Local Synthesis and Tooth Contact Analysis of Face-Milled Spiral Bevel Gears

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$K_i^{(i)}, K_{ii}^{(i)} \ (i = 1, 2)$  Principal curvatures of surface 1 or 2

$[L_{yx}]$  Matrix of coordinate transformation from system $S_x$ to system $S_y$ for free vectors

$m'_{21}(\phi_1)$  Derivative of transmission ratio

$M$  Mean contact point

$[M_{yx}]$  Matrix of coordinate transformation from system $S_x$ to system $S_y$ for position vectors

$N_i \ (i = 1, 2)$  Number of teeth of pinion ($i = 1$) or gear ($i = 2$)

$\bar{n}_p$  Unit normal vector of gear cutter surface in system $S_{p2}$

$\bar{n}$  Common unit normal at point of contact

$O_i$  Pitch cone apex of gear $i$

$O_{2R}$  Root cone apex of gear

$p$  Percentage of amount of shift along the pitch line over face width

$PW$  Point width of gear cutter

$q_i$  Cradle angle for gear $i$

$R_{cp}$  Point radius of pinion head cutter

$R_{aG}$  Gear ratio of roll

$R_{u2}$  Gear nominal cutter radius

$r_c$  Gear cutter tip radius

$\bar{r}_{p2}$  Position vector of gear cutter surface in system $S_{p2}$

$\bar{r}_i$  Position vector of tooth surface of gear $i$ represented in system $S_i$, $\bar{r}_i$ is equivalent to $[r_i]$

$\bar{r}_{h}^{(i)}$  Position vector of mean contact point in system $S_h$

$\bar{r}_h^{(PF)}$  Position vector of pinion cutter center in system $S_h$

$\bar{r}_1(\theta_F, \phi_F)$  Position vector of pinion in system $S_1$

$\bar{r}_2(\theta_G, \phi_p)$  Position vector of gear in system $S_2$
Coordinate systems originated at point of contact between $\Sigma_1$ and $\Sigma_2$

$S_a, S_b$

Coordinate system rigidly connected to the cutting machine of gear $i$

$S_{ni}$

Movable coordinate system rigidly connected to the cradle of cutting machine for gear $i$

$S_h$

Fixed coordinate system

$S_i$

Coordinate system rigidly connected to gear $i$

$(s_G, \theta_G)$

Surface coordinates of gear cutter surface

$(s_F, \theta_F)$

Surface coordinates of pinion cutter surface

$S_{ri}$

Radial setting of gear $i$

$S_x$

Auxiliary coordinate system identified by subscript $x$

$\Delta T$

Cam setting

$V_r^{(i)}$ ($i = 1, 2$)

Sliding velocity of contact point in the motion over surface $\Sigma_i$

$V_{tr}^{(i)}$ ($i = 1, 2$)

Transfer velocity of contact point in the motion with surface $\Sigma_i$

$\bar{v}_{s}^{(i)}, \bar{v}_{q}^{(i)}$ ($i = 1, 2$)

Projection of $V_r^{(i)}$ upon $\bar{e}_s$ and $\bar{e}_q$

$V_{12}$

Relative velocity at contact point

$\bar{v}_{m2}^{(p2)}$

Relative velocity in the process for gear generation represented in system $S_{m2}$

$(X_o^{(i)}, Z_o^{(i)})$

Coordinates of center of the arc blade

$X_{Bi}$

Sliding base for generation of gear $i$

$X_{Gi}$

Machine center to back for generation of gear $i$

$(X_L, RL)$

Parameters determining mean contact point

$V_G, H_G$

Vertical and horizontal adjustments for the gear drive

$Z_R$

Distance of gear root cone apex beyond pitch cone apex

$\kappa_s, \kappa_q$

Principal curvatures of surface $\Sigma_2$

$\kappa_s^{(p)}, \kappa_q^{(p)}$

Principal curvatures of surface $\Sigma_2$

$\kappa_f, \kappa_h$

Principal curvatures of surface $\Sigma_1$

$\gamma_{ni}$

Machine root angle for generation of gear $i$
\( \Gamma_i \) Pitch angle of gear \( i \)

\( \gamma_i \) Root angle of gear \( i \)

\( \alpha \) Cam guide angle

\( \alpha_G, \alpha_F \) Cutter blade angles for gear and pinion respectively

\( \rho \) Radius of circular arc

\( (\lambda, \theta_F) \) Surface coordinates of the surface of revolution generated by circular arc blade

\( \eta_i \) Direction angle of contact path on surface \( \Sigma_i \)

\( (\bar{\eta}, \bar{\zeta}) \) Unit vectors along long and short axes of contact ellipse

\( \delta \) Elastic approach

\( \delta_G \) Gear dedendum angle

\( \phi_i \) Angle of rotation of gear \( i \) in the process for generation

\( \phi'_i \) Rotation angle in meshing of gear \( i \) between the gear (2) and the pinion (1)

\( \phi_F, \phi_p \) Rotation angles of cradle in the process for pinion and gear generation, respectively

\( \psi_G \) Gear spiral angle

\( (\theta_G^*, \phi_p^*) \) Surface coordinates of gear tooth surface at mean contact point

\( \sigma^{(12)} \) Angle formed between principal directions \( \bar{e}_f \) and \( \bar{e}_s \) (in meshing and generation )

\( \Sigma_i \) Surface of gear \( i \)

\( \Sigma_F \) Pinion generating surface

\( \Sigma_p \) Gear generating surface

\( \bar{\omega}^{(1)} \ (i = 1, 2) \) Angular velocity of surface \( \Sigma_i \) (in meshing and generation )

\( \bar{\omega}^{(F)}, \bar{\omega}^{(P)} \) Angular velocity of the cradle in the process for pinion and gear generation, respectively

\( \bar{\omega}^{(P2)} \) Relative angular velocity in the process for gear generation represented
in system $S_{m2}$

$\omega^{(P1)}$

Relative angular velocity in the process of pinion generation

$\omega^{(i)}$

Angular velocity of gear $i$

$\omega^{(ij)}$

Relative angular velocity between gear $i$ and gear $j$
SUMMARY

Computerized simulation of meshing and bearing contact for spiral bevel gears and hypoid gears [1,2] is a significant achievement that could improve substantially the technology and the quality of the gears. This report covers a new approach to the synthesis of face-milled spiral bevel gears and their tooth contact analysis. The proposed approach is based on the following ideas proposed in [3] (i) application of the principle of local synthesis that provides optimal conditions of meshing and contact at the mean contact point M and in the neighborhood of M; (ii) application of relations between principle directions and curvatures for surfaces being in line contact or in point contact.

The developed local synthesis of gears provides (i) the required gear ratio at M; (ii) a localized bearing contact with the desired direction of the tangent to the contact path on gear tooth surface and the desired length of the major axis of contact ellipse at M; (iii) a predesigned parabolic function of a controlled level (8-10 arc seconds) for transmission errors; such a function of transmission errors enables to absorb linear functions of transmission errors caused by misalignment [3] and reduce the level of vibrations.

The proposed approach does not require either the tilt of the head-cutter for the process of generation or modified roll for the pinion generation. Improved conditions of meshing and contact of the gears can be achieved without the above mentioned parameters. The report is complemented with a computer program for determination of basic machine-tool settings and tooth contact analysis for the designed gears. The approach is illustrated with a numerical example.

The contents of the following sections cover the following topics:

(1). Basic ideas of local synthesis of gears and the mathematical concept of this approach (Chapter 1). The local synthesis discussed in this chapter is applicable for all types of gears and provides the optimal conditions of meshing and contact at the mean point of tangency of gear tooth surfaces.

(2). Methods for generation of the pinion and the gear and basic machine-tool settings that are
necessary for gear generation (Chapter 2).

(3). Determination of geometry of gear tooth surface, the gear mean contact point and the principal directions and curvatures at this point (Chapter 3).

(4). Application of basic principles of local synthesis for spiral bevel gears (Chapter 4).

(5). Determination of pinion machine-tool settings considering as given: (i) the gear geometry, and (ii) the conditions of meshing and contact at the mean contact point obtained from the local synthesis (Chapter 5).

(6). Computerized simulation of meshing and contact (Tooth Contact Analysis) for spiral bevel gears that have been synthesized in the previous chapters (Chapter 6).

(7). Analysis of the shift of bearing contact caused by the misalignment of gears (Chapter 7).

(8). The theory of modified roll (variation of cutting ratio in the process for generation) and mechanisms used for application of modified roll (Appendix A).

(9). Description of developed computer programs and numerical examples that illustrates the application of those programs.
1 Local Synthesis of Gears (General Concept)

1.1 Introduction

The main goals of local synthesis are to provide: (i) contact of gear tooth surfaces at the mean point of contact of gear tooth surfaces, and (ii) improved conditions of meshing within the neighborhood of the mean contact point. The local synthesis is the first stage of the global synthesis with a goal to provide improved conditions of meshing for the entire area of meshing. The criteria of conditions of meshing are the transmission errors and the bearing contact. The principles of local synthesis that are discussed in this chapter for face-milled spiral bevel gears can be applied for other types of gears as well.

1.2 Basic Linear Equations

Consider two right-handed trihedrons \( S_a(\vec{e}_f, \vec{e}_h, \vec{n}) \) and \( S_b(\vec{e}_s, \vec{e}_q, \vec{n}) \) (Fig. 1.2.1). The common origin of the trihedrons coincides with the contact point \( M \), the \( n \)-axis represents the direction of the surface unit normal, \( \vec{e}_f \) and \( \vec{e}_h \) are the unit vectors of the principal directions of surface \( \Sigma_1 \), \( \vec{e}_s \) and \( \vec{e}_q \) represent the principal directions of surface \( \Sigma_2 \), and \( \sigma^{(12)} \) is the angle formed between \( \vec{e}_f \) and \( \vec{e}_s \) (measured clockwise from \( \vec{e}_s \) to \( \vec{e}_f \) and counterclockwise from \( \vec{e}_f \) to \( \vec{e}_s \)). In reference [4] three linear equations were derived that relate the velocity \( \vec{v}_i^{(1)} \) of the contact point over surface \( \Sigma_1 \) with the principal curvatures and directions of contacting surfaces and the transfer components of velocities. These equations are:

\[
\begin{align*}
  a_{11}v_i^{(1)} + a_{12}v_q^{(1)} &= a_{13} \\
  a_{12}v_i^{(1)} + a_{22}v_q^{(1)} &= a_{23} \\
  a_{13}v_i^{(1)} + a_{23}v_q^{(1)} &= a_{33}
\end{align*}
\]

(1.2.1)

Here (see the designations in [4])
Equations (1.2.1) and (1.2.2) can be applied for two cases where: (i) surfaces $C_1$ and $C_2$ are in line contact, and (ii) the surfaces are in point contact. The instantaneous line of contact is typical for the case when the gear tooth surface $(E_l)$ is generated by the tool surface $(X_2)$. The instantaneous point of contact is typical for gears with localized bearing contact.

Line Contact

When the gear tooth surfaces are in line contact, the direction of velocity $\vec{v}_r^{(1)}$ can be varied, and equations (1.2.1) can not provide a unique solution for the unknowns $v_s^{(1)}$ and $v_q^{(1)}$. This results in that the rank of the augmented matrix

$$[A] = \begin{bmatrix}
  a_{11} & a_{12} & a_{13} \\
  a_{12} & a_{22} & a_{23} \\
  a_{13} & a_{23} & a_{33}
\end{bmatrix}$$

must be less than 2. This requirement yields
Equivalent equations are

\[
\begin{align*}
    a_{12} & = a_{11} a_{22} \\
    a_{11} a_{23} & = a_{12} a_{13} \\
    a_{12} a_{33} & = a_{13} a_{23}
\end{align*}
\]  

(1.2.4)

Using equations (1.2.5) and (1.2.2) we obtain equations that will enable us to determine \( \sigma^{(12)} \), \( \kappa_f \) and \( \kappa_h \) for \( \Sigma_1 \) considering as given \( \kappa_s \) and \( \kappa_q \) for surface \( \Sigma_2 \). The equations are:

\[
\begin{align*}
    \tan 2\sigma^{(12)} & = \frac{2a_{13}a_{23}}{a_{23}^2 - a_{13}^2 + (\kappa_s - \kappa_q)a_{33}} \\
    \kappa_f - \kappa_h & = \frac{2a_{13}a_{23}}{a_{33} \sin 2\sigma^{(12)}} \\
    \kappa_f + \kappa_h & = (\kappa_s + \kappa_q) - \frac{a_{13}^2 + a_{23}^2}{a_{33}}
\end{align*}
\]  

(1.2.6)

Equation (1.2.6) provides two solutions: \( \sigma_1^{(12)} \) and \( \sigma_2^{(12)} = \sigma_1^{(12)} + \pi/2 \) and both of them can be used for computations of \( \kappa_f \) and \( \kappa_h \) that are represented by equations (1.2.7) and (1.2.8). Fig.1.2.2 shows the orientation of two couples of unit vectors \( \hat{e}_f^{(i)} \), \( \hat{e}_h^{(i)} \) \( (i = 1, 2) \), with respect to unit vector \( \hat{e}_s \). The magnitude of principal curvature for the direction with collinear vectors \( \hat{e}_f^{(1)} \) and \( \hat{e}_h^{(2)} \) is the
same \( \kappa_f^{(1)} = \kappa_f^{(2)} \) although the notation for the unit vectors has been changed. Similarly, we can say that \( \kappa_h^{(1)} = \kappa_h^{(2)} \).

Knowing the angle \( \sigma^{(12)} \), and the unit vectors \( \vec{\varepsilon}_s \) and \( \vec{\varepsilon}_q \), the principal directions on surface \( \Sigma_1 \) can be determined with the following equations,

\[
\vec{\varepsilon}_f^{(1)} \equiv \vec{\varepsilon}_f = \cos \sigma^{(12)} \vec{\varepsilon}_s - \sin \sigma^{(12)} \vec{\varepsilon}_q
\]

(1.2.9)

\[
\vec{\varepsilon}_h^{(1)} \equiv \vec{\varepsilon}_h = \sin \sigma^{(12)} \vec{\varepsilon}_s + \cos \sigma^{(12)} \vec{\varepsilon}_q
\]

(1.2.10)

Point Contact

In the case of instantaneous point of contact, the direction of motion of the contact point over the surface is definite, equations (1.2.2) for the unknowns can provide a unique solution for the unknowns \( v_s^{(1)} \) and \( v_q^{(1)} \) and the rank of matrix \( [A] \) is 2. This yields that

\[
\begin{vmatrix}
  a_{11} & a_{12} & a_{13} \\
  a_{21} & a_{22} & a_{23} \\
  a_{31} & a_{32} & a_{33}
\end{vmatrix} = 0
\]

(1.2.11)

Equation (1.2.11) yields the following relation

\[
f(\kappa_s, \kappa_q, \kappa_f, \kappa_h, \sigma^{(12)}, m_{12}) = 0
\]

(1.2.12)

Our goal is to determine \( \kappa_f, \kappa_h \) and \( \sigma^{(12)} \) (the principal curvatures and directions of \( \Sigma_1 \)) and provide at the mean contact point (i) a certain direction of the tangent to contact path on surface
\( \Sigma_2 \), (ii) a desired length of the major axis of instantaneous contact ellipse, and (iii) a parabolic function of transmission errors. For these purpose we have to derive extra equations in addition to equation (1.2.12)

**Determination of \( m'_{21} \)**

The derivative \( m'_{21}(\phi_1) \) is the second derivative of function \( \phi_2(\phi_1) \) that is taken at the mean contact point; \( \phi_1 \) and \( \phi_2 \) are the angles of rotation of gears 1 and 2. In the case of an ideal gear train, function \( \phi_2(\phi_1) \) is linear and is represented by

\[
\phi_2 = \phi_1 \frac{N_1}{N_2} \quad (1.2.13)
\]

However, due to misalignment between the meshing gears the real function \( \phi_2(\phi_1) \) becomes a piecewise periodic function with the period equal to the cycle of meshing of a pair of teeth (Fig. 1.2.3). Due to the jump of angular velocity at the junction of cycles, the acceleration approaches to an infinitely large value and this can cause large vibration and noise. For this reason it is necessary to predesign a parabolic function of transmission error that can absorb a linear function of transmission error and reduce the jump of angular velocity and acceleration [3]. This goal (the predesign of a parabolic function) can be achieved with certain relations between the principal curvatures of contacting surfaces.

Fig. 1.2.4 shows the predesigned transmission function for the gear convex side (Fig. 1.2.4(a)) and gear concave side (Fig. 1.2.3(b)). Both functions \( \phi_2(\phi_1) \) and \( \phi_2^{(t)}(\phi_1) \) are in tangency at the mean contact point and have the same derivative \( m_{21} \), at this point.

Consider now that the predesigned transmission function is represented as

\[
\phi_2 - \phi^{(0)}_2 = F(\phi_1 - \phi^{(0)}_1) \quad (1.2.14)
\]
Here: $\phi_1^{(0)}$ and $\phi_2^{(0)}$ are the initial angles of rotation of gears 1 and 2 that provide the tangency of gear tooth surfaces at the mean contact point $M$.

Using the Taylor expansion up to the members of second order, we obtain

$$F(\phi - \phi^{(0)}) = \frac{\partial F}{\partial \phi_1}(\phi_1 - \phi^{(0)}) + \frac{1}{2} \frac{\partial^2 F}{\partial \phi_1^2}(\phi_1 - \phi^{(0)})^2$$

$$= m_{21}(\phi_1 - \phi_1^{(0)}) + \frac{1}{2} m_{21}^2(\phi_1 - \phi_1^{(0)})^2$$

(1.2.15)

where $m_{21}(\phi_1)$ is equal to $N_1/N_2$ at the mean contact point and $m_{21}^2$ is the to be chosen constant value: positive for the gear concave side, and negative for the gear convex side. The synthesized gears rotates with a parabolic function of transmission errors represented by

$$\Delta \phi_2(\phi_1) = \frac{1}{2} m_{21}'(\phi_1 - \phi_1^{(0)})^2$$

(1.2.16)

where

$$-\frac{\pi}{N_1} \leq (\phi_1 - \phi_1^{(0)}) \leq \frac{\pi}{N_1}$$

Equation (1.2.16) enables the determination of $m_{21}'$ considering as known the expected values of transmission errors.

**Relation between Directions of Paths of Contact**

We recall that velocities $\bar{v}_r^{(1)}$ and $\bar{v}_r^{(2)}$ are related by the equation [4],

$$\bar{v}_r^{(2)} = \bar{v}_r^{(1)} + \bar{v}^{(12)}$$

(1.2.17)

Directions of velocities $\bar{v}_r^{(1)}$ and $\bar{v}_r^{(2)}$ coincide with the tangents to the contact path that form angles $\eta_1$ and $\eta_2$ with the unit vector $\bar{e}_s$ (Fig. 1.2.5). Equations (1.2.17) yield
According to Fig. 1.2.5

\[ \mathbf{v}_s^{(2)} = \mathbf{v}_s^{(1)} + \mathbf{v}_s^{(12)} \]

\[ \mathbf{v}_q^{(2)} = \mathbf{v}_q^{(1)} + \mathbf{v}_q^{(12)} \]  

(1.2.18)

Third equation of system (1.2.1) and equations (1.2.18) and (1.2.19) yield

\[ \tan \eta_1 = \frac{-a_{31}v_q^{(12)} + (a_{33} + a_{31}v_s^{(12)}) \tan \eta_2}{a_{33} + a_{32}(v_q^{(12)} - v_s^{(12)}) \tan \eta_2} \]  

(1.2.20)

\[ v_s^{(1)} = \frac{a_{33}}{a_{13} + a_{23} \tan \eta_1} \]  

(1.2.21)

\[ v_q^{(1)} = \frac{a_{33} \tan \eta_1}{a_{13} + a_{23} \tan \eta_1} \]  

(1.2.22)

Prescribing a certain value for \( \eta_2 \) (choosing the direction for path of contact on \( \Sigma_2 \)), we can determine \( \tan \eta_1, v_s^{(1)} \) and \( v_q^{(1)} \). We recall that coefficients \( a_{31}, a_{32} \) and \( a_{33} \) do not depend on the to-be determined principal curvatures \( \kappa_f \) and \( \kappa_h \) and \( \sigma^{(12)} \).

Relations between the Magnitude of Major Axis of Contact Ellipse, Its Orientation and Principal Curvatures and Directions of Contacting Surfaces

Our goal is to relate parameters \( \sigma^{(12)}, \kappa_f \) and \( \kappa_h \) of the pinion surface \( \Sigma_1 \) with the length of the major axis of the instantaneous contact ellipse. This ellipse is considered at the mean contact point and the elastic approach \( \delta \) of contacting surfaces is considered as known from the experimental data. The derivation of the above mentioned relations is based on the following procedure

Step 1: Using equations (1.2.2), we obtain
Step 2: It is known from [4] that

\[ a = \sqrt{\left| \frac{\delta}{A} \right|} \]  
(1.2.24)

\[ A = \frac{1}{4} \left[ K_{\Sigma}^{(1)} - K_{\Sigma}^{(2)} - \sqrt{g_1^2 - 2g_1g_2 \cos 2\sigma + g_2^2} \right] \]  
(1.2.25)

Equation (1.2.25) yields

\[ [(a_11 + a_22 + 4A)^2 = (a_11 - a_22)^2 + 4a_{12}^2 \]  
(1.2.26)

Step 3: We may consider now a system of three linear equations in unknowns \( a_{11}, a_{12} \) and \( a_{22} \)

\[ \begin{align*}
    v_s^{(1)}a_{11} + v_q^{(1)}a_{12} &= a_{13} \\
    v_s^{(1)}a_{11} + v_q^{(1)}a_{22} &= a_{23} \\
    a_{11} + a_{22} &= K_{\Sigma}
\end{align*} \]  
(1.2.27)

Step 4: The solution of equation system (1.2.27) for the unknowns \( a_{11}, a_{12} \) and \( a_{22} \) allows to express these unknowns in terms of \( a_{13}, a_{23}, K_{\Sigma}, v_s^{(1)} \) and \( v_q^{(1)} \). Then, using equation (1.2.25) we can get the following equation for \( K_{\Sigma} \)
\[ K_\Sigma = \frac{4A^2 - (n_1^2 + n_2^2)}{2A - (n_1 \cos 2\eta_1 + n_2 \sin 2\eta_1)} \]  

(1.2.28)

Here:

\[ n_1 = \frac{a_{13}^2 - a_{23}^2 \tan^2 \eta_1}{(1 + \tan^2 \eta_1)a_{33}} \]

\[ n_2 = \frac{a_{13} \tan \eta_1 + a_{23}(a_{13} + a_{23} \tan \eta_1)}{(1 + \tan^2 \eta_1)a_{33}} \]

\[ A = \frac{a^2}{\delta} \]  

(1.2.29)

The advantage of equation (1.2.28) is that we are able to determine \( K_\Sigma \) knowing the major axis \( 2a \) of the contact ellipse and the elastic approach \( \delta \).

Step 5: The sought for principal curvatures and directions for the pinion identified with \( \kappa_f, \kappa_h \) and \( \sigma^{(12)} \) can be determined from the following equations

\[ K_\Sigma^{(1)} = K_\Sigma^{(2)} - K_\Sigma \]  

(1.2.30)

\[ \tan 2\sigma^{(12)} = \frac{2a_{22}}{g_2 - (a_{11} - a_{22})} = \frac{2n_2 - K_\Sigma \sin 2\eta_1}{g_2 - 2n_1 + K_\Sigma \cos 2\eta_1} \]  

(1.2.31)

\[ g_1 = \frac{2a_{12}}{\sin 2\sigma^{(12)}} = \frac{2n_2 - K_\Sigma \sin 2\eta_1}{\sin 2\sigma^{(12)}} \]  

(1.2.32)

\[ \kappa_f \equiv \kappa_f^{(1)} = \frac{K_\Sigma^{(1)} + g_1}{2} \]  

(1.2.33)

\[ \kappa_h \equiv \kappa_h^{(1)} = \frac{K_\Sigma^{(1)} - g_1}{2} \]  

(1.2.34)
Step 6: The orientation of unit vector $\tilde{e}_f$ and $\tilde{e}_h$ is represented with equations (1.2.9) and (1.2.10). The orientation of the contact ellipse with respect to $\tilde{e}_f$ is determined with angle $\alpha^{(1)}$ (Fig. 1.2.6) that is represented with the equations

\[
\cos 2\alpha^{(1)} = \frac{g_1 - g_2 \cos 2\sigma^{(12)}}{(g_1^2 - 2g_1g_2 \cos 2\sigma^{(12)} + g_2^2)^{\frac{1}{2}}} 
\]  
(1.2.35)

\[
\sin 2\alpha^{(1)} = \frac{g_2 \sin 2\sigma^{(12)}}{(g_1^2 - 2g_1g_2 \cos 2\sigma^{(12)} + g_2^2)^{\frac{1}{2}}} 
\]  
(1.2.36)

The minor axis of the contact $(2b)$ ellipse is determined with the equations

\[
b = \sqrt{\frac{\delta}{B}} 
\]  
(1.2.37)

\[
B = \frac{1}{4} \left[ K^{(1)}_\Sigma - K^{(2)}_\Sigma + \sqrt{g_1^2 - 2g_1g_2 \cos 2\sigma + g_2^2} \right] 
\]  
(1.2.38)

Local Synthesis Computational Procedure:

The following is an overview of the computational procedure that is to-be used for the local synthesis.

The input data are: $\kappa_s, \kappa_q, \tilde{e}_s, \tilde{e}_q, \tau^{(M)}, \omega^{(12)}, \varphi^{(12)}$ and $\delta$. The to-be chosen parameters are: $\eta_2, m_2, \text{ and } 2a$. The output data are: $\kappa_f, \kappa_h, \sigma^{(12)}, \tilde{e}_f$ and $\tilde{e}_h$.

Step 1: Choose $\eta_2$ and determine $\eta_1$ from equation (1.2.20)

Step 2: Determine $v^{(1)}_s$ and $v^{(1)}_q$ from equations (1.2.21) and (1.2.22)
Step 3: Determine $A$ from equation (1.2.29)

Step 4: Determine $K_2$ from equation (1.2.28)

Step 5: Determine $\sigma^{(12)}, \kappa_f$ and $\kappa_h$ by using the set of equations from (1.2.30) to (1.2.34)

Step 6: Determine the orientation of the contact ellipse and its minor axis by using equations from (1.2.35) to (1.2.37)

1.3 Conclusion

The contact of tooth surfaces is considered for two cases: line contact and point contact. For line contact, the principal directions and curvatures of one surface can be determined in terms of the other's knowing the relative motion between the two. For point contact, we proposed an approach for local synthesis of spiral bevel gears which enables: (i) to provide a limited level of transmission errors, (ii) optimal direction for the path of contact on gear surface $\Sigma_2$, and (iii) the guaranteed length of the major axis of contact ellipse.

The output data obtained from the procedure of local synthesis are: $\kappa_f, \kappa_h, \sigma^{(12)}, \bar{e}_f$ and $\bar{e}_h$. The machine-tool settings for the generation of the gear tooth surfaces must be carefully chosen to guarantee the above mentioned conditions of local meshing and contact.
2 Pinion and Gear Generation

2.1 Pinion Generation

To describe the pinion generation we will use the following coordinate system (Fig.2.1.1): (i) $S_{m1}$—a fixed coordinate system that is rigidly connected to the cutting machine; (ii) $S_{c1}$—a movable coordinate system that is rigidly connected to the cradle and performs rotation with the cradle about the $Z_{m1}$ axis; initially, $S_{c1}$ coincides with $S_{m1}$ (Fig.2.1.1 (b)); angle $\phi_F$ determines the current position of $S_{c1}$ (Fig.2.1.1 (c)); (iii) Coordinate systems $S_a$ and $S_b$ that are rigidly connected to the cradle and its coordinate system $S_{c1}$; systems $S_a$ and $S_b$ are used to describe the installment of the head-cutter on the cradle. Angle $q_1$ determines the orientation of $S_a$ with respect to $S_{c1}$; (iv) Coordinate system $S_F$ that is rigidly connected to the head-cutter (not shown in Fig.2.1.1); the head-cutter in the process for generation performs rotation with the cradle (transfer motion) and relative motion with respect to the cradle about an axis that passes through $O_a$; (v) Auxiliary coordinate systems $S_d$ and $S_p$ are used to describe the installment of the pinion on the cutting machine (Fig.2.1.1 and Fig.2.1.2); the pinion axis forms angle $\gamma_{m1}$ with axis $X_d$ that is parallel to $X_{m1}$. (vi) A movable coordinate system $S_1$ that is rigidly connected to the being generated pinion; the pinion rotates about the axis $X_p$ and $\phi_1$ is the current angle of pinion rotation (Fig.2.1.2).

Henceforth, we have to differentiate the parameter of motions that are performed in the process for generation and the parameters of installment of the head cutter and the pinion on the cutting machine.

In the process for generation the cradle of the cutting machine with the mounted head-cutter performs rotation with angular velocity $\bar{\omega}^{(F)}$ (Fig.2.1.2). The head-cutter performs rotational motion with respect to the cradle but this motion is not related with the process for generation and just provides the desired velocity of cutting. The being generated pinion performs rotational motion with angular velocity $\bar{\omega}^{(1)}$ (Fig.2.1.2) that is related with $\bar{\omega}^{(F)}$.

The parameters of installment of the head-cutter are: (i) the swivel angle $j$ (Fig.2.1.1) and the
tilt angle \( \hat{t} \) that is the turn angle of \( S_t \) about \( Y_t \) (Fig.2.1.3); \( S_{r1} = |O_cO_m| \) is the radial setting; \( q_1 \) is the cradle angle.

The parameters of installment of the pinion are: \( E_{m1} \)-the shortest machine center distance (Fig. 2.1.1, Fig.2.1.2); root angle \( \gamma_{m1} \); sliding base \( X_{B1} \); machine center to back \( X_{G1} \).

2.2 Gear Generation

While describing the gear generation, we will consider the following coordinate systems: (i) \( S_{m2} \) that is rigidly connected to the cutting machine; (ii) \( S_{c2} \) that is rigidly connect to the cradle, (iii) \( S_{p2} \) that is rigidly connected to the head-cutter and \( S_{c2} \); (iv) \( S_{d2} \) that is an additional fixed coordinate system rigidly connected to \( S_{m2} \); and (v) \( S_2 \) that is rigidly connected to the being generated gear.

The cradle performs rotation about the \( Z_{m2} \) axis with angular velocity \( \dot{\omega}^{(p)} \) (Fig.2.2.1). The initial and current positions of coordinate systems \( S_{c2} \) and \( S_{p2} \) with respect to \( S_{m2} \) are shown in Fig.2.2.1 (a) and Fig.2.2.1 (b), respectively.

Coordinate system \( S_{d2} \) (it is rigidly connected to \( S_{m2} \)) is used to describe the installment of the gear at the cutting machine (Fig.2.2.2(a)). In the general case apices \( O_{2R} \) and \( O_2 \) of the gear root cone and pitch cone do not coincide. Apex \( O_{2R} \) is located on axis \( X_{m2} \) of the cutting machine. The origin \( O_{d2} \) of \( S_{d2} \) coincides with the apex \( O_2 \) of the gear pitch core. Axes \( X_{d2} \) and \( X_{m2} \) form angle \( \gamma_{m2} \) which is the gear machine root angle.

Coordinate system \( S_2 \) is rigidly connected to the gear that in the process of generation performs rotation about \( X_{d2} \) with angular velocity \( \dot{\omega}^{(2)} \) (Fig.2.2.2(b)). Angle \( \phi_2 \) is the current angle of rotation of gear 2.

