FORWARD AND INVERSE KINEMATICS OF DOUBLE UNIVERSAL JOINT ROBOT WRISTS

Dr. Robert L. Williams II
Automation Technology Branch, M.S. 152D
NASA Langley Research Center
Hampton, VA 23665-5225

ABSTRACT

A robot wrist consisting of two universal joints can eliminate the wrist singularity problem found on many industrial robots. This paper presents forward and inverse position and velocity kinematics for such a wrist having three degrees of freedom. Denavit-Hartenberg parameters are derived to find the transforms required for the kinematic equations. The Omni-Wrist, a commercial double universal joint robot wrist, is studied in detail. There are four levels of kinematic parameters identified for this wrist: three forward and three inverse maps are presented for both position and velocity. These equations relate the hand coordinate frame to the wrist base frame. They are sufficient for control of the wrist standing alone.

When the wrist is attached to a manipulator arm, the offset between the two universal joints complicates the solution of the overall kinematics problem. All wrist coordinate frame origins are not coincident, which prevents decoupling of position and orientation for manipulator inverse kinematics. This is a topic for future research.

INTRODUCTION

Many current industrial robot wrists suffer from singularity limitations where at least two wrist coordinate frames align, reducing orientational freedom. Near singular positions, extremely large joint rates are required to maintain constant cartesian rates. One proposed wrist design for reducing singularities uses the universal joint to achieve roll, pitch, and yaw orientation. An overview of robot wrists, including universal joint designs, is given by Rosheim (1989). Other references present non-singular robot wrist designs, e.g., (Barker, 1986), (Milenkovic, 1987), (Rosheim, 1987), and (Trevelyan, 1986).

McKinney (1988) presents forward kinematic and resolved rate equations for single and double universal joint robot wrists. The author studies a specific double universal joint wrist, the Omni-Wrist from Ross-Hime Designs, Inc. A single universal joint wrist is attractive because its motion is purely rotational. However, the workspace is limited due to gimbal lock singularities. Also, the roll velocity of the output shaft is variable, given a constant input roll rate. Therefore, a wrist with two universal joints in series is suggested, which allows an approximately hemispherical singularity-free workspace (McKinney, 1988). Two universal joints yield a constant roll velocity ratio (Mabie and Reinholts, 1987).

The current paper presents forward and inverse kinematic position and velocity equations for control of double universal joint robot wrists. The Omni-Wrist Denavit-Hartenberg parameters are presented for double universal joint wrists. The Omni-Wrist kinematic transformations are presented. Four levels of kinematic parameters are identified, from the actuator angles to the position and orientation of the hand. Three mappings are presented for each of the forward position, inverse position, forward velocity, and inverse velocity (resolved rate) problems. These equations relate the robot hand to the robot wrist base and are sufficient for control of the wrist standing alone.

The double universal joint wrist is not purely rotational due to the offset between the two universal joints. Position and orientation trajectories thus may not be decoupled for a double universal joint wrist attached to a manipulator arm. The manipulator inverse position and velocity problems are more complicated for the double universal joint robot wrist than a purely rotational robot wrist.

SYMBOLS

\( \{ m \} \) \hspace{1cm} Cartesian coordinate frame \( m \)
\( \{ 3 \} \) \hspace{1cm} Wrist base coordinate frame
\( \{ 8 \} \) \hspace{1cm} Hand coordinate frame
\( \theta_{iA}, \theta_{iA}, \theta_{iA} \) \hspace{1cm} Actuator angles
\( \theta_{iG}, \theta_{iG}, \theta_{iG} \) \hspace{1cm} Gear ball angles
\( \theta_i, \theta_o, \theta_0 \) \hspace{1cm} Universal joint angles
\( \mathbf{X}_n \) \hspace{1cm} Homogeneous transformation matrix of \( \{ m \} \) relative to \( \{ n \} \)
\( \mathbf{R}_{i,j} \) \hspace{1cm} Rotation matrix of \( \{ m \} \) relative to \( \{ n \} \)
\( \mathbf{P}_m \) \hspace{1cm} Element \((i,j)\) of \( \mathbf{R}_{i,j} \)
\( \mathbf{X}_m \) \hspace{1cm} Unit direction vector \( X \) of \( \{ m \} \)
\( \mathbf{X}_m \) \hspace{1cm} Angular velocity of \( \{ m \} \) with respect to \( \{ 3 \} \), expressed in \( \{ m \} \)
\( \mathbf{V}_m \) \hspace{1cm} Linear velocity of \( \{ m \} \) origin with respect to \( \{ 3 \} \), expressed in \( \{ m \} \)

