A Proposed Kalman Filter Algorithm for Estimation of Unmeasured Output Variables for an F100 Turbofan Engine

A. J. Arg and Glenn B. Gilvard

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Gurbux S. Alag
PRC System Service,
Edwards, California

Glenn B. Gilyard
Ames Research Center
Dryden Flight Research Facility
Edwards, California
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Gurbux S. Alag*
PRC Systems Service
Edwards, CA

Glenn B. Gilyard**
NASA Ames Research Center
Dryden Flight Research Facility
Edwards, CA

Abstract

To develop advanced control systems for optimizing aircraft engine performance, unmeasurable output variables must be estimated. The estimation has to be done in an uncertain environment and be adaptable to varying degrees of modeling errors and other variations in engine behavior over its operational life cycle. This paper presents an approach to estimate unmeasured output variables by explicitly modeling the effects of off-nominal engine behavior as biases on the measurable output variables. A state variable model accommodating off-nominal behavior is developed for the engine, and Kalman filter concepts are used to estimate the required variables. Results are presented from nonlinear engine simulation studies as well as the application of the estimation algorithm on actual flight data. The formulation presented has a wide range of application since it is not restricted or tailored to the particular application described in the paper.

Nomenclature

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>A, B, C, D</td>
<td>system matrices in state variable representation</td>
</tr>
<tr>
<td>F, G, H</td>
<td>system matrices in state variable representation</td>
</tr>
<tr>
<td>A_J</td>
<td>nozzle area, in.</td>
</tr>
<tr>
<td>b</td>
<td>bias vector</td>
</tr>
<tr>
<td>CDF</td>
<td>component deviation factors</td>
</tr>
<tr>
<td>CIVV</td>
<td>fan inlet guide vane angle, deg</td>
</tr>
<tr>
<td>DEEC</td>
<td>digital electronic engine control</td>
</tr>
<tr>
<td>D_NO2</td>
<td>nozzle drag, lb</td>
</tr>
<tr>
<td>D_RAM</td>
<td>ram drag, lb</td>
</tr>
<tr>
<td>d</td>
<td>difference</td>
</tr>
<tr>
<td>E</td>
<td>expectation operator</td>
</tr>
<tr>
<td>EMD</td>
<td>engine model derivative</td>
</tr>
<tr>
<td>e</td>
<td>state error vector</td>
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<tr>
<td>F_G</td>
<td>gross thrust, lb</td>
</tr>
<tr>
<td>F_NP</td>
<td>net propulsive thrust, lb</td>
</tr>
<tr>
<td>I</td>
<td>identity matrix</td>
</tr>
<tr>
<td>K</td>
<td>Kalman filter gain</td>
</tr>
<tr>
<td>N1</td>
<td>fan rotor speed, rpm</td>
</tr>
<tr>
<td>N2</td>
<td>core rotor speed, rpm</td>
</tr>
<tr>
<td>P</td>
<td>Riccati matrix</td>
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<tr>
<td>P_B</td>
<td>burner static pressure, lb/in^2</td>
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<tr>
<td>PLA</td>
<td>power lever angle, deg</td>
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<tr>
<td>PSC</td>
<td>performance seeking control</td>
</tr>
<tr>
<td>PSM</td>
<td>propulsion system model</td>
</tr>
<tr>
<td>P_T1</td>
<td>compressor inlet total pressure, lb/in^2</td>
</tr>
<tr>
<td>P_T2</td>
<td>low turbine inlet pressure, lb/in^2</td>
</tr>
<tr>
<td>P_T3</td>
<td>afterburner inlet total pressure, lb/in^2</td>
</tr>
<tr>
<td>P_T4</td>
<td>nozzle throat total pressure, lb/in^2</td>
</tr>
<tr>
<td>Q</td>
<td>state noise covariance matrix</td>
</tr>
<tr>
<td>R</td>
<td>measurement noise covariance matrix</td>
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<td>R_CVV</td>
<td>compressor stator vane angle, deg</td>
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<tr>
<td>S_MF</td>
<td>fan stall margin</td>
</tr>
</tbody>
</table>

*Senior Control Engineer.
**Aerospace Engineer. Member AIAA.

