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## STRUCTURAL RELIABILITY ANALYSIS OF LAMINATED CMC COMPONENTS

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### ABSTRACT

For laminated ceramic matrix composite (CMC) materials to realize their full potential in aerospace applications, design methods and protocols are a necessity. This paper focuses on the time-independent failure response of these materials and presents a reliability analysis associated with the initiation of matrix cracking. It highlights a public domain computer algorithm that has been coupled with the laminate analysis of a finite element code and which serves as a design aid to analyze structural components made from laminated CMC materials. Issues relevant to the effect of the size of the component are discussed, and a parameter estimation procedure is presented. The estimation procedure allows three parameters to be calculated from a failure population that has an underlying Weibull distribution.

### INTRODUCTION

Structural components produced from laminated ceramic matrix composite (CMC) materials are being considered for a broad range of aerospace applications that include propulsion subsystems in the national aerospace plane, the space shuttle main engine, and advanced gas turbine engines. Specifically, composite ceramics may be used as segmented engine liners, small missile engine turbine rotors, and exhaust nozzles. These materials will improve fuel efficiency by increasing engine temperatures and pressures, which will, in turn, generate more power and thrust. Furthermore, these materials have significant potential for raising the thrust-to-weight ratio of gas turbine engines by tailoring directions of high specific reliability. The emerging composite systems, particularly those with a silicon nitride or silicon carbide matrix, can compete with metals in many demanding applications. The capabilities of laminated CMC prototypes have already been demonstrated at temperatures approaching 1400 °C, well beyond the operational limits of most metallic materials.

Adding a second ceramic phase with an optimized interface to a brittle matrix improves fracture toughness, decreases the sensitivity of the brittle ceramic matrix to microscopic flaws, and could also improve strength. The presence of fibers in the vicinity of the crack tip modifies fracture behavior by increasing the required crack driving force by several mechanisms. These mechanisms include crack pinning, fiber bridging, fiber debonding, and fiber pull-out. This increase in fracture toughness allows for "graceful" rather than catastrophic failure. A unidirectional ply loaded in the fiber direction retains substantial strength capacity beyond the initiation of matrix cracking despite the fact that neither of the constituents would exhibit such behavior if tested alone. First matrix cracking consistently occurs at strains greater than in the monolithic matrix material. As additional load is applied, the matrix tends to break in a series of cracks bridged by the ceramic fibers, until the ultimate strength of the composite is reached. The region of a typical stress-strain curve between the first matrix cracking and the ultimate tensile strength illustrates an intrinsic damage tolerance not present in monolithic ceramics.

Laminated CMC material systems have several mechanical characteristics which must be considered in the design of structural components. In regards to an individual ply, the most deleterious of these characteristics are low strain tolerance, low fracture toughness, and a large variation in failure strength in the material orientation transverse to the fiber direction. Thus analyses of components fabricated from ceramic materials require a departure from the usual deterministic philosophy of designing metallic structural components (i.e., the factor-of-safety approach). Although the so-called size effect has been reported to be non-existent in the fiber direction (see Dicarlo, 1989), the bulk strength of unidirectional-reinforced ply will decrease transverse to the fiber direction as the component volume increases. Since failure in the transverse direction will be dominated by the scatter in strength, statistical design approaches must be employed. These approaches must, on the one hand, allow for elevated strength, reduced variability in strength, and a diminished effect of bulk specimen size in the fiber direction, and, on the other hand, increased scatter in strength and effects of bulk size in the transverse

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direction. Simply stated, a reliability analysis must rationally account for material symmetry imposed by the reinforcement. Computational structural mechanics philosophies must emerge that address the issues of scatter in strength, size effect, and material anisotropy. There is a need for test-bed software programs that incorporate stochastic design protocols, that are user friendly, that are computationally efficient, and that have flexible architectures which can readily incorporate changes in design philosophy. The C/CARES (Composite Ceramics Analysis and Reliability Evaluation of Structures) program, which will be highlighted in this article, was developed to fulfill this need. C/CARES is a public domain computer algorithm, coupled to a general purpose finite element program, which predicts the fast fracture reliability of a laminated structural component under multiaxial loading conditions.