\( F_1 \) \hspace{1cm} Forward map solving \( \theta_{iG} \) given \( \theta_{iA} \), \( i=4,5,6 \)
\( F_2 \) \hspace{1cm} Forward map solving \( \theta_i \) given \( \theta_{iG} \), \( i=4,5,6 \)
\( F_3 \) \hspace{1cm} Forward map solving \( \theta_{iA} \) given \( \theta_i \), \( i=4,5,6 \)
\( F_1, F_2, F_3 \) \hspace{1cm} Inverses of \( F_3, F_2, F_1 \), respectively
\( FV_1, IV_{i=1,2,3} \) \hspace{1cm} Forward and Inverse velocity maps, defined analogously

\( \dot{\theta}_i \) \hspace{1cm} Joint rate \( i \)
\( c_i \) \hspace{1cm} \( \cos \dot{\theta}_i \)
\( s_i \) \hspace{1cm} \( \sin \dot{\theta}_i \)
\( t_i \) \hspace{1cm} \( \tan \dot{\theta}_i \)
\( L \) \hspace{1cm} Offset length between the universal joints

DOUBLE UNIVERSAL JOINT WRIST KINEMATICS

A universal joint is used to transfer rotations between intersecting shafts. Most kinematics textbooks discuss universal joints (e.g. Mabie and Reinholts, 1987). A kinematic diagram for the double universal joint robot wrist is shown in Fig. 1. The input shaft rotates about a fixed axis and the output shaft is free; thus there are five degrees of freedom. Coupling of \( \theta_5 \) and \( \theta_6 \) reduces this number to three degrees of freedom.
Figure 1 shows the initial position for all wrist coordinate frames; all universal joint angles are zero in this configuration. Frame \( \{3\} \) is the wrist base frame, fixed for this paper. Frame \( \{4\} \) rotates by \( \theta_4 \) relative to \( \{3\} \); \( \{5\} \) rotates by \( \theta_5 \) relative to \( \{4\} \); \( \{6\} \) rotates by \( \theta_6 \) relative to \( \{5\} \); \( \{7\} \) rotates by the coupled \( \theta_7 \) relative to \( \{6\} \); and the hand frame \( \{8\} \) rotates by the coupled \( \theta_8 \) relative to \( \{7\} \).

Denavit-Hartenberg Parameters

The Denavit-Hartenberg parameters for the double universal joint robot wrist of Fig. 1 are given in Table I, which follows the convention in Craig (1988).

\[ \begin{array}{cccc}
\text{i} & a_{i-1} & d_i & \theta_i \\
4 & 0 & 0 & 0 & \theta_4 + 90^\circ \\
5 & 90^\circ & 0 & 0 & \theta_5 + 90^\circ \\
6 & 0 & 0 & 0 & \theta_6 \\
7 & 0 & L & 0 & \theta_7 - 90^\circ \\
8 & -90^\circ & 0 & 0 & \theta_8 - 90^\circ \\
\end{array} \]

Forward Position

The forward solution finds \( [3T] \) given \( \theta_4, \theta_5, \theta_6 \). Equation 1 is the homogeneous transformation matrix describing the position and orientation of \( \{x\} \) with respect to \( \{\text{base}\} \) (Craig, 1988).

\[
[\text{T}]^{-1} = 
\begin{bmatrix}
\begin{array}{cccc}
\begin{array}{cccc}
\theta_0 & \begin{array}{c}
\{3\}T & \{4\}T & \{5\}T & \{6\}T & \{7\}T & \{8\}T
\end{array}
\end{array}
\end{array}
\end{bmatrix}
\]