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Introduction

Efforts to improve aircraft turbine engine efficiency have led to an increase in the number of engine control variables and a corresponding increase in the complexity of control laws. Control laws for current engines are based on classical control theory and empirical schedules for a nominal engine. Classical control theory has served well for the current and older engines. The design of future fighters as multifunction aircraft and development of integrated flight/propulsion control systems, however, require sophisticated control systems capable of obtaining the maximum performance from the engine. Optimal control techniques using modern control theory are required to obtain additional gains in engine performance. For modern aircraft, accounting for engine variations through designs based on predetermined control schedules is increasingly difficult because of the increased complexity and increased number of control effectors on the engines. Engine-to-engine component variations, engine deterioration, and off-nominal behavior are difficult to account for in the design of control system schedules.

An adaptive control algorithm, which computes optimal control trim settings for the engine while maximizing the vehicle performance for a given flight condition, accounts for these variations better than gain scheduling. Specifically, an adaptive trim control system computes and applies an incremental steady state trim to enhance the engine performance. 1

For over a decade, the National Aeronautics and Space Administration (NASA) Ames Research Center, Dryden Flight Research Facility (Ames-Dryden) has conducted a multidisciplinary flight research program on an F-15 airplane. Significant portions of this research involved the flight evaluation of advanced propulsion control concepts in programs such as digital electronic engine control (DEEC), the F100 engine model derivative (EMD), and highly integrated digital electronic control (HIDEC). 2 The increased performance and improved fuel economy demonstrated on the F-15 HIDEC research vehicle is the basis of the performance seeking control (PSC) program, which will provide additional improvements in these areas.

Ames-Dryden, McDonnell Aircraft Company, and Pratt & Whitney are currently developing and demonstrating an adaptive PSC system in flight on a NASA F-15 airplane powered by F100 EMD engines. The PSC system optimizes aircraft performance by applying adaptive trim control to the propulsion system operating in a pseudo-steady-state cruise mode. The trim schedules are determined for a highly nonlin-
ear propulsion system which has system and measure-
ment noise, unmeasurable parameters, and sensitivity
to normal deterioration over its life cycle.

Figure 1 shows the adaptive trim control structure
used for the PSC. The state variable model (SVM)
and the steady-state model (SSM) which model the dy-
amic and steady-state behavior of a nominal engine,
are key components of the system. These models are
stored onboard the aircraft in a table look-up form and
are discussed in more detail in the following section.

These models are used in formulating the propulsion
system model (PSM) which represents a small pertur-
bation model of the actual flight propulsion system.
The PSM contains relations which provide estimates
of performance measures (such as augmentor effects,
thrust, and stall margins) and constraint equations. A
linear programming algorithm is used to find the opti-
mal solution and these commands are then applied to
the engine through the DEEC.

The values of output variables, which are often not
directly measurable, are needed for the optimization
algorithm used in the PSC. These variables are esti-
eated under changing levels of engine health, man-
ufacturing differences between engines, and other off-
nominal behavior. Accommodating these performance
variations in engines has been investigated in two re-
cent studies. 3

Reference 3 presents an algorithm for estimating the
cause and level of off-nominal engine operation by us-
ing a Kalman filter algorithm to estimate five engine
factors. These five factors, referred to as component
deviation factors (CDF), compensate for off-nominal
performance. These factors were estimated by treat-
ing them as biases, and the original state vector was
augmented to give five additional states. 5 These five
factors are not explicitly used in the optimization algo-

rithm and their physical significance is unclear because
the formulation does not account for biases, prediction
errors, and Reynolds number effects. Since the coeffi-
cients with respect to the CDF parameters are required
in the Kalman filter development, the CDF formulation
requires detailed modeling of the off-nominal process.
A flight data evaluation of this algorithm is described
in Ref. 6.

In Ref. 4, a component tracking filter is used to
achieve the model accuracy required to optimize en-
gine performance. The component tracking filter com-
bines the concept of state tracking and adaptive filter-
ing to minimize engine/model mismatch. It is based on
a frequency decomposition of the differences between
the sensed engine parameters and the model values.

