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Summary

A computer code for a single-point thermodynamic analysis of hydrogen/oxygen expander-cycle rocket engines is described in this report. Full, split, and dual expander cycles are included. Heat regeneration between the turbine exhaust and the pump exhaust is allowed. The combustion process is modeled as one of chemical equilibrium in either an infinite-area or a finite-area combustor. Gas composition in the nozzle during expansion may be either equilibrium or frozen.

This report, which serves as a users guide for the computer code, describes the system, the analysis methodology, and the program input and output and contains sample calculations to show the effects of key variables such as nozzle area ratio and oxidizer-to-fuel mass ratio. For this analysis, ideal specific impulse is a function of only nozzle area ratio and oxidizer-to-fuel mass ratio and is independent of expander-cycle type, chamber pressure, regenerator effectiveness, and turbomachine efficiencies. These latter parameters primarily affect the pump exit pressure required to achieve the specified chamber pressure.

A single-point analysis is useful for determining system performance from known component performance and for performing sensitivity analyses over narrow ranges of the cycle variables. In analyzing a given engine over a wide range of cycle variables, care must be taken to use appropriate variations in component performance parameters.

Introduction

Expander-cycle engines are the primary propulsion candidates for orbital transfer missions. The advantages of an expander-cycle engine are good specific impulse, engine simplicity, and relatively low weight (ref. 1). For a topping cycle, such as the expander cycle, however, the fuel-pump discharge pressure is much higher than that for a bleed cycle, such as the gas generator cycle, and this tends to limit the achievable combustion chamber pressure.

In support of space propulsion studies being conducted at the NASA Lewis Research Center, a computer code for the single-point thermodynamic analysis of hydrogen/oxygen expander-cycle rocket engines was developed. This code is applicable to full, split, and dual expander cycles, which can include a regenerator. Vacuum specific impulse, cycle temperatures, and cycle pressures are computed. Real fluid properties and methodologies are used for all computations except combustion and expansion. Combustion and nozzle expansion are modeled with one-dimensional, ideal-gas calculations. The combustion process is one of chemical equilibrium in either an infinite-area or a finite-area chamber. Expansion in the nozzle may be either an equilibrium or a frozen chemical process.

This report presents the cycle analysis methodology and also serves as a users guide for the computer code. The expander cycles are described and the cycle analysis equations are presented. Program input and output are described and sample cases are included. The sample calculations are intended to show the effects of some of the key variables on the cycle performance.

System Description

The computer code presented herein can be used to analyze the single-point performance of full, split, and dual expander rocket engine cycles. Each of these cycles can incorporate a regenerator on the fuel side to transfer heat from the turbine exit stream to the pump exit stream. The three cycles are described in this section. The format for the system sketches is that used by Pratt & Whitney in reference 2.

The full expander cycle with a regenerator is shown in figure 1. Liquid hydrogen is pumped to high pressure and then flows through the regenerator, where heat is transferred to it from the turbine exit stream. The hydrogen is further heated as it regeneratively cools the nozzle and the combustion chamber. Most of the heated hydrogen then expands in series through the oxygen-pump and hydrogen-pump drive turbines. Alternatively, the order of the flow through the turbines can be reversed or the flow can be in parallel. Some of the heated hydrogen from the combustion chamber cooling jacket is bypassed around the turbines (and the regenerator) to provide a means for controlling turbine power. After transferring heat to the pump exit stream in the regenerator, the turbine exit flow mixes with the bypass flow, passes through the fuel shutoff valve, and enters the combustion chamber. On the oxidizer side the liquid oxygen is pumped to the pressure required to match the hydrogen pressure at the combustor. It then flows through the oxidizer control valve, which controls the oxidizer-to-fuel (o/f) ratio, and enters the combustion chamber, where combustion occurs. The combustion gases then expand through the nozzle to high velocity to provide the thrust.
A split expander cycle is shown in figure 2. This cycle derives its name from the fact that the hydrogen flow is split in the fuel-pump system. Only part of the hydrogen is pumped to maximum pressure for use as the chamber/nozzle regenerative coolant and the turbine drive fluid. This figure shows parallel flow through the turbines, which is one of the available modeling options. The remaining hydrogen flows through the jacket bypass valve, which controls the flow split, and mixes with the turbine exhaust and bypass streams before entering the combustion chamber. A regenerator can be incorporated into this cycle in a similar manner as for the full expander cycle (fig. 1).

A dual expander cycle is shown in figure 3. In this cycle each of the fluids is heated by regenerative cooling of the nozzle or the combustion chamber, or both, and used in a separate turbine to drive its own pump. Thus, the fluids can be kept separated before combustion without relying on the turbopump seals. A bypass valve controls the flow to each turbine. If desired, a regenerator can be incorporated on the fuel side of the cycle.

The method of analysis is documented later in this report. Symbols used for the analysis are defined in appendix A, and the analysis method is presented in appendix B.