The general forward kinematics solution is Eq. 2. The \( (4\times4) \) forward transform is comprised of a \( (3\times3) \) rotation matrix representing the orientation and a \( (3\times1) \) position vector locating the origin of \( \{8\} \) in \( \{3\} \). The specific terms are given in Eq. 3.

\[
[3T] = \begin{bmatrix}
2c_5c_6K1 + s_4 & 2s_5c_6K1 & -s_4K1 + c_4 & L(K1) \\
2c_5c_6K2 + c_4 & 2s_5c_6K2 & -s_4K2 + c_4 & L(K2) \\
2c_6c_5 & 2c_5c_6 & -c_6 & -2s_5c_6c_6 & Lc_6c_6
\end{bmatrix}
\]

Inverse Position

The inverse problem solves for the universal joint angles given task space input. The full \( [3T] \) cannot be specified because it has six freedoms, and the wrist only three. Due to the following constraint, which dictates that \( \{3F_k\} \) travel on the surface of a sphere of radius \( L \), \( \{3F_k\} \) cannot be the input, because it has two independent freedoms.

\[
P_2^2 + P_3^2 + P_4^2 = L^2
\]

The rotation matrix \( [3R] \) is the input to the inverse problem.

\[
[3R] = 
\begin{bmatrix}
\begin{array}{ccc}
\begin{array}{ccc}
T_{11} & T_{12} & T_{13} \\
T_{21} & T_{22} & T_{23} \\
T_{31} & T_{32} & T_{33}
\end{array}
\end{array}
\end{bmatrix}
\]

The angle \( \theta_8 \) is isolated by inverting \( [3R] \) and multiplying it on the left of both sides of Eq. 5.

\[
[3R][3R]^{-1} = [3T]^{-1} \quad [3R][3R]^{-1} = [3T]^{-1} \quad [3R][3R]^{-1} = [3T]^{-1}
\]

The angles \( \theta_8 \) and \( \theta_9 \) are eliminated from the right hand side of Eq. 6 by equating the \( (1,1) \), \( (2,2) \), and \( (3,3) \) elements, given in Eqs. 7, 8, and 9, respectively.
\[ r_{21}c_4 - r_{11}t_4 = -2c_o^2 + 1 \]
\[ -r_{13}c_4 - r_{23}t_4 = -2c_o^2 + 1 \]
\[ r_{32} = 2c_o^2 - 1 \]

Equation 7 is subtracted from Eq. 9 to eliminate \( \theta_5 \).
\[ r_{32} - r_{21}c_4 + r_{11}t_4 = 2c_o^2 - 2 \]

Equations 8 and 10 are added to remove \( \theta_5 \).
\[ E\cos \theta_4 + F\sin \theta_4 + G = 0 \]
\[ E = -(r_{13} + r_{21}) \]
\[ F = r_{11} - r_{23} \]
\[ G = r_{32} + 1 \]

Using the tan half angle substitution (Mabe and Reinholt, 1987), there are two solutions for \( \theta_4 \):
\[ \theta_{4_{1,2}} = 2\tan^{-1} \left[ \frac{-F \pm \sqrt{E^2 + F^2 - G^2}}{G - E} \right] \]

The radicand in Eq. 12 is simplified with orthonormal constraints. The columns of \([R]\) are the \( X, Y, Z \) unit vectors of \( \{8\} \) expressed in \( \{3\} \) coordinates, while the rows are those of \( \{3\} \) given in \( \{8\} \). The orthogonal constraints dictate that both the columns and rows of a rotation matrix form a dextral mutually perpendicular set. The normal constraints dictate that the length of all columns and rows is unity. With the following four constraints, \( E^2 + F^2 - G^2 = 0 \).
\[ 3 \vec{r}_3 = 3 \theta_6 \times \vec{S}_6 \]
\[ \vec{S}_3 = 1.0 \]
\[ 3 \vec{r}_6 = 1.0 \]
\[ \vec{S}_6 = 1.0 \]

