ON-ORBIT DAMAGE DETECTION AND HEALTH MONITORING OF LARGE SPACE TRUSSES - STATUS AND CRITICAL ISSUES

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Abstract

The long lifetimes, delicate nature and stringent pointing requirements of large space structures such as Space Station Freedom and geostationary Earth science platforms might require that these spacecraft be monitored periodically for possible damage to the load carrying structures. A review of the literature in damage detection and health monitoring of such structures is presented, along with a candidate structure to be used as the test bed for future work in this field. A unified notation and terminology is also proposed to facilitate comparisons between candidate methods.

Nomenclature

\[ b = \text{number of independent stiffness elements or substructures} \]
\[ m = \text{number of measured modes} \]
\[ n = \text{number of degrees of freedom} \]
\[ s = \text{number of sensors} \]
\[ [M] = \text{n x n updated mass matrix} \]
\[ [K] = \text{n x n updated stiffness matrix} \]
\[ [C]_i = \text{Connectivity matrix for mode i} \]
\[ [D]_i = \text{n x n analytical system damping matrix} \]
\[ [F] = \text{n x n frequency-stiffness sensitivity matrix} \]
\[ [K_d] = \text{n x n analytical stiffness matrix} \]
\[ [M_d] = \text{n x n analytical mass matrix} \]
\[ k_{ij} = \text{elements of stiffness matrix with largest \% adjustment} \]
\[ [\phi]_i = \text{m x 1 vector of modal coordinates for mode i} \]
\[ [q]_i = \text{nodal displacement vector} \]
\[ [\Phi] = \text{n x m updated analytical mode shape vectors} \]
\[ [\Phi_d] = \text{n x m expanded and orthogonalized mode shape vectors of undamaged structure} \]
\[ [\lambda] = \text{matrix of Lagrange multipliers} \]
\[ [\Omega] = \text{m x m diagonal matrix of squared circular frequencies of undamaged structure} \]
\[ [\Omega_d] = \text{m x m diagonal matrix of squared circular frequencies of damaged structure} \]
\[ [\Psi] = \text{n x m expanded mode shape vectors of undamaged structure} \]
\[ [\Psi_d] = \text{n x m expanded mode shape vectors of damaged structure} \]
\[ \omega_i = \text{system circular frequency for mode i} \]

Introduction

Future structures in space will be orders of magnitude larger and more complex than their predecessors. Structures such as the space station Freedom will typically be built around a large flexible frame and consist of truss members, habitat and experimental modules, flexible and articulating appendages, along with numerous utility trays and moving parts. The complexity and size of these structures, along with the need to design the spacecraft to be lightweight, strong and modular for ease of expansion, repair and modification, all require that the integrity of the structures be monitored periodically.

Several researchers have proposed methods of detecting damage to large space trusses and locating the site of this damage based on changes in the vibration frequencies and modes of the structure. A research program underway at the NASA Langley Research Center is using a hybrid-scale model of the space station as a test-bed for studying the dynamic behavior of such structures. As part of this program, researchers plan to study the implementation of an on-orbit damage location scheme. Results to date indicate that it may be possible to use the reaction control systems of the space station to perform an on-orbit modal test of the structure and extract frequency and mode shape data which might be used in this damage location technique.

The purpose of this paper is to provide a comprehensive literature review covering those damage location methods based on knowledge of the dynamic properties of the structure. Some of the most important aspects that make an on-orbit verification and identification of truss structures different from the equivalent processes on the ground are also discussed. In addition, results from the current effort have indicated the need for a generally accepted notation and terminology to allow researchers to readily compare their methods and experimental results. Finally, it is proposed that the Dynamic Scale Model Technology (DSMT) hybrid scale space station model be used as the benchmark test bed for future work.

This paper draws heavily from three sources. The Task Committee on Methods for Identification of Large Structures in Space published the report "Identification of Large Space Structures On Orbit" in 1986. This report provides an
excellent overview of many of the issues central to system identification of large space structures. The bibliography is extensive and the report contains several chapters that are of particular interest for on-orbit testing and damage location.

More recent information about on-orbit modal identification has been taken from in-house progress reports and briefings of the Modal Identification Experiment (MIE) - a research program at the NASA Langley Research Center. The MIE program is performing extensive research into procedures for carrying out an on-orbit verification and modal identification of the space station Freedom during and after each phase of assembly. Topics covered in this research include optimal sensor placement, baseline excitation definition and data acquisition.

The third key source for this paper is the research of Dr. S. W. Smith. Smith's doctoral dissertation, "Damage Detection and Location in Large Space Trusses"^3, contains a large bibliography as well as comparisons and evaluations of many of the candidate system identification methods.

**Damage Location Methodology**

**Problem Description**

For purposes of the current research, "Large Space Structures" will refer to spacecraft such as space station Freedom, geostationary platforms such as those proposed for Mission to Planet Earth, and other structures that are predominantly erectable or deployable trusses to which a variety of payloads are attached. Standard assumptions will be made, including the assumption that the structure behaves linearly. In addition, damping in the structure is assumed small and therefore a proportional damping model is used.

