ANALYSIS, PRELIMINARY DESIGN
AND SOFTWARE SYSTEMS FOR
CONTROL-STRUCTURE INTERACTION
PROBLEMS

by

K. C. Park and K. Alvin

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Analysis, Preliminary Design and Simulation Systems for Control-Structure Interaction Problems

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Final Report on Grant NAG1-1021, funded by NASA Langley Research Center
SUMMARY

This is a final report on the tasks supported by NASA Langley Research Center under Grant NAG1-1021, Analysis, Preliminary Design and Simulation Systems for Control-Structure Interaction Problems. When the proposal was submitted, it was intended to be a three-year program, with its first-year effort to be concentrated on software aspects of control-structure interaction (CSI) analysis. Due to the termination of the grant at the end of its first-year period, appropriate adjustments had to be made in order to make most out of the grant. The accomplishments from the one-year effort include: 1) the delivery of a research-level CSI analysis software that runs both on SUN Workstations as well as Alliant shared-memory parallel machines to Dr. W. K. Belvin of NASA/Langley Research Center; 2) three presentations at conferences, one in a bound book publication, the second in the 1990 AIAA Guidance and Control Conference Proceedings, and the third to appear in a NASA/DOD/JPL proceeding on system identifications. Two of these papers are being prepared for submittal to journal publications.
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Summary

This report summarizes research work on the implementation of a partitioned numerical algorithm for determining the dynamic response of coupled structure/controller/estimator finite-dimensional systems. The partitioned approach leads to a set of coupled first and second-order linear differential equations which are numerically integrated with extrapolation and implicit step methods. The present software implementation, ACSIS, utilizes parallel processing techniques at various levels to optimize performance on a shared-memory concurrent/vector processing system. The current work also generalizes the form of state estimation, whereby the Kalman filtering method is recast in a second-order differential equation equivalent to and possessing the same computational advantages of the structural equations. As part of the present implementation effort, a general procedure for the design of controller and filter gains is also implemented, which utilizes the vibration characteristics of the structure to be solved. Example problems are presented which demonstrate the versatility of the code and computational efficiency of the parallel methods is examined through runtime results for these problems. A user's guide to the ACSIS program, including descriptions of input formats for the structural finite element model data and control system definition, can be found in Appendix A. The procedures and algorithm scripts related to gain design using PRO-Matlab are included in Appendix B. In Appendix C, a stability analysis for the partitioned algorithm is presented which extends previous analysis to include observer dynamics, leading to a clearly definable stability limit. The source code for the parallel implementation of ACSIS is listed in Appendix D.

1.0 Introduction

The present work on the implementation of a partitioned transient analysis algorithm for the simulation of linear Control-Structure Interaction (CSI) problems has concentrated on four major areas. The initial software implementation emphasized the user-friendly aspect and a structural dynamics-oriented interface for experienced practitioners of finite element analysis programs. Another reason for a new implementation of the algorithm was that the initial architecture of the CS3 software testbed developed by Belvin and Park [1-2] had little provision for effective parallelization. CS3 also included extensive links to optimization and optimal control algorithms which were not central to the current work and proved to be a further hinderance. This new software implementation was designated ACSIS, for Accelerated Control-Structure Interaction Simulation, and led to significant improvements in speed for particular problems on conventional serial processors due to simpler and more economical storage of primary variables. ACSIS is also versatile in its usage of a general Timoshenko beam element with pin release capability and shear correction factor adjustment, and control system definition via a single data file. A preliminary Users Guide was developed for ACSIS, and the input formats were made compatible with pre/postprocessing software developed for Sun and Silicon Graphics computers so that
future versions using parallel techniques via element mesh domain decompositions can rely on the same X-Window-based I/O utilities. Table 1 documents runtime comparisons of ACSIS and CS3 on a sample 48 DOF problem simulated on a Sun 3/260 workstation using a floating point accelerator.

2.0 State Estimation via Second-Order Kalman Filtering

One restriction of the CS3 software testbed was the form of dynamic observer equations used in the partitioned algorithm as developed in [2]. However, Belvin and Park [3] showed that a general Kalman filtering type of state estimation was not only possible in the partitioned solution, but could be implemented at a very small additional cost in computations by a slight modification in the way the dynamical equations of the plant are cast into first-order form for filter gain design. The methods employed in [3] have been successfully implemented into ACSIS, thereby enhancing the code's ability to handle a wide variety of state estimation schemes. This is important as the application of optimal control techniques to the state estimator problem leads to this more general form, and as the restrictions of the observer used in CS3 typically meant either discarding part of the observer gain parameters, resulting in a loss in system performance.

ACSIS retains the option of using the more restricted observer form, as well as simple full state feedback (no dynamic compensation). It is clear from examining the respective equations, that for structures with stiffness-proportional damping, the only additional calculations required for the Kalman filter is the multiplication of the $L_1$ gain matrix by the predicted state estimation error vector $\gamma$ (see equations 25 of [3]). This is not a significant portion of the computations required at each integration step, as the dimension of $\gamma$ is small (number of sensors). By far the major costs are for computing an internal force of the form $Kq$, and backsolving the factored integration matrix for the estimated displacement states. This is verified numerically in the ACSIS results of Table 1 (using a restricted form of observer) and version (A1) of Table 2. The model used in Table 1 is roughly comparable to the 54 dof truss in Table 2, and both show the additional simulation time due to state estimation is roughly the same as an additional transient analysis. Finally, the Kalman filtering equations do not lead to any additional complications in parallel implementation of the overall algorithm as compared to the more restricted second-order observer developed for CS3.
3.0 Parallel Implementation of ACSIS

A primary emphasis in our work dealt with the optimization of the software implementation on a concurrent processing system. The platform chosen (primarily due to availability, initially at CU and later at NASA LaRC) was the Alliant FX/8 shared memory multiprocessor system with 8 parallel processing units and vectorization capabilities. Versions of the software have been ported to the Alliant and compiled using the Concentrix FX/Fortran optimizing compiler, which has available options for automatic vectorization and concurrency of standard, problem-independent parallel computations.

The partitioned CSI algorithm has three primary levels of parallelism in its numerics which can be exploited. At the highest level is the integration of the second-order dynamical plant (structure) and filter (estimator) equations, which as designed are of roughly equivalent size. Through the algorithm, these systems are effectively decoupled and independent at each discrete time step, and thus may be handled in parallel by invoking a compiler directive in the main program, which calls the respective subroutines simultaneously and handles re-synchronization of the execution upon their return.

At a lower level of the algorithm, the structure and state estimator equations exhibit symmetric, second-order forms typical of linear structural dynamics problems. It is well known that computations related to the formation of the internal force vector, $Kq$, can be re-implemented at an element level [4], which, through decomposition of the element mesh [5], can be handled in parallel within exclusive subdomains. This technique becomes particularly attractive for larger problems on more massively parallel systems; in the present work, the element-by-element (EBE) technique was not as effective as other types of parallelism. It should be noted here that the partitioned algorithm employs implicit integration methods, leading to systems of algebraic equations which are factored and solved using direct, rather than iterative, numerical methods. Therefore, formulation of the internal force vector is needed only in the formation of the known (right hand side) vector for integration of the plant and filter equations. The alternative to EBE computations is multiplication of the relevant displacement vector with the global stiffness matrix stored in profile (skyline) form. To "simulate" the advantages of local memory typical in large-scale parallel processing systems, the computed element stiffnesses were saved in shared memory for the EBE calculations, thus avoiding the need to recompute this data at each integration step. In addition, a low-overhead automatic element domain decomposition was provided for the parallel EBE method.

The final and lowest level of parallelism is obviously that of the basic matrix computations such as addition, multiplication, etc. These numerical operations are inherent in nearly all areas of the software implementation, including problem preprocessing. With the very capable optimizing compiler available on the Alliant system, this parallelism was exploited through vectorization and concurrency of the nominal source code using the FX/Fortran compiler run-time options -0 -DAS -alt. The performance of the resultant executable code was examined using the Alliant's profiling capabilities, and changes to the nominal
source code, remaining compliant to F77, to maximize the identifiable concurrency and thus enhance the resultant speed. This was particularly useful in the profile matrix/vector multiply operation, whose speed is critical to the overall program performance.

4.0 Problem Descriptions

Three structural dynamics problems were developed for code testing at various levels of complexity. All three problems have the following common features: simulations consisted of 1000 integration steps and employed a stiffness-proportional damping. The damping was not needed for algorithm stability, but to ensure consistency between the examples and because the existence of damping in the plant equations has a strong influence on program speed. The Kalman filter models were of second-order form [2] and equivalent in size to their respective plant models. For controlled simulations, the control system began operating after 100 integration steps, and all gain matrices were full (i.e. all model states influence all actuators and are influenced by all sensors). Additional specific information for the problems follow.

A. Axial Vibration of Elastic Bar (Spring Model)

- # Nodes: 3 free, 1 fixed
- # Elements: 3
- # Degrees of freedom: 3
- # Actuators: 1
- # Sensors: 1
- Disturbance: Initial displacement

B. Planar Vibration of Space Truss (Truss Model)

- # Nodes: 18 free
- # Elements: 33
- # Degrees of freedom: 54
- # Actuators: 4
- # Sensors: 6
- Disturbance: Bang-bang type sinusoidal applied force

C. General 3D Vibration of EPS Satellite Reboost (EPS7 Model)

- # Nodes: 97 free
- # Elements: 256
- # Degrees of freedom: 582
- # Actuators: 18
- # Sensors: 18
- Disturbance: Bang-bang type square wave applied force
As can be seen, the problem sizes are roughly three different orders of magnitude, with corresponding increases in the sizes of the control systems. Appendix B includes routines using Matlab for the design of control and filter gains which were used for the control system design of all three example problems. An illustration of the EPS model is shown in Figure 1.

5.0 Performance Assessment

Table 2 compares CPU runtimes on the Alliant computer (using the UNIX "time" command) for distinct versions of the software. Version (A1) is the nominal F77 program code compiled without any performance-enhancing options, while version (A2) invokes automatic vectorization and concurrency of low-level, problem-independent computations such as vector addition and inner products. The performance improvements are significant, especially for the large EPS7 model, where the speed-up factor is 35-37.

Version (A3) also uses the compiler options from (A2), but in addition has a compiler directive added to the main program which allows the plant and filter integration subroutines to be called in parallel. This does not affect the transient response results as that analysis option bypasses the altered code, but for controlled response there is some effect on performance. If filtering is used, which results in a significant increase in computation, the directive can lead to some increased speed as can be seen for the spring and truss problems. There can also, however, be a reduction in performance as compared to (A2) if the finite amount of processors and vector units are used in a less efficient way. This appears to be the case for the large EPS problem, where the "overhead" introduced by the directive, and its effect on processor assignment, is greater than the improvement generated by the manually-invoked parallel construct.

Table 3 shows CPU run times for ACSIS using E-B-E computations, which, as mentioned previously did not lead to better performance on the example problems using the Alliant system. This appears to be due to the lack of effective vectorization of the individual element computations when forming the internal force via the EBE method. To determine whether the EBE method effectively lead slower speeds through increased numbers of computations, version (A2) (see above) was altered by removing compiler optimization of the profile matrix/vector multiply routine (the alternate method to EBE). The resultant runtimes matched almost exactly with those of version (A4), leading to the conclusion that both methods require roughly the same amount of computations, but differ in how they can be optimized on the Alliant system. The matrix/vector multiply operation, in this environment, can exploit both vectorization and concurrency through the compiler's performance options; this can be examined in the compiler output. The EBE computations, at the element level, do not vectorize because the parts of the global displacement and force vectors being operated on per element are not contiguous. In version (A5), the elements within each subdomain of the mesh are computed in parallel, leading to some performance
Overall, versions (A2) and (A3) provide the best code performance for the hardware available. Parallelizing the observer and structure (A3) leads to mixed results; improvement for the small spring and truss problems, but not for the large EPS model. Element-by-element computations do not improve code performance over compiler optimization via vectorization and concurrency for this platform. Reimplementation of the algorithm lead to a 5:1 improvement over the CS3 testbed software on a serial computer (Table 1). Further optimization of ACSIS on the Alliant FX/8 lead to an additional 30:1 improvement in runtimes for large-order systems such as the 582 dof EPS model. Time history responses of selected variables for the example problems are shown in Figures 2 through 10. For Figures 8 through 10, the $u_x, u_y, u_z$ displacements plotted are located at node 45, which is located at the vertex of the large antenna of the EPS model (see Figure 1).

6.0 Conclusions

The present work has demonstrated the efficiency of a streamlined simulation code for the analysis of large-order CSI systems and the viability of the continuous second-order Kalman filtering equations for state estimation. These methods show the versatility of the partitioned CSI integration algorithm and the promise of its application to real-time simulation. It is evident, however, that the use of element-by-element computational techniques requires the development of innovative algorithms for effective implementation on massively parallel processing systems. Future work in this area will include integrating algorithms for on-line system identification and applying these capabilities to the problem of real-time control.

Acknowledgements

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<table>
<thead>
<tr>
<th>Simulation</th>
<th>CS3</th>
<th>ACSIS</th>
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<tr>
<td>Transient</td>
<td>439.2</td>
<td>98.8</td>
</tr>
<tr>
<td>Full State Feedback</td>
<td>688.2</td>
<td>181.5</td>
</tr>
<tr>
<td>FSFB with Observer</td>
<td>1156.7</td>
<td>282.3</td>
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Table 1: Comparison of Runtime Speeds for CS3 and ACSIS on a Sun 3/260 System

<table>
<thead>
<tr>
<th>Model</th>
<th>Problem Type</th>
<th>(A1) Nominal Code</th>
<th>(A2) Compiler Optimized</th>
<th>(A3) Parallel Observer</th>
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</thead>
<tbody>
<tr>
<td>3 DOF</td>
<td>Transient</td>
<td>6.6</td>
<td>2.1</td>
<td>2.1</td>
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<tr>
<td>Spring</td>
<td>FSFB</td>
<td>8.0</td>
<td>3.3</td>
<td>3.3</td>
</tr>
<tr>
<td></td>
<td>K. Filter</td>
<td>12.3</td>
<td>3.5</td>
<td>3.3</td>
</tr>
<tr>
<td>54 DOF</td>
<td>Transient</td>
<td>78.2</td>
<td>5.7</td>
<td>5.6</td>
</tr>
<tr>
<td>Truss</td>
<td>FSFB</td>
<td>97.1</td>
<td>9.4</td>
<td>10.2</td>
</tr>
<tr>
<td></td>
<td>K. Filter</td>
<td>170.7</td>
<td>13.0</td>
<td>10.7</td>
</tr>
<tr>
<td>582 DOF</td>
<td>Transient</td>
<td>3506.</td>
<td>98.6</td>
<td>100.3</td>
</tr>
<tr>
<td>EPS7</td>
<td>FSFB</td>
<td>7040.</td>
<td>190.2</td>
<td>294.5</td>
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<tr>
<td></td>
<td>K. Filter</td>
<td>n/a</td>
<td>284.2</td>
<td>312.5</td>
</tr>
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</table>

Table 2: CPU Results for Versions of ACSIS
<table>
<thead>
<tr>
<th>Model</th>
<th>Problem Type</th>
<th>(A4) E-B-E Computation</th>
<th>(A5) Parallel E-B-E</th>
<th>(A6) Parallel Obs. &amp; EBE</th>
</tr>
</thead>
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<td>3 DOF</td>
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<td>3.3</td>
<td>3.3</td>
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<td>Spring</td>
<td>FSFB</td>
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<td>4.4</td>
<td>4.9</td>
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<tr>
<td></td>
<td>K. Filter</td>
<td>6.6</td>
<td>5.6</td>
<td>5.0</td>
</tr>
<tr>
<td>54 DOF</td>
<td>Transient</td>
<td>31.7</td>
<td>13.0</td>
<td>13.0</td>
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<tr>
<td>Truss</td>
<td>FSFB</td>
<td>35.5</td>
<td>16.9</td>
<td>35.6</td>
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<tr>
<td></td>
<td>K. Filter</td>
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<td>27.3</td>
<td>36.2</td>
</tr>
<tr>
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<td>Transient</td>
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<td>153.9</td>
<td>n/a</td>
</tr>
<tr>
<td>EPS7</td>
<td>FSFB</td>
<td>485.9</td>
<td>245.9</td>
<td>n/a</td>
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<tr>
<td></td>
<td>K. Filter</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
</tr>
</tbody>
</table>

Table 3: CPU Results for ACSIS with EBE Computations
Figure 1: EPS Finite Element Model
Spring Model: Open Loop Transient Response

Figure 2: Spring Transient Response

Node 3, \( u_x \)
Spring Model: Full State Feedback Response

Figure 3: Spring FSFB Response
Figure 4: Spring Response w/ Filter
Truss Model: Open Loop Transient Response

Figure 5: Truss Transient Response
Figure 6: Truss FSFB Response
Figure 7: Truss Response w/Filter
EPS Model: Open Loop Transient Response

Figure 8: EPS Transient Response
Figure 9: EPS FSFB Response
EPS7 Model: Controlled Response w/Kalman Filter

Figure 10: EPS Response w/Filter
REFERENCES


APPENDIX A

ACSIS User's Manual
Introduction

ACSIS is an analysis program for full-order simulation of control-structure interaction (CSI) problems. The CSI simulation is carried out using a partitioned analysis procedure which treats the structure (or plant), the observer, and the controller/observer interaction terms as separate entities. This procedure allows ACSIS to maintain relatively small, sparse matrix equations when compared to the process of assembling the computational elements into a single set of equations of motion and solving simultaneously. Although this software can also carry out modal analysis, the transient response and CSI simulation are done in real space using the entire finite element model. (For more information, see Ref. 2 of report, p. 14)

The ACSIS program is run using two previously prepared data input files and interactive input of run options. The two data input files are the finite element input file and the controller definition file.
Interactive Options

The run options are as follows, note that no defaults exist, data must be input each time it is requested for each item requested except during a background run. Files may be given any names, example names are given only to match the truss example at the back of this manual.

Please input analysis type:

This option is for selecting modal analysis, CSI simulation or the transient response of a structure. Not all interactive inputs are required for each analysis type.

Do you wish to save an input file? (y or n)

This option creates a file which saves all the interactive input options. ACSIS can be background run with minor option changes by editing this file and then directing the screen input to this file. This file is very different for each analysis type. (To do this run with the example names after running acsis.exe once interactively, the command would be: acsis.exe <INP_truss> &.

Name of save input file? (filename)

This question asks for the name under which to save the interactive input. example name: INP_truss

Finite Element Model Input File Name (filename):

This file should contain all the finite element nodes, mesh, materials, properties, lumped inertias, fixations, and initial velocity and displacement conditions. It must be prepared in advance in the card format specified later. example name: FEM_truss

Number of modes desired?

This only appears in the modal analysis to request the number of modes to be output. The actual analysis is carried out with double this number of modes for accuracy.

Controller Definition File Name:

This only appears for the CSI simulation. The file should contain the actuator and sensor locations, control gains, and observer gains. It also must be prepared in advance in card format. example name: CON_truss

Please input type of control::

This option is for selecting the form of control law equations used in the CSI simulation. Full state feedback uses the current states of the plant (structure) to determine the control via constant gain matrices. Second-order observer uses only the L2 filter gain matrix of a Kalman filtering type of state estimation where the state variables are position and velocity. The Kalman filter option allows the use of a full set of filter gains, but the gain design must come from a alternate variable casting using position and generalized momenta.
Initial time, final time, control-on time, step size:

Data format is four columns for CSI, but only three columns for transient response since the control-on time part is deleted.

Forcing function ID, scale factor, damping coeff- a,b:

In order for any time dependent forcing function to be easily implemented despite variable time step sizes, the forcing functions must be entered into the subroutine forces.f. The format is to assign each new forcing function an ID number in a elseif statement. An example of forces.f is included at the back of the manual. Then forces.f must be recompiled into an object file and acsis.exe relinked. This is easily done by typing make which detects changes to the fortran files and does only the necessary compiling. A particular forcing function is chosen by entering its ID number in the first column, in addition the force can be scaled by a constant using the scaling factor in the second column. The Rayleigh damping coefficients a and b are the third and fourth columns.

Phase lag fix? (y or n):

Specifies whether to include an extra iteration of control and sensor state prediction at each integration step to improve accuracy. Not generally needed unless the user is investigating the source of instability in a simulation and needs to test the sensitivity of the response to the partitioned algorithm's extrapolation method.

Gain scale factors (4 total):

These scaling factors are, in order: the F1 control gain matrix (displacement), F2 control gain matrix (velocity), L1 and L2 state estimator filter gain matrix. This question appears only for CSI.

ACSI S has two types of output options. The first is the displacement or velocity motion of up to twenty separate degrees of freedom. Interactive questions are, 'Number of displacement results to output (max 10)' followed by as many 'Input node #, dof for displacement output #' as necessary. Then these repeat for velocity results. The name of the file where the output will be stored is requested by 'Output file name? (filename).' example name: OUT.truss. The second output option saves the displacement of all nodes at any time step where output is sent (see next entry). The format is suitable for animation of the entire structure. Question asks 'Animation Output? (y or n)' then for an animation filename if necessary.

Send output every how many steps?

This option affects both output options to reduce the size of the output and animation files. It causes output to only be sent after an integer number of time step iterations. To get output each time step, simply enter 1.
Finite Element Input File

The finite element input file consists of title cards followed by columns of data. The title cards can be in any order, but they must be all capitalized with the appropriate number of columns of data for each card. The program reads rows of data until encountering a new card. Data which represents an integer value may be entered with a decimal point while real data may be entered without a decimal point as necessary. Any line beginning with a * anywhere in the file is ignored and can be used to insert comments. Any blank line will result in a read error.

NODES

Each node must be defined on a separate line. Columns can be separated by any number of spaces or a comma. Data format is four columns: node number, x-coordinate, y-coordinate, z-coordinate.

TOPOLOGY

Each element must be defined on a separate line. Truss elements require two nodes then two columns of zeroes. (Truss elements would also require pin releases in ATTRIBUTES below.) Beam elements require two nodes then a third reference node representing a point in the xz plane of the beam and then a column of zeroes. Element type refers to finite element formulation. (Currently only type 1 = timeshenko beam element is now available.) Data format is six columns: element number, element type, node #1, node #2, node #3, node #4.

ATTRIBUTES

Each element is characterized by an ID number from each of the MATERIAL and PROPERTIES cards below. Each element also has six pin release codes. The first code is for longitudinal stiffness, the second code is for torsional stiffness, the third and fifth codes are bending stiffness at each end in the y direction and must be the same value, and the fourth and sixth codes are for bending stiffness in the z direction and also must be the same. (0 is stiff, 1 is released.) Data format is nine columns: element number, material type, property type, and six pin release codes.

MATERIAL

The material data is formatted in four columns: material type, Young's modulus, shear modulus and density of the material.

PROPERTIES

The properties data is formatted in six columns: property type, cross sectional area of the element, $I_y$, $I_z$, $I_y$, $I_z$, shear shape factor SSF2, shear shape factor SSF3.

FIXITY

Nodes with any fixations are defined here in the finite element file. The nodes must be entered with the fixity of all six of their DOF's, restrained or not. (1 is restrained, 0 is free) Data format is seven columns: node number, x, y, z, $\phi_x$, $\phi_y$, $\phi_z$. 

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INERTIA

All lumped inertias must be entered with each separate DOF on an individual line. Therefore a single node could take up to six lines to define. Data format is three columns: node number, DOF number (1-6), and value of inertia.

INITIAL CONDITIONS

All initial displacement and velocity conditions are entered into the finite element file. Data format is four columns: node number, DOF number (1-6), initial displacement, and initial velocity.

END
End of file.

Controller Definition Cards

The controller definition file also consists of title cards followed by data entry. Each card must be followed by the appropriate number of columns of data and in some cases the appropriate number of rows. Integers can be entered in real format and vice versa if necessary. Any blank line will result in a read error.

NACT
Number of actuators in the entire control system.

BMAT
This entry creates the actuator position matrix or B matrix. There should be one row for each actuator, data format is four columns: node number, DOF number (1-6), actuator number, and sensitivity.

NSEN
Number of sensors in the entire control system.

HDMA
This entry creates the matrix of displacement sensor locations. One row per displacement sensor, data format is four columns: node number, DOF number (1-6), sensor number, and sensitivity.

HVMA
This entry creates the matrix of velocity sensor locations. One row per velocity sensor, data format is four columns: node number, DOF number (1-6), sensor number, and sensitivity.

F1GA
This is a list of the F1 or displacement control gains. The data format is four columns: node number, DOF number (1-6), actuator number, and value of gain.
F2GA

This is a list of the F2 or velocity control gains. The data format is four columns: node number, DOF number (1-6), actuator number, and value of gain.

