND AMP OF LARGEST SIGNAL STRENGTH
DO 20 I=1,NCNT
    IF (TAMP(IN2(I)).GT.0.) ISPOL=ISPOL+1
    IF (ABS(TAMP(IN2(I))).GT. BMAX) THEN
        BMAX=ABS(TAMP(IN2(I)))
        KDF=IN2(I)
    ENDIF
20 CONTINUE
IF (KDF . EQ. 0) THEN
    WRITE (6,50) TSOLN,KDF,IN2,AZ2,R2,BMAX,SPOL,FKA,NCNT,TLAT,TLON,
*RADIUS, TAMP
    FKA=-9999.
    RETURN
ENDIF

C FIND POLARITY OF MAX AMPLITUDE (POSITIVE IFF ALL DFS POSITIVE)
SPOL=-1.
IF(ISPOL.EQ.NCNT) SPOL=1.
BMAX2= BMAX*SPOL
SIGNI=BMAX2

C FIND RANGE NORMALIZED DISTANCE
SLAT = ALATI(KDF)*DTR
SLONG = ALONGI(KDF)*DTR
XTL2= TLAT* DTR
XTL02= TLON* DTR
CALL AZRN(SLAT,SLONG,XTL2,XTL02,AZ2,R2)
FKA = R2*ERAD2*BMAX2/298.

C WRITE(6,50) TSOLN,KDF,IN2,AZ2,R2,BMAX,SPOL,FKA,NCNT,TLAT,TLON,
*C RADIUS, TAMP
50 FORMAT(/IX,'KA SOLN.. ',FI2.3#IX,614,SFI2.4/IX,'NCNT',I2,TFI0.4)
RETURN
END

FUNCTION SITERR(IDF, BRG)
C
C FUNCTION SITERR RECEIVES A DIRECTION FINDER ID AND C
C AN UNCORRECTED BEARING HAVING A KNOWN SITE ERROR WHICH C
C CAN BE FOUND BY A LINEAR INTERPOLATION PROCEDURE APPLIED C
C TO A LOOK-UP TABLE FOR THAT DF C
C
COMMON/DFSTUF/DfEI(128),NS1(128),DfRI0(128),NS10(128),
*DfR15(128),NS15(128),SQ1(128),SQ10(128),SQ15(128)
COMMON/KOUNT/RKNT, KNT2, KNT3, KNT4, NTHHR, MTNRT, DFTST, ISECCOD
DIMENSION DFI(60),DF2(60),DF3(60),DF4(60),AZM(61)
DIMENSION DFA(128),DFB(128),DFC(128),DFD(128),AZM1(129)
DATA DF1/-2.47,-1.89,-1.65,-0.13,-1.02,-2.69,-3.57,
*,-3.45,-4.00,-2.78,-7.0,.83,1.08,0.89,
*,-2.14,2.69,3.4,4.09,4.90,5.58,5.39,5.38,5.59,5.89,6.03,6.72,
*,-6.51,6.46,6.86,7.14,7.04,6.56,6.29,6.21,-4.2,5.13,
*,-4.61,4.85,5.05,3.50,3.46,2.67,2.72,7.68,-1.2,-1.21,-1.20,
*,-0.6,-0.42,-0.3,-0.4,-0.87,-1.35,-1.3,-1.65,-4.18,
*,-4.5,-3.99,-4.29,-3.53/
DATA DF2/-2.47,-1.89,-1.65,-0.13,-1.02,-2.69,-3.57,
*,-3.45,-4.00,-2.78,-7.0,.83,1.08,0.89,
*,-2.14,2.69,3.4,4.09,4.90,5.58,5.39,5.38,5.59,5.89,6.03,6.72,
*,-6.51,6.46,6.86,7.14,7.04,6.56,6.29,6.21,-4.2,5.13,
*,-4.61,4.85,5.05,3.50,3.46,2.67,2.72,7.68,-1.2,-1.21,-1.20,
*,-0.6,-0.42,-0.3,-0.4,-0.87,-1.35,-1.3,-1.65,-4.18,
*,-4.5,-3.99,-4.29,-3.53/
163
33

ISECOD = 1
WRITE (6, 33)
FORMAT (1H1)
ENDIF
NDF = 4
DO 10 I = 1, LI
IF (BRG.GE.AZM(I).AND.BRG.LT.AZM(I+1)) THEN
  C
  IF (IDF.EQ.1) THEN
    ERI = DFI (I)
    ER2 = DFI (I+1)
    AZ1 = AZM(I)
    AZ2 = AZM(I+1)
  ELSE IF (IDF.EQ.2) THEN
    ERI = DF2 (I)
    ER2 = DF2 (I+1)
    AZ1 = AZM(I)
    AZ2 = AZM(I+1)
  ELSE IF (IDF.EQ.3) THEN
    ERI = DF3 (I)
    ER2 = DF3 (I+1)
    AZ1 = AZM(I)
    AZ2 = AZM(I+1)
  ELSE IF (IDF.EQ.4) THEN
    ERI = DF4 (I)
    ER2 = DF4 (I+1)
    AZ1 = AZM(I)
    AZ2 = AZM(I+1)
  ENDIF
  ENDIF
CONTINUE
PERFORM INTERPOLATION
SLOPE = (ER2 - ERI) / (AZ2 - AZ1)
SITERR = ERI - SLOPE * (AZ2 - BRG)
C IF (IDF.GT.0) GOTO 909
C
C.. TEMPORARY CORRECTION TO DFS3 AND 1
C
C
C IF (IDF.EQ.3) THEN
SITERR = -3.54 + 6 * SIN (2 * BRG * DTR) - 5.1 * COS (2 * BRG * DTR)
ENDIF
C IF (IDF.EQ.1) SITERR = 0.
C WRITE (6, 25) IDF, BRG, SITERR
25 FORMAT (1X, I2, 2X, 2 (F6.2, 2X))
C DO 120 I = 1, LI
C WRITE (6, 35) I, DF2(I), DF3(I), DF4(I), AZM(I)
35 FORMAT (1X, I4, 4 (F6.2, 2X))
120 CONTINUE
C WRITE (6, 399)
399 FORMAT (1X, 'GOING BACK TO DFDATA')
909 RETURN
END
FUNCTION SITERC (IDF, BRG)
CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC CCCC
*-6.0,-6.5,-4.6,-3.2,-0.8,0.0,0.0,0.0,-2.,-2.,-1.5,-4.,
+10.,+3.9,5.4,6.8,3.00,-2.1,-3.2,-2.8,-3.7,-6.6,-9.70,-10.7,
+8.5,+3.5,-5.3,-3.8,-2.8,-2.3/
DATA DF3/-1.5,-1.2,-2.0,-2.0,-0.6,-1.7,-1.4,-1.3,-1.2,
-1.2,-1.1,-0.9,-0.3,1.0,3.1,1.0,2.4,-1.9,-0.8,-1.,
-1.0,-2.5,0.0,0.0,-2.0,-6.0,-12.,-11.,-10.,-10.,-9.8,
-3.8,-3.0,-1.5,-1.0,-0.66,
+0.05,0.6,1.1,1.6,2.1,2.7,3.1,3.2,3.9,4.2,0.2,-7.,-8.8,
-2.7,-2.8,-1.5,-1.3,-1.4/ 
DATA DF4/-0.4,-0.2,-0.6,0.2,0.6,1.9,2.0,2.0,2.2,2.3,4.0,
+2.0,1.5,3.2,4.1,4.4,7.5,1.5,2.5,6.4,6.4,5.9,4.5,2.0,2.0,4.4,4.1,
+3.7,3.3,2.0,3.2,1.9,1.9,1.9,1.9,1.3,
+0.68,0.56,0.62,0.65,0.68,0.68,0.71,0.71,0.71,0.71,0.71,
+1.7,1.2,1.1,0.01,0.01/
DATA AZM/0.0.,6.0.,12.0.,18.0.,24.0.,30.0.,36.0.,42.0.,48.0.,54.0.,60.0.,66.0.,72.0.,
+78.0.,84.0.,90.0.,96.0.,102.0.,108.0.,114.0.,120.0.,126.0.,132.0.,138.0.,144.0.,150.0.,
+156.0.,162.0.,168.0.,174.0.,180.0.,186.0.,192.0.,198.0.,204.0.,210.0.,216.0.,222.0.,
+228.0.,234.0.,240.0.,246.0.,252.0.,258.0.,264.0.,270.0.,276.0.,282.0.,288.0.,
+294.0.,300.0.,306.0.,312.0.,318.0.,324.0.,330.0.,336.0.,342.0.,348.0.,354.0.,360.0./
IF ( IDF. EQ. IDFTST) TRZNS
SITERC-=0
RETURN
EN_IF
SLOPE=0.