This paper presents another method of accounting
for off-nominal operation and other modeling inaccur-
cacies. Since any variation from the nominal model
would result in a change in the sensed values of the
measured outputs, the off-nominal behavior of an en-
gine is characterized in terms of these changes. Uncer-
tainties associated with any given engine will be repre-
sented as systematic errors in the sensed output param-
eters. These systematic errors will be accounted for by
augmenting the original state equation with bias states.
A Kalman filter is used to estimate the original engine
states and the bias states. The Kalman filter inputs are
measurements from standard F100 engine control in-
sertion. The auxiliary output equations for the unmeasured output variables are modified to include
the effect of the bias states.

The concept is validated by applying the developed
filter on both simulation and flight data. For the sim-
ulation data case, the output variables were estimated
by using the data from the available nonlinear engine
simulation. Both a nominal engine and an engine in
which intentional degradation was introduced to create
off-nominal behavior were considered. For the flight
data case, the estimation process was performed using
actual flight data from an F-15 aircraft. For this case,
comparative results are also presented for the proposed
algorithm and the CDF formulation. Both the sim-
ulation and flight evaluations were carried out for a
flight condition of Mach 0.90 and 30,000 ft, for a part
power setting.

**Engine Description**

The engine used in this study is the Pratt & Whitney
F100 EMD low-bypass ratio, twin spool, afterburn-
ing turbofan engine7 (Fig. 2). The engine is controlled
by a DEEC, a full-authority digital electronic control
system which performs the functions of the standard
F100 engine hydromechanical, unified fuel control,
and supervisory digital electronic engine control.

**Engine Models**

Pratt & Whitney has developed a comprehensive
nonlinear dynamic engine model, the state-of-the-art
propulsion program (SOAPP) model. This model is
the best representation of the engine and predicts engine performance with minimal error over the full power range and flight envelope and for both steady-state and transient operation. This nonlinear simulation is a high-fidelity model that represents each component in the engine and control but does not run in real time.

For real-time use, a set of linearized SVMs were developed from the SOAPP model. To cover the entire flight envelope, 49 models were developed. The model is selected as a function of burner static pressure ($PB$). These models compare well with the large scale nonlinear aerothermal model and actual engine test data, and they can be implemented efficiently in real time. Figure 3 shows a simulation model for the F100 engine based on the state variable formulation.

The SSM engine relationships and trim predictions (basepoints) are also derived from the SOAPP model. A two-dimensional table look-up scheduled on 7 values of $PB$ and 40 values of afterburner total pressure ($P_{76}$) is needed to represent the steady state information. Each SSM consists of a basepoint control vector, a basepoint output vector, and a sensitivity coefficient matrix which relates the changes in control positions to change in outputs.

The PSC algorithm requires the variables listed in Table 1, which are functions of the engine states and the input control variables. These variables include engine outputs which cannot be measured but are required to calculate performance measures of the engine. An additional set of variables, which are nonlinear functions of the unmeasured output variables, are listed in Table 2. These variables are used to predict both the engine performance and the constraints needed to develop optimal engine controllers.

Kalman Filter Concepts

The entire state vector of the system to be controlled is often assumed to be measurable. Most of the solutions to optimal control problems are obtained as a feedback law implementable only if the entire state vector is available. In most complex systems the entire state vector cannot be measured, and a suitable approximation to the state vector must be determined and substituted into the control law. The system that produces, in deterministic setting, an approximation to the state vector is called an observer. Kalman and Bucy solved the optimal observer problem in a stochastic environment, and this solution has had a tremendous impact on optimal filtering theory. The Kalman filter represents the most widely applied and demonstrably useful result to emerge from the state variable approach of "modern control theory.

The system is

$$\dot{x} = Ax + Bu + w_1$$
$$y = Cx + Du + w_2$$

Where $A$, $B$, $C$, and $D$ are system matrices in state variable representation, $x$ is the state vector, $u$ is the control input vector, $y$ is the output vector, $w_1$ is the state excitation noise, and $w_2$ is the observation or measurement noise. Both $w_1$ and $w_2$ are white, uncorrelated Gaussian processes, with intensity $Q$ and $R$ respectively.

