Elastic Response of [001]-Oriented PWA 1480 Single Crystal—The Influence of Secondary Orientation

Sreeramesh Kalluri and Ali Abdul-Aziz
Sverdrup Technology, Inc.
Lewis Research Center Group
Brook Park, Ohio

and

Michael A. McGaw
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio

Prepared for the
1991 Aerospace Atlantic Meeting
sponsored by the Society of Automotive Engineers
Dayton, Ohio, April 23–26, 1991
Elastic Response of [001]-Oriented PWA 1480 Single Crystal -

The Influence of Secondary Orientation

Sreeramesh Kalluri and Ali Abdul-Aziz
Sverdrup Technology, Inc.,
NASA Lewis Research Center Group
Cleveland, Ohio 44135

and

Michael A. McGaw
NASA Lewis Research Center
Cleveland, Ohio 44135

Abstract

The influence of secondary orientation on the elastic response of a [001]-oriented nickel-base single-crystal superalloy, PWA 1480, was investigated under mechanical, thermal, and combined thermal and mechanical loading conditions by applying finite element techniques. Elastic stress analyses were performed with a commercially available finite element code. Secondary orientation of the single-crystal superalloy was offset with respect to the global coordinate system in increments from 0° to 90° and stresses developed within the single crystal were determined for each loading condition. The results indicated that the stresses were strongly influenced by the angular offset between the secondary crystal orientation and the global coordinate system. The degree of influence was found to vary with the type of loading condition (mechanical, thermal, or combined) imposed on the single-crystal superalloy.

Introduction

Single crystal (SC) superalloys were identified as potential blade materials that warranted further development for advanced liquid propellant rocket engine turbopumps by Chandler (1). After subsequent development, materials scientists evaluated the mechanical properties of several single crystal superalloys (2,3) to find a suitable candidate replacement blade material for the turbopumps of the Space Shuttle Main Engine (SSME) and advanced rocket engines. A well characterized, nickel-base cubic single crystal superalloy, PWA 1480, was selected by Pratt & Whitney as the blade material for the Alternate Turbopump Development (ATD) program for the SSME.

Turbine blades of nickel-base single crystal superalloys (including PWA 1480) are directionally solidified along the low modulus [001] crystallographic direction to enhance thermal fatigue resistance. The directional solidification process usually generates a secondary crystallographic direction, [010] that is randomly oriented with respect to fixed geometric axes in the turbine blade. However, orientation of the secondary crystallographic direction can be controlled by using a seed crystal during solidification (4). Since single crystals exhibit anisotropic elastic behavior, the stress-strain response and dynamic characteristics of a turbine blade that is directionally cast along the [001] crystal orientation tend to vary with the orientation of the secondary crystallographic direction within the blade (5-6).

The main objective of this study was to determine the influence of secondary orientation on the elastic response of [001]-oriented, single crystal PWA 1480 under mechanical, thermal, and combined thermal and mechanical loading conditions by applying finite element stress analyses. A parametric study was conducted in which the secondary orientation angle was varied in increments of 10° from 0° to 90° for each loading condition. The stresses developed within the single crystal PWA 1480 superalloy for all the cases of the parametric study are presented.

The turbine blade of a rocket engine turbopump is schematically represented in Fig. 1. This is a hollow SC PWA 1480 turbine blade that is directionally cast such that the [001] crystallographic direction (primary orientation) is parallel to the span of the blade. In this particular design the hollow core extends below the platform into the shank region. In Fig. 1, XYZ is the global coordinate system, with Z-axis along the span of the blade, Y-axis along the chord of the blade, and X-axis along the thickness of the blade.

In a cryogenic liquid propellant rocket engine turbopump, the blade attachment region may be actively cooled. Since the turbine blade airfoil is exposed to the hot gas flow path, thermal gradients are induced both along the span of the blade as well as through the thickness, for the
full length of the hollow core. These gradients may be especially severe through the thickness (of the order of 875 °C/cm), while being milder along the span. These gradients are substantially higher than those in typical aircraft gas turbine blades. To examine the role which the secondary crystal orientation may play in such cases, an analysis of a square plate subjected to similar loadings will be employed (Fig. 1).