DTR=0.01745329
LI=60
LI1=LI-1
IF (ISECCOD.EQ.1) THEN
WRITE(6,32)
FORMAT(1BL1,5F10.2)
DO 5 I=1,LI
WRITE(6,2) I,AZM(I) ,DF2(I) ,DF3(I) ,DF4(I)
FORMAT(1X,14,F10.2)
5 CONTINUE
ISECCOD=1
WRITE(6,33)
FORMAT(1E1)
ENDIF
NDF=4
DO 10 I=1,LI
IF (BRG.GE.AZM(I) .AND. BRG.LT.AZM(I+1)) THEN
C IF (IDF.FQ.1) THEN
ER1=DF1(I)
ER2=DF1(I+1)
AZ1=AZM(I)
AZ2=AZM(I+1)
GOTO 20
ELSE IF (IDF.EQ.2) THEN
ER1=DF2(I)
ER2=DF2(I+1)
AZ1=AZM(I)
AZ2=AZM(I+1)
GOTO 20
ELSE IF (IDF.EQ.3) THEN
ER1=DF3(I)
ER2=DF3(I+1)
AZ1=AZM(I)
AZ2=AZM(I+1)
GOTO 20
ELSE IF (IDF.EQ.4) THEN
ER1=DF4(I)
ER2=DF4(I+1)
AZ1=AZM(I)
AZ2=AZM(I+1)
GOTO 20
ENDIF
ENDIF
CONTINUE
C PERFORM INTERPOLATION
C
10 DELA=AZ2-AZ1
SLOPE=(ER2-ER1)/DELA
SITERC=ER2-SLOPE*(AZ2-BRG)
IF (IDF.GT.0) GOTO 919
C TEMPORARY CORRECTION TO DF53 AND 1
C
20 IF (IDF.EQ.3) THEN
IF (BRG.GE.150. .AND. BRG.LE.180.) THEN
SITERC=SITERC-3
ENDIF
ENDIF
166
C IF,IDF.EQ.2)THEN
C IF(BRG.GE.230..AND.BRG.LE.250.)THEN
C SITERC=SITERC-6.
C ENDIF
C ENDIF
C IF(IDF.EQ.1)THEN
C IF(BRG.GE.330..AND.BRG.LE.340.)THEN
C SITERC=SITERC+3.
C ENDIF
C ENDIF
C IF(IDF.EQ.4)THEN
C IF(BRG.GE.65..AND.BRG.LE.75.)THEN
C SITERR=SITERR+1.5
C ENDIF
C ENDIF
C IF(IDF.EQ.3)THEN
C SITERR=-3.5+.6*SIN(2.*BRG*DTR)-5.1*COS(2.*BRG*DTR)
C ENDIF
C WRITE(6,25) IDF,BRG,SITERR
25 FORMAT(IX,I2,2X,2(F6.2,2X)
C DO 120 I=I,LI
C WRITE(6,35) I,DF2(I),DF3(I),DF4(I),AZM(I)
35 FORMAT(IX,I4,4(F6.2,2X}
C120 CONTINUE
C WRITE(6,399)
399 FORMAT(1X,'GOING BACK TO DFDATA')
919 RETURN
C END
FUNCTION SITER2(IDF,BRG)

COMMON/DFSTUF/DFEI(128),NSI(128),DFRI0(128),NSI0(128),
*DFRI5(128),NSI5(128),SQI(128),SQI0(128),SQI5(128)
COMMON/KOUNTR/KNTI,KNT2,KNT3tKNT4,KNTHH,MTKNT,IDFTST,ISECOD
DIMENSION DFA(60),DFB(60),DFC(60),DFD(60),AZM(61)
DIMENSION AO(4),ADI(12),AD2(12),AD3(12),AD4(12)
DATA A0/1.63,-3.33,-1.47,1.94/
DATA ADI/0.8788,-4.9730,-0.4359,0.2113,-0.2401,0.2218,
*0.4777,0.0991,0.8518,0.1006,0.4715,0.4008/
DATA AD2/-2.1636,1.6295,4.9684,-2.5906,-0.0242,1.3654,
*0.7188,0.7636,-1.6591,0.7491,0.2532,-0.1620/
DATA AD3/-0.3506,1.2770,-0.3550,-1.5981,-0.0535,-0.1957,
*0.5497,-0.1734,0.5484,0.4748,-0.2305,0.3460/
DATA AD4/2.0684,-1.2777,-0.6244,-0.6550,-0.3275,0.4715,
*0.0013,-0.5822,0.0014,-0.2151,-0.3507,-0.3409/
DATA DFA/-2.47,-1.89,-1.65,-0.13,-1.02,-2.69,-3.57,
*2.48,-0.48,-2.78,-7.0,-0.83,1.45,-1.63,1.94/
DATA DFB/-2.14,2.69,5.3,4.09,4.90,5.58,5.39,5.38,5.59,5.89,6.03,6.72,
*6.51,6.44,6.86,7.14,7.04,6.58,6.29,6.21,-4.2,5.13,
DATA DFC/-4.61,4.85,5.05,5.30,3.46,2.67,2.72,1.68,2.0,-1.21,-1.20,
*0.6,-0.42,-0.3,-0.4,-0.87,-1.35,-1.65,-4.14,
*1.3,-3.99,-4.29,-3.53/
DATA DFD/-2.47,-1.89,-1.65,-0.13,-1.02,-2.69,-3.57,
*2.48,-0.48,-2.78,-7.0,-0.83,1.45,-1.63,1.94/
DATA AZM/0.,6.,12.,18.,24.,30.,36.,42.,48.,54.,60.,66.,72.,
*78.,84.,90.,96.,102.,108.,114.,120.,126.,132.,138.,144.,150.,
*156.,162.,168.,174.,180.,186.,192.,198.,204.,210.,216.,222.,
DTR=0.01745329
LI=12
LI1=LI-1

ORIGINAL FACE IS
OF POOR QUALITY
IF (ISECOD .EQ. 2) THEN
WRITE(6,32)
32 FORMAT(IH1/X, 'SITE ERROR POLYNOMIAL'/)
WRITE(6,3) (A0(I), I=I, 4)
3 FORMAT(1X, 'AO=', 4F10.4/) 
DO 5 I=1, 12
WRITE(6,2) I, ADI(I), AD2(I), AD3(I), AD4(I)
2 FORMAT(IX, I4, 4F10.4)
5 CONTINUE
ISECOD = ISECOD - 1
WRITE(6,33)
33 FORMAT(IH1)
ENDIF
NDF = 4

C C.. CORRECTED BEARING ANGLE Y = A0 + A1 SinX + A2 CosX + ... + A11 Sin6X + A12 Cos6X
C
C STERN = 0.
CTERM = 0.
DO 20 J = 1, 6
TJ = DTR * FLOAT(J)
ITJ = (J-1) * 2 + I
ICJ = (J-1) * 2 + 2
IF (IDF .EQ. 1) THEN
STERM = AD1(ITJ) * SIN(TJ * BRG) + STERM
CTERM = AD1(ICJ) * COS(TJ * BRG) + CTERM
ENDIF
IF (IDF .EQ. 2) THEN
STERM = AD2(ITJ) * SIN(TJ * BRG) + STERM
CTERM = AD2(ICJ) * COS(TJ * BRG) + CTERM
ENDIF
IF (IDF .EQ. 3) THEN
STERM = AD3(ITJ) * SIN(TJ * BRG) + STERM
CTERM = AD3(ICJ) * COS(TJ * BRG) + CTERM
ENDIF
IF (IDF .EQ. 4) THEN
STERM = AD4(ITJ) * SIN(TJ * BRG) + STERM
CTERM = AD4(ICJ) * COS(TJ * BRG) + CTERM
ENDIF
20 CONTINUE
SITER2 = A0(IDF) + STERM + CTERM
C IF (IDF .EQ. IDFSTST) SITER2 = 0.