Program Input and Output

Input

The input data consist of namelist INPUT, which is required for the cycle calculations, and THERMO data, REACTANTS data, and namelists INPT2 and RKTINP, which are used for
the combustion and expansion calculations performed by the Gordon and McBride CEC code (refs. 3 and 4). The CEC input data are described in detail in references 3 and 4 and are described here only to the extent necessary to run this program without consulting the CEC references. Input for a sample case is presented in figure 4. The output corresponding to this sample input is presented and described in the next section.

Namelist INPUT is read from unit 2. The INPUT variables along with descriptions, units, and any special instructions are presented in the following list:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Units</th>
<th>Special Instructions</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>fuel-pump inlet pressure (saturation temperature is assumed)</td>
<td>psi</td>
<td></td>
</tr>
<tr>
<td>P12</td>
<td>oxidizer-pump inlet pressure (saturation temperature is assumed)</td>
<td>psi</td>
<td></td>
</tr>
<tr>
<td>PC</td>
<td>combustion chamber injection pressure</td>
<td>psi</td>
<td></td>
</tr>
<tr>
<td>TBP</td>
<td>fuel-turbine bypass fraction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OBP</td>
<td>oxidizer-turbine bypass fraction (dual expander only)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>JBP</td>
<td>cooling-jacket bypass fraction (split expander only)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TC</td>
<td>thrust correction factor, ratio of actual thrust to ideal thrust</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OF</td>
<td>oxidizer-to-fuel flow ratio</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ETAFP</td>
<td>fuel-pump efficiency</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ETAFT</td>
<td>fuel-pump drive turbine efficiency</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ETALP</td>
<td>oxidizer-pump efficiency</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ETALT</td>
<td>oxidizer-pump drive turbine efficiency</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ETABP</td>
<td>low-pressure, fuel-pump efficiency (split expander only)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EPS</td>
<td>regenerator effectiveness (input only for IREG = 1)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Namelist INPUT is read from unit 2. The INPUT variables ETAFP, ETAFT, ETALP, ETALT, and ETABP are described here only to the extent necessary to run this program without consulting the CEC references. Input for a sample case is presented in figure 4. The output corresponding to this sample input is presented and described in the next section.

Figure 4.—Sample input.
DPCREG  regenerator cold-side fractional pressure loss
(input only for IREG = 1)

DPHREG  regenerator hot-side fractional pressure loss
(input only for IREG = 1)

DPFI    fuel-stream (line/valve/injector) fractional pressure loss

DPLI    oxidizer-stream (line/valve/injector) fractional pressure loss

DPBI    bypass-stream (line/valve/injector) fractional pressure loss

DPLOSS nozzle/chamber cooling jacket fractional pressure loss
(full and split expanders only)

DTRISE  nozzle/chamber coolant temperature rise, °R
(full and split expanders only)

DPCHM   chamber cooling jacket (fuel) fractional pressure loss (dual expander only)

DPNOZ   nozzle cooling jacket (oxidizer) fractional pressure loss (dual expander only)

DTCHM   chamber coolant (fuel) temperature rise, °R
(dual expander only)

DTNOZ   nozzle coolant (oxidizer) temperature rise, °R
(dual expander only)

CASE    cycle-type indicator
1—full expander cycle
2—split expander cycle
3—dual expander cycle

IREG    regenerator indicator
1—no regenerator
2—fuel-side regenerator

IHO     turbine arrangement indicator (full and split expanders only)
1—series with fuel turbine followed by oxidizer turbine
2—series with oxidizer turbine followed by fuel turbine
3—parallel

IDBG    debug output indicator
0—no additional output
1—values of pressure p, absolute temperature t, and enthalpy h for each station printed during calculations for both unconverged and converged passes

The CEC input data are read from unit 5 and consist of THERMO data, REACTANTS data, and namelists INPT2 and RKTINP. The THERMO data are the least-squares, curve-fit coefficients of the thermodynamic data for the species H, HO, H₂, H₂O (vapor and liquid), H₂O₂, O, OH, O₂, and O₃. These are the species considered for the equilibrium calculations for the H₂/O₂ system. These THERMO data are used exactly as presented in figure 4. Formats are included in the table. Each time the THERMO data are read from unit 5, they are written in binary format to unit 4 and subsequently read from unit 4 for use in the program. Therefore, once the binary file exists, the THERMO data need not be included in the input data read from unit 5.

The REACTANTS data identify each of the reactant species. These data are used exactly as presented in figure 4. Formats are included in the figure.

The only variables to be included in namelist INPT2 are

P must be set equal to value of PC in namelist INPUT

MIX must be set equal to value of OF in namelist INPUT

The variables that need to be included in namelist RKTINP are

EQL  logical variable set to .TRUE. for equilibrium chemical composition in the rocket nozzle

FROZ logical variable set to .TRUE. for frozen chemical composition in the rocket nozzle (defaults to .FALSE. if FAC = .TRUE.)