The observer is

$$\dot{\hat{x}} = A\hat{x} + Bu + K[y - C\hat{x} - Du]$$

where $K$ is the Kalman filter gain.

The optimal observer problem is finding the matrix $K$ so as to minimize $E\{e^TRe\}$, where

$$e = x - \hat{x}$$

and $R$ is a positive-definite symmetric weighting matrix. In this problem, $E$ is the expectation operator and $e$ is the state error vector. If $R$ is a positive-definite matrix, the optimal observer is called nonsingular. The Kalman filter is the solution to the nonsingular optimal observer previously outlined. The optimal observer problem is solved by choosing the gain matrix.

$$K = PC^TR^{-1}$$

where $P$ is the state error covariance matrix, $E[(x - \hat{x})(x - \hat{x})^T]$, and is the solution to the matrix Riccati equation

$$\dot{P} = AP + PA^T + Q - PC^TR^{-1}CP$$

For a time invariant case, the steady state solution for $P$ is a constant matrix and is a unique nonnegative definite solution of the algebraic Riccati equation

$$0 = AP + PA^T + Q - PC^TR^{-1}CP$$
Figure 4 shows a typical Kalman filter structure used to estimate states and outputs.

**Proposed Formulation**

In Kalman filter derivation, linear models for the system dynamics and measurement relation are assumed to be adequate for developing optimal estimators. No model is perfect, and a linear model, in particular, is the result either of intentional approximation and simplification or of a lack of knowledge about the system being modeled.\(^\text{12}\) To account for degraded engine operation and modeling inaccuracies, the proposed formulation augments the output vector by adding a bias vector to represent the uncertain parameters.\(^\text{5}\) The dynamic equations can thus be expressed as

\[
\begin{align*}
\dot{x} &= Ax + Bu + w_t \\
y &= Cx + Du + b + w_2
\end{align*}
\]

where \(b\) is the bias vector. The bias vector is estimated by adjoining \(b\) to \(x\) and defining a new state vector, \(z\)

\[
z = \begin{bmatrix} x \\ \cdots \\ b \end{bmatrix}
\]

with the condition

\[
\dot{b} = 0
\]

The state equation can be rewritten as

\[
\begin{align*}
\dot{z} &= A_1 z + B_1 u + G w_1 \\
y &= C_1 z + Du + w_2
\end{align*}
\]

where

\[
A_1 = \begin{bmatrix} A & 0 \\ \cdots & \cdots \\ 0 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} B \\ \cdots \\ 0 \end{bmatrix}, \quad C_1 = [C \ 0], \quad G = \begin{bmatrix} I \\ \cdots \\ 0 \end{bmatrix}
\]

If the estimate of \(z\) is \(\hat{z}\), where

\[
\hat{z} = \begin{bmatrix} \hat{x} \\ \cdots \\ \hat{b} \end{bmatrix}
\]

then the Kalman filter estimate is given by

\[
\begin{align*}
\hat{z} &= A_1 \hat{z} + B_1 u + PC_1^T R^{-1} [y - C_1 \hat{z} - Du]
\end{align*}
\]

where \(P\) is the steady state solution to the Riccati equation

\[
0 = A_1 P + P A_1^T + C Q C^T - P C_1^T R^{-1} C_1 P
\]

The auxiliary set of unmeasured output variables (\(\hat{y}_{aux}\)) are related to the engine states and control inputs through the algebraic equation

\[
\hat{y}_{aux} = Hz + Fu
\]

Details of the state variable formulation for the F100 engine are presented in the appendix. The \(\hat{y}_{aux}\) outputs are listed in Table 1.

In spite of the mathematical formalism of the Kalman filter, engineering insight and experience is required to develop an effective operational filter algorithm. A mathematical model of both the system structure and uncertainty is inherently embodied in the Kalman filter structure. The main design problem is attaining an adequate mathematical model upon which to base the filter. Even after selecting an appropriate model, the matrices \(Q\) and \(R\) can be difficult to determine. This is done by a process called "tuning" the Kalman filter. It is a trial and error procedure for determining which matrix values yield the best estimation performance for that particular filter structure.