## STOCHASTIC DESIGN ISSUES

For a number of composite material systems, several authors (see for example Batdorf and Ghaffarian, 1984; and Wu, 1989) have reported a diminished size effect in the fiber direction; and DiCarlo (1989) has reported this effect for ceramic composites, in particular. This phenomenon is an important feature that must be addressed by any reliability model. How it is addressed depends on whether the material is modeled as a series system, a parallel system, or a combination. Current analytical practice uses finite element methods to determine the state of stress throughout the component. It is assumed that failure depends on the stress state in a component, such that deformations are not controlling design. Since failure may initiate in any of the discrete volumes (elements), it is useful to consider a component from a systems viewpoint. A component comprised of discrete volumes is a series system if it fails when one of the discrete volumes fail. This approach gives rise to weakest-link theories. In a parallel system, failure of a single element does not necessarily cause the component to fail since the remaining elements may be able to sustain the load through redistribution. The parallel system approach leads to what has been referred to in the literature as "bundle" theories.

The basic principles underlying these bundle theories were originally discussed by Daniels (1945) and Coleman (1958). Their work was extended to polymer matrix composites by Rosen (1964) and Zweben (1968). Here, a relatively soft matrix serves to transfer stress between fibers and contributes little to the composite tensile strength. Hence, when a fiber breaks the load is transferred only to neighboring fibers. Their analysis is rather complex and limited to establishing bounds on the stress at which the first fiber breaks and the stress at which all the fibers are broken. Harlow and Phoenix (1978) proposed a rather abstract approach that established a closed-form solution for all the intermediate stress levels in a two-dimensional problem, and Batdorf (1982) used an approximate solution to establish the solutions for the three-dimensional problem. Batdorf's model includes the two-dimensional model as a special case. In both of the latter two models, the authors proposed that the effective Weibull modulus increases with increasing component volume. This implies a diminished size effect. However, these current bundle theories are predicated on the fact that fibers are inherently much stronger and stiffer than the matrix. In laminated CMC materials this is not always the case. The strength and stiffness of both the fiber and matrix are usually closer in magnitude. For this reason bundle theories will not be considered in this paper.

We advocate the use of a weakest-link reliability theory for designing components manufactured from laminated CMC materials that do not exhibit strong size effects in specific directions. Assuming that a laminated structure behaves in a weakest-link manner allows a conservative estimate of structural reliability to be calculated. Thomas and Wetherhold (1990) point out that this assumption is equivalent to predicting the probability of the first matrix crack occurring in an individual ply. For most applications the design failure stress for a laminated material is assumed to coincide with this first ply matrix cracking because matrix cracking usually allows the fibers to oxidize at high temperatures, embrittling the composite.

Next, we address the righteousness of applying weakest-link theory to a material that in some sense does not exhibit size effects. In general, the mean strength of a sample population representing uniaxial tension test specimens can be obtained by integrating the probability of survival  $P_s$  with respect to the applied tensile stress: that is,

$$\bar{\sigma} = \int_0^{\infty} P_s d\sigma \quad (1)$$

Here  $\bar{\sigma}$  is the mean tensile strength,  $\sigma$  is the applied tensile stress, and  $P_s$  is the probability of survival. The form of  $P_s$  depends on the probability density function that best represents the failure data, which in turn depends on whether the structural component acts as a parallel or series system. Adopting a three-parameter Weibull cumulative distribution function, and assuming a weakest-link system (a conservative assumption) gives the following form to Eq. (1):

$$\bar{\sigma} = \int_{\gamma}^{\infty} \exp\left[-\left(\frac{\sigma - \gamma}{\beta}\right)^{\alpha} V\right] d\sigma \quad (2)$$

Here  $V$  is the volume of the tensile test specimen.  $\beta$  is the scale parameter,  $\alpha$  is the Weibull shape parameter, and  $\gamma$  is the threshold stress. This integral has the following closed-form solution (see DeSalvo 1970):

$$\bar{\sigma} = \gamma + \left[\frac{\beta}{(V)^{1/\alpha}}\right] \Gamma\left(1 + \frac{1}{\alpha}\right) \quad (3)$$

which depends on volume, the Weibull parameters, and the gamma function  $\Gamma$ . When an argument originally outlined by Jayatilaka (1979) is followed, two uniaxial tensile specimen populations with distinctly different specimen volumes will yield different mean strengths. Associating  $\bar{\sigma}_1$  with  $V_1$  gives

$$\bar{\sigma}_1 = \gamma + \left[\frac{\beta}{(V_1)^{1/\alpha}}\right] \Gamma\left(1 + \frac{1}{\alpha}\right) \quad (4)$$