FAC  logical variable specifying combustion process
      .TRUE.—finite-area combustion, which includes total-pressure loss due to heat addition
      .FALSE.—infinite-area combustion

ACAT ratio of combustor area to throat area (input only if FAC = .TRUE.)

SUPAR rocket-nozzle supersonic area ratio (exit area to throat area)

Output

Program output consists of normal output, diagnostic output, and error messages.

Normal output.—Normal output, which is written to unit 7, consists of a listing of input variable values, gas composition after both combustion and expansion, state conditions at all cycle points, and rocket performance. Figure 5 presents the normal output that corresponds to the sample input shown in figure 4.

System configuration and input values are presented descriptively in the output (fig. 5). This particular case is for a full expander cycle with regenerator. The key parameters are a combustion chamber injection pressure of 1500 psia, an oxidizer-to-fuel ratio of 6.5, and a nozzle area ratio of 1000. All the component efficiencies and pressure losses are shown.

This is specified to be a case with finite-area combustion and equilibrium expansion. Equilibrium gas compositions in terms of species mole fractions are shown for the combustor exit and the nozzle exit. The gas temperature at the nozzle exit is sufficiently low that the only species there are unburned hydrogen and steam. If the expansion were specified to be frozen, the nozzle exit composition would be the same as the combustion chamber composition.
ROCKET ENGINE CYCLE CODE
FULL EXPANDER CYCLE WITH REGENERATOR

FUEL PRESSURE = 18.0 PSIA
OXYGEN PRESSURE = 16.0 PSIA
INJECTION PRESSURE = 1500.0 PSIA
THRUST CORRECTION FACTOR = 1.00
FUEL TURBINE BYPASS RATIO = 0.05
O/F = 6.5

COMPONENT EFFICIENCIES:
H2 PUMP H2 TURBINE O2 PUMP O2 TURBINE H2 LP PUMP REGENERATOR
0.650 0.850 0.700 0.850 1.000 0.40

NOZZLE JACKET DELTA T = 650.0 R.
NOZZLE JACKET (DELTA P)/P (HOT AND COLD SIDES) = 0.04 AND 0.02
FUEL INJECTOR (DELTA P)/P = 0.15
LOX INJECTOR (DELTA P)/P = 0.35

OPTIONS:
CASE = 1 IREG = 2 IMO = 1

EQUILIBRIUM MOLE FRACTIONS IN CHAMBER AND AT EXIT:
H 0.03274 0.00000
H2 0.00010 0.00000
H20 0.69765 0.81898
H202 0.00002 0.00000
O 0.05619 0.00000
O2 0.00704 0.00000

PRESSURE
PSIA

FUEL PUMP INLET 18.00
FUEL HP PUMP EXIT : 3322.14
REGENERATOR COLD EXIT: 3254.70
COOLING JACKET EXIT : 2767.34
FUEL TURBINE EXIT : 2018.00
LOX TURBINE EXIT : 1764.76
TURBINE BYPASS : 2767.34
MIXER EXIT : 1764.76
FUEL INJECTOR EXIT : 1500.00
OXYGEN PUMP INLET : 16.00
LOX PUMP EXIT : 2507.60
LOX INJECTOR EXIT : 1500.00

EQUILIBRIUM PERFORMANCE PARAMETERS:
NOZZLE AREA RATIO = 1000
CHARACTERISTIC VELOCITY = 7476. FT/SEC.
NOZZLE EXIT MACH NUMBER = 6.912.
CHAMBER MACH NUMBER = 0.246
VACUUM SPECIFIC IMPULSE = 493.4 LBF-SEC/LBM

Figure 5.—Sample output.

Temperatures, pressures, and flow rates are shown for all cycle calculation stations. Flow rates are based on 1 lb/sec of fuel, since size effects are not considered in this single-point thermodynamic analysis. Also shown are turbomachinery power requirements. For this case, a fuel-pump exit pressure of 3322 psia is required to provide the specified combustion chamber injection pressure of 1500 psia. The finite-area combustion, which occurred at a chamber Mach number of 0.246, resulted in a reduction in chamber stagnation pressure from 1500 to 1452 psia. Rocket performance is presented in
entire computation is printed. Pressures, temperatures, and

A longer diagnostic output, as shown in figure 7, tracking the
diagnostic output is written to unit 6

terms of characteristic velocity and vacuum specific impulse,
which are 7476 ft/sec and 493 sec, respectively, for this case.

Also shown is the nozzle exit Mach number, which is 6.9.

Diagnostic output.—Diagnostic output is written to unit 6

in two forms. A tracking of the convergence of combustion
chamber injection pressure to the required input value is always
printed as shown in figure 6. If the input variable IDBG = 1,
a longer diagnostic output, as shown in figure 7, tracking the
entire computation is printed. Pressures, temperatures, and

Figure 6.—Chamber pressure convergence.

Figure 7.—Detailed diagnostic output.

enthalpies are shown for the various calculation stations, and
required-versus-calculated powers are shown for the turbines.
Figure 7 shows one fuel-pump exit pressure iteration loop.
The regenerator iteration loop is indicated as are the turbine
iterations to convergence.