A square plate with the dimensions of 25.4 by 25.4 by 3.2 mm was selected for the parametric finite element stress analyses. The crystal coordinate systems for the plate are shown in Fig. 1. The global coordinate system is obtained by rotating the crystal coordinate system about the Z-axis by an angle of θ. Since the axes Z and Zc are both oriented along the [001] crystallographic direction, the angle θ between the crystal and global coordinate systems represents the orientation of the secondary crystallographic direction. The matrix of direction cosines that relates the global coordinate system to the crystal coordinate system (Fig. 1) is as follows:

\[
\begin{bmatrix}
\cos \theta & \sin \theta & 0 \\
-\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

(1)

ELASTIC STIFFNESS COEFFICIENTS

Three independent elastic stiffness coefficients (C11, C12, and C44; Eq. (2)) are required to describe the elastic behavior of a cubic single crystal such as SC PWA 1480 within the crystallographic coordinate system (7). To describe the elastic behavior in the global coordinate system the stiffness tensor must be transformed from the crystal coordinate system to the global coordinate system (Fig. 1) according to the law of transformation of fourth rank tensors. Lieberman and Zirinsky (8) developed a simplified method for transforming the stiffness coefficients of single crystals between two coordinate systems that are related by a matrix of direction cosines (Eq. (1)). Their method was used together with Eq. (1) to obtain the elastic stiffness coefficients of a cubic single crystal in the global coordinate system. The stiffness matrix of the cubic single crystal within the global coordinate system and its relationship to the engineering stresses and engineering strains is as follows:

\[
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{zz} \\
\tau_{yx} \\
\tau_{zx} \\
\tau_{xy}
\end{bmatrix} =
\begin{bmatrix}
D_{11} & D_{12} & C_{12} & 0 & 0 & D_{16} \\
D_{12} & D_{11} & C_{12} & 0 & 0 & D_{26} \\
C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{44} & 0 \\
D_{16} & D_{26} & 0 & 0 & 0 & D_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\varepsilon_{zz} \\
\gamma_{yx} \\
\gamma_{zx} \\
\gamma_{xy}
\end{bmatrix}
\]

(2)

where,

\[
D_{11} = C_{11} - 2(C_{11} - C_{12} - 2C_{44}) x \sin^2 \theta \cos^2 \theta
\]

(3)

\[
D_{12} = C_{12} + 2(C_{11} - C_{12} - 2C_{44}) x \sin^2 \theta \cos^2 \theta
\]

(4)

\[
D_{16} = (C_{11} - C_{12} - 2C_{44})(\sin^2 \theta \cos \theta)
\]

(5)

\[
D_{26} = -D_{16}
\]

(6)

\[
D_{66} = C_{44} + 2(C_{11} - C_{12} - 2C_{44}) x \sin^2 \theta \cos^2 \theta
\]

(7)

In Eq. (2), σ_{xx}, σ_{yy}, σ_{zz} and τ_{yx}, τ_{zx}, τ_{xy} are the engineering normal and shear stresses, respectively, and ε_{xx}, ε_{yy}, ε_{zz} and γ_{yx}, γ_{zx}, γ_{xy} are the engineering normal and shear strains, respectively. Thus, Eqs. (2) to (7) define the stiffness matrix in a global coordinate system (XYZ) that is offset by an (arbitrary) angle θ to the crystal coordinate system (XcYcZc, Fig. 1).
The three independent elastic stiffness coefficients (applicable within the crystal coordinate system) for SC PWA 1480 at various temperatures are listed in Table 1. These coefficients were determined by the Engineering Division of Pratt & Whitney, United Technologies Corporation under the NASA Lewis Research Center contract NAS3-23939. The variations of stiffness coefficients $D_{11}$, $D_{12}$, $D_{16}$, $D_{26}$, and $D_{66}$ with the secondary orientation angle $\theta$, determined from Eqs. (2) to (7) for SC PWA 1480 at 38 °C, are shown in Fig. 2. It is evident from this figure that the variation in the elastic stiffness coefficients has a periodicity of 90° in $\theta$, and therefore, a range of 0 to 90° was selected for the secondary orientation angle in the parametric study.