2.3 Gear Machine Tool Settings

Gear Cutting Ratio

Fig.2.3.1 shows the sketch of the gear with noncoinciding apexes of the root and pitch cones. In the process for generation the pitch line \( O_2P \) is the instantaneous axis of rotation. It is evident
that the angular velocity of rotation in relative motion, \( \omega^{(p2)} \), must lie in the plane that is formed by vectors \( \omega^{(p)} \) and \( \omega^{(2)} \) (Fig. 2.3.2)

\[
\omega^{(p2)} = \omega^{(p)} - \omega^{(2)}
\]

(2.3.1)

The cutting gear ratio is:

\[
R_{aG} = \frac{|\omega^{(2)}|}{|\omega^{(p)}|} \frac{\cos \delta_G}{\sin \Gamma_2} = \frac{\cos (\Gamma_2 - \gamma_2)}{\sin \Gamma_2}
\]

(2.3.2)

Gear Settings

Fig. 2.3.3 shows the installment of the head-cutter. We designate the mean pitch cone distance \( O_2 P \) (Fig. 2.3.1, Fig. 2.3.3) by \( A_m \). Then we obtain (Fig. 2.3.3)

\[
H_G = A_m \cos \delta_G - R_{u2} \sin \psi_G
\]

(2.3.3)

\[
V_G = R_{u2} \cos \psi_G
\]

(2.3.4)

\[
S_{r2} = (H_G^2 + V_G^2)^{\frac{1}{2}} \quad (S_{r2} = |O_m Z_{O_2} P_2|)
\]

(2.3.5)

\[
q_2 = \sin^{-1} \frac{V_G}{S_{r2}}
\]

(2.3.6)

Here: \( \psi_G \) is the spiral angle on the root cone, \( R_{u2} \) is the mean radius of the head cutter. The sliding base \( |O_m Z_{O_2}| \) is

\[
X_{B2} = Z_R \sin \gamma_{m2}
\]

(2.3.7)
Here: $\gamma_m$ is the same as the gear root cone angle $\gamma_2$, and $Z_R$ is the distance between $O_{2R}$ and $O_2$, which are the apexes of the root cone and the pitch cone, respectively.
3 Gear Geometry

3.1 Gear Surface

The gear tooth surface is the envelope to the family of generating surfaces. We recall that the cradle carries the head-cutter that is provided with finishing blades. The blades are rotated about the axis of the head-cutter and generate two cone surfaces. Fig.3.1.1 shows one of the cones.

The family of a generating surface (the cone surface) is generated in $S_2$ while the cradle and being generated gear perform related rotations, about the $Z_{m2}$-axis and $X_2$-axis (Fig.2.2.2).

The derivation of the gear tooth surface is based on the following procedure:

Step 1: We represent the cone surface and its unit normal in system $S_{p2}$ (Fig.3.1.1) as follows

$$
\vec{r}_{p2} = \begin{bmatrix}
(r_c - s_G \sin \alpha_G \cos \theta_G) \\
(r_c - s_G \sin \alpha_G \sin \theta_G) \\
-s_G \cos \alpha_G \\
1
\end{bmatrix}
$$

(3.1.1)

$$
\vec{n}_{p2} = \frac{\vec{N}_{p2}}{|\vec{N}_{p2}|} ; \quad \vec{N}_{p2} = \frac{\partial \vec{r}_{p2}}{\partial \theta_G} \times \vec{r}_{p2} / \partial s_G
$$

(3.1.2)

i.e.,

$$
\vec{n}_{p2} = \begin{bmatrix}
-\cos \alpha_G \cos \theta_G \\
-\cos \alpha_G \sin \theta_G \\
\sin \alpha_G
\end{bmatrix}
$$

(3.1.3)

Here: $s_G$ and $\theta_G$ are the surface coordinates; $\alpha_G$ is the blade angle; $r_c$ is the radius of the head-cutter that is measured at the bottom of the blades. It is evident (Fig.3.1.2) that

$$
r_c = R_{u2} \pm \frac{PW}{2}
$$

(3.1.4)
Here: \( R_{u2} \) is the nominal radius, \( PW \) is the so called point width; the positive sign in (3.1.4) corresponds to the gear concave side and the negative sign corresponds to the gear convex side.

Equations (3.1.1) and (3.1.3) represent both generating cones with \( \alpha_G > 0 \) for the gear convex side and \( \alpha_G < 0 \) for the gear concave side.

**Step 2:** The family of generating surfaces that is generated in \( S_2 \) is represented by the following matrix equation

\[
\vec{r}_2(s_G, \theta_G, \phi_p) = [M_{2d_2}] [M_{d_2m_2}] [M_{m_2c_2}] [M_{c_2p_2}] \vec{r}_p
\]  

(3.1.5)

Here (Fig.2.2.2, Fig.2.2.1):

\[
[M_{2d_2}] = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \phi_2 & \sin \phi_2 & 0 \\
0 & -\sin \phi_2 & \cos \phi_2 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  

(3.1.6)

\[
[M_{d_2m_2}] = \begin{bmatrix}
\cos \gamma_{m_2} & 0 & \sin \gamma_{m_2} & -X_B \sin \gamma_{m_2} \\
0 & 1 & 0 & 0 \\
-\sin \gamma_{m_2} & 0 & \cos \gamma_{m_2} & -X_B \cos \gamma_{m_2} \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  

(3.1.7)
The machine root angle $\gamma_{m2}$ in equation (3.17) is equal to gear root cone angle $\gamma$.

Step 3: The derivation of the equation of meshing is based on the equation

$$\vec{n}_{m2} \cdot \vec{v}^{(p2)}_{m2} = 0$$

(3.1.10)

The subscript "m2" means that vectors in equation (3.1.10) are represented in coordinate system $S_{m2}$; $\vec{n}_{m2}$ is the unit normal to the generating surface; $\vec{v}^{(p2)}_{m2} = \vec{v}^{(p)}_{m2} - \vec{v}^{(2)}_{m2}$ is the relative (sliding) velocity. Vector $\vec{n}_{m2}$ is represented by the matrix equation

$$\vec{n}_{m2} = [L_{m2p2}]\vec{n}_{p2} = \begin{bmatrix} 1 & 0 & 0 & \cos \alpha_G \cos (\theta_G + \phi_p) \\ 0 & 1 & 0 & -\cos \alpha_G \sin (\theta_G + \phi_p) \\ 0 & 0 & 1 & \sin \alpha_G \end{bmatrix}$$

(3.1.11)

where $[L_{m2p2}]$ is the $3 \times 3$ submatrix $[M_{m2p2}]$.

We consider that the axes of rotation of the cradle and the gear intersect each other (Fig.2.2.2(a)), thus
where
\[
\vec{\omega}_{m_2}^{(q_2)} = (\vec{\omega}_{m_2}^{(p_2)} - \vec{\omega}_{m_2}^{(2)}) \times \vec{f}_{m_2} = \vec{\omega}_{m_2}^{(p_2)} \times \vec{f}_{m_2}
\] (3.1.12)

We assumed that \(|\vec{\omega}_{m_2}^{(p_2)}| = 1\) in equation (3.1.13). Equations from (3.1.10) to (3.1.13) yield the following relation
\[
s_G = \frac{A(\theta_G, \phi_p)}{B(\theta_G, \phi_p)}
\] (3.1.14)

Here
\[
A(\theta_G, \phi_p) = n_{m_2 x} [-A_1 (\sin \gamma_2 - \frac{1}{R_{aG}})] + n_{m_2 y} [X_{B_2} \cos \gamma_2 + A_2 (\sin \gamma_2 - \frac{1}{R_{aG}})] + n_{m_2 z} A_1 \cos \gamma_2
\] (3.1.15)

\[
B(\theta_G, \phi_p) = -n_{m_2 x} \sin \alpha_G \sin(\theta_G + \phi_p) + n_{m_2 y} [\sin \gamma_2 - \frac{1}{R_{aG}}] \sin \alpha_G \cos(\theta_G + \phi_p)
- \cos \alpha_G \cos \gamma_2] + n_{m_2 z} \cos \gamma_2 \sin \alpha_G \sin(\theta_G + \phi_p)
\] (3.1.16)

\[
A_1 = r_c \sin(\theta_G + \phi_p) - S_{r_2} \sin(q_2 - \phi_p)
\] (3.1.17)

\[
A_2 = r_c \cos(\theta_G + \phi_p) - S_{r_2} \cos(q_2 - \phi_p)
\] (3.1.18)

Step 4: Equations (3.1.5) and (3.1.14) considered simultaneously represent the gear surface in three-parametric form but with related parameters. Since parameter \(s_G\) in equation of meshing
(3.1.14) is linear, it can be eliminated in equation (3.1.5), and then the gear tooth surface will be represented in two-parametric form, by the vector function \( \vec{r}_2(\theta_G, \phi_p) \).

3.2 Mean Contact Point and Gear Principal Directions and Curvatures

The mean contact point \( M \) is shown in Fig.2.3.1. Usually, \( M \) is chosen in the middle of the tooth surface. The gear tooth surface and the pinion tooth surface must contact each other at \( M \).

The procedure of local synthesis discussed in section 2.1 is directed at providing improved conditions of meshing and contact at \( M \) and in the neighborhood of \( M \). The location of point \( M \) is determined with parameters \( X_L \) and \( R_L \) (Fig.2.3.1) that are represented by the following equations

\[
X_L = A_m \cos \Gamma_2 + \left( b_G - \frac{h_m + c}{2} \right) \sin \Gamma_2 \\
R_L = A_m \sin \Gamma_2 - \left( b_G - \frac{h_m + c}{2} \right) \cos \Gamma_2
\]  

(3.2.1)

(3.2.2)

Here: \( A_m \) is the pitch cone mean distance; \( h_m \) is the mean whole depth; \( b_G \) is the gear mean dedendum; \( c \) is the clearance Equations (3.2.1), (3.2.2) and vector equation \( \vec{r}_2(\theta_G, \phi_p) \) for the gear tooth surface allows to determine the surface parameters \( \theta_G^* \) and \( \phi_p^* \) for the mean contact point from the equations

\[
X_2(\theta_G^*, \phi_p^*) = X_L \\
Y_2^2(\theta_G^*, \phi_p^*) + Z_2^2(\theta_G^*, \phi_p^*) = (R_L)^2
\]  

(3.2.3)

(3.2.4)
Gear Principal Directions and Curvatures

The gear principal directions and curvatures can be expressed in terms of principal curvatures and directions of the generating surface (see chapter (13) in [4]), that is the cone surface.

**Step 1:** The cone principal directions are represented in $S_{p2}$ by the equations (see (3.1.1))

\[
\tilde{e}_{sp2}^{(p)} = \frac{\partial \tilde{r}_{p2}}{\partial s_{G}} = \begin{bmatrix} -\sin \theta_{G} & \cos \theta_{G} & 0 \end{bmatrix}^T \tag{3.2.5}
\]

\[
\tilde{e}_{qp2}^{(p)} = \frac{\partial \tilde{r}_{p2}}{\partial \theta_{G}} = \begin{bmatrix} \sin \alpha_{G} \cos \theta_{G} & - \sin \alpha_{G} \sin \theta_{G} & - \cos \alpha_{G} \end{bmatrix}^T \tag{3.2.6}
\]

The superscript “p” indicates that the cone surface $\Sigma_p$ is considered. Unit vector $\tilde{e}_{qp2}^{(p)}$ is directed along the cone generatrix and unit vector $\tilde{e}_{sp2}^{(p)}$ is perpendicular to $\tilde{e}_{qp2}^{(p)}$. The unit vectors of cone principal directions are represented in $S_{m2}$ by the equations

\[
\tilde{e}_{sm2}^{(p)} = \begin{bmatrix} -\sin(\theta_{G} + \phi_{p}) & \cos(\theta_{G} + \phi_{p}) & 0 \end{bmatrix}^T \tag{3.2.7}
\]

\[
\tilde{e}_{qm2}^{(p)} = \begin{bmatrix} -\sin \alpha_{G} \cos(\theta_{G} + \phi_{p}) & - \sin \alpha_{G} \sin(\theta_{G} + \phi_{p}) & - \cos \alpha_{G} \end{bmatrix}^T \tag{3.2.8}
\]

The cone principal curvatures are:

\[
\kappa_{s}^{(p)} = \frac{\cos \alpha_{G}}{r_{c} - s_{G} \sin \alpha_{G}} \quad \text{and} \quad \kappa_{q}^{(p)} = 0 \tag{3.2.9}
\]

**Step 2:** The determination of principal curvatures and directions for gear tooth surface $\Sigma_2$ is based on equations from (1.2.6) to (1.2.8). The superscript “2” in these equations must be changed
for "p" and superscript "1" for "2". The second derivative of cutting ratio, \( m'_{21} \equiv m'_{p2} \) is zero because the cutting ratio is constant. The principal curvatures of the gear tooth surface will be determined as \( \kappa_f \) and \( \kappa_h \). The principal directions on gear tooth surface will be represented in by \( \vec{e}_f \) and \( \vec{e}_h \) and they can be determined from equations (1.2.9) and (1.2.10). To represent in \( S_2 \) the principal directions on gear tooth surface \( \Sigma_2 \) and its unit normal we use the matrix equation that describe the coordinate transformation from \( S_{m2} \) to \( S_2 \). This equation is

\[
\bar{a}_2 = [L_{d2}][L_{d_{m2}}]\bar{a}_{m2}
\]

(3.2.10)

Here: \( \bar{a}_{m2} \) stands for vectors \( \bar{n}_{m2} \), \( \bar{e}_f_{m2} \) and \( \bar{e}_h_{m2} \), and \( \bar{a}_2 \) stands for \( \bar{n}_2 \), \( \bar{e}_{12} \) and \( \bar{e}_{112} \).
4 Local Synthesis of Spiral Bevel Gears

4.1 Conditions of Synthesis

The basic principles of local synthesis of gear tooth surfaces discussed in Section 1 will enable us to determine the principle curvatures and directions of the being synthesized pinion. Thus, we will be able to determine the required machine-tool settings for the pinion. While solving the problem of local synthesis, we will consider as known:

(i) The location of the mean contact point $M$ in a fixed coordinate system, and the orientation of the normal to gear surface $\Sigma_2$.

(ii) The principle curvatures and directions on $\Sigma_2$ at $M$. The local synthesis of gear tooth surfaces must satisfy the following requirements:

1. The pinion and gear tooth surfaces must be in contact at $M$.

2. The tangent to the contact path on the gear tooth surface must be of the prescribed direction.

3. The function of gear ratio $m_{21}(\phi_1)$ in the neighborhood of mean contact point must be a linear one, be of prescribed value at $M$ and have the prescribed value for the derivative $m'_{21}(\phi_1)$ at $M$.

The satisfaction of these requirements provides a parabolic type of function for transmission errors of the desired value at each cycle of meshing.

4. The major axis of the instantaneous contact ellipse must be of the desired value (with the given elastic approach of tooth surfaces).

4.2 Procedure of Synthesis

We will consider in this section the following steps of the computational procedure: (i) representation of gear mean contact point in a fixed coordinate system $S_h$; (ii) satisfaction of equation of meshing of the pinion and gear at the mean contact point; (iii) representation of principle directions on gear tooth surface $\Sigma_2$ in $S_h$; (iv) observation of the desired derivative $m'_{21}(\phi_1)$. (v) observation at the mean contact point of the desired direction of the tangent to the path contact on gear tooth
surface; (vi) observation at the mean contact point of the desired length of the major axis of the contact ellipse; (vi) determination of principal directions and curvatures on pinion tooth surface $\Sigma_1$ at the mean contact point.

Step 1: We set up a fixed coordinate system $S_h$ that is rigidly connected to the gear mesh housing (Fig.4.2.1(a)). In addition to $S_h$, we will use coordinate systems $S_2$ (Fig.4.2.1(a)) and $S_1$ (Fig.4.2.1(b)) that are rigidly connected to gears 2 and 1, respectively. We designate with $\phi'_2$ and $\phi'_1$ the angles of rotation of gears being in mesh. We have to emphasize that with this designation $\phi'_i (i = 1, 2)$ we differentiate the angle of gear rotation in meshing from the angle $\phi_i$ of gear rotation in the process of generation.

The orientation of coordinate system $S_h$ is based on following considerations: (i) The axes of rotation of the pinion and the gear in a drive of spiral bevel gears intersect each other. Taking into account the possible gear misalignment, we will consider that the pinion-gear axes are crossed at angle $\Gamma$ and the shortest distance is $E$. (ii) We will choose that $X_h$ coincides with the pinion axis and $O_h$ is located on the shortest distance (Fig.4.2.1(a)). (iii) Considering as given the shaft angle $\Gamma$, we will define $j_h$—the unit vector of $Y_h$—as follows

$$j_h = \frac{\bar{i}_h \times \bar{a}_h}{|\bar{i}_h \times \bar{a}_h|} \quad (4.2.1)$$

where $\bar{a}_h$ is the unit vector of gear axis that is parallel to plane $(X_h, Y_h)$.

The coordinate transformation from $S_2$ to $S_h$ is based on matrix equation

$$\bar{t}_h^{(M)} = [M_{hd}][M_{d2}]\bar{t}_2(\theta_G, \phi_p) \quad (4.2.2)$$

where $S_d$ (Fig.4.2.1) is an auxiliary fixed coordinate system. The unit normal to $\Sigma_2$ is repre-
presented in $S_h$ as

$$\vec{n}_h^{(2)} = [L_{hd}] [L_{d2}] \vec{n}_2(\theta_G, \phi_p) \quad \tag{4.2.3}$$

Here (Fig. 4.2.1)

$$[M_{d2}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -\cos \phi_2' & \sin \phi_2' & 0 \\ 0 & -\sin \phi_2' & -\cos \phi_2' & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \tag{4.2.4}$$

$$[M_{hd}] = \begin{bmatrix} \cos \Gamma & 0 & \sin \Gamma & 0 \\ 0 & 1 & 0 & E \\ -\sin \Gamma & 0 & \cos \Gamma & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \tag{4.2.5}$$

where $\Gamma$ is the shaft angle.

Equations (4.2.3), (4.2.2) and (4.2.3) enable to represent in $S_h$ the position vector and unit contact normal at $M$ by

$$\vec{r}_h^{(1)}(\theta_G^*, \phi_p^*, \phi_2') \quad \vec{n}_h^{(2)}(\theta_G^*, \phi_p^*, \phi_2') \quad \tag{4.2.6}$$

where $(\theta_G^*, \phi_p^*)$ are the surface coordinates for the mean contact point at $\Sigma_2$; the angle $\phi_2'$ of rotation of gear 2 will be determined from the equation of meshing (see below).
Step 2: The equation of meshing of pinion and gear at the mean contact point is

$$\bar{t}_h^{(2)} \cdot \bar{v}_h^{(12)} = f(\theta_G, \phi_2', \phi_2') = 0 \quad (4.2.7)$$

Here (Fig. 4.2.1)

$$\bar{v}_h^{(12)} = \left[ (\bar{\omega}_h^{(1)} - \bar{\omega}_h^{(2)}) \times \bar{r}_h^{(M)} \right] - (\bar{F} \times \bar{\omega}_h^{(2)}) \quad (4.2.8)$$

$$\bar{\omega}_h^{(1)} = [-1 \quad 0 \quad 0]^T \quad (|\bar{\omega}_h^{(1)}| = 1) \quad (4.2.9)$$

$$\bar{\omega}_h^{(2)} = \frac{N_1}{N_2} \begin{bmatrix} \cos \Gamma & 0 & \sin \Gamma \end{bmatrix}^T \quad (4.2.10)$$

since at point $M$ the angular velocity ratio is

$$\frac{\omega^{(2)}}{\omega^{(1)}} = \frac{N_1}{N_2} \quad (4.2.11)$$

Substituting equations (4.2.3), (4.2.8)-(4.2.11) in equation (4.2.7), we can solve equation (4.2.7) for $\phi_2'$. Usually equation (4.2.7) yields two solutions for $\phi_2'$ but the smaller one, say $(\phi_2')^*$, should be chosen.

Step 3: We consider as known the principal curvatures and directions on $\Sigma_2$ at any point of $\Sigma_2$, including the mean contact point (see section 3). To represent in $S_h$ the principal directions at the mean contact point, we use the matrix equation

$$\bar{a}_h^{(M)} = [L_{h\alpha}][L_{d2}]\bar{a}_2 \quad (4.2.12)$$
where $\vec{a}_2$ is the unit vector of principal directions on $\Sigma_2$ that is represented in $S_2$. The following steps of computational procedure are exactly the same that have been described in section 1.2. This procedure permit determination of the pinion principal directions and curvatures at the mean contact point.
5 Pinion Machine-Tool Settings

5.1 Introduction

We consider at this stage of investigation as known:

(i) the common position vector $r_{h}^{(i)}$ and unit normal $\vec{n}_{h}^{(i)}$ at the point of contact point $M$ of $\Sigma_2$ and $\Sigma_1$

(ii) pinion surface principal directions and curvatures at $M$.

The goal is to determine the settings of the pinion and the head-cutter that will satisfy the conditions of local synthesis. We consider that the pinion surface and the generating surface are in line contact. Henceforth, we will consider two types of the generating surface: (a) a cone surface, and (b) a surface of revolution. We consider that each side of the pinion tooth is generated separately and two head-cutters must be applied for the pinion generation.

5.2 Head-Cutter Surface

Cone Surface

The cone surface is generated by straight blades being rotated about the $z_F$-axis (Fig. 5.2.1(a)). The $\Sigma_F$ equations are represented in coordinate system $S_F$ that is rigidly connected to the head-cutter as following:

$$\vec{r}_{F} = \begin{bmatrix} (R_{cp} + s_{F} \sin \alpha_{F}) \cos \theta_{F} \\ (R_{cp} + s_{F} \sin \alpha_{F}) \sin \theta_{F} \\ -s_{F} \cos \alpha_{F} \\ 1 \end{bmatrix}$$

Here: $s_{F}$ and $\theta_{F}$ are the surface coordinates; $\alpha_{F}$ and $R_{cp}$ are the blade angle and the radius of the cone in plane $z_{F} = 0$. The blade angle $\alpha_{F}$ is standardized and is considered as known. Parameter
\( \alpha_F \) is considered as negative for the pinion convex side and \( \alpha_F \) is positive for the pinion concave side. The point radius \( R_{cp} \) is considered as unknown and must be determined later.

The unit normal to pinion tooth surface is represented as

\[
\vec{n}_F = \frac{\vec{N}_F}{|\vec{N}_F|} \quad \text{and} \quad \vec{N}_F = \frac{\partial \vec{r}_F}{\partial \theta_F} \times \frac{\partial \vec{r}_F}{\partial s_F} \tag{5.2.2}
\]

i.e.,

\[
\vec{n}_F = -[\cos \alpha_F \cos \theta_F \quad \cos \alpha_F \sin \theta_F \quad \sin \alpha_F]^T \tag{5.2.3}
\]

The principal directions on the cone surface are:

\[
\varepsilon_F^I = \frac{\partial \vec{r}_F}{\partial \theta_F} = \begin{bmatrix} -\sin \theta_F & \cos \theta_F & 0 \end{bmatrix}^T \tag{5.2.4}
\]

\[
\varepsilon_F^{II} = \frac{\partial \vec{r}_F}{\partial s_F} = \begin{bmatrix} \sin \alpha_F \cos \theta_F & \sin \alpha_F \sin \theta_F & -\cos \alpha_F \end{bmatrix}^T \tag{5.2.5}
\]

The corresponding principal curvatures are

\[
\kappa_F^I = \frac{\cos \alpha_F}{R_{cp} + s_F \sin \alpha_F} \quad \text{and} \quad \kappa_F^{II} = \frac{1}{\rho} \tag{5.2.6}
\]
We consider that the head-cutter surface $\Sigma_F$ is generated by a circular arc of radius $\rho$ by rotation about the $z_0$-axis that coincides with the $z_F$-axis of the head-cutter (Fig. 5.2.1(b)) and (Fig.5.2.1(c)). The shape of the blade is represented in $S_o$ by the vector equations

$$\overline{O_oN} = \overline{O_oC} + \overline{CN} = (X_o^{(c)} + \rho \cos \lambda)\hat{t}_o + (Z_o^{(c)} + \rho \sin \lambda)\hat{k}_o$$  \hspace{1cm} (5.2.7)

Here: $(X_o^{(c)}, Z_o^{(c)})$ are algebraic values that represent in $S_o$ the location of center $C$ of the arc; $\rho = |CN|$ is the radius of the circular arc and is an algebraic value, $\rho$ is positive when center $C$ is on the positive side of the unit normal. $\lambda$ is the independent variable that determines the location of the current point $N$ of the arc. By using the coordinate transformation from $S_o$ to $S_F$ (Fig.5.2.1(c)), we obtain the following equations of the surface of the head-cutter:

$$\bar{r}_F = \begin{bmatrix} (X_o^{(c)} + \rho \cos \lambda) \cos \theta_F \\ (X_o^{(c)} + \rho \cos \lambda) \sin \theta_F \\ Z_o^{(c)} + \rho \sin \lambda \end{bmatrix}$$  \hspace{1cm} (5.2.8)

where $\lambda$ and $\theta_F$ are the surface coordinates (independent variables).

The surface unit normal $\bar{n}_F$ is represented by the following equations

$$\bar{n}_F = \frac{\bar{N}_F}{|\bar{N}_F|}$$  \hspace{1cm} and  \hspace{1cm} $\bar{N}_F = -\frac{\partial \bar{r}_F}{\partial \theta_F} \times \frac{\partial \bar{r}_F}{\partial \lambda}$  \hspace{1cm} (5.2.9)

Then we obtain
\[ n_F = -[\cos \lambda \cos \theta_F \cos \lambda \sin \theta_F \sin \lambda]^T \tag{5.2.10} \]

The variable \( \lambda \) at the mean contact point \( M \) has the same value as the standardized blade angle \( \alpha_F \). The principal directions on the head-cutter surface are

\[ \hat{e}_I^F = \begin{bmatrix} \frac{\partial \hat{r}_F}{\partial \theta_F} \\ \frac{\partial \hat{r}_F}{\partial \lambda} \end{bmatrix} = [-\sin \theta_F \cos \theta_F 0]^T \tag{5.2.11} \]

\[ \hat{e}_{II}^F = -\frac{\partial \hat{r}_F}{\partial \lambda} = [\sin \lambda \cos \theta_F \sin \lambda \sin \theta_F - \cos \lambda]^T \tag{5.2.12} \]

The principal curvatures are

\[ \kappa_I^{(F)} = \frac{\cos \lambda}{X^{(c)}_o + \rho \cos \lambda} \quad \text{and} \quad \kappa_{II}^{(F)} = \frac{1}{\rho} \tag{5.2.13} \]

The radius \( R_{cp} \) of the head-cutter in plane (Fig. 5.2.1) can be determined from the equations

\[ R_{cp} = X^{(c)}_o + \rho \sqrt{1 - \left( \frac{Z^{(c)}_o}{\rho} \right)^2} \tag{5.2.14} \]

5.3 Observation of a Common Normal at the Mean Contact Point for Surfaces \( \Sigma_p \), \( \Sigma_2 \), \( \Sigma_F \) and \( \Sigma_1 \)

We consider that at the mean contact point \( M \) four surfaces- \( \Sigma_p, \Sigma_2, \Sigma_F \) and \( \Sigma_1 \)- must be in tangency. The contact of \( \Sigma_F \) and \( \Sigma_2 \) at \( M \) has been already provided due to the satisfaction of
their equation of meshing (3.1.10). Our goal is to determine the conditions for the coincidence at $M$ of the unit normals to $\Sigma_F, \Sigma_p$ and $\Sigma_2$. The tangency of $\Sigma_1$ with the three above mentioned surfaces will be discussed below.

We will consider the coincidence of the unit normals in coordinate system $S_{m1}$. To determine the orientation of coordinate system $S_h$ with respect to $S_{m1}$, let us imagine that the set of coordinate systems $S_h, S_1$ and $S_2$ (Fig.4.2.1) with gears 1 and 2 is installed in $S_{m1}$ with observation of following conditions (Fig.5.2.2): (i) axis $x_h$ of $S_h$ coincides with axis $x_p$ of $S_p$; (ii) coordinate system $S_1$ coincides with $S_p$ and the orientation of $S_h$ with respect to $S_1$ is designated with angle $\phi_h = (\phi'_1)^0$ where $\phi_h$ is the to be determined instalment angle. Angle $\phi_h$ will be determined from the conditions of coincidence of the unit normals to $\Sigma_F, \Sigma_2 \Sigma_p$ and $\Sigma_1$. The procedure for derivation is as follows:

Step 1: Consider that the coordinate system $S_h$ with the point of tangency of surfaces $\Sigma_2$ and $\Sigma_p$ is installed in $S_{m1}$. We may represent the surface unit normal $\vec{n}^{(2)}$ in $S_{m1}$ by using the following matrix equation (Fig. 5.2.2).

$$\vec{n}^{(2)}_{m1} = [L_{m1p}] [L_{ph}] \vec{n}^{(2)}_h = \begin{bmatrix} \cos \gamma_1 & 0 & -\sin \gamma_1 \\ 0 & 1 & 0 \\ \sin \gamma_1 & 0 & \cos \gamma_1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi_h & -\sin \phi_h \\ 0 & \sin \phi_h & \cos \phi_h \end{bmatrix} \vec{n}^{(M)}_h$$ (5.3.1)

The unit vector $\vec{n}^{(2)}_h$ has been represented by equation (4.2.3).

Step 2: The unit vector to the surface of $\Sigma_F$ of the head-cutter that generates the pinion has been represented in $S_F$ by equation (5.2.3) for a cone and equation (5.2.10) for a surface of revolution. Axes of coordinate systems $S_F$ and $S_{m1}$ have the same orientation and

$$\vec{n}^{(F)}_{m1} = \vec{n}^{(F)}_F$$ (5.3.2)

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Equations (5.3.1), (5.3.2), (5.2.3) and (5.2.10) yield the following equations

\[
\cos \theta_F = -\frac{n_{zh}^{(2)} + \sin \alpha_F \sin \gamma_1}{\cos \gamma_1 \cos \alpha_F}
\]  

(5.3.3)

\[
\cos \phi_h = \frac{a_1 n_{zh}^{(2)} + a_2 n_{zh}^{(2)}}{(n_{yh}^{(2)})^2 + (n_{zh}^{(2)})^2}
\]

\[
\sin \phi_h = \frac{a_1 n_{yh}^{(2)} - a_2 n_{zh}^{(2)}}{(n_{yh}^{(2)})^2 + (n_{zh}^{(2)})^2}
\]  

(5.3.4)

Here:

\[
a_1 = -\cos \alpha_F \sin \theta_F \quad a_2 = \cos \alpha_F \sin \gamma_1 \cos \theta_F - \sin \alpha_F \cos \gamma_1
\]  

(5.3.5)

The advantage of the proposed approach is that the coincidence of the unit normals to surfaces \(\Sigma_F, \Sigma_2, \Sigma_F\) and \(\Sigma_1\) can be achieved with standard blade angles and without a tilt of the head-cutter.

5.4 Basic Equations for Determination of Pinion Machine-Tool Settings

At this stage of investigation we will consider as known: \(\kappa_1, \kappa_{II}, \varepsilon_{Im1}, \varepsilon_{IIm1}, \tilde{n}_{m1}\) and \(\tilde{r}_{m1}^M\). It is necessary to determine: \(\kappa_I^{(F)}, \kappa_{II}^{(F)}, \sigma^{(1F)}, R_{cp}, E_{m1}, X_{BG}\), and \(m_{F1}'\). Here: \(\kappa_I^{(F)}, \kappa_{II}^{(F)}, \varepsilon_{Im1}^{(1)}\) and \(\varepsilon_{IIm1}^{(1)}\) are the principal curvatures and unit vectors of principal directions on the pinion surface that are taken at mean contact point \(M\); \(\tilde{r}_{m1}^M\) and \(\tilde{n}_{m1}^M\) are the position vector of \(M\) and the contact normal at \(M\). The subscript "m1" indicates that the vectors are represented in \(S_m1\). Designations \(\kappa_I^{(F)}\) and \(\kappa_{II}^{(F)}\) indicate the principal curvatures of the surface of the pinion head-cutter that are taken at \(M\). The angle \(\sigma^{(1F)}\) is formed by the unit vectors \(\tilde{e}_{I}^{(1)}\) and \(\tilde{e}_{I}^{(F)}\) of principal directions on \(\Sigma_1\) and \(\Sigma_F\); \(R_{cp}\) is the cutter "point radius" (Fig.5.5.1) that is measured in plane \(\varepsilon_F = 0\) and is dependent.
on $\kappa_I^{(1)}$. $E_{m1}$ and $X_{G1}$ are the pinion settings for its generation (Fig.2.1.1 and Fig.2.1.2); $m_{F1}$, which is equal to $\frac{1}{R_{cp}}$, and $m'_{F1}$ are the cutting ratio and its derivative.

We recall that the pinion surface curvatures $\kappa_I^{(1)}$ and $\kappa_{II}^{(1)}$ have been determined in the process of local synthesis. Vectors $\vec{c}_{Ih}, \vec{c}_{IIh}, \vec{r}_{h}$ have been determined in system $S_h$. To represent these vectors in $S_{m1}$ we have to apply the coordinate transformation from $S_h$ to $S_{m1}$ similar to equation (5.3.1).

\[
\tilde{a}_{m1} = [L_{m1p}][L_{ph}]\tilde{a}_h \tag{5.4.1}
\]

where $\tilde{a}_h$ represents that principal directions of the pinion surface $\vec{c}_{Ih}$ and $\vec{c}_{IIh}$, the position vector of mean contact point $\vec{r}_{h}^{(M)}$; $\tilde{a}_{m1}$ represents the corresponding vectors $\vec{c}_{Im1}, \vec{c}_{IIm1}$ and $\vec{r}_{m1}^{(m)}$.

