Probabilistic Simulation of Uncertainties in Thermal Structures

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PROBABILISTIC SIMULATION OF UNCERTAINTIES IN THERMAL STRUCTURES

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SUMMARY

Development of probabilistic structural analysis methods for hot structures is a major activity at NASA Lewis Research Center. It consists of five program elements: (1) probabilistic loads, (2) probabilistic finite element analysis, and (3) probabilistic material behavior, (4) assessment of reliability and risk, and (5) probabilistic structural performance evaluation. Recent progress includes: (1) quantification of the effects of uncertainties for several variables on High Pressure Fuel Turbopump (HPFT) blade temperature, pressure, and torque of the Space Shuttle Main Engine (SSME), (2) the evaluation of the cumulative distribution function for various structural response variables based on assumed uncertainties in primitive structural variables, (3) evaluation of the failure probability, (4) reliability and risk-cost assessment, and (5) an outline of an emerging approach for eventual hot structures certification. Collectively, the results demonstrate that the structural durability/reliability of hot structural components can be effectively evaluated in a formal probabilistic framework. In addition, the approach can be readily extended to computationally simulate certification of hot structures for aerospace environments.

INTRODUCTION

It is becoming increasingly evident that deterministic structural analysis methods will not be sufficient to properly design critical components in hot structures in general and propulsion structures in particular. These structural components are subjected to a variety of complex, and severe cyclic loading conditions, including high temperatures and high temperature gradients. Most of these are quantifiable only as best engineering estimates. These complex loading conditions subject the material to coupled nonlinear behavior which depends on stress, temperature, and time. Coupled nonlinear material behavior is nonuniform, is very difficult to determine experimentally, and it is difficult (if not impossible) to quantify deterministically. In addition, hot rotating structural components are relatively small. Fabrication tolerances on these components, which in essence are small thickness variations, can have significant effects on the component structural response. Fabrication tolerances by their very nature are statistical. Furthermore, the attachment of components in the structural system generally differs by some indeterminant degree from that assumed for designing the component. In summary, all four fundamental aspects -- (1) loading conditions, (2) materials behavior, (3) materials behavior, (4) assessment of reliability and risk, and (5) probabilistic structural performance evaluation.
geometric configuration, and (4) boundary conditions (on which structural analyses are based) -- are of random nature. The direct way to formally account for all these uncertain aspects is to develop probabilistic structural analysis methods where all participating variables are described by appropriate probability functions.

The development of the probabilistic structural analysis methodology is an on-going joint program of NASA Lewis Research Center in-house and sponsored research (ref. 1). Theoretical considerations, computer codes, and other relevant applications are described in papers presented in various recent conferences (refs. 2 to 32). Activities and progress up to June 1989 are summarized in reference 33. The objectives of this invited paper are (1) to provide a brief description of the fundamental aspects and (2) to demonstrate the application of this methodology to a specific example (the reliability/risk of turbine blade components of rocket propulsion systems). The specific example includes the four key elements required in probabilistic structural analysis of thermal structures, namely: (1) probabilistic loads, (2) probabilistic finite element analysis, (3) probabilistic description of coupled nonlinear material behavior, and (4) evaluation of reliability and risk. Throughout the specific example discussion appropriate comments are included to illustrate the generality of the method and its application to a variety of hot structures.

### FUNDAMENTAL CONSIDERATION

Central to the probabilistic structural analysis described herein is the fundamental consideration that: Uncertainties observed in the structural performance (displacements), frequencies, buckling, global fracture toughness, stresses/strains of hot structures can be quantified in terms of corresponding uncertainties in basic parameters (primitive variables). The primitive variables are those which are used to describe the structure and its respective environment. For example: (1) structural configuration, (2) boundary conditions, (3) loading conditions, and (4) material thermomechanical nonlinear behavior.

The uncertainties in these primitive variables are then integrated through structural mechanics to quantify the uncertainties in the global structural responses (example-displacements) and are decomposed to quantify the uncertainties in local responses (example-stresses/strains). The concept is schematically illustrated in figure 1. The structural component is the blade which is modelled for finite element analysis. The input uncertainties are the blade loads (centrifugal, pressure, and temperature), geometry and material variables. The output is quantification of uncertainties in structural responses or in local stresses for probable fracture initiation. Brief descriptions are given on subsequent sections on each of these as it is applied to the specific example.