FINITE ELEMENT ANALYSES

Elastic stress analyses were conducted on the plate model shown in Fig. 3 by using the MARC finite element structural analysis code (9). The finite element model of the plate contained a total of 500 isoparametric eight-noded solid elements with ten elements along the side and five elements through the thickness of the plate. The boundary conditions used were as follows: (1) the node at D was fixed in X, Y, and Z directions, (2) nodes along the edge AD were fixed in Y and Z directions, (3) nodes along the edge BC were fixed in X and Z directions and (4) nodes along edges AB and BC were fixed in the Z direction. All boundary conditions were enforced by specifying the fixed boundary option of the MARC code. A total of five different loading cases were considered in this study. In these cases mechanical, thermal and combined mechanical and thermal loads were applied separately to the model. The previously described boundary conditions were employed in all five loading cases. The first loading case involved a distributed mechanical load of 137.9 MPa along the Z-direction (Fig. 4(a)). In the second and third loading cases thermal gradients were imposed along the Z- and X-direction, respectively (Figs. 4(b) and (c)). Imposition of thermal gradients along the Z and X directions was an attempt to simulate the thermal gradients that occur along the span and through the thickness of the actual turbine blade. Finally, the fourth and fifth loading cases were obtained by combining the first loading case with the second and third loading cases, respectively.

The physical properties of SC PWA 1480 including mean coefficient of thermal expansion, thermal conductivity and specific heat were obtained from Ref. 10. For the loading cases involving thermal gradients, thermal boundary conditions represented by the temperatures in Fig. 4 were applied to the finite element model considered. These thermal boundary conditions may be representative of cryogenic rocket engine turbomachinery turbine blade applications. Thermal boundary conditions were introduced into the finite element code enforcing the assigned temperatures to the specific faces of the plate in Fig. 4, while insulating all other faces. Thermal gradients shown in Fig. 5 were used in the second and third loading cases. These data represent temperatures at the centroids of the elements. The thermal output feature of MARC code was used to generate the input subsequent stress analysis.

For each loading case in the parametric study, the secondary orientation angle $\theta$ was varied from 0° to 90° in increments of 10°. In addition, all loading cases were studied for $\theta = 45°$ to obtain the data at the symmetry location (Fig. 2). The stiffness matrices for the various secondary orientations were incorporated in the finite element analyses by programming Eq. (2) in the MARC user's subroutine HOOKLW (9). Variations in the physical and elastic properties of the material with temperature were accounted for in the analyses.

The variations in the different components of stress with secondary crystal orientation angle $\theta$ were documented for three elements shown in Fig. 3. These elements were selected to capture the extreme as well as reference stress locations for the five different loading cases considered in this study. For example, for the second loading case in which a thermal gradient was imposed along the Z-direction (Fig. 4(b)), extreme thermal stresses could be generated in faces ABCD and EFGH. Hence, the selection of element 295 (Fig. 3) which was near surface EFGH for this loading case. Similar reasoning was used in selecting element 445 (Fig. 3) for the third loading case in which a thermal gradient was imposed along the X-direction. Finally, a centrally located, element 245 (Fig. 3) was selected for the first loading case in which only the mechanical loading was imposed. Another consideration that led to the choice of these specific locations was mitigation of the constraint effects on the stresses generated within the elements.