Similarly associating  $\bar{\sigma}_2$  with  $V_2$  gives

$$\bar{\sigma}_2 = \gamma + \left[\frac{\beta}{(V_2)^{1/\alpha}}\right] \Gamma\left(1 + \frac{1}{\alpha}\right) \quad (5)$$

If the effective mean is defined as

$$(\bar{\sigma})_{\text{eff}} = \bar{\sigma} - \gamma \quad (6)$$

then the ratio of the effective mean strengths depends only on the specimen volume and the Weibull modulus: that is,

$$\frac{(\bar{\sigma}_1)_{\text{eff}}}{(\bar{\sigma}_2)_{\text{eff}}} = \frac{\bar{\sigma}_1 - \gamma}{\bar{\sigma}_2 - \gamma} = \left(\frac{V_2}{V_1}\right)^{1/\alpha} \quad (7)$$

As the Weibull modulus of a particular material increases, the ratio of the effective mean strengths approaches unity. In this situation the material exhibits no size effect (even though the distribution of failure strength may be represented by a Weibull probability density function). From a practical standpoint, doubling the specimen size of a material whose Weibull modulus is  $\approx 15$  would yield less than a 5 percent difference in the effective mean failure strengths of the two populations. We expect an elevated Weibull modulus to be associated with the strength of CMC materials in the fiber direction. Reports of an apparent lack of size effect associated with the strength in the fiber direction (see DiCarlo, 1989) could easily be an artifact of an increasing shape parameter (or small sample size). However, at this time there is an insufficient quantity of CMC failure data from which to estimate the Weibull parameters. In general, the weakest-link theory allows for diminishing size effects as the Weibull modulus increases.

#### RELIABILITY MODEL

The ongoing metamorphosis of ceramic material systems and the lack of standardized design data has in the past tended to minimize the emphasis on modeling. Many structural components fabricated from ceramic materials were designed by "trial and error," since emphasis was placed on demonstrating feasibility rather than on fully understanding the processes controlling behavior. (This is understandable during periods of rapid improvements in material properties for any system.) In predicting failure behavior, there is a philosophical division that separates analytical schools of thought into microstructural methods (usually based on principles of fracture mechanics) and phenomenological methods. Blass and Ruggles (1990) point out that analysts from the first school would design the material assuming that the constituents are distinct structural components and would consider the composite ply (or lamina) a structure in its own right. Analysts from the latter school of thought would design with the material (i.e., they would analyze structural components fabricated from the material). Rigorous fracture mechanics criteria have been proposed (e.g., Budiansky et al., 1986; and Marshall et al., 1985) that adopt the microstructural viewpoint, but since they are all deterministic criteria, they will not be considered here. Fracture mechanics has been combined with a probabilistic Weibull analysis of failure location to determine the stress-strain behavior and subsequent work of fracture for unidirectional composites (e.g., Thouless and Evans, 1988; and Sutcu, 1989). However, the focus here is first matrix cracking, and we note that mature reliability-based design methods using fracture mechanics concepts will not surface until a coherent mixed-mode fracture criterion has been proposed.

The aforementioned second school of thought represents the ply (or lamina) as a homogenized material with strength properties that are determined from a

number of well-planned phenomenological experiments. The authors currently embrace this philosophy, and there are practical reasons for initially adopting this viewpoint. We fully recognize that the failure characteristics of these composites are controlled by a number of local phenomena including matrix cracking, debonding and slipping between matrix and fibers, and fiber breakage, all of which strongly interact. Understanding the underlying analytical concepts associated with the microstructural viewpoint allows one to gain insight and intuition prior to constructing multiaxial failure theories that in some respect reflect the local behavior. Tensile failure in the fiber direction is dependent on these local mechanisms, and the future intent is to extend reliability methods to the constituent level in a rational and practical manner. However, a top-down approach, that is first proposing design models at the ply level, will establish viable and working design protocols. Initially adopting the bottom-up approach allows for the possibility of becoming mired in detail (experimental and analytical) when multiaxial reliability analyses are conducted at the constituent level.