Error messages.—The program contains three output
messages that indicate abnormal operation due to either the
nonexistence of a solution satisfying the specified input require-
ments or the inability of a convergence algorithm to find a
solution. These messages are presented in this section, and their causes are discussed. In general, when one of the following messages appears, the program input should be checked for errors:

(1) TURBINE N NOT CONVERGED!—The value of N is either 1 or 2 depending on which turbine in the sequence is not converged. This message is caused either by insufficient energy in the turbine drive fluid or by a failure in the convergence scheme. Corrective action includes reducing input chamber pressure or increasing turbine drive fluid energy, or both.

(2) NO SOLUTION FOR CHAMBER PRESSURE, MAX PC = XXXXX.—This message is caused by the maximum achievable chamber pressure (shown in error message) being less than the chamber pressure specified by program input. If the input design parameters are correct, the input chamber pressure must be reduced.

(3) CHAMBER PRESSURE SOLUTION NOT FOUND IN 100 ITERS—This message is caused by the program making 100 iterations without either finding a pump exit pressure that yields the required chamber pressure or determining that none exists. This situation has not occurred and there is no obvious reason for it to occur, but the possibility does exist.

Sample Calculations

Sample calculations were made to illustrate the effects of some key system parameters on cycle performance and operating conditions. The effects of nozzle area ratio and oxidizer-to-fuel mass ratio on ideal vacuum specific impulse were explored along with the effects of chamber pressure and regenerator effectiveness on pump exit pressure.

The effect of nozzle area ratio (i.e., the ratio of exit area to throat area) on ideal vacuum specific impulse is presented in figure 8. Ideal specific impulse increased very rapidly with increasing area ratio up to an area ratio of about 200, after which the increase tapered off. Going from an area ratio of 1000 to one of 2000 resulted in a 1-percent increase in specific impulse.

The effect of oxidizer-to-fuel mass ratio on ideal vacuum specific impulse is presented in figure 9. Maximum specific impulse occurred at an o/f ratio of about 6.5. Although combustion temperature was maximum at the stoichiometric o/f ratio of 8, the increase in reaction-product molecular weight with increasing o/f ratio caused the exit velocity, and therefore the specific impulse, to reach a maximum at an o/f ratio somewhat less than stoichiometric.

Figures 8 and 9 were discussed without reference to a specific cycle (full, split, or dual) because those results were independent of cycle type. For this analysis, ideal specific impulse is a function of only nozzle area ratio and o/f ratio and is independent of cycle type, chamber pressure, regenerator effectiveness, and turbomachine efficiencies. These latter parameters primarily affect pump exit pressure, as illustrated in the next figure.

The effects of chamber pressure and regenerator effectiveness on fuel-pump exit pressure for the full expander cycle are presented in figure 10. Increasing the chamber pressure obviously required higher pump exit pressures. Increasing the regenerator effectiveness allowed a lower pump exit pressure for a specified chamber pressure or, conversely, a higher chamber pressure for a given pump exit pressure. For example, a chamber pressure of 1500 psia required a 3860-psia pump exit pressure with no regeneration but only a 2800-psia pump exit pressure with regeneration of 0.6 effectiveness. Turbine inlet temperature increased with regeneration, thus reducing the turbine pressure drop required to provide a given power output.
Figure 10.—Effects of chamber pressure and regenerator effectiveness on fuel-pump exit pressure (full expander cycle). Oxidizer-to-fuel ratio, 6.5; nozzle area ratio, 1000.

Concluding Remarks

In a single-point thermodynamic analysis all component performance parameters (i.e., turbomachine efficiencies, heat transfer effectiveness, and fractional pressure losses) are input as single values. Such an analysis, with the component parameters remaining constant, is used to obtain system performance at a single point where the component performances are known. It is also useful for performing sensitivity analyses over narrow ranges of the cycle variables where component performance would remain essentially constant.

Over a wide range of cycle variables and system size, it would be expected that component performance would vary as the component design and off-design conditions (e.g., chamber pressure, pressure ratio) varied. Therefore, for accurate analysis of an engine over a wide range of conditions, variations in the component performance parameters, as determined externally to this code, should be included as part of the program input. Future enhancements of this code could include incorporation of algorithms to correlate component performance with the cycle variables.

Summary of Results

This report presents a single-point thermodynamic code for analyzing hydrogen/oxygen expander-cycle rocket engines. The code is capable of analyzing full, split, and dual expander cycles. Heat regeneration between the turbine exhaust and the pump exhaust is allowed. The combustion process is modeled as one of chemical equilibrium in either an infinite-area or a finite-area combustor. Expansion in the nozzle can be either equilibrium or frozen.