For the five loading cases considered in this study, the values of the six stress components at the centroid of the selected elements are listed in Table 3 for a secondary orientation angle of $\theta = 0°$. In all loading cases the stresses developed within the three selected elements were well within the elastic regime of SC PWA 1480. In loading Case 1, which involved only mechanical loading, values of the six stress components in the three selected elements were very similar. However, in loading Cases 2 and 3 which involved thermal gradients along the Z- and X-axis, respectively, elements 295 (near the hot side of a thermal gradient along Z-axis) and 445 (near the cold side of a thermal gradient along X-axis) were subjected to large normal compressive and tensile stresses, respectively, compared to the normal stresses in element 245 which was nearly at the center of the mesh. The magnitudes of the tensile normal stresses of element 445 in loading Case 3 were larger than the magnitudes of compressive normal stresses in
element 296 under loading Case 2 because the square plate was subjected to a more severe thermal gradient along the thickness (X-axis) than along the length (Z-axis). Finally, in loading Cases 4 and 5 which involved both mechanical and thermal loading, the stresses developed within the selected elements were very nearly equal to those expected from the superposition of stresses developed in the individual mechanical and thermal loadings (loading Case 1 with loading Cases 2 and 3, respectively). These results are in agreement with the principle of superposition for the linear elastic analysis.

The influence of secondary orientation on the stresses developed within the selected elements are shown in Figs. 6 to 8. For each element and loading case, the individual stress components at a given secondary orientation angle \( \theta \) were normalized with the corresponding stress components obtained for \( \theta = 0^\circ \), i.e.,

\[
g(X) = \sigma(X) / \sigma(X)_{0}
\]

\[
v(X) = \tau(X) / \tau(X)_{0}
\]

\[
w(X) = \sigma(X) / \sigma(X)_{0}
\]

and

\[
X = \tau(X) / \tau(X)_{0}
\]

The individual stress components were then plotted against the secondary orientation angle \( \theta \) for each element and loading condition. If there was no influence of secondary orientation on the elastic response of SC PWA 1480, then all six normalized stress components should remain at a fixed value of unity as \( \theta \) is varied from \( 0^\circ \) to \( 90^\circ \). Any deviation from unity is an indication that the elastic response of PWA 1480 SC is affected by the secondary orientation angle \( \theta \). The influence of secondary orientation on individual and normalized stress components under imposed mechanical loading (Case 1) is shown in Fig. 6 for the three selected elements. The variation of individual and normalized stress components with secondary orientation angle \( \theta \) are shown in Figs. 7 and 8 for thermal loads (loading Cases 2 and 3) and combined thermal and mechanical loads (loading Cases 4 and 5), respectively.

The influence of the secondary orientation angle \( \theta \) on both the individual and normalized stress components was dictated by the type (mechanical or thermal or combined) of loading imposed on the SC PWA 1480 square plate. For example, no significant influence of secondary orientation was observed for the individual stress components under mechanical loading (Fig. 6). However, in cases involving thermal gradients along X- and Z-axis the individual normal stress components exhibited significantly larger influence of secondary orientation than the individual shear stresses (Figs. 7 and 8). The normalized stress components exhibited a substantial dependence on the secondary orientation angle \( \theta \) under all five loading cases. In all of the loading cases except those involving thermal gradient along the X-axis, largest influence of secondary orientation was observed for the normalized stress component \( \sigma_{xy} / \sigma_{xy,0} \) (for the cases involving thermal gradient along the X-axis the normalized components \( \sigma_{xy} / \sigma_{xy,0} \) and \( \sigma_{x} / \sigma_{x,0} \) exhibited about the same order of variation with the secondary orientation angle \( \theta \).