For a spacecraft in orbit there are numerous mechanisms by which damage can be introduced into such a complex system. Damage scenarios ranging from radiation degradation of load carrying members, micrometeorite impact, loosening of joints due to excessive vibration, all have to be considered and studied. Some damage scenarios may lend themselves well to visual detection, while others will be invisible, on external inspection.

In general, the problem of locating a damaged site on a structure can be equated to locating regions where the stiffness or load carrying capacity has been reduced by a measurable amount. These regions might be identified by performing an on-orbit modal test using the spacecraft reaction control systems to excite the structure and produce modal response characteristics such as frequencies and mode shapes. These parameters are then compared to a baseline set of parameters. A variety of algorithms have been proposed that will trace differences in the two sets of data to specific or likely damage locations. The problem is complicated significantly by the test environment when the test is performed on orbit and these difficulties will be discussed in subsequent sections.

**Proposed Methodology**

A flow chart illustrating the approach for damage location that is being used in the DSMT program is shown in figure 1. Each of the vertical arrows represents a process that produces the result in bold text that follows. The horizontal arrows represent processes by which data is exchanged between the analytical model and the on-orbit structure. Each of the studies that is discussed uses a particular algorithm for each of these processes. The purpose of this paper is to consider each of these steps and how different investigators have chosen to attack each process. The figure depicts a general scheme for the design of a large spacecraft, the on-orbit identification of its dynamic characteristics and the algorithm for health monitoring and damage detection and location. The column on the left shows the stages through which the analytical model progresses, from an initial design model to a finite element model which has been modified using data obtained from on-orbit measurements. The column on the right shows a similar evolution of the actual hardware, from design studies and ground testing of individual components to the full on-orbit spacecraft dynamic characteristics. The boxed region is the heart of the damage location process. Two kinds of iterations occur: the first run through serves to verify the newly assembled and

![Figure 1: Damage Location Approach](image-url)
presumed undamaged spacecraft via system identification methods. During subsequent passes through this boxed portion of the flow-chart, the measured data is assumed to be from a potentially damaged spacecraft.

Based on design studies and years of concept development, an analytical model is created, usually in the form of a finite element model. This model is used to predict mode shapes and natural frequencies, as well as the forced response of the structure. Using the static and dynamic finite element solutions, components of the structure are fabricated and individually subjected to detailed ground tests. The mass, stiffness and damping matrices of the finite element model are updated based on results of the component tests. The final pre-launch version of the finite element model is used, among other things, in sensor placement studies such that the accelerometers and strain gauges and other instruments are placed in as close to an optimal configuration as possible. The sensors will be used to extract system frequencies and mode shapes once the spacecraft has been assembled in orbit.

After assembly, the structure is subjected to a series of on-orbit tests during which forced response measurements are acquired and used in a modal identification of the completed structure. Once again this data is used to update the finite element model to ensure that it accurately predicts the true response of the structure and can be used to predict response to expected operational loads and identify damage. The updated finite element model is then stored to be used as the reference (or undamaged) configuration against which a damaged model can be compared to accurately predict the location and magnitude of damage.

Research test-bed

To date most research into structural damage detection has been performed by a handful of researchers at a wide variety of sites with little or no coordination in research efforts. Many of these methods have been tested using mass-spring test models or simple planar truss models. Few of the standard test problems truly embrace the essence of large flexible structures in space and as such are poor judges of the performance of a new method. It would be considerably more beneficial for these methods to be tested on a more realistic model.

An ideal candidate for such a standard test-bed is the DSMT hybrid scale model at the NASA Langley Research Center. As described earlier the DSMT model is designed as an experimental test-bed for the space station Freedom research program. Currently, an early build configuration is suspended by cables from a 40 foot high gantry, and all the components are available to build and suspend the current version of the "assembly complete" configuration. Starting with the MB-2 (Second Mission Build) configuration, finite element models and test data for each assembly flight configuration will be available for researchers to use. Figure 2 is a finite element model of the HMB-2 (Hybrid Mission Build-2) configuration.

As part of the DSMT program, all components of the model are being tested individually. One of these components consists of eight bays of the hybrid model truss. This component is currently being used to verify the dynamic characteristics of the hybrid truss assembly and has been used in some preliminary damage detection efforts. Figure 3 is a finite element depiction of the eight bay truss.

Table 1 shows analytical frequencies for the first five modes of the eight bay truss with no damage and also frequencies for four damage cases as identified in figure 3. Damage here refers to a particular strut missing. This table demonstrates that differences in the natural frequencies can be very small indeed, and this information, by itself, is not in general sufficient to detect or locate the damage.

