Wind Tunnel Wall Effects in a Linear Oscillating Cascade

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Prepared for the
36th International Gas Turbine and Aeroengine Congress and Exposition
sponsored by the American Society of Mechanical Engineers
Orlando, Florida, June 3–6, 1991
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ABSTRACT

Experiments in a linear oscillating cascade reveal that the wind tunnel walls enclosing the airfoils have, in some cases, a detrimental effect on the oscillating cascade aerodynamics. In a subsonic flow field, biconvex airfoils are driven simultaneously in harmonic, torsion-mode oscillations for a range of interblade phase angle values. It is found that the cascade dynamic periodicity - the airfoil-to-airfoil variation in unsteady surface pressure - is good for some values of interblade phase angle but poor for others. Correlation of the unsteady pressure data with oscillating flat plate cascade predictions is generally good for conditions where the periodicity is good and poor where the periodicity is poor. Calculations based upon linearized unsteady aerodynamic theory indicate that pressure waves reflected from the wind tunnel walls are responsible for the cases where there is poor periodicity and poor correlation with the predictions.

NOMENCLATURE

\( C \)  airfoil chord
\( C_m \)  unsteady aerodynamic moment coefficient,
\[ \int_0^1 (x^*_a - x^*) \Delta C_p(x^*) dx^* \]
\( C_p \)  unsteady pressure coefficient, \( p_i/(\rho_iV_{in}^2\alpha_i) \)
\( \bar{C}_p \)  steady pressure coefficient, \( (\bar{p}_{in} - \bar{p}_o)/(\rho_iV_{in}^2) \)
\( \hat{C}_p \)  unsteady pressure influence coefficient for the \( n^{th} \) airfoil
\( \hat{e} \)  unit vector
\( k \)  reduced frequency, \( \omega C/V_{in} \)
\( \Delta C_p \)  unsteady pressure difference coefficient
\( \delta C_p \)  dynamic periodicity magnitude difference
\( \delta \phi \)  dynamic periodicity phase difference
\( \rho_{in} \)  inlet density
\( \omega \)  airfoil oscillation frequency
INTRODUCTION

Oscillating cascade experiments play an important role in the development of advanced cascade unsteady aerodynamic analyses, providing data used to both evaluate existing analyses and provide direction for future modeling efforts.

Both linear and annular cascades have been used to investigate the aerodynamics of airfoils driven in controlled, harmonic, fixed interblade phase angle traveling-wave mode oscillations. Certain advantages and disadvantages are associated with these two types of facilities. In particular, a highly two-dimensional flow field may be obtained in a linear cascade while, in an annular cascade, undesirable three-dimensional effects may be problematic. However, while the annular cascade appears to be inherently dynamically periodic, i.e., the temporal fluctuations in the flow field vary from passage to passage according to the interblade phase angle, periodicity is less certain in the linear cascade due to the finite extent of the cascade and the boundaries introduced by the wind tunnel walls. The potential for wall interference is indicated by a number of papers devoted to the effects of wind tunnel walls on a single oscillating airfoil, for example, Jones (1943) and Runyan et al. (1955).

Carta (1983) was the first researcher to investigate the dynamic periodicity of a linear oscillating cascade. His cascade consisted of 11 NACA 65-series airfoils staggered at 30 degrees with 1.33 solidity. Airfoil surface unsteady pressures were measured at reduced frequency values ranging from 0.14 to 0.30 over a wide range of interblade phase angles with a low subsonic inlet Mach number, M = 0.18. Good dynamic periodicity was generally found except for in-phase oscillations, where circumferential gradients in the phase of the unsteady pressure coefficient were found. Carta attributed this to an acoustic resonance condition in the cascade.

In the present study, the steady and unsteady aerodynamics of an oscillating, linear cascade are investigated in a low solidity configuration. The cascade solidity, 0.65, was chosen to be representative of an advanced propeller model which fluttered during wind tunnel performance tests (Mehmed et al., 1982). For an inlet Mach number of 0.55, the torsion mode biconvex airfoil oscillating cascade aerodynamics are quantified for reduced frequencies as high as 0.64 and a range of interblade phase angles. To help determine the validity of the data, an investigation is made into the steady state and dynamic periodicity of the cascade. Then the unsteady airfoil surface pressure data are correlated with the predictions of the linearized subsonic cascade. His cascade oscillation analysis of Smith (1972). Insight into the effect of the wind tunnel walls on the cascade unsteady aerodynamics is gained from the theoretical acoustic wave generation properties of an oscillating cascade.