**DISCUSSION**

In this study, the elastic stress components were normalized to determine the influence of secondary orientation angle \( \theta \), under different loading conditions. In Figs. 6 to 8 the normalized stress components indicate that, on a relative basis, all six components of stress are influenced by the secondary orientation angle, \( \theta \). However, the importance of secondary orientation effects cannot be evaluated by consideration of the variation in the normalized stress components (with \( \theta \)) alone. The magnitudes of the individual components must also be taken into consideration for accurate assessment of the influence of secondary orientation. For example, one normalized stress component, \( \sigma_{xy} / \sigma_{xy,0} \), exhibited a range as large as \( \pm 166 \) in element 295 under loading Case 2, in which thermal loading was imposed along the Z-axis (Fig. 7(a)(ii)). However, it is clear from Table 3 that the value of \( \sigma_{xy} \) at \( \theta = 0^\circ \) is only 0.005 Mpa. For this loading condition, multiplying this stress component by a factor of 166 yields 0.830 Mpa, a relatively small value compared to the other normal components of stress within this element (Fig. 7(a)(ii)). By comparison, the normalized stress component \( \sigma_{xy} / \sigma_{xy,0} \) increased only from 1.0 to 1.4 as \( \theta \) was increased from \( 0^\circ \) to \( 90^\circ \) under thermal loading along the X-axis (Fig. 7(b)(ii)). However, this represented an increase in \( \sigma_{xy} \) from 204.2 Mpa at \( \theta = 0^\circ \) to 286.3 Mpa at \( \theta = 45^\circ \) (Fig. 7(b)(ii)). Thus, the variations in the normalized stress components with \( \theta \), while indicative of the influence of secondary orientation on the elastic response of SC PWA 1480, alone are not sufficient to assess the importance of secondary orientation effects. The variation of actual magnitudes of the individual components of stress also must be considered to determine the quantitative significance of the secondary orientation effects.

Among the mechanical, thermal and combined loading cases considered in this study, the case of a thermal gradient along the X-axis (through the thickness of the square plate) produced large stresses (Table 3; Case 3, element 445 is under biaxial tension). This is due to the fact that the thermal gradient through the thickness of the plate is more severe than the thermal gradient along the length of the square plate (Fig. 5). In loading case 5, the mechanical load which was superimposed on the thermal load of Case 3, nearly doubled the \( \sigma_{xy} \) component of stress within the square plate (Table 3; Case 5, element 445).

The state of maximum biaxial tension occurred for loading Case 5 at a secondary orientation angle \( \theta = 45^\circ \) (Table 3, element 445 and Fig. 8(b)(i)). This is to be expected because in this instance the stiffness coefficient \( D_{11} \) (which reaches a maximum at \( \theta = 45^\circ \), Fig. 2) is coupled with a steep thermal gradient through the thickness of the square plate and a mechanical load along the Z-axis. Thus, large
thermal gradients perpendicular to the primary orientation [001] of the turbine blade may be expected to produce very high stresses, if they occur near the secondary orientation of $\theta = 45^\circ$. However, these stresses can be minimized by controlling the secondary orientation so that thermal gradients perpendicular to [001] occur near $\theta = 0^\circ$.

Some normalized stress components shown in Figs. 6 to 8 exhibit a lack of symmetry about the secondary orientation angle, $\theta = 45^\circ$. For example, the normalized stress component $(\sigma_x)_{0}\theta/(\sigma_{xx})_\theta$ is not symmetric about $\theta = 45^\circ$. Similar behavior is also exhibited by the normalized stress components, $(\tau_{yx})_{0}\theta/(\tau_{yx})_\theta$ and $(\tau_{xy})_{0}\theta/(\tau_{xy})_\theta$ in Fig. 6(c)(ii) and by $(\sigma_{xx})_{0}\theta/(\sigma_{xx})_\theta$, $(\tau_{yy})_{0}\theta/(\tau_{yy})_\theta$, and $(\tau_{xy})_{0}\theta/(\tau_{xy})_\theta$ in Fig. 8(b)(ii). The main reason for lack of symmetry about $\theta = 45^\circ$ is the coupling between $\sigma_{xx}$ and $\sigma_{xy}$ as well as between $\tau_{xy}$ and $\tau_{xx}$ in the single crystal stiffness matrix (Eq. (2)). The coupling elastic coefficients $D_{1e}$ and $D_{2e}$ change sign about $\theta = 45^\circ$ (Fig. 2) which creates a lack of symmetry in the stress components. A small amount of observed lack of symmetry may also have been caused by numerical round off errors. This may especially be the reason for the observed asymmetry in $(\tau_{yx})_{0}\theta/(\tau_{yx})_\theta$, because $\tau_{yx}$ has no coupling terms in the single crystal stiffness matrix.

