MODELING OF ROLLING ELEMENT BEARING MECHANICS

CONTRACT NAS 8-38607
MONTHLY TECHNICAL PROGRESS REPORT
KHK-10

AEROJET PROPULSION DIVISION
P.O. BOX 13222
SACRAMENTO, CALIFORNIA 95813
FEBRUARY 1991

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APPROVED BY:  
R. L. BICKFORD  
PROGRAM MANAGER

I. REPORTING PERIOD OF PERFORMANCE

This report covers the accounting month of February 1991. Beginning with this report, Version 2.0 of the program plan will be used as the schedule and fiscal reference. Further comments relative to this revised plan are given in the following section of this report.

This is month 10 of the 18 month Phase I as given in Version 2.0 of the program plan. With the revision, the overall program extends over 31 months.

Exact reporting period of performance is 01-19-91 to 02-15-91.

The program start date was 05-08-90.

II. PROGRAM STATUS vs. PLAN

PROGRAM PLAN REVISIONS

Beginning with this monthly report, the Version 2.0 program plan presented to the NASA technical monitor, Steve Ryan, on January 15, 1991 will be used to track progress. It is recognized that this replan has not been formally approved by NASA, especially in regards to the accelerated funding required. However, the obsolescence of the original program plan warrants use of this new plan.

The impact of switching to the Version 2.0 program plan on Phase I activities is minor. Essentially, the duration has been shortened from 19 to 18 months, and some resources have been reallocated.

It is imperative that a decision relevant to adopting the Version 2.0 program plan be made before program month 15 (July 91). At this point, the replan program starts Phase II. If necessary, the program can be stretched to avoid the overlap of development phases, however, an additional replan exercise will be necessary to accommodate this event.

PROGRAM STATUS CHARTS

Program status charts for this month follow on the next three pages. The schedule and funding profiles are for the Version 2.0 program plan. Page 2 contains the Milestone and Schedule Chart. A spreadsheet summary of the program for both salary hours and material (ODC) is given on page 3. Finally, page 4 displays the overall budget versus actuals. Only Phase I data are given, as the other two phases have not started.
# Modeling of Rolling Element Bearing Mechanics

## Version 2.0 Program Plan Phase I Milestone Schedule

Revised 02-07-91  Lyn Greenhill, Principal Investigator

<table>
<thead>
<tr>
<th>TASK</th>
<th>1990</th>
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<tr>
<td>program month</td>
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<td>I.1 LITERATURE SEARCH</td>
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<td>I.2 FORMULATE/DOCUMENT MODELS</td>
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<td>I.3 FORMUL. SOLN METHODOLOGY</td>
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<td>I.5.B TESTING</td>
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<td>I.5.D CORRELATION</td>
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<td>I.6 DEMONSTRATED USAGE</td>
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<tr>
<td>PROGRAM MANAGEMENT MONTHLY REPORTS</td>
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* 1: Theoretical developments complete/doc'd  
   2: Unverified code complete and documented  
   3: Testing completed  
   4: Verified code installed at NASA/MSFC
**PHASE I LABOR SUMMARY ENDING FEBRUARY 1991, PROGRAM MONTH # 10, VERSION 2.0 PROGRAM PLAN**

<table>
<thead>
<tr>
<th>TASK</th>
<th>DESCRIPTION</th>
<th>TOTAL HOURS BY PROGRAM MONTH</th>
<th>CUM PCT VARIANCES(%)</th>
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<td>1.5.D</td>
<td>KHK150 Correlate</td>
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<tr>
<td>1.6</td>
<td>KHK160 Demonstrate</td>
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**TOTAL MONTHLY BUDGET: 2278**

**CUMULATIVE LABOR TOTALS:**

**GROUP ACTUAL HRS:**

**NOTES:**
1. For each task, the first row are planned allocations, the second row are actuals.
2. Cumulative labor percent completion is a weighted average of all tasks combined.
3. ODC plan values do not include material burden.

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**PHASE I ODC SUMMARY ENDING FEBRUARY 1991, PROGRAM MONTH # 10, VERSION 2.0 PROGRAM PLAN**

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<thead>
<tr>
<th>TASK</th>
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<td>1.6</td>
<td>KHK160 Demonstrate</td>
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</tbody>
</table>

**TOTAL MONTHLY BUDGET:**

**CUMULATIVE ODC TOTALS:**

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**NOTES:**
1. For each task, the first row are planned allocations, the second row are actuals.
2. Cumulative labor percent completion is a weighted average of all tasks combined.
3. ODC plan values do not include material burden.
EXPLANATIONS OF SIGNIFICANT VARIANCES AND CORRECTIVE ACTION

Program Variances:

KHK120: A computer charge of $512 was made on this task during February. The code that was used (MACSYMA) is only available on the VAX network, necessitating the expenditure, although the amount was surprising.