The matrix \(R\) was determined by analysis of flight data available for the F100 engine. The elements of matrix \(Q\) were, however, selected by evaluating the performance of the Kalman filter by trial and error. Figure 5 shows the implementation process used to estimate the output variables for the F100 engine using the Kalman filter.

This proposed formulation estimates unmeasured output variables by explicitly modeling the effects of off-nominal engine behavior as biases on the measurable output variables.

**Results**

The proposed estimation algorithm was developed and evaluated for a Mach 0.90 and 30,000 ft flight condition. The algorithm was evaluated by a comparison with SOAPP simulation results and also by application to flight data. The flight data results were compared with the CDF formulation results for the same data.

**Simulation Evaluation**

The SOAPP simulation evaluations consisted of estimating the desired variables using both a nominal and
a degraded engine. In each case, the power lever angle (PLA) was held to 37° for 15 sec and then stepped up to 43° and held constant for the remainder of the run.

Measured outputs were obtained from the SOAPP simulation and were corrupted with noise, as shown in Table 3. These are typical values obtained from flight data. The measurements with noise and the values of the control variables were entered into the estimation algorithm and the desired estimates were obtained. The Kalman filter state vector, a perturbation of the steady state conditions, was initialized to zero for all states.

The algorithm needed to generate consistent state estimates which were robust with respect to the measurement covariance matrix \( Q \) (the only variable selected by trial and error). An important aspect of the development is determining unmeasured output vector, \( \hat{y}_{aux} \). Inconsistent estimates of the states would give different values of \( \hat{y}_{aux} \) for different values of \( Q \) when applied to the same data.

The state vector estimates converged to the same value for different values of \( Q \). This was evaluated for values of \( Q = I \) and \( Q = 10I \). The difference in the estimated states for \( Q = I \) and \( Q = 10I \), for a nominal engine, is shown in Fig. 6. This figure shows that the state estimates converge to the same value and the effect of change in \( Q \) on the steady-state response is minimal.

The five measured output variables obtained from the SOAPP for a nominal engine were compared with the estimates of these variables obtained from the filter (Fig. 7(a)). The prediction values subtracted from the simulated measurements were held constant throughout the run. These values were the same as the simulated measurements at the beginning of the run, accounting for the excellent comparison over the initial interval. The Kalman filter was not updated in this evaluation, so the comparisons indicate that the model is quite robust. The comparisons are very good in spite of the large change in the operating conditions. The CDF based formulation would have used five different models for the PB change of this maneuver.

Figure 7(b) shows the measurement bias estimates. As expected, they are nearly zero until the PLA is increased. As the engine attains a new operating condition, the bias parameters increase to levels which account for the effects not modeled in the SVM.

To assess the condition when significant differences exist between the measured data and the predicted data, the following nominal biases were added to the simulated flight data: \( \Delta N_1 \) (fan rotor speed) = 50.0, \( \Delta N_2 \) (core rotor speed) = 50.0, \( \Delta P_B = 2.0 \), \( \Delta T_{7.5} \) (low turbine inlet total temperature) = 30.0, and \( \Delta P_{7.5} = 0.5 \). The results of this evaluation (Fig. 8(a)) show that the tracking of the five measurements is again very good. The final values of the bias estimates (Fig. 8(b)) are the sum of biases estimated in Fig. 7(b) and the biases placed on the simulated measurements as previously listed.

In Fig. 9, estimates of the unmeasured output variables (\( \hat{y}_{aux} \)) are compared with the actual values obtained from the SOAPP. The estimates show good tracking of the simulation values.

Simulation evaluations were then carried out for a degraded engine by simultaneously introducing the following deteriorations: (a) high turbine efficiency is 2.5 percent below nominal, (b) low turbine efficiency is 2.5 percent below nominal, (c) compressor airflow deviation is 1 lb/sec less than nominal, and (d) the fan airflow deviation is 5 lb/sec less than nominal.

The results for the simulated degraded engine are presented in Fig. 10. These results are similar to the results of Fig. 7 and demonstrate the adaptability and robustness of the proposed estimator to degraded engine performance. Again, the Kalman filter was not updated during the evaluation and the predicted constant values subtracted from the simulated data were the same as those for an engine that was not degraded.