OSCIILLATING CASCADE FACILITY

The NASA Lewis Transonic Oscillating Cascade, Figure 1, combines a linear cascade wind tunnel capable of inlet flow approaching Mach one with a high-speed airfoil drive system which imparts torsion-mode oscillations to the cascaded airfoils at specified interblade phase angles and realistic high reduced frequency values.

AIRFOILS AND INSTRUMENTATION

Figure 2 illustrates the airfoil and cascade geometry which is summarized in Table 1. Four uncambered, 7.6% thick biconvex airfoils are used to create a low solidity (C/S = 0.65) cascade. The stagger angle is 45 degrees and the airfoils oscillate about the midchord.

Airfoils instrumented with static pressure taps are used to measure the airfoil surface steady pressure distributions. There are sixteen chordwise measurement locations, with a higher density in the leading edge region used to capture the higher gradients there. Rows of sidewall static pressure taps located upstream and downstream of the cascaded airfoils are used to determine the mean inlet and exit pressures.
response, which varies with the airfoil velocity magnitude, is dominated by the acceleration response. Thus calibration data were obtained to correct the oscillating airfoil pressure data for acceleration effects.

The time-variant position of the reference oscillating airfoil is determined by a capacitance-type proximity sensor which produces a voltage proportional to the air gap between it and an adjacent object. This sensor is positioned to face a six cycle sinusoidally-shaped cam mounted on the airfoil drive camshaft so as to be in phase with the reference airfoil motion.

DATA ACQUISITION AND ANALYSIS

Conventional instrumentation is used to quantify the steady flow field. An average of the upstream sidewall static pressures along with the atmospheric pressure (total) are used to calculate the inlet Mach number. Steady flow airfoil surface static pressures are calculated from an average of approximately 100 samples. The steady pressure coefficient is defined in Equation 1.

\[ \bar{C}_p(x) = \frac{p_{in} - p_0(x)}{\rho_{in}V_{in}^2} \]  

\( p_{in} \) is the mean inlet static pressure, \( p_0 \) is the time-average airfoil surface static pressure at the chordwise coordinate \( x \), and \( p_{in} \) and \( V_{in} \) are the inlet values of density and velocity.

Unsteady signals are recorded on magnetic tape for post-experiment processing. During tape playback, the signals are simultaneously digitized at rates sufficient to capture at least three harmonics of the oscillation frequency, with 32,768 samples taken per channel. Each channel of data is divided into contiguous blocks, each block typically with 2048 samples, and then Fourier decomposed to determine the first harmonic of each block of data. The first harmonic pressure of each block is referenced to the airfoil motion by subtracting from it the phase of the first harmonic motion signal of the same block. Once all of the blocks from a channel are decomposed in this manner, the first harmonic block results are averaged and the complex-valued acceleration response is subtracted vectorially. Statistical analysis of the block results is used to estimate uncertainties for the average first harmonic values.

In these experiments, the motion of the \( n^{th} \) airfoil is defined by the change in the incidence angle with time:

\[ \alpha(t) = \alpha_0 + \alpha_1 \Re\{e^{i(\omega t + \beta)}\} \]  

where \( \alpha_0 \) is the mean incidence angle, \( \alpha_1 \) is the torsional oscillation amplitude, \( \omega \) is the frequency, \( \beta \) is the interblade phase angle and \( \Re \) denotes the real part.

The complex-valued unsteady pressure coefficient is defined as

\[ C_p(x) = \frac{P_1(x)}{\rho_{in}V_{in}^2 \alpha_1}. \]
\( p_l \) is the first harmonic airfoil surface static pressure. The dynamic pressure difference coefficient is the difference between the lower and upper surface unsteady pressure coefficients:

\[
\Delta C_p = C_{p_l} - C_{p_u}.
\]  (4)

**LINEARIZED ANALYSIS**

The experimental dynamic pressure difference coefficient data are correlated with the predictions of a computer program published by Whitehead (1987) which is based on the analysis of Smith (1972). This is a semi-analytical technique for determining the unsteady forces on an infinite cascade of flat plate airfoils subject to harmonic disturbances in an inviscid, isentropic, subsonic, unsteady flow. The analysis assumes that the airfoils are at zero mean incidence and the unsteadiness creates small disturbances to a uniform mean flow, resulting in a linear system of equations with constant coefficients. Additional analytical results which will be used in the results section are derived below.