This report, which serves as a users guide for the computer code, describes the system, the analysis methodology, and the input and output. Sample calculations are included to show the effects of some key variables. These sample calculations showed

1. That ideal specific impulse for this analysis is a function of only nozzle area ratio and oxidizer-to-fuel mass ratio and is independent of expander-cycle type, chamber pressure, regenerator effectiveness, and turbomachine efficiencies

2. That cycle type, chamber pressure, regenerator effectiveness, and turbomachine efficiencies primarily affect the pump exit pressure required to achieve the specified chamber pressure

This single-point analysis is useful for determining system performance from known component performance and for performing sensitivity analyses over narrow ranges of cycle conditions. For the analysis of an engine over a wide range of cycle variables, appropriate variations in component performance should be included.
Appendix A
Symbols

\( H \) GASPLUS function for enthalpy, Btu/lb
\( h \) enthalpy, Btu/lb
\( o/f \) oxidizer-to-fuel mass flow ratio
\( P \) power, Btu/sec
\( p \) pressure, psi
\( S \) GASPLUS function for entropy, Btu/(lb)(°R)
\( s \) entropy, Btu/(lb)(°R)
\( T \) GASPLUS function for temperature, °R
\( t \) absolute temperature, °R
\( w \) mass flow rate, lb/sec
\( x \) mass fraction
\( \epsilon \) regenerator effectiveness
\( \eta \) efficiency

Subscripts:
\( b \) bypass
CEC referring to CEC code
\( f \) fuel
\( fcj \) fuel cooling jacket
\( fp \) fuel pump
\( ft \) fuel turbine
\( fib \) fuel turbine bypass
\( id \) ideal
\( inj \) injection
\( jbp \) jacket bypass
\( loss \) loss
\( lp \) low pressure
\( o \) oxidizer
\( ocj \) oxidizer cooling jacket
\( op \) oxidizer pump
\( ot \) oxidizer turbine
\( otb \) oxidizer-turbine bypass
\( rc \) regenerator cold side
\( rh \) regenerator hot side
\( sat \) saturated
\( t \) turbine
\( 0-19 \) station identification (see figs. 1-3)
Appendix B
Analysis Method

This analysis makes use of the GASPLUS properties program (ref. 5) available at NASA Lewis. The thermodynamic property functions obtained from GASPLUS and incorporated in the computer program are hereinafter expressed in the form

\[ y = F(z_1, z_2) \]  \hspace{1cm} (I)

where \( y \) is the value of the function \( F \), and \( z_1 \) and \( z_2 \) are the independent variables. These are real-fluid properties and are used for all computations except those for combustion and nozzle expansion. The previously referenced ideal-gas equilibrium analysis code (CEC code) of Gordon and McBride (refs. 3 and 4) is used for the combustion and expansion analyses.

The analysis method for the full expander cycle, as shown in figure 1, is presented in full. Then, the modifications required for the split and dual expander cycles are presented. The calculation procedure for these cycles, particularly for the fuel system, is iterative, involving two or three levels of iteration depending on whether there is regeneration. For the oxidizer system, only the dual expander cycle requires such iteration. A calculation logic diagram illustrating the nature of this iterative procedure for the fuel system is shown in figure 11.

Figure 11.—Fuel-system calculation logic.
Full Expander

Starting on the oxidizer side, oxygen enters the system at input pressure $p_{12}$ and saturation temperature $t_{12}$, where

$$t_{12} = T_{sat}(p_{12})$$

Oxidizer enthalpy, entropy, and flow rate are then given by

$$h_{12} = H(t_{12}, p_{12})$$
$$s_{12} = S(t_{12}, p_{12})$$

and

$$w_{12} = (o/f)w_1$$

where $o/f$ is the input oxidizer-to-fuel mass flow ratio and $w_1$ is the fuel mass flow rate, whose value is set to unity for this analysis.

In the oxidizer pump the oxygen is pumped to pressure $p_{3}$, where

$$p_{3} = \frac{p_{13}}{1 - \frac{\Delta p_{o, loss}}{p_{13}}}$$

The combustion chamber injection pressure $p_{inj}$ and the oxygen-stream pressure loss ratio $\Delta p_{o, loss}/p_{13}$, which represents the sum of the line, valve, and injector pressure losses, are both input. Pump exit conditions are then determined from

$$h_{13, id} = H(s_{12}, p_{13})$$
$$\Delta h_{op} = \frac{h_{13, id} - h_{12}}{\eta_{op}}$$
$$h_{13} = h_{12} + \Delta h_{op}$$
$$t_{13} = T(h_{13}, p_{13})$$