L1GA

This is a list of the state estimator L1 filter gains. The data format is four columns: node number, DOF number (1-6), actuator number, and value of gain.

L2GA

This is a list of the state estimator L2 filter gains. The data format is four columns: node number, DOF number (1-6), actuator number, and value of gain.

END

End of file.

Examples

This section includes all the files and procedures needed to run all three analysis on a simple elastic bar problem. The naming of the files is a simple and easy to remember system, however no particular format is necessary.

The finite element file was created simply by typing the node locations, connectivity (topology), etc. with a text editor.

File: FEM_spring

* MESH *

NODES
1  0.00  0.00  0.00
2  1.00  0.00  0.00
3  2.00  0.00  0.00
4  3.00  0.00  0.00

TOPOLOGY
1  1  1  2  0  0
2  1  2  3  0  0
3  1  3  4  0  0

ATTRIBUTES
1  1  1  0  1  1  1  1  1
2  1  1  0  1  1  1  1
3  1  1  0  1  1  1  1

MATERIAL
1  1000.  0.0  0.0

PROPERTIES
The modal analysis only requires the number of modes desired in addition to the finite element file. The file INP_springO documents the interactive inputs used in the modal analysis. The results are saved in file EIG_spring.

File: INP_springO

-1
n
   * ACSIS input file, two lines above are
   * analysis type and save input file. Do
   * not change them by editing this file.
   * Finite element input file? (filename)

FEM_spring
   * Number of modes desired?
   3
   * Output file? (filename)

EIG_spring

File: EIG_spring

SUBSPACE ITERATION ROUTINE

NB OF EIGENVALUES= 3
NB OF VECTOR= 3
NB OF DOF= 3
TOLERANCE= 1.000E-04

NB OF RIGID MODES= 0

ITERATION NO 1
To run the transient response of the structure, a forcing function or initial condition would be needed to excite the structure. An initial condition would be added to the finite element file. A forcing function must be added to forces.f with a new ID number, then this number given as interactive input. In this case, an initial displacement acts on the second degree of freedom, which is defined in the finite element model input file. The file INP_spring1 documents the interactive inputs used in the transient analysis.

File: INP_spring1

-3

* ACSIS input file, two lines above are
* analysis type and save input file. Do
* not change them by editing this file.
* Finite element input file?(filename)

FEM_spring
  * Initial, final, step size?
  0.000000000 1.000000000 0.001000000
  * Forcing function, scale f, damping a, b?
  0 0.0000000 0.000000000 0.00002000

OUT_spring
  * Number of displacement outputs?
  1
  3 1
  * Number of velocity outputs?
In addition to the finite element file, the interactive input, and an excitation, CSI simulation requires a controller definition file. A full state feedback controller for the truss structure is defined in CON_spring.

File: CON_spring

NACT
1
BMAT
3 1 1 1.0
F1GAIN
2 1 1 193.31415000000
3 1 1 1574.63150000000
4 1 1 -1669.04600000000
F2GAIN
2 1 1 20.7742560000000
3 1 1 27.7904030000000
4 1 1 10.7802120000000
NSEN
1
HVMAT
3 1 1 1.0
L1GAIN
2 1 1 8.79289090000000D-02
3 1 1 -4.88079010000000D-02
4 1 1 3.01027350000000D-03
L2GAIN
2 1 1 1.038093120000000D-01
3 1 1 6.1410130000000
4 1 1 -3.0608725000000
END

Then if acsis.exe is run interactively and the input is saved, the file INP_spring2 can be produced. The file could be edited to change the input files, the output file, the forcing function ID, the length of simulation, control-on time, etc. Then to run it again, type acsis.exe<INP_spring2> scr &. The scr is a scratch file which will store the screen output.

File: INP_spring2

-2
* ACSIS input file, two lines above are
  * analysis type and save input file. Do
  * not change them by editing this file.
  * Finite element input file?(filename)

FEM_spring
  * Controller file name?(filename)

CON_spring
  * Please input type of control:
    3
    * Initial, final, control-on, step size?
      0.00000000 1.00000000 0.10000000 0.00100000
    * Forcing function, scale f, damping a, b?
      0 0.000000 0.00000000 0.00002000
    * Phase lag fix?(y or n)

OUT_spring
  * Number of displacement outputs?
    1
    3 1
  * Number of velocity outputs?
    0
  * Send output every how many steps?
    1
  * Send animation output?(y or n)
APPENDIX B

Matlab Procedure and Scripts
for Controller and Kalman Filter
Gain Designs
Introduction

In order to retain simplicity in the ACSIS code for parallel implementation purposes, no control system design algorithms were included. Instead, using modal data output from the eigenmode analysis module of ACSIS, a procedure was developed using the Pro-Matlab and its Control System Toolbox, which includes algorithm scripts for optimal control solutions via the solution of an algebraic Ricatti equation. In order to accommodate large order dynamical systems, the design is accomplished in the uncoupled normal modes domain, using the available lowest eigenmodes from ACSIS.

The procedure begins by copying and editing the mode data output from ACSIS (see the listing for EIG_spring in Appendix A) into readable variables for input to Matlab. The typical approach was to create one file as a Matlab script (i.e. a "*.m" file), with the eigenvalues, eigenvectors, and mass/dof data from ACSIS at the beginning, followed by actuator and sensor influence matrices (related to the physical degrees of freedom), the objective function weighting matrices, and the function which calls other Matlab scripts to determine the solution. An example of the above input for the spring problem described in Appendix A is in file EIG_spring.m, which is listed below. Compare this the ACSIS mode data output shown in Appendix A to see how the editing was accomplished, and the additional control design data added.

File: EIG_spring.m

```
lam = [ 1980.6
        15550.
        32470. ];
vl=[1 2.3305 1.8689 -1.0372 0. 0.
    2 1.8689 -1.0372 2.3305 0. 0.
    3 1.0372 -2.3305 -1.8689 0. 0. ];
m=[ 2 1 3 1.00000000000000D-01
    3 1 2 1.00000000000000D-01
    4 1 1 1.00000000000000D-01];
t=[vl(:,2:4); qbazeros(3,1); qb(2,1)=1.0; hd=zeros(1,3); hv=zeros(1,3); hv(1,2)=1.0; qv=[1; 5; 1]; q=diag([qv;qv]); r=.0001*eye(1); [f1,f2]=nlqr(lam,t,m,qb,q,r,3); qv=[1;1;1]; q=diag([qv;qv]); r=100*eye(1); [k1,k2]=mkf(lam,t,m,hd,hv,q,r,3); save f1out f1 /ascii
save f2out f2 /ascii
```
The scripts `mlqr.m` and `mkf.m` were written to accept as input the vector of eigenvalues, the eigenvector matrix (orthogonal vectors stored in columns), a matrix of node, component, d.o.f, mass data, and the actuator (or sensor) influence matrix and weighting matrices. The scripts output the gain results in four-column arrays, with one gain per row, and the corresponding node number, displacement component, and actuator (or sensor) identification. The top-level problem script (listed above) then saves the output in external files and the analysis is complete. The design scripts (listed below) also include checks on controllability and observability of the system based on the modal data and influence matrices defined, and produce plots of the closed loop poles resulting from the gain design. This aids the analyst in assessing the expected performance (and stability) of the exact system before moving the data back to ACSIS for simulation.

The script `contrank.m` finds the rank of the controllability matrix through iterative rank calculations of submatrices so as to avoid the illconditioning experienced in the full matrix. This is both faster and more accurate for determining whether a particular actuator placement has full control of the included structural modes.

**File: mlqr.m**

```matlab
function [Flout,F2out]=mlqr(lam,t,m,qb,Q,R,nmode)

% Controller gain design for second-order
% structural system via given eigenmodes.
% Gains are transformed to be coefficients
% of structural variables (disp, velocity);
% i.e. plant is second-order, of size ndof.

% Arguments:
% lam: vector of eigenvalues (nmode x 1)
% t: matrix of eigenvectors (ndof x nmode)
% m: mass diagonal and dof mapping info (ndof x 4)
% qb: actuator position influence matrix (nact x ndof)
% Q: optimal design state weighting matrix (2*nmode x 2*nmode)
% R: optimal design feedbk weight. matrix (nact x nact)
% Flout: F1 gain matrix for 2nd-order plant
% F2out: F2 gain matrix for 2nd-order plant

% Written by K.F. Alvin

format short e
nmodmax=length(lam);
[ndof,nact]=size(qb);

% Variables:
% mass: mass matrix
```

```
% A: state transition matrix
% B: actuator influence matrix
% G: control gain matrix

massd(m(:,3)) = m(:,4);

mass = diag(massd);

A = [zeros(nmode), eye(nmode); -1*diag(lam(1:nmode)), zeros(nmode)];
Amax = [zeros(nmodmax), eye(nmodmax); -1*diag(lam), zeros(nmodmax)];

B = [zeros(nmode, nact); t(:, 1:nmode)*qb];
Bmax = [zeros(nmodmax, nact); t'*qb];

disp('Number of structural modes and actuators used: ')
disp([nmode, nact])
disp('Rank of the controllability matrix: ')
disp(contrank(A, B))
disp('Determining controller gains for given system... ')
G = lqr(A, B, Q, R);

wmax = 1.1*max(sqrt(lam));
axis([-wmax, wmax, -wmax, wmax]);
plot(eig(A-B*G), '*')
grid
title('Roots of controlled system')
hold
pause

% partition gain matrix

G1 = G(:, 1:nmode);
G2 = G(:, nmode+1:nmode+nmode);

% Transform resultant gain matrices for use in
% partitioned csi algorithm using second-order
% structure(plant) equations.

disp('Mapping gains back to physical domain... ')
f1 = G1*t(:, 1:nmode)'*mass;
f2 = G2*t(:, 1:nmode)'*mass;

% find modal damping ratios of controller for calculated gains:

Gmax = [f1*t, f2*t];
lambda = eig(Amax - Bmax*Gmax);
plot(lambda, '*')
pause

nfreq = sqrt(real(lambda).^2 + imag(lambda).^2);
mdamp = real(lambda)/nfreq;
disp('Resultant modal damping ratios for controller: ')
disp([' Damping ', ' Damped Freq (rad/s) '])
disp([mdamp, nfreq.*sqrt(1-mdamp.^2), lambda])
bdamp = max(-2*mdamp./nfreq);
disp('Estimated minimum stiffness damping coefficient necessary')
disp(' to stabilize residual modes due to gain roundoff accumulation: ')
disp(bdamp)
disp('Writing gains in node correspondence output form... ')

34
Flout=zeros(nact*ndof,4);
F2out=zeros(nact*ndof,4);
for i=1:nact;
    kmin=(i-1)*ndof+1;
    kmax=i*ndof;
    Flout(kmin:kmax,:)=[m(:,1:2),i*ones(ndof,1),fl(i,m(:,3))']
    F2out(kmin:kmax,:)=[m(:,1:2),i*ones(ndof,1),f2(i,m(:,3))]';
end;
disp('Finished mlqr')

File: mkf.m

function[L1out,L2out]=mkf(lam,t,m,hd,hv,Q,R,nmode)

% Kalman filter design for second-order
% structural system via given eigenmodes
% and transformed to independent
% displacement/gen. momentum variable
% casting for partitioned csi transient
% analysis. See Belvin/Park paper for
% filter variable definitions.
%
% Arguments:
% lam: vector of eigenvalues (nmode x 1)
% t: matrix of eigenvectors (ndof x nmode)
% m: mass diagonal and dof mapping info (ndof x 4)
% hd: sensor position influence matrix (nsen x ndof)
% hv: velocity position influence matrix (nsen x ndof)
% Q: optimal design state weighting matrix (2*nmode x 2*nmode)
% R: optimal design feedbk weight. matrix (nsen x nsen)
% L1out: L1 gain matrix for 2nd-order filter
% L2out: L2 gain matrix for 2nd-order filter
%
% Written by K.P. Alvin

format short e
nmodmax=length(lam);
[nsen,ndof]=size(hd);

% Variables:
% mass: mass matrix
% A: state transition matrix
% G: noise influence matrix
% C: output influence matrix
% K: filter gain matrix

massd(m(:,3))=m(:,4);
mass=diag(massd);
A=[zeros(nmode),eye(nmode);-1*diag(lam(1:nmode)),zeros(nmode)];
Amax=[zeros(nmodmax),eye(nmodmax);-1*diag(lam),zeros(nmodmax)];
G=eye(nmode+nmode);
C=[Hd(:,:nmode),hv(:,:nmode)];
disp('Number of structural modes and sensors used:');
disp([nmode,nsen])
disp('Rank of the observability matrix:')
disp(contran(A,C))

disp('Determining filter gains for given system...')
K=lqe(A,G,C,Q,R);

% partition gain matrix
K1=K(:,1:nmode);
K2=K(:,nmode+1:nmode+nmode);
Kmax=[K1;zeros(nmodmax-nmode,nsen);K2;zeros(nmodmax-nmode,nsen)];

% find modal damping ratios of filter for calculated gains:
lambda=eig(Amax°Kmax*Cmax);
plot(lambda,'*')
hold
al=real(lambda);
bimag(lambda);
disp('Resultant modal damping ratios for filter:')
disp(['Damping ','Freq (rad/s) '])
disp([ai./sqrt(ai.^2+b1.^2), b1])

% Transform resultant gain matrices for use in
% partitioned csi algorithm using second-order
% Kalman filter approach.
% disp('Mapping gains back to physical domain...')
ll=ll(:,1:nmode)*K1;
l2=ll(:,1:nmode)*K2;
disp('Writing gains in node correspondence output form...')
Llout=zeros(nsen*ndof,4);
L2out=zeros(nsen*ndof,4);
for i=1:nsen;
    kmin=(i-1)*ndof+1;
    kmax=i*ndof;
    Llout(kmin:kmax,:)=[m(:,1:2),i*ones(ndof,1),ll(:,3),i];
    L2out(kmin:kmax,:)=[m(:,1:2),i*ones(ndof,1),l2(:,3),i];
end;
disp('Finished mkf')

File: contrank.m

function maxrank=contrank(a,b)
maxrank=0;
[nstate,nact]=size(b);
i=0;
mat=b;
newrank=rank(mat);
while newrank > maxrank
    maxrank=newrank;
i=i+1;
    mat=[mat,(a^i)*b];
newrank=rank(mat);
if newrank=nstate
    maxrank=newrank;
end
end

Unfortunately, the external files created from Matlab with the gain results are written completely in terms of real numbers, whereas the first three columns are actually to be read by ACSIS as integers (they are used as indices). A separate utility was written to convert the format of these files; the source code is listed below. On Unix systems, the user simply assigns the standard input to be the current data file created by Matlab, and gives another file name for the standard output. The code is basically just a filter to change the three columns of indices to integers. The output can then be pasted directly into the control definition file used by ACSIS.

File: convcont.f

program convcont

    parameter (NMAX=100000)
    real*8 f(NMAX),v(4)
    integer node(NMAX),dof(NMAX),act(NMAX)

    n=0

100   read (*,*,err=200,end=200) (v(i),i=1,4)
    n=n+1
    node(n)=int(v(1))
    dof(n) =int(v(2))
    act(n) =int(v(3))
    f(n) =v(4)
    goto 100

200   do 300 i=1,n
        print *, node(i),dof(i),act(i),f(i)
300    continue
end
APPENDIX C

Stability Analysis of a CSI Partitioned Simulation Algorithm with State Estimator
The equation of the open-loop plant without passive damping in modal second-order form is

$$\ddot{q} + \omega^2 q = u$$  \hspace{1cm} (1)

The controller uses a second-order observer to estimate the plant state, along with a full-state feedback control gain design.

$$u = - (\eta \omega^2 p + \zeta \omega \dot{p})$$
$$\ddot{p} + \omega^2 p = u + \xi \gamma$$
$$\gamma = z - \dot{p}$$
$$z = \dot{q}$$  \hspace{1cm} (2)

where \( q \) and \( p \) are the plant and estimator states, respectively, \( u \) is the control force, \( \gamma \) is the state estimation error, \( z \) is the sensor output, and \( \eta, \zeta, \xi \) are gain coefficients for position and velocity feedback, and the estimator filter.

The partitioned analysis procedure uses a stabilized form of the control law and estimation error determination to reduce inaccuracies associated with the extrapolation of variables in the controller force prediction. A first-order filtering is achieved by taking the time derivative of (2a,c),

$$\dot{u} = -\eta \omega^2 \dot{p} - \zeta \omega \ddot{p}$$
$$\dot{\gamma} = \dot{z} - \ddot{p}$$  \hspace{1cm} (3)

and then embedding the equations of motion through substitution for \( \ddot{p} \). This leads to the following two coupled, first-order differential equations for the prediction of the control force \( u \) and state error \( \gamma \).

$$\begin{cases}
\dot{u} \\
\dot{\gamma}
\end{cases} + \begin{bmatrix}
\zeta \omega & \zeta \xi \omega \\
1 & \xi
\end{bmatrix} \begin{bmatrix}
u \\
\gamma
\end{bmatrix} = \begin{bmatrix}0 \\
\dot{z}
\end{bmatrix} + \begin{bmatrix} \zeta \omega^3 \\
\omega^2
\end{bmatrix} \begin{bmatrix} p \\
0
\end{bmatrix} - \begin{bmatrix} \eta \omega^2 \\
0
\end{bmatrix} \ddot{p}$$  \hspace{1cm} (4)

Time discretization of (4) using an implicit midpoint rule leads to the following coupled difference equation:

$$\begin{bmatrix}
1 + \delta \zeta \omega & \delta \zeta \xi \omega \\
\delta & 1 + \delta \xi
\end{bmatrix} \begin{bmatrix} u^{n+\frac{1}{2}} \\
\gamma^{n+\frac{1}{2}}
\end{bmatrix} = \begin{bmatrix} 0 \\
x_p^{n+\frac{1}{2}}
\end{bmatrix} + \begin{bmatrix} \delta \zeta \omega^3 - \eta \omega^2 \\
\delta \omega^2
\end{bmatrix} \begin{bmatrix} p_p^{n+\frac{1}{2}} \\
0
\end{bmatrix} - \begin{bmatrix} \zeta \omega \\
1
\end{bmatrix} \dot{p}^n$$  \hspace{1cm} (5)

where \( \delta \equiv \) half-step size = \( \frac{h}{2} \). Solving this equation requires knowledge of the plant (to obtain sensor output) and observer states. These values are extrapolated as:

$$p_p^{n+\frac{1}{2}} = p^n + \delta \dot{p}^n$$
$$x_p^{n+\frac{1}{2}} = q_p^{n+\frac{1}{2}} = q^n$$  \hspace{1cm} (6)
Using these equations to obtain \( u^{n+\frac{1}{2}} \) and \( \gamma^{n+\frac{1}{2}} \) allows the plant and observer equations to be solved independently. Midpoint time integration of (1) and (2b) leads to the following equations:

\[
(1 + \delta^2 \omega^2) q^{n+\frac{1}{2}} = \delta^2 u^{n+\frac{1}{2}} + q^n + \delta \dot{q}^n \\
(1 + \delta^2 \omega^2) p^{n+\frac{1}{2}} = \delta^2 u^{n+\frac{1}{2}} + p^n + \delta \dot{p}^n + \delta^2 \xi \gamma^{n+\frac{1}{2}} \\
\dot{q}^{n+\frac{1}{2}} = \frac{1}{\delta} \left( q^{n+\frac{1}{2}} - q^n \right) \\
\dot{p}^{n+\frac{1}{2}} = \frac{1}{\delta} \left( p^{n+\frac{1}{2}} - p^n \right) \\
q^{n+1} = 2q^{n+\frac{1}{2}} - q^n \\
p^{n+1} = 2p^{n+\frac{1}{2}} - q^n \\
\dot{q}^{n+1} = 2\dot{q}^{n+\frac{1}{2}} - \dot{q}^n \\
\dot{p}^{n+1} = 2\dot{p}^{n+\frac{1}{2}} - \dot{q}^n
\] (7)

Computational stability of the modal form of the CSI partitioned equations of motion using the aforementioned time discretization can be assessed by seeking a nontrivial solution of

\[
\begin{pmatrix}
q^{n+1} \\
p^{n+1} \\
\dot{q}^{n+1} \\
\dot{p}^{n+1}
\end{pmatrix} = \lambda \begin{pmatrix}
q^n \\
p^n \\
\dot{q}^n \\
\dot{p}^n
\end{pmatrix}
\] (8)

such that

\[|\lambda| \leq 1\] (9)

for stability. Substituting (8) into (5-7), we obtain

\[Jx = 0\] (10)

where

\[
x_1 = [p_p^{n+\frac{1}{2}} \quad u^{n+\frac{1}{2}} \quad p^{n+\frac{1}{2}} \quad \dot{p}^{n+\frac{1}{2}} \quad p^n \quad \dot{p}^n]^T
\]

\[
x_2 = [z_p^{n+\frac{1}{2}} \quad \gamma^{n+\frac{1}{2}} \quad q^{n+\frac{1}{2}} \quad \dot{q}^{n+\frac{1}{2}} \quad q^n \quad \dot{q}^n]^T
\]

\[J = \begin{bmatrix}
J_{11} & J_{12} \\
J_{21} & J_{22}
\end{bmatrix}
\] (11)
A nontrivial solution to (10) is found from
\[ \det J = 0 \] (16)

which leads to the characteristic equation
\[
((1 - \delta \xi) (1 + \delta^3 \zeta \omega^3 - \delta^2 \eta \omega^2) + \delta \xi (1 + \delta^2 \omega^2)) z^4 \\
+ (\delta \zeta \omega (1 - \delta \xi) + \delta \xi (1 + \delta^3 \zeta \omega^3 - \delta^2 \eta \omega^2)) z^3 \\
((1 - \delta \xi) (\delta^2 \omega^2 + \delta^2 \eta \omega^2) + \delta^2 \omega^2 (1 + \delta^3 \zeta \omega^3 - \delta^2 \eta \omega^2) \\
+ \delta \xi (\delta \zeta \omega + \delta^2 \omega^2 + \delta^4 \omega^4)) z^2 \\
(\delta^2 \omega^2 (\delta \zeta \omega) + \delta \xi (\delta^2 \omega^2 + \delta^2 \eta \omega^2)) z \\
+ \delta^2 \omega^2 (\delta^2 \omega^2 + \delta^2 \eta \omega^2) = 0
\] (17)

where

\begin{align*}
J_{11} &= \begin{bmatrix}
1 & 0 & 0 & 0 & -1 & -\delta \\
(\eta \omega^2 - \delta \zeta \omega^3) & (1 + \delta \zeta \omega) & 0 & 0 & 0 & 0 \\
0 & -\delta^2 & (1 + \delta^2 \omega^2) & 0 & -1 & -\delta \\
0 & 0 & -\frac{1}{\delta} & 1 & \frac{1}{\zeta} & 0 \\
0 & 0 & 0 & -2 & \lambda + 1 & 0 \\
0 & 0 & 0 & 0 & -2 & \lambda + 1 \\
\end{bmatrix} \\
J_{12} &= \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & \delta \zeta \omega & 0 & 0 & 0 & 0 \\
0 & -\delta^2 \xi & 0 & 0 & 0 & 0 \\
0 & -\delta \xi & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} \\
J_{21} &= \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
-\delta \omega^2 & \delta & 0 & 0 & 0 & 1 \\
0 & -\delta^2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} \\
J_{22} &= \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & -1 \\
-1 & (1 + \delta \xi) & 0 & 0 & 0 & 0 \\
0 & 0 & (1 + \delta^2 \omega^2) & 0 & -1 & -\delta \\
0 & 0 & -\frac{1}{\delta} & 1 & \frac{1}{\zeta} & 0 \\
0 & 0 & 0 & -2 & 0 & \lambda + 1 \\
0 & 0 & 0 & 0 & -2 & \lambda + 1 \\
\end{bmatrix}
\]
Thus, a test of the polynomial equation for possible positive real roots by the Routh-Hurwitz criterion indicates that the partitioned approach as applied to the modal equations give a computationally stable solution for no velocity feedback $\zeta = 0$ provided

$$h \leq \frac{2}{\sqrt{\eta \omega}}$$  \hspace{1cm} (19)

$$h \leq \frac{2}{\xi}$$  \hspace{1cm} (20)
APPENDIX D

ACESIS Source Code
File: Makefile

.SUFFIXES: .f .o

FFLAGS =

.f.o:
  fortran -c $(FFLAGS) $*.f

OBJS = acsis.o acssout.o addstf.o
       beam3d.o forces.o input.o
       pmvxml.o prepfem.o profile.o
       read.o solver.o mophlag.o
       zerevot.o lu.o prepcon.o
       control.o secorder.o measure.o
       eigens.o singeig.o animout.o
       stiffrc.o estifvm.o renum.o

acsis.exe: $(OBJS)
  fortran -o acsis.exe $(FFLAGS) $(OBJS)

File: shared.inc

shared.inc (ACSIS database)

Argument definitions:
adamp: Rayleigh damping coefficient alpha
b: actuator location matrix (packed storage)
brow: row number of corresponding real value in b
bcol: column number of corresponding real value in b
bval: number of nonzero values in b
bdamp: Rayleigh damping coefficient beta
confile: controller input file name
cotype: id for type of control
corxys: array of the x,y, and z components of each node
delta: one-half time step
delat: one-half time step squared
e: control prediction integration coefficient matrix
ean: array of element material types
eoa: observer construct matrix S in vector form
eprop: array of element property types
ee: structure construct matrix S (M+delta*D+delsq*K)
etype: array of element types
ei: array of element numbers for domain decomposition
f: vector of applied forces
fi: control gain matrix
f2: control gain matrix
femfile: finite element input file name
forceid: identification number of forcing function
gamma: state correction force
go: RHS vector for control prediction module
gk: RHS vector for Kalman filter momentum eqn
go: RHS vector for observer module
gs: RHS vector for structure module
h: time step size
hd: displacement sensor location matrix (packed storage)
bdrov: row number of corresponding real value in bd
bdcol: column number of corresponding real value in bd
bdval: number of nonzero values in bd
hv: velocity sensor location matrix (packed storage)
hvrow: row number of corresponding real value in hv
hvcol: column number of corresponding real value in hv
hvval: number of nonzero values in hv
id: DOF mapping array: id(comp,node #)=Global DOF #
ix: array of element connectivity and orientation
inertia: array of concentrated inertias or lumped masses
jdiag: array of diagonal element addresses
l1: State estimator filter gain matrix
l2: State estimator filter gain matrix
mask: mass matrix K in reduced vector form
mat: array of different materials
mlen: length of global matrices in profile vector storage
nact: actual number of actuators
nacti: actual number of actuators and sensors
ndisout: number of displacement results to output
ndof: actual number of degrees of freedom
ndomain: actual number of element domains (for dom. decomp.)
nel: actual number of elements
neld: array of number of elements in each domain
nnp: actual number of nodes
nsem: actual number of sensors
nvelout: number of velocity results to output
nolag: logical flag to signal corrector loop in measurement
outfile: output file name
outlabel: array of output data requested
pin: array of element pin release codes
pivot: Column pivoting info from FACTA
prop: array of different properties
q: generalized displacement vector
q0: initial displacement condition
qalpha: gain scale factor for f1
qalphao: gain scale factor for l1
qbeta: gain scale factor for f2
qbetao: gain scale factor for l2
qdot: velocity vector
qdot0: initial velocity condition
qe: state estimator displacement vector
qdot: state estimator velocity vector
r: Solution vector of control module {u,gamma}
scalef: Scaling factor for forcing function
stiff: stiffness matrix K in reduced vector form
t0: initial time
tc: control-on time
tf: final time
Parameter definitions

**PARAMETER**
- **MAXACT**: max. # of actuators
- **MAXCSI**: max. combined # of actuators and sensors
- **MAXDAT**: max. # of materials and properties
- **MAXDOF**: max. # of degrees of freedom
- **MAXDOM**: max. # of decomposition domains
- **MAELE**: max. # of elements
- **MAXLEN**: max. length of global vectors in reduced form
- **MAINODE**: max. # of nodes