**Wave Generation by an Oscillating Cascade**

The linearized conservation equations for mass and momentum may be expressed as

\[
\frac{\partial q}{\partial t} + A \frac{\partial q}{\partial \xi} + B \frac{\partial q}{\partial \eta} = 0
\]  (5)

where \( q \) is the matrix of perturbation variables

\[
q = \begin{pmatrix} p_1 \\ u_1 \\ v_1 \end{pmatrix}
\]  (6)

and the coefficient matrices \( A \) and \( B \) are constant, depending on the uniform mean flow.

\[
A = \begin{pmatrix} \rho_o & \rho_o \alpha^2 & 0 \\ 1/\rho_o & \rho_o & 0 \\ 0 & 0 & \rho_o \end{pmatrix}
\]  (7)

\[
B = \begin{pmatrix} \nu_o & 0 & \rho_o \alpha^2 \\ 0 & \nu_o & 0 \\ 1/\rho_o & 0 & \nu_o \end{pmatrix}
\]  (8)

\( \nu_o \) and \( \nu_o \) are the \( \xi \) and \( \eta \) components of the mean flow velocity, \( \rho_o \) is the mean density and \( \alpha_o \) is the mean speed of sound.

For an infinite cascade of equally-spaced airfoils oscillating harmonically at a fixed interblade phase angle, the dependent variables depend harmonically on the spatial position and time. Thus the perturbations are expressed as

\[
q = \begin{pmatrix} p_1 \\ u_1 \\ v_1 \end{pmatrix} = \begin{pmatrix} p_1 \\ u_1 \\ v_1 \end{pmatrix} e^{(\omega \xi + \xi \cdot \eta \xi \cdot \eta)} = \overline{q} e^{(\omega \xi + \xi \cdot \eta \xi \cdot \eta)}
\]  (9)

where \( \omega \) and \( \mu \) are the axial and circumferential wavenumbers and the quantities with overbars are complex. Substituting these perturbation expressions into Equation 5 and differentiating results in

\[
(\omega I + l A + m B)\overline{q} = 0
\]  (10)

where \( I \) is the identity matrix.

Equation 10 may be rearranged in the form of an eigenvalue problem, Equation 11, with the axial wavenumber \( l \) being the eigenvalue.

\[
( -A^{-1}(\omega I + m B) - l I ) \overline{q} = 0
\]  (11)

The eigenvalues are

\[
l = - \frac{(\omega + m \nu_o)}{\nu_o}
\]  (12)

and

\[
l = \frac{\nu_o (\omega + \nu_o m) \pm \alpha_o \nu_o (\omega + \nu_o m)^2 - (\alpha_o^2 - \nu_o^2) m^2}{\alpha_o^2 - \nu_o^2}
\]  (13)

As shown by Smith, the first eigenvalue, Equation 12, corresponds to convection of vorticity by the mean flow with no associated pressure fluctuations. This solution is of no further interest for this application. However, the eigenvalues of Equation 13, corresponding to irrotational pressure perturbations, are of interest.

The tangential wave number must satisfy cascade dynamic periodicity according to the interblade phase angle \( \beta \). Hence

\[
m S = \beta + 2 \pi r
\]  (14)

where \( S \) is the cascade spacing and \( r \) is an integer specifying the mode of the wave.

The nature of the acoustic waves produced by the cascade depends upon the term under the radical in Equation 13. Let \( \delta \) be that term:

\[
\delta = (\omega + \nu_o m)^2 - (\alpha_o^2 - \nu_o^2) m^2.
\]  (15)

When \( \delta = 0 \), the acoustic resonance condition, only one wave is created which propagates in the circumferential direction. The resonant values of the interblade phase angle are determined by solving Equation 15, with \( \delta = 0 \), for \( m \), then using Equation 14 to determine \( \beta \) for \( r = 0 \). The result is:

\[
\beta^* = \frac{2 k M S}{C(1 - M^2)} ( M \sin (\alpha_o + \gamma) + \sqrt{1 - M^2 \cos^2 (\alpha_o + \gamma)}).
\]  (16)