In this study the influence of secondary orientation was studied under purely elastic conditions and steady state thermal gradients. In actual service, turbine blades are subjected to severe thermal transients due to start up and shutdown, which can produce localized plasticity and creep in the turbine blades. The influence of secondary orientation on the stresses produced within the actual blade under conditions of plasticity and creep will be different from the influence determined under fully elastic conditions. To determine the influence of secondary orientation on the stresses developed within the turbine blades, nonlinear, transient thermal structural analyses must be conducted with appropriate boundary conditions.

CONCLUSIONS

A parametric study was conducted with elastic finite element analyses to determine the influence of secondary orientation angle on the elastic behavior of a [001]-oriented nickel-base single crystal superalloy, PWA 1480 under mechanical, thermal and combined mechanical and thermal loads. The secondary orientation angle was varied from $0^\circ$ to $90^\circ$ and the stresses developed within a square plate of SC PWA 1480 under different loading conditions were computed. The following conclusions were drawn from the parametric study.

1. Under the loading conditions studied, the normalized stress components showed a substantial variation with the secondary orientation angle. However, the normalized stress components should be considered along with the magnitudes of the individual stress components to determine the significance of secondary orientation in engineering design.

2. The type of loading (mechanical, thermal, or combined) imposed on the square plate dictated the variation in the individual stress components with the secondary orientation angle.

3. For the same temperature boundary conditions, thermal gradients through the thickness of the square plate produced higher stresses than thermal gradients along the length (primary orientation of [001]) of the square plate. This was due to a higher thermal gradient through the thickness of the plate. If it is required to impose thermal gradients perpendicular to the primary [001] orientation, a secondary orientation of $0^\circ$ tends to minimize the thermal stresses developed within the plate, whereas a secondary orientation of $45^\circ$ tends to produce much larger thermal stresses.

REFERENCES


### TABLE 1. - INDEPENDENT ELASTIC STIFFNESS COEFFICIENTS FOR SC PWA 1480

<table>
<thead>
<tr>
<th>Temperature, °C</th>
<th>( C_{11} ), GPa</th>
<th>( C_{12} ), GPa</th>
<th>( C_{44} ), GPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>-18</td>
<td>252</td>
<td>163</td>
<td>131</td>
</tr>
<tr>
<td>38</td>
<td>250</td>
<td>163</td>
<td>129</td>
</tr>
<tr>
<td>93</td>
<td>248</td>
<td>161</td>
<td>128</td>
</tr>
<tr>
<td>149</td>
<td>246</td>
<td>160</td>
<td>126</td>
</tr>
<tr>
<td>204</td>
<td>244</td>
<td>159</td>
<td>124</td>
</tr>
<tr>
<td>260</td>
<td>242</td>
<td>158</td>
<td>123</td>
</tr>
<tr>
<td>316</td>
<td>240</td>
<td>157</td>
<td>121</td>
</tr>
<tr>
<td>371</td>
<td>238</td>
<td>157</td>
<td>119</td>
</tr>
<tr>
<td>427</td>
<td>235</td>
<td>156</td>
<td>117</td>
</tr>
<tr>
<td>482</td>
<td>233</td>
<td>154</td>
<td>115</td>
</tr>
<tr>
<td>538</td>
<td>230</td>
<td>154</td>
<td>113</td>
</tr>
<tr>
<td>593</td>
<td>228</td>
<td>152</td>
<td>111</td>
</tr>
<tr>
<td>649</td>
<td>225</td>
<td>152</td>
<td>109</td>
</tr>
<tr>
<td>704</td>
<td>223</td>
<td>151</td>
<td>107</td>
</tr>
<tr>
<td>760</td>
<td>219</td>
<td>150</td>
<td>105</td>
</tr>
<tr>
<td>816</td>
<td>217</td>
<td>150</td>
<td>102</td>
</tr>
<tr>
<td>871</td>
<td>213</td>
<td>149</td>
<td>100</td>
</tr>
<tr>
<td>927</td>
<td>210</td>
<td>148</td>
<td>97</td>
</tr>
<tr>
<td>982</td>
<td>206</td>
<td>146</td>
<td>95</td>
</tr>
<tr>
<td>1038</td>
<td>201</td>
<td>145</td>
<td>92</td>
</tr>
<tr>
<td>1093</td>
<td>197</td>
<td>143</td>
<td>88</td>
</tr>
<tr>
<td>1149</td>
<td>192</td>
<td>143</td>
<td>85</td>
</tr>
<tr>
<td>1204</td>
<td>186</td>
<td>142</td>
<td>81</td>
</tr>
</tbody>
</table>