Therefore, given \([R]\), there is one solution for \( \theta_4 \) (two repeated roots), from Eq. 12.
\[ \theta_4 = 2\tan^{-1} \left[ \frac{-F}{G - E} \right] \]

With \( \theta_4 \) solved, the left hand side of Eq. 6 is known. The next step is to isolate and solve \( \theta_6 \).
\[ [S] (R(\theta_6)^{-1}) [S] (R(\theta_4)^{-1}) [S] R(\theta_6) \] \[ = [S] R(\theta_6) \] \[ = [S] R(\theta_6) \] \[ = [S] R(\theta_6) \]

The (2,2) elements of Eq. 15 are equated to solve \( \theta_6 \).
\[ \theta_6 = \tan^{-1} \left( \frac{r_{13}c_4 - r_{23}t_4}{r_{32}} \right) \]

Both solutions from the inverse tangent function are mathematically valid, due to symmetry: \( \theta_6 \) (\( \frac{\pi}{2} < \theta_6 < \frac{3\pi}{2} \)) and \( \theta_6 + \pi \). When \( \theta_4 \) and \( \theta_6 \) are known the left hand side of Eq. 15 is known. Angle \( \theta_6 \) is solved by equating the (3,3) elements of Eq. 15.
\[ 2 \theta_6 = \cos^{-1} (r_{13}c_4 + r_{23}t_4) \]

The inverse cosine function solution is \( \pm 2 \theta_6 \). This ambiguity is resolved by determining which sign satisfies the (1,2) terms of Eq. 15.
\[ 2 \theta_6 = (r_{23}c_4 - r_{13}t_4) \theta_5 - r_{32} \theta_5 \]
The inverse velocity problem is solved by inverting Eq. 21.

\[
\{\dot{T}\} = \left[J_R^{-1}\right] \{\dot{o}_R\}
\]

\[
J_R^{-1} = \begin{bmatrix}
t_6 & 1 & -\frac{1}{c_5} \\
s_6 t_6 & c_5 t_6 & 1 - \frac{1}{c_5} \\
c_5 - \frac{1}{c_5} & -s_5 & \frac{1}{2}s_5 t_6
\end{bmatrix}
\]  

(24)

The wrist singularity conditions are found by setting the determinant to zero.

\[
|J_R| = 4c_5 s_5^2 = 0
\]  

(25)

The double universal joint robot wrist is singular when \(\theta_6 = \pm \frac{\pi}{5}\) or \(\theta_6 = \pm \frac{\pi}{6}\).

**OMNI-WRIST KINEMATICS**

The Omni-Wrist by Ross-Hime Designs, Inc. is a double universal joint robot wrist. Figure 2 displays a section view of the Omni-Wrist. Planetary gears transfer the first universal joint rotations \(\theta_5\) and \(\theta_6\) to the second universal joint.

The rotational axes for \(\theta_5\) and \(\theta_6\) are moving. The Omni-Wrist has outer and inner bevel gear bails to transfer rotations from two actuators fixed in the wrist base to the angles \(\theta_5\) and \(\theta_6\), to avoid moving actuators. No intermediate gear ball is required for \(\theta_4\) because it rotates about an axis fixed in the wrist base. In addition to the outer and inner gear bails, helical gear trains are used to reduce the speed and amplify the torque for each of the three actuators.

Referring to Fig. 2, actuator 1 drives \(\theta_4\). The inner gear ball rotates in the plane of the paper; the outer rotates about a perpendicular axis. The inner gear ball angle, rotated by actuator 2, equals \(\theta_5\) when \(\theta_4 = \theta_5 = 0\). Actuator 3 rotates the outer gear ball, whose angle equals \(\theta_6\) when \(\theta_4 = \theta_5 = 0\). In general, the inner and outer gear bails combine to yield \(\theta_5\) and \(\theta_6\).

![Figure 2 Omni-Wrist Section View](image)

The roll angle \(\theta_4\) is continuous and bidirectional. The inner and outer gear ball angles are limited to ±45°. These limits apply to \(\theta_4\) when \(\theta_4 = \theta_5 = 0\), and to \(\theta_6\) when \(\theta_4 = \theta_6 = 0\). When these angles are not zero, the limits on \(\theta_5\) and \(\theta_6\) are more restrictive.