C WRITE(6,25) IDF, BRG, SITER2
25 FORMAT(IX, I2, 2X, 2(F6.2, 2X))
C DO 120 I = 1, LI
C WRITE(6,35) I, DFI(I), DF2(I), DF3(I), DF4(I), AZM(I)
35 FORMAT(1X, I4, 4(F6.2, 2X))
120 CONTINUE
C WRITE(6,399)
399 FORMAT(1X, 'GOING BACK TO DFDATA')
909 RETURN
END

FUNCTION STDEVB_IDF, BRG)

CCCCCCCCCCCCCCCCCCCCCC
C
FUNCTION STDEVB RECEIVES A DIRECTION FINDER ID AND AN
BEARING ANGLE HAVING A KNOWN SITE ERROR WHICH IS USED TO
COMPUTE THE POLYNOMIAL FORM OF THE BEARING
STANDARD DEVIATION AS A FUNCTION OF ANGLE FOR USE IN PPX
C
CCCCCCCCCCCCCCCCCCCCCC
C
COMMON/DFSTUF/DFEI(128), NSI(128), DFR10(128), NS10(128),
* DFR15(128), NS15(128), SQI(128), SQ10(128), SQ15(128)
COMMON/KOUNTR/KNT1, KNT2, KNT3, KNT4, KNTHM, MTHT, IDFSTST, ISECOD
DIMENSION SDA(60), SDB(60), SDC(60), SDD(60), AZM(61)
DIMENSION BO(4), BD1(12), BD2(12), BD3(12), BD4(12)
DATA BO/1.63, -3.33, -1.47, 1.94/
DATA BD1/0.8788, -4.9730, -0.4359, 0.2113, -0.2401, 0.2218,
* -0.4777, 0.0991, 0.8518, 0.1006, 0.4715, 0.4008/
DATA BD2/-2.1636, 1.6295, 4.9684, -2.5906, -0.0242, 1.3654,
* -0.7188, 0.7636, -1.6591, 0.7491, 0.2532, -0.1620/
DATA BD3/-0.3506, 1.2770, -0.3550, -1.5981, -0.0535, -0.1957,
* 0.4947, -0.1734, 0.5484, 0.4748, -0.2303, 0.3460/
DATA BD4/-0.3506, 1.2770, -0.3550, -1.5981, -0.0535, -0.1957,
* 0.4947, -0.1734, 0.5484, 0.4748, -0.2303, 0.3460/
DATA BD2/-2.1636, 1.6295, 4.9684, -2.5906, -0.0242, 1.3654,
* -0.7188, 0.7636, -1.6591, 0.7491, 0.2532, -0.1620/
DATA BD3/-0.3506, 1.2770, -0.3550, -1.5981, -0.0535, -0.1957,
* 0.4947, -0.1734, 0.5484, 0.4748, -0.2303, 0.3460/
DATA BD4/-0.3506, 1.2770, -0.3550, -1.5981, -0.0535, -0.1957,
* 0.4947, -0.1734, 0.5484, 0.4748, -0.2303, 0.3460/
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```fortran
DATA SDB/-2.1,-1.4,-1.6,-1.1,-0.2,0.01,-0.15,-0.21,-0.46,-0.74,-0.61,0.96,0.2,-1.7,-3.8,-5.5,-6.1,-6.6,-7.2,-5.6,-5.0,-6.8,-9.9,-12.2,-14.0,-14.8,-13.5,-12.6,-9.9,-8.0,-6.5,-4.6,-3.2,-0.8,-3.9,2.8,2.5,1.8,1.3,-6.4,1.5,2.5,3.9,5.4,6.8,3.0,-2.1,-3.2,-2.8,-3.7,-6.6,-9.7,-8.5,-6.1,-5.3,-3.8,-2.8,-2.3/ 
DATA SDC/-1.5,-1.2,-2.0,-2.0,-0.6,-1.7,-1.4,-1.3,-1.2,-1.1,-0.9,-0.3,0.1,0.3,0.5,-0.3,-1.9,-0.8,0.4,-0.2,-1.5,-2.4,-3.3,-4.3,-5.2,-6.1,-7.0,-8.0,-8.9,-9.8,-8.9,-8.0,-7.1,-6.2,-5.3,-4.4,-5.5,-2.0,-1.5,-1.0,-0.46,-0.05,0.6,1.1,1.6,2.1,2.6,3.1,3.0,2.9,2.1,0.2,-0.7,-0.8,-2.7,-2.6,-1.5,-1.3,-1.2/ 
DATA SDD/-0.4,-0.7,-0.6,0.24,0.67,1.9,2.0,0.2,0.2,2.2,2.2,1.8,5.2,4.1,4.7,5.1,5.2,5.6,5.4,5.9,5.0,4.7,4.4,4.1,3.7,3.2,3.0,2.7,1.9,1.0,2.2,6.4,1.3,2.5,2.6,2.3,1.9,1.3,1.6,6.2,-6.2,-3.1,0.9,-3.6,1.1,1.6,2.1,2.6,3.1,3.0,2.9,2.1,0.2,-0.7,-0.8,-2.7,-2.6,-1.5,-1.3,-1.2/ 
DTR=0.01745329
LI=L1=1 
IF (ISECOD.EQ.2) THEN 
   WRITE(6,32) 
   32 FORMAT(1H1/IX, 'SITE ERROR ST. DEV. POLYNOMIAL'/) 
   WRITE(6,3) (B0(I),I=1,4) 
   3 FORMAT(1X, 'B0=' , 4F10.4/) 
   DO 5 I=1,12 
      WRITE(6,2) I,BDI(I),BD2(I),BD3(I),BD4(I) 
      2 FORMAT (1X, I4,4F10.4) 
   5 CONTINUE 
   ISECOD=1 
   WRITE(6,33) 
   33 FORMAT(1H1) 
ENDIF 
NDF=4 
C.C. CORRECTED BEARING ANGLE Y=A0+A1SINX+A2COSX+...+A11SIN6X+A12COS6X 
C STERM=0. 
C CTERM=0. 
C DO 20 J=1,6 
TJ=DTR*FLOAT(J) 
ITJ=(J-1)*2+1 
ICTJ=(J-1)*2+2 
IF (IDF.EQ.1) THEN 
   STERM=BD1(ITJ)*SIN(TJ*BRG)+STERM 
   CTERM=BD1(ICTJ)*COS(TJ*BRG)+CTERM 
   20 CONTINUE 
ENDIF 
IF (IDF.EQ.2) THEN 
   STERM=BD2(ITJ)*SIN(TJ*BRG)+STERM 
   CTERM=BD2(ICTJ)*COS(TJ*BRG)+CTERM 
ENDIF 
IF (IDF.EQ.3) THEN 
   STERM=BD3(ITJ)*SIN(TJ*BRG)+STERM 
   CTERM=BD3(ICTJ)*COS(TJ*BRG)+CTERM 
ENDIF 
IF (IDF.EQ.4) THEN 
   STERM=BD4(ITJ)*SIN(TJ*BRG)+STERM 
   CTERM=BD4(ICTJ)*COS(TJ*BRG)+CTERM 
ENDIF 
C WRITE(6,25) IDF, BRG, STDEVB 
25 FORMAT (IX, I2,2X, 2 (F6.2,2X)) 
C DO 120 I=1,LI 
C WRITE(6,35) I,DF2(I),DF3(I),DF4(I),AZM(I) 
35 FORMAT(I4, I4,4 (F6.2,2X)) 
120 CONTINUE 
909 RETURN 
C SUBROUTINE FFIX(BRGID,TEMPBR,TM) 
C THIS PROGRAM IS A VECTOR APPROACH TO DF FIXING 
C MODIFIED AT MSFC TO COMPUTE OPTIMAL LLP SOLNS FROM DF DATA 
C site errors removed from data (SJG/01-05-86) 
C IMPLICIT DOUBLE PRECISION(A-H,O-Z) 
C DIMENSION TEMPER(4),LOD(4),LOG(4),LOM(4),LAD(4),LAM(4),LAS(4) 
```

DIMENSION BRGSV(50)
DIMENSION ISUB(50), IUSE(50)
CHARACTER*1 M, S, E, W, NORS, EORW
INTEGER KNAME(50), BRGID(4)
DIMENSION SIGX(4), TM(4)
DOUBLE PRECISION NX, NY, NZ
INTEGER SEQ1, SEQ2
DOUBLE PRECISION INVAR
INTEGER SAVE
DIMENSION SINT(32)
EQUIVALENCE (SINT(1), STAZ(1)), (NCT, NBR), (IERR, IRES)
DIMENSION XHIS0(30)
COMMON K, NST, WBRF, INVAR(50), COST(50),
* ISING(50), COST(50),
* IST(1), IST(2),
* BRNX(50), BRNY(50), BRNZ(50), IL(50), SAVE(50), SWT(50),
* CHISQ(30), DTR, WT(50), TX, TY, TZ, E1, E2, E3,
* C11, C12, C13, C12, C23
COMMON/SOLN/SNKA, SSGNL, TLAT, TLON,
* IYDSN, IYR, NON, NDAY, NRS, NBR, IRES, NEVT, SMA, SMI
COMMON/KOUNTR/KNTI,
DATA N,S,E,W/’N’,’S’,’E’,’W’/
DATA KNAME/1, 2, 3, 4, 46/’0’/
DATA LOD/86, 86, 87, 87/.
* LOM/40, 04, 26, 52/.
* LOS/09, 37, 38, 54/.
* LAD/34, 35, 35, 34/.
* LAM/38, 23, 50, 43/.
* LAS/57, 57, 15, 00/. C
****** USING 20 PER CENT TABLES ******
DATA XHIS/1.642, 3.219, 4.642, 5.985, 7.289, 8.585, 9.803, 11.030,
C22. 760, 23. 900, 25. 038, 26. 171, 27. 301, 28. 429, 29. 553, 30. 675, 31. 795,
D32. 912, 34. 027, 35. 139, 36. 250/. C
IF(NBR.EQ.0) GOTO 869
IF (NBR. EQ. 2 ) THEN
WRITE(6, 1191) NBR, BRGID, TEMPBR, TM(1)
FORMAT (1X, ’NULL BRGID FOR NBR m’, 4FI2.6, 2X, 2(F12.3, 2X))
ENDIF
ENDIF
DTR = .01745329
TX = 0.
TY = 0.
TZ = 0.
E1 = 0.
E2 = 0.
E3 = 0.
C11 = 0.
C22 = 0.
C33 = 0.
C13 = 0.
C12 = 0.
C21 = 0.
WBRF = 0.
IF(LISH = 0)
DO 101 I = 1, 30
101 CHISQ(I) = XHIS(I)
C NST IS THE NUMBER OF STATIONS IN THE TABLE
C READ (5, 1) NST
C 1 FORMAT (I3)
NCT = 4
DO 20 I = 1, NST
20 READ (5, 2) XNAME(I), LAD, LAM, LOD, LOM, LOS, SIGX(I)
2 FORMAT(A2, 614, IX, F5.1)
SLAT = LAD(I) + (LAM(I) + LOS(I) / 60. / 60.
SISLON = LOD(I) + (LAM(I) + LOS(I) / 60. / 60.