### TABLE 2. - PHYSICAL PROPERTIES OF SC PWA 1480

[From Ref. (9).]

<table>
<thead>
<tr>
<th>Temperature, °C</th>
<th>Thermal coefficient of expansion, ( \alpha ), m/m/°C</th>
<th>Thermal conductivity, ( k ), W/m °C</th>
<th>Specific heat, ( c ), kJ/kg °C</th>
</tr>
</thead>
<tbody>
<tr>
<td>93</td>
<td>0.0000107</td>
<td>9.2</td>
<td>0.406</td>
</tr>
<tr>
<td>204</td>
<td>0.000111</td>
<td>10.8</td>
<td>0.440</td>
</tr>
<tr>
<td>316</td>
<td>0.000115</td>
<td>12.4</td>
<td>0.465</td>
</tr>
<tr>
<td>427</td>
<td>0.000119</td>
<td>13.7</td>
<td>0.477</td>
</tr>
<tr>
<td>538</td>
<td>0.000124</td>
<td>15.3</td>
<td>0.481</td>
</tr>
<tr>
<td>649</td>
<td>0.000129</td>
<td>16.9</td>
<td>0.490</td>
</tr>
<tr>
<td>760</td>
<td>0.000134</td>
<td>18.5</td>
<td>0.511</td>
</tr>
<tr>
<td>871</td>
<td>0.000140</td>
<td>20.0</td>
<td>0.569</td>
</tr>
<tr>
<td>982</td>
<td>0.000147</td>
<td>21.5</td>
<td>0.687</td>
</tr>
<tr>
<td>1093</td>
<td>0.000157</td>
<td>22.8</td>
<td>0.917</td>
</tr>
</tbody>
</table>

### TABLE 3. - STRESSES DEVELOPED IN SINGLE CRYSTAL PWA 1480 PLATE UNDER MECHANICAL, THERMAL AND COMBINED LOADING CASES SECONDARY ORIENTATION ANGLE, \( \beta = 0^\circ \)