There are four levels of Omni-Wrist kinematic parameters: 1) Actuator angles (\(\theta_{4A}, \theta_{5A}, \theta_{6A}\)); 2) Gear ball angles (\(\theta_{4G}, \theta_{5G}, \theta_{6G}\)); 3) Universal joint angles (\(\theta_4, \theta_5, \theta_6\)); and 4) Hand coordinate frame \([\mathbf{T}]\).

All angles are zero in the initial position.

**Omni-Wrist Position Kinematics**

Figure 3 describes the three forward and inverse position mappings between the four levels of Omni-Wrist kinematic parameters. The overall forward position problem finds \([\mathbf{T}]\) given the actuator angles, using maps \(F1, F2,\) and \(F3\). The inverse position problem finds the actuator angles given \([\mathbf{T}]\) via the maps \(I1, I2,\) and \(I3\).

![Figure 3 Position Mappings](image)

Maps \(F3\) and \(I1\) are the general wrist solutions, Eq. 3 and Eqs. 14, 16, and 17, respectively. The remaining maps are developed in this section.

**Position Maps \(F1\) and \(I3\)**

The gear ball angles are related to the actuator angles by gear trains. Forward map \(F1\) is given in Eq. 26.

\[
\begin{align*}
\theta_{4G} &= N_1 \theta_{4A} \\
\theta_{5G} &= N_2 \theta_{5A} \\
\theta_{6G} &= N_3 \theta_{6A}
\end{align*}
\]  

(26a)

For the Omni-Wrist, \(N_1 = \frac{-21}{172}, N_2 = \frac{-1}{205},\) and \(N_3 = \frac{-1}{290}\). The map \(F3\) is the inverse of Eq. 26.

**Position Maps \(F2\) and \(I2\)**

The kinematic relationships between the gear ball angles and the universal joint angles are coupled and transcendental. McKinney (1988) solved a problem equivalent to \(I2\); the map \(F2\) was not solved.
Two coordinate frames are introduced to determine the kinematic relationships between the gear ball and universal joint angles. The \( \{IGD\} \) frame is attached to the inner gear ball, and \( \{OGD\} \) is attached to the outer gear ball, as shown in Fig. 4. Both origins are collocated with the origin of \( \{3\} \). The inner gear ball rotates by angle \( \theta_{4G} \) about \( \theta_sG \) and \( \theta_{OGD} \) are coincident.

\[
\theta_s = \tan^{-1}(u) \quad u = \left[ \frac{c_6 \cos \theta_{6G} + c_4 \cos \theta_{3G}}{c_4 \sin \theta_{3G} + c_6 \sin \theta_{6G}} \right]
\]

The sin\( \theta_{6G} \) term is eliminated from Eqs. 30a and \( b \) to solve for \( \theta_6 \).

The inverse cosine function solution is \( \pm \theta_6 \). Since both results are potentially in the motion range of the Omni-Wrist, this ambiguity must be resolved by choosing the \( \theta_6 \) sign which satisfies Eq. 30b. Map F2 is unique.

The mapping F2 solves for the gear ball angles given the universal joint angles. The \( \theta_{4G} \) mapping is Eq. 29. The remaining gear ball angles are found by dividing Eqs. 28b and 28a by Eq. 28c.

**Omniv-Wrist Velocity Kinematics**

Figure 5 shows the three forward and inverse maps relating the four levels of Omni-Wrist velocity parameters. The forward velocity problem finds the cartesian rates given the actuator joint rates, using maps \( FV1, FV2 \), and \( FV3 \). The inverse velocity problem accepts \( \{\theta_{4G}\} \) and calculates the actuator joint rates via maps \( IV1, IV2 \), and \( IV3 \).

The velocity maps \( FV3 \) and \( IV1 \) are Eqs. 21 and 22, and Eq. 24, respectively. The remaining Omni-Wrist velocity solutions are presented below.