SLAT = SLAT - DTR
C LONGITUDE INTERNAL TO THE PROGRAM IS NEGATIVE FOR WEST AND
C POSITIVE FOR EAST. DATA SUBMITTED AND PRINTED USES THE OPOSITE
C CONVENTIONS
C SISLON = -SISLON + DTR
COSG(I) = DCOS(SISLON)
SING(I) = DSIN(SISLON)
COST(I) = DCOS(SLAT)
C COMPUTE THE STATION VECTOR (STAX, STAY, STAZ)
STAX(I) = COST(I) * COSG(I)
STAY(I) = COST(I) * SING(I)
STAZ(I) = DSIN(SLAT)
### 32

**WRITE**(6,324) **STAX**(I),**STAY**(I),**STAZ**(I)

**FORMAT**(1X,3(F12.4,1X))

*****CALCULATE INVERSE STATION VARIANCE HERE***

**C**

**###|TE INVERSE STATION VARIANCE HERE###**

**C**

**###|SIGX IS THE STANDARD DEVIATION IN DEGREES OF BEARINGS FROM THIS###**

**DF ID#'-I. IT HAS LITTLE OR NO IMPACT ON THE SOLN, BUT THE**

**###|CONFIDENCE RADII ARE DIRECTLY PROPORTIONAL TO THE VALUE OF###**

**SIX. IN ADDITION, SOME DISTANT SOLNS MAY NOT BE COMPUTED**

**WHEN SIGX IS LARGE (SAY 1 DEG WITH SOLN 5000 KM FROM DF).**

**SIGX(I)=1.5**

**C**

**WRITE**(6,3) **KNAME**(I),**LAT**(I),**LAM**(I),**LAS**(I)

**C**

**##]])*LOG**(I),**LOM**(I),**LOS**(I),**SIGX**(I)**

**C**

**FORMAT**(1X,12,2X,F.5)**

**20 CONTINUE**

**108 FORMAT(A3)**

**INDEX=10**

**900 CONTINUE**

**DO 301 IX=1,50**

**IUSE**(IX) =0

**301 ISUB**(IX)=0

**C**

**###|NBR IS THE NUMBER OF BEARINGS IN THIS FLASH###**

**C**

**IFLSH=IFLSH+1**

**DO 40 J=1,NBR**

**READ**(9,225) **BRGID**,**TEMPBR**(J)

**225 FORMAT(1X,111,1X,F6.2)**

**C**

**WRITE**(6,241) **BRGID**,**TEMPBR**(J)

**C241 FORMAT(1X,1A1,1X,F10.4)**

**DO 38 I=1,NBR**

**C**

**WRITE**(6,241) **BRGID**,**KNAME**(I)

**646 FORMAT(1X,2(I2,IX))**

**IF(BRGID(J).EQ.KNAME(I)) GO TO 39**

**39 ISTA**(J)=I

**ISTA**(J)=I

**INVAR**(J)=1/(DTR**SIGX**(I))**2**

**BRG**=**TEMPBR**(J)**DTR**

**BRS**(J)**=**TEMPBR**(I)

**CBRG**=**DCOS**(BRG)

**SBRG**=**DSIN**(BRG)

**###|BEARING PLANE NORMAL VECTOR###**

**###|COMPUTE THE BEARING VECTOR AS THE CROSS PRODUC**

**###|IT OF THE BEARING PLANE NORMALS AND THE SITE VECTORS###**

**CBRG**=**SING**(I)**-SBRG**

**SINT**(I)***COSG**(I)

**BRNY**(J)**=-**CBRG**

**COSG**(I)**-SBRG**

**SINT**(I)***SING**(I)

**BRNZ**(J)**=**SBRG**

**###|NXSX**(J)**=**BRNY**(J)***STAZ**(I)**-BRNZ**(J)***STAY**(I)**

**###|NXSY**(J)**=**BRNZ**(J)***STAX**(I)**-BRNX**(J)***STAZ**(I)**

**###|NXSZ**(J)**=**BRNX**(J)***STAY**(I)**-BRNY**(J)***STAX**(I)**

**CONTINUE**

**40 CONTINUE**

**C**

**USE EXHAUSTIVE REJECTION FOR 10 OR LESS BEARINGS**

**USE SEQUENTIAL REJECTION FOR MORE THAN 10 BEARINGS**

**IF (NBR .LE.10) GO TO 50**

**CALL SEQ**

**GO TO 60**

**50 K = NBR**

**CALL EXH**

**60 IF (INES .NE. 1) GO TO 800**

**NREJ IS THE NUMBER OF REJECTED BEARINGS**

**C**

**K IS THE NUMBER OF BEARINGS USED IN THE FIX**

**NREJ = NBR -K**

**C**

**WRITE**(6,699) **TLAT**,**TLON**

**699 FORMAT(1I,2(F10.6,**LON**=',F10.6)**

**TLON**=**TLON**

**NORS=N**

**EORW=E**
SUBROUTINE EXH

IMPLICIT DOUBLE PRECISION(A-H,O-Z)

DOUBLE PRECISION NXSX,NXSY,NXSZ,

DOUBLE PRECISION INVAR

INTEGER SAVE

DIMENSION KLIM(10)

COMMON K,NST,WNBW,INVAR(50),COST(50),

1 NXSX(50),NYSX(50),NXSZ(50),

2 BRNX(50),BRNY(50),BRNZ(50),IL(50),SAVE(50),SWT(50),

3 CHISQ(30),DTR,WT(50),TX,TY,TZ,E1,E2,E3,

4 C11,C12,C13,C14,C23

COMMON/SOLN/SNKA, S SGNL, TLAT, TLON,

5 RADIUS, AREA, TSOLN,

*IYDSN, IYR, MND, NDAY, NRS, NBR, IRES, NEVT, SMA, SMI, ORIEN

COMMON/KOUNTR/KNTI, KNT2, KNT3, KNT4, KNTH, MPKN, IDFTST, ISECODE

EQUIVALENCE (NCNT,NBR), (IERR, IRES)

DATA KLIM/3,3,3,3,3,3,4,4,5,5/

DO 330 IP=1,4

C WRITE(6, 33) STAX(IP),STAY(IP),STAZ(IP)

33 FORMAT(1X,3(F12.4,1X),' IN EXH')

330 CONTINUE

10 CONTINUE

DO 15 I = 1, K

ORIGINAL PAGE IS OF POOR QUALITY
CONTINUE
DO 15 I = 1, K

IL(I) = I
15 CONTINUE
REJ = 1000.

IS STATION UNIQUE?

DO 20 I = 1, K
20 CONTINUE

SET INITIAL WEIGHTS AND CALL BPE

CONTINUE
DO 25 I = 1, K
25 CONTINUE

CALL BPE(2)

IS BEARING SET BETTER THAN PREVIOUS BEST?

CONTINUE
IF (WBRF .GE. REJ) GO TO 40

SAVE THIS CASE

CONTINUE
DO 30 I = 1, K
30 CONTINUE

REJ = WBRF

THIS FORCES FFIN TO GET SOLN IN ONE PASS IF THERE
ARE THREE OR LESS BEARINGS IN THE FIX.

IF (K .LE. 2) GO TO 60

FIND NEXT COMBINATION

I = K
CONTINUE
IF (IL(I) .LT. NBR - K + 1) GO TO 45
IF (I .LE. 1) GO TO 50
40 I = I - 1
41 CONTINUE
IF (I .LE. 1) GO TO 50
I = I - 1
GO TO 41
45 J = I
46 IL(I) = IL(J) + 1
IF (I .GE. K) GO TO 20
J = I
I = I + 1
GO TO 46

ALL COMBINATIONS COMPLETE

WBRF = REJ
IF (WBRF .LT. CHISQ(K - 2)) GO TO 60

TRY SMALLER K

IF (K .LE. KLIM(NBR)) GO TO 90
K = K - 1
GO TO 10

OUTPUT RESULTS

CONTINUE
DO 61 I = 1, K
61 CONTINUE

CALL BPE(I)
IRES = 1
CALL CONFID
GO TO 999
90 IRES = 2
999 RETURN
END

SUBROUTINE SEQ
IMPLICIT DOUBLE PRECISION(A-H,O-Z)
DOUBLE PRECISION NXSX,NXSY,NXSZ
DOUBLE PRECISION INVAR
INTEGER SAVE
COMMON K,NST,WERF,INVAR(50),COST(50),
1 SING(50),COSG(50),STAX(50),STAY(50),STAZ(50),
2 BRNX(50),BRNY(50),BRNZ(50),IL(50),SAVE(50),SMT(50),
3 NXSX(50),NXSX(50),NXSZ(50),
4 CHISQ(30),DTH,WT(50),TX,TY,TZ,E1,E2,E3,
5 C11,C22,C33,C12,C13,C23
COMMON/SOLN/SNKA,SSGNL,TLAT,TDON,RADIUS,AREA,TSONL,
*YDSN,LYR,MON,HDAY,MRK,NBR,IRE,HEMT,SM,SM,OREM
COMMON/KOUNTR/KNTR1,KNTR2,KNTR3,KNTR4,KNT5,KNT6,KNT7,KNT8,
*IDFTST,ISECID,EQUIVALENCE (NCMT,NBR), (IERR,IRES)
X = NBR
DO 15 I=1,K
IL(I) = I
15 CONTINUE
DO 20 I=1,K
WT(I) = INVAR(I)
20 CONTINUE
21 CONTINUE
CALL BPE(2)

C TEST RESULTS
M = K-2
IF (K .GE. 33) GO TO 25
IF (WERF .LT. CHISQ(M)) GO TO 50
GO TO 26
25 RM-M
XCHI = DSQRT(2.0*WERF)-DSQRT(2.0*RM-I.)
IF (XCHI .LT. 0.842) GO TO 60
CONTINUE
NOT ACCEPTABLE - FIND BEARING TO REMOVE
IF (X .LE. (NBR+I)/2.) GO TO 900
EMAX = 0.