There is a great deal of intrinsic variability in the strength of each brittle constituent of a ceramic matrix composite, but depending on the composite system, the transverse matrix cracking strength may either be deterministic or probabilistic. Statistical models are a necessity for those composite systems which exhibit any scatter in the initiation of first matrix cracking. We treat it in a probabilistic fashion, requiring that deterministic strength be a limiting case that is readily obtainable from the proposed reliability model. Predicting the reduction in reliability due to loads in the fiber direction addresses an upper bound for ply reliability in a structural design problem. Conversely, a tensile load applied transverse to the fiber direction results in failure behavior similar to a monolithic ceramic, which corresponds to the lower bound of ply reliability. Thus multiaxial design methods must be capable of predicting these two bounds as well as account for the reduction in reliability due to an in-plane shear stress, and compressive stresses in the fiber direction and transverse to the fiber direction.

A number of macroscopic theories exist that treat unidirectional composites as homogenized, anisotropic materials. These methods use phenomenological strength data directly without hypothesizing specific crack shapes or distributions. Theories of this genre generally are termed noninteractive if individual stress components are compared to their strengths separately. In essence, failure mechanisms are assumed not to interact, and this results in component reliability computations that are quite tractable. Work by Thomas and Wetherhold (1990), Duffy and Arnold (1990), Duffy and Manderscheid (1990), and Duffy et al. (1990) are representative of multiaxial noninteractive reliability models for anisotropic materials. In addition Wu (1989), and Hu and Goetschel (1989) have proposed simpler unidirectional reliability models for laminated composites that can be classified as noninteractive. Alternatively, one can assume that for multiaxial states of stress, failure mechanisms interact and depend on specific stochastic combinations of material strengths. Usually a failure criterion is adapted from existing polymer matrix design technologies. The probability that the criterion has been violated for a given stress state is computed using Monte Carlo methods (de Roo and Paluch, 1985) or first-order-second-moment (FOSM) methods (Yang, 1989, and Miki et al., 1989). The interactive approach often results in computationally intensive reliability predictions.

In this paper a noninteractive phenomenological approach has been chosen such that a unidirectional ply is considered a two-dimensional structure, assumed to

have five basic strengths (or failure modes). They include a tensile and compressive strength in the fiber direction, a tensile and compressive strength in the direction transverse to the fiber direction, and an in-plane shear strength. In addition each ply is discretized into individual sub-ply volumes. For reasons discussed in the previous section we assume that failure of a ply is governed by its weakest link (or sub-ply volume). Under this assumption, events leading to failure of a given link do not affect other links (see, for example, Batdorf and Heinisch, 1978; Wetherhold, 1983; and Cassenti, 1984); thus the reliability of the *i*th ply is given by the following expression

$$R_i = \exp\left(-\int_V \psi_i \, dV\right) \quad (8)$$

where *V* is the component volume. Here,  $\psi_i(x_j)$  is the failure function per unit volume at position  $x_j$  within the ply, given by

$$\begin{aligned} \psi_i = & \left(\frac{\langle\sigma_1 - \gamma_1\rangle}{\beta_1}\right)^{\alpha_1} + \left[\frac{|\tau_{12} - \gamma_2|}{\beta_2}\right]^{\alpha_2} + \left[\frac{\langle\sigma_2 - \gamma_3\rangle}{\beta_3}\right]^{\alpha_3} \\ & + \left[\frac{\langle(-1)(\sigma_1 + \gamma_4)\rangle}{\beta_4}\right]^{\alpha_4} + \left[\frac{\langle(-1)(\sigma_2 + \gamma_5)\rangle}{\beta_5}\right]^{\alpha_5} \end{aligned} \quad (9)$$

The  $\alpha$ 's associated with each term in Eq. (9) correspond to the Weibull shape parameters, the  $\beta$ 's correspond to Weibull scale parameters, and the  $\gamma$ 's correspond to the Weibull threshold stresses. In addition,  $\sigma_1$  and  $\sigma_2$  represent the in-plane normal stresses that are aligned with and transverse to the fiber direction, respectively. Also,  $\tau_{12}$  is the in-plane shear stress. The normal stresses appear twice and this allows for different failure modes to emerge in tension and compression. Note that the brackets indicate a unit step function: i.e.,

$$\langle x \rangle = x \cdot u[x] = \begin{cases} x & x > 0 \\ 0 & x \leq 0 \end{cases} \quad (10)$$

Inserting Eq. (9) into the volume integration given by Eq. (8) yields the reliability of the *i*th ply, and the probability of first ply failure for the laminate is given by the expression

$$P_{\text{fpf}} = 1 - \prod_{i=1}^n R_i \quad (11)$$

where *n* is the number of plies.