### PROBABILISTIC SIMULATION OF LOADS

The fundamental assumption for the probabilistic simulation of loads is that each individual load condition is the probabilistic time synthesis of four primitive parts: (1) steady state, (2) periodic, (3) random, and (4) spike. Each of these parts, except random, is described by a deterministic portion and a probabilistic perturbation about this deterministic portion. The resulting distribution is similar to the schematic, in figure 1 upper left and as described in reference 33. One justification for describing each loading condition in terms of primitive parts is...
that experts, over the years, have developed good judgment on the ranges of the perturbations about nominal (deterministic) conditions. A computer code with dedicated expert systems (Composite Load Spectra) has been developed to synthesize these four parts by using (1) available data from various SSME engines, (2) probability theory, and (3) expert opinion, as depicted schematically in figure 2.

The results from the application of the Composite Load Spectra computer code to probabilistically simulate loads for two blades are summarized in table I. The first 5 lines are uncertainties in some of the engine specific primitive variables, while the last four are the predicted loads. The comparisons of predicted loads with the measured data are in very good agreement considering the large number of primitive variables (47) required to synthesize these loads. The conclusion is that methods can be developed and are available to probabilistically synthesize complex load conditions for hot structures.

PROBABILISTIC FINITE ELEMENT STRUCTURAL ANALYSIS

The fundamental assumption for developing probabilistic finite element structural analysis (PFEM) is that the uncertainties in each primitive structural variable can be represented by a probabilistic distribution. Primitive structural variables are those which are used to describe a structure such as: (1) stiffness, (2) strength, (3) thickness and tolerance, (4) spatial location, (5) attachment, and (6) various nonlinear material dependencies (temperature, stress, time, etc) typical to the schematic in figure 1 upper right (ref. 33). Subsequently, the uncertainties in the load conditions (described by the composite load spectra) and the uncertainties in the primitive structural variables are computationally synthesized by performing probabilistic finite element structural analysis to simulate uncertainties in the structural response of a specific SSME structural component. The structural response such as displacement, frequencies, buckling loads, and global fracture toughness as was already mentioned, is generally described in terms of cumulative probability distribution.

PFEM has been formalized and integrated into a computer code identified as NESSUS (Numerical Evaluation of Stochastic Structures Under Stress). NESSUS is driven by an expert system. A schematic diagram of NESSUS is shown in figure 3. Representative results obtained using NESSUS for one of the blades are shown in figure 4 for stress at two different points. The distributions assumed for the primitive variables are listed in table II in addition to the blade loads from the CLS computer code. The combined effects of the primitive variable uncertainties on effective effect stress are shown in figure 4 in terms of cumulative probability distribution (CPD). The information generated during the generation of the CPD can also be used to evaluate the sensitivities as shown in figure 4 for each point. The significant point is that the PFEM yields a wealth of information which can be used to evaluate: (1) the uncertainties in the local structural response, and (2) the sensitivities which can be used to adjust the design for enhanced probability of success. The important conclusion is that probabilistic finite element methods can be developed and are available to quantify uncertainties in the structural performance of hot structures. In addition the sensitivities in the primitive variables that influence this performance can be evaluated and ranked.
PROBABILISTIC SIMULATION OF MATERIAL BEHAVIOR

The fundamental assumptions to probabilistically simulate nonlinear material behavior are: (1) a relationship for material behavior can be developed in terms of primitive variables affecting this behavior and (2) the uncertainties in the primitive variables can be described by assumed distributions (ref. 33). A multifactor interaction model (MFIM) for this relationship is shown in figure 5, where its constituent primitive variables are identified.

This MFIM is applied to the blade to probabilistically determine: (1) the resistance curve for damage (crack) initiation and (2) damage propagation and its effects on global structural response. The inputs are summarized in table III. The results for the most probable damage path, with a probability of occurrence of 0.0002, and the respective degradation in frequencies are shown in figure 6. The important observation is that the uncertainties in damage initiation, propagation and subsequent effects on structural performance can be probabilistically simulated by the methodology described herein. It is worthy of note that this methodology can readily be incorporated to monitor the in-service health of aerospace hot structures (provide suitable monitoring devices are available).