When \( \delta > 0 \), both values of \( l \) as specified by Equation 13 are real, thus there are two acoustic waves which propagate undiminished, one going upstream and the other downstream. This behavior is termed superresonant when the mean flow field is subsonic.
The final case is \( \delta < 0 \), the subsonic cascade subresonant condition. \( l \) is now complex, and may be expressed as \( l = l^r + il^i \) with the real and imaginary parts determined from Equation 13 to be

\[
l^r = \frac{u_0(\omega + u_0 m)}{\alpha_0^2 - u_0^2} \tag{17}
\]

and

\[
l^i = \frac{\alpha_0}{\omega - u_0} \sqrt{[\omega + u_0 m]^2 - (\alpha_0 - u_0^2) m^2} \tag{18}
\]

In this case, the pressure perturbation is of the form

\[
p_1 - \bar{p}_1 = e^{i(\omega r + (\xi + \eta) t + m n)}
\]

\[
- \bar{p}_1 e^{-il^i t} e^{i(\omega r + (\xi - \eta) t + m n)} \tag{19}
\]

Thus the wave decays exponentially with \( \xi \), depending upon the imaginary part of \( l^i \):

\[
\left| \frac{p_1(\xi)}{\bar{p}_1} \right| = e^{-|l^i| \xi} \tag{20}
\]

where, from Equation 19, \( |\bar{p}_1| \) is equal to \( |p_1(\xi = 0)| \). The absolute value of the exponent disallows the non-physical case of an amplifying wave.

The axial and tangential wave numbers specify the acoustic wave propagation direction relative to \( \xi^r, \eta^r \) coordinates which are parallel to the fixed \( \xi, \eta \) coordinate system but moving with the mean flow velocity (Whitehead, 1987). Relative to the \( \xi^r, \eta^r \) coordinates, the waves propagate at the angle

\[
\theta^r = \tan^{-1} \left( \frac{\eta}{-\xi^r} \right) \tag{21}
\]

with the speed of sound. The wave propagation vector \( V_p \) in the \( \xi, \eta \) coordinate system is therefore the sum of the steady flow velocity vector and the wave propagation vector relative to the moving coordinate system,

\[
V_p = (u_0 + \alpha_0 \cos \theta^r) \bar{e}_\xi + (v_0 + \alpha_0 \sin \theta^r) \bar{e}_\eta. \tag{22}
\]

It follows that the direction of propagation \( 0 \) in the \( \xi, \eta \) coordinate system is

\[
0 = \tan^{-1} \left( \frac{v_0 + \alpha_0 \sin \theta^r}{u_0 + \alpha_0 \cos \theta^r} \right). \tag{23}
\]

### Influence Coefficient Technique

In this analysis, the superposition principle is valid since the system of governing small disturbance equations is linear. The unsteady pressure difference coefficient on a reference airfoil (airfoil 0) for fixed values of the inlet Mach number, reduced frequency and cascade geometry may be expressed as the sum of the effects of the oscillations of the reference airfoil itself and the other airfoils in the cascade, Equation 24.

\[
\Delta C_p(x, \beta) = \sum_{n=1}^{N} \Delta C_p^n(x) e^{i n \beta} \tag{24}
\]

where the complex-valued influence coefficient \( \Delta C_p^n \) expresses the influence that the oscillations of airfoil \( n \) have on the pressure difference coefficient of the reference airfoil. \( N = \infty \) for an infinite cascade.

By inversion of Equation 24, an expression for the influence coefficients is

\[
\Delta C_p^n(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \Delta C_p(x, \beta)e^{-i n \beta} d\beta. \tag{25}
\]

Unsteady aerodynamic influence coefficients may thus be determined from oscillating cascade analyses by integrating the analytical predictions over the complete interblade phase angle interval. Using these influence coefficients for a finite value of \( N \) in Equation 24 then enables analytical results for a finite number of airfoils oscillating in an infinite cascade to be determined.

### RESULTS

The effect of wind tunnel walls on the aerodynamics of a low solidity linear oscillating cascade is investigated using experimental and analytical techniques. For a mean inlet Mach number of 0.55 and 2 degrees mean incidence, the airfoil surface steady pressure coefficient distributions are presented first, followed by dynamic periodicity data and correlation of the unsteady pressure difference coefficient data with linearized analysis predictions. Linearized analysis is then used to ascertain the effects of the wind tunnel walls on the cascade unsteady aerodynamics.