Now our goal, as it was mentioned above, is to determine $\kappa_I^{(F)}, \kappa_{II}^{(F)}, \sigma^{(1F)}, E_{m1}, X_{G1}, m_{F1}$ and $m'_{F1}$. We recall that vectors $\vec{c}_I^{(1)}$ and $\vec{c}_{II}^{(1)}$ are known from the local synthesis, and $\vec{c}_I^{(F)}$ and $\vec{c}_{II}^{(F)}$ become known from equations (5.2.4) and (5.2.5) for straight blade, and from equations (5.2.11) and (5.2.12) for curved blade, after the coincidence of the contact normal to surfaces $\Sigma_2, \Sigma_p$ and $\Sigma_F$ is provided. Thus parameter $\sigma^{(1F)}$ can be determined from the equations

\[
\begin{align*}
\sin \sigma^{(1F)} &= \vec{n}_{m1}^{(M)} \cdot (\vec{c}_{Im1}^{(1)} \times \vec{c}_{Im1}^{(F)}) \\
\cos \sigma^{(1F)} &= \vec{c}_{Im1}^{(1)} \cdot \vec{c}_{Im1}^{(F)}
\end{align*} \tag{5.4.2}
\]

According to Fig. 5.5.1, since the $Z_{m1}$-axis is parallel to $Z_F$-axis, surface parameter $s_F$ for the cone surface at mean point can be determined as:

\[
s_F^* = \frac{|Z_{m1}|}{\cos \alpha_F} \tag{5.4.3}
\]
Parameter $\kappa_{II}^{(F)}$ is equal to zero for a cone surface of the head-cutter and it must be chosen for a head-cutter with a surface of revolution. Then, the number of remaining parameters to-be determined becomes equal to five and they are: $\kappa_{I}^{(F)}$, $E_{m1}$, $X_{G1}$, $m_{1F}$ and $m_{F1}'$.

It will be shown below that we can derive only four equations for determination of the unknowns of the output data. Therefore one more parameter has to be chosen, and this is $m_{F1}'$—the modified roll. Usually, it is sufficient to choose $m_{F1}' = 0$, but the more general case with $m_{F1}' \neq 0$ is considered in this report as well.

The to be derived equations are as follows,

$$n_{m1}^{(M)} \cdot v_{m1}^{(1F)} = 0 \tag{5.4.4}$$

$$a_{11}a_{22} = a_{12}^2 \tag{5.4.5}$$

$$a_{11}a_{23} = a_{12}a_{13} \tag{5.4.6}$$

$$a_{12}a_{33} = a_{13}a_{23} \tag{5.4.7}$$

Equation (5.4.4) is the equation of meshing of the pinion and head-cutter that is applied at the mean contact point. Equations from (5.4.5) to (5.4.7) come from the conditions of existence of instantaneous line contact between $\Sigma_1$ and $\Sigma_F$. The coefficients $a_{ij}$ in equation (5.4.5)-(5.4.7) are represented as follows,

$$a_{11} = \kappa_{I}^{(F)} - \kappa_{I}^{(1)} \cos^2 \sigma^{(1F)} - \kappa_{II}^{(1)} \sin^2 \sigma^{(1F)} \tag{5.4.8}$$
\[
a_{12} = a_{21} = \frac{\kappa_I^{(1)} - \kappa_{II}^{(1)}}{2} \sin 2\sigma(1F)
\]

\[
a_{13} = a_{31} = -\kappa_I^{(F)} v_I^{(1F)} - \left[ \omega(1F) \, \bar{n} \, e_I^{(F)} \right]
\]

\[
a_{22} = \kappa_{II}^{(F)} - \kappa_I^{(1)} \sin^2 \sigma(1F) - \kappa_{II}^{(1)} \cos^2 \sigma(1F)
\]

\[
a_{23} = a_{32} = -\kappa_{II}^{(F)} v_{II}^{(1F)} - \left[ \omega(1F) \, \bar{n} \, e_{II}^{(F)} \right]
\]

\[
a_{33} = \kappa_I^{(F)} \left( v_I^{(1F)} \right)^2 + \kappa_{II}^{(F)} \left( v_{II}^{(1F)} \right)^2 - \left[ \bar{n} \, \omega(1F) \, \bar{v}^{(1F)} \right]
\]

- \left[ \bar{n}_{m1} \, \omega(1) \, \bar{v}_{tr}^{(1F)} \right] + \frac{\left( \bar{\omega}(1)^{2} \right)}{m_{F1}^{2}} m_{F1} \left( \bar{n} \cdot \bar{v}_{tr}^{2} \right)

Vectors in equation of meshing (5.4.4) can be represented as follows

\[
\bar{\omega}_{m1}^{(1)} = \begin{bmatrix} \cos \gamma_1 & 0 & \sin \gamma_1 \end{bmatrix}^T \quad (|\omega^{(1)}| = 1)
\]

\[
\bar{\omega}_{m1}^{(F)} = \frac{1}{R_{ap}} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}
\]

where \( R_{ap} \), which is equal to \( \frac{1}{m_{F1}} \), is the ratio of roll.

\[
\bar{\omega}_{m1}^{(1F)} = \bar{\omega}_{m1}^{(1)} - \bar{\omega}_{m1}^{(F)}
\]

\[
\bar{v}_{m1}^{(1F)} = \bar{v}_{tr}^{(1)} - \bar{v}_{tr}^{(F)}
\]
I-Yml sin yl 4) -jM) = xml sin71 zrn1 cos yl vtr earm x Tm1 1 5.5 Determination of Cutter Point Radius

Step.1: Equations (5.4.3), (5.4.6) and (5.4.7) yield the following expression for \( \kappa_I^{(F)} \)

\[
\kappa_I^{(F)} = \frac{\kappa_I^{(1)} \kappa_{II}^{(1)} + \kappa_{II}^{(F)} \left( \kappa_I^{(1)} \cos^2 \sigma^{(1F)} + \kappa_{II}^{(1)} \sin^2 \sigma^{(1F)} \right)}{\kappa_{II}^{(F)} - \kappa_I^{(1)} \sin^2 \sigma^{(1F)} - \kappa_{II}^{(1)} \cos^2 \sigma^{(1F)}}
\]  

(5.5.1)

Step 2: According to Meusnier's theorem, the cutter radius \( R_m \) at the mean contact point is (Fig.5.5.1)

\[
R_m = \frac{\cos \alpha_F}{|\kappa_I^{(F)}|}
\]  

(5.5.2)

As shown in Fig.5.2.1, the cutter point radius can be determined for a straight blade cutter as follows,

\[
R_{cp} = R_m - s_F \sin \alpha_F
\]  

(5.5.3)

For the arc blade, the location of the center of the arc can be determined in \( S_0 \) by following equations,
\[ X_o^{(c)} = R_m - \rho \cos \alpha_F \] (5.5.4)

\[ Z_o^{(c)} = R_m - \rho \sin \alpha_F \] (5.5.5)

Knowing \( X_o^{(c)} \) and \( Z_o^{(c)} \), we can determine the point radius for the arc blade by equation (5.2.14).

In order to find the position vector of the center of the head cutter, we define the following two vectors in \( S_{m1} \) as shown in Fig. 5.5.1.

\[ \bar{\rho}^{(c)} = \bar{n} \cos \alpha_F - \bar{e}_{II}^{(F)} \sin \alpha_F \] (5.5.6)

\[
\bar{\rho}^{(c)} = \begin{bmatrix}
\cos \theta_F & -\sin \theta_F & 0 & 0 \\
\sin \theta_F & \cos \theta_F & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
X_o^{(c)} \\
0 \\
Z_o^{(c)} \\
1
\end{bmatrix}
= \begin{bmatrix}
X_o^{(c)} \cos \theta_F \\
X_o^{(c)} \sin \theta_F \\
Z_o^{(c)} \\
1
\end{bmatrix}
\] (5.5.7)

where, \( \rho^{(c)} \) is a unit vector directed from the blade tip \( M_o \) to the cutter center \( O_F \), and \( \rho^{(c)} \) is a position vector directed from \( O_F \) to the arc center \( C \). Referring to Fig.5.2.2 and Fig.5.5.2, the position vector of the cutter center \( O_F \) with respect to \( O_h \), \( \bar{r}_h^{(O_F)} \) can be determined in system \( S_{m1} \) as follows,

For straight blade:

\[
\bar{r}_h^{(O_F)} = \bar{r}_{m1}^{(M)} - S_F \bar{e}_{II}^{(F)} + R_m \rho^{(c)}
\] (5.5.8)
For arc blade:

\[ \tau_h^{(OF)} = \tau_{m1}^{(M)} + \rho \tilde{n}_{m1} - \rho^{(c)} \]  \hspace{1cm} (5.5.9)

It can be verified that the \( Z_{m1} \) component of \( \tau_h^{(OF)} \) is zero, since equations (5.4.3) and (5.5.5) are observed. It is worth to mention that \( \theta_{F}^* \) and \( s_{F}^* \) are the surface coordinates where the contact is at mean point. The values of \( \theta_{F}^* \) and \( s_{F}^* \) will serve as the initial guess in tooth contact analysis.

### 5.6 Determination of \( m_{F1} \equiv \frac{1}{R_{ap}}, E_{M1} \) and \( X_{G1} \)

The determination of cutting ratio \( R_{ap} \), settings \( E_{m1} \) and \( X_{G1} \) is based on application of equations (5.4.2), (5.4.4) and (5.4.5).

**Initial Derivations**

It is obvious that equation of meshing (5.4.4) is satisfied at point \( M \) if the relative velocity \( \vec{v}^{(F1)} \) lies in plane that is tangent to the contacting surfaces at \( M \). Thus, if velocity \( \vec{v}^{(F1)} \) satisfies the equation,

\[ \vec{v}^{(F1)} = \psi_{I}^{(F1)} \vec{e}_{I}^{(F)} + \psi_{II}^{(F1)} \vec{e}_{II}^{(F)} \] \hspace{1cm} (5.6.1)

it means that equation of meshing (5.4.4) is also satisfied. Assuming that vectors of equation (5.6.1)) are represented in coordinate system \( S_{m1} \), we obtain
For further derivations we will use the following expressions for $a_{13}$ and $a_{23}$.

\[ a_{13} = \kappa_{I}^{(F)} y_{I}^{(F)} + M_{11} t_{1} + M_{12} \]
\[ a_{23} = \kappa_{II}^{(F)} y_{II}^{(F)} + M_{21} t_{1} + M_{22} \]  

(5.6.3)

Here,

\[ M_{11} = n_{m1X} e_{I1mY}^{(F)} - n_{m1Y} e_{I1mX}^{(F)} \]
\[ M_{12} = -\cos \gamma_{1} [n_{m1Y} e_{I1mZ}^{(F)} - n_{m1Z} e_{I1mY}^{(F)}] \]
\[ M_{21} = n_{m1X} e_{II1mY}^{(F)} - n_{m1Y} e_{II1mX}^{(F)} \]
\[ M_{22} = -\cos \gamma_{1} [n_{m1Y} e_{II1mZ}^{(F)} - n_{m1Z} e_{II1mY}^{(F)}] \]  

(5.6.4)

\[ t_{1} = m_{F1} - \sin \gamma_{1} \]  

(5.6.5)

Using equations (5.6.2), (5.6.3) and (5.4.16), we obtain

\[ y_{I}^{(F)} e_{I1m2}^{(F)} + y_{II}^{(F)} e_{II1m2}^{(F)} + Y_{m1} \cos \gamma_{1} = 0 \]  

(5.6.6)

Following Derivations
Step 1: Expressions for $v_I^{(F1)}$ and $v_{II}^{(F1)}$. Equations (5.6.6) and (5.4.4) represent a system of two linear equation in unknowns $v_I^{(F1)}$ and $v_{II}^{(F1)}$. The solution of these equations for the unknowns yields:

\[ v_I^{(F1)} = L_{21} t_1 + L_{22} \]  \hspace{1cm} (5.6.7)

\[ v_{II}^{(F1)} = L_{11} t_1 + L_{12} \]  \hspace{1cm} (5.6.8)

Here,

\[ L_{21} = \frac{e_{IIm1Z}^{(F)} (a_{11} M_{21} - a_{12} M_{11})}{a_{12} \kappa_I^{(F)} e_{IIm1Z}^{(F)} + a_{11} \kappa_{II}^{(F)} e_{IIm1Z}^{(F)}} \]  \hspace{1cm} (5.6.9)

\[ L_{22} = \frac{e_{IIm1Z}^{(F)} (a_{11} M_{22} - a_{12} M_{12}) - a_{11} \kappa_{II}^{(F)} Y_{m1} \cos \gamma_1}{a_{12} \kappa_I^{(F)} e_{IIm1Z}^{(F)} + a_{11} \kappa_{II}^{(F)} e_{IIm1Z}^{(F)}} \]  \hspace{1cm} (5.6.10)

\[ L_{11} = \frac{-e_{IIm1Z}^{(F)} (a_{11} M_{21} - a_{12} M_{11})}{a_{12} \kappa_I^{(F)} e_{IIm1Z}^{(F)} + a_{11} \kappa_{II}^{(F)} e_{IIm1Z}^{(F)}} \]  \hspace{1cm} (5.6.11)

\[ L_{12} = \frac{-e_{IIm1Z}^{(F)} (a_{11} M_{22} - a_{12} M_{12}) - a_{11} \kappa_{II}^{(F)} Y_{m1} \cos \gamma_1}{a_{12} \kappa_I^{(F)} e_{IIm1Z}^{(F)} + a_{11} \kappa_{II}^{(F)} e_{IIm1Z}^{(F)}} \]  \hspace{1cm} (5.6.12)

Step 2: Expression for $\dot{v}^{(F1)}$

Substituting the above equation in equation (5.6.2), we obtain
Here:

\[
\begin{align*}
\theta^{(F1)} &= \begin{bmatrix} X_{11}t_1 + X_{12} \\ X_{21}t_1 + X_{22} \\ X_{31}t_1 + X_{32} \end{bmatrix} \\
X_{11} &= L_{21}e^{(F)}_{I_m1X} + L_{11}e^{(F)}_{IIm_1X} \\
X_{12} &= L_{22}e^{(F)}_{I_m1X} + L_{12}e^{(F)}_{IIm_1X} \\
X_{21} &= L_{21}e^{(F)}_{I_m1Y} + L_{11}e^{(F)}_{IIm_1Y} \\
X_{22} &= L_{22}e^{(F)}_{I_m1Y} + L_{12}e^{(F)}_{IIm_1Y} \\
X_{31} &= L_{21}e^{(F)}_{I_m1Z} + L_{11}e^{(F)}_{IIm_1Z} \\
X_{32} &= L_{22}e^{(F)}_{I_m1Z} + L_{12}e^{(F)}_{IIm_1Z}
\end{align*}
\]

Step 3: Expression for \( \varphi^{(F1)}_{tr} \)

Equations (5.4.16) and (5.4.18) yield

\[
\varphi^{(F)}_{tr} = \begin{bmatrix} X_{11}t_1 + X_{13} \\ X_{21}t_1 + X_{23} \\ X_{31}t_1 + X_{33} \end{bmatrix}
\]

Here,

\[
\begin{align*}
X_{13} &= X_{12} - Y_m \sin \gamma \\
X_{23} &= X_{22} + X_m \sin \gamma - Z_m \cos \gamma \\
X_{33} &= X_{32} + Y_m \cos \gamma
\end{align*}
\]
Step 4: Expressions for triple products in equation (1.2.2) for $a_{33}$

\[
[n \bar{\omega}^{(F_1)} \bar{\varphi}^{(F_1)}] = E_{11}t_1^2 + E_{12}t_1 + E_{13} \quad (5.6.17)
\]

where

\[
E_{11} = n_{m1}X_{11} + n_{m1}X_{21}
\]
\[
E_{12} = n_{m1}X_{12} - n_{m1}X_{22} - n_{m2}X_{21} \cos \gamma_1 + n_{m1}X_{31} \cos \gamma_1
\]
\[
E_{13} = -(n_{m1}X_{22} - n_{m1}X_{32}) \cos \gamma_1
\]

\[
[n \bar{\omega}^{(1)} \bar{\varphi}^{(F)}] = Y_{21}t_1 + Y_{22} \quad (5.6.19)
\]

where

\[
Y_{21} = -n_{m1}X_{21} \sin \gamma_1 + n_{m1}Y_{X_{11}} \sin \gamma_1 - X_{31} \cos \gamma_1 + n_{m2}X_{21} \cos \gamma_1
\]
\[
Y_{22} = -n_{m1}X_{22} \sin \gamma_1 + n_{m1}Y_{X_{13}} \sin \gamma_1 - X_{33} \cos \gamma_1
\]

\[
[n \bar{\omega}^{(F)} \bar{\varphi}^{(1)}] = Y_{11}t_1 + Y_{12} \quad (5.6.21)
\]

where

\[
Y_{11} = -n_{m1}Y_{X_{m1}} \sin \gamma_1 - Z_{m1} \cos \gamma_1 - n_{m1}Y_{m1} \sin \gamma_1
\]
\[
Y_{12} = \sin \gamma_1 Y_{11}
\]

Step 5: Expression for the last term in equation (1.2.2) for $a_{33}$.

We have to differentiate between two derivatives: $m'_2$ and $m'_F$. The first one, $m'_2$, is applied to provide a parabolic function of transmissions errors for the case of meshing of the generated
pinion and the gear. Such a function is very useful because it will allow to absorb linear functions of transmission errors caused by the gear misalignment. The other derivative, \( m'_{F_1} \), means that the cutting ratio in the process for pinion generation is not constant and it is just an additional parameter of machine-tool settings.

In the approach proposed in this research project it is not required to have modified roll. However the use of such parameter in the more general case with \( m'_{F_1} \neq 0 \) is also included to offer an extra choice. After some derivations, we obtain

\[
\frac{(\omega^{(1)})^2}{\omega^{(2)}} m'_{F_1} (\tilde{n} \cdot \dot{v}^{(F)}) = Z_{11}t_1^2 + Z_{12}t_1 + Z_{13}
\]

(5.6.23)

Here

\[
\begin{align*}
Z_{11} &= (2C)(n_{m_1X}X_{11} + n_{m_1Y}X_{21} + N_{m_1Z}X_{31}) \\
Z_{12} &= (2C)(n_{m_1X}X_{13} + n_{m_1Y}X_{23} + n_{m_1Z}X_{33} + \sin \gamma_1(n_{m_1X}X_{11} + N_{m_1Y}X_{21} + N_{m_1Z}X_{31})) \\
Z_{13} &= (2C)\sin \gamma_1(n_{m_1X}X_{13} + n_{m_1Y}X_{32} + n_{m_1Z}X_{33})
\end{align*}
\]

(5.6.24)

where

\[
2C = \frac{m'_{F_1}}{(m_{F_1})^2}
\]

(5.6.25)

**Step 6: Final expression for \( a_{33} \).**

Using the expressions received in steps 4 and 5, we obtain the following expression for \( a_{33} \)

\[
a_{33} = Z_1t_1^2 + Z_2t + Z_3
\]

(5.6.26)
where,

\[ Z_1 = \kappa_I^{(F)} L_{21}^2 + \kappa_{II}^{(F)} L_{11} - E_{11} + Z_{11} \]  \hspace{1cm} (5.6.27)

\[ Z_2 = 2\kappa_I^{(F)} L_{21} L_{22} + 2\kappa_{II}^{(F)} L_{11} L_{12} - E_{12} - Y_{21} + Y_{11} + Z_{12} \]  \hspace{1cm} (5.6.28)

\[ Z_3 = \kappa_I^{(F)} L_{22}^2 + \kappa_{II}^{(F)} L_{12}^2 - E_{13} - Y_{22} + Y_{12} + Z_{13} \]  \hspace{1cm} (5.6.29)

**Step 7:** New representations of coefficients \( a_{13} \) and \( a_{23} \).

Equations (5.6.3), (5.6.7) and (5.6.8) yield

\[
\begin{align*}
  a_{13} &= N_{21} t_1 + N_{22} \\
  a_{23} &= N_{11} t_1 + N_{12}
\end{align*}
\]  \hspace{1cm} (5.6.30)

Here;

\[
\begin{align*}
  N_{11} &= \kappa_{II}^{(F)} L_{11} + M_{21} \\
  N_{12} &= \kappa_{II}^{(F)} L_{12} + M_{22} \\
  N_{21} &= \kappa_I^{(F)} L_{21} + M_{11} \\
  N_{22} &= \kappa_I^{(F)} L_{22} + M_{12}
\end{align*}
\]  \hspace{1cm} (5.6.31)

**Step 8:** Derivation of squared equation for \( t_1 \).

Equations (5.6.30) and (5.4.5) yield

\[
a_1 t_1^2 + a_2 t_1 + a_3 = 0 \]  \hspace{1cm} (5.6.32)

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where,

\[
\begin{align*}
    &a_1 = a_{12}Z_1 - N_{21}N_{11} \\
    &a_2 = a_{12}Z_2 - (N_{21}N_{12} + N_{22}N_{11}) \\
    &a_3 = a_{12}Z_3 - N_{22}N_{12}
\end{align*}
\]  

(5.6.33)

Solving equation (5.6.32), we obtain

\[
t_1 = \frac{-a_2 \pm \sqrt{a_2^2 - 4a_3a_1}}{2a_1}
\]  

(5.6.34)

There are two solutions for \(t_1\) and we can choose one of them. If the tilt and the modified roll are not used, it can be proven that in this case \(a_1\) becomes equal to zero and equation (5.6.32) yields

\[
t_1 = -\frac{a_3}{a_2}
\]  

(5.6.35)

knowing \(t_1\), the ratio of roll may be easily determined as

\[
\begin{align*}
    &m_{F1} = t_1 + \sin \gamma_1 \\
    &R_{ap} = \frac{1}{m_{F1}}
\end{align*}
\]  

(5.6.36)

According to equations (5.4.19) and (5.6.15), the blank offset and machine center to back can be determined by

\[
E_{m1} = \frac{Y_{m1}m_{F1} + X_{11}t_1 + X_{13}}{m_{F1}}
\]  

(5.6.37)
Knowing $E_{m1}$ and $X_{G1}$, we may represent the position vector of the center of head-cutter with respect to the cradle center as follows,

\[ X_{G1} = \frac{X_{21}t_1 + X_{23} - x_{m1}m_{F1}}{m_{F1} \cos \gamma_1} \quad (5.6.38) \]

In practice, the position of the center of the head cutter is defined by radial setting $S_{r1}$ and cradle angle $q_1$, which may be determined by the following equations,

\[ S_{r1} = \sqrt{(X_{m1}^{(OF)})^2 + (Y_{m1}^{(OF)})^2} \]
\[ q_1 = \sin\left(\frac{Y_{m1}}{S_{r1}}\right) \]

Thus, the sliding base $X_{B1}$ may be determined as,

\[ X_{B1} = -X_{G1} \sin \gamma_1 \quad (5.6.41) \]
6 Tooth Contact Analysis

6.1 Introduction

The tooth contact analysis (TCA) is directed at simulation of meshing and contact for misaligned gears and enables to determine the influence of errors of manufacturing, assembly and shaft deflection. The basic equations for TCA are as follows:

\[ \tau_h^{(1)}(\theta_F, \phi_F, \phi'_1) = \tau_h^{(2)}(\theta_G, \phi_p, \phi'_2) \] \hspace{1cm} (6.1.1)

\[ \hat{n}_h^{(1)}(\theta_F, \phi_F, \phi'_1) = \hat{n}_h^{(2)}(\theta_G, \phi_p, \phi'_2) \] \hspace{1cm} (6.1.2)

Equations (6.1.1) and (6.1.2) describe the continuous tangency of pinion and gear tooth surfaces \( \Sigma_1 \) and \( \Sigma_2 \). The subscript \( h \) indicates that the vectors are represented in fixed coordinate system \( S_h \). The superscripts 1 and 2 indicate the pinion tooth surface \( \Sigma_1 \) and gear tooth surface \( \Sigma_2 \), respectively. Vector equation (6.1.1) describes that the position vectors of a point on \( \Sigma_1 \) and a point on \( \Sigma_2 \) coincide at the instantaneous point of contact \( M \); vector equation (6.1.2) describes that the surface unit normals coincide at \( M \).

Parameters \( \theta_F \) and \( \phi_F \) represent the surface coordinates for \( \Sigma_1 \); \( \theta_G \) and \( \phi_p \) are the surface coordinates for \( \Sigma_2 \). Parameters \( \phi'_1 \) and \( \phi'_2 \) represent the angles of rotation of the pinion and gear being in mesh.

Two vector equations (6.1.1) and (6.1.2) are equivalent to five independent scaler equations in six unknowns, which are represented as

\[ f_i(\theta_F, \phi_F, \phi'_1, \theta_G, \phi_p, \phi'_2) = 0 \hspace{1cm} (i = 1, 2, \cdots, 5) \] \hspace{1cm} (6.1.3)
The continuous solution of equations (6.1.3) means determination of five functions of a parameter chosen as the input one, say $\phi'_1$. Such functions are:

$$\theta_F(\phi'_1), \ \phi_F(\phi'_1), \ \theta_G(\phi'_1), \ \phi_p(\phi'_1), \ \phi_2(\phi'_1),$$

(6.1.4)

In accordance with the theorem of Implicit Function System Existence [4], solution (6.1.4) exists if at any iteration the following requirements are observed:

(i) There is a set of parameters

$$P(\theta_F, \ \phi_F, \ \theta_G, \ \phi_p, \ \phi_2)$$

(6.1.5)

that satisfies equations (6.1.1) and (6.1.2)

(ii) The Jacobian that is taken with the above mentioned set of parameters and with $\phi'_1$ as an independent variable, differs from zero, i.e.

$$D(f_1, f_2, f_3, f_4, f_5) \neq 0$$

(6.1.6)

The solution of the system (6.1.3) of nonlinear equations is based on application of a subroutine, such as DNEQNF of the IMSL software package. The first guess for the starting the iteration process is based on the data that are provided by the local synthesis.

The tooth contact analysis output data, functions (6.1.4), enable to determine the contact path on the tooth surface, the so called line of action, and the transmission errors.

The contact path on pinion tooth surface is determined in $S_1$ by the following functions
Similarly, the contact path on gear tooth surface is represented by functions

\[ \tilde{r}_1(\theta_F, \phi_F, \phi'_1), \quad \theta_F(\phi'_1), \quad \phi_F(\phi'_1) \]  

(6.1.7)

Similarly, the contact path on gear tooth surface is represented by functions

\[ \tilde{r}_2(\theta_G, \phi_p, \phi'_2), \quad \theta_G(\phi'_2), \quad \phi_p(\phi'_2) \]  

(6.1.8)

Function \( \phi'_2(\phi'_1) \) relates the angles of rotation of the gear and the pinion being in mesh. Deviations of \( \phi'_2(\phi'_1) \) from the theoretical linear function represent the transmission errors (see section 6.4). TCA is accomplished by the following procedure: (i) derivation of gear tooth surface, (ii) derivation of pinion tooth surface, (iii) determination of transmission errors, and (iv) determination of bearing contact as the set of instantaneous contact ellipses.

6.2 Gear Tooth Surface

The gear tooth surface \( \Sigma_2 \) and the surface unit normal have been represented in \( S_2 \) by equations (3.1.5) and (3.2.10), where \( \theta_G \) is the parameter of generating cone and \( \phi_p \) is the rotational angle of the cradle. Coordinate system \( S_2 \) is rigidly connected to the gear. To represent the gear tooth surface \( \Sigma_2 \) and its unit normal in fixed coordinate system \( S_h \) we can use the following matrix equations:

\[ \tilde{r}_h(2)(\theta_G, \phi_p, \phi'_2) = [M_{h2}(\phi'_2)] \tilde{r}_2(\theta_G, \phi_p) \]  

(6.2.1)

\[ \tilde{n}_h(2)(\theta_G, \phi_p, \phi'_2) = [L_{h2}(\phi'_2)] \tilde{n}_2(\theta_G, \phi_p) \]  

(6.2.2)
6.3 Pinion Tooth Surface

We will consider two cases for generation of pinion tooth surface: (i) by a cone, and (ii) by a surface of revolution that is formed by rotating curved blades.

Generation by a Cone Surface

Step 1: We recall that the generating cone surface and the surface unit normal has been represented in \( S_F \) by equations (5.2.1) and (5.2.3).

\[
\vec{r}_F = \begin{bmatrix}
(R_{cp} + s_F \sin \alpha_F) \cos \theta_F \\
(R_{cp} + s_F \sin \alpha_F) \sin \theta_F \\
-s_F \cos \alpha_F \\
1
\end{bmatrix}
\]

\[ (6.3.1) \]

\[
\vec{n}_F = \begin{bmatrix}
-\cos \alpha_F \cos \theta_F \\
-\cos \alpha_F \sin \theta_F \\
-s_F \cos \alpha_F
\end{bmatrix}
\]

\[ (6.3.2) \]

where \( S_F \) and \( \theta_F \) are the surface coordinates.

Step 2: During the process for generation the cradle with the mounted cone surface performs a rotational motion about the \( Z_{m1} \)-axis and a family of cone surfaces with parameter \( \phi_F \) is generated in \( S_{m1} \). This family is represented in \( S_{m1} \) by the matrix equation

\[
\vec{r}_{m1}(s_F, \theta_F, \phi_F) = [M_{m1c1}(\phi_F)] \vec{r}_{c1}(s_F, \theta_F)
\]

\[ (6.3.3) \]
where

\[ \tilde{r}_{c1} = \tilde{r}_F + \left[ S_{r1} \cos q_1 - S_{r1} \sin q_1 \ 0 \right]^T \]  \hspace{1cm} (6.3.4)

The position vector \( \tilde{r}_{c1} \) represents a point of the cone surface in coordinate system \( S_{c1} \); \( S_{r1} \) and \( q_1 \) are the settings of the head-cutter center \( O_F \) in \( S_{m1} \).

Matrix \([M_{m1c1}]\) is (Fig.2.1.1)

\[
[M_{m1c1}] = \begin{bmatrix}
\cos \phi_F & \sin \phi_F & 0 & 0 \\
-\sin \phi_F & \cos \phi_F & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  \hspace{1cm} (6.3.5)

The unit normal at a point of the generating surface \( \Sigma_F \) is represented in \( S_{m1} \) by

\[ \tilde{n}_{m1}(\theta_F, \phi_F) = [L_{m1c1}(\phi_F)]\tilde{n}_{c1}(\theta_F) \]  \hspace{1cm} (6.3.6)

where \( \tilde{n}_{c1} \equiv \tilde{n}_F \).