Table 2 contains the mode shape coefficients at node number 1 for the first bending mode and the same four damage cases. This table clearly indicates that the mode shapes can be drastically

![Figure 2. DSMT Hybrid Scale HMB-2 Model](image_url)

![Figure 3. Cantilevered Eight Bay Truss With Damage](image_url)
affected by different damage cases and therefore may contain more useful information for determining damage location than the frequencies alone. Besides the modes and frequencies, the kinetic and strain energies for each mode also contain important information that will help in locating damage. The DSMT program will investigate the use of all these quantities.

In preparation for using the full DSMT model to study the damage location problem, the methods will first be developed using the eight bay truss. Since this is a clean truss that demonstrates many of the characteristics that will be found on large space structures it is an ideal candidate for a preliminary test-bed for damage location studies. Modal tests of each of the DSMT configurations will provide the baseline mass and stiffness matrices for tests with missing members or loose joints. The models will also be used to perform excitation and sensor placement studies in conjunction with the on-orbit Modal Identification Experiment. Data from all of these tests will be available to researchers in industry and academia to use as verification tools for new methods or new applications for existing methods.

**Analytical Modelling**

Truss structures can be modeled as discrete structural dynamic systems and thus the analytical model of such a structure consists of linear second order differential equations. These equations are often represented by their finite element formulation

\[
[M]\{\ddot{x}\} + [D]\{\dot{x}\} + [K]\{x\} = \{f(t)\} \tag{1}
\]

For design studies a detailed finite element model containing on the order of hundreds of elements and thousands of degrees of freedom is created to study the static behavior of the structure. It is often computationally inefficient to use the entire finite element model for a dynamic analysis which requires the repeated solution of the eigenvalue equation:

\[
([K] - \omega^2 [M])\{\phi\}_i = 0 \tag{2}
\]

There are several ways to reduce the size of the model to increase the solution speed. Some of the more common methods in use and being improved are static and dynamic reduction methods; equivalent and continuum modelling; and component modal synthesis. These, and other reduction techniques often require significantly less computational effort, but introduce assumptions and idealizations that almost always reduce the accuracy of the results.

**Ground Test Methods**

The size and cost of structures such as the space station make it prohibitive, if not impossible, to perform full scale system testing during the design phase. In fact the space station will be the first manned spacecraft that will not have undergone full vibration testing on the ground prior to launch. It is, however, possible to test individual components and sub-assemblies of the structure, along with tests of fully mated scale models. The component tests can be used to improve the analytical models at the component level (for example, accurate mass and damping properties). The scale models can be used to explore excitation and sensor placement methods, along with component synthesis techniques.

**Scale Model Techniques**

Scale model techniques allow for insightful tests to be conducted on smaller versions of the spacecraft. Letchworth and McGowan discuss the DSMT program at the NASA Langley Research Center, which has as its objective the

![Table 1: Natural Frequencies for Eight Bay Truss (Frequencies in Hz)](data:image/png;base64,iVBORw0KGgoAAAANSUhEUgAAAIcAAAAHCAIAAAD8z8MgAAAABJRU5ErkJggg==)

![Table 2: Mode Shape Coefficients at Node 1](data:image/png;base64,iVBORw0KGgoAAAANSUhEUgAAAIgAAAAHCAIAAAD8z8MgAAAABJRU5ErkJggg==)
Development of a verified capability for predicting the on-orbit structural dynamic behavior of large, multi-bodied, joint-dominated, articulated, flexible space structures.

The approach in the DSMT program is to use data from ground tests of fully mated, near-replica, dynamic scale models to improve theoretical analyses and thereby improve the ability to predict full-scale on-orbit structural dynamics. The DSMT program is currently using a hybrid scale model of the space station, where hybrid refers to the 1/10th scale overall geometry and 1/5th scale dynamic properties.

Component Mode Synthesis

A large number of papers discuss the analytical aspects of the Component Mode Synthesis method (CMS) that was first proposed by Hurty in 1965 and further developed by Hou (1969), Craig and Bampton (1976) and Hale and Meirovich (1982). Very few papers, however, have discussed the use of CMS in experimental testing. Martinez and his colleagues discuss the use of experimental mode data and Baker identified several shortcomings of some of the standard CMS methods when used in experimental modal analysis. Still, very little exists in the literature concerning the synthesis of structural response of systems based on experimentally determined component modes and frequency data. The DSMT program has initiated a series of vibration tests on each of the scale model components and major substructures. The data from these tests will be used to evaluate the existing CMS methods and if necessary develop more realistic and applicable approaches.

For the space station it is likely that each stand-alone component such as the solar arrays, radiators, equipment pallets and habitation modules, will be thoroughly tested prior to launch and assembly. The dynamic characteristics of each component will be well known individually, but not as a part of the assembled structure. The task of ground-test synthesis methods will be to accurately predict the response of the spacecraft when each of these components is mated to the truss and other structural members of the infrastructure.