**Velocity Maps FV1 and IV3**

The map \( FV1 \) is a time derivative of Eqs. 26; \( IV3 \) is the inverse of Eqs. 34.

\[
\dot{\theta}_{4G} = N_1 \dot{\theta}_{4A} \\
\dot{\theta}_{4G} = N_2 \dot{\theta}_{4A} \\
\dot{\theta}_{4G} = N_3 \dot{\theta}_{4A}
\]
Velocity Maps $FV_2$ and $IV_2$

The map $FV_2$ is a time derivative of $I_2$, Eqs. 29, 31, and 32. The angular rate $\dot{\theta}_0$ is required for the $\dot{\theta}_5$ calculation.

$$\dot{\theta}_4 = \dot{\theta}_{4|2}$$

$$\dot{\theta}_0 = \frac{1}{\sqrt{1 - \omega^2}} \frac{du}{dt}$$

$$\frac{du}{dt} = (B + 2\dot{\theta}_4|_2 - As_2\dot{\theta}_{4|2} + c_2^2) + c_4^2 A + c_4 s_4; A$$

$$A = Q \left[ s_4 c_6 s_6 c_6 c_6 + c_6 s_6 c_6 + \frac{c_6 s_6}{M} M \right]$$

$$\dot{\theta}_b = \frac{1}{\sqrt{1 - \omega^2}} \frac{dv}{dt}$$

$$\frac{dv}{dt} = \frac{1}{c_5} \left[ C_0 \dot{\theta}_0 + C \right]$$

$$\dot{\theta}_5 = \frac{1}{M} \left[ \frac{c_6 s_6 c_6 c_6 c_6 + c_6 s_6 c_6 + c_6 s_6}{M} M \right]$$

The inverse map $IV_2$ is a time derivative of $I_2$, Eqs. 33a and 33b. The mapping for $\dot{\theta}_{4|2}$ is Eq. 35.

$$\dot{\theta}_{5|3} = \frac{1}{1 + \omega^2} \frac{dw}{dt}$$

$$\frac{dw}{dt} = -\frac{1}{c_6} (c_4 s_6 + s_4 s_6) \dot{\theta}_4 + \frac{1}{c_6^2} (c_4 + s_4 s_6) \dot{\theta}_6 - \frac{1}{c_6 c_6} c_4 \dot{\theta}_6$$

$$\dot{\theta}_{6|3} = \frac{1}{1 + q^2} \frac{dq}{dt}$$

$$\frac{dq}{dt} = \frac{1}{c_5} (c_4 s_6 - s_4 s_6) \dot{\theta}_4 + \frac{1}{c_6} (c_4 + c_6 s_6 s_6) \dot{\theta}_6 - \frac{1}{c_6 c_6} c_4 \dot{\theta}_6$$

The derivatives Eqs. 36, 38, and 39 hold for the angle range $\pm \frac{\pi}{2}$ to $\frac{\pi}{2}$. The sign of $\dot{\theta}_3$ in Eq. 37 is positive when $-\pi < \dot{\theta}_3 < 0$.

**EXAMPLES**

This section presents two examples to demonstrate the equations derived in this paper. The first example deals with the forward and inverse position and velocity problems for the general double universal joint robot wrist mechanism. The second presents forward and inverse position and velocity results for the Omni-Wrist. The dimensions used in this section are mm, degrees, $m^2$, and $m^4$.

**Example 1**

**Forward Position**

Given $\theta_4 = 120.0^\circ, \theta_6 = -25.0^\circ, \theta_0 = 10.0^\circ$, and $L = 41$, $[^T]\!R$ is calculated using Eq. 3.

$$[^T]\!R = \begin{bmatrix} -0.494 & -0.798 & -0.345 & -18.3 \\ -0.452 & -0.103 & 0.886 & -2.4 \\ -0.743 & 0.593 & -0.310 & 36.6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

**Inverse Position**

Given $[^T]\!R$ from Eq. 40 three universal joint angles are calculated with Eqs. 14, 16, and 17; the four solutions are formed from Table II.