RISK-COST ASSESSMENT

The methodology described previously can be extended to perform reliability and risk-cost assessments. In order to accomplish this, (1) the cost for component flight readiness needs to be quantified and (2) the cost as a consequence of failure must be established. Assumed aspects of both of these have been integrated into the methodology described herein (ref. 34). Application of this methodology to the blade in the previous section yields the results summarized in figure 7.

It can be seen in figure 7 that: (1) the probability of damage initiation can be evaluated versus fatigue cycles and (2) the total cost, which is used to assess risk, is evaluated versus fatigue cycles. The important observation from the aforementioned discussion is that the reliability and risk-cost of hot structures can be assessed using the probabilistic methods of the type described herein. The implications are far-reaching because these methods can be applied to existing hot structures to evaluate their risk for continuing service as well as those on the design board and those still in the conceptual phase.

RELIABILITY/CERTIFICATION -- AN EMERGING APPROACH

The collective observations from the previous discussion lead to an emerging approach to computationally simulate hot structures reliability, risk, component qualification, and eventually vehicle structure certification. The general steps for this emerging approach are outlined as follows:

1. Develop a hot structural component/vehicle (global) analysis model.

2. Conduct probabilistic structural analysis (PSA) of the types described herein.

3. Identify the critical component/vehicle areas from the results of PSA.
4. Perform global/local PSA to evaluate nonlinear effects and to locate probable sites of damage initiation.

5. Determine the most probable damage propagation path.

6. Evaluate probable structural degradation along this path.

7. Establish probable path extent for violation of specified structural performance criteria (for example, 10-percent reduction in the first vibration mode).

8. Assess corresponding reliability and risk and decide on their acceptability.

9. Schedule inspection intervals and retirement for cause criteria based on the results of items 5 to 7.

10. Verify with probabilistically selected (using respective sensitivities) critical hot-structures components and prototype tests.

11. Design a suitable in-service health monitoring system using the results from items 8 and 9 above in order to ascertain that the component/structure will meet the acceptable reliability and risk.

CONCLUSIONS

A methodology has been developed for the formal probabilistic quantification of uncertainties in the structural performance of aerospace hot structures. The key elements in this methodology are: (1) probabilistic load simulation, (2) probabilistic finite element analysis, (3) probabilistic simulation of thermomechanical nonlinear material behavior, and (4) risk-cost assessment. This methodology is described herein in terms of fundamental aspects and application to a specific structural component which is a turbopump blade of the Space Shuttle Main Engine (SSME). The specific example illustrates how the uncertainties in all the basic parameters (primitive variables) for loads, structure and material behavior are incorporated in order to probabilistically simulate the uncertainties in the structural response (global and local). Also, the example illustrates how the reliability and risk-cost can be assessed. Collectively, the summary of the fundamental considerations and the results from the specific example demonstrate that a formal methodology is available to evaluate the reliability and risk-cost of hot structures in aerospace environments. In addition, an emerging approach is outlined which can be used to computationally qualify and eventually certify hot structures.

REFERENCES


<table>
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<tr>
<th>Condition</th>
<th>HPOTP</th>
<th></th>
<th>HPFTP</th>
<th></th>
</tr>
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<tr>
<td></td>
<td>Speed</td>
<td>Turbine temperature</td>
<td>Speed</td>
<td>Turbine discharge temperature</td>
</tr>
<tr>
<td></td>
<td>Calculated, rpm</td>
<td>Measured, rpm</td>
<td>Calculated, °R</td>
<td>Measured, °R</td>
</tr>
<tr>
<td>Hardware - 2σ random</td>
<td>294</td>
<td>53</td>
<td>288</td>
<td>65</td>
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<tr>
<td>Test - 2σ random</td>
<td>210</td>
<td>157</td>
<td>306</td>
<td>20</td>
</tr>
<tr>
<td>Total random</td>
<td>360</td>
<td>165</td>
<td>554</td>
<td>70</td>
</tr>
<tr>
<td>Low NPSP - det</td>
<td>620</td>
<td>225</td>
<td>56</td>
<td>52</td>
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<tr>
<td>High NPSP - det</td>
<td>-317</td>
<td>-219</td>
<td>-94</td>
<td>-62</td>
</tr>
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<td>Range = random + det</td>
<td>1 660</td>
<td>475</td>
<td>1 260</td>
<td>114</td>
</tr>
<tr>
<td>Max</td>
<td>28 090</td>
<td>1630</td>
<td>35 742</td>
<td>1740</td>
</tr>
<tr>
<td>Nom</td>
<td>28 100</td>
<td>1374</td>
<td>35 130</td>
<td>1688</td>
</tr>
<tr>
<td>Min</td>
<td>27 430</td>
<td>1155</td>
<td>34 482</td>
<td>1625</td>
</tr>
</tbody>
</table>