```plaintext
PARAMETER(MAXDOF=3000, MAXACT=50, MAXCSI=100)
PARAMETER(MAXNODE=1000, MAXELE=3000, MAXDAT=100)
PARAMETER(MAXLEN=200000, MAXDOM=50, MAXNZV=200)
```

**REAL**
- `t0, tf, tc, h, delta, delsq, qalpha, qbeta, qalphao, qbetao`
- `q(MAXDOF), qdot(MAXDOF), ge(MAXDOF), qdot(MAXDOF)`
- `u(MAXACT), gamma(MAXACT), f(MAXACT), r(MAXCSI)`
- `es(MAXLEN), eo(MAXLEN), ec(MAXCSI, MAXCSI)`
- `gs(MAXDOF), go(MAXDOF), gc(MAXCSI), scalef, pe(MAXDOF)`
- `eoxyz(3, MAXHODE), mat(6, MAXDAT), prop(10, MAXDAT)`
- `q0(6, MAXNODE), qdot0(6, MAXDOF), inertia(6, MAXNODE)`
- `b(MAXNZV), hd(MAXNZV), hv(2000), astifm(78, 500)`
- `f(MAXACT, MAXDOF), f2(MAXACT, MAXDOF)`
- `l(MAXACT, MAXDOF), l2(MAXACT, MAXDOF)`

**INTEGER**
- `etype(MAXELE), ix(4, MAXELE), emat(MAXELE), forceid`
- `eprop(MAXELE), px(6, MAXELE), id(6, MAXNODE)`
- `mask(MAXNODE), contype, brow(MAXNZV), bcol(MAXNZV)`
- `hdrow(MAXNZV), hdcol(MAXNZV), hvrow(MAXNZV)`
- `hvcfle(MAXNZV), bval, hdval, hvval`
- `mdof, nact, nsen, ncsi, mlen, jdiag(MAXDOF), nnp, nel, nei`g`
- `neld(MAXDOM), elnum(MAXELE, MAXDOM), eldom(MAXELE), ndomain`
- `outlabel(40), ndisout, nvelout, pivot(MAXCSI)`
- `iapdjc, MAXLEN), icount(MAXNODE+1), perm(MAXNODE)`
- `xls(MAXNODE)`

**LOGICAL**
- `animate, nolag`

**CHARACTER**
- `32 femfile, confile, outfile, animfile`

**COMMON**
- `/FILES/ femfile, confile, outfile, outlabel, ndisout, nvelout, animfile, animate, nolag`
- `/TIMERS/ t0, tf, tc, h, delta, delsq`
- `/STATES/ q, qdot, ge, qdot, u, gamma, f, r, pe`
- `/FEMDAT/ mass, stif, adamp, bdamp, coxyz, mat, prop, q0, qdot0, inertia, scalef`
- `/INTEGR/ es, g, ge, gc, gk, pivot`
- `/DIMENS/ ndof, nact, nsen, ncsi, mlen, jdiag, nnp, nel, nei`
- `/CONDAT/ b, hd, hv, f1, f2, 11, 12, qalpha, qbeta, qalphao, qbetao`
- `/ELEDOM/ estifm, ndomain, neld, elnum, eldom`
- `/INTEGER/ forceid, etype, ix, emat, eprop, pint, id, mask, contype, brow, bcol, hdrow, hdcol, hvrow, hvcfle, bval, hdval, hvval`
- `/RESEQH/ iapdjc, icount, perm, xls`

File: acsis.f
C=Program ACSIS
C=Purpose Accelerated CSI Simulation
C=Author K. Alvin
C=Date May 1990
C=Block Fortran

C........................................................................C
C
C
C

C Purpose: 2nd Order Accelerated CSI Simulation
C
C
C

C........................................................................C

program ACSIS
C

GET SHARED DATA FILE

include 'shared.inc'

C LOCAL VARIABLES

real*8 t,z(MAXACT)

integer n,m,runtype,outskip

C LOGIC

call INPUT(runtype,outskip)

if (runtype) 100,200,300

C EIGENMODE ANALYSIS

100 continue

call PREPFEM

call EIGENS

goto 999

C CSI SIMULATION

200 call PREPFEM

call PREPCON

n = 0
m = 0
print *, 'Finished Preprocessing . . . starting simulation'
print *, 'Time =', t0
CALL ACSISOUT(t0)

do 250 t=t0,tf,h

   call FORCES(t+h/2)

C Predict CSI coupling variables u and gamma
if (t .ge. tc) then
    call MEASURE(z)
call CONTROL(z)
if (nolag) then
    call NOPHLAG(z)
call CONTROL(z)
endif
endif

C Structure and observer set up for parallel execution

CVD$ CECALL
    do 275 i=1,2
C
    Integrate Observer Equations

    if ((i .eq. 1).and.(t .ge. tc)) then
        if (contype .eq. 0) then
            call SECORDER(mass, stif, adamp, bdamp, f, go, so, q, qdot,
                            delta, dlsq, jdiag, ndof, MAXDOF)
        elseif (contype .eq. 1) then
            call KFILTER
        endif
    endif

    Integrate Structure Equations

    elseif (i .eq. 2) then
        call SECORDER(mass, stif, adamp, bdamp, f, ge, es, q, qdot,
                        delta, dlsq, jdiag, ndof, MAXDOF)
    endif

275 continue

C PRINT TIME EACH 100 iterations

    n = n + 1
    m = m + 1
    if (n .ge. 100) then
        print *, 'Time = ', t+h
        n = 0
    endif
    if (m .ge. outskip) then
        call ACSISOUT(t+h)
        write(24,'(40fi12.8)') t, z(i), i=1, nsen
        m = 0
    endif

250 continue
goto 999

C TRANSIENT RESPONSE

300 call PREPFEM

    n = 0
    m = 0
    print *, 'Finished Preprocessing . . . starting simulation'
    print *, 'Time = ', t0
call ACSISOUT(t0)
do 360 t=t0,tf,h
call FORCES(t+h/2)
call ZEROVECT(gs,ndof)
call SECORDER(mass, stif, adaamp, bdamp, f, gs, es, q, qdot, 
delta, deltaq, jdiag, ndof, MAXDOF)
PaiBT TIME EACH 100 iterations
 m = m + 1
 m = m + 1
 if (m .ge. 100) then
 print * , 'Time ' , t+h
 m = 0
 endif
 if (m .ge. outskip) then
 call ACSISOUT(t+h)
m = 0
 endif
350 continue
999 stop end

File: acsisout.f

C=Module ACSISOUT
C=Purpose Write desired output from ACSIS for plotting, etc.
C=Author K. Alvin
C=Date May 1990
C=Block Fortran
C
---------------------------------------------
C
Subroutine ACSISOUT

C Purpose:
TThis subroutine outputs formatted displacement and velocity
results at a given time for plotting time histories. The
desired output variables are defined in outlabel().

C
---------------------------------------------
C
Arguments
C t - time

subroutine ACSISOUT(t)
include 'shared.inc'
real*8 t
C LOCAL VARIABLES

integer i

C LOGIC

write(13,'(40f12.8)') t,(q(id(outlabel(i+10),outlabel(i))),
i=1,ndisout),(qdot(id(outlabel(i+30),outlabel(i+20))),
i=i1,nvelout)

write(23,'(40f12.8)') t,(qe(id(outlabel(i+10),outlabel(i))),
i=1,ndisout),(qedot(id(outlabel(i+30),outlabel(i+20))),
i=i1,nvelout)

write(25,'(40f12.8)') t,(u(i),i=1,nact),(gamma(i),i=1,nseq)

if (animate) call ANIMOUT(q,id,nnp,t,15)

return

cend

File: addstf.f

C=Module ADDSTF
C=Purpose Assemble Global stiffness matrix
C=Author who knows
C=Update January 1989, by E. Pramono
C=Block Fortran

subroutine ADDSTF(sk,lm,bk,jdiag,nseq)

C PURPOSE:

C THIS SUBROUTINE ASSEMBLES THE ELEMENT STIFFNESS MATRICES C

C INTO THE COMPACTED GLOBAL STIFFNESS VECTOR.

C ARGUMENTS:

C sk - ELEMENT STIFFNESS MATRIX
C lm - LOCATION VECTOR FOR ELEMENT STIFFNESS MATRIX
C bk - COMPACTED GLOBAL STIFFNESS VECTOR
C jdiag - VECTOR OF DIAGONAL ELEMENT ADDRESSES
C nseq - NUMBER OF DEGREES OF FREEDOM PER ELEMENT

C ARGUMENTS

real*8 sk(nseq,nseq), bk(1)
integer lm(18), jdiag(1)
C integer lm(18), jdiag(1), nseq

C LOCAL ARGUMENTS

integer i, j, k, l, m

C ASSEMBLE GLOBAL STIFFNESS AND LOAD ARRAYS

do 20 j = 1, nseq
  k = lm(j)
  if (k .eq. 0) goto 20
1 = jdiag(k) - k
1 = jdiag(k+1) - k
do 10 i = 1, nseq

m = lm(i)
if(m .gt. k .OR. m .eq. 0) goto 10
m = 1 + m
bk(m) = bk(m) + sk(i,j)
10 continue
20 continue
C return
end
C=End Fortran

File: beam3d.f

C=Module BEAM3D
C=Purpose Construct 3-d Timoshenko beam element stiffness and lumped mass
C=Author K. Alvin
C=Date May 1990
C=Block Fortran

subroutine BEAM3D(n,ni,nj,nk,xyz,emod,gmod,rho,area,ssf2,ssf3,
& jtor,i2,i3,ipin,sk,sm)
C ARGUMENTS:
C n Element ID Number
C ni Node ID Number at End i
C nj Node ID Number at End j
C xyz Node Location Array
C emod Material Elastic Modulus (Young's Modulus)
C gmod Material Modulus of Rigidity (Shear Modulus)
C rho Material Mass Density
C area Element Cross-sectional area
C ssf2 Shear shape factor in element x2 direction
C ssf3 Shear shape factor in element x3 direction
C jtor Torsional constant J
C i2 Area moment of inertia about element x2 axis
C i3 Area moment of inertia about element x3 axis
C ipin Pin release codes: 0=Fixed, 1=Freed
  (1) Axial
  (2) Torsional
  (3) End A rotation about x2 axis
  (4) End A rotation about x3 axis
  (5) End B rotation about x2 axis
  (6) End B rotation about x3 axis
C sk Element Stiffness Matrix
C sm Element Mass Matrix

integer n,ni,nj,nk,ipin(1)
real*8 xyz(3,1),emod,gmod,rho,area,ssf2,ssf3,jtor,i2,i3
real*8 sk(12,1),sm(12,1)

C LOCAL VARIABLES:

integer i,j
real*8 dc(3,3),length,rlength,kc(10),mc(3)
C LOGIC

C Find Element Length

    length = 0.0d0
    do 10 i = 1,3
        dc(1,i) = xyz(i,nj) - xyz(i,ni)
    length = length + dc(1,i)**2.
    continue
    length = sqrt(length)
    if (length .eq. 0.0d0) then
        print *, 'BAR2D: Zero element length; n= ',n
        return
    endif

C Find direction cosines for xi,x2,x3 element axes

    do 15 i=1,3
        dc(1,i) = dc(1,i)/length
        if (ak .eq. 0) then
            dc(2,i) = 0.0
        else
            dc(2,i) = xyz(i,nk) - xyz(i,ni)
        endif
        continue
    if (ak .eq. 0) dc(2,3) = 1.0
    dc(3,1) = dc(1,2)*dc(2,3) - dc(2,2)*dc(1,3)
    dc(3,2) = dc(2,1)*dc(1,3) - dc(1,1)*dc(2,3)
    dc(3,3) = dc(1,1)*dc(2,2) - dc(2,1)*dc(1,2)
    rlength = sqrt(dc(3,1)**2 + dc(3,2)**2 + dc(3,3)**2)
    if (rlength .ne. 0.) goto 17
    dc(2,2) = 1.0
    dc(2,3) = 0.0
    goto 16
    17 do 18 i=1,3
        dc(3,i) = dc(3,i)/rlength
        continue
    dc(2,1) = dc(3,2)*dc(1,3) - dc(1,2)*dc(3,3)
    dc(2,2) = dc(1,1)*dc(3,3) - dc(3,1)*dc(1,3)
    dc(2,3) = dc(3,1)*dc(1,2) - dc(1,1)*dc(3,2)

C Compute various stiffness constants, accounting for pin codes

    if (ipin(1) .eq. 0) then
        kc(1) = area*emod/length
    else
        kc(1) = 0.0d0
    endif
    if (ipin(6) .eq. 0) then
        if (ipin(6) .eq. 0) then
            kc(2) = area*gmod*ssf2/length
            kc(6) = i3*emod/length
            kc(7) = kc(2)*length/2.0d0
            kc(9) = kc(7)*length/2.0d0
        else
            print *, 'STAR3D: Pin code error, x3 direction, el #',n
endif
else
  if (ipin(6) .eq. 0) then
    print *, 'BEAM3D: Pin code error, x3 direction, el #', n
  else
    kc(2) = 0.0d0
    kc(6) = 0.0d0
    kc(7) = 0.0d0
    kc(9) = 0.0d0
  endif
endif

if (ipin(3) .eq. 0) then
  if (ipin(6) .eq. 0) then
    kc(3) = area*mod*ssf3/length
    kc(5) = i2*mod/length
    kc(8) = kc(3)*length/2.0d0
    kc(10) = kc(8)*length/2.0d0
  else
    print *, 'BEAM3D: Pin code error, x2 direction, el #', n
  endif
else
  if (ipin(6) .eq. 0) then
    print *, 'BEAM3D: Pin code error, x2 direction, el #', n
  else
    kc(3) = 0.0d0
    kc(5) = 0.0d0
    kc(8) = 0.0d0
    kc(10) = 0.0d0
  endif
endif

if (ipin(2) .eq. 0) then
  kc(4) = jtor*mod/length
else
  kc(4) = 0.0d0
endif

mc(1) = area*rho*length/2.0d0
mc(2) = i2*rho*length/2.0d0
mc(3) = i3*rho*length/2.0d0

sk(1,1) = kc(1)*dc(1,1)*dc(1,1) + kc(2)*dc(2,1)*dc(2,1) + kc(3)*dc(3,1)*dc(3,1)
sk(1,2) = kc(1)*dc(1,1)*dc(1,2) + kc(2)*dc(2,1)*dc(2,2) + kc(3)*dc(3,1)*dc(3,2)
sk(1,3) = kc(1)*dc(1,1)*dc(1,3) + kc(2)*dc(2,1)*dc(2,3) + kc(3)*dc(3,1)*dc(3,3)
sk(1,4) = kc(7)*dc(2,1)*dc(3,1) - kc(8)*dc(3,1)*dc(2,1)
sk(1,5) = kc(7)*dc(2,1)*dc(3,2) - kc(8)*dc(3,1)*dc(2,2)
sk(1,6) = kc(7)*dc(2,1)*dc(3,3) - kc(8)*dc(3,1)*dc(2,3)
sk(1,7) = -sk(1,1)
sk(1,8) = -sk(1,2)
sk(1,9) = -sk(1,3)
sk(1,10) = sk(1,4)
sk(1,11) = sk(1,5)
sk(1,12) = sk(1,6)
sk(2,2) = kc(1)*dc(1,2)*dc(1,2) + kc(2)*dc(2,2)*dc(2,2) + kc(3)*dc(3,2)*dc(3,2)
\[ \text{sk}(2,3) = \text{k}(1)\text{dc}(1,2)\text{dc}(1,3) + \text{kc}(2)\text{dc}(2,2)\text{dc}(2,3) \]
\[ \text{sk}(2,4) = \text{k}(7)\text{dc}(2,2)\text{dc}(3,1) - \text{kc}(8)\text{dc}(3,2)\text{dc}(2,1) \]
\[ \text{sk}(2,5) = \text{k}(7)\text{dc}(2,2)\text{dc}(3,2) - \text{kc}(8)\text{dc}(3,2)\text{dc}(2,2) \]
\[ \text{sk}(2,6) = \text{k}(7)\text{dc}(2,2)\text{dc}(3,3) - \text{kc}(8)\text{dc}(3,2)\text{dc}(2,3) \]
\[ \text{sk}(2,7) = -\text{sk}(1,2) \]
\[ \text{sk}(2,8) = -\text{sk}(2,2) \]
\[ \text{sk}(2,9) = -\text{sk}(2,3) \]
\[ \text{sk}(2,10) = \text{sk}(2,4) \]
\[ \text{sk}(2,11) = \text{sk}(2,5) \]
\[ \text{sk}(2,12) = \text{sk}(2,6) \]
\[ \text{sk}(3,3) = \text{k}(1)\text{dc}(1,3)\text{dc}(1,3) + \text{kc}(2)\text{dc}(2,3)\text{dc}(2,3) \]
\[ \text{sk}(3,4) = \text{k}(7)\text{dc}(2,3)\text{dc}(3,1) - \text{kc}(8)\text{dc}(3,3)\text{dc}(2,1) \]
\[ \text{sk}(3,5) = \text{k}(7)\text{dc}(2,3)\text{dc}(3,2) - \text{kc}(8)\text{dc}(3,3)\text{dc}(2,2) \]
\[ \text{sk}(3,6) = \text{k}(7)\text{dc}(2,3)\text{dc}(3,3) - \text{kc}(8)\text{dc}(3,3)\text{dc}(2,3) \]
\[ \text{sk}(3,7) = -\text{sk}(1,3) \]
\[ \text{sk}(3,8) = -\text{sk}(2,3) \]
\[ \text{sk}(3,9) = -\text{sk}(3,3) \]
\[ \text{sk}(3,10) = \text{sk}(3,4) \]
\[ \text{sk}(3,11) = \text{sk}(3,5) \]
\[ \text{sk}(3,12) = \text{sk}(3,6) \]
\[ \text{sk}(4,4) = \text{k}(4)\text{dc}(1,1)\text{dc}(1,1) + (\text{kc}(10)\text{kc}(5))\text{dc}(2,1)\text{dc}(2,1) \]
\[ \quad + (\text{kc}(9)\text{kc}(6))\text{dc}(3,1)\text{dc}(3,1) \]
\[ \text{sk}(4,5) = \text{k}(4)\text{dc}(1,1)\text{dc}(1,2) + (\text{kc}(10)\text{kc}(5))\text{dc}(2,1)\text{dc}(2,2) \]
\[ \quad + (\text{kc}(9)\text{kc}(6))\text{dc}(3,1)\text{dc}(3,2) \]
\[ \text{sk}(4,6) = \text{k}(4)\text{dc}(1,1)\text{dc}(1,3) + (\text{kc}(10)\text{kc}(5))\text{dc}(2,1)\text{dc}(2,3) \]
\[ \quad + (\text{kc}(9)\text{kc}(6))\text{dc}(3,1)\text{dc}(3,3) \]
\[ \text{sk}(4,7) = -\text{sk}(1,4) \]
\[ \text{sk}(4,8) = -\text{sk}(2,4) \]
\[ \text{sk}(4,9) = -\text{sk}(3,4) \]
\[ \text{sk}(4,10) = -\text{k}(4)\text{dc}(1,1)\text{dc}(1,1) + (\text{kc}(10)\text{kc}(5))\text{dc}(2,1)\text{dc}(2,1) \]
\[ \quad + (\text{kc}(9)\text{kc}(6))\text{dc}(3,1)\text{dc}(3,1) \]
\[ \text{sk}(4,11) = -\text{k}(4)\text{dc}(1,1)\text{dc}(1,2) + (\text{kc}(10)\text{kc}(5))\text{dc}(2,1)\text{dc}(2,2) \]
\[ \quad + (\text{kc}(9)\text{kc}(6))\text{dc}(3,1)\text{dc}(3,2) \]
\[ \text{sk}(4,12) = -\text{k}(4)\text{dc}(1,1)\text{dc}(1,3) + (\text{kc}(10)\text{kc}(5))\text{dc}(2,1)\text{dc}(2,3) \]
\[ \quad + (\text{kc}(9)\text{kc}(6))\text{dc}(3,1)\text{dc}(3,3) \]
\[ \text{sk}(5,5) = \text{k}(4)\text{dc}(1,2)\text{dc}(1,2) + (\text{kc}(10)\text{kc}(5))\text{dc}(2,2)\text{dc}(2,2) \]
\[ \quad + (\text{kc}(9)\text{kc}(6))\text{dc}(3,2)\text{dc}(3,2) \]
\[ \text{sk}(5,6) = \text{k}(4)\text{dc}(1,2)\text{dc}(1,3) + (\text{kc}(10)\text{kc}(5))\text{dc}(2,2)\text{dc}(2,3) \]
\[ \quad + (\text{kc}(9)\text{kc}(6))\text{dc}(3,2)\text{dc}(3,3) \]
\[ \text{sk}(5,7) = -\text{sk}(1,5) \]
\[ \text{sk}(5,8) = -\text{sk}(2,5) \]
\[ \text{sk}(5,9) = -\text{sk}(3,5) \]
\[ \text{sk}(5,10) = \text{sk}(4,11) \]
\[ \text{sk}(5,11) = -\text{k}(4)\text{dc}(1,2)\text{dc}(1,2) + (\text{kc}(10)\text{kc}(5))\text{dc}(2,2)\text{dc}(2,2) \]
\[ \quad + (\text{kc}(9)\text{kc}(6))\text{dc}(3,2)\text{dc}(3,2) \]
\[ \text{sk}(5,12) = -\text{k}(4)\text{dc}(1,2)\text{dc}(1,3) + (\text{kc}(10)\text{kc}(5))\text{dc}(2,2)\text{dc}(2,3) \]
\[ \quad + (\text{kc}(9)\text{kc}(6))\text{dc}(3,2)\text{dc}(3,3) \]
\[ \text{sk}(6,6) = \text{k}(4)\text{dc}(1,3)\text{dc}(1,3) + (\text{kc}(10)\text{kc}(5))\text{dc}(2,3)\text{dc}(2,3) \]
\[ \quad + (\text{kc}(9)\text{kc}(6))\text{dc}(3,3)\text{dc}(3,3) \]
\[ \text{sk}(6,7) = -\text{sk}(1,6) \]
\[ \text{sk}(6,8) = -\text{sk}(2,6) \]
\[ \text{sk}(6,9) = -\text{sk}(3,6) \]
\[ \text{sk}(6,10) = \text{sk}(4,12) \]
\[ \text{sk}(6,11) = \text{sk}(5,12) \]
\[ \text{sk}(6,12) = -\text{k}(4)\text{dc}(1,3)\text{dc}(1,3) + (\text{kc}(10)\text{kc}(5))\text{dc}(2,3)\text{dc}(2,3) \]
\[ \quad + (\text{kc}(9)\text{kc}(6))\text{dc}(3,3)\text{dc}(3,3) \]
\[ \text{sk}(7,7) = \text{sk}(1,1) \]
sk(7,8) = sk(1,2)
sk(7,9) = sk(1,3)
sk(7,10) = -sk(1,4)
sk(7,11) = -sk(1,5)
sk(7,12) = -sk(1,6)
sk(8,8) = sk(2,2)
sk(8,9) = sk(2,3)
sk(8,10) = -sk(2,4)
sk(8,11) = -sk(2,5)
sk(8,12) = -sk(2,6)
sk(9,9) = sk(3,3)
sk(9,10) = -sk(3,4)
sk(9,11) = -sk(3,5)
sk(9,12) = -sk(3,6)
sk(10,10) = sk(4,4)
sk(10,11) = sk(4,5)
sk(10,12) = sk(4,6)
sk(11,11) = sk(5,5)
sk(11,12) = sk(5,6)
sk(12,12) = sk(6,6)

do 50 i = 1,12
do 55 j = 1,12
  sm(i,j) = 0.d0
55  continue
50  continue

C Row-sum rotated mass matrix to re-diagonalize

sm(1,1) = mc(1)
sm(2,2) = mc(1)
sm(3,3) = mc(1)
sm(4,4) = mc(2)*((dc(1,1)*dc(1,1)+dc(2,1)*dc(2,1)) +
  . mc(3)*((dc(1,1)*dc(1,1)+dc(3,1)*dc(3,1)) +
  . mc(2)*((dc(1,1)*dc(1,2)+dc(2,1)*dc(2,2)) +
  . mc(3)*((dc(1,1)*dc(1,2)+dc(3,1)*dc(3,2)) +
  . mc(2)*((dc(1,1)*dc(1,3)+dc(2,1)*dc(2,3)) +
  . mc(3)*((dc(1,1)*dc(1,3)+dc(3,1)*dc(3,3)) +

C.sm(4,5) = mc(2)*((dc(1,1)*dc(1,2)+dc(2,1)*dc(2,2)) +
  . mc(3)*((dc(1,1)*dc(1,2)+dc(3,1)*dc(3,2)) +
  . mc(2)*((dc(1,1)*dc(1,3)+dc(2,1)*dc(2,3)) +
  . mc(3)*((dc(1,1)*dc(1,3)+dc(3,1)*dc(3,3)) +
  . sm(5,4) = mc(2)*((dc(1,1)*dc(1,2)+dc(2,1)*dc(2,2)) +
  . mc(3)*((dc(1,1)*dc(1,2)+dc(3,1)*dc(3,2)) +
  . mc(2)*((dc(1,1)*dc(1,3)+dc(2,1)*dc(2,3)) +
  . mc(3)*((dc(1,1)*dc(1,3)+dc(3,1)*dc(3,3)) +
  . sm(5,5) = mc(2)*((dc(1,2)*dc(1,2)+dc(2,2)*dc(2,2)) +
  . mc(3)*((dc(1,2)*dc(1,2)+dc(3,2)*dc(3,2)) +
  . mc(2)*((dc(1,2)*dc(1,3)+dc(2,2)*dc(2,3)) +
  . mc(3)*((dc(1,2)*dc(1,3)+dc(3,2)*dc(3,3)) +
  . sm(5,6) = mc(2)*((dc(1,2)*dc(1,3)+dc(2,2)*dc(2,3)) +
  . mc(3)*((dc(1,2)*dc(1,3)+dc(3,2)*dc(3,3)) +
  . mc(2)*((dc(1,2)*dc(1,3)+dc(2,2)*dc(2,3)) +
  . mc(3)*((dc(1,2)*dc(1,3)+dc(3,2)*dc(3,3)) +
  . sm(6,6) = mc(2)*((dc(1,3)*dc(1,3)+dc(2,3)*dc(2,3)) +
  . mc(3)*((dc(1,3)*dc(1,3)+dc(3,3)*dc(3,3)) +

55
Subroutine FORCES

Purpose:
Returns force from stored function at any given time.
The forcing functions are hardwired by the user. The
function is selectable at program execution using the
forcing function ID, which by convention is the statement
label used in branching.

subroutine FORCES(time)

include 'shared.inc'
real*8 time, pi

pi = 3.1415926
call ZEROVECT(f,ndof)

101 if (forceid .eq. 101) then
    if (time .le. .02) then
        f(id(2,15)) = 100.*(1.-cos(2.*pi*time/.02))
    endif
    if (time .lt. .02) then
        f(id(2,95)) = 100.*(1.-cos(2.*pi*time/.02))
    endif
    if ((time .gt. .01) .and. (time .lt. .02)) then
        f(id(2,95)) = 100.*(1.-cos(2.*pi*time/.01))
    endif
102 elseif (forceid .eq. 102) then
    if (time .lt. .1) then
        f(id(2,95)) = 0.
    elseif (time .eq. .1) then
        f(id(2,95)) = 0.
    else
        f(id(2,95)) = 0.
    endif
103 elseif (forceid .eq. 103) then
    if (time .le. .01) then
        f(id(1,15)) = 100
    endif
104 elseif (forceid .eq. 104) then
    if ((time .gt. 0) .and. (time .lt. .17)) then
        f(id(3,125)) = 10
    elseif ((time .gt. .17) .and. (time .lt. 1.0)) then
        f(id(3,125)) = -10
    endif
105 elseif (forceid .eq. 105) then
    if (time .le. .01) then
        f(id(2,9)) = 100.*(1.-cos(2.*pi*time/.01))
    elseif (time .le. .02) then
        f(id(2,9)) = 100.*(cos(2.*pi*time/.01)-1.)
    endif
    do 10 i = 1, ndof
        f(i) = scalef * f(i)
10    continue
    return
end

File: input.f

C=Module INPUT