This reliability model can be readily integrated with laminate analysis options available in several commercial finite element codes. A preliminary version of a public domain computer algorithm (C/CARES) that is coupled with MSC/NASTRAN has been developed at NASA Lewis Research Center to perform this analysis. A simple benchmark application illustrates the approach. A thin-wall tube is subjected to an internal pressure and an axial compressive load. The component is fabricated from a three-ply laminate, with a 90°/θ/90° layup. Here angle  $\theta$  is measured relative to the longitudinal axis of the tube (see Fig. 1). An arbitrary internal pressure of 4.25 MPa and an axial compressive stress of

87.5 MPa were applied to the tube. The Weibull parameters were also arbitrarily chosen (see Table 1). Note that the threshold stresses are taken as zero for simplicity. In design, setting the threshold stresses equal to zero would represent a conservative assumption. The overall component reliability is depicted as a function of the midply orientation angle ( $\theta$ ) in Fig. 2. The ply orientation has a decided effect on component reliability, as expected. Similar studies could demonstrate the effects of component geometry, ply thickness, load, and/or Weibull parameters on component reliability. Hence, the C/CARES code allows the design engineer a wide latitude to optimize a component relative to a number of design parameters.

## PARAMETER ESTIMATION

We anticipate that laminated CMC materials will exhibit threshold behavior, at least in the fiber direction. Hence, a three-parameter Weibull distribution is used in the stochastic failure analysis of the components. The threshold stress parameter is included to allow for zero probability of failure when the load is below a predetermined level. The three-parameter distribution has been somewhat ignored due to difficulties encountered in extracting the parameters from experimental data. Several authors (including Weibull, 1939; Weil and Daniel, 1964; and Schneider and Palazotto, 1979) have proposed estimation methods for the three-parameter distribution. For various reasons, these techniques have not been widely accepted. However, Cooper (1988) recently proposed a nonlinear regression method to estimate parameters. Regression analysis postulates a relationship between two variables. In an experiment, typically one variable can be controlled (the independent variable) while the response variable (or dependent variable) is uncontrolled. In simple failure experiments the material dictates the strength at failure, indicating that the failure stress is the response variable. The ranked probability of failure  $\mathcal{P}_i$  can be controlled by the experimentalist since it is functionally dependent on the sample size *N*. If the observed failure stresses ( $\sigma_1, \sigma_2, \sigma_3, \dots, \sigma_N$ ) are placed in ascending order, then

$$\mathcal{P}_i(\sigma_i) = \frac{i - 0.3}{N + 0.4} \quad (12)$$

Clearly one can influence the ranked probability for a given stress level by increasing or decreasing the sample size. Cooper's procedure adopts this philosophy, and the specimen failure stress is treated as the dependent variable. The associated ranked probability of failure is assumed to be the independent variable. The basic three-parameter Weibull expression for probability of failure can be expressed as

$$\tilde{\sigma}_i = \tilde{\gamma} + \tilde{\beta} \left[ \ln\left(\frac{1}{1 - \mathcal{P}_i}\right) \right]^{1/\tilde{\alpha}} \quad (13)$$

where  $\tilde{\sigma}_i$  is an estimate of the dependent variable, and  $\tilde{\gamma}$ ,  $\tilde{\beta}$ , and  $\tilde{\alpha}$  are estimates of the threshold parameter, the characteristic strength, and the shape parameter, respectively. Defining

$$\delta_i = \tilde{\sigma}_i - \sigma_i \quad (14)$$

as the *i*th residual, where as before  $\sigma_i$  is the *i*th failure stress, then

$$\sum_{i=1}^N (\delta_i)^2 = \sum_{i=1}^N (\tilde{\gamma} + \tilde{\beta} W_i^{1/\tilde{\alpha}} - \sigma_i)^2 \quad (15)$$

where we adopt Cooper's notation and take

$$W_i = \ln\left(\frac{1}{1 - \mathcal{P}_i}\right) \quad (16)$$

Setting the partial derivatives of the sum of the squares of the residuals with respect to  $\tilde{\gamma}$ ,  $\tilde{\beta}$ , and  $\tilde{\alpha}$  equal to zero yields the following three expressions:

$$\tilde{\beta} = \frac{N \left[ \sum_{i=1}^N \sigma_i (W_i)^{1/\tilde{\alpha}} \right] - \left[ \sum_{i=1}^N \sigma_i \right] \left[ \sum_{i=1}^N (W_i)^{1/\tilde{\alpha}} \right]}{N \sum_{i=1}^N (W_i)^{2/\tilde{\alpha}} - \left[ \sum_{i=1}^N (W_i)^{1/\tilde{\alpha}} \right] \left[ \sum_{i=1}^N (W_i)^{1/\tilde{\alpha}} \right]} \quad (17)$$