ACTION: This expenditure will be covered with other ODC funds. No further use of the code will be authorized.

KHK400: Replan activities associated with contract requirements resulted in 27 hours being spent.

ACTION: It appears that with the replan effort, this activity may be underfunded. It will be monitored over the course of the next few months to determine if the initial estimates are adequate and what additional allocation will be needed to balance expenditures with budget.
II. TECHNICAL PROGRESS

Work Package KHK110 (Literature Search)

We have obtained the remaining documentation on computer codes RAPIDREB and SHABERTH, thus concluding this task. A complete list of references for the quasi-static solution are given in the Appendix.

Work Package KHK120 (Formulate/Document Models)

The classical bearing mechanics work of Jones [1] and Harris [2] were studied in detail, primarily with reference to angular contact ball bearings. All of the equations related to bearing geometry, relative motions of rolling elements, and internal rolling-element load distribution were rederived to determine implicit assumptions in the formulations. These derivations were facilitated by using the symbolic analysis code MACSYMA.

A preliminary assessment of some of the assumptions inherent in traditional bearing mechanics, relative to structural flexibility analysis, is as follows:

1. Except for Hertzian contact between the balls and races, all the bearing components (inner ring, outer ring and housing) are considered rigid. Elastic displacements of these components, as well as local deformations of a hollow shaft, could significantly change the quasi-static load distribution between the components.

2. The azimuth angle defining ball location ($\psi_j$) is considered constant, and the rigid-body small displacements of the inner ring are described by only five degrees of freedom (DOF). With general elastic deformations of the bearing components, small displacements could occur in the tangential (azimuth) direction, and the rigid-body small displacements of the inner ring could require a sixth DOF at the shaft/inner ring interface (i.e., $\psi_j$ not constant).

3. Some contributions to inner ring rigid-body motion, which may be of the same order as elastic deflections, are neglected. These contributions become evident when the coordinate system fixed at the shaft center line is allowed to move with the shaft relative to inertial space. The use of the moving coordinate system will facilitate the proposed study of enforced motion in Phase II.

4. The initial radial distance from the shaft center to the inner race curvature center ($R_i$) is used as a constant to calculate the forces and moments at the bearing center for comparison with the known applied forces and moments. The actual coordinates from the ball/inner race contact points to the fixed point of force and moment application should properly be used.

5. The inner race and outer race curvature centers necessarily move with their respective races as part of a rigid body. These curvature centers, required to determine normal force directions at the ball/race contact points, must be considered part of the flexible inner and outer rings for the subsequent quasi-static analysis.

* Numbers in brackets denote references given in Section IV of the report
6. Traction forces applied to the ball by relative fluid motion are neglected. Only the friction forces needed to react the gyroscopic moment are included in the force equilibrium equations.

7. The concept of race-ball control utilized by Jones [1], in which spin occurs only at the outer race or at the inner race, is not supported by subsequent work such as Harris [2] and Walters [3]. According to these sources, inner-race control is impossible but outer-race control can be approximated in certain situations. Usually ball spin occurs at both races. We will formulate the bearing mechanics equations without reliance on this assumption.

At the time these traditional formulations were developed, such assumptions were probably necessary to obtain a timely solution with the computational power available. The figure on page 8, taken from Harris [2] page 256, can be used to illustrate some of these common assumptions in the kinematic analysis of ball bearings.

In the unloaded condition, the initial position of the inner and outer raceway groove curvature centers and the ball center are collinear (and coplanar), separated by the distance $BD$. In terms of the inner and outer raceway curvatures ($f_i$ and $f_o$), $BD$ may be expressed as:

$$BD = (f_o + f_i - 1)D$$

where $D$ is the ball diameter. This dimension implies that the internal clearance in the bearing has been absorbed by axial translation of the inner race.

Assuming a fixed outer raceway groove curvature center, the application of static load and centrifugal force results in radial and axial (but not tangential) motion of the inner raceway groove curvature center, denoted by quantities $\Delta r$ and $\Delta z$. In terms of the unknown rigid body relative radial, axial, and angular displacements of the inner and outer rings, the motion is expressed as:

$$\Delta r = \delta_r \cos \psi_j$$

$$\Delta z = \delta_z + \theta R_i \cos \psi_j$$

The figure illustrates the key dimension $R_i$, the initial radius of the inner raceway groove curvature center, which is used as a constant in the Jones and Harris force analyses. Note that the figure and these equations express the relative motion of the inner and outer rings in terms of three degrees of freedom. In actuality, both Jones and Harris utilize five degrees of freedom in their complete analysis, resolving the motion into a Cartesian coordinate system with only the angular position in the axial direction fixed (again, no tangential motion allowed).