#### Steady State Aerodynamics

For a linear cascade to be a valid simulation of a turbomachine blade row, the cascade must exhibit good passage-to-passage periodicity for the steady flow field. Thus airfoil surface steady pressure coefficient distributions are obtained for multiple passages in the low solidity cascade. In Figure 3, steady pressure coefficient data are presented for the center cascade.

![Figure 3 Airfoil surface steady pressure coefficient distributions, M=0.55, \( \alpha_0=2 \) degrees](image)
passage and the two adjacent passages. Good cascade periodicity is apparent, with the only significant differences found at the leading edge of the airfoil upper surface.

The upper surface pressure coefficient distribution peaks near the leading edge and the pressure difference tends toward zero near the trailing edge. There is negligible loading beyond 50% of chord. Using the method of Kline and McClintock (1953), a 95% confidence interval of ±0.003 is estimated for these pressure coefficients. The mean exit region static pressure divided by the inlet total pressure was 0.8251.

**Unsteady Aerodynamics**

Figure 4 illustrates typical subsonic cascade behavior in terms of the wave propagation modes predicted by the linearized analysis. Acoustic resonances at positive and negative interblade phase angle values, $\beta^*$, bracket the wave-propagating superresonant region which includes $\beta = 0$. When $\beta > \beta^*$ or $\beta < \beta^*$, the cascade is subresonant and the waves decay. Included in this figure are the resonant values of interblade phase angle, Equation 16, for the experimental conditions.

The unsteady aerodynamic experiments discussed herein include subresonant and superresonant values of $\beta$. Airfoil surface unsteady pressure distributions are obtained for interblade phase angle values of 0, 45, -45, 90, -90 and 180 degrees at reduced frequency values of 0.40 and 0.64. Sample time-variant pressure signals presented in Figure 5 along with the resulting pressure spectra illustrate the dominance of the first harmonic component as typically found in the signals. 95% confidence intervals of ±5% in magnitude and ±3 degrees in phase are estimated for the mean value of the unsteady pressure coefficients.

**Dynamic Periodicity.** Oscillating cascade data were first obtained by positioning the two instrumented airfoils to measure the unsteady pressures on the airfoil surfaces which define the center passage of the cascade. The dynamic periodicity of the cascade was investigated by subsequently positioning the instrumented airfoils to measure the opposite surfaces of the two center airfoils positions, thus giving dynamic pressure measurements for both surfaces of the two most centrally located airfoils.

First harmonic unsteady pressure coefficient periodicity data are presented in Figure 6 for $k=0.40$ and $\beta=-45$ degrees. To simplify the discussion of these results, the two instrumented airfoils will be referred to as A and B as labeled in the figure. The data indicate that the dynamic periodicity is excellent in both magnitude and phase for the airfoil upper surface. Although the lower surface periodicity is good, the magnitudes tend to be larger on airfoil A, particularly over the forward half of the airfoil. There are also small but noticeable phase differences in the midchord region on the lower surface.

To aid the presentation of the periodicity data, two new quantities are defined. The dynamic periodicity magnitude and phase differences, $\delta C_{p}$ and $\delta \phi$, are defined in Equations 26.
and 27 for each airfoil surface. Ideally, both of these quantities will be zero.

\[
\delta C_p = \frac{|C_p^4| - |C_p^b|}{\frac{1}{2}(|C_p^4| + |C_p^b|)}_{x/C=0.12} 
\]

(26)

\[
\delta \phi = \phi_p^4 - \phi_p^b 
\]

(27)

Figure 7 Dynamic periodicity difference, \(k=0.40\), \(\beta = -45\) degrees

Figure 7 presents the dynamic periodicity data determined from the data presented in Figure 6. Both the excellent upper surface periodicity and defects in the lower surface periodicity are clearly revealed.

The oscillating cascade periodicity is now investigated as a function of reduced frequency and interblade phase angle using the quantities \(\delta C_p\) and \(\delta \phi\). Reduced frequency crossplots of the periodicity data for \(k=0.40\) and 0.64 are presented in Figures 8 through 13 for several values of interblade phase angle. Beginning with \(\beta = 0^\circ\), Figure 8, the dynamic periodicity is generally poor in both magnitude and phase for these superresonant conditions, and reduced frequency has little effect on the results. The dynamic periodicity is improved somewhat for \(\beta = 45^\circ\), Figure 9, but the small values of \(\delta \phi\) occurring over the forward half of the airfoil are in contrast to the very large values of \(\delta \phi\) at 60 and 75% of chord on the upper surface. Reduced frequency again has little effect on the results even though \(k=0.40\) corresponds to a subresonant condition and \(k=0.64\) is superresonant. In contrast, the periodicity is generally good for \(\beta = -45^\circ\), Figure 10, where both reduced frequencies are subresonant. There, all of the values of \(\delta \phi\) are...
Figure 10 Dynamic periodicity difference, $\beta = -45$ degrees