C=Purpose Input data parameters for ACSIS
C=Author K. Alvin
Subroutine INPUT

Argument definitions

runtype - ID of analysis run type
savint - variable to control creation of input file
runfile - variable indicates if run is from input file
comment - dummy name for comment input lines
outskip - number of steps to skip before sending output

subroutine INPUT(runtype,outskip)

include 'shared.inc'
integer runtype, outskip.
character*1 savint,runfile,temp
character*48 comment,inpfie

C PRZHT USB READ START-UP

print *, '2nd Order Accelerated CSI Simulation (ACSIS)'
print *
pint *, 'Please input analysis type: '
pint *= '1. Eigenmode Analysis'
pint *= '2. CSI Simulation'
pint =,' 3. Transient Response'
pint *=
read *, runtype

-run options and input file setup

runfile = 'n'
if (runtype .lt. 0) then
  runtype = -1 * runtype
  runfile = 'y'
endif
print *, 'Do you wish to save an input file? (y or n)'
read 21, savint
20 format (a32)
21 format (ai)
if (savint .eq. 'y') then
  print *, 'Name of save input file? (filename)'
  read 20, inpfie
  open(16, file=inpfie)
  runtype = -1 * runtype
  write(16,'(i2)') runtype
  runtype = -1 * runtype
  write(16,'(a47)') '* ACSIS input file two lines above are'
  write(16,'(a48)') 'analysis type and save input file. Do'
  write(16,'(a48)') 'not change them by editing this file.'
endif
runtype = runtype - 2

if (runfile .eq. 'y') then
   do 30 i = 1,4
        read 20, comment
   continue
endif

print *, 'Finite Element Model Input File Name (filename)'
read 20, femfile
open(11,file=femfile)
if (savin .eq. 'y') then
   write(16,'(a47)') ' Finite element input file?(filename)'
   write(16,'(a32)') femfile
endif
if (runtype) 100,200,300
C EIGENMODE INPUTS

100 print *, 'Number of modes desired:'
   if (runfile .eq. 'y') read 20, comment
   read *, neig
   if (runfile .eq. 'y') read 20, comment
   print *, 'Output File Name:'
   read 20, outfile
   open(13,file=outfile)
   if (savin .eq. 'y') then
      write(16,'(a35)') ' Number of modes desired?'
      write(16,'(a14)') neig
      write(16,'(a33)') ' Output file?(filename)'
      write(16,'(a32)') outfile
   endif

   call READFEM

   goto 1000

C CSI INPUTS

200 print *, 'Controller Definition File Name:'
   if (runfile .eq. 'y') read 20, comment
   read 20, confile
   open(12,file=confile)
   print *, 'Please input type of control:'
   print *
   print *, '  1. Full State Feedback'
   print *, '  2. Luenberger Observer (Li=0)'
   print *, '  3. Kalman Filter'
   print *
   if (runfile .eq. 'y') read 20, comment
   read *, contype
   contype = contype - 2
   print *, 'Initial time, final time, control-on time, step size:'
   if (runfile .eq. 'y') read 20, comment
   read *, t0,tf,tc,h
   print *, 'Forcing function ID, scale factor, damping coeff- a,b:'
   if (runfile .eq. 'y') read 20, comment
   read *, forceid,scalef,adamp,bdamp
   print *, 'Phase lag fix?(y or n):'
   if (runfile .eq. 'y') read 20, comment
read 2i, temp
if (temp .eq. 'y') nolag = .true.
if (temp .eq. 'n') nolag = .false.
print *, 'Gain scale factors (4 total):'
if (runfile .eq. 'y') read 20, comment
read *, qalpha, qbeta, qalphao, qbetao
if (savinv .eq. 'y') then
  write(16, '(a42)') '* Controller file name?(filename)
  write(16, '(a42)') confile
  write(16, '(a42)') '* Please input type of control: ' 
  write(16, '(i10)') contype + 2
  write(16, '(a60)') '* Initial, final, control-on, step size?'
  write(16, '(4f14.8)') t0, tf, tc, h
  write(16, '(a60)') '* Forcing function, scale f, damping a,b?'
  write(16, '(i4,f15.6,2f12.8)') forcoid, scalef, adamp, bdamp
  write(16, '(a60)') '* Phase lag fix?(y or n)?
  if (.not. nolag) write(16, '(a1)') 'y'
  endif
  endif
  call READFEM
  goto 999

C TRANSIENT RESPONSE INPUTS

300 print *, 'Initial time, final time, step size:'
if (runfile .eq. 'y') read 20, comment
read *, to, tf, h
print *, 'Forcing function ID, scale factor, damping coeff- a,b:'
if (runfile .eq. 'y') read 20, comment
read *, forcoid, scalef, adamp, bdamp
if (savinv .eq. 'y') then
  write(16, '(a48)') '* Initial, final, step size?'
  write(16, '(3f14.8)') t0, tf, h
  write(16, '(a49)') '* Forcing function, scale f, damping a,b?'
  write(16, '(i4,f15.6,2f12.8)') forcoid, scalef, adamp, bdamp
endif
  call READFEM
  goto 999

C OUTPUT OPTIONS

999 print *, 'Output File Name:'
if (runfile .eq. 'y') read 20, comment
read 20, outfile
open(13, file=outfile)
print *, 'Number of displacement results to output (max 10):'
if (runfile .eq. 'y') read 20, comment
read *, ndisout
do 500 i=1, ndisout
  print *, 'Input node #, dof for displacement output #:', i
  read, outlabel(i), outlabel(i+10)
  continue
print *, 'Number of velocity results to output (max 10):'
if (runfile .eq. 'y') read 20, comment
read *, nvelout
do 600 i = 1, nvelout
  print *, 'Input node * do for velocity output?', i
  read *, outlabel(i+20), outlabel(i+30)
600 continue
  print *, 'Send output every how many steps?'
if (runfile .eq. 'y') read 20, comment
read *, outskip
if (savin .eq. 'y') then
  write(16, '(a38)') 'Output file name?(filename)'
  write(16, '(a32)') outfile
  write(16, '(a42)') 'Number of displacement outputs?'
  write(16, '(i4)') ndisout
  do 650 i = 1, ndisout
    write(16, '(2i8)') outlabel(i), outlabel(i+10)
 650 continue
  write(16, '(a38)') 'Number of velocity outputs?'
  write(16, '(i4)') nvelout
  do 660 i = 1, nvelout
    write(16, '(2i8)') outlabel(i+20), outlabel(i+30)
 660 continue
  write(16, '(a38)') 'Send output every how many steps?'
  write(16, '(13)') outskip
endif
C ANIMATION OPTION
  print *, 'Animation Output? (y or n):'
if (runfile .eq. 'y') read 20, comment
read 21, temp
if (temp .eq. 'y') animate = .true.
if (temp .eq. 'n') animate = .false.
if (animate) then
  print *, 'Animation file name (filename)'
  if (runfile .eq. 'y') read 20, comment
  read 20, animfile
  open (15, file=animfile)
endif
if (savin .eq. 'y') then
  write(16, '(a41)') 'Send animation output?(y or n)'
  if (animate) write(16, '(a1)') 'y'
  if (.not. animate) write(16, '(a1)') 'n'
  if (animate) then
    write(16, '(a31)') 'Animation file name?'
    write(16, '(a32)') animfile
  endif
endif
delta = h/2.
delsq = delta**2
1000 return
end

File: pmvnum.f
Subroutine PMVMDL

Purpose:
This subroutine multiplies a matrix in vector form and a vector.

Arguments
a - matrix in vector form
b - vector
c - result vector
neq - order of vector and square matrix
jdiag - array of diagonal addresses for a

subroutine PMVMDL(a,jdiag,b,neq,c)

recursive subroutine  PMVMDL(a,jdiag,b,neq,c)

real*8 a(1), b(i), c(i)
integer jdiag(i), neq

do 100 i=1,neq
   c(i) = a(jdiag(i))*b(i)
100 continue

do 200 i=2,neq
   do 300 j=jdiag(i-1)+1,jdiag(i)-1
      k = jdiag(i) - j
      c(i) = c(i) + a(j)*b(i-k)
   300 continue
200 continue

do 250 i=2,neq
   do 400 j=jdiag(i-1)+1,jdiag(i)-1
      k = jdiag(i) - j
      c(i-k) = c(i-k) + a(j)*b(i)
400 continue
250 continue

return
end

Subroutine PMVHAD

Purpose:
Multiply a matrix in vector form and a vector and add the resultant vector multiplied by a constant to a second vector.
multiplied by a second vector

Arguments:
- a - matrix in vector form
- b - vector to be multiplied with matrix
- c - resultant vector to be added
- fact1 - constant multiplier of matrix and first vector
- fact2 - constant multiplier of second vector
- jdiag - array of diagonal addresses for matrix
- neq - order of vectors and matrix

subroutine PMVMAD(a,jdiag,b,neq,fact1,c,fact2)

recursive subroutine PMVMAD(a,jdiag,b,neq,fact1,c,fact2)

real*8 a(1), b(1), c(1), fact1, fact2
integer jdiag(1), neq

do 100 i=1,neq
   c(i) = fact2*c(i) + fact1*a(jdiag(i))*b(i)
100 continue

do 200 i=2,neq
   do 300 j=jdiag(i-1)+1,jdiag(i)-1
      k = jdiag(i) - j
      c(i) = c(i) + fact1*a(j)*b(i-k)
300 continue
200 continue

do 250 i=2,neq
   do 400 j=jdiag(i-1)+1,jdiag(i)-1
      k = jdiag(i) - j
      c(i-k) = c(i-k) + fact1*a(j)*b(i)
400 continue
250 continue

return
end

File: prepfem.f

Subroutine PREPFEN

Purpose:
This subroutine prepares the finite element mass,
stiffness, and S matrices in reduced profile vector form
C
C Local variables:
C
C ak Element Stiffness matrix
C am Element Mass Matrix
C lm Local/Global DOF Mapping vector
C nseq Number of element degrees of freedom
C em,ep Material and Property id $ for element
C
C subroutine PREPFEM

include 'shared.inc'
C LOCAL VARIABLES

parameter (MAISEQ=24)
real*8 ak(MAISEQ,MAISEQ),am(MAISEQ,MAISEQ),me,mc
integer lm(MAISEQ),nseq,em,ep

call RENUM
C Set up skyline storage profile for global matrices

call PROFILE(ix,id,jdiag,nmp,nel,4,6,mlen,ndof,mask)
C Perform automatic domain decomposition

call DOMDEC
C Check size of skyline profile against storage limitation

if (mlen .gt. MA1MLEH) then
  print*, 'PREPFEM: error, global matrix exceeded max. size'
endif
C Zero Global Matrices prior to assembly

call ZEROVECT(stif,mlen)
call ZEROVECT(mass,mlen)
C ASSEMBLE EACH ELEMENT MASS AND STIFFNESS

do 100 n=1,nel
  do 20 k=1,4
    j=ix(k,n)
    if (((etype(n).eq.1).and.(k.gt.2)) j = 0
    do 30 i=1,6
      kk=6*(k-1) + i
      if (j .ne. 0) then
        lm(kk) = id(i,j)
      else
        lm(kk) = 0
      endif
    30 continue
  20 continue

64
if (etype(n) .eq. 1) then
    nseq = 12
    em = emat(n)
    ep = eprop(n)
    call BEAM3D(n,ix(1,n),ix(2,n),ix(3,n),coxyz,mat(1,em),
        mat(2,em),mat(3,em),prop(i,ep),prop(5,ep),prop(6,ep),
        prop(2,ep),prop(3,ep),prop(4,ep),pin(1,n),sk,sm)
  elseif (ix(1,n) .ne. 0) then
    print*, 'PREPFEF:Element type not found,n=','etype=",
endif

C ADD ELEMENT TO GLOBAL MASS AND STIFFNESS

    call ADDSTF(sk,lm,stif,jdiag,nseq)
    call ADDSTF(em,lm,mass,jdiag,nseq)

C SAVE THE ELEMENT STIFFNESS FOR E-BY-E COMPUTATIONS

    call SAVESE(sk,n,nseq)

100 continue

C ADD LUMPED INERTIAS TO GLOBAL MASS:

    do 125 i=1,nmp
        do 130 j=1,6
            if (id(j,i) .eq. 0) goto 130
            k=diag(id(j,i))
            mass(k) = mass(k) + inertia(j,i)
        130 continue
    125 continue

C ASSEMBLE AND FACTORIZE es (S MATRIX)

    mc = 1. + delta*adamp
    kc = delta*b damp + delsq
    do 200 i=1,mnm
        es(i) = mc*mass(i) + kc*stif(i)
    200 continue
    call SOLVER(es,gs,jdiag,ndof,1)

C INITIALIZE DISPLACEMENT AND VELOCITY VECTORS

    do 300 i = 1, nmp
        do 350 j = 1,6
            if (id(j,i) .ne. 0) then
                q(id(j,i)) = q0(j,i)
                qdot(id(j,i)) = qdot0(j,i)
            endif
        350 continue
    300 continue

return
end

subroutine SAVESK(sk,n,nseq)

include 'shared.inc'
real*8 sk(nseq,1)
integer n,nseq

k=0
do 10 j=1,nseq
   do 20 i=1,j
      k=k+1
      esteem(k,n)=sk(i,j)
   20 continue
10 continue

return
end

subroutine DOMDEC

include 'shared.inc'

logical nchk,ndchk(MAXNODE,MAXDOM)
integer ndom

do 10 j=1,MAXDOM
   neld(j)=0
   do 20 i=1,nmp
      ndchk(i,j)=.false.
   20 continue
10 continue

ndomain=0

do 100 n=1,nel

   ndom=0
   nchk=0
   do 200 while (nchk.eq.0)
      ndom=ndom+1
      if (ndom.gt.ndomain). ndomain=ndom
      nchk=1
      if (ndchk(ix(1,n),ndom)). nchk=0
      if (ndchk(ix(2,n),ndom)). nchk=0
      if (nchk.eq.1) then
         neld(n)=ndom
         ndchk(ix(1,n),ndom)=.true.
         ndchk(ix(2,n),ndom)=.true.
      endif
   200 continue

   neld(ndom)=neld(ndom)+1
   elnum(neld(ndom),ndom)=n
100 continue

return
end

File: profile.f
subroutine PROFILE(ix, id, jdiag, nnp, nel, ndof, nad, neq, mask)

PURPOSE:

THIS SUBROUTINE COMPUTES THE NUMBER OF EQUATIONS REQUIRED TO SOLVE THE PROBLEM BY ELIMINATING RESTRAINED DEGREES OF FREEDOM FROM THE SYSTEM OF EQUATIONS. KNOWING THE EQUATION NUMBERS CORRESPONDING TO THE NODAL DEGREES OF FREEDOM, THE DIAGONAL ELEMENT LOCATIONS CAN BE COMPUTED FOR STORING THE GLOBAL STIFFNESS MATRIX IN COMPACTED VECTOR FORM.

ARGUMENTS

integer ix(nen), id(ndof), jdiag
integer nnp, nel, nad, neq, mask

LOCAL ARGUMENTS

integer i, j, k, l, m, n, ji, kl, li, ml

SET UP EQUATION NUMBERS

neq = 0
do 30 m = 1, nnp
   do 20 n = 1, ndof
      j = id(m, mask(n))
      if (j .eq. 1) goto 10
      neq = neq + 1
      id(m, mask(n)) = neq
      jdiag(neq) = 0
      goto 20
   10     id(m, mask(n)) = 0
   20 continue
30 continue

COMPUTE COLUMN HEIGHTS

do 80 n = 1, nel
   do 70 m = 1, nen
      ml = ix(m, n)
      if (ml .le. 0) goto 70
      do 60 l = 1, ndof
         li = id(l, ml)
         if (li .eq. 0) goto 60
         do 60 k = 1, nen
            kl = ix(k, n)
            if (kl .le. 0) goto 50
            do 40 j = 1, ndof
               jl = id(j, kl)
               if (jl .eq. 0) goto 40
C COMPOTE DIAGONAL FOOTERS

nad = 1
jdiag(i) = 1
if (aeq .eq. 1) return
do 90 n = 2, neq
   jdiag(n) = jdiag(n) + jdiag(n-1) + 1
90 continue
nad = jdiag(neq)
C return
end
C=End Fortran

File: read.f

C=Module READ
C=Author K. Alvin
C=Date May 1990
C=Block Fortran
C-------------------------------------
C
C Subroutine READFEM
C
C Purpose:
   This subroutine reads the data file for the finite element model.
C-------------------------------------
C
C Arguments
C  ctype - stores code for type of line
C
C subroutine READFEM
C
C GLOBALS
C
include 'shared.inc'
C
C LOCALS
C
integer j,n,ctype,GETTYPE
character*132 aline
real*8 in
C
C INITIALIZE SIZE OF PROBLEM
C IDENTIFY CARD TYPE AND ASSIGN INPUT

10  read(i1,1000,,end=9999) aline
100 ctype = GETTYPE(aline)
   if (ctype) 10,10,150
150  if (aline(1:4) .eq. 'NODE') goto 200
   if (aline(1:4) .eq. 'TOPO') goto 300
   if (aline(1:4) .eq. 'ATTR') goto 400
   if (aline(1:4) .eq. 'MATE') goto 500
   if (aline(1:4) .eq. 'PROP') goto 600
   if (aline(1:4) .eq. 'FIXI') goto 700
   if (aline(1:4) .eq. 'INIT') goto 800
   if (aline(1:4) .eq. 'FERN') goto 900
   if (aline(1:4) .eq. 'END') goto 10
   if (aline(1:4) .eq. 'HOT') goto 10
   print *, 'READPROC: Unrecognized card type; ',aline(1:4)
   goto 10
C READ NODES

200 read(i1,1000,endpoint=9999) aline
200 ctype = GETTYPE(aline)
   if (ctype) 200,250,100
250  read(aline,*), n, (coxyz(j,n),j=1,3)
   if (n .gt. nnp) nnp = n
   goto 200
C READ TOPOLOGY

300 read(i1,1000,endpoint=9999) aline
300 ctype = GETTYPE(aline)
   if (ctype) 300,350,100
350  read(aline,*), n, etype(n), (ix(j,n),j=1,4)
   if (n .gt. nel) nel = n
   goto 300
C READ ATTRIBUTES

400 read(i1,1000,endpoint=9999) aline
400 ctype = GETTYPE(aline)
   if (ctype) 400,460,100
450  read(aline,*), n,emat(n), eprop(n), (pin(j,n),j=1,6)
   if (edom(n) .gt. ndomain) ndomain = edom(n)
   goto 400
C READ MATERIAL

500 read(i1,1000,endpoint=9999) aline
500 ctype = GETTYPE(aline)
   if (ctype) 500,550,100
550  read(aline,*), n, (mat(j,n),j=1,3)
   goto 500
C READ PROPERTIES
read(11,1000,end=9999) aLINE
CTYPE = GETTYPE(ALINE)
IF (CTYPE) 600,850,100
650 read(ALINE,*) n,(PROP(J,N),J=1,6)
goto 600
C READ FIXITY
700 read(11,1000,end=9999) aLINE
CTYPE = GETTYPE(ALINE)
IF (CTYPE) 700,750,100
750 read(ALINE,*) n,(ID(J,N),J=1,6)
goto 700
C READ INITIAL CONDITIONS
800 read(11,1000,end=9999) aLINE
CTYPE = GETTYPE(ALINE)
IF (CTYPE) 800,850,100
850 read(ALINE,*) n,j,q0(j,n),qDOT0(j,n)
goto 800
C READ INERTIA
900 read(11,1000,end=9999) aLINE
CTYPE = GETTYPE(ALINE)
IF (CTYPE) 900,950,100
950 read(ALINE,*) n,j,inertia(j,n)=inertia(j,n)*IN
GOTO 900
1000 FORMAT(A132)
9999 CONTINUE
RETURN
END

Subroutine READCON

Purpose:
This subroutine reads the actuator and sensor locations and the gains for the control system

Arguments
CTYPE - stores code for type of line

Abbreviations
NACT - number of actuators
NSEN - number of sensors
BMAT - locations of actuators
HDMA - array of displacement sensor locations
HVMA - array of velocity sensor locations
FIGA - control gain matrix
C F2GA - control gain matrix
C L1GA - state estimator filter gain matrix
C L2GA - state estimator filter gain matrix

subroutine READCON
include 'shared.inc'

C LOCALS
real*8 val
integer j,n,ctype,GETTYPE
character*132 aline

bval = 0
hdval = 0
hval = 0
hdbval = 0

C IDENTIFY CARD TYPE AND ASSIGN INPUT
10 read(12,1000,end=9999) aline
100 ctype = GETTYPE(aline)
   if (ctype) 10,10,150
150 if (aline(1:4) .eq. 'NACT') goto 200
   if (aline(1:4) .eq. 'NSEN') goto 300
   if (aline(1:4) .eq. 'BNAT') goto 400
   if (aline(1:4) .eq. 'HDMA') goto 500
   if (aline(1:4) .eq. 'HVMA') goto 600
   if (aline(1:4) .eq. 'F1GA') goto 700
   if (aline(1:4) .eq. 'F2GA') goto 800
   if (aline(1:4) .eq. 'L2GA') goto 900
   if (aline(1:4) .eq. 'L1GA') goto 1100
   if (aline(1:4) .eq. 'END ') goto 10
   print *,'READCON: Unrecognized card type; ',aline(1:4)
goto 10

C READ INPUT CARDS
200 read(12,1000,end=9999) aline
250 read(aline,*) nact
   goto 200
300 read(12,1000,end=9999) aline
350 read(aline,* ) nsen
   goto 300
400 read(12,1000,end=9999) aline
450 read(aline,* ) i,j,n,val
   bval = bval + 1
   b(bval) = val
   brow(bval) = id(j,i)
bcol(bval) = n
goto 400

500 read(12,1000, end=9999) aline
ctype = GETTYPEn(aline)
if (ctype) 500,550,100

550 read(aline,*) i,j,n,val
hdval = hdval + 1
hd(hdval) = val
hdrow(hdval) = n
hdcol(hdval) = id(j,i)
goto 500

600 read(12,1000, end=9999) aline
ctype = GETTYPEn(aline)
if (ctype) 600,650,100

650 read(aline,*) i,j,n,val
hvval = hvval + 1
hv(hvval) = val
hvrow(hvval) = n
hvcol(hvval) = id(j,i)
goto 600

700 read(12,1000, end=9999) aline
ctype = GETTYPEn(aline)
if (ctype) 700,750,100

750 read(aline,*) i,j,n,val
f1(a, id(j,i)) = qalpha=val

goto 700

800 read(12,1000, end=9999) aline
ctype = GETTYPEn(aline)
if (ctype) 800,850,100

850 read(aline,*) i,j,n,val
f2(a, id(j,i)) = qbeta=val

goto 800

900 read(12,1000, end=9999) aline
ctype = GETTYPEn(aline)
if (ctype) 900,950,100

950 read(aline,*) i,j,n,val
l2(id(j,i),n) = qbetao=val

goto 900

1100 read(12,1000, end=9999) aline
ctype = GETTYPEn(aline)
if (ctype) 1100,1150,100

1150 read(aline,*) i,j,n,val
l1(id(j,i),n) = qalphao=val

goto 1100

1000 format(a132)
9999 continue
return
end

C
C
C

72
Function GETTYPE

Purpose:
This function identifies whether a line is a character input, data input or comment.