DO 30 I=1,K
IB = IL(I)
FIND THE BEARING HAVING THE LARGEST WEIGHTED ERROR TO THE
C CURRENT BPE( X )
X = WT(IB)*BRNX(IB)*TX+BRNY(IB)*TY+BRNZ(IB)*TZ**2
IF(0.LT.TX*XNXS(IB)+TY*XNSY(IB)+TZ*XNSZ(IB)) GO TO 60
X = 2*WT(IB)-X
30 WRITE(6,500) (IL(I), 'REJECTED BEARING ID=',I4)
CONTINUE
K = K-1
DELETE THE REJECTED BEARING FROM THE LIST
DO 35 I=IS,K
IL(I) = IL(I+1)
35 CONTINUE
IS STATION UNIQUE
I1 = ISTA(IL(1))
DO 40 I = -2,K
IF (ISTA(IL(I)) .NE. I1) GO TO 21
CONTINUE

C NO FIX
900 IRES = 2
GO TO 999
50 CONTINUE
CALL BPE(1)
IRES=1
CALL CONFID
999 RETURN
END

SUBROUTINE BPE(ITER)
IMPLICIT DOUBLE PRECISION(A-H,O-Z)
DOUBLE PRECISION NXSX,NXSY,NXSZ
DOUBLE PRECISION INVAR
INTEGER SAVE

ORIGINAL PAGE IS OF POOR QUALITY

174
COMMON K,NST,WRBF,INVAR(50),COST(50),
  1 SING(50),COSG(50),STAX(50),STAY(50),ISTA(50),
  2 BRNX(50),BRNY(50),BRNZ(50),IL(50),SAVE(50),SWT(50),
  3 NXX(50),NXXS(50),NXXZ(50),
  4 CHISO(50),DTR,MT(50),TX,TY,TZ,E1,E2,E3,
  5 C11,C12,C13,C12,C23
COMMON/SOLN/SNKA,SSGNL,TLAT,TLON,RADIUS,AREA,TSOLN,
  *IYDSN, IYR,
COR_ION/KOUNTR/KNT1,E1Q2,KNT3,KNT4,KNTRR,
  MTKNT,IDFTST,IDSECOD
EQUIVALENCE (NCNT,NBR), (IERR, IRES)
DATA RFN/1.
DO 100 KK=1,ITER
  C11= 0.
  C22= 0.
  C13= 0.
  C12= 0.
  C23= 0.
DO 6 I=1,K
   J = IL(I)
   BUILD THE CMATRIC AS THE PRODUCT OF THE 3 BY N MATRIX OF THE
   WEIGHTED BEARING PLANE NORMAL VECTORS(BRNX,BRNY,BRNZ) AND ITS TRANSPOSE
   C11=C11+WT(J)*BRNX(J)**2
   C22=C22+WT(J)*BRNY(J)**2
   C13=C13+WT(J)*BRNZ(J)**2
   C12=C12+WT(J)*BRNX(J)*BRNY(J)
   C13=C13+WT(J)*BRNX(J)*BRNZ(J)
   C23=C23+WT(J)*BRNY(J)*BRNZ(J)
6 CONTINUE
CALL EIGEN(C11,C22,C33,C12,C13,C23,E1,E2,E3, TX,TY,TZ)
WRITE (6,509) C11, C22,C33
509 FORMAT(IX,'C11,C22,C33, ',3(2X, FI6.4))
WRITE (6,512) TX, TY, TZ,E1
512 FORMAT(1X, 'X, Y, Z-', 4 (2X, F12.7) )
WRITE (6,513) E1
513 FORMAT(' #-', F12.7)
NEED ANTIPODE?
   ICT=0
DO 10 I=1,K
   J = IL(I)
   IF (0 .LE. TX*NXX(J)+TY*NXXS(J)+TZ*NXXZ(J) ) ICT = ICT+1
10 CONTINUE
   IF ICT IS K ALL BEARINGS ARE FORWARD
   IF ICT IS 0 ALL BEARINGS ARE BACKWARD
   IF (K .LE. 2*ICT) GO TO 11
   TX = -TX
   TY = -TY
   TZ = -TZ
11 CONTINUE
DO 20 I=1,K
   J = IL(I)
   L = ISTA(J)
   TDOTS IS THE COSINE OF THE DISTANCE OF THE BPE TO THE SITE
   TDOTS = TX*STAX(L)+TY*STAY(L)+TZ*STAZ(L)
   IF (TDOTS .GT. .99999) TDOTS = .99999
   IF (TDOTS .LT. -.99999) TDOTS = -.99999
INSERT RANGE WEIGHTING FUNCTION HERE
NOTE RFN=1.0 GIVES BEST (SMALLEST) ERROR ELLIPSE FOR LLP
THE COMPLICATED RFN RELATION ACCOUNTS FOR INCREASED VARIANCE
IN THE BEARINGS DUE TO SKY WAVE EFFECTS WHEN SOLN CLOSE TO
THE STATIONS (HIGH INCIDENCE ANGLE TO IONOSPHERE)- THIS IS
AN IMPORTANT CONSIDERATION IN HP DIRECTION FINDING, NOT HERE.
   RGE = 34.44*DACOS(TDOTS)
WRITE (6,222) RGE,TDOTS
222 FORMAT(1X,'RGE AND TDOTS ARE ',2F12.3)
   IF (RGE .LT. 0. ) GO TO 137
   IF (RGE .LT. 1. ) GO TO 137
   RFN = .285714+RGE*.07142856
   GO TO 138
   RFN = 3.-RGE* (.402-RGE*.0204)
   MODIFY THE WEIGHTS BY THE RANGE TO THE CURRENT BPE
   SEE STANSFIELD(1947) FOR WEIGHT
   WT(J) = INVAR(J)/(RFN**2*(1.-TDOTS**2))
20 CONTINUE
100 CONTINUE
SUBROUTINE EIGEN (D11, D22, D33, D12, D13, D23, E1, E2, E3, TX, TY, TZ)

IMPLICIT DOUBLE PRECISION (A-H, O-Z)

C11 = D11
C22 = D22
C33 = D33
C12 = D12
C13 = D13
C23 = D23
B = C11 + C22 + C33
C = C11 * (C22 + C33) + C22 * C33 - (C12^2 + C23^2 + C13^2)
A = C12 / C11
D = (C22 - C12 * A) * (C13 - C13 * C11 - (C23 - C13 * A) * 2)

IF (D .GT. 0) GO TO 13
EI = 0
RETURN
CONTINUE
13 X = 0
DO 10 I = 1, 10
FX = X^3 - B * X^2 + C * X - D
FP = 3 * X^2 - 2 * B * X + C
XN = X - FX / FP
IF (XN .EQ. 0) GO TO 12
IF (ABS ((XN - X) / XN) .LT. 0.001) GO TO 12
X = XN
10 CONTINUE
RETURN
END

SUBROUTINE EVECT (EI, C11, C22, C33, C12, C13, C23, TX, TY, TZ)

IMPLICIT DOUBLE PRECISION (A-H, O-Z)

D1 = (C11 - EI) * (C22 - EI) - C12^2
IF (ABS (D1) .LE. .0001) GO TO 10
X = C13 * (EI - C22) + C12 * C23
Y = (EI - C11) * C23 + C13 * C12
Z = D1
GO TO 50
10 D2 = (C11 - EI) * (C33 - EI) - C13^2
IF (ABS (D2) .LE. .0001) GO TO 20
X = (EI - C33) * C12 + C13 * C23
Y = D2
Z = (EI - C11) * C23 + C13 * C12
GO TO 50
20 X = (C22 - EI) * (C33 - EI) - C23^2
Y = (EI - C33) * C12 + C23 * C13
Z = (EI - C22) * C13 + C23 * C12
50 TEMP = DSQRT (X^2 + Y^2 + Z^2)
TX = X / TEMP
TY = Y / TEMP
TZ = Z / TEMP
RETURN
END

SUBROUTINE CONFID

COMPUTES THE CONFIDENCE ELLIPSE FOR GIVEN ANGLE ERROR
WHERE THE ANGLE STANDARD DEVIATION IS IN DEGREES
RETURNS ERROR RADIUS AND AREA FOR THIS ELLIPSE.

IMPLICIT DOUBLE PRECISION(A-H,O-Z)
DOUBLE PRECISION NXSX,NXSY,NXSZ
DOUBLE PRECISION SAVE

COMMON X,NST,WRFJ,INVAR(50),COST(50),
1 SING(50),COGET(50),STAX(50),STAZ(50),ISTA(50),
2 BNN(50),BNNY(50),BNNZ(50),IL(50),SAVE(50),SWT(50),
3 NXS(50),NXSY(50),NXSZ(50),
4 CHISQ(30),DTR,WT(50),TX,TY,TZ,E1,E2,E3,
5 C11,C22,C33,C12,C23

COMMON/SOLN/SNKA,SSGNL,TLAT,TLON,RADIUS,AREA,T5OLJ,
1 YDSM,JYR,HON,NDAY,NRS,NNR,IRES,KEVT,SHA,SMI,SMI/OIEN
COMMON/KOUNTR/KNTI,KNT2,KNT3,KNT4,KNTHH,MTKNT,ISDTS,ISEC01

FACT = EARTH RADIUS * DSQRT(-2 LN (1-P))
FACT IS IN UNITS OF NAUT. MI.; FOR P=.5, FACT=4055
DATA FACT /4055./
MISS=10000.