We recall that the generating cone surface is a ruled developed surface and the surface unit normal does not depend on \( s_F \) (Parameter \( s_F \) determines the location of a point on the cone generatrix.) Matrix \([L_{m1c1}]\) is the 3 x 3 rotational part of \([L_{m1c1}]\) and is represented as follows,

\[
[L_{m1c1}] = \begin{bmatrix}
\cos \phi_F & \sin \phi_F & 0 \\
-\sin \phi_F & \cos \phi_F & 0 \\
0 & 0 & 1
\end{bmatrix}
\]  \hspace{1cm} (6.3.7)
Step 3: Equation of meshing of the head-cutter cone with the pinion tooth surface. The equation of meshing is considered with vectors that are represented in $S_{m1}$. Thus:

\[
\vec{n}_{m1}^{(F)} \cdot \vec{v}_{m1}^{(1F)} = 0 \tag{6.3.8}
\]

Here: $\vec{v}^{(F1)}$ is the sliding (relative) velocity represented as follows

\[
\vec{v}_{m1}^{(1F)} = (\vec{\omega}_{m1}^{(1)} - \vec{\omega}_{m1}^{(F)}) \times \vec{r}_{m1} + \vec{R}_{m1} \times \vec{\omega}_{m1}^{(1)} \tag{6.3.9}
\]

While deriving equation (6.3.9), we have taken into account that vector of angular velocity $\vec{\omega}^{(1)}$ of pinion rotation does not pass through the origin $O_{m1}$ of $S_{m1}$; $\vec{R}_{m1}$ represents the position vector that is drawn from $O_{m1}$ to a point of line of action of $\vec{\omega}^{(1)}$; $\vec{R}_{m1}$ can be represented as (Fig.2.1.2):

\[
\vec{R}_{m1} = [X_{G1} \cos \gamma_1 \quad -E_{m1} \quad X_{G1} \sin \gamma_1]^T \tag{6.3.10}
\]

vectors $\vec{\omega}^{(1)}$ and $\vec{\omega}^{(F)}$ are represented in $S_{m1}$ as follows

\[
\vec{\omega}_{m1}^{(1)} = [\cos \gamma_1 \quad 0 \quad \sin \gamma_1]^T \quad (|\vec{\omega}^{(1)}| = 1) \tag{6.3.11}
\]

\[
\vec{\omega}_{m1}^{(F)} = \frac{1}{R_{ap}} [0 \quad 0 \quad 1]^T \quad (\frac{1}{R_{ap}} = \frac{\omega^{(F)}}{\omega^{(1)}}) \tag{6.3.12}
\]

Equations from (6.3.8) to (6.3.12) yield
Here:

\[
T_1 = X_{nm1}(-E_{m1} \sin \gamma_1 - A_1(\sin \gamma_1 - m_{F1})) + Y_{nm1}(X_{B1} \cos \gamma_1 + A_2(\sin \gamma_1 - m_{F1})) + Z_{nm1}(E_{m1} \cos \gamma_1 + A_1 \cos \gamma_1)
\] (6.3.14)

\[
T_2 = X_{nm1}(\sin \gamma_1 - m_{F1}) \sin \alpha_F \sin(\theta_F + \phi_F) - Y_{nm1}[(\sin \gamma_1 - m_{F1}) \sin \alpha_F \cos(\theta_F + \phi_F)
- \cos \alpha_F \cos \gamma_1] - Z_{nm1} \cos \gamma_1 \sin \alpha_F \sin(\theta_F + \phi_F)
\] (6.3.15)

where

\[
\begin{align*}
A_1 &= R_{cp} \sin(\theta_F + \phi_F) + S_{r1} \sin(-q_1 + \phi_F) \\
A_2 &= R_{cp} \cos(\theta_F + \phi_F) + S_{r1} \cos(-q_1 + \phi_F)
\end{align*}
\] (6.3.16)

**Step 4: Two-parametric representation of surface of action**

The surface of action is the set of instantaneous lines of contact between the generating cone surface and the pinion tooth surface that are represented in the fixed coordinate system \(S_{m1}\). The surface of action is represented by equations (6.3.3) and (6.3.13) being considered simultaneously. These equations represent the surface of action by three related parameters. Taking into account equation (6.3.13), we can eliminate \(s_F\) and represent the surface of action in two-parametric form by
The common normal to contacting surfaces has been already represented in two-parametric form by equations (6.3.6).

**Generation by a Surface of Revolution**

*Step 1:* The shape of the blades is a circular arc (Fig.5.5.1) and such blades generate a surface of revolution by rotation about the head-cutter axis.

The position-vector of the center of the generating arc is represented in \( S_{m1} \) by the equation

\[
\hat{\rho}_{m1}^{(c)}(\theta_F, \phi_F) = [M_{m1c1} \{ \hat{\rho}^{(c)} + [ S_{r1} \cos q_1 - S_{r1} \sin q_1 \; 0]^T \} \] (6.3.18)

where,

\[
[M_{m1c1}] = \begin{bmatrix}
\cos \phi_F & \sin \phi_F & 0 & 0 \\
- \sin \phi_F & \cos \phi_F & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \] (6.3.19)

and \( \hat{\rho}^{(c)} \) has been expressed by equation (5.5.7).

*Step 2:* We will need for further transformations the following equations

\[
\bar{e}_{lm1} = [L_{m1c1}] \bar{e}_{F1}^{(F)} = \begin{bmatrix}
- \sin(\theta_F + \phi_F) \\
\cos(\theta_F + \phi_F) \\
0
\end{bmatrix} \] (6.3.20)
and

\[ \vec{\tau}_{m1} = \begin{bmatrix} -\cos(\theta_F + \phi_F) \\ -\sin(\theta_F + \phi_F) \\ 0 \end{bmatrix} \]

(6.3.21)

Here: \( \vec{e}^{(F)}_{Im_1} \) is the unit vector of principal direction \( I \) on the head-cutter surface and \( \vec{\tau}_{m1} \) is a unit vector that is perpendicular to \( \vec{e}^{(F)}_{Im_1} \) and the axis of the head-cutter (Fig. 6.3.1)

**Step 3:** To simplify the equation of meshing we will represent it by the following equation

\[ \vec{n}_{m1} \cdot \vec{v}^{(1F,C)}_{m1} = 0 \]

(6.3.22)

where \( \vec{v}^{(1F,C)}_{m1} \) is the relative velocity of the center of the circular arc that generates the head-cutter surface of revolution. The proof that (6.3.22) is indeed the equation of meshing is based on the following considerations:

(i) The relative velocity for a point of the head-cutter surface is represented by equation (6.3.9), given as

\[ \vec{v}^{(1F)}_{m1} = (\vec{\omega}_{m1}^{(1)} - \vec{\omega}^{(F)}_{m1}) \times \vec{r}_{m1} + R_{m1} \times \vec{\omega}_{m1}^{(1)} \]

(6.3.23)

We can represent position vector \( \vec{r}_{m1} \) for a point \( M \) as

\[ \vec{r}_{m1} = \vec{r}^{(C)}_{m1} + \rho \vec{n}_{m1} \]

(6.3.24)
where $\rho$ is the radius of the arc blade.

While deriving equation (6.3.24), we have taken into account that a normal to the head-cutter surface passes through the current arc center $C$; the sign of $\rho$ depends on how the surface unit normal is directed with respect to the surface.

Then, we may represent the equation of meshing as follows

$$
\tilde{v}_{m1}^{(F)} \cdot \tilde{n}_{m1}^{(F)} = \left\{ (\tilde{\omega}_{m1}^{(1)} - \tilde{\omega}_{m1}^{(F)}) \times [ (\tilde{r}_{m1} + \rho \tilde{n}_{m1}^{(F)}) + (\tilde{R}_{m1} \times \tilde{\omega}^{(1)}) ] \right\} \cdot \tilde{n}_{m1}^{(F)}
$$

$$
= [(\tilde{\omega}_{m1}^{(1)} - \tilde{\omega}_{m1}^{(F)}) \times [(\tilde{r}_{m1} + (\tilde{R}_{m1} \times \tilde{\omega}^{(1)}) ] \cdot \tilde{n}_{m1}^{(F)}
$$

$$
= \tilde{v}_{m1}^{(1F,C)} \cdot \tilde{n}_{m1}^{(F)} = 0
$$

(6.3.25)

Thus, equation (6.3.22) is proven.

**Step 4:** It follows from equation (6.3.22) that vector $\tilde{v}_{m1}^{(1F,C)}$ belongs to a plane that is parallel to the tangent plane $T$ to the head-cutter surface (Fig.6.3.2). This means that if vector $\tilde{v}_{m1}^{(1F,C)}$ is translated from point $C$ to $M$ it will lie in plane $T$. The unit vector $\tilde{e}_{m1}^{(F)}$ lies in plane $T$ already.

Then, we may represent the unit normal $\tilde{n}_{m1}$ by the equation

$$
\tilde{n}_{m1}(\theta_F, \phi_F) = \frac{\tilde{e}_{m1} \times \tilde{v}_{m1}^{(1F,C)}}{|\tilde{e}_{m1} \times \tilde{v}_{m1}^{(1F,C)}|}
$$

(6.3.26)

where $\tilde{v}_{m1}^{(1F,C)}$ is represented as follows,

$$
\tilde{v}_{m1}^{(F)} =  \tilde{\omega}^{(F)} \times \tilde{\rho}_{m1}^{(C)}
$$

(6.3.27)
\[ \vec{v}_{m1}^{(1)} = \vec{\omega}^{(1)} \times \left\{ \rho_{m1}^{(C)} + [-X_{G1} \cos \gamma_1 \ E_{m1} \ - \ X_{G1} \sin \gamma_1]^T \right\} \]  

(6.3.28)

\[ \vec{v}_{m1}^{(1F,C)} = \vec{v}_{m1}^{(1)} - \vec{v}_{m1}^{(F)} \]  

(6.3.29)

The advantage of vector equation (6.3.26) is that the surface unit normal at the point of contact is represented by a vector function of two parameters only, \( \theta_F \) and \( \phi_F \); this vector function does not contain the surface parameter \( \lambda \).

The order of co-factors in vector equation must provide that the direction of \( \vec{n}_{m1} \) is toward the axis of the head-cutter. The direction of \( \vec{n}_{m1} \) can be checked with the dot product

\[ \Delta = \vec{n}_{m1} \cdot \vec{\tau}_{m1} \]  

(6.3.30)

The surface unit normal has the desired direction if \( \Delta > 0 \). In the case when \( \Delta < 0 \), the desired direction of \( \vec{n}_{m1} \) can be observed just by changing the order of co-factors in equation (6.3.26).

To determine parameter \( \lambda \) for the current point of contact we can use the equation,

\[ \cos \lambda = \vec{n}_{m1} \cdot \vec{\tau}_{m1} \]  

(6.3.31)

**Step 5:** Our final goal is the determination in \( S_{m1} \) of a position vector of a current point of contact of surfaces \( \Sigma_F \) and \( \Sigma_1 \). This can be done by using the equation,

\[ \vec{r}_{m1}(\theta_F, \phi_F) = \rho_{m1}^{(C)} - \rho \vec{n}_{m1} \]  

(6.3.32)
where \( \rho \) is the radius of the circular arc.

Finally, the pinion tooth surface may be determined in \( S_1 \) as the set of contact points. Thus:

\[
\vec{r}_1(\theta_F, \phi_F) = [M_{1p}][M_{pm_1}]\vec{r}_{m_1}(\theta_F, \phi_F)
\]  
(6.3.33)

The unit normal to surface \( \Sigma_1 \) is determined in \( S_1 \) with the equation

\[
\vec{n}_1(\theta_F, \phi_F) = [L_{1p}][L_{pm_1}]\vec{n}_{m_1}(\theta_F, \phi_F)
\]  
(6.3.34)

Here: \( \vec{r}_{m_1}(\theta_F, \phi_F) \) and \( \vec{n}_{m_1}(\theta_F, \phi_F) \) have been represented by equations (6.3.17) and (6.3.6) for straight blade cutter and by equations (6.3.26) and (6.3.32) for curved blade cutter. Here (Fig. 2.1.2):

\[
[M_{pm_1}] = \begin{bmatrix}
\cos \gamma_1 & 0 & \sin \gamma_1 & -X_{G1} \sin \gamma_1 \\
0 & 1 & 0 & E_{m_1} \\
-\sin \gamma_1 & 0 & \cos \gamma_1 & -(X_{G1} \sin \gamma_1 + X_{B1}) \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  
(6.3.35)

\[
[M_{1p}] = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \phi_1 & \sin \phi_1 & 0 \\
0 & -\sin \phi_1 & \cos \phi_1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  
(6.3.36)
where $\phi_1$ is the angle of the pinion rotation in the process for generation. Angles $\phi_1$ and $\phi_F$ (the angle of rotation of the cradle) are related as follows:

(i) in the case when the modified roll is not used and $R_{ap}$ is constant, we have

$$\phi_1 = R_{ap}\phi_F$$  \hspace{1cm} (6.3.37)

(ii) when the modified roll is used, $\phi_1$ is represented by the Taylor's series

$$\phi_1 = f(\phi_F) = R_{ap}(\phi_F - C\phi_F^2 - D\phi_F^3 - E\phi_F^4 - F\phi_F^5)$$  \hspace{1cm} (6.3.38)

where $C, D, E$ and $F$ are the coefficients of Taylor's series of generation motion (see Appendix B).

Step 7: The tooth contact analysis, as it was mentioned above, is based on conditions of tangency of the pinion and gear surfaces that are considered in the fixed coordinate system $S_h$ (see section 6.1). To represent the pinion tooth surface and the surface unit normal in $S_h$ we use the matrix equations

$$\vec{r}_h^{(1)} = [M_{h1}]\vec{r}_1^{(1)}(\theta_F, \phi_F)$$  \hspace{1cm} (6.3.39)

$$\vec{n}_h^{(1)} = [L_{h1}]\vec{n}_m^{(1)}(\theta_F, \phi_F)$$  \hspace{1cm} (6.3.40)

Here:
where $\phi'_1$ is the angle of rotation of the pinion being in mesh with the gear.

6.4 Determination of Transmission Errors

The function of transmission errors is determined by the equation

$$\delta(\phi'_1) = [\phi'_2 - (\phi'_2)^0] - \frac{N_1}{N_2}[\phi'_1 - (\phi'_1)^0]$$

(6.4.1)

Here: $(\phi'_i)^0$ $(i = 1, 2)$ is the initial angle of gear rotation with which the contact of surfaces $\Sigma_1$ and $\Sigma_2$ at the mean contact point is provided. Linear function

$$\frac{N_1}{N_2}[\phi'_1 - (\phi'_1)^0]$$

(6.4.2)

provides the theoretical angle of gear rotation for a gear drive without misalignments. The range of $\phi'_2$ is determined as follows:

$$[\frac{\pi}{N_2} \leq \phi'_2 \leq (\phi'_2)^0 + \frac{\pi}{N_2}]$$

(6.4.3)

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The function of transmission errors is usually a piecewise periodic function with period equal to \( \phi_i = \frac{2\pi}{N_i} \) (Fig.6.4.1). The purpose of synthesis for spiral bevel gears is to provide that the function of transmission errors will be of a parabolic type and of a limited value \( \delta' \) (Fig.6.4.1).

The tooth contact analysis enables to simulate the influence of errors of assembly of various types, particularly, when the center of the bearing contact is shifted in two orthogonal directions (see section 7).

### 6.5 Simulation of Contact

#### Mapping of Contact Path into a Two-Dimensional Space

It was mentioned above that the contact path on the pinion and gear tooth surfaces is determined with functions (6.1.7) and (6.1.8), respectively. For the purpose of visualization, the contact path on the gear tooth surface is mapped onto plane \((X_c, Y_c)\) that is shown in Fig.6.5.1. The \(X_c\)-axis is directed along the root cone generatrix and \(Y_c\) is perpendicular to the root cone generatrix and passes through the mean contact point (Fig. 6.5.1).

Consider that a current contact point \(N^*\) is represented in \(S_2\) (Fig.6.5.2) by coordinates: \(X_2(\phi_2^*, RL'(\phi_2^*))\) where \(\phi_2^*\) is the angle of rotation of the gear and \(RL' = |\overline{EN}| = (Y^2 + Z^2)^{\frac{1}{2}}\). Axis \(X_2\) belongs to plane \((X_c, Y_c)\) (Fig.6.5.2). While mapping the contact path onto plane \((X_c, Y_c)\), we will represent its current point \(N^*\) by \(N\) that can be determined by coordinates \(X_2\) and \(RL'\), where \(RL' = |\overline{EN}| = |\overline{EN^*}|\) (Fig.6.5.3). The coordinates of mean contact point \(M\), \(XL\) and \(R\), have been previously determined by equations (3.2.1) and (3.2.2). Drawing of Fig.6.5.3 yield

\[
\overline{O_cN} = \overline{O_cO_2} + \overline{O_2E} + \overline{EN}
\]

(6.5.1)

Here:
\[ \overline{O_cO_2} = \overline{O_cK} + \overline{KO_2} \]  
(6.5.2)

\[ \overline{KO_2} = -RL \cos(\gamma_k - \gamma_2) \overline{\alpha_c} \]  
(6.5.3)

where \( \gamma_k \) is determined by:

\[ \gamma_k = \tan^{-1}\left( \frac{RL}{XL} \right) \]  
(6.5.4)

Equations from (6.5.1) to (6.5.8) yield

\[ \overline{O_cK} = -|\overline{O_cO_2}| \sin \gamma_2 \overline{\alpha_c} \]  
(6.5.5)

\[ \overline{O_2E} = X_2 \cos \gamma_2 \overline{\alpha_c} - X_2 \sin \gamma_2 \overline{J_c} \]  
(6.5.6)

\[ \overline{EN} = RL \left( \sin \gamma_2 \overline{\alpha_c} + \cos \gamma_2 \overline{J_c} \right) \]  
(6.5.7)

\[ \overline{O_cN} = X_c \overline{\alpha_c} + Y_c \overline{J_c} \]  
(6.5.8)

\[
\begin{align*}
X_c &= X_2(\phi'_2) \cos \gamma_2 + RL'(\phi'_2) \sin \gamma_2 - [(XL)^2 + (RL)^2]^\frac{1}{2} \cos(\gamma_k - \gamma_2) \\
Y_c &= X_2(\phi'_2) \sin \gamma_2 + RL'(\phi'_2) \cos \gamma_2 - Z_R \sin \gamma_2
\end{align*}
\]  
(6.5.9)

Contact Ellipse
Theoretically, the tooth surfaces of the pinion and the gear are in point contact. However, due to the elastic deformation of tooth surfaces their contact will be spread over an elliptic area. The dimensions and orientation of the instantaneous contact ellipse depend on the elastic approach $\delta$ of the surfaces and the principal curvatures and the angel $\sigma^{(12)}$ formed between principal directions $e_I^{(1)}$ and $e_I^{(2)}$ of the surfaces. The elastic approach depends on the magnitude of the applied load. The value of $\delta$ can be taken from experimental results and this will enable us to consider the determination of the instantaneous contact ellipse as a geometric problem. Usually, the magnitude $\delta$ is taken as $\delta = 0.00025$ inch.

In our approach the curvatures and principal directions of the pinion and the gear are determined with the principal curvatures and directions of the generating tools and parameters of relative motion in the process for generation.

**Gear Tooth Principal Curvatures and Directions**

The procedure for determination of gear tooth principal curvatures and directions was described in section 1.2. Knowing functions $\theta_p(\phi_2'), \phi_p(\phi_2')$ from the TCA procedure of computation, we are able to determine the position vector $\vec{r}_{m2}(\theta_p(\phi_2'), \phi_p(\phi_2'))$ and the surface unit normal $\vec{n}_{m2}(\theta_p(\phi_2'), \phi_p(\phi_2'))$ for an instantaneous point of contact. The principal directions and curvatures for the generating surface can be determined from equations (5.2.4), (5.2.5) and (5.2.6). The parameters of relative motions in the process for generation can be determined with equations (3.1.12) and (3.1.13).

**Pinion Tooth Principal Curvatures and Directions**

As it was mentioned above, the pinion tooth surface can be generated by a cone or by a surface of revolution. The derivation of principal curvatures and directions on the pinion tooth surface is based on relations between principal curvatures and directions between mutually enveloping surfaces $\Sigma_F$ of the head- cutter and $\Sigma_1$ of the pinion. The procedure of derivation is as follows:

**Step 1:** We represent in $S_{m1}$ the principal directions on the head- cutter surface $\Sigma_F$ using the following equations

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Step 2: Parameters of relative motion in the process for pinion generation have been represented by equations (5.4.14) to (5.4.19). The derivative of cutting ratio, $m'_{F1}$, is equal to zero for the case when the modified roll is not used, and can be determined when the modified roll is applied as follows (see the Appendix)

$$m'_{F1} = \frac{d^2 \phi_F}{d\phi_1^2} = \frac{f''(\phi_F)}{[f'(\phi_F)]^3}$$  \hfill (6.5.11)

where,

$$f'(\phi_F) = R_ap(1 - 2C \phi_F - 3D \phi_F^2 - 4E \phi_F^3 - 5F \phi_F^4)$$

$$f''(\phi_F) = -R_ap(2C + 6D \phi_F + 12E \phi_F^2 + 20F \phi_F^3)$$  \hfill (6.5.12)

Step 3: Now, since the principal curvatures and directions on $\Sigma_F$ are known and the relative motion is also known, we can determine for each point of contact path the principal curvatures $\kappa_I$ and $\kappa_{II}$ of the pinion tooth surface $\Sigma_1$, the angle $\sigma^{(F1)}$ and the principal directions $\varepsilon^{(1)}_{Im_1}, \varepsilon^{(1)}_{IIm_1}$ on $\Sigma_1$. We use for this purpose equations (1.2.6) to (1.2.10). The principal directions on $\Sigma_1$ can be represented in coordinate system $S_h$ by the matrix equation (Fig.5.2.2),
Orientation and Dimensions of the Instantaneous Contact Ellipse

Knowing the principal directions and principal curvatures for the contacting surfaces at each point of contact path, we can determine the half-axes $a$ and $b$ of the contact ellipse and angle $\alpha^{(1)}$ of the ellipse orientation (Fig.6.5.4). The procedure of computation is as follows [4]:

**Step 1: Determination of $a$ and $b$**

\[
A = \frac{1}{4} \left[ K^{(1)}_\Sigma - K^{(2)}_\Sigma - \sqrt{g_1^2 - 2g_1g_2\cos2\sigma + g_2^2} \right] \tag{6.5.14}
\]

\[
B = \frac{1}{4} \left[ K^{(1)}_\Sigma - K^{(2)}_\Sigma + \sqrt{g_1^2 - 2g_1g_2\cos2\sigma + g_2^2} \right] \tag{6.5.15}
\]

\[
a = \sqrt{\left| \frac{\delta}{A} \right|} \tag{6.5.16}
\]

\[
b = \sqrt{\left| \frac{\delta}{B} \right|} \tag{6.5.17}
\]

where,

\[
K^{(i)}_\Sigma = K^{(i)}_I + K^{(ii)}_I \quad g_i = K^{(i)}_I - K^{(ii)}_I \quad (i = 1, 2) \tag{6.5.18}
\]

**Step 2: Determination of $\sigma^{(12)}$** (Fig.6.5.4)
Step 3: Determination of $\alpha^{(1)}$

Angle $\alpha^{(1)}$ determines the orientation of the long axis of the contact ellipse with respect to $\ell_{hI}$ (Fig. 6.5.4) and is one of the angles determined by the following equations,

$$
\tan 2\alpha^{(1)} = \frac{g_2 \sin 2\sigma^{(12)}}{g_1 - g_2 \cos 2\sigma^{(12)}}
$$

(6.5.20)

Step 4: The orientation of unit vectors $\bar{\eta}$ and $\bar{\zeta}$ of long and short axes of the contact ellipse (Fig. 6.5.4) with respect to the pinion principal directions is determined with the equations

$$
\bar{\eta}_h = \ell_{hI}^{(1)} \cos \alpha^{(1)} - \ell_{hII}^{(1)} \sin \alpha^{(1)}
$$

(6.5.21)

$$
\bar{\zeta}_h = \ell_{hI}^{(1)} \sin \alpha^{(1)} + \ell_{hII}^{(1)} \cos \alpha^{(1)}
$$

(6.5.22)

Step 5: In order to visualize the contact ellipse we represent its axes of contact ellipse in plane $(X_c, Y_c)$ (Fig. 6.5.1), using the following equations

$$
\bar{\eta}_2 = [L_2h] \bar{\eta}_h \quad \bar{\zeta}_2 = [L_2h] \bar{\zeta}_h
$$

(6.5.23)

where
The unit vectors of axes of contact ellipse form in plane $(X, Y)$ the following angles with the $X,-$axis (the generatrix of the root cone):

$\sin I' = \frac{1}{L_2h}$

$\cos I' = \frac{L_2h}{L_2h}$

Axes of the contact ellipse form with the gear axes the following angles:

\[ \begin{bmatrix} L_{2h} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\cos \phi' & -\sin \phi' \\ 0 & \sin \phi' & -\cos \phi' \end{bmatrix} \begin{bmatrix} \cos \Gamma & 0 & -\sin \Gamma \\ 0 & 1 & 0 \\ \sin \Gamma & 0 & \cos \Gamma \end{bmatrix} \tag{6.5.24} \]

The unit vectors of axes of contact ellipse form in plane $(X_c, Y_c)$ the following angles with the $X_c$-axis (the generatrix of the root cone):

$\triangle \eta = \arccos(\vec{n}_2 \cdot \vec{i}_2)$

$\triangle \zeta = \arccos(\vec{\zeta}_2 \cdot \vec{i}_2)$

The unit vectors of axes of contact ellipse form in plane $(X, Y)$ the following angles with the $X_c$-axis (the generatrix of the root cone):

$\tau_1 = \triangle \eta - \gamma_2 \quad \tau_2 = \triangle \zeta - \gamma_2$ \tag{6.5.26}
7 V and H check

The purpose of the so called V and H check is the computer aided simulation of the shift of the bearing contact to the toe and to the hill of the gear. The gear quality is judged with the sensitivity of the shape of the contact pattern and the change in the level of transmission errors to the above-mentioned shift of contact.

7.1 Determination of V and H values

Fig.7.1.1 shows the initial position $M$ of contact point (it is the mean contact point), and the new position $M^*$ of the contact point). The shift of the contact pattern was caused by the deformation under the load. Coordinates $XL$ and $RL$ determines the location of $M$. For the following derivations we will use the following notations.

(i) $PF = A - A^*$ is the shift of the center of bearing contact, where $F$ is the tooth length measuring along the pitch line; $p$ is an algebraic value, that is positive when $A^* < A$ and the shift is performed to the toe as shown in Fig.7.1.1. Usually, $p$ is equal to 0.25.

(ii) $\delta_G$ and $\alpha_G$ are the gear dedendum and addendum angle.

(iii) $PD = b_G$ and $P^*D^* = b_G^*$ are the gear dedendums that are measured in sections $I$ and $I^*$.

(iv) $h_m = BD$ and $h^* = B^*D^*$ are the gear tooth heights.

(v) $\Gamma_2$ is the pitch cone angle

The determination of V and H for point contact $M^*$ is based on the following procedure.

Step 1: Determination of $XL^*$ and $RL^*$.

Fig.7.1.1 results in:

$$h^* = h_m - pF(\tan \delta_G + \tan \alpha_G)$$  \hspace{1cm} (7.1.1)
\[ b_G^* = b_G - PF \tan \delta_G \]  

(7.1.2)

where \( b_G^* = P^*D^* \) and \( b_G = PD \)

We assume that \( M^*D^* = \frac{h^* + c}{2} \) and \( MD = \frac{h + c}{2} \), where \( c \) is the clearance.

Taking into account that

\[ \overline{O_2M^*} = \overline{O_2P^*} + \overline{P^*M^*} \]  

(7.1.3)

we obtain

\[ XL^* = A^* \cos \Gamma_2 + P^*M^* \sin \Gamma_2 = A^* \cos \Gamma_2 + \left( b_G^* - \frac{h^* + c}{2} \right) \sin \Gamma_2 \]  

(7.1.4)

\[ RL^* = A^* \sin \Gamma_2 + P^*M^* \cos \Gamma_2 = A^* \sin \Gamma_2 - \left( b_G^* - \frac{h^* + c}{2} \right) \cos \Gamma_2 \]  

(7.1.5)

The surface coordinates \((\theta_G^*, \phi_p^*)\) can be determined by solving the following two equations,

\[ X_2(\theta_G^*, \phi_p^*) = XL^* \]  

(7.1.6)

\[ [Y_2(\theta_G^*, \phi_p^*)]^2 + [Z_2(\theta_G^*, \phi_p^*)]^2 = (RL^*)^2 \]  

(7.1.7)
Step 2: Determination of $V$ and $H$

We introduce the shift of the bearing contact in coordinate system $S_h$ by $V$ and $H$ that are directed along the shortest distance between the pinion and gear axes, and the pinion axis, respectively (Fig.7.1.2). $V$ is positive when the gear is shifted apart from the pinion in $Y_h$ direction, $H$ is positive when the pinion is withdrawn. It is obvious that

$$[\tilde{\pi}_h^{(2)}]^* = \tilde{\pi}_h^{(2)} + V\tilde{j}_h \quad (7.1.8)$$

$$[\tilde{\pi}_h^{(1)}]^* = \tilde{\pi}_h^{(1)} + H\tilde{j}_h \quad (7.1.9)$$

Here: $\tilde{\pi}_h^{(i)}$ ($i = 1, 2$) is the position vector for the initial point of contact, $[\tilde{\pi}_h^{(i)}]^*$ ($i = 1, 2$) is the position vector for the shifted contact point; $\tilde{i}_h, \tilde{j}_h$ and $\tilde{k}_h$ are the unit vectors of coordinate axes $S_h$.

Equations of tangency at the new contact point provide

$$[\tilde{\pi}_h^{(2)}(\theta_G^*, \phi_p^*, \phi_2')]^* = [\tilde{\pi}_h^{(1)}(\theta_F^*, \phi_F^*, \phi_1')]^* \quad (7.1.10)$$

$$[\tilde{n}_h^{(2)}(\theta_G^*, \phi_p^*, \phi_2')]^* = [\tilde{n}_h^{(1)}(\theta_F^*, \phi_F^*, \phi_1')]^* \quad (7.1.11)$$

Gear surface coordinates $\theta_G^*$ and $\phi_p^*$ can be determined from equations (7.1.6) and (7.1.7). Equations (7.1.10) and (7.1.11) yield

$$V = [Y_h^{(2)}(\theta_G^*, \phi_p^*, \phi_2')]^* - [Y_h^{(1)}(\theta_F^*, \phi_F^*, \phi_1')]^* \quad (7.1.12)$$
Equations from (7.1.12) to (7.1.16) represent a system of five independent equations in six unknowns: $V$, $H$, $\phi_2$, $\phi_1'$, $\theta_F$ and $\phi_F$. The sixth independent equation, that is required for the solution of unknowns, can be derived based on the condition that the equation of meshing must be satisfied with the designed gear ratio, i.e.,

\[
\bar{\nu}_{h}^{(2)} \cdot \bar{\nu}_{h}^{(12)} = f(\theta_{G}^{*}, \phi_{G}^{*}, \phi_{2}', \phi_{1}', \theta_{F}, \phi_{F}, V, H) = 0
\] (7.1.17)

In solving the above system, we first solve a sub-system composed of equations (7.1.14), (7.1.16) and (7.1.17) for $\phi_2', \phi_F$ and $\theta_F$, and then calculate the values of $\phi_1'$, $V$ and $H$ directly, by equations (7.1.12), (7.1.13) and (7.1.15).
7.2 Tooth Contact Analysis for Gears with Shifted Center of Bearing Contact

After the determination of parameters V and H, the tooth contact analysis for gears with shifted center of bearing contact can be performed similarly to the analysis described in sections 6.4. and 6.5. The initial guess for the first iteration in the procedure of computations is provided by the set of six unknowns obtained in section 7.1.
Appendix A

Generation with Modified Roll

1 Introduction

Modification of roll or sometimes called modified roll means that the cutting ratio is not constant but varied in the process for generation. The variable cutting ratio—the variable ratio of roll—can be provided by a cam mechanism of the transmission of the cutting machine or by the servo-motors of a computer controlled cutting machine. According to the developments of Gleason, the TCA program can analyze the process for generation up to members of the fifth order. However, due to the limitations caused by application of cam mechanisms only the parameters up to the third order are controllable in the process for generation.

The modified roll is an additional parameter for the synthesis of spiral bevel gears. In our approach the synthesis of spiral bevel gears can be performed, as it was mentioned above, with a constant cutting ratio. However, we consider in this section the application of modified roll as well to provide a broader point of view on synthesis of spiral bevel gears.

2 Taylor Series for the Function of Generation Motion

According to the practice of Gleason, the kinematic relation between the angles of rotation of the workpiece and the cradle is represented by a Taylor's series up to fifth order. To the knowledge of the authors, Gleason has never published any materials related to the kinematics of the modified
roll. However, Professor Zheng had done a good job in deciphering Gleason's mechanisms for modified roll and represented the kinematic relations in his valuable book [5].

Consider that the angles of rotation of the pinion and the cradle are related by a nonlinear function

$$\phi_1 = f(\phi_F) \quad f \in C^K \quad (K \geq 3)$$  \hspace{1cm} (A.1)

We assume that $\phi_1 = 0$ at $\phi_F = 0$ and represent $f(\phi_F)$ in the neighborhood of $\phi_F = 0$ by the Taylor series as follows,

$$\phi_1 = f'(0)\phi_F + \frac{1}{2!} f''(0)\phi_F^2 + \cdots$$  \hspace{1cm} (A.2)

Taking into account that

$$\frac{d\phi_1}{d\phi_F} = f'(\phi_F)$$  \hspace{1cm} (A.3)

We obtain

$$f'(0) = \left. \frac{\omega^{(1)}}{\omega(F)} \right|_{\phi_F=0} = R_{ap}$$  \hspace{1cm} (A.4)

where $R_{ap}$ is the ratio of roll.