Assuming the pressure loss $\Delta p_{o, loss}$ to occur at constant enthalpy, the oxygen enthalpy and temperature after injection into the combustion chamber at pressure $p_{inj}$ are

$$h_{o, inj} = h_{13}$$

and

$$t_{o, inj} = T(h_{o, inj}, p_{inj})$$

On the fuel side, hydrogen enters the system with flow rate $w_1$ at input pressure $p_1$ and saturation temperature $t_1$, where

$$t_1 = T_{sat}(p_1)$$

Fuel enthalpy and entropy are then given by

$$h_1 = H(t_1, p_1)$$
$$s_1 = S(t_1, p_1)$$

The hydrogen is pumped to pressure $p_2$ in the fuel pump. A converged value of $p_2$ must be determined iteratively on the basis of the calculated hydrogen pressure after injection matching the input value of combustion chamber injection pressure $p_{inj}$. On the basis of an initially estimated value of $p_2$, the fuel-pump exit conditions are determined as

$$h_{2, id} = H(s_1, p_2)$$
$$\Delta h_{fp} = \frac{h_{2, id} - h_1}{\eta_{fp}}$$
$$h_2 = h_1 + \Delta h_{fp}$$
$$t_2 = T(h_2, p_2)$$

The thermal performance of the regenerator is specified by its effectiveness, which is an input and is defined as

$$\epsilon = \frac{w_{2}(h_3 - h_2)}{w_{19}(h_{19} - h_2)} = \frac{h_{19} - h_2}{h_{19} - h_2}$$

where the hot-side flow rate $w_{19}$ is less than the cold-side flow rate $w_2$ as a result of the fuel-turbine bypass ratio $x_{f tb}$, which is input.

$$w_{19} = (1 - x_{f tb})w_2$$

Since the value of $h_3$ depends on the value of $h_{19}$, which is unknown at this point, these values must be determined iteratively. With an initial estimate for $h_{19}$, we get

$$h_3 = h_2 + \epsilon \left( \frac{w_{19}}{w_2} \right) (h_{19} - h_2)$$

and

$$p_3 = p_2 \left( 1 - \frac{\Delta p_{r,r}}{p_2} \right)$$

where the regenerator cold-side pressure loss ratio $\Delta p_{r,r}$ is input. Then

$$t_3 = T(h_3, p_3)$$
Note that with no regenerator (i.e., \( \epsilon = 0 \) and \( \Delta p_{r}/p_{2} = 0 \)), the variable values at station 3 are equal to those at station 2.

The hydrogen is then used to regeneratively cool the nozzle and the combustion chamber. Fuel cooling-jacket temperature rise \( \Delta t_{fj} \) and pressure loss ratio \( \Delta p_{fj}/p_{3} \) are input, and the cooling-jacket exit conditions are given by

\[
t_{4} = t_{3} + \Delta t_{fj} \\
p_{4} = p_{3}\left(1 - \frac{\Delta p_{fj}}{p_{3}}\right) \\
h_{4} = H(t_{4},p_{4}) \\
s_{4} = S(t_{4},p_{4})
\]

Some of the hydrogen leaving the cooling jacket is bypassed around the turbines, and the hydrogen flow rate entering the turbine system is the same as that previously determined for the regenerator hot side. That is,

\[
w_{5} = w_{19} \quad (29)
\]

The state conditions at station 5 are the same as those at station 4.

There are three options for specifying the turbine system configuration: (1) series with oxidizer-pump drive turbine first, as shown in figure 1, (2) series with fuel-pump drive turbine first, and (3) parallel, as shown in figure 2. For any of these configurations the turbine power requirements are

\[
P_{\beta} = w_{2} \Delta h_{\beta} \quad (30) \\
P_{\alpha} = w_{12} \Delta h_{\gamma}
\]

and the turbine specific works are

\[
\Delta h_{\beta} = \frac{P_{\beta}}{w_{\beta}} \quad (32) \\
\Delta h_{\alpha} = \frac{P_{\alpha}}{w_{\alpha}} \quad (33)
\]

For the series arrangements

\[
w_{\beta} = w_{\alpha} = w_{5} \quad (34)
\]

For the parallel arrangement the flow \( w_{5} \) is split between the two turbines on the basis of equal pressure ratios (i.e., equal ideal specific works). This yields

\[
\Delta h_{i.d,\beta} = \Delta h_{i.d,\alpha} \quad (35)
\]

With the required specific work now determined for each of the turbines, the solution procedure is the same for each of the two turbines in any of the three arrangements. An iteration to determine turbine pressure ratio is required, since GASPLUS does not provide a means for calculating pressure with enthalpy and entropy given as the independent variables. Using the turbine arrangement in figure 1 as an example, a pressure ratio \( p_{6}/p_{5} \) across the oxidizer turbine is assumed and

\[
P_{6} = p_{5}\left(\frac{p_{6}}{p_{5}}\right) \quad (41) \\
h_{6,i.d} = H(s_{5},p_{6}) \quad (42) \\
\Delta h_{\alpha} = \eta_{\alpha}(h_{5} - h_{6,i.d}) \quad (43)
\]

The turbine pressure ratio is varied and equations (41) to (43) are repeated until the calculated value of turbine specific work \( \Delta h_{\alpha} \) from equation (43) matches the required value from equation (33). Then,

\[
h_{6} = h_{5} - \Delta h_{\alpha} \quad (44) \\
t_{6} = T(h_{6},p_{6}) \quad (45) \\
s_{6} = S(t_{6},p_{6}) \quad (46)
\]

The fuel-turbine inlet conditions at station 7 are the same as the oxidizer-turbine exit conditions at station 6. A similar procedure for the fuel turbine then yields \( p_{5}, h_{5}, \) and \( t_{5} \). In the parallel arrangement the inlet conditions for the fuel turbine are those at station 5 rather than those at station 7.