Measured: Measured variation for phase II test set.
Hardware: Variations engine hardware.
Test: Initial lost conditions - inlet temperatures and mixture ratio.
Det: Duty cycle effects of inlet pressures plus correlated 2σ variations of cavitation.
### TABLE II. - RANDOM VARIABLES CONSIDERED AND THEIR STATISTICS

<table>
<thead>
<tr>
<th>Number</th>
<th>Random variable</th>
<th>Type</th>
<th>Affected FEM quantities</th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>Material axis Z</td>
<td>Material orientation</td>
<td>Anisotropic</td>
<td>-0.087266 radian</td>
<td>0.067544</td>
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<tr>
<td>2</td>
<td>Material axis Y</td>
<td>Material orientation</td>
<td>material</td>
<td>-0.034907</td>
<td>0.067544</td>
</tr>
<tr>
<td>3</td>
<td>Material axis X</td>
<td>Effects</td>
<td>Orientation angles</td>
<td>-0.052360</td>
<td>0.067544</td>
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<tr>
<td>4</td>
<td>Elastic modulus</td>
<td>Material properties</td>
<td>Elastic constants</td>
<td>18.38E6 psi</td>
<td>0.4595E6</td>
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<tr>
<td>5</td>
<td>Poisson's ratio</td>
<td></td>
<td>0.386</td>
<td>0.00965</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Shear modulus</td>
<td></td>
<td>18.63E6 psi</td>
<td>0.046575E6</td>
<td></td>
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<tr>
<td>7</td>
<td>Geometric lean</td>
<td>Geometrical variations</td>
<td>Node coordinates</td>
<td>0 deg</td>
<td>0.14 deg</td>
</tr>
<tr>
<td>8</td>
<td>Geometric tilt</td>
<td></td>
<td>0 deg</td>
<td>0.14 deg</td>
<td></td>
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<tr>
<td>9</td>
<td>Geometric twist</td>
<td></td>
<td>0 deg</td>
<td>0.30 deg</td>
<td></td>
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<tr>
<td>10</td>
<td>Mixture ratio</td>
<td>System independent loads</td>
<td>Pressure, temperature, centrifugal force</td>
<td>6.0</td>
<td>0.02</td>
</tr>
<tr>
<td>11</td>
<td>Fuel inlet pressure</td>
<td></td>
<td>30.0 psi</td>
<td>5.00</td>
<td></td>
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<tr>
<td>12</td>
<td>Oxidizer inlet pressure</td>
<td></td>
<td>100.00 psi</td>
<td>26.00</td>
<td></td>
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<tr>
<td>13</td>
<td>Fuel inlet temperature</td>
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<td>37 °F</td>
<td>0.50</td>
<td></td>
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<tr>
<td>14</td>
<td>Oxidizer inlet temperature</td>
<td></td>
<td>164 °F</td>
<td>1.33</td>
<td></td>
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<tr>
<td>15</td>
<td>Pump efficiency</td>
<td>Component independent loads</td>
<td>Pressure, temperature, centrifugal force</td>
<td>1.00</td>
<td>0.008</td>
</tr>
<tr>
<td>16</td>
<td>Head coefficient</td>
<td></td>
<td>1.024</td>
<td>0.008</td>
<td></td>
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<tr>
<td>17</td>
<td>Coolant seal leakage</td>
<td>Local effects</td>
<td>Temperature</td>
<td>1.00</td>
<td>0.10</td>
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<tr>
<td>18</td>
<td>Hot gas seal leakage</td>
<td></td>
<td>1.00</td>
<td>0.05</td>
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</table>