Without losing generality of the solution, we can take $\omega^{(1)} = 1$ and then obtain
Differentiation of equation (A.5) yields

\[ f'(\phi_F)\frac{d\phi_F}{dt} = 1 \]  \hspace{1cm} (A.5)

Equation (A.6) yields

\[ f''(\phi_F)(\frac{d\phi_F}{dt})^2 = f'(\phi_F)\frac{d^2\phi_F}{dt^2} \]  \hspace{1cm} (A.6)

Equation (A.6) yields

\[ \frac{a_2}{\omega_F^2} = -\frac{f''(\phi_F)}{f'(\phi_F)} \]  \hspace{1cm} (A.7)

where \( a_2 = \frac{d^2\phi_F}{dt^2} \) is the angular acceleration of the cradle.

Equation (A.7) with new designations can be represented as follows

\[ 2C = \frac{a_2}{\omega_F^2} = -\frac{1}{R_{ap}}f''(0) \]  \hspace{1cm} (A.8)

Similar differentiation of higher order of equation (A.3) yields:

\[ \phi_1 = R_{ap}(\phi_F - C\phi_F^3 - D\phi_F^5 - E\phi_F^7 - F\phi_F^9) \]  \hspace{1cm} (A.9)

Here:
Unfortunately, function $f(\phi_F)$ cannot be represented in explicit form for certain cutting machines, for instance, for the Gleason spiral bevel grinder. For such a case we will consider the following auxiliary expressions

\[ 2C = -\frac{1}{R_{ap}} f''(0) \]

\[ 6D = -\frac{1}{R_{ap}} f'''(0) \]

\[ 24E = -\frac{1}{R_{ap}} f^{iv}(0) \]

\[ 120F = -\frac{1}{R_{ap}} f^{v}(0) \]

Unfortunately, function $f(\phi_F)$ cannot be represented in explicit form for certain cutting machines, for instance, for the Gleason spiral bevel grinder. For such a case we will consider the following auxiliary expressions

\[ a_3 = \frac{d^3 \phi_F}{dt^3}, \quad 6CX = \frac{a_3}{\omega_F^3} \quad (A.10) \]

\[ a_4 = \frac{d^4 \phi_F}{dt^4}, \quad 24DX = \frac{a_4}{\omega_F^4} \quad (A.11) \]

\[ a_5 = \frac{d^5 \phi_F}{dt^5}, \quad 120EX = \frac{a_5}{\omega_F^5} \quad (A.12) \]

Then, differentiating equation (A.6) and taking $\phi_F = 0$, we may obtain the following equations

\[ 6D = 6CX - 3(2C)^2 \quad (A.13) \]
The procedure for determination of coefficients $C$, $D$, $E$ and $F$ for the Taylor's series (A.9) when function $f_1(\phi_F)$ cannot be represented in explicit form is as follows:

**Step 1:** Differentiate the implicit equation that relates $\phi_F$ and $\phi_1$ up to five times and then find $\omega_F, a_2, a_3, a_4, a_5$ in terms of $\phi_1$ and $\phi_2$ at $\phi_1 = \phi_F = 0$.

**Step 2:** Considering $\phi_1 = \phi_F = 0$, find $6CX, 24DX, 120EX$ by equations (A.10) – (A.12).

**Step 3:** Find $2C, 6D, 24E, 120F$ by equation (A.8), (A.13) – (A.15).

3 Synthesis of Gleason's Cam

**Introduction**

Gleason's cam mechanism, as shown schematically in Fig. A.3.1, is an ingenious invention that has been proposed and developed by the engineers of the Gleason Works. The mechanism transforms rotation of the cam about $O_q$ into rotation of the cradle about $O_c$. The rotation of the cam about $O_q$ is related with the rotation of the pinion being generated, but the angles of cam rotation and pinion rotation, $\phi_q$ and $\phi_1$, are related by a linear function when there is no cam settings.

To authors' knowledge, the engineers of the Gleason Works have not published the principles of synthesis and analysis of this mechanism. However, H. Cheng [6], Zheng [5] have made good contributions to the deciphering of this mechanism. The following is a systematic representation of synthesis and analysis of Gleason's mechanism.

The purpose of cam synthesis is to obtain the shape of the cam, considering that the angles of rotation of the cam and the cradle are related by a linear function, $\phi_c(\phi_2)$. However, this function

\[
24E = 24DX + (2C)[15(2C)^2 - 10(6CX)]
\]  

(A.14)

\[
120F = 120EX - 15(2C)(24DX) + 105(2C)^2[6CX - (2C)^2] - 10(6CX)^2
\]  

(A.15)
can be modified into a nonlinear function by changing the location of the designed cam with respect

to \( O_q \) and the orientation of the cam guides that are installed on the cradle. Fig. A.4.1 shows the
settings of the cam mechanism with the designed shape: (i) the cam is translated along the line
\( O_c O_q \) an amount \( \Delta T \); (ii) and then, the cam guides are rotated about the cam rotation center
and formed angle \( \alpha \) with \( O_c O_q \). It is obvious that the cam mechanism with the settings \( \Delta T \) and
\( \alpha \) will transform rotation about \( O_q \) to \( O_c \) with a nonlinear function between the angles of rotation
of the cam and the cradle. The deviation of this function from a linear one depends on settings of
the cam mechanism and will be discussed in section A.4.

**Coordinate Systems**

While considering the synthesis of the cam mechanism, we will use three coordinate systems:
the movable coordinate systems \( S_c \) and \( S_q \) that are rigidly connected to the cradle and the cam,
and \( S_f \) that is the fixed coordinate system (Fig. A.3.2).

**Equation of Meshing, Contact Point in \( S_c \)**

Assuming that the transformation of motion is performed with constant ratio of angular veloc-
ities and in the same direction, we can determine the location of instantaneous center of rotation,
\( I \), in coordinate system \( S_f \) by using the equation (Fig. A.3.2)

\[
\frac{\omega_q}{\omega_c} = \frac{E + r_u}{r_u}
\]  

(A.16)

Where, \( E \) in the distance between the cradle center \( O_c \) and the cam rotation center \( O_q \), \( r_u \) is the
so-called pitch radius of the cam.

The location of instantaneous point of contact \( M \) on the guides can be determined by using
the theorem of planar gearing [4]. According to this theorem the common normal to the guides
and the cam at the point of their contact must pass through the instantaneous center of rotation
\( I \). Thus, contact point \( M \) and the unit normal at \( M \) are represented in \( S_c \) as follows
Here: \( b \) is an algebraic value (\( b \) is positive if the left side of guides is considered and \( b \) is negative if the right side of guides is considered); \( u \) is a variable parameter that is determined with the equation

\[
u = (E + r_u) \cos \theta_c - E
\]  \hfill (A.20)

Equations (A.17) and (A.20) yield

\[
\vec{r}_c(\theta_c) = [-b \quad f(\theta_c) \quad 0 \quad 1]^T
\]  \hfill (A.21)

where

\[
f(\theta_c) = E - (E + r_u) \cos \theta_c
\]  \hfill (A.22)

**Shape of the Cam**

The shape of the cam is a planar curve that is represented in \( S_q \) by the matrix equation
Here: coordinate system $S_p$ is an auxiliary fixed coordinate system (Fig. A.3.2). Matrices in equation (A.23) are represented as follows

$$
\vec{r}_q(\theta_c) = [M_{qp}][M_{pf}][M_{fc}]\vec{r}_c(\theta_c)
$$

(A.23)

The normal to the cam shape is represented by the matrix equation

$$
\vec{n}_q(\theta_c) = [L_{qp}][L_{pf}][L_{fc}]\vec{n}_c(\theta_c)
$$

(A.27)
Here: \([L_{pf}]\) is the identity matrix and is the \((3 \times 3)\) submatrix of the respective matrix \([M]\). We consider that the shape of the cam and its normal depend on the generalized parameter \(\theta_q\) only since

\[
\theta_c = \left(\frac{r_u}{E + r_u}\right)\theta_q
\]

(A.28)

The final equations of the cam and its normal are represented as follows:

\[
\vec{\pi}_q = \begin{bmatrix}
-b \cos\left(\frac{E}{E + r_u} \theta_q\right) + (r_u + E) \cos\left(\frac{r_u}{E + r_u} \theta_q\right) \sin\left(\frac{E}{E + r_u} \theta_q\right) - E \sin \theta_q \\
-b \sin\left(\frac{E}{E + r_u} \theta_q\right) - (r_u + E) \cos\left(\frac{r_u}{E + r_u} \theta_q\right) \cos\left(\frac{E}{E + r_u} \theta_q\right) - E \cos \theta_q \\
0 \\
1
\end{bmatrix}
\]

(A.29)

\[
\vec{\pi}_q = \begin{bmatrix}
\cos\left(\frac{E}{E + r_u} \theta_q\right) \\
\sin\left(\frac{E}{E + r_u} \theta_q\right) \\
0
\end{bmatrix}
\]

(A.30)

4 Cam Analysis

The cam analysis is directed at the determination of function \(\theta_c^x(\theta_c^y)\) for a cam and guides with modified settings. The analysis is based on simulation of tangency of the designed cam with the cradle guides taking into account the settings of the cam and the guides.

Coordinate Systems and Coordinate Transformation

Coordinate systems \(S_d\), \(S_e\) and \(S_c\) are rigidly connected to the guides and the cradle (Fig. A.4.1(a)). The guides after rotation about \(O_c\) form angle \(\alpha\) with the \(y_c\)-axis.
Coordinate systems $S_q$, $S_n$ and $S_m$ are rigidly connected to the cam. The settings of the cam with respect to $S_m$ are determined by $\Delta T$ and angle $\alpha$.

The cam and the cradle perform rotations about $O_m$ and $O_f$, respectively (Fig. A.4.2). The conditions of continuous tangency mean that the designed cam and the guides have a common normal and a common position vector at every instant in $S_f$.

A current point $N$ of the guide is determined in $S_f$ with the equation (Fig. A.4.1 and Fig. A.4.4):

\[
\vec{r}_f^{(1)} = [M_{fc}][M_{oe}][M_{od}]\vec{r}_d
\]  
(A.31)

where

\[
\vec{r}_d = [-b - \lambda 0 1]^T
\]  
(A.32)

The unit normal is determined in $S_f$ as follows

\[
\vec{n}_f^{(1)} = [L_{fc}][L_{oe}][L_{od}]\vec{n}_d
\]  
(A.33)

where

\[
\vec{n}_d = [1 0 0]^T
\]  
(A.34)

Here:
A current point of the cam and the unit normal at this point are represented in $S_f$ by the equations (Fig. A.4.1(b), Fig. A.4.2).

\[
[M_{fc}] = \begin{bmatrix}
\cos \theta_c^* & -\sin \theta_c^* & 0 & 0 \\
\sin \theta_c^* & \cos \theta_c^* & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  
(A.35)

\[
[M_{ce}] = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & -E \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  
(A.36)

\[
[M_{cd}] = \begin{bmatrix}
\cos \alpha & \sin \alpha & 0 & 0 \\
-\sin \alpha & \cos \alpha & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  
(A.37)

\[
\vec{r}_f^{(2)} = [M_{fp}^*][M_{pm}^*][M_{mn}^*][M_{nq}^*]\vec{r}_q
\]  
(A.38)

\[
\vec{n}_f^{(2)} = [L_{fp}^*][L_{pm}^*][L_{mn}^*][L_{nq}^*]\vec{n}_q
\]  
(A.39)

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Equations (A.38) and (A.23) yield

\[ \tau_f^{(2)} = [M_{fp}^*][M_{pm}^*][M_{mn}^*][M_{rq}^*][M_{qp}][M_{pf}][M_{fc}][\bar{r}_c] \]  

(A.40)

where

\[ \bar{r}_c = [-b - u 0 1]^T \]  

(A.41)

Matrices \([M_{qp}], [M_{pf}]\) and \([M_{fc}]\) have been represented by equations (A.24), (A.25) and (A.26), respectively. Matrices \([M_{fp}^*], [M_{pm}^*], [M_{mn}^*]\) and \([M_{rq}^*]\) are represented as follows (Fig. A.4.1(b), Fig. A.4.2):

\[
[M_{fp}^*] = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & -E \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]  

(A.42)

\[
[M_{pm}^*] = \begin{bmatrix}
\cos \theta_q^* & -\sin \theta_q^* & 0 & 0 \\
\sin \theta_q^* & \cos \theta_q^* & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]  

(A.43)

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Equations (A.39) and (A.27) yield

$$\tilde{n}_f^{(2)} = [L_{jp*}] [L_{pm*}] [L_{mn*}] [L_{nq}] [L_{pq}] [L_{pf*}] [L_{fc}] \tilde{n}_c$$

(A.46)

Here:

$$[\tilde{n}_c] = [1 \ 0 \ 0]$$

(A.47)

Matrices $[L^*]$ and $[L]$ are $3 \times 3$ submatrices of matrices $[M^*]$ and $[M]$.

Equations of Tangency

The tangency of cam and guides with modified settings is represented by equations

$$\tilde{\tau}_f^{(1)}(\theta_c^*, \theta_c, \alpha) = \tilde{\tau}_f^{(2)}(\theta_q^*, \theta_q, \Delta T, \alpha)$$

(A.48)
\[ \hat{p}_{ij}^{(1)}(\theta^*_c, \theta^*_c, \alpha) = \hat{p}_{ij}^{(2)}(\theta^*_q, \theta^*_q, \alpha) \]  

(A.49)

We recall that vector equations (A.48) and (A.49) yield a system of only three independent equations in four unknowns: \( \theta^*_c, \theta^*_q, \theta_c, \) and \( \lambda; \) setting parameters \( \Delta T \) and \( \alpha \) are considered as given; \( \theta_q \) and \( \theta_c \) are related with equation (A.28) and \( \theta_q \) is considered as a generalized parameter. Our goal is to determine the function that relates angles of rotation of the cam and the cradle, \( \phi^*_q \) and \( \phi^*_c, \) and the parameters of settings \( \alpha \) and \( \Delta T, \) i.e. the function

\[ F(\theta^*_q, \theta^*_c, \Delta T, \alpha) = 0 \]  

(A.50)

Equality of Contact Normal–Satisfaction of Equation (A.49)

Equations (A.49), (A.33) and (A.46) yield

\[ [L^*_q][A] = [L^*_p][A][L^*_q][L^*_c] \]  

(A.51)

Then we obtain

\[ [B][A] = [A][c] \]  

(A.52)

Here:
Matrices (A.52) are rotational matrices that describe rotation about axes of the same orientation. This means that we can change the order of co-factor matrices and

\[
[B] = [L_{fc}]^*[L_{pm}] = \begin{bmatrix}
\cos(\theta_q^* - \theta_c^*) & \sin(\theta_q^* - \theta_c^*) & 0 \\
-\sin(\theta_q^* - \theta_c^*) & \cos(\theta_q^* - \theta_c^*) & 0 \\
0 & 0 & 1
\end{bmatrix}
\] (A.53)

\[
[C] = [L_{qp}]^*[L_{fc}] = \begin{bmatrix}
\cos(\theta_q - \theta_c) & -\sin(\theta_q - \theta_c) & 0 \\
\sin(\theta_q - \theta_c) & \cos(\theta_q - \theta_c) & 0 \\
0 & 0 & 1
\end{bmatrix}
\] (A.54)

\[
[A] = [L_{ed}] = [L_{pq}]^* = \begin{bmatrix}
\cos \alpha & \sin \alpha & 0 \\
-\sin \alpha & \cos \alpha & 0 \\
0 & 0 & 1
\end{bmatrix}
\] (A.55)

This yields that

\[
[B][A] = [C][A]
\] (A.56)

This yields that

\[
[C]^{-1}[B][A] = [A]
\] (A.57)

\[
[C]^{-1}[B] = [I]
\] (A.58)

where [I] is a unitary matrix and
\[ [B] = [C] \quad (A.59) \]

Equation (A.59) yields

\[ \theta_q^* - \theta_c^* = -(\theta_q - \theta_c) = \frac{E}{E + r_u} \theta_q \quad (A.60) \]

since

\[ \theta_c = \frac{E}{E + r_u} \theta_q \quad (A.61) \]

Equality of Position Vectors—Satisfaction of Equation (A.48)

Equations (A.48), (A.31) and (A.38) yield

\[ [M_{pm}^*]^{-1}[M_{fp}^*]^{-1}[M_{f_c}^*][M_{c_e}][M_{c_d}]F_d = [M_{mn}^*][M_{na}^*][M_{at}][M_{f_f}][M_{f_c}]F_c \quad (A.62) \]

After transformations we obtain

\[ [Q]F_d = [S]F_c \quad (A.63) \]

Here:
The rotational $3 \times 3$ submatrices of $[Q]$ and $[S]$ are equal due to the equality of contact normals (see equation (A.52). The elements of $[Q]$ and $[S]$ are represented by

$$
\begin{bmatrix}
a_{11} & a_{12} & 0 & a_{14} \\
a_{21} & a_{22} & 0 & a_{24} \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

(A.64)

$$
\begin{bmatrix}
a_{11} & a_{12} & 0 & a_{14}^* \\
a_{21} & a_{22} & 0 & a_{24}^* \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

(A.65)

The rotational $3 \times 3$ submatrices of $[Q]$ and $[S]$ are equal due to the equality of contact normals (see equation (A.52). The elements of $[Q]$ and $[S]$ are represented by

$$
a_{11} = \cos \eta \quad a_{12} = \sin \eta \quad a_{21} = -a_{12} \quad a_{22} = a_{11}
$$

(A.66)

where $\eta = \theta_q^* - \theta_c^* + \alpha$, and

$$
a_{14} = E[-\sin(\theta_q^* - \theta_c^*) + \sin \theta_q^*], \quad a_{24} = E[-\cos(\theta_q^* - \theta_c^*) + \cos \theta_q]
$$

(A.67)

$$
a_{14}^* = -E[\sin(\theta_q - \alpha) - \Delta T \sin \alpha], \quad a_{24}^* = E[\cos(\theta_q - \alpha) - \Delta T \cos \alpha]
$$

(A.68)

Matrix equation (A.62) yields the following system of two linear equations

$$
a_{12}(u - \lambda) + a_{14} - a_{14}^* = 0, \quad a_{22}(u - \lambda) + a_{24} - a_{24}^* = 0
$$

(A.69)
Eliminating \((u - \lambda)\), we obtain

\[(a_{14} - a_{14}^*)a_{22} - (a_{24} - a_{24}^*)a_{11} = 0\]  \hspace{1cm} (A.70)

Equations (A.70), (A.66), (A.67) and (A.68) results in

\[F(\theta^*_q, \theta^*_c, \Delta T, \alpha) = \sin\left[\frac{r_u}{E}(\theta^*_q - \theta^*_c)\right] + \frac{\Delta T}{E}\sin(\theta^*_q - \theta^*_c) - \sin(\theta^*_c - \alpha) - \sin \alpha = 0\]  \hspace{1cm} (A.71)

Equation (A.71) represents in implicit form the displacement function for the cam mechanism with settings \(\alpha\) and \(\Delta T\). It is easy to be verified, that equation (A.71) with \(\Delta T = 0, \alpha = 0\) represents the linear function,

\[\theta_q = \frac{E + r_u}{r_u} \theta_c\]  \hspace{1cm} (A.72)

For Gleason’s grinder, \(E\) is equal to 15 inch. According to Gleason’s practice, the sense of rotation of the cradle and the cam is opposite to the assumption in the derivation in this report. Without loss of generality, by substituting \(\theta^*_q = -\theta^*_q\) and \(\theta^*_c = -\theta^*_c\) with \(E = 15\) in equation (A.71) we obtain the final expression of the relation between \(\theta^*_c\) and \(\theta^*_q\) as follows,

\[\sin(\theta^*_c + \alpha) - \sin \alpha + \frac{\Delta T}{15} \sin(\theta^*_c - \theta^*_q) + \sin \frac{r_u}{15}(\theta^*_c - \theta^*_q) = 0\]  \hspace{1cm} (A.73)
5 Determination of Coefficients of the Taylor’s Series

The determination of the coefficients of the Taylor’s Series for generation motion with modified roll is a lengthy process. Gleason provides its customers with computer program which can select the cams with settings and analyze the effects of the modified roll. However Gleason’s program is a black box with no explanation for the determination of the coefficients of the Taylor’s Series. Valuable contribution to the understanding of Gleason’s program has been made by C.Q. Zheng [5]. For reader’s convenience, a series of derivations are represented in this section, which coincide with the equations in [5] except some printing errors.

In the process of generation, the cam rotates at a constant angular velocity. Without loss of generality, we assume that the cam rotates with unitary velocity, i.e. \( \frac{d\theta_q^*}{dt} = 1 \). Using the procedure discussed in section A.2, we differentiate equation (A.73) five time as follows,

\[
\omega_c^* \cos(\theta_c^* + \alpha) + (\omega_c^* - 1) \left[ \frac{\Delta T}{15} \cos(\theta_c^* - \theta_q^*) + \frac{r_u}{15} \cos \frac{r_u}{15} \sin(\theta_c^* - \theta_q^*) \right] = 0 \quad (A.74)
\]

\[
a_2 \cos(\theta_c^* + \alpha) + (\omega_c^*)^2 \sin(\theta_c^* + \alpha)
+ a_2 \left[ \frac{r_u}{15} \cos \frac{r_u}{15} (\theta_c^* - \theta_q^*) + \frac{\Delta T}{15} \cos(\theta_c^* - \theta_q^*) \right]
- (\omega_c^* - 1)^2 \left[ \left( \frac{r_u}{15} \right)^2 \sin \frac{r_u}{15} (\theta_c^* - \theta_q^*) \right] + \frac{\Delta T}{15} \sin(\theta_c^* - \theta_q^*) = 0 \quad (A.75)
\]

\[
a_3 \cos(\theta_c^* + \alpha) - 3a_2(\omega_c^*)^2 \sin(\theta_c^* + \alpha) - (\omega_c^*) \cos(\theta_c^* + \alpha)
+ a_3 \left[ \frac{r_u}{15} \cos \frac{r_u}{15} (\theta_c^* - \theta_q^*) + \frac{\Delta T}{15} \cos(\theta_c^* - \theta_q^*) \right]
- 3a_2(\omega_c^* - 1)^2 \left[ \left( \frac{r_u}{15} \right)^2 \sin \frac{r_u}{15} (\theta_c^* - \theta_q^*) \right] + \frac{\Delta T}{15} \sin(\theta_c^* - \theta_q^*) = 0 \quad (A.76)
\]
\[ a_4 \cos(\theta_c^* + \alpha) - 4a_3 \omega_c^* \sin(\theta_c^* + \alpha) - 3a_2^2 \sin(\theta_c^* + \alpha) \]

\[-6(\omega_c^*)^2a_2 \cos(\theta_c^* + \alpha) + (\omega_c^*)^4 \sin(\theta_c^* + \alpha) \]

\[-4a_3(\omega_c^* - 1)[(\frac{r_u}{15})^2 \sin \frac{r_u}{15} (\theta_c^* - \theta_q^*) + \frac{\Delta T}{15} \cos(\theta_c^* - \theta_q^*)] \]

\[-3a_2^2[(\frac{r_u}{15})^2 \sin \frac{r_u}{15} (\theta_c^* - \theta_q^*) + \frac{\Delta T}{15} \sin(\theta_c^* - \theta_q^*)] \]

\[+ a_4[\frac{r_u}{15} \cos \frac{r_u}{15} (\theta_c^* - \theta_q^*) + \frac{\Delta T}{15} \cos(\theta_c^* - \theta_q^*)] \]

\[-6a_2(\omega_c^* - 1)^2[(\frac{r_u}{15})^3 \cos \frac{r_u}{15} (\theta_c^* - \theta_q^*) + \frac{\Delta T}{15} \cos(\theta_c^* - \theta_q^*)] \]

\[(\omega_c^* - 1)^3[(\frac{r_u}{15})^4 \sin \frac{r_u}{15} (\theta_c^* - \theta_q^*) + \frac{\Delta T}{15} \sin(\theta_c^* - \theta_q^*)] = 0 \quad (A.77) \]

\[ a_5 \cos(\theta_c^* + \alpha) - [10a_4(\omega_c^*)^2 + 15a_2^2 \omega_c' + (\omega_c^*)^5] \cos(\theta_c^* + \alpha) \]

\[-[5a_4\omega_c^* + 10a_2a_3 - 10a_2(\omega_c^*)^3] \sin(\theta_c^* + \alpha) \]

\[+ a_5[\frac{r_u}{15} \cos \frac{r_u}{15} (\theta_c^* - \theta_q^*) + \frac{\Delta T}{15} \cos(\theta_c^* - \theta_q^*)] \]

\[-5a_4(\omega_c^* - 1)[(\frac{r_u}{15})^2 \sin \frac{r_u}{15} (\theta_c^* - \theta_q^*) + \frac{\Delta T}{15} \sin(\theta_c^* - \theta_q^*)] \]

\[-10a_3(\omega_c^* - 1)^2[(\frac{r_u}{15})^3 \cos \frac{r_u}{15} (\theta_c^* - \theta_q^*) + \frac{\Delta T}{15} \cos(\theta_c^* - \theta_q^*)] \]

\[-10a_2a_3[(\frac{r_u}{15})^2 \sin \frac{r_u}{15} (\theta_c^* - \theta_q^*) + \frac{\Delta T}{15} \sin(\theta_c^* - \theta_q^*)] \]

\[-15a_2^2(\omega_c^* - 1)[(\frac{r_u}{15})^3 \cos \frac{r_u}{15} (\theta_c^* - \theta_q^*) + \frac{\Delta T}{15} \cos(\theta_c^* - \theta_q^*)] \]

\[+ 10a_2(\omega_c^* - 1)^3[(\frac{r_u}{15})^4 \sin \frac{r_u}{15} (\theta_c^* - \theta_q^*) + \frac{\Delta T}{15} \sin(\theta_c^* - \theta_q^*)] \]

\[+(\omega_c^* - 1)^5[(\frac{r_u}{15})^5 \cos \frac{r_u}{15} (\theta_c^* - \theta_q^*) + \frac{\Delta T}{15} \cos(\theta_c^* - \theta_q^*)] = 0 \quad (A.78) \]

At \( \theta_c^* = \theta_q^* = 0 \), we can determine \( \omega_c^*, a_2, a_3, a_4, a_5 \) from above expressions as follows,

\[ \omega_c^* = \frac{r_u + \Delta T}{15 \cos \alpha + r_u + \Delta T} \quad (A.79) \]
\[ a_2 = \frac{15 \sin \alpha}{15 \cos \alpha + r_u + \Delta T} (\omega_c^*)^2 \]  
\[ a_3 = \frac{3a_2 \omega_c^* \sin \alpha + (\omega_c^*)^3 \cos \alpha + (\omega_c^* - 1)^3 (\frac{r_u^3}{15^3} + \frac{\Delta T}{a_5})}{\cos \alpha + \frac{r_u + \Delta T}{15}} \]  
\[ a_4 = \frac{6a_2 (\omega_c^*)^2 \cos \alpha + [4a_3 \omega_c^* + 3a_2^2 - (\omega_c^*)^4] \sin \alpha + 6a_2 (\omega_c^* - 1)^2 (\frac{r_u^3}{15^3} + \frac{\Delta T}{a_5})}{\cos \alpha + \frac{r_u + \Delta T}{15}} \]  
\[ a_5 = \frac{1}{\cos \alpha + \frac{r_u + \Delta T}{15}} \{[10a_3 (\omega_c^*)^2 + 15a_2^2 \omega_c^* - (\omega_c^*)^5] \cos \alpha \] 
\[ + [5a_4 \omega_c^* + 10a_2 a_3 - 10a_2 (\omega_c^*)^3] \sin \alpha + [10a_3 (\omega_c^* - 1)^2 \] 
\[ + 15a_2^2 (\omega_c^* - 1)^2 (\frac{r_u^3}{15^3} + \frac{\Delta T}{a_5}) - (\omega_c^* - 1)^3 (\frac{r_u^3}{15^3} + \frac{\Delta T}{a_5}) \} \]  

Using equation (A.8) and (A.10) - (A.12), we obtain

\[ R_{ac} = \frac{1}{\omega_c^*} = 1 + \frac{15 \cos \alpha}{r_u + \Delta T} \]  
\[ 2C = \frac{R_{ac} - 1}{R_{ac}} \tan \alpha \]  
\[ 6CX = \frac{1 + 3(2C) \tan \alpha + \frac{(1 - R_{ac})^3}{15 \cos \alpha} (\frac{r_u^3}{15^3} + \Delta T)}{1 + \frac{r_u + \Delta T}{15 \cos \alpha}} \]  
\[ 24DX = \frac{1}{\cos \alpha + \frac{r_u + \Delta T}{15}} \{6(2C) \cos \alpha + [4(6CX) + 3(2C)^2 - 1] \sin \alpha \] 
\[ + 6(2C) (1 - R_{ac})^2 (\frac{r_u^3}{15^3} + \frac{\Delta T}{15}) \} \]
120EX = \frac{1}{\cos \alpha + \frac{\Delta T}{15}} \{[10(6CX) + 15(2C)^2 - 1] \cos \alpha \\
+ [5(24DX) + 10(2C)(6CX) - 10(2C)] \sin \alpha \\
+ [10(6CX)(1 - R_{ac})^2 + 15(2C)^2(1 - R_{ac}) \left( \frac{r_u^5}{15^5} + \frac{\Delta T}{15} \right) \\
- (1 - R_{ac})^5 \left( \frac{r_u^5}{15^5} + \frac{\Delta T}{15} \right) \} \tag{A.88}

Knowing \( R_{ac}, 2C, 6CX, 24DX \) and 120EX, we can determine 6D, 24E and 120F by equations (A.13) – (A.15).

6 Selection of Cams and Cam Settings

In order to provide the desired low transmission errors and bearing contact, the ratio of roll \( R_{ap} \) and second ratio of roll \( (2c) \), which are determined by the local synthesis, must be applied for the grinder. Due to the structure of Gleason’s grinder, the ratio of roll, \( R_{ap} \) is related to \( R_{ac} \) as follows,

\[ R_{ac} = m_i m_c R_{ap} \tag{A.89} \]

Here, \( R_{ac} \) is the transmission ratio between the cam and the cradle, as determined by equation (A.84). \( m_c \) is a fixed gear ratio and is equal to 1 in Gleason’s grinder; \( m_i \) is the gear ratio from the workpiece to the cam and is determined as,

\[ m_i = \frac{n}{n_i} \tag{A.90} \]
here, \( n \) is the number of teeth of the workpiece and \( n_i \) is the index internal, i.e. the gear tooth number skipped over in indexing.

From equation (A.80) and (A.89), we obtain

\[
\alpha = \tan^{-1}\left(\frac{(2c)R_{ac}}{R_{ac} - 1}\right)
\]

\[
r_u + \Delta T = \frac{15 \cos \alpha}{R_{ac} - 1}
\]

The cams and their pitch radii \( r_u \) are tabulated. A cam with pitch radius closest to \( (r_u + \Delta T) \) calculated by equation (A.92) should be selected. After the cam with pitch radius \( r_u \) is selected the corresponding setting, \( \Delta T \), can then be determined as:

\[
\Delta T = \frac{15 \cos \alpha}{R_{ac} - 1} - r_u
\]

In some cases, it is also necessary to control 6\( CX \). In order to satisfy \( R_{ac}, 2C \) and 6\( CX \), the value of \( n_i \) can be used together with \( \Delta T \) and \( \alpha \). Since \( n_i \) must be an integral number it is difficult to obtain an accurate solution. But by careful selection of cams and index interval \( n_i \), a practical engineering solution is often achievable.

When the cam and its settings are selected, it is then necessary to determine the coefficients of the Taylor's Series of the generation motion and carry out the TCA to see how the higher order coefficients (i.e., \( 6D, 24E \) and \( 120F \)) affect the transmission errors and bearing contact. If the result of TCA are satisfactory, then the gears can be ground by the selected cam and cam settings.
Appendix B

Description of Program and Numerical Example

Input and Output of Program

The research project is complemented by a computer program, which can be used for the determination of machine tool settings through the method of local synthesis and simulate the transmission errors and bearing contact through TCA. The input data to the program include four parts.