TLAT = DASIN(TZ)/DTR
IF (ABS(TX).LE.1.E-6) GO TO 10
TLON = ATAN(TY/TX)/DTR
IF (TX .LE. 0.) TLON = TLON + 180.
IF (TLON .GT. 180.) TLON = TLON - 360.
GO TO 15
TLON = 90.
IF (TY .LE. 0.) TLON = -90.
CONTINUE

CII = 0.
C22 = 0.
C33 = 10.
C12 = 0.
C13 = 0.
C23 = 0.
DO 30 I=1,K
J = ISTA(IL(I))

C COMPUTE THE BEARING PLANE NORMAL VECTORS(BX,BY,BZ) TO THE BPE
BX = STAX(J)*TZ-STAZ(J)*TY
BY = STAZ(J)*TX-STAX(J)*TZ
BZ = STAX(J)*TY-STAY(J)*TX
WRITE(6,222) COF,BX,BY,BZ,J,STAX(J),STAY(J),STAZ(J),TX,TY,TZ
C222 FORMAT(/1X,'COF ETC ',4F8.4,I2,2X,6(F8.4,2X))
BSQAR=BX**2+BY**2+BZ**2
IF(BSQAR.EQ.0.)THEN
IRES=0
250 ENDIF
COF = WT(IL(I))/BSQAR
C COMPUTE THE WEIGHTED C MATRIX
C11 = C11+COF*BX**2
C22 = C22+COF*BY**2
C33 = C33+COF*BZ**2
C12 = C12+COF*BX*BY
C13 = C13+COF*BX*BZ
C23 = C23+COF*BY*BZ
30 CONTINUE
B = (C11+C22+C33)/2.
C = (C11*(C22+C33)+C22*C33-(C12**2+C13**2+C23**2))/2.
D = DSQRT(B**2-C)
E2 = B-D
E3 = B+D
C COMPUTE THE CONFIDENCE REGION PARAMETERS
SMA = FACT/DSQRT(E2)
SMI = FACT/DSQRT(E3)
CALL EVECT(E2,C11,C22,C33,C12,C13,C23,X,Y,Z)
IF ((X*TY-Y*TX).GT.0.) Z = -Z
TEMP = Z/DSQRT(1.-T2**2)
IF (1. .GE. ABS(TEMP)) GO TO 20
ORIEN = 0.
GO TO 22
CONTINUE
ORIEN = DACOS(TEMP)/DTR
22 CONTINUE
AREA = 3.14159*SMA*SMI
C CIRCULAR REGION APPROXIMATION (NOT OUTPUT IN THIS VERSION)
RATIO=(SMI/SMA)
RADIUS=SMA*(.29294*(RATIO**2)-.063163*RATIO+.76996)
250 RETURN
END
SUBROUTINE NETFIX (I1, A$1, I2, A$2, XTL, XTLO)

THIS PROGRAM COMPUTES A TWO STATION FIX

INPUT: XSL1 = LAT OF STATION 1 (DEGREES)
       XSL01 = LON OF STATION 1 (DEGREES)
       AZ1 = OBSERVED BEARING (DEGREES)
       XSL2 = LAT OF STATION 2 (DEGREES)
       XSL02 = LON OF STATION 2 (DEGREES)
       AZ2 = OBSERVED BEARING (DEGREES)

OUTPUT: XTL = LAT OF TARGET (DEGREES)
         XTLO = LON OF TARGET (DEGREES)

MODIFIED FOR USE ON EADS
JAN 1987

IMPLICIT DOUBLE PRECISION(A-H,O-Z)
DOUBLE PRECISION LAD(4), LAM(4), LAS(4), LOD(4), LO(4), LOS(4)
DATA LAD/34., 35., 35., 34./
A, LAM/38., 23., 50., 43./
B, LAS/57., 57., 15., 00./
C, LOD/86., 86., 87., 87./
D, LO/40., 26., 52., 20./
E, LOS/09., 38., 54./
XSL1=LAD(I)+LAM(I)+LAS(I)/60.
XSL01=LOD(I)+LOM(I)+LOS(I)/60.
XSL2=LAD(I)+LAM(I)+LAS(I)/60.
XSL02=LOD(I)+LOM(I)+LOS(I)/60.
WRITE(6,901) XSL1, XSL01, AZ1, XSL2, XSL02, AZ2
901 FORMAT (IX, ' NETFIX... XSL1, XSL01, AZ1, XSL2, XSL02, AZ2 ',6(F10.4,1X) )

PI=3.14159265359
RAD=57.295795131
XSL1=XSL1/RAD
XSL01=XSL01/RAD
XSL2=XSL2/RAD
XSL02=XSL02/RAD

COMPUTE RANGE (RADIANS) BETWEEN SITES
AZ1=AZ1/RAD
AZ2=AZ2/RAD
WRITE(6,19) XSL1, XSL01, XSL2, XSL02, AZ12, RN
19 FORMAT(IX, 'BEFORE AZRN ',6F12.4)
CALL AZRN(XSL1, XSL01, XSL2, XSL02, AZ12, RN)

COMPUTE INCLUDED ANGLES
ANG1=ABS(AZ12-AZ1)
IF(ANG1.GT.PI) ANG1=2.*PI-ANG1
ANG2=ABS(AZ12-AZ2)
IF(ANG2.GT.PI) ANG2=2.*PI-ANG2

COMPUTE 1/2 SUM AND DIFFERENCE ANGLES
SUM=0.5*(ANG2+ANG1)
DIFF=0.5*(ANG2-ANG1)

SET UP FOR HAPIER ANALOGY SOLUTION
SINS=DSIN(SUM)
SIND=DSIN(DIFF)
COS=DCOS(SUM)
COSD=DCOS(DIFF)
TANR=DTAN(0.5*RN)

COMPUTE LENGTH OF SIDE OPPOSITE STATION 2
ALN=DATAN(TANR*SIND/SINS)+DATAN(TANR*COSD/COS)

COMPUTE LOCATION OF TARGET
CALL LOCLO(XSL1, XSL01, ALN, AZ1, XTL, XTLO)
XTL=XTL*RN
XTLO=XTLO*RN
WRITE(6,46) XTL, XTLO
46 FORMAT(IX, ' NETFIX... XTL, XTLO ',F12.7,2X,F12.7)
RETURN
END
SUBROUTINE LOCLO(XLA, XLOT, RT, AT, XLATD, XLOTD)

CC LOCLO COMPUTES THE LATITUDE AND CC
CC LONGITUDE OF A POINT GIVEN:
CC XLA - LAT OF REFERENCE PT (RAD) CC
CC XLO - LONG OF REF PT (RAD) CC
CC RT = GCB FROM REF PT TO TARGET CC
CC AT = OBSERVED AZIMUTH (RAD) CC
CC OUTPUT: XLATD = LAT OF PT IN QUESTION (RAD) CC
CC XLOTD = LON OF PT IN QUESTION (RAD) CC

C
IMPLICIT DOUBLE PRECISION(A-H,O-Z)
PI=3.1415926535
XAT=AT
XLO=XLOT
IF(AT.GT.PI) XAT=DABS(2.*PI-AT)
XLATL=DCOS(DCOS(XLARL)*DCOS(RT)+DSIN(XLARL)*DSIN(RT)*DCOS(XAT))
IF(XLATL.LE.0.) XLATL=PI+XLATL
XLO=2.*XLATL
BETAL=ATAN2((DSIN(.5*(RT-XLARL))*DCOS(.5*XAT)),(DSIN(.5*XAT)))
BETAL=ATAN2((DCOS(.5*(RT-XLARL))*DCOS(.5*XAT)),(DCOS(.5*XAT)))
GAM=BETAL+BETAL
IF(GAM.LT.0.0) GAM=(2.*PI)+GAM
XLOTD=XLO+GAM
IF( (XLOTD+XLOT).LT.0.) XLOTD=2.*PI-XLOTD
RETURN
END

SUBROUTINE AZRN(XSL, XSLO, XTL, XTLO, AZ, R)

CC THIS ROUTINE COMPUTES THE RANGE AND AZIMUTH CC
CC FROM ONE GEOGRAPHICAL COORDINATE TO ANOTHER CC
CC INPUT: XSL = LAT OF POINT S (RADIANS) CC
CC XSLO = LON OF POINT S (RADIANS) CC
CC XTL = LAT OF POINT T (RADIANS) CC
CC XTLO = LON OF POINT T (RADIANS) CC
CC OUTPUT: R = RANGE (RADIANS) CC
CC AZ = AZIMUTH (RADIANS) CC

C
IMPLICIT DOUBLE PRECISION(A-H,O-Z)
C
C INITIAlIZE EAST WEST FLAG
C WRITE(6,33) XSL,XSLO,XTL,XTLO,AZ,R
33 FORMAT(IX,'AZRN INPUT.. ',6F12.6)
IWE=0
PI=3.1415926535
C
A : POLAR ANGLE OF SITE AND TARGET
A=DABS(XSLO-XTLO)
C
BRANCH IF A IS GREATER THAN PI RADIANS
IF(A.GE.PI) GO TO 10
A=2.*PI-A
IWE=1
C
DS = DISTANCE (RAD) FROM SITE TO NORTH POLE
DS=PI/2.-XSL
C
DT=DISTANCE (RAD) FROM TARGET TO NORTH POLE
DT=PI/2.-XTL
C
AZ = .5*(AZ-B)
AZ1=ATAN2(DCOS(.5*A)*DSIN(.5*(DT-DS)), DSIN(.5*A)*DSIN(.5*(DT+DS)))

AZ2 = 0.5*(AZ+B)
AZ2 = ATAN2(DCOS(.5*A)*DCOS(.5*(DT-DS)), DSIN(.5*A)*DCOS(.5*(DT+DS)))
AZ = AZ1 + AZ2

USE LAW OF COSINES FOR SIDES

R = DCOS(DS)*DCOS(DT) + DSIN(DS)*DSIN(DT)*DCOS(A)

DISTANCE CANNOT BE NEGATIVE

IF(R.LT.0.) R = R

Determine which way target is from site

IF((IEAWE.EQ.1) .AND. (XSLO.LT.XTLO)) AZ = 2.*PI - AZ
IF((IEAWE.EQ.0) .AND. (XTLO.LT.XSLO)) AZ = 2.*PI + AZ

AZ CANNOT BE GT 2*PI OR LT 0.