```
function GETTYPE(string)

GLOBALs
character*132 string

LOCALs
integer GETTYPE,ctype(10)
character*1 head(10)

data head /!'.'@'.'$'.'#'.'&'.'*','.'c'.' '/
data ctype /-1,-1,-1,-1,-1,-1,-1,-1,-1,0/

LOGIC
GETTYPE=1
do 100 i=1,10
  if (string(i:i) .eq. head(i)) GETTYPE=ctype(i)
100  continue

return
end
```

File: solver.f

C=Module SOLVER
C=Purpose Solves the system of linear symmetric equations
C=Author who knows
C=Date
C=Update January 1989 by E. Pramono
C=Block Fortran

```c
SUBROUTINE SOLVER(BK,BR,JDIAG,NEQ,IFLAG)

recursive SUBROUTINE SOLVER(BK,BR,JDIAG,NEQ,IFLAG)
```

C= PURPOSE:

C= THIS SUBROUTINE SOLVES THE SYSTEM OF LINEAR SYMMETRIC

C= EQUATIONS IN VECTOR FORM USING THE CROUT REDUCTION

C= METHOD.

C= ARGUMENTS:

C= BK   - GLOBAL STIFFNESS EQUATIONS IN VECTOR FORM
C= BR   - GLOBAL LOAD VECTOR
C= JDIAG - LOCATION VECTOR FOR DIAGONALS IN [BK]
C= NEQ  - NUMBER OF EQUATIONS
C= IFLAG - FLAG INDICATING WHICH FUNCTION IS TO BE PERFORMED
C= 1  -> FORWARD REDUCTION
C= 2  -> BACKWARD SUBSTITUTION
```

73
C ARGUMENTS

REAL*8 BK(1), BR(1)
INTEGER JDIAG(1), NEQ, IFLAG

C LOCAL VARIABLES

REAL*8 ZERO, EZERO, TOL, DAVAL, DOT, D, RDD, DD
INTEGER LDFLAG, JR, J, JD, JH, IS, IE, K, JDT
INTEGER JJ, ID, I, IR, IH

JJ = 6

C C C C C C C C

C NEW PARAMETERS

C C C C

ZERO = 0.0D0
EZERO = 0.3D-14
TOL = 0.5D-7
LDFLAG = 0

C C

C FACTOR BK TO UT=D=U OR REDUCE R

C JR = 0
DO 70 J = 1, NEQ
 JD = JDIAG(J)
 JH = JD - JR
 IS = J - JH + 2
 IF (JH = 2) 60, 30, 10

10 IF (IFLAG .NE. 1) GOTO 50
 IE = J - 1
 K = JR + 2
 ID = JDIAG(IS-1)

C IF DIAGONAL IS ZERO COMPUTE A NORM FOR SINGULARITY TEST

C JDT = JDIAG(IE) + 1
 IF (BK(JD) .EQ. ZERO .AND. IFLAG .EQ. 1) THEN
 CALL DATEST (BK(JDT), JH-2, DAVAL)
 END IF

C REDUCE ALL EQUATIONS EXCEPT FIRST ROW AND DIAGONAL

DO 20 I = IS, IE
 IR = ID
 ID = JDIAG(I)
 IH = MIN0(ID-IR-1, I-IS+1)
 IF (IH .GT. 0) BK(K) = BK(K) - DOT(BK(K-IH), BK(ID-IH), IH)
 K = K + 1
20 CONTINUE

C REDUCE FIRST ROW AND DIAGONAL
c
30 IF (IFLAG .NE. 1) GOTO 50
IR = IR + 1
IE = JD - 1
K = J - JD
DD = BK(JD)
DO 40 I = IR, IE
   ID = JDIAG(K+I)
   IF (BK(ID) .EQ. 0.0) GOTO 40
   D = BK(I)
   BK(I) = BK(I)/BK(ID)
   BK(JD) = BK(JD) - D*BK(I)
40 CONTINUE
C
C CHECK FOR POSSIBLE ERRORS AND PRINT WARNINGS
C
   RDD = BK(JD)
   IF (DABS(RDD) .LT. TOL*DABS(DD)) WRITE (JJ,2000) J
   IF (DD .LT. ZERO .AND. RDD .GT. ZERO) WRITE (JJ,2001) J
   IF (DD .GT. ZERO .AND. RDD .LT. ZERO) WRITE (JJ,2001) J
   IF (DABS(RDD) .LT. EZERO) WRITE (JJ,2002) J
C
C COMPLETE RANK TEST FOR A ZERO DIAGONAL TEST
C
   IF (DD .EQ. ZERO .AND. JH .GT. 0) THEN
      IF (DABS(RDD) .LT. TOL*DAVAL) WRITE (JJ,2003) J
   END IF
C
C REDUCE RIGHT HAND SIDE
C
50 IF (IFLAG .EQ. 2) BR(J) = BR(J) - DOT(BK(JR*1), BR(IS-1), JH-1)
60 JR = JD
70 CONTINUE
C
C CHECK FOR ZERO LOAD VECTOR
C
   IF (LDFLAG .EQ. 0) WRITE(JJ,2004)
C
C BACK SUBSTITUTE
C
   J = NEQ
   JD = JDIAG(J)
90 D = BR(J)
   J = J - 1
IF (J .LE. 0) RETURN
JR = JDIAG(J)
IF (JD - JR .LE. 1) GOTO 110
IS = J - JD + JR + 2
K = JR - IS + 1
DO 100 I = IS, J
   BR(I) = BR(I) - BK(I+K)*D
100 CONTINUE
110 JD = JR
GOTO 90
C C ---------------
C WARNING FORMATS
C ---------------
C 2000 FORMAT('!! WARNING !! 1 - IN SOLVER, LOSS OF AT LEAST 7 DIGITS'
   + /18X, 'IN REDUCING DIAGONAL OF EQUATION:',4X,15)
C 2001 FORMAT('!! WARNING !! 2 - IN SOLVER, SIGN OF DIAGONAL CHANGED'
   + /18X, 'WHEN REDUCING EQUATION:',15X,15)
C 2002 FORMAT('!! WARNING !! 3 - IN SOLVER, REDUCED DIAGONAL IS ZERO'
   + /18X, 'FOR EQUATION:',2SX,15)
C 2003 FORMAT('!! WARNING !! 4 - IN SOLVER, RANK FAILURE FOR A ZERO'
   + /18X, 'UNREDUCED DIAGONAL IN EQUATION:',7X,15)
C 2004 FORMAT('!! WARNING !! 5 - IN SOLVER, ZERO LOAD VECTOR')
C END
C=End Fortran
C=Module DATEST
C=Block Fortran
C SUBROUTINE DATEST(A,JH,DAVAL)
  C LOCAL SUBROUTINE DATEST(A,JH,DAVAL)
  C SUBROUTINE DATEST(A,JH,DAVAL)
C+-----------------------------------------------
C C TEST FOR RANK
C
C C INPUTS:
C A(J) - COLUMN OF UNREDUCED ELEMENTS IN ARRAY
C JH - NUMBER. OF ELEMENTS IN COLUMN
C
C C OUTPUTS:
C DAVAL - SUM OF ABSOLUTE VALUES
C
C C ARGUMENTS
C
C REAL*8 A(1), DAVAL
C INTEGER JH
C
C LOCAL ARGUMENTS
C
C INTEGER J
C
C DAVAL = 0.00D
C DO 10 J = 1, JH
   DAVAL = DAVAL + DABS(A(J))
10 CONTINUE
C
C RETURN
C END

76
SUBROUTINE HOPHLAG(ZP)

REAL*8 ZP(L)

INCLUDE 'SHARED.INC'

SUBROUTINE HOPHLAG(ZP)

REAL*8 ZP(L)

INCLUDE 'SHARED.INC'

INTEGER N

INTEGER I

DOT = 0.0

DO 10 I = 1, N

DOT = DOT + A(I)*B(I)

10 CONTINUE

RETURN

END
LOCAL VARIABLES

integer i
real*8 v(MAXDOF), delbeta

LOGIC
ADD APPLIED FORCES TO RHS AND PREPARE MASS MULTIPLIER

do 10 i=1,ndof
   gs(i) = gs(i) + f(i)
   v(i) = (1. + delta*adamp)*q(i) + delta*qdot(i)
10 continue

SOLVE FOR RIGHT HAND SIDE, gs

do 77 i = 1,ndof
   gs(i) = delsq*gs(i) + v(i)*mass(jdiag(i))
77 continue

if (bdamp .ne. 0.) then
   delbeta = delta*bdamp
   call PHNMAD(stif,jdiag,q,ndof,delbeta,gs,i,do)
endif

SOLVE FOR DISPLACEMENT, q, USING RHS AND MATRIX S

call SOLVER(es,gs,jdiag,ndof,2)

do 100 i=1,ndof
   v(i) = (gs(i)-q(i))/delta
100 continue

call ZEROVECT(zp,nsen)

do 200 jj = 1,hdval
   i = hdrow(jj)
   j = hdcol(jj)
   zp(i) = zp(i) + hd(jj)*gs(j)
200 continue

do 250 jj = 1,hvval
   i = hvrow(jj)
   j = hvcol(jj)
   zp(i) = zp(i) + hv(jj)*v(i)
250 continue

return
end

File: zerovect.f

C=Module ZEROVECT
C=Purpose Initialize vector of given length to zero
C=Author K. Alvin
C=Date May 1990
C=-----------------------------------------------

78
Subroutine ZEROVECT

subroutine ZEROVECT(v,n)

real*8 v(i)
integer n

do 100 i=1,n
   v(i) = 0.d0
100 continue

return
end

File: lu.f

SUBROUTINE LUFACT(A,N,PIVOT,DET,IER,NMAX)

SUBROUTINE FACTOR USES GAUSSIAN ELIMINATION WITH
PARTIAL PIVOTING AND IMPLICIT SCALING TO DETERMINE
THE L=U DECOMPOSITION OF A SQUARE MATRIX "A" OF
ORDER N. THE ALGORITHM ALSO FINDS THE DETERMINANT
OF "A". UPON COMPLETION, THE ELEMENTS OF THE UPPER
TRIANGULAR MATRIX "U" ARE CONTAINED IN THEIR RESPECTIVE
LOCATIONS IN MATRIX "A". THE ELEMENTS OF MATRIX "L"
ARE CONTAINED IN THE LOWER TRIANGULAR PORTION OF "A",
BUT ARE SCRAMBLED WITH RESPECT TO "U" BECAUSE OF ROW
INTERCHANGE OPERATIONS NOT PERFORMED ON THE ELEMENTS
OF "L". THE VECTOR PIVOT (SEE BELOW) MUST BE USED TO
UNSCRAMBLE "L" IF IT IS TO BE USED FOR OTHER OPERATIONS.

VARIABLES: A=FULL SQUARE MATRIX (DOUBLE PRECISION)
N=ORDER OF MATRIX A (INTEGER)
PIVOT=VECTOR CONTAINING A RECORD OF
ROW INTERCHANGES. THE INTEGER
VALUE PIVOT(K) IS THE ROW WHICH
WAS INTERCHANGED WITH ROW K AT
FORWARD ELIMINATION STEP K. (INTEGER)
DET=DETERMINANT OF MATRIX A (DOUBLE PRECISION)
IER=ERROR FLAG. IF IER=1, THE MATRIX A WAS FOUND
TO BE SINGULAR, AND THE ROUTINE WAS EXITED. IF
IER=0, THE DECOMPOSITION WAS SUCCESSFUL.

INTEGER PIVOT(1),IER,N,I,J,K,IO,NMAX
REAL*8 A(NMAX,1),S(1000),C(1000),DET,TEMP
DET=1.000

FIND THE ROW NORMALIZING COEFFICIENTS S(I) FOR IMPLICIT SCALING.
EXIT ROUTINE IF ANY S(I)=0.0

79
DO 100 I=1,N
S(I)=0.000
DO 110 J=1,N
IF (ABS(A(I,J)).GT.S(I)) S(I)=ABS(A(I,J))
110 CONTINUE
IF (S(I).EQ.0) THEN
IER=1
DET=0.000
RETURN
END IF
100 CONTINUE

C**************************************************************************
C(K)=0.000
DO 130 I=K,N
TEMP=ABS(A(I,K)/S(I))
IF (TEMP.GT.C(K)) THEN
C(K)=TEMP
IO=I
END IF
130 CONTINUE
PIVOT(K)=IO

C**************************************************************************
C EXIT ROUTINE IF ALL VALUES IN COLUMN K AT OR BELOW
C THE MAIN DIAGONAL ARE EQUAL TO 0.0
C
IF (C(K).EQ.0.0) THEN
IER=1
DET=0.000
RETURN
END IF

C**************************************************************************
C INTERCHANGE ROWS IO AND K FOR COLUMNS K TO N. SKIP IF IO=K.
C SET DET=-DET IF ROWS ARE INTERCHANGED.
C
IF (IO.EQ.K) GOTO 150
DET=-1.000=DET
DO 140 J=K,N
TEMP=A(K,J)
A(K,J)=A(IO,J)
A(IO,J)=TEMP
140 CONTINUE
ELIMINATE COLUMN K BELOW MAIN DIAGONAL BY MULTIPLYING
ROW K FROM COLUMN K TO N BY A(I,K)/A(K,K) AND SUBTRACTING
FROM ROW I. STORE THE MULTIPLIER FOR ROW I IN THE ELIMINATED
COLUMN K. MULTIPLY THE RUNNING PRODUCT DET BY DIAGONAL ELEMENT A(K,K).

DO 160 I=K+1,N
   A(I,K) = A(I,K)/A(K,K)
   DO 170 J=K+1,N
      A(I,J) = A(I,J) - A(I,K) * A(K,J)
   170 CONTINUE
160 CONTINUE

DET = DET * A(K,K)
120 CONTINUE

CHECK LAST ROW/COLUMN FOR SINGULARITY. IF THERE IS NO ERROR,
COMPLETE CALCULATION OF THE DETERMINANT, SET THE ERROR FLAG
TO INDICATE NORMAL COMPLETION, AND EXIT.

IF (A(N,N) .EQ. 0.0) THEN
   IER = 1
   DET = 0.0DO
   RETURN
END IF
DET = DET - A(N,N)
IER = 0
RETURN
END

SUBROUTINE LUSOLVE(A,N,B,PIVOT,NMAX)

INTEGER PIVOT(1),N,I,J,K,NMAX
REAL*8 A(NMAX,1),B(1),TEMP
DO 100 K=1,N-1
   IF (PIVOT(K) .EQ. K) GOTO 110
   TEMP = B(K)
   B(K) = B(J)
   B(J) = TEMP
110 DO 120 I=K+1,N
   B(I) = B(I) - A(I,K) * B(K)
120 CONTINUE
100 CONTINUE

B(N) = B(N)/A(N,N)
DO 130 I=N-1,-1,-1
   DO 140 J=I+1,N
      B(J) = B(J) - A(I,J) * B(I)
140 CONTINUE
   B(I) = B(I)/A(I,I)
130 CONTINUE
RETURN
END
C=END FORTRAN
C=DECK FACTA
C= PURPOSE - Factors the A matrix as L U = A, with partial pivoting
C=AUTHOR W K BELVIN, Sept. 24, 1987
C

---

Input
amat---[n I n] matrix to be factored, destroyed on output
np------problem size

Output
amat-----contains the LU decomposition

---

subroutine FACTA amat, np, nrow, lp

real*8 amat(*), eta
integer lp(*)

50 lp(i)=i

Find largest pivot.

do 200 m=k, np
   if (abs(amat((lp(m)+(k-1)*nrow)) .gt. amax) then
      amax=abs(amat((lp(m)+(k-1)*nrow))
      mmax=m
   endif

200 continue

l=lp(k)
lp(k)=lp(mmax)
lp(mmax)=1

do 400 i=k+1, np
   eta=amat(lp(i)+(k-1)*nrow)/amat(lp(k)+(k-1)*nrow)
   amat(lp(i)+(k-1)*nrow)=eta
   amat(lp(i)+(k-1)*nrow)=amat(lp(i)+(j-1)*nrow)-
      eta*amat(lp(k)+(j-1)*nrow)

400 continue

C END FORTRAN
C=DECK LUSOLV
C= PURPOSE - Solve L U x = b,
C=AUTHOR W K BELVIN, Sept. 24, 1987
C
C
First solves $L y = b$, then $U x = y$

Input:
- $amat$ - [n x n] matrix factored by FACTA
- $np$ - problem size
- $lp$ - pointer vector based on pivoting
- $rhs$ - RHS of equation

Output:
- $amat$ - contains the LU decomposition
- $x$ - the solution vector

```
subroutine LUSOLV amat(*) x(*) rhs(*)
   real*8 amat(*), x(*), rhs(*)
   integer lp(*)
   do 50 i=1, np
      x(i)=rhs(i)
   50 continue
   ! Solve Lower System-----------------------------------------------
   ! Outer loop
   do 100 k=1, np
      rhs(k)=x(lp(k))
      if (rhs(k) .eq. 0.) go to 100
   100 continue
   ! Inner loop
   do 200 i=k+1, np
      x(lp(i))=x(lp(i))-amat(lp(i)+(k-1)*nrow)*rhs(k)
   200 continue
   100 continue
   ! Solve Upper System-----------------------------------------------
   !
   do 300 k=np, 1, -1
      x(k)=rhs(k)/amat(lp(k)+(k-1)*nrow)
      do 400 i=1, k-1
         rhs(i)=rhs(i)-x(k)*amat(lp(i)+(k-1)*nrow)
      400 continue
   300 continue
   return
end
```

File: prepcon.f

C=Module PREPCON
C=Purpose Preprocess control analysis module for ACSIS
C=Author K. Alvin
C=Date June 1990
C=Block Fortran
C
C Subroutine PREPCON

83
subroutine PREPCON

include 'shared.inc'

C LOCAL VARIABLES

real*8 mc,kc
integer ier

C LOGIC

call READCON

C Form Control Prediction Integration Coefficient Matrix

C
C contype = -1 : Full State Feedback
C contype = 0 : Luenberger Observer
C contype = +1 : Kalman Filter

do 10 i = 1,nact + nsen
    do 20 j = 1,nact + nsen
        ec(i,j) = 0.0
    20 continue
10 continue

if (contype) 100,200,400

100 ncsi = nact

    do 110 i = 1,nact
        do 120 jj = 1,bval
            j = bcol(jj)
            k = brow(jj)
            ec(i,j) = ec(i,j) + delta*f2(i,k)*b(jj)/mass(jdiag(k))
        120 continue
    110 continue

goto 600

200 ncsi = nact + nsen

    do 210 i = 1,nact
        do 220 jj = 1,bval
            j = bcol(jj)
            k = brow(jj)
            ec(i,j) = ec(i,j) + delta*f2(i,k)*b(jj)/mass(jdiag(k))
        220 continue
    210 continue

    do 240 j = nact+1,ncsi
        do 250 k = 1,nof
            ec(i,j) = ec(i,j) + delta*f2(i,k)*l2(k,j-nact)
        250 continue
240 continue

84
250     continue
240     continue
210     continue
260     do ii = 1, hvval
240       i = nact + hvrow(ii)
260       do jj = 1, bval
210         j = bcol(jj)
240         k = brow(jj)
210           if (hvcol(ii) .ne. k) goto 270
240             ec(i,j) = ec(i,j) + delta*hv(ii)*b(jj)/mass(jdiag(k))
270     continue
280     do jj = aact+1, ncsi
210       k = hvcol(ii)
240       ec(i,j) = ec(i,j) + delta*hv(ii)*l2(k,j-nact)
290     continue
280     continue

400     ncsi = nact + nsen
470     do i = 1, nact
480       do jj = 1, bval
470         j = bcol(jj)
480         k = brow(jj)
470           ec(i,j) = ec(i,j) + delta*f2(i,k)*b(jj)/mass(jdiag(k))
480       continue
490     do k = 1, ndof
500       do jj = aact+1, ncsi
510         k = hvcol(ii)
500           ec(i,j) = ec(i,j) + delta*fv(i,k)*l2(k,j-nact)/mass(jdiag(k))
510       continue
520     continue
530     do ii = 1, hvval
540       i = nact + hvrow(ii)
530       do jj = 1, bval
540         j = bcol(jj)
530         k = brow(jj)
540           if (hvcol(ii) .ne. k) goto 530
530             ec(i,j) = ec(i,j) + delta*hv(ii)*b(jj)/mass(jdiag(k))
550     continue
560     do jj = aact+1, ncsi
570       k = hvcol(ii)
560         ec(i,j) = ec(i,j) + delta*hv(ii)*l2(k,j-nact)/mass(jdiag(k))
550     continue
580     continue

600     continue

610     do i = 1, ncsi
620       ec(i,i) = ec(i,i) + 1.0d0
610     continue
620     continue

C FACTORIZATION ec
299
85

85

85
if (ier .eq. 1) then
  print *, 'PREPCON: Singular Matrix for Control Integration'
endif

C Form Observer Integration Coefficient Matrix, eo

mc = 1. * delta=adamp
kc = delta=bdamp + delsq
do 1200 i=1,mlen
    eo(i) = mc*mass(i) + kc*stif(i)
1200 continue

C FACTORIZE eo

call SOLVER(eo,go,jdiag,ndof,1)

C Initialize Observer States

call ZEROVECT(qe,ndof)
call ZEROVECT(qidot,ndof)
call ZEROVECT(ps,ndof)

return
end

File: control.f

C=Module CONTROL
C=Author K. Alvin
C=Date May 1990
C=Block Fortran

C Subroutine CONTROL

C Purpose:
This subroutine carries out the numeric integration of one time
step of the control system.