Part 1. Blank Data

TN1: pinion number of teeth
TN2: gear number of teeth
C: shaft offset (zero for spiral bevel gear)
FW: width of gear
GAMMA: shaft angle
MCD: mean Cone distance
RGMA1: pinion root cone angle
B1: pinion spiral angle
B2: gear spiral angle
RGMA2: gear root cone angle
FGMA2: gear face cone angle
PGMA2: gear pitch cone angle
D2R: gear root cone apex beyond pitch apex
D2F: gear face cone apex beyond pitch apex
ADD2: gear mean addendum
DED2: gear mean dedendum
WD: whole depth
CC: clearance
DEL: elastic approach (experiment datum)

Part 2. Cutter Specifications
RU2: gear nominal cutter radius
PW2: point width of gear cutter
ALP2: blade angle of gear cutter

Part 3. Parameters of Synthesis Condition
FI21: derivative of transmission ratio, negative for gear convex side and positive for gear concave side. The range is $-0.008 < FI21 < 0.008$.
KD: percentage of the half long axes of contact ellipse over face width. $KD = 0.15 - 0.20$.
ETAG: direction angle of contact path. For right hand gear, $-80^\circ \leq ETAG \leq 0^\circ$ for gear convex side and $-80^\circ \leq ETAG \leq 0^\circ$ for gear concave side; For left hand gear, $0^\circ \leq ETAG \leq 80^\circ$ for gear convex side and $-80^\circ \leq ETAG \leq 0$ for gear concave side. When ETAG is close to zero, the contact path is along the tooth height, when the magnitude is increased, the contact path will have bias in and reach almost longitudinal direction if ETAG is close to 90 degrees.
GAMA1: pinion machine root angle, which is the same as the pinion root angle if no tilt is used.
RHO: radius of the arc blade if curved blade is used, which can be any values when curved blade is not used.
C2: second order ratio of roll if modified roll is used. C2 is zero without modified roll.
ALP1: pinion cutter blade angle. ALP1 is positive for gear convex side and negative for gear concave side. ALP1 can be the same as ALP2. For better result, it is suggested for pinion concave side the magnitude of ALP1 is smaller than ALP2 and for pinion convex side, the magnitude of ALP1 is larger than ALP2. (As shown in the example).

TN1I: number of teeth skipped over indexing. TN1I is only used in modified roll, the ratio between TN1I and TN1 must not be an integer.

Part 2. Control Codes

JCL: JCL control V and H check, $JCL = 1$ means no V-H check.

JCH: For right hand gear, set $JCH = 1$, for left hand gear set $JCH = 2$.

JCC: For straight blade, set $JCC = 1$, for curved blade set $JCC = 2$.

TL1, TL2: Extra points on contact path, both should be less or equal than 2.

The program output includes: (1) the machine-tool settings for gear and pinion; (2) the transmission error; (3) the contact path; (4) the length and orientation of the long axes of the contact ellipse; and bearing contact at toe and heel position.

Numerical Example

The model used in this report is the spiral bevel drive with the shaft angle of 90 degrees. In the numerical example, modified roll and curved blade for generation of gears were not used since favorable results were attained without them. The list of the blank data and machine tool settings are tabulized in the attached tables.

The TCA results with V-H check are shown through Fig. B.1 through Fig. B.6. The V and H values shown in the figures are of $\frac{1}{1000}$ inch. It is shown also that the transmission errors are very small and the bearing contact is stable for both side at the three positions, toe, mean and heel.
BLANK DATA

<table>
<thead>
<tr>
<th></th>
<th>PINION</th>
<th>GEAR</th>
</tr>
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<tbody>
<tr>
<td>NUMBER OF TEETH:</td>
<td>11</td>
<td>41</td>
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<tr>
<td>PRESSURE ANGLE:</td>
<td>20°</td>
<td></td>
</tr>
<tr>
<td>SHAFT ANGLE:</td>
<td>90°</td>
<td></td>
</tr>
<tr>
<td>MEAN SPIRAL ANGLE:</td>
<td>35.0°</td>
<td></td>
</tr>
<tr>
<td>HAND OF SPIRAL:</td>
<td>LF</td>
<td>RH</td>
</tr>
<tr>
<td>OUTER CONE DISTANCE:</td>
<td></td>
<td>90.07</td>
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<tr>
<td>FACE WIDTH:</td>
<td></td>
<td>27.03</td>
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<tr>
<td>WHOLE DEPTH:</td>
<td>8.11</td>
<td>8.11</td>
</tr>
<tr>
<td>CLEARANCE:</td>
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<td>0.81</td>
</tr>
<tr>
<td>ADDENDUM:</td>
<td>5.24</td>
<td>2.061</td>
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<tr>
<td>DEEDENDUM:</td>
<td>2.87</td>
<td>6.05</td>
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<tr>
<td>PITCH ANGLE:</td>
<td>15°1'</td>
<td>74°59'</td>
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<tr>
<td>ROOT ANGLE:</td>
<td>13°20'</td>
<td>70°39'</td>
</tr>
<tr>
<td>FACE ANGLE:</td>
<td>19°21'</td>
<td>76°40'</td>
</tr>
</tbody>
</table>

GEAR CUTTER SPECIFICATIONS

<p>| | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>BLADE ANGLE:</td>
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</tr>
<tr>
<td>CUTTER DIAMETER:</td>
<td>152.40</td>
</tr>
<tr>
<td>POINT WIDTH:</td>
<td>2.79</td>
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</tbody>
</table>
GEAR MACHINE TOOL SETTINGS

RADIAL SETTING($s$): 70.43577
CRADLE ANGLE($q$): 62.3981°
MACHINE CENTER TO BACK($x_G$): 0.00
SLIDING BASE($x_B$): 0.00
RATIO OF ROLL($r_a$): 1.032397
BLANK OFFSET($E_m$): 0.0
MACHINE ROOT ANGLE($\gamma_m$): 70.65°

PINION MACHINE TOOL SETTINGS

<table>
<thead>
<tr>
<th></th>
<th>CONVEX</th>
<th>CONCAVE</th>
</tr>
</thead>
<tbody>
<tr>
<td>CUTTER BLADE ANGLE:</td>
<td>21.5°</td>
<td>18.5°</td>
</tr>
<tr>
<td>CUTTER POINT RADIUS:</td>
<td>80.4876</td>
<td>71.7222</td>
</tr>
<tr>
<td>RADIAL SETTING($s$):</td>
<td>71.55166</td>
<td>69.04316</td>
</tr>
<tr>
<td>CRADLE ANGLE($q$):</td>
<td>59.4638°</td>
<td>64.0624°</td>
</tr>
<tr>
<td>MACHINE CENTER TO BACK($x_G$):</td>
<td>1.08497</td>
<td>-1.58960</td>
</tr>
<tr>
<td>SLIDING BASE($x_B$):</td>
<td>-0.25021</td>
<td>0.36659</td>
</tr>
<tr>
<td>RATIO OF ROLL($r_a$):</td>
<td>3.898097</td>
<td>3.788604</td>
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<tr>
<td>BLANK OFFSET($E_m$):</td>
<td>-2.56862(Up)</td>
<td>2.19033(Down)</td>
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<tr>
<td>MACHINE ROOT ANGLE($\gamma_m$):</td>
<td>13.333°</td>
<td>13.333°</td>
</tr>
</tbody>
</table>
REFERENCES

References


Fig. 1.2.1 Unit Vectors of Principal Directions
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Fig. 1.2.4 Parabolic Type of Transmission Errors
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Fig. 2.1.2 Pinion Generation: Additional Coordinate Systems
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Fig. 2.2.2 Gear Generation: Additional Coordinate Systems
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CAM PROFILE AND ITS MOTION

\[ b = 5.5, \ E = 15, \ \ t_n = 3.6230 \]

Fig. A.3.1 Cam Profiles and Guides
Fig. A.3.2 Schematic of Motions of Cam and Guides without Modified Settings
Fig. A.4.1 Coordinate Systems for Cam Mechanism with Modified Settings
Fig. A.4.2 Transmission of Cam Rotation into Cradle Rotation by Modified Settings
Transmission Error in Meshing Period
Gear Convex Side - Mean

Fig. B.1 Contact Pattern and Transmission Errors at Gear Mean Position of Convex Side
Transmission Error in Meshing Period
Gear Convex Side Toe
V = 9; H = 0.0

Fig. B.2 Contact Pattern and Transmission Errors at Gear Toe Position of Convex Side
Fig. B.3 Contact Pattern and Transmission Errors at Gear Heel Position of Convex Side
Transmission Error in Meshing Period
Gear Concave Side MEAN
\( V = 0; H = 0 \)

Fig. B.4 Contact Pattern and Transmission Errors at Gear Mean Position of Concave Side
Transmission Error in Meshing Period
Gear Concave Side - TOE
V = -11; H = 3

Fig. B.5 Contact Pattern and Transmission Errors at Gear Toe Position
of Concave Side
Transmission Error in Meshing Period
Gear Concave Side - Heel
V = 9 : H = -3

Fig. B.6 Contact Pattern and Transmission Errors at Gear Heel Position of Concave Side
THIS PROGRAM IS TO DERIVE THE MACHINE TOOL SETTINGS FOR PINION GENERATION & TEST THE RESULTS

IMPLICIT REAL*8(A-H,O-Z)
REAL*8 KS,KQ,KF,KH,KFI,KFII,KD,KF,KH,med
REAL*8 M11,M12,M13,L11,L12,L13,L14,N21,N22,M23,L21,L22,L23,L24,
&N11,N12,N21,N22
REAL*8 xi(5),x(5),f(5)
EXTERNAL FCN1,FCN2,FCN,FCNM,FCNR,FCNMR
DIMENSION CH(3),P(3),E1EF(3),ESN(3),EQN(3),W1VT2(3),WV12(3),
$W2VT1(3),EFIIH(3),EHIH(3),RH(3),GNH(3),E2I1H(3),E2IH(3),PI2P(20),
&EI1H(3),EI1IH(3),EI1H(3),EI2H(3),GN(3),EFE1(3),
&ERR(20),xcp(20),ycp(20),AX1(20),AX2(20),ANG1(20),ANG2(20)
COMMON/A1/CNST,TN1,TN2,C,FW,GAMMA,xi,r1,med
COMMON/A2/B1,RGM1,PGM1,D1R,D1F,ADD1,D1D1
COMMON/A3/B2,RGM2,PGM2,D2R,D2F,ADD2,D2D2,WD,CC,D2P
COMMON/A4/SR2,Q2,RC2,PW2,XB2,XG1,EM2,GM3,CR2,ALP2,PHI2,PHI2P
COMMON/A5/SG,XX,YM,ZM,XNH,YNM,ZNM,X2M,Y2M,Z2M,Y2M,ZN2M,
&XNH2,YNH2,ZNH2,XXH,YXH,ZXH
COMMON/A6/ES(3),E0(3),CN(3),W1(3),W2(3),W12(3),VT1(3),VT2(3),
SV12(3),KS,KQ,KH,KF,EF(3),EH(3),SIGSF,PI21
COMMON/A7/SR1,Q1,Ref,PW1,XB1,XG1,EM1,GM1,CR1,ALP1,PHI1,PHI1P
COMMON/A8/SF1,XM1,YM1,ZM1,XNH1,YNM1,ZNH1,XH1,YH1,ZH1
COMMON/A9/PHI2P0,OX,OZ,XO,ZO,RHO,ALP,V,H,CR1T,PCR1T
COMMON/A10/K11,K12,K21,K22,DEL,E11H,E11IH,E2IH,EZH,NH,
&A2P,B2P,TAT1,TAT2,A2L,B2L
COMMON/A11/RAM,PSI1,C2,D6,E24,F120,CX6,DX24,EX120,RU1,DEL,TUP,
SR1,CPF,DPF,EPF,FPF
CNST=DARCOS(-1.0D00)/180.0D00

C
C... INPUT THE CONTROL CODES
C
C  IF V AND H CHECK IS NOT DESIRED, SET JCN = 1
C  DO NOT SET JCN TO BE 3
C
JCN=2
C
FOR RIGHT HAND GEAR JCH=1, FOR LEFT HAND GEAR JCH = 2
C
JCH=1
C
FOR STRAIGHT BLADE JCC=1, FOR CURVED BLADE JCC=2
C
JCC=1
C
TL1 AND TL2 ARE NUMBER OF EXTRA POINT ON CONTACT PATH WHICH SHOULD NOT BE LARGER THAN 2
C
TL1=1.0
TL2=1.0
C
C... INPUT BLANK DATA OF GEAR AND PINION
C
TN1=11.0
TN2=41.0
C=0.0

142
FW=27.03
GAMMA=90.0°CNST
MCD=76.56
RGMA1=13.3333°CNST
B1=35.0°CNST
B2=35.0°CNST
RGMA2=70.6500°CNST
FGMA2=76.6667°CNST
PGMA2=74.9833°CNST
D2R= 0.0
D2F=0.0
ADD2=2.06
DED2=6.05
WD= 8.11
CC=0.81
DEL=0.00025°25.4

C
C... INPUT NORMAL RADIUS OF GEAR CUTTER AND POINT WIDTH, BLADE ANGLE
C
RU2=152.4000/2.0
PW2=2.79
ALP2=20.0°CNST
DC2=2.0°RU2

C
C... INPUT THE SYNTHESIS CONDITION PARAMETERS AND PINION CUTTER BLADE ANGLE, ALP1 (FOR GEAR CONVEX, ALP1>0, FOR GEAR CONCAVE ALP1<0)
C
**** GEAR CONVEX SIDE
C
FI21=-0.0008
KD=0.180
ETAG=-65.0°CNST
GAMA1=13.3333°CNST
RHO= 250.0
C2= 0.00
ALP1=18.500°CNST
TN11=8.0

C...
SGN=DSIN(ALP1)/DABS(DSIN(ALP1))
KSIDE=0
C
C...
C
GOTO 1989
C
**** GEAR CONCAVE SIDE
C
1990 CONTINUE
FI21= 0.0008
KD=0.180
ETAG=65.0°CNST
GAMA1=13.3333°CNST
RHO= 200.0
C2= 0.00
ALP1=-21.50°CNST
TN11=8.0

C...
SGN=DSIN(ALP1)/DABS(DSIN(ALP1))
KSIDE=1
jcl=2
C
C... INPUT GEAR MACHINE TOOL SETTINGS
C
1989 Q2= 52.6589*CNST
SR2=3.8872*25.4
XG2=0.0
XB2=-0.0333*25.4
CR2=.9772974
C
RAG=1.0/CR2
C
GAMA2=RGMA2
C
EM2=0.0
C
RC2=RU2-SGN*PW2/2.0
C
ALP2= SGN*ALP2
C
C
C... CALCULATE GEAR MACHINE TOOL SETTINGS
C
1989 hg=mcD*cos(pgma2-rgma2)-ru2*sin(b2)
vg=ru2*cos(b2)
q2=datan(vg/hg)
sr2=dbsqrt(hg2+vg2)
xg2=0.0
GAMA2=RGMA2
xb2=d2r*sin(gama2)
EM2=0.0
rag=dcos(pgma2-rgma2)/dsin(pgma2)
rc2=1.0/rag
RC2=RU2-SGN*PW2/2.0
ALP2= SGN*ALP2
C
C... DELT IS THE CAM SETING
C
DELT=0.0
C
C... DEFINE THE MEAN CONTACT POINT
C
V=0.000
H=0.000
FA=FGMA2-PGMA2
RA=PGMA2-RGMA2
HM=CC+WD-0.5*FD*DTAN(FA)+DTAN(RA))
DED2=DED2-0.5*FD*DTAN(RA)
XL=MCRDCOS(PGMA2)+(DED2-HM/2.0)*DSIN(PGMA2)
RL=MCRDSIN(PGMA2)-(DED2-HM/2.0)*DCOS(PGMA2)
C
AGL=DATAN(RL/XL)
OX=DSQRT(XL*2+RL*2)*DCOS(AGL-RGMA2)
OY=-D2R*DSIN(RGMA2)
C
WRITE(9,11) OX,OY,XL,RL
C
C
C... FIND SURFACE COORDINATES OF THE MEAN CONTACT POINT
C
ERRREL=0.1D-10
N=2
ITMAX=200
IF (JCH.EQ.1) THEN
Q2=Q2
XI(1)=270.0°+CNST+B2
ELSE
XI(1)=B2
END IF
XI(2)=0.0
CALL DNEQNF(FCN1,ERRORL,N,ITMAX,Xi,X,FNORM)
TH=X(1)
PH=X(2)
ST=DSIN(TH)
CT=DCOS(TH)
SH=DSIN(PH)
CS=DCOS(PH)
SP=DSIN(ALP2)
CP=DCOS(ALP2)
SM=DSIN(GAMA2)
CM=DCOS(GAMA2)
THIG=TH
WRITE(9,11) Xn2M,Yn2M,Zn2M,ZNM,YNM,ZNM
WRITE(9,11) XM,YM,ZM,sg,hm
DEFINE VECTORS TO COMPUTE THE SECOND ORDER PROPERTY OF GEAR
C
ES(1)=DSIN(TH-PH)
ES(2)=-DCOS(TH-PH)
ES(3)=0.0
EQ(1)=SP*DCOS(TH-PH)
EQ(2)=-SP*DSIN(TH-PH)
EQ(3)=CP
CN(1)=XNM
CN(2)=YNM
CN(3)=ZNM
KS=CP/(RC2-SG*SP)
KQ=0.0
W1(1)=-CM
W1(2)=0.0
W1(3)=-SM
W2(1)=0.0
W2(2)=0.0
W2(3)=-CR2
VT1(1)=YM*SM+EM2*SM
VT1(2)=XM*SM+ZM-XB2*CM
VT1(3)=YM*CK-EM2*CM
VT2(1)=YM*CR2
VT2(2)=-XM*CR2
VT2(3)=0.0
DO 10 I=1,3
W12(I)=W1(I)-W2(I)
VT2(I)=VT1(I)-VT2(I)
10 CONTINUE
C
C... FIND THE PRINCIPAL DIRECTION AND CURVATURES AT MEAN POINT
C
PI21=0.0
CALL CURVAL
C
WRITE(9,12) KF,KH,SIGSF
C 12 FORMAT(3X,3(C14.7,2X))
K21=KF
K211=KH
PH21=PH/CR2
sh2=dsin(phi2)
\[ \text{ch2} = \cos(\text{phi2}) \]
\[ xX = \cos(e1) + \sin(e3) \]
\[ yY = \sin(e2) \]
\[ zZ = -\sin(e1) + \cos(e3) \]
\[ e1 = xX \]
\[ e2 = \text{CH2} \times \cos(yY) - \text{SH2} \times zZ \]
\[ e3 = \text{SH2} \times \cos(yY) + \text{CH2} \times zZ \]
\[ \text{WRITE} (9, 11) \; \text{xx}, \text{yy}, \text{zz} \]
\[ xX = \cos(e1) + \sin(e3) \]
\[ yY = \sin(e2) \]
\[ zZ = -\sin(e1) + \cos(e3) \]
\[ e1 = xX \]
\[ e2 = \text{CH2} \times \cos(yY) - \text{SH2} \times zZ \]
\[ e3 = \text{SH2} \times \cos(yY) + \text{CH2} \times zZ \]
\[ \text{WRITE} (9, 11) \; e1, e2, e3 \]
\[ \text{WRITE} (9, 11) \; \text{eh1}, \text{eh2}, \text{eh3} \]
\[ \text{CHP} = \cos(x1) \]
\[ \text{SHP} = \sin(x1) \]
\[ \text{CM} = \cos(G) \]
\[ \text{SM} = \sin(G) \]
\[ XX = e1 \]
\[ YY = -e2 \times \cos(e3) + e3 \times \sin(e3) \]
\[ ZZ = -e1 \times \cos(e3) - e3 \times \sin(e3) \]
\[ EF(1) = XX \times \cos(ZZ) + ZZ \times \sin(ZZ) \]
\[ EF(2) = YY \]
\[ EF(3) = -XX \times \sin(ZZ) - ZZ \times \cos(ZZ) \]
\[ \text{ETAG} = \text{CNST} + \text{SIGSF} + \text{ETAG} \]
\[ \text{LOCAL SYNTHESIS AT MEAN CONTACT POINT} \]
\[ \text{RH(1)} = \text{XH2} \]
\[ \text{RH(2)} = \text{YH2} \]
\[ \text{RH(3)} = \text{ZH2} \]
\[ \text{GNH(1)} = \text{XNH2} \]
\[ \text{GNH(2)} = \text{YNH2} \]
\[ \text{GNH(3)} = \text{ZNH2} \]
RELATIVE MOTION PARAMETERS IN GEAR & PINION MESHING PROCESS

\[
R12 = \frac{TN1}{TN2}
\]

\[
W1(1) = -1.000
\]

\[
W1(2) = 0.000
\]

\[
W1(3) = 0.000
\]

\[
W2(1) = R12 \times CMM
\]

\[
W2(2) = 0.000
\]

\[
W2(3) = -R12 \times SMM
\]

\[
w12(1) = W1(1) - W2(1)
\]

\[
w12(2) = W1(2) - W2(2)
\]

\[
w12(3) = W1(3) - W2(3)
\]

\[
VT1(1) = 0.000
\]

\[
VT1(2) = ZH2
\]

\[
VT1(3) = -YH2
\]

\[
VT2(1) = R12 \times (YH2 - C) \times SMM
\]

\[
VT2(2) = -R12 \times (XH2 \times SMM + ZH2 \times CMM)
\]

\[
VT2(3) = R12 \times (YH2 - C) \times CMM
\]

\[
v12(1) = VT1(1) - VT2(1)
\]

\[
v12(2) = VT1(2) - VT2(2)
\]

\[
v12(3) = VT1(3) - VT2(3)
\]

\[
WRITE(9,3) \text{ V12(1), V12(2), V12(3)}
\]

C3 FORMAT(5X,3G14.7,/)
W1TN=0.0D00
W2TN=0.0D00
VT2N=0.0D00

DO 1 I=1,3
V12S= V12(I)*EZIH(I)+V12S
V12Q= V12(I)*EZIIH(I)+V12Q
WNES= W12(I)*ESN(I)+WNES
WNEQ= W12(I)*EIQ(I)+WNEQ
VWN =GNH(I)*WV12(I)+VWN
W1TN=GNH(I)*WVT2(I)+W1TN
W2TN=GNH(I)*WVT1(I)+W2TN
VT2N = GNH(I)*VT2(I)+VT2N
1 CONTINUE
C WRITE(9,6) V12S,V12Q
C 6 FORMAT(5X,2G14.7,/) C...
C...
C...

COMPUTER THE COEFFICIENTS A13,A23,A33
C...

B13=-K2I*V12S-WNES
B23=-K2II*V12Q-WNEQ
B33=K1I*V12S**2+K2II*V12Q**2-VWN-W1TN+W2TN+VT2N*FI21*TN2/TN1
C...

LOCAL SYNTHESIS OF MESHING AT MEAN CONTACT POINT
C...

DL=KD*FW
SIGK2= K2I+K2II
SIGG2= K2I-K2II
A=DEL/DL**2
T1=-B13*V12Q+(B33+B13*V12S)*DTAN(ETAP)
T2=B33+B13*(V12Q-V12S)*DTAN(ETAP)
ETAP=DATAN(T1/T2)
VS1 =B33/(B13+B23*DTAN(ETAP))
AM1=DSIN(ETAP)**2
AM2=-DSIN(2.0D00*ETAP)/2.0D00
AN1=(B13-B23*DTAN(ETAP))/(1.0D00+DTAN(ETAP)**2)*VS1
AN2=(B13*DTAN(ETAP)+B23)/(1.0D00+DTAN(ETAP)**2)*VS1
SGN=DSIN(ALP1)/DABS(DSIN(ALP1))
A=A*SGN
SIGK=-(4.0D00*A**2-(AN1**2+AN2**2))/2.0D00*A-(AN1*DCOS(2.0D00*
&ETAP)+AN2*DSIN(2.0D00*ETAP)))
SIGK1= SIGK2-SIGK
T1= 2.0D00*AN2-SIGK*DSIN(2.0D00*ETAP)
T2=SIGK2-2.0D00*AN1+SIGK*DCOS(2.0D00*ETAP)
SIG12=.50D00*DATAN(T1/T2)
SIGG1=(2.0D00*AN2-SIGK*DSIN(2.0D00*ETAP))/DSIN(2.0D00*SIG12)
KII=(SIGK1+SIGG1)/2.0D00
KIII=(SIGK1-SIGG1)/2.0D00
C...
C...

WRITE(9,11) ETAP,KII,KIII
C WRITE(9,11) SIGK,SIGG2,SIGK1,SIGG1
C WRITE(9,8) T1,T2
C 8 FORMAT(5X,3G14.7)
C...
C...

PRINCIPLE DIRECTIONS OF PINION SURFACE AT POINT M
C...
DO 15 I=1,3
E11H(I)= DCOS(SIG12)*E21H(I)-DSIN(SIG12)*E21H(I)
E11H(I)= DSIN(SIG12)*E21H(I)+DCOS(SIG12)*E21H(I)
15 CONTINUE
C WRITE(9,11) E11H(1),E11H(2),E11H(3)
C WRITE(9,11) E11IH(1),E11IH(2),E11IH(3)
C...
C... COINCIDE THE NORMALS OF CUTTER AND THE PINION SURFACES
C...
SM1=DSIN(GAMA1)
CM1=DCOS(GAMA1)
SP=DSIN(ALP1)
CP=DCOS(ALP1)
T1=-(XNH2+SP*SM1)
T2= CP*CM1
IF (JCH.EQ.1) THEN
THF=DARCOS(T1/T2)
ELSE
THF=360.0*CNST-THF
END IF
BA1=-CP*DSIN(THF)
BA2= CP*DSIN(GAMA1)*DCOS(THF)-SP*DCOS(GAMA1)
TT=-(XNH2*ZNH2+ZNH2**2)/T1
CSH=-(BA1*YNH2+BA2*ZNH2)/TT
SNH=(BA2*YNH2-BA1*ZNH2)/TT
PHIH=2.0*DATAN2(SNH,(1.0D00+CSH))
C WRITE(9,8) THF,PHIH
C...
C... FIND THE PRINCIPAL DIRECTIONS OF PINION GENERATING SURFACE
C
EFI(1)=-DSIN(THF)
EFI(2) = DCOS(THF)
EFI(3) = 0.0D00
EFI1(1)= SP*DCOS(THF)
EFI1(2) = SP*DSIN(THF)
EFI1(3)=-CP
C WRITE(9,11) EFI(1),EFI(2),EFI(3)
C WRITE(9,11) EFI1(1),EFI1(2),EFI1(3)
C...
C... FIND THE PINION PRINCIPAL DIRECTIONS IN SYSTEM SM1
C
XX= E11H(1)
YY= DCOS(PHIH)*E11H(2)+DSIN(PHIH)*E11H(3)
ZZ=DSIN(PHIH)*E11H(2)+DCOS(PHIH)*E11H(3)
E11(1)= CM1*XX-SM1*ZZ
E11(2)= YY
E11(3)= SM1*XX+CM1*ZZ
C
XX= E11IH(1)
YY= DCOS(PHIH)*E11IH(2)+DSIN(PHIH)*E11IH(3)
ZZ=DSIN(PHIH)*E11IH(2)+DCOS(PHIH)*E11IH(3)
E11I(1)= CM1*XX-SM1*ZZ
E11I(2)= YY
E11I(3)= SM1*XX+CM1*ZZ
C...
C... FIND THE UNIT NORMAL IN SYSTEM SM1
C
XX= XNH2
YY= DCOS(PHIH)*YNH2+DSIN(PHIH)*ZNH2

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EXPRESS THE POSITION VECTOR IN SM1

XX = XH2
YY = DCOS(PHIH)*YH2+DSIN(PHIH)*ZH2
ZZ = DSIN(PHIH)*YH2+DCOS(PHIH)*ZH2
RX = CM1*XX-SM1*ZZ
RY = YY
RZ = SM1*XX+CM1*ZZ

XX = CP*DCOS(THF)
YY = CP*DSIN(THF)
ZZ = SP

WRITE(9,11) EFI(I),EII(1),EII(2),EII(3)
WRITE(9,11) EIII(1),EIII(2),EIII(3)
WRITE(9,11) GN(1),GN(2),GN(3)
WRITE(9,11) XX,YY,ZZ
WRITE(9,11) RX,RY,RZ

DO 20 I=1,3
XX = EFI(I)
EII(I) = EFI(I)
EIII(I) = XX
20 CONTINUE

WRITE(9,11) EFI(I),EII(1),EII(2),EII(3)
WRITE(9,11) EIII(1),EIII(2),EIII(3)

FIND THE ANGLE FORMED BETWEEN PRINCIPAL CURVATURES

EFE1(1) = EFI(2)*EII(3)-EII(3)*EII(2)
EFE1(2) = -EFI(1)*EII(3)+EII(3)*EII(1)
EFE1(3) = EFI(1)*EII(2)-EII(2)*EII(1)
T1 = 0.0D00
T2 = 0.0D00
DO 30 I=1,3
T2 = EFI(I)*EII(I)+T2
T1 = GN(I)*EFE1(I)+T1
30 CONTINUE
SIGF1 = 2.0D00*DATAN(T1,1.0+T2)

FIND THE CURVATURE OF PINION GENERATION SURFACE AT MEAN POINT

IF (JCC.EQ.1) THEN
KFI = 0.0
B12 = 0.5D00*(KII-K111)*DSIN(-2.0D00*SIGF1)
B11 = KFI-K111*DCOS(SIGF1)**2-K111*DSIN(SIGF1)**2
TKK = K111*DSIN(SIGF1)**2+K111*DCOS(SIGF1)**2
KFI1 = (B12**2+B11*TKK)/B11
WRITE(9,11) SIGF1,KFI1

FIND THE CUTTER POINT RADIUS AND ITS CENTER

SF = SG*DCOS(ALP2)/DCOS(ALP1)
SF = DABS(RZ)/DCOS(ALP1)
RCF = CP/DABS(KFI1)-SF*SP
DO 40 I=1,3
P(I) = GN(I) * CP - EFI(I) * SP

CONTINUE
RCX = RX - SF * EFI(1) + RCF * P(1)
RCY = RY - SF * EFI(2) + RCF * P(2)
RCZ = RZ - SF * EFI(3) + RCF * P(3)

WRITE (9, 11) RCF, SF
WRITE (9, 11) RCX, RCY, RCZ
ELSE


KFI = 1.0 / RHO
B12 = 0.5 * 0.0001 *(KII - K11) * DSIN (-2.0 * 0.0001 * SIGF1)
B11 = KFI - K11 * DCOS (SIGF1) ** 2 - K111 * DSIN (SIGF1) ** 2
TKK = K111 * DSIN (SIGF1) ** 2 + K111 * DCOS (SIGF1) ** 2
KFI = (B12 ** 2 * B11 ** T) / B11
WRITE (9, 11) SIGF1, KFI
DBT = -RZ
RM = CP / DABS (KFI)
ZO = -(DBT + RHO * SP)
XO = RM - RHO * CP
RCF = XO + RHO * DSQRT (1.0 - (ZO / RHO) ** 2)
RCX = RHO * GN(1) - XO * DCOS (THF) + RX
RCY = RHO * GN(2) - XO * DSIN (THF) + RY
RCZ = RHO * GN(3) - ZO + RZ
WRITE (9, 777) XO, ZO
WRITE (9, 11) SIGF1, KFI
WRITE (9, 11) RCF, RM, DBT
WRITE (9, 11) RCX, RCY, RCZ
END IF

THE FOLLOWING IS TO FIND THE CUTTING RATIO

CSM = CM
SM = SM
T1X = EFI(1)
T1Y = EFI(2)
T1Z = EFI(3)
T2X = EFI(1)
T2Y = EFI(2)
T2Z = EFI(3)
XN = GN(1)
YN = GN(2)
ZN = GN(3)
RCX = RX
RCY = RY
RCZ = EZ
WRITE (9, 11) RCX, RCY, RCZ
WRITE (9, 11) T1X, T1Y, T1Z
WRITE (9, 11) T2X, T2Y, T2Z
WRITE (9, 11) XN, YN, ZN
WRITE (9, 11) RCX, RCY, RCZ
THE FOLLOWING IS TO DETERMINE DELTA, EM, AND IFM

M11 = XN * T1Y - YN * T1X
M12 = -CSM * (YN * T1Z - ZN * T1Y)
M21 = XN * T2Y - YN * T2X
M22 = -CSM * (YN * T2Z - ZN * T2Y)
WRITE (9, 11) M11, M12, M21, M22
L11 = (B12/B11*M11 - M21) / KF1I
L12 = (B12/B11*M12 - M22) / KF1I