Flight Data Evaluation

The Kalman filter formulation was also evaluated on flight data obtained on the NASA F-15 research aircraft. The flight data was obtained at Mach 0.90, an altitude of 30,000 ft, and a PLA of 43.5°. The time history of the test data (Fig. 11) starts with no bleed air being extracted from the test engine. Approximately 40 sec into the run, the pilot manually changed the bleed switch to extract all the aircraft bleed air requirements from the test engine. This maneuver was designed to simulate a change in engine operating efficiency. The engine control system increased fuel flow (\( W_P \)) to maintain the scheduled fan speed, resulting in an increase in \( T_{7.5} \). After holding this bleed condition for approximately 70 sec, the bleed was again switched back to the initial no bleed air condition.
The Kalman filter estimation results are shown in Fig. 12. Figure 12(a) shows that the filter tracks the flight measurements accurately. Initial discrepancies occur because the bias estimates start at zero; however, this startup transient is brief, with good tracking occurring in approximately 20 sec. Although the tracking quality is slightly worse at the time the bleed switching occurs, the filter rapidly adapts to the simulated change in engine efficiency. The bias estimates, shown in Fig. 12(b), converge rapidly to steady-state values as the engine state is changed from one condition to another. The initial startup transient could be minimized by initializing the bias estimates with the actual values of the biases for the given flight condition.

Figure 13 shows the results from the proposed formulation compared with the corresponding results from the CDF formulation. The results were obtained using the flight data shown in Fig. 11. The results show that the performance obtained by the proposed method compares favorably with the CDF procedure. A significantly improved startup transient performance is evident. Figure 14 presents similar comparisons for the estimates of normally unmeasured output variables. Figure 14(a) shows the estimate of compressor inlet total temperature \(T_{I_{2.5}}\) and the measured values. The superiority of the proposed formulation is clearly evident, if the measurement of \(T_{I_{2.5}}\) is considered reliable. Figure 14(b) shows the comparative estimates of corrected fan airflow \(W_{CFAN}\). The values are comparable, with better transient performance for the proposed formulation.

Concluding Remarks

An approach has been proposed to estimate the unmeasured or auxiliary output variables of a turbofan F100 engine by using Kalman filter concepts. The approach is based on explicitly modeling the effects of off-nominal engine behavior as biases on the measured output variables. Results are presented for estimates of the output variables and are compared with values obtained from detailed nonlinear simulation of the engine. The evaluation was carried out for both a nominal engine and an engine in which intentional deterioration was introduced. The proposed filter was also evaluated for output estimation using actual F-15 flight data.

The formulation is robust with respect to the value of state covariance matrix \(Q\). A critical component of the performance seeking control (PSC) problem for the F100 engine is determining consistent values for auxiliary output variables. Consistent estimates for the states were obtained for different values of \(Q\) and thus consistent estimates of the auxiliary output variables are ensured.

The proposed estimation algorithm was able to accurately predict the values of the output variables for the simulation studies for both nominal and degraded engine conditions. The proposed algorithm has been validated by comparing its estimates with the values from the detailed nonlinear simulation, and it has performed well on flight data. A comparative study of the proposed algorithm results with component deviation factors (CDF) results gave additional proof of the validity of the concept. Unlike the CDF method, the proposed algorithm does not require detailed modeling of the engine degradation process. This formulation has a wide range of application because it is not restricted or tailored to the particular application described in this paper.