### TABLE III. - PRIMITIVE VARIABLE PROBABILITY DISTRIBUTIONS FOR PROBABILISTIC MATERIAL PROPERTY MODEL

<table>
<thead>
<tr>
<th>Variable</th>
<th>Distribution type</th>
<th>Mean</th>
<th>Standard deviation</th>
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<tr>
<td></td>
<td></td>
<td></td>
<td>Value</td>
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<tr>
<td>$T_F$</td>
<td>Normal</td>
<td>2750 °F</td>
<td>51.4 °F</td>
</tr>
<tr>
<td>$T_O$</td>
<td>Normal</td>
<td>68 °F</td>
<td>2.04 °F</td>
</tr>
<tr>
<td>$S_F$</td>
<td>Normal</td>
<td>212.0 ksi</td>
<td>10.6 ksi</td>
</tr>
<tr>
<td>$\sigma_o$</td>
<td>Constant</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$N_{WF}$</td>
<td>Lognormal</td>
<td>$10^8$</td>
<td>$5 \times 10^6$</td>
</tr>
<tr>
<td>$N_{MO}$</td>
<td>Lognormal</td>
<td>$10^3$</td>
<td>50</td>
</tr>
<tr>
<td>$n$</td>
<td>Normal</td>
<td>0.25</td>
<td>--------</td>
</tr>
<tr>
<td>$p$</td>
<td>Normal</td>
<td>0.25</td>
<td>--------</td>
</tr>
<tr>
<td>$q$</td>
<td>Normal</td>
<td>0.25</td>
<td>--------</td>
</tr>
</tbody>
</table>
Turbine blade loading

Nessus turbine blade coarse model

Geometry and material behavior model

Centrifugal

Input variables

Pressure and temperature

Figure 1.—Probabilistic simulation of component reliability using CLS coupled with PSAM and PMBM.

LeRC contacts:
CLS - Composite Load Spectra - Rocketdyne
PSAM - Probabilistic Structural Analysis Methods - SWRI
PMBM - Initial assessment - In-house

Figure 2.—Composite load spectra simulation using expert systems.

Composite load spectra expert system

LDEXPT design
Load expert system design philosophy
- Rule base production system
  - If then rules
- Simple inference scheme
  - Inference net (decision tree)
- Sophisticated probabilistic methods
  - Discrete probability distribution
  - Monte Carlo
  - Barrier crossing
- Powerful knowledge base
  - Influence coefficient
  - Scaling coefficients
  - Duty cycle load profiles
  - Engine configuration and geometry data
  - Raw engine flight and test data

Input

User specified requirement

Rule base

Driver (user interface module)

Data base

Output

Composite load spectra

• Probable damage initiation
- Resistance scatter
Figure 3.—Computational simulation of probabilistic structural response using Nessus.

Figure 4.—Probabilistic component stress analysis.
\[ M_p = M_p^0 \left( \frac{T_f - T}{T_f - T_0} \right)^n \left( \frac{S_f - \sigma}{S_f - \sigma_0} \right)^p \left( \log \frac{N_{MF}}{N_{MO}} - \log \frac{N_M}{N_M^0} \right)^q \]

**Primitive variables**
- \( M_p \): Degraded material property
- \( T \): Temperature
- \( S \): Stress
- \( N_M \): Mechanical cycles

**Subscripts**
- \( F \): Final characteristic value
- \( O \): Reference property

**Figure 5.**—Generic probabilistic material property degradation model in terms of primitive variables.

**Figure 6.**—Natural frequencies decrease as fracture progresses.
Figure 7.—Probabilistic risk-cost assessment.
**Probabilistic Simulation of Uncertainties in Thermal Structures**

**Abstract**

Development of probabilistic structural analysis methods for hot structures is a major activity at Lewis Research Center. It consists of five program elements: (1) probabilistic loads, (2) probabilistic finite element analysis, and (3) probabilistic material behavior, (4) assessment of reliability and risk, and (5) probabilistic structural performance evaluation. Recent progress includes: (1) quantification of the effects of uncertainties for several variables on High Pressure Fuel Turbopump (HPFT) blade temperature, pressure, and torque of the Space Shuttle Main Engine (SSME), (2) the evaluation of the cumulative distribution function for various structural response variables based on assumed uncertainties in primitive structural variables, (3) evaluation of the failure probability, (4) reliability and risk-cost assessment, and (5) an outline of an emerging approach for eventual hot structures certification. Collectively, the results demonstrate that the structural durability/reliability of hot structural components can be effectively evaluated in a formal probabilistic framework. In addition, the approach can be readily extended to computationally simulate certification of hot structures for aerospace environments.

**Key Words**

Structural analysis; Finite elements; Strength degradation; Fracture mechanics; Failure probability; Structural durability; Fatigue life; Probability distributions

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