L21 = -TZZ/T1Z*L11
L22 = -TZZ/T1Z*L12 - RYC*CSM1 / T1Z
DTT* B12*KFI*TZZ + B11*KFI*T1Z
L11 = -T1Z*(B11*M21 - B12*M11) / DTT
L12 = (-B12*KFI*RYC*CSM1 - T1Z*(B11*M22 - B12*M12)) / DTT
L21 = TZZ*(B11*M21 - B12*M11) / DTT
L22 = (-B12*KFI*RYC*CSM1 + TZZ*(B11*M22 - B12*M12)) / DTT

WRITE(^,^^)
L11, L12, L21, L22

X11 = L21*T1X + L11*T2X
X12 = L22*T1X + L12*T2X
X21 = L21*T1Y + L11*T2Y
X22 = L22*T1Y + L12*T2Y
X31 = L21*T1Z + L11*T2Z
X32 = L22*T1Z + L12*T2Z

E11 = YN*X11 - XN*X21
E12 = YN*X12 - XN*X22 - ZN*X21*CSM1 + YN*X31*CSM1
E13 = -(ZN*X22 - YN*X32) * CSM1
Y11 = XN*(RXC*SNM1 - RZC*CSM1) - YY*RYC*SNM1
Y12 = YY*SNM1
X13 = X12 - RYC*SNM1
X14 = X22 + RYC*SNM1 - RZC*CSM1
X33 = X32 + RYC*CSM1
Y21 = XN*X21*SNM1 + ZN*X21*CSM1 + YN*(X11*SNM1 - X31*CSM1)

THE EFFECT OF SECOND ORDER RATIO OF ROLL ON A33

TM1 = XN*X11 + YN*X21 + ZN*X31
TM2 = XN*X13 + YN*X23 + ZN*X33

WRITE(9,163) TM1, TM2

C163 FORMAT(2X, 'TM1, TM2 ', 2(2X, G14.7))

Z1 = C2*TM1
Z2 = C2*(TM2*SNM1 * TM1)
Z3 = C2*SNM1 * TM2

WRITE(^,^^)
B12, Z1, M11, N11

AA = B12*Z1 - N21*N11
BB = B12*Z2 - N21*N12 - N22*N11
CC = B12*Z3 - N22*N12

WRITE(9,11) AA, BB, CCC
IF (AA.GT. 0.000001) GOTO 1949
T1 = -CCC/BB
GOTO 1950

1949 T1 = (-BB + DSQRT(BB**2 - 4.0D00*AA*CCC)) / (2.0D00*AA)
1950 FM1 = T1*SNM1
CR1=FM1
RAP=1.0/CR1
VF3=X31*T1+X32+RCY*CSM1

THE DETERMINATION OF E\(M\) AND DELTA

E\(M\)=\((X11*T1+RCY*FM1+X13)/FM1
XG1=(X21*T1-RCX*FM1+X23)/(FM1*CSM1)

RCX=RCX+XG1*CSM1
RCY=RCY-EM1
RCZ=RCZ+XG1*SNM1

V1=RCY
H1=RCX
XB1=RCZ
SR1=DSQRT(V1**2+H1**2)
Q1=-DARSIN(V1/SR1)

DETERMINE THE CAM SETTING

RA1=1.0/CR1
RAM=TN1/TN1*RA1
PSI1=DATAN(C2*RAM/(RAM-1.0))
RUP=15.0*DCOS(PSI1)/(RAM-1.0)
RU1=RUP

CALL CAM
WRITE(9,191) RUP
WRITE(9,191) RU1
C 191 FORMAT(15,2X,2(2X,G14.7))
WRITE(9,199) RA1,C2,D6,E24,F120
WRITE(9,199) RA1,CPF,DPF,EPF,FPF
C 199 FORMAT(2X,'CAM1
XGl,XBl,Vl,Hl
FM1=
EM1,SR1,Q1,3(3X,G14.7))
WRITE(9,45) XGl,XBl,Vl,Hl

IF (KSIDE.EQ.0.0) THEN
WRITE(9,131)
131 FORMAT(/2X,'********************************************************************'),/
& 2X,'* OUTPUT FOR GEAR CONVEX SIDE',/
& 2X,'********************************************************************'),/
ELSE
WRITE(9,331)
331 FORMAT(/2X,'********************************************************************'),/
& 2X,'* OUTPUT FOR GEAR CONCAVE SIDE',/
& 2X,'********************************************************************'),/
END IF
WRITE(9,13)
WRITE(9,115) DC2,PW2,ALP2
115 FORMAT(/2X,'GEAR CUTTER SPICIFICATIONS '*'//
& 2X,'GEAR CUTTER DIAMETER : DC2 = G14.7,/
$ 2X,'CUTTER POINT WIDTH : PW = G14.7,/
& 2X,'CUTTER BLADE ANGLE : PH12 = G14.7,//)
WRITE(9,3)
3 FORMAT(/2X,'BASIC GEAR MACHINE-TOOL SETTINGS '*'//
& 2X,'BASIC GEAR MACHINE-TOOL SETTINGS '*'//)
WRITE(9,4) Q2,SR2,XG2,XB2,EM2,GAMA2,RAG
4 FORMAT(/2X,'BLADE ANGLE : ALP1 = G14.7,/
& 2X,'POINT RADIUS : RCF = G14.7,/
$ 2X,'RADIAL SETTING : SR1 = G14.7,/
$ 2X,'MACHINE CENTER TO BACK : XB1 = G14.7,/
$ 2X,'SLIDING BASE : EM1 = G14.7,/
$ 2X,'MACHINE ROOT ANGLE : GAMA1 = G14.7,/
$ 2X,'RATIO OF ROLL : RAP = G14.7,//)
WRITE(9,6)
6 FORMAT(/2X,'BASIC PINION MACHINE-TOOL SETTINGS '*'//
& 2X,'BASIC PINION MACHINE-TOOL SETTINGS '*'//)
WRITE(9,7) ALP1,RCF,Q1,SR1,XG1,XB1,EM1,GAMA1,RAP
7 FORMAT(/2X,'BLADE ANGLE : ALP1 = G14.7,/
& 2X,'POINT RADIUS : RCF = G14.7,/
$ 2X,'RADIAL SETTING : SR1 = G14.7,/
$ 2X,'MACHINE CENTER TO BACK : XB1 = G14.7,/
$ 2X,'SLIDING BASE : EM1 = G14.7,/
$ 2X,'MACHINE ROOT ANGLE : GAMA1 = G14.7,/
$ 2X,'RATIO OF ROLL : RAP = G14.7,//)
ELSE
GOTO 1919
END IF
1919 CONTINUE
WRITE(9,16)
16 FORMAT(/2X,'CAM SETTINGS AND COEFFICIENTS OF TAYLOR SERIES*'//
& 2X,'CAM SETTINGS AND COEFFICIENTS OF TAYLOR SERIES*'//)
WRITE(9,17) PSI1,RUP,DELT,RA1,C2,D6,E24,F120
17 FORMAT(/2X,'GUIDE ANGLE : PSI1 = G14.7,/
$ 2X,'CAM PITCH RADIUS : RUP = G14.7,/
& 2X,'CAM SETTING : DELT = G14.7,/
& 2X,'1ST ORDER COEFFICIENT : RA1 = G14.7,/
& 2X,'2ND ORDER COEFFICIENT : C2 = G14.7,/
& 2X,'3RD ORDER COEFFICIENT : D6 = G14.7,/
& 2X,'4TH ORDER COEFFICIENT : E24 = G14.7,/
& 2X,'5TH ORDER COEFFICIENT : F120 = G14.7,//)
C
C... CALL TCA
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DEFINE THE INITIAL POINT

XI(1) = THIG
XI(2) = 0.000000
XI(3) = THF
XI(4) = 0.0
XI(5) = 0.00

FIND THE INITIAL CONTACT POINT

5555 N = 5
ERRREL = 0.1D-10
ITMAX = 200
PHI2P = PHI2P0
IF (JCC.EQ.1) THEN
CALL DNEQNF (FCN, ERRREL, N, ITMAX, XI, X, FNORM)
ELSE
CALL DNEQNF (FCNR, ERRREL, N, ITMAX, XI, X, FNORM)
END IF

PHI1P0 = X(5)

PHI2P1 = PHI2P0 - 180.0° CNST / (6.0° TN2) - TL1° 180.0° CNST / (6.0° TN2)
PHI2P2 = PHI2P0 + 180.0° CNST / (6.0° TN2) + TL2° 180.0° CNST / (6.0° TN2)
KK = 1
PHI2P = PHI2P1

CONTINUE
IF (JCC.EQ.1) THEN
CALL DNEQNF (FCN, ERRREL, N, ITMAX, XI, X, FNORM)
ELSE
CALL DNEQNF (FCNR, ERRREL, N, ITMAX, XI, X, FNORM)
END IF

XI(1) = X(1)
XI(2) = X(2)
XI(3) = X(3)
XI(4) = X(4)
XI(5) = X(5)

find the transmission error

ERRR = PHI2P - PHI2P0 - TN1° TN2° (X(5) - PHI1P0)
ERRR = PHI2P - PHI2P0 + TN1° TN2° (X(5) - PHI1P0)
ERR (KK) = 3600.0° ERRR / CNST
PI2P(KK) = PHI2P

computer the contact path

xlc = x2m
rlc = dsqrt(y2m**2 + z2m**2)
xcp(KK) = xlc * dcos(rgma2) + rlc * dsin(rgma2) + ox
ycz(KK) = -xlc * dsin(rgma2) + rlc * dcos(rgma2) + oy

COMPUTER THE PRINCIPAL DIRECTIONS AND CURVATURES OF GEAR

TH = X(1)
PH=X(2)
ST=DSIN(TH)
CT=DCOS(TH)
SH=DSIN(PH)
CS=DCOS(PH)
SP=DSIN(ALP2)
CP=DCOS(ALP2)
SH=DSIN(GAMA2)
CH=DCOS(GAMA2)

C.... DEFINE VECTORS TO COMPUTER THE SECOND ORDER PROPERTY OF GEAR

C

ES(1)=-DSIN(TH-PHI)
ES(2)= DCOS(TH-PHI)
ES(3)= 0.0
EQ(1)=-SP*DCOS(TH-PHI)
EQ(2)=-SP*DSIN(TH-PHI)
EQ(3)= CP
CN(1)=XNM
CN(2)=YNM
CN(3)=ZNM
KS=CP/(RC2-SG*SP)
KQ=0.0
W1(1)=-CM
W1(2)= 0.0
W1(3)=-SM
W2(1)= 0.0
W2(2)= 0.0
W2 (3)=-CR2
VT1 (1) = YM*SM+EM*SM
VT1 (2) =-XM*SM+(ZM-XB2)*CM
VT1 (3) =-YM*CM-EM*CM
VT2(1) = YM*CR2
VT2(2) =-XM*CR2
VT2(3) = 0.0
DO 110 I=1,3
W12(I)=W1(I)-W2(I)
V12(I)=VT1(I)-VT2(I)
110 CONTINUE

C

P121=0.0
CALL CURVAl
K2I=KF
K2II=KH
PHI2=PH/CR2
sh2=dsin(phi2)
ch2=dcos(phi2)
xX= CM*ef(1)+SM*ef(3)
yY= ef(2)
zZ=-SM*ef(1)+CM*ef(3)
ef(1)=xx
ef(2)= CH2*yY-SH2*zZ
ef(3)= SH2*yY+CH2*zZ

C

xX= CM*eh(1)+SM*eh(3)
yY= eh(2)
zZ=-SM*eh(1)+CM*eh(3)
eh(1)=xx
eh(2)= CH2*yY-SH2*zZ

156
\[ eh(3) = \text{SH2}^2yY + \text{CH2}^2zZ \]

```plaintext
CHP = \text{DCOS} (\text{PHI2P})
SHP = \text{DSIN} (\text{PHI2P})
CMM = \text{DCOS} (\text{GAMMA})
SMM = \text{DSIN} (\text{GAMMA})
XX = ef (1)
YY = -ef (2) * CHP + ef (3) * SHP
ZZ = -ef (2) * SHP - ef (3) * CHP
E2IH(1) = XX * CMM + ZZ * SMM
E2IH(2) = YY
E2IH(3) = -XX * SMM + ZZ * CMM
```

```plaintext
XX = \text{eh} (1)
YY = \text{eh} (2) * CHP + \text{eh} (3) * SHP
ZZ = \text{eh} (2) * SHP - \text{eh} (3) * CHP
E2IIH(1) = XX * CMM + ZZ * SMM
E2IIH(2) = YY
E2IIH(3) = -XX * SMM + ZZ * CMM
```

```plaintext
\text{COMPUTER THE PRINCIPAL DIRECTIONS AND CURVATURES OF PINION}
```

```plaintext
TH1 = X(3)
PH1 = X(4)
STP = \text{DSIN} (\text{TH1} + \text{PH1})
CTP = \text{DCOS} (\text{TH1} + \text{PH1})
IF (JCC.EQ.1) THEN
SP1 = \text{DSIN} (\text{ALP1})
CP1 = \text{DCOS} (\text{ALP1})
ELSE
SGN = ALP1 / DABS (ALP1)
ALP = SGN * ALP
SP1 = \text{DSIN} (ALP)
CP1 = \text{DCOS} (ALP)
END IF
SM1 = \text{DSIN} (\text{GAMA1})
CM1 = \text{DCOS} (\text{GAMA1})
```

```plaintext
\text{DEFINE VECTORS TO COMPUTER THE SECOND ORDER PROPERTY OF PINION}
```

```plaintext
ES(1) = -STP
ES(2) = CTP
ES(3) = 0.0
EQ(1) = SP1 * CTP
EQ(2) = SP1 * STP
EQ(3) = -CP1
CN(1) = XNM1
CN(2) = YNM1
CN(3) = ZNM1
IF (JCC.EQ.1) THEN
KS = CP1 / (RCF + SF * SP1)
KQ = 0.0
ELSE
KS = \text{DCOS} (ALP) / (RHO * \text{DCOS} (ALP) + XO)
KQ = 1.0 / RHO
END IF
W1(1) = CM1
W1(2) = 0.0
W1(3) = SM1
W2(1) = 0.0
```

157
W2(2) = 0.0
W2(3) = CRT
VT1(1) = -YM1*SM1 - EM1*SM1
VT1(2) = XM1*SM1 - (ZM1 - XB1)*CM1
VT1(3) = YM1*CM1 + EM1*CM1
VT2(1) = -YM1*CR1T
VT2(2) = XM1*CR1T
VT2(3) = 0.0
DO 210 I=1,3
W12(I) = W1(I) - W2(I)
V12(I) = VT1(I) - VT2(I)
210 CONTINUE

C
PI21 = PCR1T
CALL CURVAL
C
WRITE(9,12) KF, KH, SIGSF
KI1 = KF
KI1I = KH
C
PH1 = PH1/CR1
SH1 = DSIN(PH1)
CH1 = DCOS(PH1)
XX = CM1*EF(1) + SM1*EF(3)
yY = ef(2)
ZZ = -SM1*EF(1) + CM1*EF(3)
ef(1) = xx
EF(2) = CH1*YY + SH1*ZZ
EF(3) = -SH1*YY + CH1*ZZ
C
XX = CM1*EH(1) + SM1*EH(3)
yY = eh(2)
ZZ = -SM1*EH(1) + CM1*EH(3)
eh(1) = xx
EH(2) = CH1*YY + SH1*ZZ
EH(3) = -SH1*YY + CH1*ZZ
C
CH1P = DCOS(X(5))
SH1P = DSIN(X(5))
E1IH(1) = EF(1)
E1IH(2) = CH1P*EF(2) - SH1P*EF(3)
E1IH(3) = SH1P*EF(2) + CH1P*EF(3)
E1IH(1) = EH(1)
E1IH(2) = CH1P*EH(2) - SH1P*EH(3)
E1IH(3) = SH1P*EH(2) + CH1P*EH(3)
DO 109 I=1,3
E1IH(I) = -E1IH(I)
E1IH(I) = -E1IH(I)
109 CONTINUE
C
C
COMPUTER THE DIMENSION AND ORIENTATION OF THE CONTACT ELLIPSE
C
GNH(1) = XNH2
GNH(2) = YNH2
GNH(3) = ZNH2
CALL ELLIP
AX1(KK) = A2L
AX2(KK) = B2L
ANG1(KK) = TAU1R
ANG2(KK) = TAU2R
KK=KK+1
PHI2P=PHI2P+180.0*CNST/(TN2*6.0)
IF(PHI2P.LE.(PHI2P+0.0001)) GOTOC 333
C...
C...
WRITE(9,441)
441 FORMAT(/,'*****************************************************************',
&' /,'** TRANSMISSION ERROR IN A MESHING PERIOD **',
&' /,'*****************************************************************',/)
C
DO 444 I=1,KK-1
PHI2P(I)=PHI2P(I)/CNST
WRITE(9,555) PHI2P(I),ERR(I)
555 FORMAT(3X,3(G14.7,3X))
444 CONTINUE
C
WRITE(9,551)
551 FORMAT(/,'*****************************************************************',
&' /,'** CONTACT PATH FOR A PAIR OF TEETH IN MESH **',
&' /,'*****************************************************************',/)
DO 666 I=1,KK-1
WRITE(9,747) XCP(I),YCP(I)
747 FORMAT(3X,2(G14.7,3X))
666 CONTINUE
C
WRITE(9,661)
661 FORMAT(/,'*****************************************************************',
&' /,'** DIMENSION AND ORIENTATION OF CONTACT ELLIPSE **',
&' /,'*****************************************************************',/)
DO 888 I=1,KK-1
WRITE(9,889) AX1(I),ANG1(I),AX2(I),ANG2(I)
889 FORMAT(3X,4(G14.7,3X))
888 CONTINUE
C
IF(JCL.EQ.1) GOTOL111
IF(JCL.EQ.3) GOTOL113
C...
V AND H CHECK FOR TOE POSITION
C
HMT=WD+CC-3.0/4.0*FW*(DTAN(FA)+DTAN(RA))
DED2T=DED2-3.0/4.0*FW*DTAN(RA)
TMCD=MCD-0.25*FW
XL=TMCD*DCOS(PGMA2)+(DED2T-HMT/2.0)*DSIN(PGMA2)
RL=TMCD*DSIN(PGMA2)-(DED2T-HMT/2.0)*DCOS(PGMA2)
C...
FIND THE MEAN CONTACT POINT ON THE GEAR SURFACE
C
ERRREL=0.1D-7
N=2
IF (JCH.EQ.1) THEN
XI(1)=270.0*CNST+B2
ELSE
XI(1)=B2
END IF
XI(2)=90.0*CNST-B2
CALL DNEQNF(FCN1,ERRREL,N,ITMAX,XI,X,FNORM)
TH=X(1)
PH=X(2)
ZY1=X(1)
ZY2=X(2)
N=3
ERRREL=0.1D-10
ITMAX=200
XI(1)=0.0
XI(2)=THF
XI(3)= 0.0
IF (JCC.EQ.1) THEN
CALL DNEQNF(FCNM,ERRREL,N,ITMAX,XI,X,FNORM)
ELSE
CALL DNEQNF(FCNMR,ERRREL,N,ITMAX,XI,X,FNORM)
END IF
PHI2PO=X(1)
XI(1)=ZY1
XI(2)=ZY2
XI(3)=X(2)
XI(4)=X(3)
XI(5)=PHI1P
WRITE (9,149)
149 FORMAT (/6X,' ',149)
& 6X,'* V AND H CHECK AT TOE POSITION **',/ 149
& 6X,'-----------------------------------------------------------------','/ 149
WRITE(9,139) V,H
139 FORMAT (/4X, '*** V = ',G14.7,' *** H = ',G14.7//)
C...
JCL=3
GO TO 5555
C... V AND H CHECK FOR HEEL POSITION
C C1113 HMH=W+CCC-1.0/4.0*FW*(DTAN(FA)+DTAN(RA))
C DED2H=DED2-1.0/4.0*FW*DTAN(RA)
C HMCD=MCD+0.25*FW
C1113 HMH=W+CC-0.16*FW*(DTAN(FA)+DTAN(RA))
C DED2H=DED2-0.16*FW*DTAN(RA)
HMCD=MCD+0.16*FW
XL=HMCD*DCOS(PGMA)+ (DED2H-HMH/2.0)*DSIN(PGMA2)
RL=HMCD*DSIN(PGMA2)+ (DED2H-HMH/2.0)*DCOS(PGMA2)
ERRREL=0.1D-7
N=2
ITMAX=200
IF (JCH.EQ.1) THEN
XI(1)=270.0*CNST+B2
ELSE
C XI(1)=90.0*CNST-B2
C XI(1)=B2
C... FIND THE V AND H VALUE FOR HEEL POSITION
C N=3
ERRREL=0.1D-10
ITMAX=200
XI(1)=0.00
XI(2)=THF
XI(3)=0.0
IF (JCC.EQ.1) THEN
CALL DNEQNF(FCNM,ERRREL,N,ITMAX,XI,X,FNORM)
END IF
PHI2PO=X(1)
XI(1)=ZY1
XI(2)=ZY2
XI(3)=X(2)
XI(4)=X(3)
XI(5)=PHI1P
WRITE (9,11) PHI2PO,PHI1P
WRITE (9,159)
159 FORMAT(/6X,'*********************************************************************/,
& 6X,'* V AND H CHECK AT HEEL POSITION */,
& 6X,'*********************************************************************/)
WRITE (9,169) V,H
169 FORMAT(/4X,'*** V = ',G14.7,'*** H = ',G14.7//)
C...
JCL=1
GOTO 5555
1111 CONTINUE
IF (KSIDE.EQ.0) GOTO 1990
STOP
END
C...
FCN1 IS TO FIND THE MEAN CONTACT POINT
C
SUBROUTINE FCN1(X,F,N)
IMPLICIT REAL*(A-H,O-Z)
INTEGER N
REAL*8 X(N),F(N),mcd
COMMON/A1/CNST,TN1,TN2,C,FW,GAMMA,x1,r1,mcd
COMMON/A3/B2,RGMA2,FGMA2,PGMA2,D2R,D2F,ADD2,DED2,WD,CC,D2P
COMMON/A4/SR2,Q2,RC2,PW2,XB2,XG2,EM2,GAMA2,CR2,ALP2,PHI2,PHI1P
COMMON/A5/SG,XM,YM,ZM,XNM,YNM,ZNM,X2M,Y2M,Z2M,XN2M,YN2M,ZN2M,
&XNH2,YNH2,ZNH2,XH2,YH2,ZH2
TH=X(1)
PH=X(2)
SP=DSIN(ALP2)
CP=DCOS(ALP2)
SM=DSIN(GAMA2)
CM=DCOS(GAMA2)
STP=DSIN(TH-PH)
CTP=DCOS(TH-PH)
XNM=-CP*CTP
YNM=-CP*STP
ZNM=SP
AA1=RC2*STP+SR2*DSIN(-Q2-PH)
AA2=RC2*CTP+SR2*DCOS(-Q2-PH)
AX=-EM2*SM
AY=XB2*CM
AZ=EM2*CM
C...
FIND SG
C

T1 = XNM*(AX-AA1*(SM-CR2))+YNM*(AY+AA2*(SK-CR2))+ZNM*(AZ+AA1*CM)
T2 = XNM*(SM-CR2)*SP*STP+YNM*((SM-CR2)*SP*CTP-CP*CM)+ZNM*CM*SP*STP
SC = T1/T2
XH = (RC2-SG*SP)*CTP+SR2*DCOS(-Q2-PH)
YN = (RC2-SG*SP)*STP+SR2*DSIN(-Q2-PH)
ZM = -SG*CP
C
XM = SG/ST2
YM = SG/ST1
ZM = SG/CP
XM = SG/SR*(SM-XN)-XG-XP-SM
YM = SG/SR*(SM-YN)-XP-SM
ZM = SG/SR*(SM-ZN)
X2M = XH
Y2M = CH2*YM+SM2
Z2M = SH*ZM+CM2
XN2M = XN
YN2M = CH2*YN+CM2
Z2M = SH*YN+CM2
F(1) = X2M-XL
F(2) = Y2M*Z2M-RL
C
RETURN
END

C
SUBROUTINE CURVAL IS TO COMPUTE THE CURVATURE OF THE
GENERATED SURFACE
C
SUBROUTINE CURVAL
IMPLICIT REAL*8(A-H,O-Z)
REAL*8 KS,KQ,KF,KH
DIMENSION ESN(3),EQN(3),W1VT1(3),W1VT2(3),W2VT1(3)

C
ESN(1) = CN(2)*ES(3)-CN(3)*ES(2)
ESN(2) = -(CN(1)*ES(3)-CN(3)*ES(1))
ESN(3) = CN(1)*ES(2)-CN(2)*ES(1)
C
EQN(1) = CN(2)*EQ(3)-CN(3)*EQ(2)
EQN(2) = -(CN(1)*EQ(3)-CN(3)*EQ(1))
EQN(3) = CN(1)*EQ(2)-CN(2)*EQ(1)
C
W1VT1(1) = W1(2)*VT1(3)-W1(3)*VT2(2)
W1VT1(2) = -(W1(1)*VT1(3)-W1(3)*VT2(1))
W1VT1(3) = W1(1)*VT2(2)-W1(2)*VT2(1)
C
W2VT1(1) = W2(2)*VT1(3)-W2(3)*VT1(2)
W2VT1(2) = -(W2(1)*VT1(3)-W2(3)*VT1(1))
W2VT1(3) = W2(1)*VT1(2)-W2(2)*VT1(1)
C
W12(1) = W12(2)*V12(3)-W12(3)*V12(2)
W12(2) = -(W12(1)*V12(3)-W12(3)*V12(1))
W12(3) = W12(1)*V12(2)-W12(2)*V12(1)
C...
V12S=0.0
V12Q=0.0
WNES=0.0
WNEQ=0.0
VWN=0.0
W1TN=0.0
W2TN=0.0
VT2N=0.0
C...
DO 1 I=1,3
V12S= V12(I)*ES(I)+V12S
V12Q= V12(I)*EQ(I)+V12Q
WNES= W12(I)*ESN(I)+WNES
WNEQ= W12(I)*EQN(I)+WNEQ
VWN= CN(I)*VW12(I)+VWN
W1TN= CN(I)*W1VT2(I)+W1TN
W2TN= CN(I)*W2VT1(I)+W2TN
VT2N= CN(I)*VT2(I)+VT2N
1 CONTINUE
C...
COMPUTER THE CURVATURE OF THE GENERATED SURFACE
C...
A13=-KS*V12S-WNES
A23=-KQ*V12Q-WNEQ
A33=KS*V12S**2+KQ*V12Q**2-VWN-W1TN+W2TN+PI21*VT2N/W2(3)
T1=2.0D00*A13*A23
T2=A23**2-A13**2+(KS-KQ)**A33
SIGIF=0.5D00*DATAN2(T1,T2)
KF=0.5D00*(KS+KQ)-0.5D00*(A13**2+A23**2)/A33
&+A13*A23/(A33*DSIN(2.0D00*SIGIF))
KH= KF-2.0D00*A13*A23/(A33*DSIN(2.0D00*SIGIF))
SIGSF=SIGIF
DO 2 I=1,3
EF(I)= DCOS(SIGIF)*ES(I)-DSIN(SIGIF)*EQ(I)
EH(I)= DSIN(SIGIF)*ES(I)+DCOS(SIGIF)*EQ(I)
2 CONTINUE
RETURN
END
C...
FCN2 IS TO FIND THE INITIAL GEAR ROTATIONAL ANGLE
C
SUBROUTINE FCN2(X,F,N)
INTEGER N
REAL*8 X(N),F(N)
COMMON/A1/CNST,TN1,TN2,C,FW,GAMMA,XL,RL,MCD
COMMON/A5/SG,XM,YNM,XNM,YNM,ZNM,XM2M,YN2M,ZN2M,
&XNH2,YNH2,ZNH2,XH2,YH2,ZH2
CM=DCOS(Gamma)
SM=DSIN(Gamma)
CHP=DCOS(X(I))
SHP=DSIN(X(I))
XX= X2M
YY=-Y2M*CHP+Z2M*SHP
ZZ=-Y2M*SHP-Z2M*CHP
XH2= XX*CM+ZZ*SM
YH2= YY+C
ZH2=-XX*SM+ZZ*CM
C... THE FOLLOWING IS THE TCA SUBROUTINE FOR CURVED BLADE
C...
SUBROUTINE FCNR (X, F, N)
IMPLICIT REAL*8 (A-H, O-Z)
REAL*8 X(N), F(N)
DIMENSION CH(3), P(3), EI1F(3), ESN(3), EQN(3), WV12(3),
SW2VT1(3), EFIH(3), EFI1H(3), RH(3), GHN(3), EIH(3), EIIH(3),
& EI1H(3), EI1IH(3), EI1(3), EI11(3), CN(3), EFE1(3),
& ERR(20), XP(20), YP(20)
COMMON/A1/CNST, TN1, TN2, C, FW, GAMMA, x1, r1, mcd
COMMON/A2/B1, RGMA1, PGMA1, D1R, D1F, ADD1, DED1
COMMON/A3/B2, RGMA2, PGMA2, D2R, D2F, ADD2, DED2, WD, CC, D2P
COMMON/A4/SR2, Q2, RC2, PW, XB2, XG2, EM2, GAMA2, CR2, ALP2, PHI2, PHI2P
COMMON/A5/SG, XM, YM, ZM, XM1, YM1, ZM1, XNM, YNM, ZNM, XNH, YNH, ZNH,
& XNH2, YNH2, ZNH2, XH2, YH2, ZH2
SV12(3), KS, KQ, KF, EF(3), EH(3), SIGSF, PI21
COMMON/A7/SR1, Q1, RF, PW1, XB1, XG1, EM1, GAM1, CR1, ALP1, PHI1, PHI1P
COMMON/A8/ST, XM1, YM1, ZM1, XM1, YM1, ZM1, XNM1, YNM1, ZNM1,
& XNH1, YNH1, ZNH1, XH1, YH1, ZH1
COMMON/A9/PHI2PO, QR, O2, XO, XO, RO, ALP, V, H, CR1T, FCR1T
COMMON/A11/RAM, P1ST, C2, D2, E24, F120, CX24, DX24, EX120, RUI, DELT, RUP,
$RA1, CPF, DPF, EPF, PF
TH=X(1)
PH=X(2)
SP=DSIN(ALP2)
CP=DCOS(ALP2)
SM=DSIN(GAMA2)
CM=DCOS(GAMA2)
STP=DSIN(TH-PH)
CPT=DCOS(TH-PH)
XNH=-CP*CTP
YNM=-CP*STP
ZNM=SP
AA1=RC2*STP+SR2*DSIN(-Q2-PH)
AA2=RC2*CPT+SR2*DCOS(-Q2-PH)
AX=-EM2*SM
AY=XB2*CM
AZ=XM2*CM
C... FIND SG
C
T1= XNH*(AX-AA1*(SM-CR2)) + YNH*(AY+AA2*(SM-CR2)) + ZNH*(AZ+AA1*CM)
T2= XNH*(SM-CR2)**SP*STP+YNH*((SM-CR2)**SP*CPT-CP*CM) + ZNH*CM*SP*STP
SG=T1/T2
XM = (RC2-SP)^CTP+SR2^DCOS(-Q2-PH)
YM = (RC2-SP)^STP+SR2^DSIN(-Q2-PH)
ZN = -SG^CP
C
XM = SG^SP*CTP+AA2
C
YM = SG^SP*STP+AA1
C
ZN = -SG^CP
C
xX = CM^XM+SM^ZM-XG2-XB2^SM
yY = YM+EM2
zZ = SM^XM+CM^ZM-XB2^CM
XN = CM^XNM+SM^ZNM
YN = YNM
ZN = SM^XNM+CM^ZNM
PHI2 = PH/CR2
sh2 = dsin(phi2)
ch2 = dcos(phi2)
X2M = xX
Y2M = CH2*yY-SH2*zZ
Z2M = SH2*yY+CH2*zZ
XN2M = XN
YN2M = CH2*YN-SH2*ZN
ZN2M = SH2*YN+CH2*ZN
CMN = D(COS(GAMMA))
SNM = D(SIN(GAMMA))
CHP = D(COS(PHI2P))
SHP = D(SIN(PHI2P))
XX = X2M
YY = -Y2M^CHP+Z2M^SHP
ZZ = -Y2M^SHP-Z2M^CHP
XH2 = XX^CMM+ZZ^SMM
YH2 = YY+C+V
ZH2 = -XX^SMM+ZZ^CMM
C...
XX = XN2M
YY = -YN2M^CHP+ZN2M^SHP
ZZ = -YN2M^SHP-ZN2M^CHP
XNH2 = XX^CMM+ZZ^SMM
YNH2 = YY
ZNH2 = -XX^SMM+ZZ^CMM
C...
DEFINE THE PINION SURFACE
C
TH1 = X(3)
PH1 = X(4)
SM1 = D(SIN(GAMA1))
CM1 = D(COS(GAMA1))
STP = D(SIN(TH1+PH1))
CTP = D(COS(TH1+PH1))
C
C...
FIND CRIT, PF, PPF, PCR1T
C
DDD = DBS(PH1)
IF (DDD.LE.0.001) GOTO 6
PHI1 = RA1*(PH1-CPF*PH1**2-DFP*PH1**3-EPF*PH1**4-FPF*PH1**5)
PF = RA1*(1.0-2.0*CPF*PH1-3.0*DFP*PH1**2
-4.0*EPF*PH1**3-5.0*FPF*PH1**4)
PPF = RA1*(2.0*CPF+6.0*DFP*PH1+12.0*EPF*PH1**2+20.0*FPF*PH1**3)
CR1T = 1.0/PF
PCR1T = -PPF/PF**3
GOTO 7
6
PH1 = RA1*PH1
CR1T=CR1
PCR1T=2.0*CPF/(RA1**2)
CONTINUE
CR1T=CR1
PCR1T=0.000
CONTINUE

FIND THE NORMAL OF THE EQUIDISTANCE SURFACE

XMO= XO*CTP+SR1*DCOS(-Q1+PH1)
YMO= XO*STP+SR1*DSIN(-Q1+PH1)
ZMO= ZO
V1X=-YMO*SM1-EM1*SM
V1Y= XMO*SM1-(ZMO-XB)*CM1
V1Z= YMO*CM1+EM1*CM1
V2X=-YMO*CR1T
V2Y= XMO*CR1T
V2Z= 0.0
VX=V1X-V2X
VY=V1Y-V2Y
VZ=V1Z-V2Z
TX=-CTP
TY=-STP
TZ=0.0
FX= STP
FY=-CTP
FZ=0.0
XNN= FY*VZ-FZ*VX
YNN= FZ*VX-FX*VY
ZNN= FX*VY-FY*VX
DDD=DSQRT(XNN**2+YNN**2+ZNN**2)
XNM1=XNN/DDD
YNM1=YNN/DDD
ZNM1=ZNN/DDD
DT=TX*XNM1+TY*YNM1+TZ*ZNM1
IF(DT.GE.0.0) GOTO 10
XNM1=-XNM1
YNM1=-YNM1
ZNM1=-ZNM1
CONTINUE

XM1= XMO-RHO*XNM1
YM1= YMO-RHO*YNM1
ZM1= ZMO-RHO*ZNM1
ALP=DARCOS(TX*XNM1+TY*YNM1+TZ*ZNM1)
xX= CM1*XM1+SM1*ZM1-XG1-XB1*SM1
yY= YM1+EM1
zZ=-SM1*XM1+CM1*ZM1-XB1*CM1
XN1=CH1*XNM1+SM1*ZNM1
YN1=YNM1
ZN1=SM1*XNM1+CM1*ZNM1

PH11=PH1/CR1
sh1=dsin(ph11)
ch1=dcos(ph11)
X1M= xX
Y1M= CH1*yY+SH1*zZ
Z1M=-SH1*yY+CH1*zZ
XN1M= XN1
YN1M= CH1*YN1+SH1*ZN1
ZN1M=-SH1*YN1+CH1*ZN1
PH1P=X(5)
sh1P=dsin(ph1P)
CH1 = DCOS(PI/2)
XH1 = X1M + H
YH1 = CH1 * Y1M - SH1 * Z1M
ZH1 = SH1 * Y1M + CH1 * Z1M
XNH1 = XN1M
YNH1 = CH1 * YN1M - SH1 * ZN1M
ZNH1 = SH1 * YN1M + CH1 * ZN1M
F(1) = XH2 - XH1
F(2) = YH2 - YH1
F(3) = ZH2 - ZH1
F(4) = XNH2 - XNH1
F(5) = ZNH2 - ZNH1
RETURN
END

C...
THE FOLLOWING IS THE SUBROUTINE FOR STRAIGHT BLADE
C...