Excitation Placement

In ground tests of components or scale models, the source of transient excitations for modal tests are generally electrodynamic shakers. The excitations can be applied at any location on the structure in order to excite particular modes of interest. The excitations that will be used for modal testing on orbit will be provided by the reaction control system (RCS) thrusters. These will be positioned along the structure in predetermained locations. In order to perform a modal test for system identification or damage location purposes the thrusters will be fired in a particular sequence so as to excite a given set of modes. The placement of the thrusters will be determined by the control system needs and the set of excitable modes may not be optimal for structural dynamic parameter identification. Damage location or system identification algorithms must take these factors into consideration.

Researchers at NASA Langley Research Center have initiated studies to determine the best way to excite the modes of interest on a large space structure. A recent MIE project review presented results of an excitation sensitivity study which evaluated various excitation parameters such as the type, duration, direction and size of the excitation, to aid in designing the baseline excitation for modal identification. The results of this study indicate that a random forcing function will provide the best performance for modal identification. In addition, the forcing functions observed during a nominal reboost maneuver are suitable for some modal ID tests, but not as good as the random forcing function.

Sensor Placement

In general only a subset of the desired number of sensors will be available for use in system identification and health monitoring. Thus the question of sensor placement becomes one of paramount importance in designing an on-orbit modal test program. The system has to be designed with built in redundancy to compensate for the likelihood of sensor malfunction as well as poor initial location. Ground tests have the advantage of allowing for rapid replacement of sensors whereas if a sensor goes off-line in space, the identification system will be required to function correctly without the sensor for some (extended) period of time before it can be replaced or repaired. In addition, optimal sensor placement is still an art form even on the ground. It is likely that even after considerable effort has been made to locate the best positions for all the sensors, initial verification once on orbit will show that some of the sensors are located near zero displacement locations (nodes) of important modes. These sensors will thus be useless until some later time when they can be relocated. The design of the system has to take all of this into account in order to perform as required.

Kammer presents the Effective Independence (Eft) method for sensor placement which is based upon ranking the contribution of each candidate sensor location to the linear independence of the corresponding target modes. In an iterative fashion, locations which do not contribute significantly to the independent information contained within the target modes are removed. Within a relatively small number of iterations, the initial candidate set of sensor locations can be reduced to the allotted number in a suboptimal manner.

Lim extends the Eft method of Kammer to provide a systematic procedure to define candidate target modes and to optimize the sensor placement to recover these modes. The important modes are defined using the strain and kinetic energy of each mode from a finite element modal analysis along with
The process of obtaining modal data from an orbiting spacecraft is portrayed in Figure 4. The on-orbit modal test will use the reaction control system (RCS) thrusters to excite a set of modes. The response of the structure to this excitation will be recorded in the form of acceleration time histories at \( s \) locations on the structure. The raw data from the accelerometers will be filtered and conditioned and then converted to digital form. In general this data will then be transmitted via data relay satellites such as the TDRSS family, to ground stations where it can be processed.

Cooper and Johnson\(^\text{13}\) describe the ongoing development of the Space Station Freedom on-orbit Modal Identification Experiment. The MIE is being designed to obtain a sufficient quality and quantity of on-orbit, time domain test data composed of measurable accelerations taken over sufficient time and adequately distributed spatially to realize important modes for a sequence of intermediate and final build configurations of space station Freedom. Design of the experiment is being performed jointly at NASA Langley Research Center, McDonnell Douglas Space Systems Company and The Structural Dynamics Research Corporation. To date, the feasibility of performing an on-orbit modal identification has been established and a detailed experimental design is underway. Simulations have been performed using an excitation based on a reboost of the station. Ground test validation is expected using the DSMT scale model.

Madden and Wilhelm\(^\text{14}\) describe the Space Station Data Management System (DMS) architecture. The backbone of the architecture is a token ring fiber optic network connecting processing, data storage, data acquisition and workstation nodes. The DMS will provide no serious obstacles or changes to normal data acquisition techniques, but limited data storage and processing capacity will require that data be transmitted via relay satellite to ground processing stations. There are several key differences in obtaining modal data from a laboratory experiment compared with acquiring the same data from an orbiting spacecraft.

Soucy and Deering\(^\text{15}\) discuss how the operating conditions for the data acquisition equipment affects the quality of the data in modal testing. Concerns raised include effects of different excitation levels, exciter locations, methods of fastening the transducer cables to the test structure, methods of exciter suspension, and excitation configurations.

**Mode Expansion and Orthogonalization/ Test-Analysis Model Reduction**

For a structure in orbit, a predetermined number, \( s \), of sensors will be permitted, whereas the full finite element model that will be verified has \( n \) degrees of freedom. Current thinking in the design of space station Freedom calls for \( 100 - 200 \) accelerometers while the structural model contains well over \( 1000 \) degrees of freedom.

With fewer measurements than physical degrees of freedom, the mode shape vectors from the finite element analysis cannot be compared directly with those obtained from the test. There are two possible ways to eliminate this problem. In the first approach, each of the test modes is expanded from the \( s \)
measured dofs to an approximation of the \( n \) physical dofs using the FEM mass and stiffness matrices. The disadvantage of using this method is that any errors in the FEM model are included in the expanded test modes and thus corrupt the test data, perhaps leading to errors in the system identification process. To date this has been the method used in most damage location research.