**Table III Inverse Position Solutions**

<table>
<thead>
<tr>
<th>Solution</th>
<th>$\theta_4$</th>
<th>$\theta_6$</th>
<th>$\theta_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>120.0</td>
<td>-25.0</td>
<td>10.0</td>
</tr>
<tr>
<td>2</td>
<td>120.0</td>
<td>-25.0</td>
<td>190.0</td>
</tr>
<tr>
<td>3</td>
<td>120.0</td>
<td>155.0</td>
<td>-10.0</td>
</tr>
<tr>
<td>4</td>
<td>120.0</td>
<td>155.0</td>
<td>170.0</td>
</tr>
</tbody>
</table>

**Forward Velocity**

Given $\dot{\theta}_4 = 1.0, \dot{\theta}_6 = 2.0, \dot{\theta}_0 = 3.0$, and $L = 41$, $[^r\!\omega_8]$ and $[^r\!v_8]$ are calculated using Eqs. 21 and 22.

$$[^r\!\omega_8] = \begin{bmatrix} 4.4 \\ 3.7 \end{bmatrix}$$

$$[^r\!v_8] = \begin{bmatrix} -80.0 \\ -100.0 \end{bmatrix}$$

**Inverse Velocity (Resolved Rate)**

Given $[^r\!\omega_8]$ from Eq. 41, $\dot{\theta}_4 = 1.0, \dot{\theta}_6 = 2.0, \dot{\theta}_0 = 3.0$, are calculated using Eq. 24.

**Example 2**

**Forward Position**

Given the actuator angles, the gear bail angles, universal joint angles, and $[^T]\!R$ are calculated successively, using maps $F_1, F_2$, and $F_3$. Example 1 presents the $F_3$ result.
Inverse Position

Given $\theta_R$ from Eq. 40, the universal joint, gear bail, and actuator angles are calculated using $I_1$, $I_2$, and $I_3$. Example 1 presents $I_1$. Considering angular limits, only the first solution in Table III is reachable. The inverse maps $I_2$ and $I_3$ are the reverse of maps $F_2$ and $F_1$ in Eq. 42, respectively.

Forward Velocity

Given the actuator rates, the gear bail, universal joint, and cartesian rates are calculated with the mappings $FV_1$, $FV_2$, and $FV_3$. Example 1 gives $FV_3$.

Inverse Velocity (Resolved Rate)

Given $\omega_\omega$ from Eq. 41, the universal joint, gear bail, and actuator rates are found, using $IV_1$, $IV_2$, and $IV_3$. Example 1 presents $IV_1$. The mappings $IV_2$ and $IV_3$ are the reverse of $FV_2$ and $FV_1$ in Eq. 43, respectively.

CONCLUSION

This paper presents kinematic equations for control of a double universal joint robot wrist. The forward and inverse position and velocity problems were solved. The Omni-Wrist equations were developed in detail. This wrist has four levels of kinematic parameters. Three forward and inverse position and velocity maps relating these parameters were presented. These equations relate the hand coordinate frame to the wrist base coordinate frame, and are sufficient for controlling the wrist standing alone. All pertinent kinematic equations were derived; any specific control algorithm will not require all of the equations. All Omni-Wrist solutions are unique. The Omni-Wrist is completely singularity-free throughout its range of motion.

The equations of this paper have been verified by computer simulation. As demonstrated by the examples, the inverse solutions validate the forward solutions. Experimental work using the Omni-Wrist is planned to further validate the equations.

The offset, $L$, between the two universal joints complicates the inverse kinematics problems when the double universal joint robot wrist is attached to a manipulator arm. The wrist coordinate frames are not all collocated, which prevents decoupling of the hand coordinate frame position and orientation. For a three degree of freedom manipulator arm carrying the double universal robot wrist, the inverse position problem involves six transcendental equations, coupled in the six unknowns. The associated Jacobian matrix is fully populated, which means the hand linear velocity depends on the wrist rates in addition to the first three joint rates. The kinematics of a manipulator using the double universal joint robot wrist is a subject for future research.

REFERENCES