Using the deflections due to load and speed given in equation (2), coupled with the unloaded dimension $BD$ from (1), the relative axial and radial motion of the loci of inner and outer raceway groove curvature centers at any ball position is:

$$A_{1j} = BD \sin \alpha^0 + \Delta z$$

$$A_{2j} = BD \cos \alpha^0 + \Delta r$$

where $\alpha^0$ is the initial contact angle. Since the external forces and/or displacements exerted on the inner race are known, and the internal forces generated by the ball rotation (centrifugal and gyroscopic) can be calculated, the solution to equation (3) yields the full specification of the displacement in the traditional bearing mechanics analysis.
In order to obtain the relative curvature center displacements, the kinematic equations expressed by (3) were reformulated by Jones and Harris for the ball center displacements $X_{1j}$ and $X_{2j}$ and inner and outer ring contact angles $\alpha_{ij}$ and $\alpha_{ij}$, and solved using a Newton-Raphson iteration. Actually, the contact angles are not obtained directly; the iterative solution obtains values for $X_{1j}$, $X_{2j}$, $\delta_{ij}$, and $\delta_{ij}$, where the latter two quantities are the inner and outer ring contact deformations. These hertzian deflections are the only flexibility in the analysis. As was discussed in the work of Davis and Vallance [4], this limited compliance can result in significant differences in contact angle and ball load distributions compared to a fully flexible ring analysis.

Once these four terms are obtained, forces and moments satisfying static equilibrium can be computed, although both Jones and Harris use $R_i$ as the moment arm. This means that the moments exerted by the inner ring on the shaft use a lever arm foreshortened by the $\Delta r$ displacement. The static equilibrium forces are combined with the solution for $X_{1j}$, $X_{2j}$, $\delta_{ij}$, and $\delta_{ij}$ to calculate the primary unknown relative displacements $\delta_r$, $\delta_r$, and $\theta$ using iterative refinement.

The Newton-Raphson iteration scheme used by Jones and Harris requires numerous partial derivatives of some rather complicated expressions. If our analysis indicates that these terms must be rederived, we found that the use of MACSYMA can significantly reduce the time and labor required to obtain such quantities.
Work Package KHK130 (Formulate/Document Solution Methodology)

We had intended to begin work on this task by examining the codes we have obtained. This effort was deferred until we could adequately review the documentation obtained on the two primary bearing mechanics programs, RAPIDREB and SHABERTH, which we received this month.

Work Package KHK400 (Meetings/Technical Reporting)

The Version 2.0 program plan was formally submitted through contractual channels for approval by NASA. D'Jack Klingler (Aerojet) sent Joyce Mallory (NASA) two memos, one describing the revised program plan, and the other documenting a phone call in regard to personnel changes.

Work Package KHK410 (Cost Reporting)

Monthly report and support of replan activities.

Work Package KHK420 (Program Management)

Support of replan activity.
III. WORK FOR NEXT REPORTING PERIOD

Work Package KHK120 (Formulate/Document Models)

We will conclude work on the study of the theoretical aspects of ball and roller bearing kinematics and dynamics as well as the development of theory to support the incorporation of race flexibility into bearing mechanics software.

Work Package KHK130 (Formulate/Document Solution Methodology)

We will review the theoretical manuals obtained for the primary bearing mechanics analysis codes, RAPIDREB and SHABERTH, primarily to determine the applicability of the codes to support the incorporation of flexibility, enforced motion, and transient analyses. Where possible, we will compare the quasi-static solution approach in the codes to the methodology we have developed, in order to determine the amount of modification required.

Work Package KHK400 (Meetings/Technical Reporting)

Monthly report activity.

Work Package KHK410 (Cost Reporting)

Monthly report activity.

Work Package KHK420 (Program Management)

Monthly report activity.
IV. REFERENCES


Appendix A

List of Quasi-Static Analysis Technical References


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List of Quasi-Static Analysis Technical References


Rolling element bearings provide the primary mechanical interface between rotating and non-rotating components in the high performance turbomachinery of the Space Shuttle Main Engine (SSME). Knowledge of bearing behavior under various loading and environmental conditions is essential to predicting and understanding the overall behavior of turbopumps, including rotordynamic stability, critical speeds, and bearing life. The objective of this work is to develop mathematical models and computer programs to describe the mechanical behavior of ball and cylindrical roller bearings under the loading and environmental conditions encountered in the SSME and future high performance rocket engines. This includes characteristics such as nonlinear load/motion relationships, stiffness and damping, rolling element loads for life prediction, and roller and cage stability.

The work is divided into three phases; I: Quasi-static Equilibrium Analysis, II: Time Varying Loading and Dissipative Force Analysis, and III: Constrained Dynamic Analysis. The work emphasizes inclusion of race and bearing support flexure and clearances effects by finite element techniques.