IF(AZ.GE.2.*PI) AZ = AZ - 2.*PI
IF(AZ.LT.0.) AZ = AZ

CONVERT RADIIANS TO KM

AZ = 2.*PI - AZ
RETURN

END

SUBROUTINE YDDMY(SYD, DAY, MON, YR)

IMPLICIT INTEGER (A-Z)
DIMENSION DN(12,2)
DATA DN/31,59,90,120,151,181,212,243,273,304,334,365,
* 31,60,91,121,152,182,213,244,274,305,335,366/

YY = INT(SYD/1000)
DDD = MOD(SYD, 1000)
ILY = 1
IF (MOD(YY, 4) .EQ. 0) ILY = 2
DO 20 ID = 1, 12
IF (DDD.LT.DN(ID, ILY)) GOTO 21
20 CONTINUE

MON = ID
DAY = DDD
IF (MON.GT.1) DAY = DDD - DN(MON - 1, ILY)
YY = YY
WRITE(6,1000) SDY, MON, DAY, YR
100 FORMAT(1X, 'YYDDD', 4I6)
RETURN

END

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APPENDIX C: CLUSTER ANALYSIS SOFTWARE

The cluster analysis software consists of programs to read the lightning data from a disk file, convert the lightning locations from spherical earth coordinates to rectangular coordinates within a user defined region from a user defined reference point, compute an extrapolation vector, compute seed points for input to the K-means algorithm, and write the results to an output file.
CLUSTERING ALGORITHM FOR LTG DATA AND FCSTS OF Y/N AHEAD CC
GRID. READ FROM STAT2A WITH X,Y DATA, NOT LDATA CC

IMPLICIT DOUBLE PRECISION (A-H,O-Z)

COMMON/SOLN/NEVT, TSOLN, IYR, MON, NDAY, TLAT, TLON, SNKA, SSGNL, NRS, SNUM(100,100), AGRID(100,100), ANUM(100,100), A(8000,2)

INTEGER SNUM, ANUM

INTEGER IFRQ, IPRINT, IWT, KK, LDCM, LDSWT, LDX, MAXIT, JCOL, NOBS, NV, NVAR

INTEGER IC(NOBS), IND(NVAR), NOB1, NV1

REAL CM(KK, 'VAR), NC(KX), SWT(KK, NVAR), WSS(KK), SMTRIX(NOBS, SNUM, ANUM)

CHARACTER ILABEL(NOBS), ICHR(26)

EXTERNAL KMEAN, WRIRN

DATA ICHR/'A','B','C','D','E','F','G','H','I','J','K','L','M','N','O','P','Q','R','S','T','U','V','W','X','Y','Z'/

DATA IND/I, 2/

DATA CN(I, I), CM(I, 2), CM(2, 1), CM(2, 2)/10., 60., 0., 0./

DATA CM(3, 1), CM(3, 2), CM(4, 1), CM(4, 2)/-10., 60., 0., 0./

DATA CM(5, 1), CM(5, 2), CM(6, 1), CM(6, 2)/-50, -30./

DATA FL_LAT, FL2LON, D_/0.

DATA CP2LAT, CP2LON/0.

DATA BMALAT, BNALON/0.

DATA CP4LAT, CP4LON/0.

DATA ATIME, BTIME, BTIME/223000., 223500., 223443./

DATA A_, BLAT, ALAT, BLON/34.0, 36.5, 86.0/
Y = COS(A) * R
IF (Y .LE. 100) GOTO 10
X = X + DX
Y = Y + DY
SMTRIX(KNUMB+1,1) = X
SMTRIX(KNUMB+1,2) = Y
XB = XB + X
YB = YB + Y
KNUMB = KNUMB + 1
GOTO 10

DO 60 CONTINUE
WRITE (6, 2211) NOBS, NVAR, LDX, IFREQ, IWT, (IND(J), J=1, NVAR),
  
K, MAXIT
2211 FORMAT (1H1, 1X, 'NOBS ETC...', 918/)
C
C.. ASSEMBLE 5 MIN GRIDS
C
37 DO 40 I = 1, KNUMB
      J1 = 20 + INT(SMTRIX(I, 1)/10.)
      J2 = 20 - INT(SMTRIX(I, 2)/10.)
      LGRID(IL, JL, KZ) = LGRID(IL, JL, KZ) + 1
WRITE (6, 2882) I, KNUMB, IL, JL, KZ
WRITE (6, 2883) (SMTRIX(I, JJ), JJ = 1, 2), LGRID(IL, JL, KZ)

2882 FORMAT (9X, I4, 2X, I6, 2X, 5 (I4, 2X))
2883 FORMAT (ZX, 2 F8.2, 2X, I6)
40 CONTINUE
C
C.. FIND MEAN X, Y OF DATA SET FOR SYSTEM MOTION VECTOR
C
XBAR(KZ) = XB/KNUMB
YBAR(KZ) = YB/KNUMB
WRITE (6, 2900) ATIME, BTIME, KNUMB, KZ, XBAR(KZ), YBAR(KZ), DX, DY, DT
DO 80 I = 1, 40
      J1 = 40 - I + 1
WRITE (6, 2902) J1, (LGRID(J1, JJ), JJ = 1, 40)

80 CONTINUE
C
C.. CALL MEAN(XBAR) x (CM(IK, JJ), JJ = 1, 2)
C
2213 DO 2230 IK = 1, K
WRITE (6, 2215) IK, (CM(IK, JJ), JJ = 1, 2)
2215 CONTINUE
C
IF (NOBS .NE. -1) STOP

CALL KMEAN(NOBS, JCOL, NVAR, SMTRIX, LDX, IFREQ, IWT, IND, KK, MAXIT,
  CM, LDCM, SWT, LDSWT, IC, NC, WSS)
CALL WRRRN('CM', KK, NVAR, CM, LDCM, 0)
CALL WRRRN('SWT', KK, NVAR, SWT, LDSWT, 0)
CALL WRRRN('IC', 1, NOBS, IC, 1, 0)
CALL WRRRN('NC', 1, KK, NC, 1, 0)
CALL WRRRN('WSS', 1, KK, WSS, 1, 0)
DO 2230 I = 1, NOBS
      ILABEL(I) = ICHR(IC(I))
C
WRITE (6, 2244) I, IC(I), ICHR(IC(I)), ILABEL(I)
C2244 FORMAT (1X, I6, 2X, 2F10.2, 2X, I4, 2X, A1)
2230 CONTINUE
WRITE (6, 6000)

6000 FORMAT (1H1/6X, 'I', 6X,'X','X', 'Y', 'X', 'Y','X', 'Y', 'X', 'Y'/)
DO 4000 I = 1, NOBS
WRITE (6, 4005) I, (SMTRIX(I, JJ), JJ = 1, NV), IC(I), ILABEL(I)
C
WRITE (6, 4005) I, (SMTRIX(I, JJ), JJ = 1, NV), IC(I), ILABEL(I)

4000 FORMAT (1X, I6, 2X, 2F10.2, 2X, I4, 2X, A1)
KZ = KZ + 1
XB = 0.
YB = 0.
ATIME = ATIME + TINCR
BTIME = BTIME + TINCR
STOP

993 STOP
END
THIS ROUTINE COMPUTES THE RANGE AND AZIMUTH FROM ONE GEOGRAPHICAL COORDINATE TO ANOTHER.

INPUT:  
XSL = LAT OF POINT S (DEGREES)  
XSLO = LON OF POINT S (DEGREES)  
XTL = LAT OF POINT T (DEGREES)  
XTLO = LON OF POINT T (DEGREES)

OUTPUT:  
R = RANGE (KM)  
AZ = AZIMUTH (RADIANS)

SUBROUTINE AZRN(XSL, XSLO, XTL, XTLO, AZ, R)
IMPLICIT DOUBLE PRECISION(A-H, O-Z)

IEAWE = 0
PI = 3.1415926535

A = DABS(XSLO - XTLO)

IF(A .LE. PI) GO TO 10
A = 2. * PI - A
IEAWE = I

DS = DISTANCE (RAD) FROM SITE TO NORTH POLE
DS = PI/2. - XSL

DT = DISTANCE (RAD) FROM TARGET TO NORTH POLE
DT = PI/2. - XTL

AZ1 = .5 * (AZ - B)
AZ1 = ATAN2(DCOS(.5*A) * DSIN(.5*(DT-DS)), DSIN(.5*A) * DSIN(.5*(DT+DS)))
AZ2 = ATAN2(DCOS(.5*A) * DCOS(.5*(DT-DS)), DSIN(.5*A) * DSIN(.5*(DT+DS)))
AZ = AZ1 + AZ2

USE LAW OF COSINES FOR SIDES
R = DCOS(DCOS(DS) * DCOS(DT) + DSIN(DS) * DSIN(DT) * DCOS(A))

DISTANCE CANNOT BE NEGATIVE
IF(R.LT.0.) R = R

DETERMINE WHICH WAY TARGET IS FROM SITE
IF((IEAWE .EQ. 1) .AND. (XSLO .LT. XTL)) AZ = 2.*PI - AZ
IF((IEAWE .EQ. 0) .AND. (XTLO .LT. XSLO)) AZ = 2.*PI - AZ

AZ CANNOT BE GT 2*PI OR LT 0.