References


Appendix—State Variable Auxiliary Output Estimation Formulation for an F100 Engine

For the system being considered, the complete state variable model is

\[
\delta z = A_1 \delta z + B_1 \delta u + G w_1
\]
\[
\delta y = C_1 \delta z + D \delta u + w_2
\]

where 1 indicates augmented system matrices, and

\[
\delta z = \begin{bmatrix}
\delta N_1 \\
\delta N_2 \\
\delta T_{MT} \\
N_1 \\
N_2 \\
P_{T_1} \\
PB \\
T_{T_1, 2}
\end{bmatrix}
\delta u = \begin{bmatrix}
\delta W_F \\
\delta A_I \\
\delta CIVV \\
\delta RCVV
\end{bmatrix}
\delta y = \begin{bmatrix}
\delta N_1 \\
\delta N_2 \\
\delta P_{T_1} \\
\delta PB \\
\delta T_{T_1, 2}
\end{bmatrix}
\]

where \(T_{MT}\) is the turbine metal temperature, \(b\) denotes the output bias term, \(CIVV\) is the fan inlet guide vane angle, \(RCVV\) is the compressor stator vane angle, and

\[
A_1 = \begin{bmatrix}
A & 0 \\
\cdots & \cdots \\
0 & 0
\end{bmatrix}
B_1 = \begin{bmatrix}
B \\
\cdots \\
0
\end{bmatrix}
\]
\[
C_1 = \begin{bmatrix}
C & I
\end{bmatrix}
G = \begin{bmatrix}
I \\
\cdots \\
0
\end{bmatrix}
\]

where \(A, B, C,\) and \(D\) are constant perturbation matrices, numerically derived from the SOAPP, \(w_1\) is the state noise with covariance \(Q,\) and \(w_2\) is the measurement noise with covariance \(R.\) The elements of \(R\) are obtained from \textit{a priori} flight data, while those of \(Q\) are selected by trial and error.

The auxiliary set of unmeasured output variables \((\hat{y}_{aux})\) listed in Table 1, is given by

\[
\hat{y}_{aux} = H \delta z + F \delta u + y_t
\]

where

\[
H = [H_1 : H_2]
\]

and \(H_2\) reflects the effect of estimated biases and its elements are derived from the SVM, \(H_1\) and \(F\) are perturbation matrices derived from the SOAPP, and \(y_t\) is the vector of predicted trim values for the auxiliary output variables, which is obtained from the SVM.
Table 1. Linear auxiliary output variables, PSC algorithm requirements.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>$P_{T2.5}$</td>
<td>compressor inlet total pressure</td>
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<tr>
<td>$P_B$</td>
<td>burner static pressure</td>
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<tr>
<td>$P_{T6}$</td>
<td>afterburner inlet total pressure</td>
</tr>
<tr>
<td>$T_{T3}$</td>
<td>compressor inlet total temperature</td>
</tr>
<tr>
<td>$T_{T4}$</td>
<td>burner inlet total temperature</td>
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<td>$T_{T4.5}$</td>
<td>burner exit total temperature</td>
</tr>
<tr>
<td>$T_{T5}$</td>
<td>low turbine inlet total temperature</td>
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<td>$WCF_{fan}$</td>
<td>corrected fan air flow</td>
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<tr>
<td>$WC_{HPC}$</td>
<td>corrected compressor air flow</td>
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Table 2. Nonlinear engine variables.

<table>
<thead>
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<td>$D_{NOZ}$</td>
<td>nozzle drag</td>
</tr>
<tr>
<td>$D_{RAM}$</td>
<td>ram drag</td>
</tr>
<tr>
<td>$F_G$</td>
<td>gross thrust</td>
</tr>
<tr>
<td>$F_{NP}$</td>
<td>net propulsive force</td>
</tr>
<tr>
<td>$P_{T7}$</td>
<td>nozzle throat total pressure</td>
</tr>
<tr>
<td>$SM_{F}$</td>
<td>fan stall margin</td>
</tr>
<tr>
<td>$SM_{HC}$</td>
<td>high compressor stall margin</td>
</tr>
<tr>
<td>$T_{T7}$</td>
<td>nozzle throat total temperature</td>
</tr>
</tbody>
</table>

Table 3. Measurement noise statistics.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_1$</td>
<td>7 rpm</td>
</tr>
<tr>
<td>$N_2$</td>
<td>7 rpm</td>
</tr>
<tr>
<td>$P_{T6}$</td>
<td>0.3 lb/in</td>
</tr>
<tr>
<td>$P_{T4}$</td>
<td>0.6 lb/in</td>
</tr>
<tr>
<td>$T_{T4.5}$</td>
<td>4 °R</td>
</tr>
</tbody>
</table>