$$\tilde{\gamma} = \frac{\left[ \sum_{i=1}^N (W_i)^{2/\tilde{\alpha}} \right] \left[ \sum_{i=1}^N \sigma_i \right] - \left[ \sum_{i=1}^N \sigma_i (W_i)^{1/\tilde{\alpha}} \right] \left[ \sum_{i=1}^N (W_i)^{1/\tilde{\alpha}} \right]}{N \sum_{i=1}^N (W_i)^{2/\tilde{\alpha}} - \left[ \sum_{i=1}^N (W_i)^{1/\tilde{\alpha}} \right] \left[ \sum_{i=1}^N (W_i)^{1/\tilde{\alpha}} \right]} \quad (18)$$

and

$$\sum_{i=1}^N \sigma_i (W_i)^{1/\tilde{\alpha}} \ln(W_i) = \tilde{\gamma} \sum_{i=1}^N (W_i)^{1/\tilde{\alpha}} \ln(W_i) + \tilde{\beta} \sum_{i=1}^N (W_i)^{2/\tilde{\alpha}} \ln(W_i) \quad (19)$$

in terms of the parameter estimates. The solution of this system of equations is iterative. One assumes an initial value for  $\tilde{\alpha}$  (a small value, usually equal

to 1), computes  $\tilde{\beta}$  from Eq. (17) and  $\tilde{\gamma}$  from Eq. (18). These values of the parameter estimates are then inserted into Eq. (19), and this expression is checked to see if it satisfies some predetermined tolerance. If Eq. (19) is not satisfied,  $\tilde{\alpha}$  is increased and a new iteration is conducted. This procedure continues until a set of parameter estimates are determined that satisfy Eqs. (17) to (19).

Currently we do not have enough CMC failure data to estimate Weibull parameters for a given material orientation. So to illustrate the technique, parameter estimates were determined for two-parameter and three-parameter distributions from a failure population representing a monolithic ceramic (alumina) reported by Quinn (1989). The failure data and specimen geometry are shown in Table 2 and Fig. 3. Figure 4 is a plot of probability of failure versus failure stress for the data. The straight line represents the two-parameter fit to the data, using Quinn's (1989) values for the shape and scale parameters. The nonlinear curve represents the three-parameter fit to the data. Note that the three-parameter distribution is more efficient in

predicting the failure data in the high reliability region.

## SUMMARY AND FUTURE DIRECTIONS

In this paper we discuss stochastic issues related to size effects in the fiber direction of a unidirectional CMC material. In addition, we present a reliability model along with a simple application that highlighted the C/CARES computer algorithm. (This public domain algorithm is capable of predicting component reliability from the state of stress and temperature distribution within the component.) The authors anticipate that CMC materials will exhibit threshold behavior, hence a nonlinear regression analysis was outlined to determine three parameters for a Weibull distribution from failure data.

Ceramic material systems will play a significant role in future elevated-temperature applications. To this end, there are a number of issues that must be addressed by the structural mechanics research community. We begin by pointing out that total failure of an individual ply effectively reduces the overall laminate stiffness. This causes local redistribution of the load to adjacent layers. In addition, delamination between plies relaxes the constraining effects among layers, allowing in-plane strains to vary in a stepwise fashion within a laminate. These effects require the development of rational load redistribution schemes. It is also apparent that before ceramics are used as structural components in harsh service environments, thoughtful consideration should be given to reliability degradation due to time-dependent phenomena. Thus, issues germane to component life, such as cyclic fatigue and creep behavior, must be addressed analytically. Computational strategies are needed to extend current methods of analysis from subcritical crack growth and creep rupture to laminated CMC materials that are subject to multiaxial states of stress.

An important aspect that has not been addressed in detail is the effect of a rising R-curve behavior, where fracture toughness is a function of crack size. Clearly fiber-toughened matrices have process zones around the crack tip. Within this zone, energy dissipates locally, which develops a damage tolerance by increasing the resistance to crack growth with crack extension. Failure of materials exhibiting R-curve behavior would depend on the rate at which resistance increases with crack growth. During crack extension this behavior would modify the strength distribution. Modeling efforts by Kendall et al. (1986) and others have accounted for this behavior in monolithic ceramics, and it is reasonable to extend their work to ceramic composite material systems. Furthermore, if ceramic materials mimic ductile failure locally, cyclic fatigue may become a design issue. Under cyclic loads, the process zone advances as the crack tip extends; therefore, brittle fracture mechanics may need to be modified to account for pseudoductile fracture. Hence, application of modified metallic fatigue analyses may be a distinct possibility.