IF(AZ .GT. 2.*PI) AZ = AZ - 2.*PI
IF(AZ .LT. 0.) AZ = -AZ

CONVERT RADIANS TO KM
AZ = 2.*PI - AZ
R = R * 6370.
RETURN
SEED ALGORITHM FOR LTG DATA AND FCSTS OF Y/N AHEAD

Program GRID21

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
COMMON/SOLN/NEVT, TSOLN, IYR, MON, NDAY, TLAT, TLON, SNKA, SSGNL, NRS, SMA, SMI, ORIEN, RADIUS, AREA
COMMON/SETGRD/NROW, NCOL, IMI, DPT, IRADI, INCR, X, Y, AZ
COMMON/CMI/LGRID(50,50,40), LNUM(100,100), SGRID(100,100), SNUM(100,100), AGGRID(100,100), ANUM(100,100), AGRID(100,100)

INTEGER SNUM, ANUM
INTEGER IFRQ, IPRINT, IIIT, KK, LDE, LDSWT, LDX, MAXIT
DIMENSION IN2(5), CBRG2(4), XBAR(40), YBAR(40), LSEED(40,40,40)
INTEGER IC(8000), IND(2), NOBI, NV, NVAR
REAL CM(80,2), NC(80), SWT(80,2), WSS(80), SMTRIX(8000,2)
equiVALENCE (LDCM, KK), (LDSWT, KK), (LDX, NOBS)
EXTERNAL KMEAN, WRI RN, WRRRN
DATA IND/1,2/
DATA IFRQ, IPRINT, IWT, MAXIT, JCOL, NV, NVAR/0,0,0,0,2,2,2/
DATA CM(I, I), CM(I, 2), CM(2, i), CM(2,2)/-152.2,-49.3,-18.8,-27.2/
DATA CM(3,1),CM(3,2),CM(4,1),CM(4,2)/-89.0,-5.3,-49.9,-20.1/
DATA CM(5,1),CM(5,2),CM(6,1),CM(6,2)/-72.2,15.1,-54.3,53.8/
DATA CM(7,1),CM(7,2),CM(8,1),CM(8,2)/-23.2,19.5,29.5,129.0/
DATA FL2LAT, FL2LON, DTR/0.604817,1.515038,0.017453/
DATA CP2LAT, CP2LON/0.608223,1.515513/
DATA BNALAT, BNALON/0.632682,1.510932/
DATA CP4LAT, CP4LON/0.606008,1.515445/
DATA ATIME,BTIME,BT_ME2/214500.,215000.,223443.0/
DATA ALAT, BLAT, ALON, BLON/34.0,36.5,86.0,90.0/

C ADD SYSTEM MOTION VECTOR HERE
C DT=0 GIVES OBSERVED DATA IF DX, DY USED THEN
C DT WILL BE 5-MIN TRANSLATION OF DATA AND CLUSTERS, DX, DY ARE
C SYSTEM DISPLACEMENT VECTORS FOR 5-MIN INTERVAL FOR EACH DT
C DT=2 IS 10 MIN ETC...

ISTOP=1
TINC=5000.
KZ=1
DT=0.
DX4.45*DT
DY=-7.95*DT
WRITE(6,2233) ATIME, BTIME, DT, DX, DY
2233 FORMAT(1HI/1X, 'TIME INTERVAL PROCESSED =',2XI0.2,' DT, DX, DY=',2F8.2/)
DO 5 LC=1,KK
CM(LC,1)=CM(LC,1)+DX
CM(LC,2)=CM(LC,2)+DY
5 CONTINUE
XB=0.
YB=0.
IDAY=15
KNUMB=0
IRADII=200
INCR=5
NROW=(IRADII*2)/INCR
NCOL=1
IMI=1
WRITE(6,2211) NOBS, NVAR, LDX, NOBS, NVAR, LDX, NOBS
2211 FORMAT(1HI1X, 'NOBS ETC... ',9I8/)
C.. ASSEMBLE 5 MIN GRIDS
C
DO 40 I=1,KNUMB
   JL=20+INT(SMTRIX(I,1)/10.)
   IL=20+INT(SMTRIX(I,2)/10.)
   LGRID(IL,JL,KZ)=LGRID(IL,JL,KZ)+1
   LSEED(IL,JL,KZ)=LGRID(IL,JL,KZ)
40 CONTINUE
C
C.. FIND SET OF SEEDS FROM DENSITY GRID
C
DO 90 IL=2,39
   DO 90 JL=2,39
      N1=LSEED(IL-1,JL-1,KZ)
      N2=LSEED(IL-1,JL,   KZ)
      N3=LSEED(IL-1,JL+1,KZ)
      N4=LSEED(IL ,JL-1,KZ)
      N5=LSEED(IL ,JL   ,KZ)
      N6=LSEED(IL ,JL+1,KZ)
      N7=LSEED(IL+1,JL-1,KZ)
      N8=LSEED(IL+1,JL,   KZ)
      N9=LSEED(IL+1,JL+1,KZ)
      N10=LGRID(IL,JL,KZ)
      N11=LGRID(IL+1,JL,KZ)
      N12=LGRID(IL-1,JL,KZ)
      N13=LGRID(IL,JL-1,KZ)
      N14=LGRID(IL,JL+1,KZ)
      N15=LGRID(IL+1,JL-1,KZ)
      N16=LGRID(IL+1,JL+1,KZ)
      NS=LSEED(IL,JL,KZ)
      N5=N5
      N6=N6
      N7=N7
      N8=N8
      N9=N9
      N10=N10
      N11=N11
      N12=N12
      N13=N13
      N14=N14
      N15=N15
      N16=N16
         * OR.N5.LE.N7.OR.N6.LE.N8.OR.N5.LE.N9) LSEED(IL,JL,KZ)=0
         * LSEED(IL,JL,KZ)=0
90 CONTINUE
WRITE(6,190) ATIME,BTIME
190 FORMAT(1H1/IX,'TIME INTERVAL',2F10.2/2X,'FIRST GUESS FOR SEEDS'/)
   DO 95 IJ=1,40
      WRITE(6,2902) IJ,(LSEED(IJ,JJ,KZ),JJ=1,40)
95 CONTINUE
WRITE(6,2904) (I,I=1,40)
C
C.. CONVERT GRID POINT INDEXES INTO X,Y POINTS FOR SEEDING ClUSTERS
C
KK=0
   DO 110 I=1,40
      DO 110 J=1,40
         IF(LSEED(I,J,KZ).GT.0)
            THEN
               K1=KK+1
               CM(K1,1)=FLOAT(J-20)*10.+DX
               CM(K1,2)=FLOAT(20-I)*10.+DY
            ENDIF
      END DO
      WRITE(6,2220) I,J,K1,CM(K1,1:2)
      WRITE(6,2902) I,J,(LGRID(I,J,JJ,KZ),JJ=1,40)
      WRITE(6,2904) (I,I=1,40)
      KK=KK+1
110 CONTINUE
C
C.. FIND MEAN X,Y OF DATA SET FOR SYSTEM MOTION VECTOR
C
XBAR(KZ)=XB/KNUMB
   YBAR(KZ)=YB/KNUMB
WRITE(6,2900) ATIME,BTIME,KNUMB,KZ,XBAR(KZ),YBAR(KZ),DX,DY,DT
2900 FORMAT(1H1/IX,'ATIME',BTIME,'KNUMB',KZ,'XBAR',YBAR,'DX','DY','DT',
      * ' XBAR=','YBAR=',DAType=,'INTERVAL=',F10.2,'XBAR=',F10.2,'YBAR=',F10.2,'DX=',F8.2,'DY=',F8.2/)
C
DO 80 IJ=1,40
   WRITE(6,2902) IJ,(LGRID(IJ,JJ,KZ),JJ=1,40)
2902 FORMAT(1X,I2,40(1X,I2))
80 CONTINUE
C
WRITE(6,2904) (I,I=1,40)
2904 FORMAT(3X,40(1X,I2))
C
WRITE(6,1299) NOBS,KK,XBAR(KZ),YBAR(KZ),DX,DY,ATIME,BTIME
1299 FORMAT(1X,'NOBS',SEEDS=',I4,ZX,'XBAR=',I4,'YBAR=',I4,'DX=',F8.2,'DY=',F8.2,'INTERVAL=',
      * 'F12.3X','F12.3'
      IF(ISTOP.EQ.I) STOP 993
993 STOP
END
BIBLIOGRAPHY


PREDICTING THUNDERSTORM EVOLUTION
USING GROUND-BASED LIGHTNING DETECTION NETWORKS

By Steven J. Goodman

The information in this report has been reviewed for technical content. Review of any information concerning Department of Defense or nuclear energy activities or programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

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