<table>
<thead>
<tr>
<th>Case</th>
<th>Type of loading</th>
<th>Element</th>
<th>( \sigma_{xx} ), MPa</th>
<th>( \sigma_{yy} ), MPa</th>
<th>( \sigma_{zz} ), MPa</th>
<th>( \tau_{xy} ), MPa</th>
<th>( \tau_{yz} ), MPa</th>
<th>( \tau_{zx} ), MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Mechanical (Z-axis)</td>
<td>245</td>
<td>0.655</td>
<td>1.310</td>
<td>136.9</td>
<td>0.001</td>
<td>0.316</td>
<td>-0.047</td>
</tr>
<tr>
<td></td>
<td></td>
<td>295</td>
<td>0.012</td>
<td>0.053</td>
<td>137.9</td>
<td>0.001</td>
<td>-0.003</td>
<td>-0.006</td>
</tr>
<tr>
<td></td>
<td></td>
<td>445</td>
<td>-0.090</td>
<td>1.234</td>
<td>136.6</td>
<td>0.005</td>
<td>0.385</td>
<td>-0.192</td>
</tr>
<tr>
<td>2</td>
<td>Thermal (Z-axis)</td>
<td>245</td>
<td>-0.043</td>
<td>-19.44</td>
<td>-20.34</td>
<td>19.98</td>
<td>-0.017</td>
<td>-5.475</td>
</tr>
<tr>
<td></td>
<td></td>
<td>295</td>
<td>-19.44</td>
<td>-20.34</td>
<td>-1.88</td>
<td>0.005</td>
<td>0.043</td>
<td>-0.034</td>
</tr>
<tr>
<td>3</td>
<td>Thermal (X-axis)</td>
<td>245</td>
<td>0.034</td>
<td>3.020</td>
<td>6.847</td>
<td>1.496</td>
<td>-0.270</td>
<td>-12.41</td>
</tr>
<tr>
<td></td>
<td></td>
<td>445</td>
<td>0.993</td>
<td>204.2</td>
<td>130.0</td>
<td>1.214</td>
<td>15.56</td>
<td>-3.744</td>
</tr>
<tr>
<td>4</td>
<td>Mechanical and thermal (Z-axis)</td>
<td>245</td>
<td>-14.47</td>
<td>20.29</td>
<td>156.9</td>
<td>0.018</td>
<td>-5.157</td>
<td>-3.343</td>
</tr>
<tr>
<td></td>
<td></td>
<td>295</td>
<td>-19.43</td>
<td>-20.29</td>
<td>136.0</td>
<td>0.006</td>
<td>0.040</td>
<td>-0.041</td>
</tr>
<tr>
<td>5</td>
<td>Mechanical (Z-axis) and thermal (X-axis)</td>
<td>245</td>
<td>0.687</td>
<td>4.330</td>
<td>143.7</td>
<td>1.496</td>
<td>0.046</td>
<td>12.46</td>
</tr>
</tbody>
</table>
Figure 1.—Schematics of SSME turbine blade and the square plate considered for the analysis.

Figure 2.—Variation of elastic stiffness coefficients of SC PWA 1480 with the secondary orientation angle $\theta$. 

XYZ : Global co-ordinate system. 
X_c Y_c Z_c : Crystal co-ordinate system.
Figure 3.—Finite element mesh of the square plate.
(a) Distributed mechanical load along Z-axis.

(b) Thermal gradient along Z-axis.

(c) Thermal gradient along X-axis.

Figure 4.—Loading cases imposed on the square plate.

Figure 5.—Temperature distributions calculated from the heat transfer analyses.
Figure 6.—Influence of secondary orientation $\theta$ on individual and normalized stress components under imposed mechanical loading.
Figure 7.—Variation of normalized stress components with secondary orientation angle \( \theta \) for thermal loads.

Figure 8.—Variation of normalized stress components with secondary orientation angle \( \theta \) for combined mechanical and thermal loads.
Elastic Response of [001]-Oriented PWA 1480 Single Crystal—The Influence of Secondary Orientation

Sreeramesh Kalluri, Ali Abdul-Aziz, and Michael A. McGaw

National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135-3191


The influence of secondary orientation on the elastic response of a [001]-oriented nickel-base single-crystal superalloy, PWA 1480, was investigated under mechanical loading conditions by applying finite element techniques. Elastic stress analyses were performed with a commercially available finite element code. Secondary orientation of the single-crystal superalloy was offset with respect to the global coordinate system in increments from 0° to 90° and stresses developed within the single crystal were determined for each loading condition. The results indicated that the stresses were strongly influenced by the angular offset between the secondary crystal orientation and the global coordinate system. The degree of influence was found to vary with the type of loading condition (mechanical, thermal, or combined) imposed on the single-crystal superalloy.