In addition, recent progress in processing ceramic material systems has not been matched by mechanical testing efforts. There is a definite need for experiments that support the development of reliability models. Initially this effort should include experiments that test the fundamental concepts (e.g., quantifying the size effect in the fiber direction) within the framework of current stochastic models. For example, probing experiments could be conducted along various biaxial load paths to establish level surfaces of reliability in a particular two-dimensional stress space (similar to probing yield surfaces in metals). One

could then verify such concepts as the maximum stress response which is often assumed in the noninteractive reliability models proposed for these materials. After establishing a theoretical framework, characterization tests should then be conducted to provide the functional dependence of model parameters with respect to temperature and environment. Finally data from structural tests that are multiaxial (and possibly nonisothermal) would be used to challenge the predictive capabilities of models through comparison to benchmark response data. These tests involve nonhomogeneous fields of stress, deformation, and temperature, and would include two-bar tests as well as plate and shell structures. Results from structural testing provide feedback for subsequent modification, but ad hoc models result in the absence of structured interaction between the experimentalist and the theoretician. The validity of these models are then forever open to question. Furthermore, we cannot over-emphasize that this kind of testing supports methods for designing components, not the materials. Currently this effort is hampered by the quality and scarcity of data (note the lack of failure data necessary to estimate composite Weibull parameters). Finally, ceramic properties pertinent to structural design (which include stochastic parameters) vary with test methods. The mechanics research community is beginning to realize this, and a consensus is beginning to form regarding standards. However, we wish to underscore the fundamental need for experimental programs that are relevant to structural mechanics issues.

In closing, we recognize that when failure is less sensitive to imperfections in the material, stochastic methods may not be as essential. Yet, trends in design protocols are moving in the direction of probabilistic analyses (even for metals) and away from the simplistic safety-factor approach. In this sense, brittle ceramics will serve as prototypical materials in the study and development of reliability models that will act as the basis of future design codes.

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TABLE 1. - COMPOSITE WEIBULL PARAMETERS FOR THIN-WALL TUBE

[Weibull threshold stress,  $\gamma_i \equiv 0$ .]

Index <sup>a</sup>	Type and direction of stress	Weibull parameters	
		Shape, $\alpha_i$	Scale, $\beta_i$
1	Normal tensile stress in fiber direction	25	450
2	In-plane shear stress	22	420
3	Normal tensile stress transverse to fiber direction	10	350
4	Normal compressive stress in fiber direction	35	4500
5	Normal compressive stress transverse to fiber direction	30	3500

<sup>a</sup>Indices correspond to subscripts in Eq. (9).

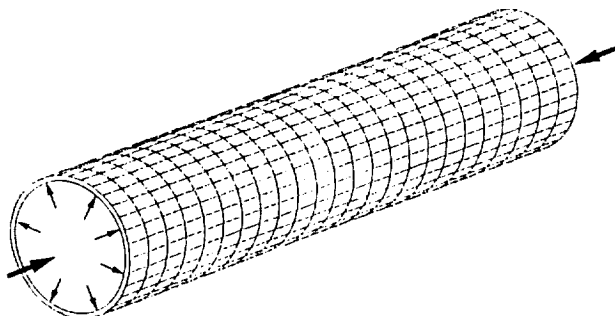


Fig. 1. Finite element mesh of thin-wall tube with three-ply (90°/θ/90°) layup. Internal pressure, 4.25 MPa; axial compressive stress, 87.5 MPa.