C Arguments.
qe - estimated displacement vector at half time step
qidot - estimated velocity vector at half time step
pp - generalized momentum (f-D*qidot-K*qep)
z - measured sensor output

subroutine CONTROL(z)
include 'shared.inc'
real*8 z(1)

C LOCAL VARIABLES

real*8 qe(MAIDOF),qidot(MAIDOF),pp(MAIDOF),z(MAIDOF)
LOGIC

Form RHS of Control Prediction Equation Set

contype = -1 : Full State Feedback
contype = 0 : Luenberger Observer with Li = 0
contype = +1 : Kalman Filter w/generalized momentum variable

call ZEROVECT(gc,n_csi)

if (contype) 100,200,300

100 continue

do 110 i = 1,ndof
   q_e(i) = q(i)
   qedot(i) = qdot(i)
110 continue

200 continue

do 210 i = 1,ndof
   qep(i) = qe(i) + delta*qedot(i)
   qedotp(i) = qedot(i)
   pp(i) = f(i) - mass(jdiag(i))*adamp*qedotp(i)
   v(i) = qep(i) + bdamp*qedotp(i)
210 continue

call PHVMAD(stif,jdiag,v,ndof,-1.d0,pp,1.d0)

do 220 i=1,nact
   do 230 j = 1,ndof
      gc(i) = gc(i) - a(i,j)*qep(j) - f2(i,j)*(qedot(j) +
             delta*pp(j)/mass(jdiag(j))
   230 continue
220 continue

if (n_csi .eq. nact) goto 600

do 240 i=nact+1,n_csi
   k = i - nact
   gc(i) = z(k)
240 continue

do 245 ii = 1,hdval
   i = hdrow(ii) + nact
   j = hdcol(ii)
   gc(i) = gc(i) - hd(ii)*qep(j)
245 continue

do 250 ii = 1,hvval
   i = hvrow(ii) + nact
   j = hvcol(ii)
   gc(i) = gc(i) - hv(ii)*(qedot(j) + delta*pp(j)/mass(jdiag(j)))
250 continue

600 continue

300 continue

do 310 i = 1,ndof
qep(i) = qe(i)
pp(i) = \( f(i) - \text{mass}(j\text{diag}(i))\text{adamp}\cdot qe(i)/\text{delta} \)
v(i) = (1 + \text{bdamp}/\text{delta})\cdot qe(i)

continue

call PHNAD(stif,jdiag,v,n dof,-1.d0,pp,1.d0)

do 320 i=1,nact
   do 330 j = 1,n dof
      gc(i) = gc(i) - f1(i,j)\cdot qe(p(j)) - f2(i,j)\cdot (pe(j) +
      \text{delta}\cdot pp(j))/\text{mass}(j\text{diag}(j))
   continue
330 continue
320 continue

do 340 i=nact+1,n csi
   k = i - nact
   gc(i) = z(k)
340 continue

do 350 ii = 1,hdval
   i = hdrow(ii) + n act
   j = hdcol(ii)
   gc(i) = gc(i) - hd(ii)\cdot qe(j)
350 continue

do 360 ii = 1,hvval
   i = hvrow(ii) + n act
   j = hvcol(ii)
   gc(i) = gc(i) - \text{hv}(ii)\cdot (pe(j) + \text{delta}\cdot pp(j))/\text{mass}(j\text{diag}(j))
360 continue

C FIND r, CONTROL AND STATE CORRECTION FORCES

600 call LUSOLV(ec,ncsi,MAXCSI,pivot,gc,r)

do 610 j=1,nact
   u(j) = r(j)
610 continue

C FIND CONTROL CONTRIBUTION TO RHS VECTOR FOR OBSERVER AND STRUCTRE

710 i=1,n dof
   gs(i) = 0.d0
   go(i) = 0.d0
   gk(i) = 0.d0
710 continue

720 jj=1,bval
   i = brow(jj)
   j = bcol(jj)
   gs(i) = gs(i) + b(jj)\cdot u(j)
   go(i) = go(i) + b(jj)\cdot u(j)
   gk(i) = gk(i) + b(jj)\cdot u(j)
720 continue

if (contype .eq. 0) then
   do 725 i = 1,n dof
      do 730 j=1,nsen
         go(i) = go(i) + \text{mass}(j\text{diag}(i))\cdot \text{12}(i,j)\cdot \text{gamma}(j)
570 continue
725 continue
730 continue

88


File: secorder.f

C=Module SECORDER
C=Author K. Alvin
C=Date May 1990
C=Block Fortran

C Subroutine SECORDER

C Purpose:
C
C Solves the second-order dynamical equation:
C
C \( Mx'' + Dx' + Kx = f + g \)
C
C at time (n+1) given \( f(n+1/2) \), \( g(n+1/2) \) and \( x, x' \) at \( n \) by
C the midpoint implicit integration rule. The step size is \( 2\delta \).
C
D is of the form \( (\alpha M + \beta K) \), \( f \) is an applied force,
C and \( g \) is assumed to be other applied force from a feedback
C control loop. The matrix \( E \) is the factored form of the
C integration coefficient matrix: \( E = (M + \delta E + \delta^2 K) \).

C Arguments:
C
C m - matrix M
C k - matrix K
C alpha - scalar alpha
C beta - scalar beta
C f - Force vector \( f(n+1/2) \)
C g - Feedback force vector \( g(n+1/2) \)
C e - matrix E
C x - Variable vector \( x(n) \)
C xd - Variable vector \( x'(n) \)
C delta - Half of integration time step
C delsq - \( \delta^2 \)
C jdiag - Diagonal location pointer for \( M, K, E \) matrices
C ndof - Number of equations and length of \( q \)
C v - mass multiplier for RHS preparation
C delbeta - delta * beta

C subroutine SECORDER(m,k,alpha,beta,f,g,e,x,xd,
C . delta,delsq,jdiag,ndof,MAXDOF)

recursive subroutine SECORDER(m,k,alpha,beta,f,g,e,x,xd,
C . delta,delsq,jdiag,ndof,MAXDOF)

C ARGUMENTS
real*8 m(i),k(i),alpha,beta,f(i),g(i),e(i)
real*8 x(i),xd(i),delta,delsq
integer jdiag(i),ndof,MAXDOF

C LOCAL VARIABLES

integer i
real*8 v(3000),delbeta

C LOGIC
C ADD APPLIED FORCES TO RHS AND PREPARE MASS MULTIPLIER

do 10 i=1,ndof
   g(i) = g(i) + f(i)
   v(i) = (1. + delta=alpha)*x(i) + delta*xd(i)
10 continue

C SOLVE FOR RIGHT HAND SIDE, g

do 77 i=1,ndof
   g(i) = delsq*g(i) + v(i)*m(jdiag(i))
77 continue

if (beta .ne. 0.) then
   -delbeta = delta*beta
C Activate EBE computations for internal force by using STIFFRC
C subroutine. Otherwise use PHVMAD (profile matrix/vector mult-add
C
call PHVMAD(x,jdiag,xd,ndof,delbeta,g,1.d0)
call STIFFRC(x,delbeta,g)
endif

C SOLVE FOR DISPLACEMENT, q, USING RHS AND MATRIX E

call SOLVER(e,g,jdiag,ndof,2)

do 100 i=1,ndof
   xd(i) = 2.*((g(i) - x(i))/delta - xd(i)
   x(i) = 2.*g(i) - x(i)
100 continue

return
end

File: measure.f
Subroutine MEASURE

Purpose:
The subroutine stores new measured sensor data by using the
previous displacement and velocity vectors at the sensor
locations.

Arguments
zp - measured sensor data array

subroutine MEASURE(zp)

include 'shared.inc'
real*8 zp(1)
call ZEROVECT(zp,nsen)
do 100 jj = 1,hdval
   i = hdrow(jj)
   j = hdcol(jj)
   zp(i) = zp(i)*hd(jj)*qdot(j)
100 continue
do 200 jj = 1,hvval
   i = hvrow(jj)
   j = hvcol(jj)
   zp(i) = zp(i)*hv(jj)*qdot(j)
200 continue
return
end

File: eigens.f

Subroutine EIGENS

Purpose Find Eigenmodes given Mass, Stiffness Matrices
Author K. Alvin
Date March 1990
Block Fortran

C subroutine EIGENS

C COMMON AND GLOBALS
include 'shared.inc'

C LOCAL VARIABLES

parameter(NVM=100,MMNVM=MAIDOF=NVM,MMCHV=NVM=(NVM+1)/2)
integer is1(MAIDOF),nvec,i,j,k,kk,nt
real*8 vl(MMNVM),vr(MMNVM),a(AMM),xx(MMNVM),amm(MMCHV)
real*8 xx(NVM,NVM),eig(NVM),eigold(NVM)
real*8 toleig,toljac,omega,fxz

toleig=.0001
out = 13

toljac = toleig
nvec = min(2*nv,100)

C SET UP ISL VECTOR

is1(1) = 1
do 50 j=2,ndof
   is1(j) = j - jdiag(j) + jdiag(j-1) + 1
50 continue

C CALL EIGENSOVER

call SSPACE(stif,mas,vl,xx,amm,xx,eig,eigold,is1,
   jdiag,nvec,ndof,teiej,toljac,out)

C WRITE OUTPUT

write(out,*) 'EIGEN ANALYSIS RESULTS:'
write(out,*)
write(out,*) ' MODE EIGENVALUE FREQUENCY FREQUENCY'
write(out,*)
do 100 i=1,nei
   omega = dsqrt(eigv(i))
   fuz = omega/(2*3.141592654)
   write(out,'(i5,3(x,gl2.5))') i,eigv(i),omega,fuz
100 continue

write(out,*) 'EIGENVECTORS:'
do 200 j=1,nei,5
   write(out,*)
do 300 i=1,ndof
      k = ndof*(j-1) + i
      write(out,'(i5,5(x,g12.5))') i,(vr(kk),kk=k,k+4=ndof,ndof)
300 continue
200 continue

write(out,*) 'MASS MATRIX DIAGONAL:'
do 400 i=1,nn
   do 450 j=1,6
      if (id(j,i).ne.0) then
         write(out,*) i,j,id(j,i),mass(jdiag(id(j,i)))
      endif
450 continue

92
CONTINUE
RETURN
END FORTRAN

FILE: SINGEIG.F

SUBROUTINE SSPACE (AK,AM,VL,VR,AKK,AMM,XX,EIGV,EIGOLD,ISL, 1 IDIAG,NEIG,NVEC,NDOF,TOLEIG,TOLJAC,NW)

C-------------------------------
C input:
C AK : stiffness matrix (profile values) (NDOF)
C AM : consistent mass matrix (profile values) (NDOF)
C ISL : stores in position "i" the row # of tip of
C column "i" (NDOF)
C IDIAG : position of diagonal terms in profile (NDOF)
C NEIG : # of required eigenvalues
C NVEC : # of subspace vectors
C NDOF : # of degrees of freedom
C TOLEIG : tolerance for eigenvalues convergence
C TOLJAC : tolerance for Jacobi convergence
C NW : logical unit number for output

C output:
C VL(NDOF,NVEC) : working array
C - VL(I..1..NRMOD) : AM times rigid modes
C - VL(NRMOD..NVEC) : subspace at the previous step
C VR(NDOF,NVEC) : eigen-vectors
C - VR(I..1..NRMOD) : rigid modes
C - VR(NRMOD..NVEC) : subspace at this step (eigenvectors)
C AKK(NVEC*(NVEC+1)/2) : stiffness matrix in the subspace
C AMM(NVEC*(NVEC+1)/2) : consistent mass matrix in the subspace
C XX(NVEC*NVEC) : subspace eigenvectors
C EIGV(NVEC) : current eigenvalues
C - EIGV(I..NRMOD) : 0 eigen-values
C - EIGV(NRMOD..NVEC) : following eigenvalues > 0
C EIGOLD(NVEC) : same as EIGV
C-------------------------------

IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION AK(1),AM(1),VL(NDOF,NVEC),VR(NDOF,NVEC),AKK(1), 1 AMM(1),XX(NVEC*NVEC),EIGV(1),EIGOLD(1),ISL(1),IDIAG(1)
C WRITE (NW,1003) NEIG,NVEC,NDOF,TOLEIG
C CALL INVECT (AK,AM,VL,VR,IDIAG,NDOF,NVEC)
C CALL FACT (AK,IDIAG,ISL,NDOF,NW)
C CALL NULL (AK,AM,VL,VR,IDIAG,ISL,NDOF,NRMOD)
C NMOD : # of rigid modes
C
WRITE (NW,1004) NMOD
BSUB=MVEC-NMOD
C
CALL ORTHO (VR,VL,HDOF,NMOD,NVEC)
C
WIT=0
WITM=16
WITMAX=16
HVEC1=NVEC-1
DO 5 I=1,NVEC
5 EIGOLD(I)=0.0
C
500 WIT=WIT+1
WRITE (NW,1000) WIT
C
CALL SOLVES (AK,VL(1,NMOD+1),VR(1,NMOD+1),IDIA,ISL,HDOF,NSUB)
CALL ORTHO (VL,VR,HDOF,NMOD,NVEC)
C
I,J=0
DO 10 J=1,NDOF
10 CONTINUE
C
CALL MDLT (AM,VB(1,HMOD+1),VL(1,NVEC),ISL,IDIA,IDIV,NSUB)
C
I,J=0
DO 20 J=1,NMOD+1,NVEC
20 CONTINUE
C
CALL JACOBI (AKK,AMM,XI,EIGV(NMOD+1),HSMAX,TOLJAC,HSUB)
C
ORDER EIGENVALUES & EIGENVECTORS
C
30 IS=0
DO 40 I=1,NMOD+1,NVEC+1
   IF (EIGV(I+1).GE.EIGV(I)) GO TO 40
   IS=1
40 TR=EIGV(I+1)
   EIGV(I+1)=EIGV(I)
94
EIGV(I) = TR
DO 41 J = 1, NSUB
   TR = XX(J + (I - HRMOD) * NSUB)
   XX(J + (I - HRMOD) * NSUB) = XX(J + (I - HRMOD - 1) * NSUB)
   XX(J + (I - HRMOD - 1) * NSUB) = TR
41 CONTINUE
IF (IS.EQ.1) GO TO 30

SUBSPACE CONVERGENCE TEST

ICONV = 0
DO 60 I = HRMOD + 1, NVEC
   TR = DABS((EIGOLD(I) - EIGV(I)) / EIGV(I))
   EIGOLD(I) = EIGV(I)
   EIGV(I) = TR
   IF (TR.GT.TOLEIG.AND.I.LE.HEIG) ICONV = 1
60 CONTINUE
WRITE (NW, 1001) (EIGV(I), I = 1, NVEC)
IF (ICONV.EQ.0) GO TO 100

IF (NIIT.LE.NITMAX) GO TO 70
WRITE (NW, 1002)
GO TO 100

UPDATE EIGEN VECTORS

70 DO 80 I = 1, NDOP
   DO 80 J = 1, NSUB
      TR = 0.0
      DO 81 K = 1, NSUB
         TR = TR + VR(I, K + HRMOD) * XX(K + (J - 1) * MSUB)
      81 CONTINUE
   80 CONTINUE
DO 90 I = 1, NDOP
   VR(I, J) = VL(I, J)
90 DO 100 J = 1, NVEC
GO TO 500

CALCULATE FINAL EIGENVECTORS

100 DO 110 I = 1, NDOP
   DO 110 J = 1, NSUB
      TR = 0.0
      DO 111 K = 1, NSUB
         TR = TR + VL(I, K + HRMOD) * XX(K + (J - 1) * MSUB)
      111 CONTINUE
   110 CONTINUE
DO 120 I = 1, NVEC
120 EIGV(I) = EIGOLD(I)

RETURN

1000 FORMAT (6X, 12HITERATION NO., IS)
1001 FORMAT (6X, 1PE10.3))
1002 FORMAT (6X, 24HVE ACCEPT CURRENT VALUES)
1003 FORMAT (/2X, 'SUBSPACE ITERATION ROUTINE/' / 'NB OF EIGENVALUES=', IS, 'NB OF VECTOR=', IS, 'NB OF DOF=', IS, 'TOLERANCE=', 1PE10.3/)
1004 FORMAT ('NB OF RIGID MODES=', IS/)

95
END

SUBROUTINE INVECT (AK,AM,VR,IDIAG,NDOF,NVEC)
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION AK(1),AM(1),VR(1),VR(NDOF,NVEC),IDIA(1)

ND=NDOF/NVEC

DO 10 I=1,NDOF
   XI=IDIAG(I)
   VR(I,1)=AM(XI)
   VL(I)=AM(XI)/AM(XI)
   DO 10 J=2,NVEC
       VR(I,J)=0.0

10 CONTINUE

RETURN
END

SUBROUTINE MLT (AM,VR,VL,ISL,IDIAG,NVEC,NDOF)
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION AM(1),VR(NDOF,NVEC),VL(1),VR(NDOF,NVEC),IDIA(1)

DO 500 IV=1,NVEC
DO 100 I=1,NDOF
   TR=0.0
   IK=IDIAG(I)
   IX=ABS(ISL(I))
   KK=I
   DO 110 K=IK,1
      TR=TR*AM(IK)*VL(K,IV)
      IK=IK-1
      KK=KK-1
   110 CONTINUE

IF (I.EQ.NDOF) GO TO 99

96
DO 120 I=1,N
   IF (AK(I).LT.ZERO .OR. AM(I).LT.ZERO) GO TO 900
   DO 20 J=1,N
      XX(I,J)=ZERO
   20 CONTINUE
10 CONTINUE
C
C SET COUNTER
C
NSWEEP=0
NR=N-1
DO 500 J=1,NR
   JK=J+1
   DO 500 K=1,N
      JJ=J+(J-1)/2+1
      KK=K+(K-1)/2+1
      EPSAK=(AK(J*K)-AK(J)*AK(K))/AK(J)*AK(K)
      EPSAM=(AM(J*K)-AM(J)*AM(K))/AM(J)*AM(K)
      IF (EPSAK.GT.EPS .AND. EPSAM.GT.EPS) GO TO 500
C
C CALCULATE ROTATION ELEMENTS
C
AKK=AK(K*K)-AM(K*K)-AK(JK)*AK(JK)
AJJ=AK(JJ)-AM(JJ)-AK(JK)*AK(JK)
AB=(AK(JJ)*AM(KK)-AK(KK)*AM(JJ))/TWO
CHECK=AB+ACK*AJJ
IF (CHECK.LT.ZERO) GO TO 900
500 CONTINUE
RETURN
END

SUBROUTINE JACOBI (AK,AM,XX,EIGV,NSWAN,TOL,N,NW)
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION iK(l),AM(l),ZX(N,N).EIGV(l)
C
C INITIALIZE
C
ZERO=0.0
TWO=2.0
DO 10 I=1,N
   II=I*(I-1)/2+1
   IF (AK(II).LE.ZERO .OR. AM(II).LE.ZERO) GO TO 900
   EIGV(I)=AK(II)/AM(II)
   DO 20 J=1,N
      XX(I,J)=ZERO
   20 CONTINUE
10 CONTINUE

C
C CHECK IF ZEROING IS REQUIRED
C
EPS=0.01**NSWEEP
EPS2=EPS*EPS
DO 150 J=1,NR
   JK=J+1
   DO 150 K=1,N
      JJ=J+(J-1)/2+1
      KK=K+(K-1)/2+1
      EPSAK=(AK(J*K)-AK(J)*AK(K))/AK(J)*AK(K)
      EPSAM=(AM(J*K)-AM(J)*AM(K))/AM(J)*AM(K)
      IF (EPSAK.LT.EPS .AND. EPSAM.LT.EPS) GO TO 150
C
C CALCULATE ROTATION ELEMENTS
C
AKK=AK(K*K)+AM(K*K)-AK(JK)*AK(JK)
AJJ=AK(JJ)-AM(JJ)-AK(JK)*AK(JK)
AB=(AK(JJ)+AM(KK)-AK(KK)+AM(JJ))/TWO
CHECK=AB+ACK*AJJ
IF (CHECK.LT.ZERO) GO TO 900
150 CONTINUE
RETURN
D1 = AB + SQCH
D2 = AB - SQCH
DEN = D1
IF (DABS(D2) .GT. DABS(D1)) DEN = D2
IF (DEN .NE. ZERO) GO TO 45
CA = ZERO
CG = -AK(JK)/AK(KK)
GO TO 60
CA = AKK/DEN
CG = -AJJ/DEN

PERFORM GENERALIZED ROTATION

50  JP1 = J + 1
   JN1 = J - 1
   KP1 = K + 1
   KM1 = K - 1
IF (JN1 .LT. 1) GO TO 70
DO 60 I = JN1, JN1
   IJ = J - JN1/2 + I
   IK = K - KM1/2 + I
   AKJ = AK(IJ)
   AKK = AK(IK)
   AMJ = AK(IJ)
   AMK = AK(IK)
   AK(IJ) = AKJ + CG * AKK
   AM(IJ) = AMJ + CG * AMK
   AK(IK) = AKK + CA * AKJ
   AM(IK) = AMK + CA * AMJ
CONTINUE
60  DO 80 I = KP1, KP1
   IJ = (I - 1)/2 + J
   IK = (I - 1)/2 + K
   AKJ = AK(IJ)
   AMJ = AM(IJ)
   AKK = AK(IK)
   AMK = AM(IK)
   AK(IJ) = AKJ + CG * AKK
   AM(IJ) = AMJ + CG * AMK
   AK(IK) = AKK + CA * AKJ
   AM(IK) = AMK + CA * AMJ
CONTINUE
70  DO 80 I = JP1, JP1
   JI = I - JP1/2 + J
   IK = (K - 1)/2 + I
   AKJ = AK(JI)
   AMJ = AM(JI)
   AKK = AK(JK)
   AMK = AM(JK)
   AK(JI) = AKJ + CG * AKK
   AM(JI) = AMJ + CG * AMK
   AK(JK) = AKK + CA * AKJ
   AM(JK) = AMK + CA * AMJ
CONTINUE
80  CONTINUE
90  IF (JP1 .GT. KM1) GO TO 110
DO 100 I = JP1, KM1
   JI = I - JP1/2 + J
   IK = (K - 1)/2 + I
   AKJ = AK(JI)
   AMJ = AM(JI)
   AKK = AK(JK)
   AMK = AM(JK)
   AK(JI) = AKJ + CG * AKK
   AM(JI) = AMJ + CG * AMK
   AK(JK) = AKK + CA * AKJ
   AM(JK) = AMK + CA * AMJ
CONTINUE
100 CONTINUE
110  AKK = AK(KK)
    AMK = AM(KK)
    AKJ = AK(JJ)