In the second approach the FEM mass and stiffness matrices are reduced to the test degrees of freedom using a transformation. The reduced-order model is known as a test-analysis model (TAM), and results in an \( s \times s \) mass matrix and an \( s \times m \) matrix of mode shape vectors that can be directly compared with the test mode shapes. There are several advantages to this approach. First, the test and analysis data remain separate throughout the process, thus eliminating the contamination problem. Secondly, the TAM development process can be used to identify dynamically important degrees of freedom which should be considered as possible locations for sensors as described above. Finally, the TAM and test data are at most \( s \) dimension matrices which can be considerably smaller than the full \( n \) dimensional analytical matrices. This is likely to result in considerable savings in computational effort.

**Mode Expansion and Orthogonalization**

As mentioned previously, most of the work by Smith and others in damage location has used the first of these approaches and expanded orthogonalized the test modes. Their results have been well documented and will not be repeated here in any detail. Beattie and Smith\textsuperscript{16} summarize these methods as applied to structural identification problems and recast many of them in a more unified form and suggest modifications that improve many of them. The most commonly referenced methods are the expansion method introduced by Berman and Nagy\textsuperscript{17}, and the orthogonalization technique introduced by Baruch and Bar Itzhack\textsuperscript{18}. Berman and Nagy use a "dynamic" expansion technique for the reordered, partitioned eigenvalue problem:

\[
\begin{bmatrix}
K_1 & K_2 \\
K_2^T & K_4
\end{bmatrix}
\begin{bmatrix}
M_1 & M_2 \\
M_2^T & M_4
\end{bmatrix}
\begin{bmatrix}
\phi_1 \\
\phi_2
\end{bmatrix}
= 0
\]  
(3)

where \( \{\phi_1\} \) represents the \( s \) measured modal displacements and \( \{\phi_2\} \) represents the unknown mode shape information at the remaining \( n-s \) dofs. Rearranging of the second equation permits a solution for the unmeasured dofs:

\[
\{\phi_2\}_i = \left(K_4 - \omega_i^2 M_4\right)^{-1} \left(\omega_i^2 M_2^T - K_2^T\right)\{\phi_1\}_i
\]  
(4)

as long as the first term on the right hand side of (4) is nonsingular for each of the \( i = 1, 2, \ldots, m \). For each of these modes the measured data is individually expanded to produce a set of mode vectors \( \{\psi\}_{1}^{n \times m} \) that will not, in general, be orthogonal to the system mass matrix. Baruch and Bar Itzhack's optimal orthogonalization technique can then be used to adjust the expanded modes in order to satisfy the orthogonality relationship. This technique forces the expanded modes to satisfy the orthogonality relationship in an optimal way. These corrected modes, now orthogonal to the mass matrix, are closest to the expanded modes in a weighted Euclidean sense.

The system equivalent reduction and expansion (SEREP) method was presented by O'Callahan, Avitable and Riemer\textsuperscript{19}. Independently, Kammer\textsuperscript{20} presented a modal reduction technique for test-analysis-model (TAM) development which leads to the same expansion process in which the mode shape vector, in the physical coordinates, is expressed as a linear combination of the \( m \) analytical model mode shapes predicted by a finite element analysis:

\[
\{\phi_i\} = \left[\Phi_{\alpha}\right]_i \{q\}_i
\]  
(5)

Note that (5) has been partitioned into the measured and unmeasured dofs. Using a generalized inverse, the first of eqs. (5) is solved for the vector of modal coordinates in terms of the measured dofs.

\[
\{q\}_i = \left[\Phi_{\alpha}\right]^{T}\left[\Phi_{\alpha}\right]^{-1}\{\phi_i\}_i = \left[\Phi_{\alpha}^{e}\right]^{-1}\{\phi_i\}_i
\]  
(6)

Equation (6) is then substituted back into the unpartitioned form of (5) and solved for the expanded modal vector:

\[
\{\phi_i\}_i = \left[\Phi_{\alpha}^{e}\right]\left[\Phi_{\alpha}^{e}\right]^{-1}\{\phi_i\}_i \quad i = 1, \ldots, m
\]  
(7)

Once again, the mode shape vectors \( \{\phi_i\}_i \) are expanded one at a time and the set is not necessarily orthogonal with respect to the mass matrix. Baruch and Bar Itzhack's optimal orthogonalization technique (or any other orthogonalization approach) can be used to orthogonalize the expanded mode shape vectors.