Figure 11 Dynamic periodicity difference, $\beta = 90$ degrees

Figure 12 Dynamic periodicity difference, $\beta = -90$ degrees

Figure 13 Dynamic periodicity difference, $\beta = 180$ degrees
acceptably small as are the values of $\delta C_p$ except for the lower surface $k=0.40$ results. When $\beta = 90^\circ$, the periodicity is poor in both magnitude and phase for both subresonant conditions, Figure 11. But for $\beta = -90^\circ$, Figure 12, dynamic periodicity is generally good for these subresonant conditions except for the upper surface magnitude difference at 12% of chord. Finally, for out-of-phase oscillations (subresonant), Figure 13, the magnitude differences are small, but the phase differences are not. Again, reduced frequency has little effect on the results.

To summarize these data, the dynamic periodicity is largely a function of the interblade phase angle, and the periodicity is acceptable only for $\beta = -45^\circ$ and $\beta = -90^\circ$. For both values of reduced frequency, these values of $\beta$ are predicted to be subresonant. However, subresonance does not guarantee good dynamic periodicity: for example, the periodicity is poor for $\beta = 90^\circ$, a subresonant value of the interblade phase angle for both values of reduced frequency.

**Correlation with Linearized Analysis.** The experimental dynamic pressure difference coefficient data are correlated with the predictions of the linearized unsteady cascade analysis for an infinite number of airfoils. In addition to the infinite cascade predictions, influence coefficient predictions for the effects of 5 oscillating airfoils ($N=2$ in Equation 24) will be presented.

For a reduced frequency of 0.64 and a range of interblade phase angle values, Figures 14 through 19 present correlations of the experimentally-determined airfoil surface unsteady pressure difference coefficient distributions with the linearized analysis predictions. For conditions where the cascade dynamic periodicity is good, $\beta = -45^\circ$ and $\beta = -90^\circ$, the correlations between the analytical results and the experimental data are good, Figures 14 and 15. Conversely, the data-analysis correlations are generally poor for conditions where the periodicity is poor, $\beta = 45^\circ, 90^\circ$ and $180^\circ$, Figures 16 through 18. At those conditions, the phase angle data-analysis correlation is consistently poor, with the experimentally-determined phases leading the predictions except near the trailing edge. An exception is $\beta = 0^\circ$, Figure 15.
19, for which the data-analysis correlation is good despite poor
dynamic periodicity.

The analytical influence coefficient predictions for $N = 2$ are
generally in very good agreement with the predictions for an
infinite cascade. This indicates that only a few oscillating airfoils
are required to model an infinite cascade under these conditions.
To consider this further, the imaginary part of the unsteady aerodynamic moment coefficient is presented as a function of
interblade phase angle in Figure 20. Linearized analysis results
are shown for $N = \infty$, $N = 6$ and $N = 2$. The predictions for
$N = 2$ and $N = 6$ are in very good agreement with the infinite
cascade results except in the vicinity of the acoustic resonances.
At those points, a large number of terms in the Fourier series are
required to describe the rapidly changing moment coefficient.
This indicates that acoustic resonances will not occur in linear
oscillating cascade experiments due to the limited number of
airfoils.

**Wind Tunnel Wall Effects.** The cascade dynamic periodicity and correlation of the experimental data with the linearized analysis have been shown to vary greatly with
interblade phase angle. How these correlations can be very
good for some interblade phase angles but poor for others leads
one to question the effects of the wind tunnel walls on these
results. In this section, linearized analysis is used to gain insight
into oscillating cascade/wind tunnel wall interactions.

The calculated direction of acoustic wave propagation \( \theta \) is
shown in Figure 21 for the low solidity cascade geometry with

\[ M = 0.55 \text{ and } k = 0.64. \]

For any one value of \( \beta \) in the
superresonant region, two waves are produced, one traveling
upstream (in the \(-\xi\) direction) and the other going downstream
(in the \(\xi\) direction). Outside this region, the oscillating cascade
produces subresonant waves which travel downstream.
Acoustic resonances occur at the boundaries between the
subresonant and superresonant regions, with pressure
disturbances propagating along the cascade in the \( \pm \eta \)
directions.