Where the turbines are in series, the regenerator hot-side inlet conditions (station 19) are equal to the turbine-system exit conditions (station 8). With the turbines in parallel the
exit flows are mixed such that

\[ p_{19} = p_6 = p_8 \]  
\[ h_{19} = \frac{w_6 h_6 + w_8 h_8}{w_5} \]  
\[ t_{19} = T(h_{19}, p_{19}) \]

The calculated value of \( h_{19} \) is compared with the estimate used for the regenerator cold-side evaluation in equation (22), and equations (22) to (48) are iterated to convergence. Regenerator hot-side exit conditions can then be determined from the following:

\[ p_9 = p_{19} \left( 1 - \frac{\Delta p}{p_{19}} \right) \]  
\[ h_9 = h_{19} - \epsilon (h_{19} - h_2) \]  
\[ t_9 = T(h_9, p_9) \]

The regenerator hot-side exit flow is then mixed with the turbine bypass flow

\[ w_{10} = w_5 + x_{bh} w_2 = w_2 \]  
\[ p_{10} = p_9 \]  
\[ h_{10} = \frac{w_5 h_5 + x_{bh} w_2 h_4}{w_{10}} \]  
\[ t_{10} = T(h_{10}, p_{10}) \]

The fuel-stream pressure loss ratio \( \Delta p_{f,loss}/p_{10} \), which is an input representing the sum of the line, valve, and injector pressure losses, is assumed to occur between the mixer and the combustion chamber. A combustion chamber injection pressure for the fuel is then determined as

\[ p_{f,inj} = p_{10} \left( 1 - \frac{\Delta p_{f,loss}}{p_{10}} \right) \]  
\[ h_{f,inj} = h_{10} \]

and

\[ t_{f,inj} = T(h_{f,inj}, p_{f,inj}) \]

The combustion and nozzle expansion analyses are performed by using the program CEC, described in references 3 and 4, as a subroutine. All the capabilities of the CEC program are available to this analysis (i.e., infinite-area or finite-area combustion, equilibrium or frozen expansion). The analysis methods are presented in the references and are not described herein. The CEC program uses an ideal-gas enthalpy reference temperature of 536.67 °R, which differs from the GASPLUS reference temperatures. Therefore, the CEC reactant enthalpies must be corrected. Furthermore, the enthalpy gained by the hydrogen during regenerative cooling of the nozzle and the combustion chamber must be accounted for. This is modeled herein by subtracting that enthalpy gain from the hydrogen enthalpy. Therefore, the CEC reactant enthalpies are

\[ h_{f,CEC} = h_{f,inj} - H(536.67 °R, 14.696 \text{ psia}) - (h_3 - h_1) \]

\[ h_{o,CEC} = h_{o,inj} - H(536.67 °R, 14.696 \text{ psia}) \]

It is recognized that the CEC program thermodynamic data are for normal hydrogen whereas the rocket-engine fuel is parahydrogen. Over the temperature range of combustion and expansion, the difference in the computed value of ideal specific impulse would be very small, if not insignificant. The ideal specific impulse computed by the CEC program is then multiplied by the thrust correction factor to yield the actual specific impulse.

**Split Expander**

The split expander cycle, shown in figure 2, differs from the full expander cycle in that the hydrogen flow is split in the fuel-pump system with part of it bypassing directly to the combustion chamber. In the low-pressure fuel pump all of the hydrogen is pumped to intermediate pressure \( p_{15} \), where

\[ p_{15} = \frac{p_{inj}}{1 - \frac{\Delta p_{bh,loss}}{p_{15}}} \]

The combustion chamber injection pressure \( p_{inj} \) and the hydrogen-bypass-stream pressure loss ratio \( \Delta p_{bh,loss}/p_{15} \), which represents the sum of the line, valve, and injector pressure losses, are both inputs. Pump exit conditions are

\[ h_{15,ad} = H(s, p_{15}) \]
\[
\Delta h_{lp,lp} = \frac{h_{15,lp} - h_1}{\eta_{hp,lp}} \\
\]
\[
h_{15} = h_1 + \Delta h_{hp,lp} \\
\]
\[
t_{15} = T(h_{15}, p_{15}) \\
\]
\[
s_{15} = S(t_{15}, p_{15}) \\
\]

The flow leaving the low-pressure fuel pump is split as specified by the input bypass fraction \(x_{hp}\). An amount, termed jacket bypass flow \(w_{15}\), where
\[
w_{15} = x_{hp}w_1 \\
\]
bypasses the regenerator, the nozzle/chamber cooling jacket, and the turbines and rejoins the main flow downstream in the mixer. The flow rate and enthalpy leaving the mixer are
\[
w_{10} = w_5 + x_{hp}w_2 + w_{15} \\
\]
and
\[
h_{10} = \frac{\frac{w_3h_9}{w_{10}} + x_{hp}w_2h_4 + w_{15}h_{15}}{w_{10}} \\
\]

For this cycle the corrected fuel enthalpy for the CEC program is
\[
h_{j, CEC} = h_{j, inj} - H(536.67 \text{ } ^\circ \text{R}, 14.696 \text{ psia}) - (1 - x_{hp})(h_4 - h_3) \\
\]

The analysis of the high-pressure fuel pump is the same as that for the full expander cycle except that the pump inlet conditions are those for station 15 rather than station 1.