Recently, Beattie and Smith\textsuperscript{21} have developed a simultaneous expansion/orthogonalization (SEO) technique based on the Orthogonal Procrustes problem from computational linear algebra. A subspace defined by the set of measured dofs \( \{\phi_i\}_i \) is compared to a subspace defined by mode shapes from the analytical model \( \{\Phi_{\alpha}\}_i \). The matrix that rotates \( \{\Phi_{\alpha}\}_i \) to be as close as possible to \( \{\phi_i\}_i \) is found to be the modal coordinate matrix \( [Q] \). When \( [Q] \) is applied to the full analytical model mode shapes \( \{\Phi_{\alpha}\}_i \), an estimate of the full mode shapes corresponding to the measured data is produced. The resulting expanded mode shape vectors are orthogonal with respect to the mass matrix. This method is computationally more efficient than the dynamic expansion and SEREP. Evaluation of the above three methods shows that performance of the SEO technique is comparable or superior to that of the other techniques in the test examples that were used. Once again,
of damage detection studies, the important point of this section is that at the end of the process, the algorithm has a full \( n \times m \) set (expansion) or a reduced \( s \times m \) set of modal displacement vectors with which to perform the system identification and model verification.

**System Identification and Model Verification**

For the purposes of structural dynamics, "system identification" is the process of using a limited number of measurements to identify the modes and frequencies of the structure and to update the analytical model of the system to duplicate the measured response. This analytical model can then be used to predict the structures response to future inputs. There are numerous approaches to system identification each of which differs mainly in the mathematical processing of the incomplete set of measurements so as to accurately infer the structural parameters of interest. Figure 6 shows the three states during the system identification process. The matrices with subscript \( a \) represent the \( n \times n \) analytical matrices which are then updated using the expanded and orthogonalized modes \( \{\Phi_a\} \). The resulting \( n \times n \) "updated" matrices are used in any subsequent analyses as the baseline, undamaged model of the structure.

Smith and Hendricks\(^3\) carried out detailed evaluations of several candidate identification methods with respect to their suitability for damage detection applications. In a more recent paper, Smith and Beattie\(^{16}\) provide a detailed mathematical background of the matrix update problem, specifically related to structural identification. They review some of the most popular methods of matrix adjustment, place them in a modern framework and link some of them to well known problems in linear algebra and optimization. The most popular and applicable system identification methods are described below.

Baruch\(^{25}\) introduced a stiffness update method which was later used by Berman and Nagy\(^{26}\) in developing the Analytical Model Improvement (AMI) method. The first step in the method corresponds to expanding and orthogonalizing the measured mode shape vectors as described by equations 3-6 above. Given the expanded modal matrix and the analytical mass matrix, (which is assumed to be accurate), an improved stiffness matrix is sought which minimizes the difference between the updated matrix and the analytical matrix, while satisfying the eigenvalue equation, the orthogonality relation, and symmetry. The minimization is carried out using a Lagrange multiplier formulation. The method does not require iteration or eigenanalysis and is thus suitable for large models. The major drawback to this method is that it does not take into account the connectivity of the structure and could thus result in an updated stiffness matrix that has unrealistic load paths. Despite this drawback the method is still quoted often in the literature.

**Model Reduction**

There are several methods for reducing the mass and stiffness matrices of an \( n \times n \) finite element model to the \( s \times s \) matrices of a TAM. Freed\(^{22}\) provides a detailed comparison of the four most commonly used reduction methods. He compares the static (Guyan) reduction, Improved Reduced System (IRS) method, Modal Reduction, and Hybrid reduction methods for accuracy and robustness. He concludes that the choice of method is very dependant on the configuration and the modelling uncertainty and no one method is better than the others in all cases. In general, the process of reducing a stiffness (or mass) matrix from a FEM to a TAM is based on the following transformation:

\[
[K_i] = [T]^T[K][T]\]

(8)

The simplest and most common TAM procedure uses the standard Guyan reduction method, where the transformation matrix only involves the re-ordered stiffness matrix. The IRS method developed by O'Callahan\(^{24}\) improves upon the Guyan reduction by including mass effects in the development of the transformation matrix. In the Modal reduction method, Kammer uses the FEM mode shapes as interpolation functions in the development of the transformation matrix and does not use the original mass or stiffness matrices. Kammer has extended the capabilities of the modal TAM method by developing the Hybrid TAM, which combines the accuracy of the modal TAM with the robustness of the Guyan reduction. The transformation matrix for the hybrid TAM is a combination of the static and modal transformation matrices.

\[
[T_{\text{HED}}] = [T_{\text{Static}}] + [T_{\text{BED}} - T_{\text{Static}}]P\]

(9)

where \( P \) is an oblique projector matrix formed from the FEM mode shapes.