Values of the initial magnitude of the outgoing pressure
disturbance, computed using Whitehead's computer program,
are presented in Figure 22 in the format of an unsteady pressure
coefficient magnitude at the leading edge of the cascade,
\( |C_p(\xi = 0)| \). Peaks in the largest initial disturbance amplitudes
are found in the vicinity of the acoustic resonances, \( \beta = \beta^* \).
Outside the near-resonance regions, relatively large amplitudes
occur at positive subresonant values of the interblade phase
angle, \( \beta > \beta^* \), but in general the amplitudes are of the same
order of magnitude throughout the interblade phase angle range.

The disturbances will propagate or decay exponentially with
distance from the cascade according to the imaginary part of the
axial wavenumber, \( l' \), Equation 20. As shown in Figure 23,
\( l' = 0 \) in the superresonant region, hence superresonant
disturbances propagate away from the cascade without
attenuation. Outside this region, \( l' \) is nonzero and increases
monotonically with \( |\beta| \), thus the subresonant waves decay
exponentially with distance away from the cascade.

Now the interaction of the predicted waves with the wind
tunnel walls is considered. For \( \beta = -45^\circ \) and \( \beta = -90^\circ \),
interblade phase angles where the data-analysis correlation is
good, decaying waves traveling at \( 0 \approx 31^\circ \) with
\( |C_p(\xi = 0)| \approx 0.5 \) are predicted. As shown in the schematic
of Figure 24, these waves are directed at the cascade upper wall.
The waves are assumed to reflect specularly from the wall so
that the reflected disturbances propagate away from the cascade
and thus will have no effect on the oscillating cascade aerodynamics.

The data-analysis correlation is also good for \( \beta = 0^\circ \), a
superresonant condition, but this is considered fortuitous
because the cascade periodicity is poor. The upstream-traveling
wave for this condition, Figure 24, is directed at the cascade upper wall
so that reflected waves travel back into the cascade and potentially affect the cascade unsteady aerodynamics and
dynamic periodicity.

The data-analysis correlation is poor for \( \beta = 45^\circ \), the other
superresonant condition. In this case, the downstream-traveling
wave is predicted to interfere with the cascade after reflection off
the lower wind tunnel wall, Figure 24.

Decaying waves are predicted for \( \beta = 90^\circ \) and \( \beta = 180^\circ \),
interblade phase angles for which there is poor correlation.
which reflected off the wind tunnel upper wall in a direction away from the cascade, so that these reflected waves did not interfere with the cascade unsteady aerodynamics. At those values of interblade phase angle where the periodicity was poor, the analysis often indicated that waves were reflecting off the wind tunnel walls back into the cascade, and therefore interfered with the cascade unsteady aerodynamics.

To make this a reliable facility for the investigation of oscillating cascade aerodynamics, the effects of the wind tunnel walls must be reduced. An effort is currently under way to replace the solid tunnel walls in the vicinity of the cascade with acoustically-treated walls as developed to reduce aircraft gas turbine engine noise (Groeneweg and Rice, 1987). The effectiveness of the acoustic treatment will be investigated in part by repeating the experiments reported upon herein.

ACKNOWLEDGEMENTS

This research was supported by the Propeller and Acoustics Technology Branch and the Turbomachinery Technology Branch of the Propulsion Systems Division at the NASA Lewis Research Center. The assistance of Vincent Verhoff, Robert Olsey and Bernadette Orahoske in completion of the experiments is gratefully acknowledged.

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Experiments in a linear oscillating cascade reveal that the wind tunnel walls enclosing the airfoils have, in some cases, a detrimental effect on the oscillating cascade aerodynamics. In a subsonic flow field, biconvex airfoils are driven simultaneously in harmonic, torsion-mode oscillations for a range of interblade phase angle values. It is found that the cascade dynamic periodicity—the airfoil-to-airfoil variation in unsteady surface pressure—is good for some values of interblade phase angle but poor for others. Correlation of the unsteady pressure data with oscillating flat plate cascade predictions is generally good for conditions where the periodicity is good and poor where the periodicity is poor. Calculations based upon linearized unsteady aerodynamic theory indicate that pressure waves reflected from the wind tunnel walls are responsible for the cases where there is poor periodicity and poor correlation with the predictions.