SUBROUTINE FCN(X,F,N)
IMPLICIT REAL*8(A-H,O-Z)
REAL*8 X(N), F(N)
DIMENSION CH(3), P(3), ELEF(3), ESN(3), ESNQ(3), W1VT2(3), W1VT1(3),
       SW2VT1(3), EFIH(3), EIIH(3), RH(3), GNH(3), EZIH(3), EZIIH(3),
       ZEIH(3), EIIH(3), EIFI(3), EIII(3), EFI(3), EIII(3), GN(3), EIFI(3),
       ERR(20), XP(20), YP(20)
COMMON/A1/CNST, TN1, TN2, C, FW, GAMMA, x1, ri, mc
COMMON/A2/B1, RMA1, PMA1, D1R, D1F, ADD1, DED1
COMMON/A3/B2, RMA2, PMA2, D2R, D2F, ADD2, DED2, WD, CC, D2P
COMMON/A4/SR2, Q2, RC2, PW2, XB2, XC2, EN2, GAMA2, CR2, ALP2, PHI2, PHI12F
COMMON/A5/SG, XM, YM, ZM, XN, YN, ZN, X2M, Y2M, Z2M, XN2M, YN2M, ZN2M,
       &XNH2, YNH2, ZNH2, XH2, YH2, ZH2
       SV12(3), KS, KQ, KF, K3, EF(3), EH(3), SIGSF, PI21
COMMON/A7/SR1, Q1, RCF, PW1, XB1, XC1, EM1, GAMA1, CR1, ALP1, PHI1, PHI12P
COMMON/A8/SF, XM1, YM1, ZM1, XN1, YN1, ZN1, X1M, Y1M, Z1M,
       &XNH1, YNH1, ZNH1, XH1, YH1, ZH1
COMMON/A9/PHI2P0, OX, OZ, XO, ZO, RHO, ALP, V, H, CRLT, PRC1T
COMMON/A11/RAM, PSI1, C2, D6, E24, F120, CX6, DX24, EX120, RU1, DELT, RUP,
       SR1, CPF, DP1, EP1, FP1
TH = X(1)
PH = X(2)
SP = DSIN(ALP2)
CP = DCOS(ALP2)
SM = DSIN(GAM2A)
CM = DCOS(GAM2A)
STP = DSIN(TH-PH)
CTP = DCOS(TH-PH)
XNH = -CP * CT1
YNH = -CP * CT2
ZNH = SP
AA1 = RC2 * STP + SR2 * DSIN(-Q2-PH)
AA2 = RC2 * CTP + SR2 * DCOS(-Q2-PH)
AX = -EM2 * SM
AY = XB2 * CM
AZ = EM2 * CM

C...
FIND SG
C...
T1 = XNH * (AX - AA1 * (SM - CR2)) + YNH * (AY + AA2 * (SM - CR2)) + ZNH * (AZ + AA1 * CM)
T2 = -XNH * (SM - CR2) * SP * STP + YNH * ((SH - CR2) * SP * CTP - CP * CM) + ZNH * CM * SP * STP
SG = T1 / T2

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XM= (RC2-SC*SP)*CTP+SR2*DCOS(-Q2-PH)
YM= (RC2-SC*SP)*STP+SR2*DSIN(-Q2-PH)
ZM=-SG*CP

C XM=-SG*SP*CTP+AA2
C YM=-SG*SP*CTP+AA1
C ZM=-SG*CP

xx= CM*XM+SM*ZM-XG2-XB2*SM
yy= YM+EM2
zz=-SM*XM+CM*ZM-XB2*CM
XN= CM*XNM+SM*ZNM
YN= YNM
ZN=-SM*XNN+CM*ZNM

PHI2=PH/CR2
sh2=dsin(phi2)
ch2=dcos(phi2)
X2M= xx
Y2M= CH2*YY-SH2*zz
Z2M= SH2*YY+CH2*zz
XN2M= XN
YN2M= CH2*YN-SH2*ZN
ZN2M= SH2*YN+CH2*ZN

C X2M= CM*XNM+SM*ZNM
YN= YNM
ZN=-SM*XNM+CM*ZNM

PHI2=PH/CR2
sh2=dsin(phi2)
ch2=dcos(phi2)
X2M= xx
Y2M= CH2*YY-SH2*zz
Z2M= SH2*YY+CH2*zz
XN2M= XN
YN2M= CH2*YN-SH2*ZN
ZN2M= SH2*YN+CH2*ZN
CM2=DCOS(GAMMA)
SM2=DSIN(GAMMA)
CHP=DCOS(PHI2P)
SHP=DSIN(PHI2P)

C Y2M= CH2*YY-SH2*zz
Z2M= SH2*YY+CH2*zz
XN2M= XN
YN2M= CH2*YN-SH2*ZN
ZN2M= SH2*YN+CH2*ZN

C XN2M= CM*XNM+SM*ZNM
YN= YNM
ZN=-SM*XNM+CM*ZNM

XX= XN2M
YY=-YN2M*CHP+ZM*SHP
ZZ=-YN2M*SHP-ZM*CHP
XH2= XX*CM2+ZZ*SM2
YH2= YY+C+V
C YH2= YY+C-V
ZH2=-XX*SM2+ZZ*CM2

C...
XX= XN2M
YY=-YN2M*CHP+ZM*SHP
ZZ=-YN2M*SHP-ZM*CHP
XH2= XX*CM2+ZZ*SM2
YH2= YY
ZH2=-XX*SM2+ZZ*CM2

C...
TH1=X(3)
PH1=X(4)
SP1=DSIN(-ALP1)
CP1=DCOS(-ALP1)
SM1=DSIN(GAMA1)
CM1=DCOS(GAMA1)
STP=DSIN(TH1+PH1)
CSP=DCOS(TH1+PH1)
XNM1=-CP1*CTP
YNM1=-CP1*STP
ZNM1= SP1
AB1=RCP*STP+SR1*DSIN(-Q1+PH1)
AB2=RCP*CTP+SR1*DCOS(-Q1+PH1)
AX=EM1*SM1
AYX=XB1*CM1
AZZ=EM1*CM1

C...
FIND SF,CR1T,PF,PPF,PCR1T
C
DDD=DABS(PHI)
IF(DDD.LE.0.001) GOTO 6
PHI1=RA1*(PH1-CPF*PH1**2-DFP*PH1**3-EPF*PH1**4-CPF*PH1**5)
FF=RA1*(1.0-2.0*CPF*PH1**3-3.0*DFP*PH1**2
$-4.0*EPF*PH1**3-5.0*PF*PH1**4)
PPF=RA1*(2.0*CPF+6.0*DFP*PH1+12.0*EPF*PH1**2+20.0*PF*PH1**3)
CR1T=1.0/PF
PCR1T=PPF/PF**3
GOTO 7
6 PHI1=RA1*PHI
CR1T=CR1
PCR1T=2.0*CPF/(RA1**2)
7 CONTINUE
C
CR1T=CR1
C
PCR1T=0.000
T1=XNM1*(AXX-AB1*(SM1-CR1T))+
&YNM1*(AYY+AB2*(SM1-CR1T))+ZN11*(AZZ+AB1*CM1)
T2=XNM1*(SM1-CR1T)*SP1*STP+
&YNM1*(SM1-CR1T)*SP1*CTP-CP1*CM1)+ZN11*CM1*SP1*STP
SF=T1/T2
C
XM1=(RCF-SF*SP1)*CTP+SR1*DCOS(-Q1+PHI)
YM1=(RCF-SF*SP1)*STP+SR1*DSIN(-Q1+PHI)
ZM1=-SF*CP1
xx=CM1*XM1+SM1*ZM1-XG1-XB1*SM1
yY=YM1+EM1
zZ=-SM1*XM1+CM1*ZM1-XB1*CM1
XN1=CM1*XNM1+SM1*ZN11
YN1=YNM1
ZN1=-SM1*XNM1+CM1*ZN11
C
PHI1=PHI1/CR1
sh1=dsin(phi1)
chl=dcos(phi1)
XL1=xx
YL1=CH1*yY+SH1*zZ
ZL1=-SH1*yY+CH1*zZ
XN11=XL1
YN11=CH1*YN1+SH1*ZN11
ZN11=-SH1*YN1+CH1*ZN11
C
WRITE(9,111) X1M,Y1M,Z1M
PH1P=X(5)
sh1P=dsin(phi1P)
chlP=dcos(phi1P)
XH1=X1M+H
YH1=CH1*Y1M-SH1*Z1M
ZH1=SH1*Y1M+CH1*Z1M
XNH1=XN1M
YNH1=CH1*YN1M-SH1*ZN1M
ZNH1=SH1*YN1M+CH1*ZN1M
F(1)=XH2-XH1
F(2)=YH2-YH1
F(3)=ZH2-ZH1
F(4)=XNH2-XNH1
F(5)=YNH2-YNH1
RETURN
END
C...
************************************************************
C...
C... SUBROUTINE ELLIP IS TO DETERMINE THE SIZE AND ORIENTATION
C...

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SUBROUTINE ELLIP
IMPLICIT REAL*8(A-H,O-Z)
REAL*8 R5,Q1,K11,K1II,K2I,K2II
DIMENSION R0(3),ETA2(3),ZETA2(3),E1E2(3),ETA(3),ZETA(3)
DIMENSION E1IH(3),E1IIH(3),E2IH(3),E2IIH(3),GNH(3)
COMMON/A1/CNST,TN1,TN2,C,FW,GAMMA,x1,r1,mc
COMMON/A3/B2,RGMAT,FMAT,D2R,D2F,ADD2,DED2,WD,CC,D2P
COMMON/A4/SR2,Q2,RC2,PW2,XB2,XG2,EM2,GAMMA2,CR2,ALP2,PHI2,PH12P
COMMON/A5/SG,XM,YM,ZM,XNM,YNM,ZNM,X2M,Y2M,Z2M,XN2M,YN2M,ZN2M,
&XNH2,YNH2,ZNH2,XH2,YH2,ZH2
COMMON/A9/PH12P0,OX,OZ,XO,ZO,RHO,ALP1,PHI,CR1,PCR1T
COMMON/A10/KII1,K1II,K2II,K2II,DEL,E1IH,E1IIH,E2IH,E2IIH,GNH,
CNST=3.14159-1.0D00/180.00

E1E2(1)=E1IH(2)*E2IH(3)-E1IH(3)*E2IH(2)
E1E2(2)=-(E1IH(1)*E2IH(3)-E1IH(3)*E2IH(1))
E1E2(3)=E1IH(1)*E2IH(2)-E1IH(2)*E2IH(1)

T1=0.0
T2=0.0
DO 1 I=1,3
T1=E1IH(I)*E2IH(I)+T1
T2=GNH(I)*E1E2(I)+T2
1 CONTINUE

T1=T1+1.0D00
SIG12=2.0D00*DATAN2(T2,T1)

SK1=K1I+K1II
SK2=K2I+K2II
SG1=K1I-K1II
SG2=K2I-K2II

T1=SIG12*DCOS(2.0D00*SIG12)
T2=SIG2*DSIN(2.0D00*SIG12)
T3=DSQRT(SIG12**2+SIG2**2-2.0D00*SIG12**2)*DCOS(2.0D00*SIG12)

TX=T2/T3
TY=T1/T3+1.0D00
ALP12=DATAN2(TX,TY)

THE DIRECTION AND LENGTH OF THE AXES OF CONTACT ELLIPSE

DEL=0.00700D00
AL=0.25D00*(SK1-SK2-T3)
BL=0.25D00*(SK1-SK2+T3)
WRITE(9,5) SIG12,AL,BL
5 FORMAT(2X,'SIG12, AL, BL = ',3(2X,G14.7))
AL=DABS(AL)
BL=DABS(BL)
A2L=2.0D00*DSQRT(DEL/AL)
B2L=2.0D00*DSQRT(DEL/BL)

DO 2 I=1,3
ETA(I)=DCOS(ALP12)*E1IH(I)-DSIN(ALP12)*E1IIH(I)
2 CONTINUE
ZETA(I) = DSIN(ALP12) * E11H(I) + DCOS(ALP12) * E111H(I)
CONTINUE

DETERMINE THE PROJECTION OF CONTACT ELLIPS IN AXIAL SECTION

CHP = DCOS(PHI2P)
SHP = DSIN(PHI2P)
CMH = DCOS(GAMMA)
SNM = DSIN(GAMMA)

XX = ETA(I) * CMH - ETA(3) * SMM
YY = ETA(2)
ZZ = ETA(I) * SMM + ETA(3) * CMH
ETA2(1) = XX
ETA2(2) = -YY * CHP - ZZ * SHP
ETA2(3) = YY * SHP - ZZ * CHP

XX = ZETA(1) * CMH - ZETA(3) * SMM
YY = ZETA(2)
ZZ = ZETA(1) * SMM + ZETA(3) * CMH
ZETA2(1) = XX
ZETA2(2) = -YY * CHP - ZZ * SHP
ZETA2(3) = YY * SHP - ZZ * CHP

R0(2) = Y2M / DSQRT(Z2M**2 + Y2M**2)
R0(3) = Z2M / DSQRT(Z2M**2 + Y2M**2)
R0(1) = 0.0D00

TAU1 = DATAN2(T11, ZETA2(1))
TAU2 = DATAN2(T12, ETA2(1))

A2P = A2L * ZETA2(1) / DCOS(TAU1)
B2P = B2L * ETA2(1) / DCOS(TAU2)

TAU1R = (TAU1 - RGMA2) / CNST
TAU2R = (TAU2 - RGMA2) / CNST
RETURN
END

THE FOLLOWING IS THE V-H CHECK PROGRAM FOR CURVED BLADE

SUBROUTINE FCNRMX(X,F,N)
IMPLICIT REAL*8(A-H,0-Z)
REAL*8 X(N),F(N)
COMMON/A1/CNST,TN1,TN2,C,FW,GAMMA,x1,rl,md
COMMON/A5/SG,XM,YM,ZM,XNH,YNH,ZNH,X2M,Y2M,Z2M,XN2M,YN2M,ZN2M,
   LXNH2,YNH2,ZNH2,XH2,YH2,ZH2
COMMON/A7/SR1,Q1,ReF,PW1,XB1,XG1,EM1,GaMA1,CR1,ALP1,PHI1,PHI1P
COMMON/A9/PHI2P0,OX,OZ,XO,ZO,RHO,ALP,V,H,CR1T,PCR1T
COMMON/A11/RAM,PS11,C2,D6,E24,F120,CX6,DX24,EX120,RU1,DELT,RUP,
   SR1,CPP,DPF,EPF,FPF
CM = DCOS(GAMMA)
SM=DSIN(GAMMA)

CHP=DCOS(PHI2PO)

C

SHP=DSIN(PHI2PO)

C

CHP=DCOS(X(I))

SHP=DSIN(X(I))

XX= X2M

YY=-2M*CHP+Z2M*SHP

ZZ=-2M*SHP-Z2M*CHP

XH2= XX*CM+ZZ*SM

YH2= YY+C

ZH2=-XX*SM+ZZ*CM

C...

XX= XN2M

YY=-YN2M*CHP+ZN2M*SHP

ZZ=-YN2M*SHP-ZN2M*CHP

XNH2= XX*CM+ZZ*SM

YNH2= YY

ZNH2=-XX*SM+ZZ*CM

C... DEFINE THE PINION SURFACE

C

TH1=X(2)

PHI1=Y(3)

SM=DSIN(GAMA1)

CM1=DCOS(GAMA1)

C

STP=DSIN(TH1+PH1)

CTP=DCOS(TH1+PH1)

C

C... FIND CR1T,PF,PPF,PCR1T

C

DDD=DABS(PHI1)

IF(DDD.LE.0.001) GOTO 6

PHI1=RA1*(PHI1*-CPF1**2*DPF**PHI1**3-EPF**PHI1**4-PPF**PHI1**5)

PF=RA1*(1.0-2.*CPF1**PHI1**3-0.0000000000000000001)

PPF=-RA1*(2.0*CPF1+6.0*DPF**PHI1+12.0*EPF**PHI1+20.0*PF1**3)

CR1T=1.0/PF

PCR1T=-PPF/PPF**3

GOTO 7

6

PHI1=RA1*PHI1

CR1T=CR1

PCR1T=2.0*CPF1/(RA1**2)

GOTO 7

7 CONTINUE

C

FIND THE NORMAL OF THE EQUIDISTANCE SURFACE

C

XMO= X0*CTP+SR1*DCOS(-Q1+PHI1)

YMO= X0*STP+SR1*DSIN(-Q1+PHI1)

ZMO= ZO

V1X=-YMO*SM1-EM1*SM1

V1Y= XMO*SM1-(ZMO-XB1)*CM1

V1Z= YMO*CM1+EM1*CM1

V2X=-YMO*CR1T

V2Y= XMO*CR1T

V2Z= 0.0

VX=V1X-V2X

VY=V1Y-V2Y

VZ=V1Z-V2Z

TX=-CTP

TY=-STP
TZ=0.0
FX= STP
FY=-CTP
FZ=0.0
XNN= FY*VZ-FZ*VX
YNN= FZ*VX-FX*VZ
ZNN= FX*VY-FY*VX

DDD=DSQRT(XNN**2+YNN**2+ZNN**2)

XNM1=XNN/DDD
YNM1=YNN/DDD
ZNMI=ZNN/DDD

DT=TX*XNM1+TY*YNM1+TZ*ZNMI

IF(DT.GE.0.0) GOTO 10

XNM1=-XNM1
YNM1=-YNM1
ZNMI=-ZNMI

CONTINUE

XM1= XMO-RHO*XNM1
YM1= YMO-RHO*YNM1
ZM1= ZMO-RHO*ZNMI

ALP=DARCOS(TX*XNM1+TY*YNM1+TZ*ZNMI)

XX= CM1*XNM1+SM1*ZM1-XG1-XB1*SM1
YY= YM1+EM1
ZZ= -SM1*XNM1+CM1*ZM1-XB1*CM1

XT1=CH1*XNN1+SM1*ZNMI
YT1=YNMI
ZT1=-SM1*XNM1+CM1*ZNMI

PHI1=PH1/CR1
sh1=dsin(phi1)
ch1=dcos(phi1)

X1M= xx
Y1M= CH1*yY+SH1*zZ
Z1M= -SH1*yY+CH1*zZ

XT1= XN1
YT1= CH1*YN1+SH1*ZN1
ZT1= -SH1*YN1+CH1*ZN1

C...

TT=YN1M**2+ZN1M**2

SHIP= (-ZN1M*YNH2+YN1M*ZNH2)/TT
CHIP= ( YN1M*YNH2+ZN1M*ZNH2)/TT

PHIP=2.0D00*DATAN2(SHIP,(1.0D00+CHIP))

C...

XH1= X1M
YH1= CH1*Y1M-SH1*Z1M
ZH1= SH1*Y1M+CH1*Z1M

XNH1= XN1M
YNH1= CH1*YN1M-SH1*ZN1M
ZNH1= SH1*YN1M+CH1*ZN1M

V=-(YH2-YH1)

H=XH2-XH1

C...

F(1)=ZH2-ZH1
F(2)=XNH2-XNH1

C F(2)=YNH2**2+ZNH2**2-TT

C
C

R12=TN1/TN2

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C...
THE FOLLOWING IS THE V-H CHECK SUBROUTINE FOR STRAIGHT BLADE
C...

SUBROUTINE FCNM(X,F,N)
IMPLICIT REAL*8(A-H,O-Z)
real*8 x(N),f(N)
COMMON/A1/CNST,TH1,TH2,C,FW,GAMMA,x1,PL,med
COMMON/A5/SG,XM,YN,ZM,XNM,YNM,ZNM,X2M,Y2M,Z2M,XN2M,YN2M,ZN2M,
&XNH2,YNH2,ZNH2,XH2,YH2,ZH2
COMMON/A7/SRI,Pl,ref,Pl1,Pl2,Pl3,Pl4,Pl5,Pl6,Pl7,Pl8,Pl9,Pl10,Pl11,Pl12
COMMON/A9/PHI1,PHI2,PHI3,PHI4,PHI5,PHI6,PHI7,PHI8,PHI9,PHI10,PHI11,PHI12
COMMON/A11/PHI13,PHI14,PHI15,PHI16,PHI17,PHI18,PHI19,PHI20,PHI21,PHI22,PHI23
COMMON/A21/RAH,PSI1,PSI2,PSI3,PSI4,PSI5,PSI6,PSI7,PSI8,PSI9,PSI10,PSI11,PSI12
COMMON/A22/PSI13,PSI14,PSI15,PSI16,PSI17,PSI18,PSI19,PSI20,PSI21,PSI22,PSI23
COMMON/A23/PSI24,PSI25,PSI26,PSI27,PSI28,PSI29,PSI30,PSI31,PSI32,PSI33,PSI34,PSI35,PSI36
COMMON/A24/PSI37,PSI38,PSI39,PSI40,PSI41,PSI42,PSI43,PSI44,PSI45,PSI46,PSI47,PSI48,PSI49,PSI50
CM=DCOS(GAMMA)
SM=DSIN(GAMMA)
CHP=DCOS(X(1))
SHP=DSIN(X(1))
XX=X2M
YY=-Y2M*CHP+Z2M*SHP
ZZ=-Y2M*SHP-Z2M*CHP
XH2=XX*CM+ZZ*SM
YH2=YY+C
ZH2=-XX*SM+ZZ*CM
C...
XX=XN2M
YY=-YN2M*CHP+ZN2M*SHP
ZZ=-YN2M*SHP-ZN2M*CHP
XNH2=XX*CM+ZZ*SM
YNH2=YY
ZNH2=-XX*SM+ZZ*CM
C...
C...
DEFINE THE PINION SURFACE
C...
C
TH1=X(2)
PH1=X(3)
SP1=DSIN(-ALP1)
CP1=DCOS(-ALP1)
SM1=DSIN(GAMA1)
CM1=DCOS(GAMA1)
STP=DSIN(TH1+PH1)
CTP=DCOS(TH1+PH1)
XNM=CP1*CTP
YNM=CP1*STP
ZNM=SP1
AB1=RCF*STP+SR1*DSIN(-Q1+PH1)
AB2=RCF*CTP+SR1*DCOS(-Q1+PH1)
AXX=-EM1*SM1
AYY=XB1*CM1
AZZ=EM1*CM1
C... FIND SF, CR17, PF, PPF, PCR17
C
PH1 = RA1*(PH1-CPF*PH1**2-EPF*PH1**3-EPF*PH1**4-FPF*PH1**5)
PF = RA1*(1.0-2.0*CPF-PH1**2+3.0*DPF*PH1**2)
$-4.0*EPF*PH1**3+5.0*FPF*PH1**4)
PPF = RA1*(2.0*CPF+6.0*DPF*PH1+12.0*EPF*PH1**2+20.0*FPF*PH1**3)
CR17 = 1.0/PF
PCR17 = -PPF/PPF**3
C
T1 = XNH1*(AXX-AB1*(SM1-CR17)) +
& YNH1*(AYY-AB2*(SM1-CR17)) + ZNH1*(AZZ+AB1*CM1)
T2 = XNH1*(SM1-CR17)**SP1*STP +
& YNH1*(SM1-CR17)**SF1*CTP-CP1*(CM1)+ZNH1*CM1**SP1*STP
SF = T1/T2
XM1 = (RCF-SF*SP1)*CTP+SR1*DCOS(-Q1+PH1)
YM1 = (RCF-SF*SP1)*STP+SR1*DSIN(-Q1+PH1)
ZM1 = SF**CP1
xX = CM1*XM1+SM1*ZM1-XG1-XB1*SM1
yY = YM1*EM1
zZ = -SM1*XM1+CM1*ZM1-XB1*CM1
XN1 = CM1*XNH1+SM1*ZNH1
YN1 = YNH1
ZN1 = -SM1*XNH1+CM1*ZNH1
C
PH1 = PH1/CR1
sh1 = dsin(phil)
ch1 = dcos(phil)
X1M = xX
Y1M = CH1*yY+SH1*zZ
Z1M = -SH1*yY+CH1*zZ
XN1 = XN1
YN1 = CH1*YN1+SH1*ZN1
ZN1 = -SH1*YN1+CH1*ZN1
TT = YN1**2+ZN1**2
SH1P = (-ZN1/YNH2+YN1/ZNH2)/TT
CH1P = (YN1/YNH2+ZNH2/ZNH2)/TT
PH1P = 2.0DO00**DATAN2(SH1P,(1.0DO00+CH1P))
C
SH1P = DSIN(PHI1P)
C
CH1P = DDCOS(PHI1P)
X1H = X1M
Y1H = CH1P*Y1M-SH1P*Z1M
Z1H = SH1P*Y1M+CH1P*Z1M
XNH1 = XN1M
YNH1 = CH1P*YN1M-SH1P*ZN1M
ZNH1 = SH1P*YN1M+CH1P*ZN1M
V = -(YH2-YH1)
H = XH2-XH1
C...
F(1) = ZH2-ZH1
F(2) = XNH2-XNH1
C
F(2) = YNH2**2+ZNH2**2-TT
C
C...
R12 = TN1/TN2
V12X = 0.0-YH2*SM*R12
V12Y = ZH1+R12*(XH2*SM+ZH2*CM)
V12Z = -YH1-R12*YH2*CM
C
V12X = -(YH2-(C-V))**SM*R12
C
V12Y = XH2*SM*R12+(1.0+R12*CM)*ZH2
C
V12Z = -YH2*(1.0+R12*CM)+(C-V)*CM*R12

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F(3)=XNH2*V12X+YNH2*V12Y+ZNH2*V12Z
RETURN
END

SUBROUTINE CAM IS FOR THE COEFFICIENTS OF GENERATION MOTION

SUBROUTINE CAM
IMPLICIT REAL*8(A-H,O-Z)
COMMON/A11/RAM,PSI1,C2,D6,E24,F120,CX6,DX24,EX120,RU1,DELT,RUP,
SRA1,CPF,DPF,EPF,PPF
T1=1.0+3.0*C2*DTAN(PSI1)
&+(1.0-RAM)**3*(RU1**3/15.0**2+DELT)/(15.0*DCOS(PSI1))
T2=1.0+(RU1+DELT)/(15.0*DCOS(PSI1))
CX6=T1/T2
&+(4.0*CX6+3.0*C2**2-1.0)*DSIN(PSI1)
6.0*C2*(1.0-RAM)**2*(RU1**3/15.0**3+DELT/15.0)
&+6.0*C2*(1.0-RAM)**2*(RU1**3/15.0**3+DELT/15.0)
T2=DCOS(PSI1)+(RU1+DELT)/15.0
DX24= T1/T2
&+(4.0*CX6+15.0*C2**2-1.0)*DCOS(PSI1)
&+(10.0*CX6*(1.0-RAM)**2 &+15.0*C2**2*(1.0-RAM))**(RU1**3/15.0**3+DELT/15.0)
&+(1.0-RAM)**5*(RU1**5/15.0**5+DELT/15.0)
T2=DCOS(PSI1)+(RU1+DELT)/15.0
EX120=T1/T2
D6=CX6-3.0*C2**2
E24=DX24+C2*(15.0*C2**2-10.0*CX6)
F120=EX120-15.0*C2**2*DX24+105.0*C2**2*(CX6+C2**2)-10.0*CX6**2
CPF=C2/2.0
DPF=D6/6.0
EPF=E24/24.0
FPF=F120/120.0
RETURN
END
**Abstract**

Computerized simulation of meshing and bearing contact for spiral bevel gears and hypoid gears is a significant achievement that could improve substantially the technology and the quality of the gears. This report covers a new approach to the synthesis of face-milled spiral bevel gears and their tooth contact analysis. The proposed approach is based on the following ideas: (i) application of the principle of local synthesis that provides optimal conditions of meshing and contact at the mean contact point M and in the neighborhood of M; (ii) application of relations between principle directions and curvatures for surfaces being in line contact or in point contact. The developed local synthesis of gears provides (i) the required gear ratio at M; (ii) a localized bearing contact with the desired direction of the tangent to the contact path on gear tooth surface and the desired length of the major axis of contact ellipse at M; (iii) a predesigned parabolic function of a controlled level (8-10 arc seconds) for transmission errors; such a function of transmission errors enables to absorb linear functions of transmission errors caused by misalignment and reduce the level of vibrations. The proposed approach does not require either the tilt of the head-cutter for the process of generation or modified roll for the pinion generation. Improved conditions of meshing and contact of the gears can be achieved without the above mentioned parameters. The report is complemented with a computer program for determination of basic machine-tool settings and tooth contact analysis for the designed gears. The approach is illustrated with a numerical example.