Fig. 1 The performance seeking control adaptive control system.
Fig. 2 The F100 engine and sensor locations.
Fig. 3 The F100 engine simulation based on the state variable model.
\[
\begin{align*}
\hat{x} &= \begin{bmatrix} N_1, N_2, TMT, N_{1b}, N_{2b}, P_{T6b}, P_{Tb}, T_{T4.5b} \end{bmatrix}^T \\
u &= \begin{bmatrix} W_F, A_J, CIVV, RCVV \end{bmatrix}^T \\
y &= \begin{bmatrix} N_1, N_2, PB, P_{Tb}, T_{T4.5} \end{bmatrix}^T \\
\hat{y}_{aux} &= \begin{bmatrix} P_{T_{2.5}}, T_{T_{2.5}}, T_{T_3}, T_{T4}, T_{T6}, WCFAN, WHPCZ \end{bmatrix}^T \\
\hat{y}_{N.L.} &= \begin{bmatrix} SM_F, SM_{HC}, DRAM, DNOZ, FN, FP, F_G, T_T, P_T \end{bmatrix}^T
\end{align*}
\]

Fig. 4 The Kalman filter structure.

Fig. 5 Modified estimation process using the proposed Kalman filter.
Fig. 6 The F100 engine simulation state estimates for a nominal engine at $Q = 1$ and $Q = 101$, PLA increased from $37^\circ$ to $43^\circ$ at 15 sec.
Fig. 6 Concluded.
(a) Measured and estimated engine outputs.

Fig. 7 The F100 engine simulation parameters for a nominal engine, with PLA increased from $37^\circ$ to $43^\circ$ at 15 sec.
(b) Bias estimates.

Fig. 7 Concluded.
(a) Measured and estimated engine outputs.

Fig. 8 The F100 engine simulation parameter estimates with biased measurements for a nominal engine, with PLA increased from 37° to 43° at 15 sec.
(b) Bias estimates.

Fig. 8 Concluded.
Fig. 9 The F100 engine simulation auxiliary output estimates for a nominal engine, with PLA increased from 37° to 43° at 15 sec.
Fig. 10 The F100 engine simulation parameter estimates for a deteriorated engine, with PLA increased from 37° to 43° at 15 sec.
(b) Bias estimates.

Fig. 10 Concluded.
Fig. 11 The F-15 airplane measured engine parameters during compressor bleed variations at Mach 0.90, an altitude of 30,000 ft, and PLA = 43°.
(b) Measured control variables.

Fig. 11 Concluded.
Fig. 12 The F100 engine parameter estimates from the flight data in Fig. 11.
(b) Bias estimates.

Fig. 12 Concluded.
Fig. 13 Proposed formulation estimated outputs from flight data compared with CDF formulation estimates from flight data.
Fig. 14 The proposed formulation and the CDF formulation engine parameter estimates from flight data compared with measured engine parameters.
(b) Corrected fan airflow estimates.

Fig. 14 Concluded.
# A Proposed Kalman Filter Algorithm for Estimation of
Unmeasured Output Variables for an F100 Turbofan Engine

**Gurbux S. Alag and Glenn B. Gilyard**

**National Aeronautics and Space Administration**
Washington, DC 20546-0001

**Technical Memorandum**

**Abstract**

To develop advanced control systems for optimizing aircraft engine performance, unmeasurable output variables must be estimated. The estimation has to be done in an uncertain environment and be adaptable to varying degrees of modeling errors and other variations in engine behavior over its operational life cycle. This paper presents an approach to estimate unmeasured output variables by explicitly modeling the effects of off-nominal engine behavior as biases on the measurable output variables. A state variable model accommodating off-nominal behavior is developed for the engine, and Kalman filter concepts are used to estimate the required variables. Results are presented from nonlinear engine simulation studies as well as the application of the estimation algorithm on actual flight data. The formulation presented has a wide range of application since it is not restricted or tailored to the particular application described in the paper.

**Key Words**

Aircraft engine performance, Off-nominal engine performance, Kalman filter, Bias estimation, Unmeasurable output estimation

**Distribution Statement**

Unclassified-Unlimited

**Subject category**

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