98
AMJ = AM(JJ)
AK(KK) = AKK + TWO*CA*AK(JK) + CA*CA*AKJ
AM(KK) = AMK + TWO*CA*AM(JK) + CA*CA*AMJ
AK(JJ) = AKJ + TWO*CG*AK(JK) + CG*CG*AKK
AM(JJ) = AMJ + TWO*CG*AM(JK) + CG*CG*AMK
AK(JK) = ZERO
AM(JK) = ZERO

C
C UPDATE EIGENVECTOR FOR THIS ROTATION
C
DO 120 I = 1, N
XX(I) = IX(I, J)
XX(K) = IX(I, K)
XX(I, J) = XX(I) + CG*XX(K)
XX(I, K) = XX(K) + CA*XX(I)
120 CONTINUE

C
C UPDATE EIGENVALUES & CHECK CONVERGENCE
C
MT = N*(N+1)/2
ICONV = 0
DO 160 I = 1, N
II = I*(I-1)/2 + 1
IF (AK(II) .LE. ZERO .OR. AM(II) .LE. ZERO) GO TO 900
TR = AK(II) / AM(II)
DEN = (TR - EIGV(I)) / TR
EIGV(I) = TR
IF (DABS(DEN) .GT. TOL) ICONV = 1
160 CONTINUE
IF (ICONV .EQ. 1) GO TO 499

C
C CHECK OFF DIAGONAL TERMS
C
EPS = TOL = TOL
DO 170 J = 1, N
IIK = J + 1
DO 170 K = IIK, N
JJ = J*(J-1)/2 + J
KK = K*(K-1)/2 + K
JK = K*(K-1)/2 + J
EPSAK = (AK(JK) = AK(JK)) / (AK(JJ) = AK(KK))
EPSAM = (AM(JK) = AM(JK)) / (AM(JJ) = AM(KK))
IF (EPSAK .LT. EPS .AND. EPSAM .LT. EPS) GO TO 170
GO TO 499
170 CONTINUE

C
C SCALE EIGENVECTORS
C
DO 180 I = 1, N
II = I*(I-1)/2 + 1
AKK = DSQRT(AM(II))
DO 180 J = 1, N
XX(J, I) = XX(J, I) / AKK
180 CONTINUE
C
RETURN
C
499 IF (NSWEEP .LE. NSWAX) GO TO 500
WRITE (NW,1000)
GO TO 179
C
900 WRITE (NW,1001)
STOP
C
1000 FORMAT (8X,3AHNO CONVERGENCE AT NSMAX ITERATIONS)
1001 FORMAT (8X,46HERROR IN JACOBI : MATRIX NOT POSITIVE DEFINITE)
C
END
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
SUBROUTINE SOLVES (AK,VL,VR,IDIAG,ISL,NDOF,NVEC)
C
PURPOSE : SOLVES THE SINGULAR PROBLEM : AK x VL = VR
C
THE SINGULAR COLUMNS INTO AK ARE INDEXED BY
C
THE NEGATIVE VALUES OF ISL, THE CORRESPONDING
C
TERMS OF THE SOLUTION ARE PUT TO 0
C
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION AK(1),VL(NDOF,NVEC),VR(NDOF,NVEC),IDIAG(1),ISL(1)
C
DO 500 IV=1,NVECC
   50 DO 60 I=1,NDOF
      VL(I,IV)=VR(I,IV)
   60 CONTINUE
C
BACKSUBSTITUTE
C
DO 100 IC=2,NDOF
   10 TR=0.0
   IC1=IC-1
   IM1=IDIAG(IC)-IC
   IX=ISL(IC)
   IF (IX.LE.0) THEN
      VL(IC,IV)=0.
      GOTO 100
   ENDIF
   IF (IX.GT.IC1) GO TO 100
   DO 120 K=IK,IC1
      TR=TR+AK(IN1+K)*VL(K,IV)
   120 CONTINUE
   VL(IC,IV)=VL(IC,IV)-TR
   100 CONTINUE
C
SOLVE DU=U
C
DO 150 IC=1,NDOF
   150 VL(IC,IV)=VL(IC,IV)/AK(IDIAG(IC))
C
BAKSUBSTITUTE
C
IIC=NDOF
DO 200 IC=2,NDOF
   200 TR=VL(IIC,IV)
   IC1=IIC-1
   IX=ISL(IIC)
   IM1=IDIAG(IIC)-IIC
   VL(IIC,IV)=VL(IIC,IV)-TR

100
IF ((IK.GT.IC1).OR.(IK.LE.0)) GO TO 221
DO 220 K=IK,IC1
   VL(K,IV)=VL(K,IV)-AK(IM1+K)*TR
220 CONTINUE
221 IIC=IIC-1
200 CONTINUE
500 CONTINUE
RETURN
END

SUBROUTINE FACT (AK,IDIAG,ISL,HDOF,NW)
C
C PURPOSE : LDLT DECOMPOSITION OF THE POSITIVE SEMI-DEFINITE
C MATRIX AK, THE SINGULAR COLUMNS OF AK ARE INDEXED
C BY A NEGATIVE VALUE OF IDIAG>
C
C
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION AK(J),IDIAG(I),ISL(1)
C
C DETERMINE MIN & MAX
C
TR=DABS(AK(1))
Amin=TR
Amax=TR
DO 10 IL=1,HDOF
   TR=DABS(AK(IDIAG(1)))
   IF (TR.LT.AMIN) AMIN=TR
   IF (TR.GT.AMAX) AMAX=TR
10 CONTINUE
ZERO=(AMAX+AMIH)*1.0D-10
C
C LOOP OVER COLUMN
C
DO 100 IC=1,HDOF
   MIC=ISL(IC)
   IC1=IC-1
   MIC1=MIC+1
   IN2=IDIAG(IC)-IC
   IF ((MIC.LT.1).OR.(MIC.GT.IC)) GO TO 901
C
C CALCULATE GS
C
   IF (MIC1.GT.IC1) GO TO 150
   DO 120 IL=MIC1,IC1
      IF (IDIAG(IL).LT.0) THEN
         AK(IN2+IL)=0
      GOTO 120
   ENDIF
   MIL=ISL(IL)
   IL1=IL-1
   MIN=MAX0(0,MIL,MIC)
   IN1=IDIAG(IL)-IL
   TR=0.0
   IF (MIN.GT.II1) GO TO 120
   DO 130 K=MIM,IL1
      TR=TR+AK(IN1+K)*AK(IN2+K)
130 CONTINUE

101
IN=IN2+IL
AK(IN)=AK(IN)-TR

CONTINUE
C
CALCULATE L&D
C
TR=0.0
IF (MIC.GT.IC1) GO TO 201
DO 200 IL=MIC,IC1
 IF (IDIAG(IL).LT.0) GOTO 200
 AG=AK(IN2+IL)
 AL=AG/AK(IDIAG(IL))
 AK(IN2+IL)=AL
 TR=TR+AL*AG
200 CONTINUE
201 IN=IDIAG(IC)
 AK(IN)=AK(IN)-TR
 IF (AK(IN).LT.ZERO) IDIAG(IC)=-IDIAG(IC)
100 CONTINUE
RETURN

901 WRITE (NW,1001)
STOP

1001 FORMAT (6X,29HSTOP ERROR IN IDIAG VECTOR)
1010 FORMAT (6X,'CONDITIONING OF THE STIFFNESS MATRIX'/2X,
 1 'MIN DIAG TERM=',IP*1.3,' MAX DIAG TERM=',E10.3)
END

SUBROUTINE NULL(AK,AM,VL,VR,IDIAG,ISL,NDOF,NMOD)
C
C PURPOSE : CALCULATE THE HULL SPACE OF AK AND PUT AN
C ORTHONORMALISED BASE OF THIS SPACE INTO THE
C HRHOD FIRST VECTORS OF VR.
C VL = AM x VR AFTER EXECUTION
C
C IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION AK(1),AM(1),IDIAG(1),ISL(1),VL(NDOF,*),VR(NDOF,*)
C
STORE THE SINGULAR COLUMNS INTO THE BEGINNING OF VR
C
THE SINGULAR EQUATION ARE NOW INDEXED BY NEGATIVE VALUES
INTO ISL INSTEAD OF IDIAG
C
NMOD=0
DO 1 IC=1,NDOF
 IF (IDIAG(IC).GT.0) GOTO 1
 IDIAG(IC)=-IDIAG(IC)
 NMOD=NMOD+1
 MIC=ISL(IC)
 IN=IDIAG(IC)-IC
 DO 2 K=1,MIC-1
 2 VR(K,NMOD)=0.
 DO 3 K=MIC,IC-1
   VR(K,NMOD)=AK(IN+K)
   AK(IN+K)=0.
3 CONTINUE
ISL(IC)=ISL(IC)
VR(IC,HRMOD)=-1.
AK(IDIAG(IC))=1.
DO 4 K=IC+1,HRDF
VR(K,HRMOD)=0.
1 CONTINUE

BAKSUBSTITUTE

DO 200 N=1,HRMOD
IIC=HDGF
DO 200 IC=2,HDGF
TR=VR(IIC,N)
IIC=IIC-1
IK=ISL(IIC)
INI=IDIAG(IIC)-IIC
IF ((IK.GT.IC1).OR.(IK.LE.0)) GO TO 221
DO 220 K=IK,IC1
VR(K,N)=VR(K,N)-AK(INI+K)*TR
220 CONTINUE
221 IIC=IIC-1
200 CONTINUE

ORTHOGONALISATION

DO 10 N=1,HRMOD
DO 20 K=1,N-1
TR=0.
DO 30 I=1,NDGF
TR=TR+VR(I,K)*VR(I,N)
30 CONTINUE
DO 40 I=1,NDGF
VR(I,N)=VR(I,N)-TR*VR(I,K)
40 CONTINUE
CALL MULTI(AH,VL(1,N),VR(1,N),ISL,IDIAH,1,NDGF)
TR=0.
DO 50 I=1,NDGF
TR=TR+VR(I,N)*VL(I,N)
50 CONTINUE
TR=1/SQRT(TR)
DO 60 I=1,NDGF
VR(I,N)=VR(I,N)*TR
VL(I,N)=VL(I,N)*TR
60 CONTINUE
10 CONTINUE

RETURN
END

SUBROUTINE ORTHO(VL,VR,HDGF,HRMOD,NVEC)

PURPOSE : ORTHOGONALISE THE NSUB LAST COLUMNS OF VL (LAST
EVALUATED SOLUTION) WITH RESPECT TO THE NULL SPACE
OF A, FOR THE AN SCALAR PRODUCT

IMPLICIT REAL*8 (A-H,O-Z)
INTEGER HDGF,HRMOD,NVEC
DIMENSION VL(NDGF,NVEC),VR(NDGF,NVEC)
NSUB=NVEC-HRMOD
```fortran
DO 1 J=1,MSUB
   DO 2 I=1,MRMOD
      S=0.
   DO 3 K=1,MDDF
      S=S+VL(K,1)*VL(K,MRMD+J)
   DO 4 L=1,MDGF
      VL(L,MRMD+J)=VL(L,MRMD+J)-S*VR(L,1)
   CONTINUE
1 CONTINUE
RETURN
END

File: animout.f

C=Module ANIMOUT
C=Author K. Alvin
C=Date June 1990
C=Block Fortran
C-----------------------------------------------------
C subroutine ANIMOUT
C-----------------------------------------------------
C subroutine ANIMOUT(q,id,nnp,time,out)
   real*8 q(i),time,v(3)
   integer out,id(6,M,nnp,i,k
   write(out,'(g15.5)') time
   do 100 i=1,nnp
       do 200 k=1,3
          if (id(k,i) .ne. 0) then
             v(k) = q(id(k,i))
          else
             v(k) = 0.
          endif
       200 continue
       write(out,1000) (v(k),k=1,3)
   100 continue
   1000 format(3g15.5)
   return
end

File: stiffrc.f

C subroutine STIFFRC(v,fact,kq)
```
recursive subroutine STIFFRC(v, fact, kq)
real*8 v(1), fact, kq(1)
include 'shared.inc'

C ASSEMBLE EACH ELEMENT MASS AND STIFFNESS

do 100 ii=1, ndomain

CVD$ CMCALL
   do 110 jj=1, neld(ii)
       n = elnum(jj, ii)
       call ELEFRC(v, fact, kq, n)
   continue

100 continue

return
end

C subroutine ELEFRC(v, fact, kq, n)

recursive subroutine ELEFRC(v, fact, kq, n)

real*8 v(1), fact, kq(1)
integer n

include 'shared.inc'

C LOCAL VARIABLES

parameter(MAISEQ=24)
real*8 sk(MAISEQ, MAISEQ)
integer lm(MAISEQ, nseq)

do 20 k=1, 4
   j=ix(k, n)
   if ((etype(n).eq.1).and.(k.gt.2)) j = 0
   do 30 i=1, 6
      kk=6*(k-1) + i
      if (j.ne. 0) then
         lm(kk) = id(i, j)
      else
         lm(kk) = 0
      endif
   30 continue

20 continue

nseq=12

call LOADSK(sk, n, nseq)

call ESTIFVM(sk, lm, nseq, v, kq, fact)

return
end
subroutine LOADSK(sk,n,nseq)
recursive subroutine LOADSK(sk,n,nseq)
include 'shared.inc'
real*8 sk(nseq,1)
integer a,nseq
k=0
do 10 j=1,nseq
do 20 i=1,j
   k=k+1
   sk(i,j)=estifm(k,n)
   sk(j,i)=sk(i,j)
20 continue
10 continue
return
end

File: estifvm.f

C subroutine ESTIFVM(sk,lm,nseq,v,kq,fact)
recursive subroutine ESTIFVM(sk,lm,nseq,v,kq,fact)
C ARGUMENTS
real*8 sk(nseq,1),v(1),kq(1),fact
integer lm(1),nseq

do 20 j = 1, nseq
   k = lm(j)
   if (k .eq. 0) goto 20
   do 10 i = 1, nseq
      m = lm(i)
      if (m .eq. 0) goto 10
      kq(m) = kq(m) + sk(i,j)*v(k)*fact
10   continue
20   continue

return
end
C=End Fortran

File: renum.f

C=DECK RENUM
C= Purposes- RENUMBERS THE GRID POINTS TO MINIMIZE PROFILE STORAGE
C= Author: W K BELYIN and DUC NGUYEN 7-5-90
C
subroutine RENUM
C
include 'shared.inc'
c Initialize vector
c maxtry=2*nnp/3 +1
c
nterms=nnp=maxtry
do 22 j=1,nterms
22 iadjcy(j)=0
do 10 i=1,nnp
10 icount(i)=0
c
1 1 i=1,nnp
nodea=ix(1,i)
nodeb=ix(2,i)
   if ((nodea.eq. 0).or.(nodeb .eq. 0)) go to 1
   icount(nodea)=icount(nodea)+1
   icount(nodeb)=icount(nodeb)+1
   i=icount(nodea)
b=icount(nodeb)
   if (ia.gt.maxtry .or. ib.gt.maxtry) go to 345
   locate=(nodea-1)*maxtry+ia
   iadjcy(locate)=nodeb
   locate=(nodeb-1)*maxtry+ib
   iadjcy(locate)=nodea
1 continue

c
ii=0
   do 37 i=1,nterms
      if ( iadjcy(i) .eq. 0 ) go to 37
      ii=ii+1
      iadjcy(ii)=iadjcy(i)
37 continue

c
last=0
   do 2 last=last+icount(i)
2 continue
   jj=icount(i)
   icount(i)=1
   do 3 i=2,nnp+1
      kk=icount(i)
      icount(i)=icount(i-1)+jj
      jj=kk
3 continue
   go to 556
345 write(6,555)
555 format(2x, 'error in dimension for MAITRY !! ')
556 continue

c call GENRCM(nnp,icount,iajncy,perm,mask,xls)
return
end

subroutine genrcm(neqns, xadj, adjncy, perm, mask, xls)
c......reference: computer solution of large sparse positive definite
    .... systems, alan george & joseph w-h liu
    ......(prentice-hall,inc.,englewood cliffs,NJ 07632)
    integer adjncy(i),mask(i),perm(i),xls(i)
integer xadj(1), ccsizex, i, neqns, nlvl, nnum, root

do 100 i=1, neqns
  mask(i)=1
  100 continue

num=1

do 200 i=1, neqns
  if( mask(i).eq.0 ) go to 200
  root=1
  call fnroot(root, xadj, adjncy, mask, nlvl, xls, ls)
  call rcm(root, xadj, adjncy, mask, perm(num), ccsizex, xls)
  num=num+ccsizex
  if(num.gt.neqns) go to 987
  200 continue

c...... perm(new node)=old node

c...... now, mask(old node)= new node

987 continue

do 11 new=1, neqns
  iold=perm(new)
  mask(iold)=new
  c
  write(6,*) 'iold,mask(iold) = ',iold,mask(iold)
  11 continue

return

end

subroutine fnroot(root, xadj, adjncy, mask, nlvl, xls, ls)
integer adjncy(1), ls(1), mask(1), xls(1)

integer xadj(1), ccsizex, jstrt, k, kstop, kstrt,

$ mindeg, nabor, ndeg, nlvl, node, numlvl, root

call rooted(root, xadj, adjncy, mask, nlvl, xls, ls)
ccsizex=xls(nlvl+1)-1
if(nlvl.eq.1 .or. nlvl.eq.ccsizex) return

100 jstrt=xls(nlvl)
  mindeg=ccsizex
  root=ls(jstrt)
  if(ccsizex.eq.jstrt) go to 400
  do 300 j=jstrt, ccsizex
    node=ls(j)
    ndeg=0
    kstrt=xadj(node)
    kstop=xadj(node+1)-1
    do 200 k=kstrt, kstop
      nabor=adjncy(k)
      if( mask(nabor) .gt. 0) ndeg=ndeg+1
      200 continue
    if(ndeg.ge.mindeg) go to 300
    root=node
    mindeg=ndeg
  300 continue
  do 400 j=ls(jstrt), xls(nlvl)
    if(j.lt.nlvl) return
    alvl=numlvl
    if(nlvl.lt.ccsizex) go to 100
    return

end

subroutine rcm(root, xadj, adjncy, mask, perm, ccsizex, deg)
integer adjncy(1), deg(1), mask(1), perm(1)

integer xadj(1), ccsizex, fnbr, i, j, jstrt, k, l, lbegin,

$ lbr, lperm, lvlend, nbr, node, root

108
call degree(root,xadj,adjncy,mask,deg,ccsize,perm)
mask(root)=0
if(ccsize.le.1) return
lvlen=0
labr=1
100 labr=labr+1
lvlen=lvlen+1
do 600 i=labr,lvlen
  node=perm(i)
  jstrt=xadj(node)
  jstop=xadj(node)+1
  fnbr=labr+1
  do 200 j=jstrt,jstop
     nbr=adjncy(j)
     if(mask(nbr).eq.0) go to 200
     fnbr=labr+1
     mask(nbr)=0
     perm(labr)=nbr
   200 continue
   if(fnbr.ge.lnbr) go to 600
   k=fnbr
   300 l=k
   k=k+1
   nbr=perm(k)
   400 if(l.lt.fnbr) go to 500
   lperm=perm(l)
   if( deg(lperm).le.deg(nbr) ) go to 500
   perm(l+1)=lperm
   l=l+1
   go to 400
   500 perm(l+1)=nbr
   if(k.lt.lnbr) go to 300
   600 continue
   if(lnbr.gt.lvlen) go to 100
   k=ccsize/2
   l=ccsize
   do 700 i=1,k
      lperm=perm(l)
      perm(l)=perm(i)
      perm(i)=lperm
      l=l-1
   700 continue
   return
end

subroutine rootls(root,xadj,adjncy,mask,nlv1,xls,ls)
integer adjncy(l),ls(l),mask(l),xls(l)
integer xadj(1),i,j,jstrt,jstop,labr,ccsize,lvlen,
lsize,nbr,alvl,root

mask(root)=0
ls(1)=root
nlv1=0
lvlen=0
ccsize=1
200 labr=lvlen+1
lvlen=lvlen+1
nlv1=nlv1+1
xls(nlv1)=labr
do 400 i=labr,lvlen
node=ls(i)
jstart=xadj(node)
jstop=xadj(node+1)-1
if(jstop.lt.jstart) go to 400
do 300 j=jstart,jstop
  nbr=adjncy(j)
  if( mask(nbr).eq.0 ) go to 300
  ccsizer=ccsize+1
  ls(ccsize)=nbr
  mask(nbr)=0
300 continue
400 continue
lsize=ccsize-lvlend
if(lsize.gt.0) go to 200
xls(nlivel+1)=lvlend+1
xadj(root)=xajx(node)
deg(root)=deg(1)
ls(root)=0
lsize=1
$100 lbegin=1lvlend+1
lvlend=ccsize
do 400 i=lbegin,lvlend
  node=ls(i)
  jstart=xadj(node)
jstop=iabs( xadj(node+1) ) -1
  ideg=0
  if(jstop.lt.jstart) go to 300
  do 200 j=jstart,jstop
     nbr=adjncy(j)
     if( mask(nbr).eq.0 ) go to 200
     ideg=ideg+1
     if(xadj(nbr).lt.0) go to 200
     xadj(nbr)=-xadj(nbr)
     ccsizer=ccsize+1
     ls(ccsize)=nbr
200 continue
300 deg(node)=ideg
400 continue
lsize=ccsize-lvlend
if(lsize.gt.0) go to 100
do 500 i=1,ccsize
  node=ls(i)
  xadj(node)=xadj(node)
500 continue
return
end

 subroutine degree(root,xadj,adjncy,mask,deg,ccsize,ls)
 integer adjncy(i),deg(i),ls(i),mask(i)
 integer xadj(i),ccsize,i,ideg,j,jstop,jstart,
 $100 lbegin,livelend,lsize,nbr,root
 xls(1)=root
 xadj(root)=-xadj(root)
 lvlend=0
 ccsizer=1
 do 400 i=lbegin,livelend
  node=ls(i)
  jstart=xadj(node)
jstop=iabs( xadj(node+1) ) -1
  ideg=0
  if(jstop.lt.jstart) go to 300
  do 200 j=jstart,jstop
     nbr=adjncy(j)
     if( mask(nbr).eq.0 ) go to 200
     ideg=ideg+1
     if(xadj(nbr).lt.0) go to 200
     xadj(nbr)=-xadj(nbr)
     ccsizer=ccsize+1
     ls(ccsize)=nbr
200 continue
300 deg(node)=ideg
400 continue
lsize=ccsize-livelend
if(lsize.gt.0) go to 100
do 500 i=1,ccsize
  node=ls(i)
  xadj(node)=xadj(node)
500 continue
return
end

 File: kfilter.f
C subroutine KFILTER

recursive subroutine KFILTER

include 'shared.inc'

do 10 i = 1,ndof
    go(i) = delsq*(f(i)+go(i))+delta*pe(i)+mass(jdiag(i))*qe(i)
    gk(i) = delta*(f(i)+gk(i))*pe(i)
    continue

call SOLVER(eq,go,jdiag,ndof,2)

C Activate EBE computations for internal force by using STIFFRC
C subroutine. Otherwise use PMVMAD (profile matrix/vector mult-add

C call PMVMAD(stif,jdiag,go,ndof,-delta,ck,1.d0)
call STIFFRC(go,-delta,gk)

do 100 i = 1,ndof
    qe(i) = 2.*go(i) - qe(i)
    pe(i) = 2.*gk(i) - pe(i)
    continue

call SOLVER(eq,go,jdiag,ndof,2)

return
end
SECOND-ORDER DISCRETE KALMAN FILTERING EQUATIONS FOR CONTROL-STRUCTURE INTERACTION SIMULATIONS

by

K. C. Park, W. K. Belvin and K. F. Alvin

March 1991

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Second-Order Discrete Kalman Filtering Equations for Control-Structure Interaction Simulations†

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Abstract

A general form for the first-order representation of the continuous, second-order linear structural dynamics equations is introduced in order to derive a corresponding form of first-order continuous Kalman filtering equations. Time integration of the resulting first-order Kalman filtering equations is carried out via a set of linear multistep integration formulas. It is shown that a judicious combined selection of computational paths and the undetermined matrices introduced in the general form of the first-order linear structural systems leads to a class of second-order discrete Kalman filtering equations involving only symmetric, sparse \( N \times N \) solution matrices. The present integration procedure thus overcomes the difficulty in resolving the difference between the time derivative of the estimated displacement vector \( \frac{d}{dt} \hat{x} \) and the estimated velocity vector \( \hat{\dot{x}} \) that are encountered when one attempts first to eliminate \( \hat{\dot{x}} \) in order to form an equivalent set of second-order filtering equations in terms of \( \frac{d}{dt} \hat{x} \). A partitioned solution procedure is then employed to exploit matrix symmetry and sparsity of the original second-order structural systems, thus realizing substantial computational simplicity heretofore thought difficult to achieve.