Mode shape expansion and orthogonalization and model reduction are essentially mathematical problems that will most likely see improvement in the next few years. For the purpose however, this comparison was performed on simplistic models and may not carry over to larger, more complex models.
Kabe\textsuperscript{27} presented a technique which preserves the zero-non-zero pattern of the original stiffness matrix in the updated result, precluding unrealistic load paths in the updated model. The stiffness matrix adjustment (KMA) method uses the analytical stiffness matrix $[K_{a}]$, the assumed known mass matrix $[M_{a}]$, the $n \times m$ matrix of expanded mode shapes $[\Phi_{e}]$ and the connectivity of the original stiffness matrix (assumed to be correct), to produce the adjusted stiffness matrix. As in the AMI method, a Lagrange multiplier formulation is used, but the error matrix that is minimized is defined such that it is independent of the system mass and the stiffness coefficient magnitudes

$$E = \sum_{i=1}^{n} \sum_{j=1}^{m} \left( \frac{K_{ij} - K_{ij}^{*}}{K_{ij}} \right)^{2}$$

(10)

In addition, the adjusted stiffness matrix is related to the original stiffness matrix by the following relation

$$[\tilde{K}] = [K_{a}] \ast [\gamma] \rightarrow \tilde{K}_{ij} = K_{ij} \gamma_{ij}$$

(11)

where the operator $\ast$ defines an element-by-element multiplication that ensures the zero-non-zero pattern is maintained. Lagrange multipliers are used to expand the error function to include the eigenvalue equation and symmetry constraints. Since the expanded and orthogonalized modes are being used, solving for the Lagrange multipliers will result in an adjusted model that exactly reproduces the measured modes. Once these Lagrange multipliers have been found, the adjusted stiffness matrix obtained by the KMA method is given by

$$[\tilde{K}] = [K_{a}] - \frac{1}{\delta} ([K_{a}] \ast [K_{a}]) \ast \left( [\lambda][\Phi_{e}^{T}] + [\Phi_{e}][\lambda]^{T} \right)$$

(12)

where $[\lambda]$ is the $n \times m$ matrix of Lagrange multipliers. This method works very well for small systems and produces an updated stiffness matrix that has the same connectivity as the analytical model, and exactly reproduces the measured modes and frequencies. Unfortunately it is computationally inefficient for large problems.

The Projector Matrix method (PMM) presented by Kammer\textsuperscript{28} is a reformulation of Kabe’s method. As such it preserves the connectivity of the original model in the optimally adjusted stiffness matrix, but requires less matrix manipulation prior to solution and less overall computational effort. The method uses projector matrix theory and the Moore-Penrose generalized inverse to correct the analytical stiffness matrix. Although this formulation is better than the original, the PMM is still inefficient for very large problems.

Chen and Garba\textsuperscript{29} proposed a method to calculate the sensitivity of the eigenvalues and eigenvectors to the structural model parameters such as mass and stiffness matrix elements. These sensitivities are in the form of the Jacobian matrix. The advantages of the method are the applicability to large complex structures without the requirement of apriori knowledge of the analytical expressions for the mass and stiffness matrices, and a cost effective approach for the recomputation of the eigendata. This method also allows the use of other measurements such as modal forces, kinetic energy distribution, and strain energy distributions in the estimation procedure.

Very recently Smith and Beattie\textsuperscript{30} have proposed a variety of secant-method adjustment techniques which appear to be superior in many ways to previous methods. Much of the work of these authors has been formulated specifically with the damage location problem in mind and thus appear to be the most likely to succeed. In addition to proposing new methods, references 16 and 30 reformulate the AMI and KMA methods to show that they are in fact similar to the secant-methods that they propose. These methods have only recently been published and have not been tested against any standard problems, but appear to be very promising as damage location tools.

### Model Comparison and Damage Location

Once the spacecraft is in orbit and the structural parameters have been identified, periodic monitoring of the load carrying properties of the truss will be carried out. The ideal time for this to occur would be during a reboost firing of the RCS thrusters which would be used as the excitation source for the modal identification process. The data acquired from such a test would then be expanded and orthogonalized as described above and the resulting mode shape vectors and frequencies will be taken to be the current or "damaged" model. This damaged model, along with the updated finite element model obtained before, will be used in the damage location process. This process searches for a stiffness matrix that maintains the zero-non-zero pattern of the updated stiffness matrix, and thus does not introduce unrealistic load paths, while reproducing the modes observed during the test. This is almost a repetition of the system identification process except that instead of

![Figure 7: Damage Location](image-url)
updating the analytical model with the new information, the process seeks out the elements of the stiffness matrix that change the most in order to produce the observed results. These elements are the most likely contributors to any damage. Once these stiffness matrix elements have been identified a physical map of the geometry can be used to determine which elements of the structure are likely contributors to the damage. This technique is predicated on the assumption that the undamaged mass matrix was unchanged from the original observation.

Smith uses Kabe's stiffness matrix adjustment method to indicate which elements of the stiffness matrix have changed by the largest percentage. Two lists of stiffness matrix elements that receive the most adjustment in the identification process are produced by searching the percent-adjusted stiffness values using one filtering threshold for the diagonal terms and a second threshold for the off diagonal terms. Two thresholds are used because the diagonal and off-diagonal terms are affected differently by the damage. Using matrix graph theory the two lists combine to form a "damage subgraph" of the connectivity matrix, thus identifying the truss element(s) with reduced stiffness. Simulations with 'exact' (i.e. no noise) data show that the algorithm locates many, but not all possible damaged members. For the majority of the damaged members in the test cases, damage was detected easily. However, insensitivity in the structure response prevents the location of damage to certain truss members. The work described here will be extended using the DSMT hybrid scale model.