**Dual Expander**

The fuel-system analysis for the dual expander cycle, shown in figure 3, is the same as that for the full expander cycle except for the simplification of having only one turbine in the system. Therefore, the variable values at stations 6 to 8 and 19 are identical.

The oxidizer system is similar to the fuel system except that there is no regenerator. The oxygen-pump exit pressure \(p_{13}\) must be determined iteratively in the same manner as the hydrogen-pump exit pressure. An estimated value of \(p_{13}\) is used to determine oxidizer-pump exit conditions at station 13 in the same manner as for the full expander cycle. The oxygen is then used to regeneratively cool the nozzle. The cooling-jacket temperature rise \(\Delta t_{oj}\) and the pressure loss ratio \(\Delta p_{oj}/p_{13}\) are input, and the nozzle cooling-jacket exit conditions are given by
\[
t_{16} = t_{13} + \Delta t_{oj} \\
\]

\[
p_{16} = \frac{p_{13}(1 - \frac{\Delta p_{oj}}{p_{13}})}{p_{16}} \\
\]
\[
h_{16} = H(t_{16}, p_{16}) \\
\]
\[
s_{16} = S(t_{16}, p_{16}) \\
\]

Some of the oxygen leaving the cooling jacket is bypassed around the turbine, and the flow through the turbine is
\[
w_{17} = w_{12}(1 - x_{ob}) \\
\]

where \(x_{ob}\) is the input oxidizer-turbine bypass ratio. The oxidizer-turbine required specific work is
\[
\Delta h_{ot} = \frac{p_{ot}}{w_{17}} = \frac{w_{12}\Delta h_{ot}}{w_{17}} \\
\]

and a turbine pressure ratio \(p_{17}/p_{16}\) is estimated. Then,
\[
p_{17} = p_{16}\left(\frac{p_{17}}{p_{16}}\right) \\
\]
\[
t_{17, id} = H(s_{16}, p_{17}) \\
\]

\[
\Delta h_{ot} = \eta_{ot}(h_{16} - h_{17, id}) \\
\]

the turbine pressure ratio is varied, and equations (78) to (80) are repeated until the calculated turbine specific work \(\Delta h_{ot}\) from equation (80) matches the required value from equation (77). The turbine-exit enthalpy and temperature are then given by
\[
h_{17} = h_{16} - \Delta h_{ot} \\
\]
\[
t_{17} = T(h_{17}, p_{17}) \\
\]

The turbine flow and the bypass flow then mix
\[
w_{18} = w_{12} \\
\]
\[
p_{18} = p_{17} \\
\]
\[
h_{18} = \frac{w_{17}h_{17} + x_{ob}w_{12}}{w_{18}} \\
\]
\[
t_{18} = T(h_{18}, p_{18}) \\
\]

The oxidizer-stream pressure loss ratio \(\Delta p_{ot,low}/p_{18}\) is assumed to occur between the mixer and the combustion chamber. A combustion chamber injection pressure for the oxygen is then
determined as

\[ p_{o, \text{inj}} = p_{18} \left(1 - \frac{\Delta p_{p, \text{inj}}}{p_{18}}\right) \]  \hspace{1cm} (87)

and compared with the input value of combustion chamber injection pressure \( p_{\text{inj}} \). The oxidizer-pump exit pressure \( p_{11} \) is varied until the calculated and input injection pressures match. Then,

\[ h_{o, \text{inj}} = h_{18} \]  \hspace{1cm} (88)

and the corrected oxygen enthalpy for use in the CEC program is

\[ h_{o, \text{CEC}} = h_{o, \text{inj}} - H(536.67 \text{ °R}, 14.696 \text{ psia}) - (h_{18} - h_{13}) \]  \hspace{1cm} (90)
References

**Title and Subtitle**


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**Abstract**

This analysis and this computer code apply to full, split, and dual expander cycles. Heat regeneration from the turbine exhaust to the pump exhaust is allowed. The combustion process is modeled as one of chemical equilibrium in an infinite-area or a finite-area combustor. Gas composition in the nozzle may be either equilibrium or frozen during expansion. This report, which serves as a users guide for the computer code, describes the system, the analysis methodology, and the program input and output. Sample calculations are included to show effects of key variables such as nozzle area ratio and oxidizer-to-fuel mass ratio.

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