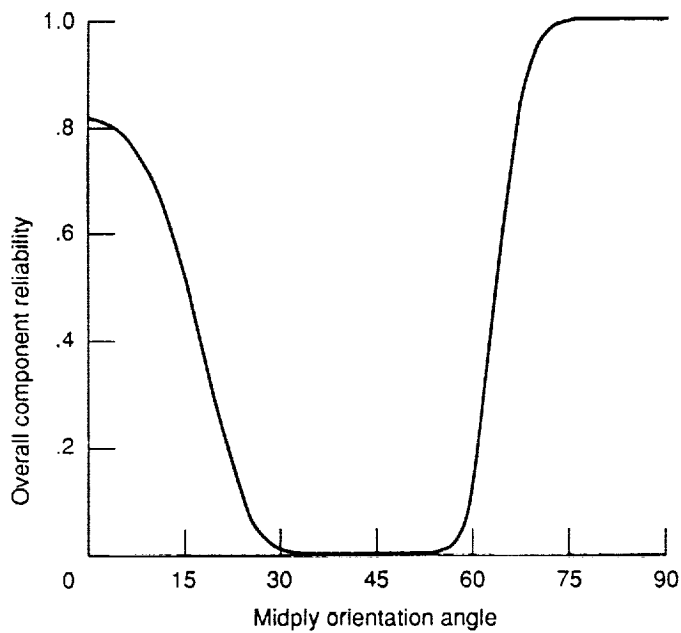


Fig. 2. Component reliability versus midply orientation angle for thin wall tube.

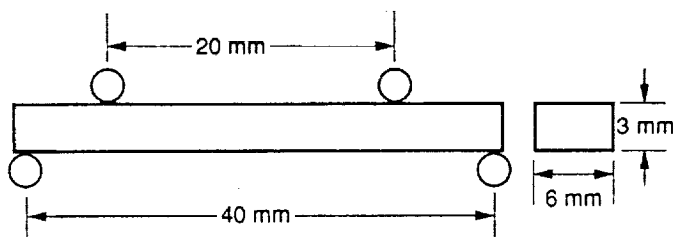


Fig. 3. Monolithic alumina specimen geometry.

	Number of parameters	Shape, $\alpha$	Weibull parameters Scale, $\beta$	Threshold stress, $\gamma$
—	3	1.15	803.41	298.48
- - -	2	13.2	376.0	
○	Data (35 points)			

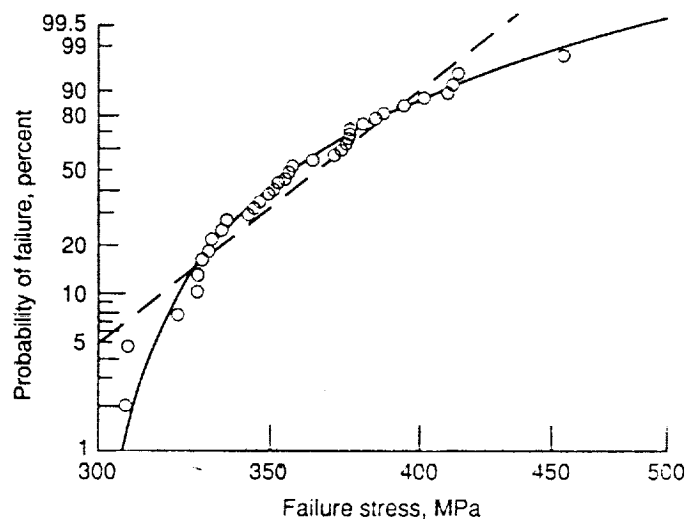


Fig. 4. Two-parameter and three-parameter distributions determined from the alumina failure.

TABLE 2. - MONOLITHIC ALLUMINA FAILURE DATA<sup>a</sup>

Specimen number	Stress, MPa	Specimen number	Stress, MPa	Specimen number	Stress, MPa	Specimen number	Stress, MPa
1	307	10	337	19	357	28	385
2	308	11	343	20	364	29	388
3	322	12	345	21	371	30	395
4	328	13	347	22	373	31	402
5	328	14	350	23	374	32	411
6	329	15	352	24	375	33	413
7	331	16	353	25	376	34	415
8	332	17	355	26	376	35	456
9	335	18	356	27	381		

<sup>a</sup>For specimen shown in Fig. 3.

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16. Abstract For laminated ceramic matrix composite (CMC) materials to realize their full potential in aerospace applications, design methods and protocols are a necessity. This paper focuses on the time-independent failure response of these materials and presents a reliability analysis associated with the initiation of matrix cracking. It highlights a public domain computer algorithm that has been coupled with the laminate analysis of a finite element code and which serves as a design aid to analyze structural components made from laminated CMC materials. Issues relevant to the effect of the size of the component are discussed, and a parameter estimation procedure is presented. The estimation procedure allows three parameters to be calculated from a failure population that has an underlying Weibull distribution.					
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