Chen and Garba postulate that the location of damage can be found by identifying those degrees of freedom whose kinetic energies are different from those of the undamaged system. Rearranging equation (2) and premultiplying by a diagonal modal matrix \([\Phi]\) results in the kinetic energy on the right-hand side and the strain energy on the left

\[
\Phi_i^T[K_i,j]\Phi_j = \omega^2 [\Phi_i] [M_i] [\Phi_j] \quad (13)
\]

Damage is located by determining the elements connected to the dof with the largest changes in both the kinetic and strain energies. Chen and Garba go even further and propose a method to quantify the extent of the damage. Using what they call the "connectivity matrix of each measured mode", they formulate a constrained optimization problem which seeks the changes in the stiffness matrix elements by minimizing the Euclidean norm of these changes subject to the connectivity and eigenvalue constraints:

\[
E = \frac{1}{2}\|\Delta k_{ij}\| + \{\lambda\}^T \{Y\} - [C]\{\Delta k_{ij}\} \quad (14)
\]

where \([\Phi]\) is the matrix of Lagrange multipliers and

\[
[Y] = \left( [\Omega^2] - [K] \right) [\Phi_i] \quad (15)
\]

The optimal solution consists of a vector \([\Delta k_{ij}]\) containing the changes in each of the independent stiffness matrix elements

\[
\{\Delta k_{ij}\} = [C]^T([C][C^T])^{-1}\{Y\} \quad (16)
\]

This list can be searched using a variety of threshold techniques to determine the subset \([k_{ij}^*]\) of the stiffness elements that change the most. Using the connectivity information the location of these elements in the structure is easy to determine. The difficulty lies in determining which structural elements contribute to the change in stiffness. The \([\Delta k_{ij}]\) are stiffness changes at nodes. In a typical truss structure each node is shared by several elements and it is difficult to determine which of the elements contributes the most to the damage. The method is demonstrated with some success using a simple spring mass model and a more complicated truss. The computation requirements for the method are quite high but the method shows considerable promise if the final element location can be resolved. Application of the method to a more complicated structure such as the DSMT model would be a suitable test of its performance.

Hajela and Soeiro present a slightly different measure of damage and use an unconstrained minimization approach to locate the damaged members. In addition, they use both the static and modal response of the structure in their identification and location scheme. Damage is represented by a reduction in the elastic properties of the materials. The stiffness of each member is expressed in terms of each of the elastic and geometric properties

\[
[K] = [K(A,I,J,L,t,E,G)] \quad (17)
\]

The net changes in these properties are lumped into a single design variable \(d_i\) in the unconstrained minimization problem in which the difference between the analytical and measured response is minimized. The response in question can be the dynamic or the static response or a combination of the two. Three different implementations of this approach are discussed, each offering different approximation or modelling strategies to combat the high computational demands of the method for large models. In the first approach a select group of dominant variables are allowed to vary with the others fixed. The Broyden-Fletcher-Goldfarb-Shanno variable metric method is used for function minimization. The set of variables is refined after a prescribed number of cycles and the dominant variables are modified based on the results. The second approach used equivalent reduced order models of the structure to first locate the approximate region of damage. The full model is then solved considering only the parameters \(d_i\) corresponding to the members of the damaged region(s). The third approach uses substructuring techniques in conjunction with reduced order models. The regions containing the damage are solved separately with appropriate boundary conditions using an equation error approach to the system identification. The method was applied to a series of "representative" truss structures and results were favorable with more research pending. Once again, the performance of the method as
knowledge representation are shown to be advantageous for identifying damage in flexible structures. A finite model space is classified by levels of abstraction into a search tree, over which a variant of best-first search is used to identify the model whose stored response most closely matches that of the input. Following this output-error approach in model space, an equation-error approach is used for numerical parameter identification.

Concluding Remarks

A review of the literature in the fields of structural identification, mode shape expansion and orthogonalization, and damage location has been presented. An aggressive research program in health monitoring and damage location in large space structures is being developed at the NASA Langley Research Center. Use of the DSMT hybrid scale model for damage location research and as a universal test-bed for other damage location methods is proposed as a means to meaningfully compare and evaluate existing and newly developed methods. Issues concerning on-orbit data acquisition, data accuracy and quality have been identified as requiring extensive research in the context of structural dynamic system identification. Other key issues have also been discussed briefly.

References


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On-Orbit Damage Detection and Health Monitoring of Large Space Trusses - Status and Critical Issues

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The long lifetimes, delicate nature and stringent pointing requirements of large space structures such as Space Station Freedom and geostationary Earth sciences platforms might require that these spacecraft be monitored periodically for possible damage to the loadcarrying structures. A review of the literature in damage detection and health monitoring of such structures is presented, along with a candidate structure to be used as the testbed for future work in this filed. A unified notation and terminology is also proposed to facilitate comparisons between candidate methods.