† An earlier version of the present paper without numerical experiments was presented at the AIAA Guidance and Control Conference, Portland, Ore., 20–22 August 1990, Paper No. AIAA 90-3387.

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Introduction

Current practice in the design, modeling and analysis of flexible large space structures is by and large based on the finite element method and the associated software. The resulting discrete equations of motion for structures, both in terms of physical coordinates and of modal coordinates, are expressed in a second-order form. As a result, the structural engineering community has been investing a considerable amount of research and development resources to develop computer-oriented discrete modeling tools, analysis methods and interface capabilities with design synthesis procedures; all of these exploiting the characteristics of second-order models.

On the other hand, modern linear control theory has its roots firmly in a first-order form of the governing differential equations, e.g., (Kwakernaak and Sivan, 1972). Thus, several investigators have addressed the issues of interfacing second-order structural systems and control theory based on the first-order form (Hughes and Skelton, 1980; Arnold and Laub, 1984; Bender and Laub, 1985; Oshman, Inman and Laub, 1987; Belvin and Park, 1989, 1990). As a result of these studies, it has become straightforward for one to synthesize non-observer based control laws within the framework of a first-order control theory and then to recast the resulting control laws in terms of the second-order structural systems.

Unfortunately, controllers based on a first-order observer are difficult to express in a pure second-order form because the first-order observer implicitly incorporates an additional filter equation (Belvin and Park, 1989). However a recent work (Juang and Maghami, 1990) has enabled the first-order observer gain matrices to be synthesized using only second-order equations. To complement the second-order gain synthesis, the objective of the present paper is to develop a second-order based simulation procedure for first-order observers. The particular class of first-order observers chosen for study are the Kalman Filter based state estimators as applied to second-order structural systems. The procedure permits simulation of first-order observers with nearly the same solution procedure used for treating the structural dynamics equation. Hence, the reduced size of system matrices and the computational techniques that are tailored to sparse second-order structural systems may be employed. As will be shown, the procedure hinges on discrete time integration formulas to effectively reduce the continuous time Kalman Filter to a set of second-order difference equations.

The paper first reviews of the conventional first-order representation of the continuous second-order structural equations of motion. An examination of the corresponding first-order Kalman filtering equations indicates that, due to the difference in the derivative of the estimated displacement \( \dot{x} \) and the estimated velocity \( \ddot{x} \), transformation of the first-order observer into an equivalent second-order observer requires the time derivative of measurement data, a process not recommended for practical implementation.
Next, a transformation via a generalized momentum is introduced to recast the structural equations of motion in a general first-order setting. It is shown that discrete time numerical integration followed by reduction of the resulting difference equations circumvents the need for the time derivative of measurements to solve Kalman filtering equations in a second-order framework. Hence, the Kalman filter equations can be solved using a second-order solution software package.

Subsequently, computer implementation aspects of the present second-order observer are presented. Several computational paths are discussed in the context of discrete and continuous time simulation. For continuous time simulation, an equation augmentation is introduced to exploit the symmetry and sparsity of the attendant matrices by maintaining state dependant control and observer terms on the right-hand-side (RHS) of the filter equations. In addition, the computational efficiency of the present second order observer as compared to the first order observer is presented.

**Continuous Formulation of Observers for Structural Systems**

Linear, second-order discrete structural models can be expressed as

\[MX + Dx + Kx = Bu + Gw, \quad x(0) = x_0, \quad \dot{x}(0) = \dot{x}_0\]

with the associated measurements

\[z = H_1 x + H_2 \dot{x} + \nu\]

where \(M, D, K\) are the mass damping and stiffness matrices of size \((N \times N)\); \(x\) is the structural displacement vector, \((N \times 1)\); \(u\) is the active control force \((m \times 1)\); \(B\) is a constant force distribution matrix \((N \times m)\); \(z\) is a set of measurements \((r \times 1)\); \(H_1\) and \(H_2\) are the measurement distribution matrices \((r \times N)\); \(Z_1\) and \(Z_2\) are the control feedback gain matrices \((m \times N)\); \(w\) and \(\nu\) are zero-mean, white Gaussian processes with their respective covariances \(Q\) and \(R\); and the superscript dot designates time differentiation. In the present study, we will restrict ourselves to the case wherein \(Q\) and \(R\) are uncorrelated with each other and the initial conditions \(x_0\) and \(\dot{x}_0\) are also themselves jointly Gaussian with known means and covariances.

The conventional representation of (1) in a first-order form is facilitated by

\[
\begin{cases}
  x_1 = x \\
  x_2 = \dot{x} = \dot{x}_1 \\
  M \ddot{x}_2 = M \ddot{x} = Bu + Gw - D x_2 - K x_1
\end{cases}
\]
which, when cast in a first-order form, can be expressed as

\[
\begin{align*}
E \dot{q} &= Fq + \dot{B}u + \ddot{G}w, \\
q &= (x_1 \ x_2)^T, \\
z &= Hq + \nu
\end{align*}
\]  

(4)

where

\[
E = \begin{bmatrix} I & 0 \\ 0 & M \end{bmatrix}, \quad F = \begin{bmatrix} 0 & I \\ -K & -D \end{bmatrix}, \\
\dot{B} = \begin{bmatrix} 0 \\ B \end{bmatrix}, \quad \ddot{G} = \begin{bmatrix} 0 \\ G \end{bmatrix}
\]

(5)

It is well-known that the Kalman filtering equations (Kalman, 1961; Kalman and Bucy, 1963) for (4) can be shown to be (Arnold and Laub, 1984):

\[
E \dot{q} = Fq + \dot{B}u + EPH^TR^{-1}z
\]

(6)

where

\[
z = z - Hq, \quad P = \begin{bmatrix} U & S^T \\ S & L \end{bmatrix}, \quad \dot{q} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}, \quad \ddot{q} = \begin{bmatrix} \ddot{x} \\ \ddot{z} \end{bmatrix}
\]

(7)

in which \( U \) and \( L \) are positive definite matrices and the matrix \( P \) is determined by the Riccati equation (Kwakernaak and Sivan, 1972; Arnold and Laub, 1984)

\[
E \ddot{P}E^T = FPS^T + ESP^T - EPH^TR^{-1}HP^T + \ddot{G}Q\dot{G}^T
\]

(8)

The inherent difficulty of reducing the first-order Kalman filtering equations given by (6) to second order form can be appreciated if one attempts to write (6) in a form introduced in (3):

\[
\begin{align*}
\{ & a) \quad \dot{x}_1 = \dot{x} \\
& b) \quad \dot{x}_2 = \ddot{x} = \dot{x}_1 - L_1\ddot{z} \\
& c) \quad M\ddot{x}_2 = -D\ddot{x}_2 - K\dot{x}_1 + Bu + ML_2\ddot{z}
\end{align*}
\]

(9)

where

\[
L_1 = (H_1U + H_2S)^TR^{-1}, \quad L_2 = (H_1S^T + H_2L)^TR^{-1}
\]

Note from (9b) that \( \dot{x}_2 \neq \dot{x}_1 \). In other words, the time derivative of the estimated displacement (\( \dot{x} \)) is not the same as the estimated velocity (\( \dot{x}_1 \)); hence, \( \dot{x}_1 \) and \( \dot{x}_2 \) must be treated as two independent variables, an important observation somehow overlooked in Hashemipour and Laub (1988).

Of course, although not practical, one can eliminate \( \dot{x}_2 \) from (9). Assuming \( \dot{x}_1 \) and \( \dot{x}_2 \) are differentiable, differentiate (9b) and multiply both sides by \( M \) to obtain

\[
M\ddot{x}_1 = M\ddot{x}_2 + ML_1\ddot{z}
\]

(10)
Substituting $M \ddot{z}_2$ from (9c) and $\dot{z}_2$ from (9b) in (10) yields

$$M \ddot{x}_1 = -D(\dot{x}_1 - L_1 \dot{z}) - K \dot{x}_1 + Bu + ML_2 \ddot{z} + ML_1 \dot{z}$$

(11)

which, upon rearrangements, becomes

$$M \ddot{x}_1 + D \dot{x}_1 + K x_1 = Bu + ML_2 \ddot{z} + ML_1 \dot{z} + DL_1 \dot{z}$$

(12)

There are two difficulties with the above second-order observer. First, the numerical solution of (12) involves the computation of $x_1$ when rate measurements are made. The accuracy of this computation is in general very susceptible to errors caused in numerical differentiation of $\dot{x}_1$. Second, and most important, the numerical evaluation of $\dot{z}$ that is required in (12) assumes that the derivative of measurement information is available which should be avoided in practice. We now present a computational procedure that circumvents the need for computing measurement derivatives and that enables one to construct observers based on the second-order models.

Second-Order Transformation of Continuous Kalman Filtering Equations

This section presents a transformation of the continuous time first-order Kalman filter to a discrete time set of second-order difference equations for digital implementation. The procedure avoids the need for measurement derivative information. In addition, the sparsity and symmetry of the original mass, damping and stiffness matrices can be maintained. Prior to describing the numerical integration procedure, a transformation based on generalized momenta is presented which is later used to improve computational efficiency of the equation solution.

Generalized Momenta

Instead of the conventional transformation (3) of the second-order structural system (1) into a first-order form, let us consider the following generalized momenta (Jensen, 1974; Felippa and Park, 1978):

$$\begin{align*}
\begin{cases}
  a) & x_1 = x \\
  b) & x_2 = AM \dot{x}_1 + C x_1
\end{cases}
\end{align*}$$

(13)

where $A$ and $C$ are constant matrices to be determined. Time differentiation of (13b) yields

$$\dot{x}_2 = AM \ddot{x}_1 + C \dot{x}_1$$

(14)
Substituting (1) via (13a) into (14), one obtains

\[ \dot{x}_2 = A(Bu + Gw) - (AD - C)\dot{x}_1 - AKx_1 \]  

(15)

Finally, pairing of (13b) and (15) gives the following first-order form:

\[
\begin{bmatrix}
AM & 0 \\
AD - C & I
\end{bmatrix}
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix}
+ 
\begin{bmatrix}
C & -I \\
AK & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
A(Bu + Gw)
\end{bmatrix}
\]

(16)

The associated Kalman filtering equation can be shown to be of the following form:

\[
\begin{bmatrix}
AM & 0 \\
AD - C & I
\end{bmatrix}
\begin{bmatrix}
\dot{\tilde{x}}_1 \\
\dot{\tilde{x}}_2
\end{bmatrix}
+ 
\begin{bmatrix}
C & -I \\
AK & 0
\end{bmatrix}
\begin{bmatrix}
\tilde{x}_1 \\
\tilde{x}_2
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
ABu
\end{bmatrix}
+ 
\begin{bmatrix}
AM \\
AD - C & I
\end{bmatrix}
\begin{bmatrix}
\tilde{L}_1 \\
\tilde{L}_2
\end{bmatrix}
\]

(17)

where

\[ \tilde{L}_1 = (\tilde{H}_1 U + \tilde{H}_2 S)TR^{-1} \]

\[ \tilde{L}_2 = (\tilde{H}_1 ST + \tilde{H}_2 L)TR^{-1} \]

and \( \tilde{H}_1 \) and \( \tilde{H}_2 \) correspond to a modified form of measurements expressed as

\[ z = H_1 x + H_2 \dot{x} = \tilde{H}_1 x_1 + \tilde{H}_2 x_2 \]  

(18)

Clearly, as in the conventional first-order form (9), \( \dot{x}_1 \) and \( \dot{x}_2 \) in (17) are now two independent variables. Specifically, the case of \( A = M^{-1} \) and \( C = 0 \) corresponds to (3) with \( x_2 = \dot{x}_1 \). However, as we shall see below, the Kalman filtering equations based on the generalized momenta (13) offer several computational advantages over (3).

**Numerical Integration**

At this juncture it is noted that in the previous section one first performs the elimination of \( \dot{x}_1 \) in order to obtain a second-order observer, then performs the numerical solution of the resulting second-order observer. This approach has the disadvantage of having to deal with the time derivative of measurement data. To avoid this, we will first integrate numerically the associated Kalman filtering equation (17).
The direct time integration formula we propose to employ is a mid-point version of the trapezoidal rule:

\[
\begin{align*}
\begin{cases}
\mathbf{a)} & \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}^{n+1/2} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}^n + \delta \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}^{n+1/2} \\
\mathbf{b)} & \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}^{n+1} = 2 \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}^{n+1/2} - \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}^n
\end{cases}
\end{align*}
\]

(19)

where the superscript \(n\) denotes the discrete time interval \(t^n = nh\), \(h\) is the time increment and \(\delta = h/2\).

Time discretization of (17) by (19a) at the \(n + 1/2\) time step yields

\[
\begin{align*}
\begin{bmatrix} AM & 0 \\ AD - C & I \end{bmatrix} \begin{bmatrix} \tilde{x}_1^{n+1/2} \\ \tilde{x}_2^{n+1/2} \end{bmatrix} + \delta \begin{bmatrix} C & -I \\ AK & 0 \end{bmatrix} \begin{bmatrix} \tilde{x}_1^{n+1/2} \\ \tilde{x}_2^{n+1/2} \end{bmatrix} \\
\delta \begin{bmatrix} AM & 0 \\ AD - C & I \end{bmatrix} \begin{bmatrix} \tilde{L}_1 \\ \tilde{L}_2 \end{bmatrix} \tilde{z}^{n+1/2} + \delta \begin{bmatrix} 0 \\ ABu^{n+1/2} \end{bmatrix}
\end{align*}
\]

(20)

The above difference equations require the solution of matrix equations of \(2N\) variables, namely, in terms of the two variables \(\tilde{x}_2^{n+1/2}\) and \(\tilde{x}_1^{n+1/2}\), each with a size of \(N\). To reduce the above coupled equations of order \(2N\) into the corresponding ones of order \(N\), we proceed in the following way by exploiting the nature of parametric matrices of \(A\) and \(C\) as introduced in (13). To this end, we write out (20) as two coupled difference equations as follows:

\[
\begin{align*}
AM(\tilde{x}_1^{n+1/2} - \tilde{x}_1^n) + \delta(C\tilde{x}_1^{n+1/2} - \tilde{x}_2^{n+1/2}) \\
= \delta AM\tilde{L}_1\tilde{z}^{n+1/2}
\end{align*}
\]

(21)

\[
\begin{align*}
(AD - C)(\tilde{x}_1^{n+1/2} - \tilde{x}_1^n) + (\tilde{x}_2^{n+1/2} - \tilde{x}_2^n) + \delta AK\tilde{x}_1^{n+1/2} \\
= \delta(AD - C)\tilde{L}_1\tilde{z}^{n+1/2} + \delta\tilde{L}_2\tilde{z}^{n+1/2} + \delta ABu^{n+1/2}
\end{align*}
\]

(22)

Multiplying (22) by \(\delta\) and adding the resulting equation to (21) yields

\[
\begin{align*}
A(M + \delta D + \delta^2 K)\tilde{x}_1^{n+1/2} &= (AM + \delta(AD - C))\tilde{x}_1^n + \delta\tilde{x}_2^n \\
+ \{\delta AM\tilde{L}_1 + \delta^2(AD - C)\tilde{L}_1 + \delta^2\tilde{L}_2\}\tilde{z}^{n+1/2} + \delta^2 ABu^{n+1/2}
\end{align*}
\]

(23)

Of several possible choices for matrices \(A\) and \(B\), we will examine

\[
\begin{cases}
\mathbf{a)} & A = I, \quad C = D \\
\mathbf{b)} & A = M^{-1}, \quad C = 0
\end{cases}
\]

(24)
The choice of (24a) reduces (23) to:

\[(M + \delta D + \delta^2 K)\ddot{x}_1^{n+1/2} = M\ddot{x}_1^n + \delta\ddot{x}_2^n + \delta^2 Bu^{n+1/2}\]
\[+ \delta(M\ddot{L}_1 + \delta\ddot{L}_2)\ddot{z}^{n+1/2}\]  

(25)

so that once \(\ddot{x}_1^{n+1/2}\) is computed, \(\ddot{x}_2^{n+1/2}\) is obtained from (22) rewritten as

\[\ddot{x}_2^{n+1/2} = \ddot{x}_2^n + \delta\hat{g}^n - \delta K\ddot{x}_1^{n+1/2}\]

(26)

where

\[\hat{g}^n = Bu^{n+1/2} + \ddot{L}_2\ddot{z}^{n+1/2}\]

(27)

which is already computed in order to construct the right-hand side of (25). Hence, \(K\ddot{x}_1^{n+1/2}\) is the only additional computation needed to obtain \(\ddot{x}_2^{n+1/2}\). It is noted that neither any numerical differentiation nor matrix inversion is required in computing \(\ddot{x}_2^{n+1/2}\). This has been achieved through the introduction of the general transformation (13) and the particular choice of the parameter matrices given by (24a).

On the other hand, if one chooses the conventional representation (24b), the solution of \(\ddot{x}_1^{n+1/2}\) is obtained from (23)

\[(M + \delta D + \delta^2 K)\ddot{x}_1^{n+1/2} = (M + \delta D)\ddot{x}_1^n + \delta M\ddot{x}_2^n\]
\[+ \delta((M + \delta D)\ddot{L}_1 + \delta M\ddot{L}_2)\ddot{z}^{n+1/2} + \delta^2 Bu^{n+1/2}\]  

(28)

Once \(\ddot{x}_1^{n+1/2}\) is obtained, \(\ddot{x}_2^{n+1/2}\) can be computed either by

\[\ddot{x}_2^{n+1/2} = (\ddot{x}_1^{n+1/2} - \ddot{x}_1^n)/\delta - \ddot{L}_1\ddot{z}^{n+1/2}\]

(29)

which is not accurate due to the numerical differentiation to obtain \(\ddot{x}_1^{n+1/2}\), or by (22)

\[\ddot{x}_2^{n+1/2} = \ddot{x}_2^n + \delta\hat{g}^n - \delta M^{-1}K\ddot{x}_1^{n+1/2} - \]
\[M^{-1}D(\ddot{x}_1^{n+1/2} - \ddot{x}_1^n) + \delta M^{-1}D\ddot{L}_1\ddot{z}^{n+1/2}\]

(30)

which involves two additional matrix-vector multiplications, when \(D \neq 0\), as compared with the choice of \(A = I\) and \(C = D\). Thus (24a) is the preferred representation in a first-order form of the second-order structural dynamics equations (1) and is used in the remainder of this work.
Decoupling Of Difference Equations

We have seen in the previous section, instead of solving the first-order Kalman filtering equations of $2n$ variables for the structural dynamics systems (1), the solution of the implicit time-discrete observer equation (25) of $n$ variables can potentially offer a substantial computational saving by exploiting the reduced size and sparsity of $M, D$ and $K$. This assumes that $z^{n+1/2}$ and $u^{n+1/2}$ are available, which is not the case since at the $n^{th}$ time step

$$u^{n+1/2} = -\bar{Z}_1 \dot{z}_1^{n+1/2} - \bar{Z}_2 \dot{z}_2^{n+1/2}$$

$$z^{n+1/2} = z^{n+1/2} - \bar{H}_1 \dot{z}_1^{n+1/2} - \bar{H}_2 \dot{z}_2^{n+1/2}$$

requires both $\dot{z}_1^{n+1/2}$ and $\dot{z}_2^{n+1/2}$ even if $z^{n+1/2}$ is assumed to be known from measurements or by solution of (1). Note in (32), the control gain matrices are transformed by

$$\bar{Z}_1 = Z_1 - Z_2 M^{-1} A^{-1} C, \quad \bar{Z}_2 = Z_2 M^{-1} A^{-1}$$

There are two distinct approaches to uncouple (25) and (26) as described in the following sections.

**Discrete Time Update**

Equations (31) and (32) can be approximated using

$$\dot{z}^{n+1/2} \approx z^n - \bar{H}_1 \dot{z}_1^n - \bar{H}_2 \dot{z}_2^n$$

$$u^{n+1/2} \approx -\bar{Z}_1 \dot{z}_1^n - \bar{Z}_2 \dot{z}_2^n$$

This approximation leads to a discrete time update of the control force and state correction terms which is analogous to that which exists in experiments where a finite bandwidth of measurement updates occurs. For discrete time approximation, the step size $h = t^{n+1} - t^n$ should be chosen to match the time required to acquire, process and output a control update.

Discrete time simulation is quite simple to implement as the control force and state corrections are treated with no approximation on the right-hand-side (RHS) of (25) and (26). Should continuous time simulation be required, a different approach is necessary.

**Continuous Time Update**

To simulate the system given in (25) and (26) in continuous time, strictly speaking, one must rearrange (25) and (26) so that the terms involving $\dot{z}_1^{n+1/2}$ and $\dot{z}_2^{n+1/2}$ are augmented
to the left-hand-side (LHS) of the equations. However, this augmentation into the solution matrix \((M + \delta D + \delta^2 K)\) would destroy the computational advantages of the matrix sparcity and symmetry. Thus, a partitioned solution procedure has been developed for continuous time simulation as described in (Park and Belvin, 1991). The procedure, briefly outlined herein, maintains the control force and state correction on the RHS of the equations as follows.

First, \(\dot{x}_1^{n+1/2}\) and \(\dot{x}_2^{n+1/2}\) are predicted by

\[
\begin{align*}
\dot{x}_1^{n+1/2} &= \dot{x}_1^n, \\
\dot{x}_2^{n+1/2} &= \dot{x}_2^n
\end{align*}
\tag{35}
\]

However, instead of direct substitution of the above predicted quantity to obtain \(u_p^{n+1/2}\) and \(z_p^{n+1/2}\) based on (31) and (32), equation augmentations are introduced to improve the accuracy of \(u_p^{n+1/2}\) and \(z_p^{n+1/2}\). Of several augmentation procedures that are applicable to construct discrete filters for the computations of \(u^{n+1/2}\) and \(z^{n+1/2}\), we substitute (26) into (31) and (32) to obtain

\[
\begin{align*}
\begin{cases}
\dot{u}^{n+1/2} &= -\ddot{Z}_1 \ddot{x}_1^{n+1/2} - \ddot{Z}_2 (\dot{x}_2^n - \delta K \ddot{x}_1^{n+1/2} + \\
&\quad \quad \delta Bu^{n+1/2} + \delta \ddot{L}_2 \dddot{z}^{n+1/2}) \\
\dot{z}^{n+1/2} &= z^{n+1/2} - \ddot{H}_2 \ddot{x}_1^{n+1/2} - \\
&\quad \quad \ddot{H}_2 (\dot{x}_2^n - \delta K \ddot{x}_1^{n+1/2} + \delta Bu^{n+1/2} + \delta \ddot{L}_2 \dddot{z}^{n+1/2})
\end{cases}
\end{align*}
\tag{36}
\]

Rearranging the above coupled equations, one obtains

\[
\begin{align*}
\begin{bmatrix}
(I + \delta \dddot{Z}_2 B) & \delta \dddot{Z}_2 \dddot{L}_2 \\
\delta \dddot{H}_2 B & (I + \delta \dddot{H}_2 \dddot{L}_2)
\end{bmatrix}
\begin{bmatrix}
\dot{u}^{n+1/2} \\
\dot{z}^{n+1/2}
\end{bmatrix}
= 
\begin{bmatrix}
-\dddot{Z}_2 \dddot{x}_2^n - (\dddot{Z}_1 - \delta \dddot{Z}_2 K) \ddot{x}_1^{n+1/2} \\
z^{n+1/2} - \dddot{H}_2 \dddot{x}_2^n - (\dddot{H}_1 - \delta \dddot{H}_2 K) \ddot{x}_1^{n+1/2}
\end{bmatrix}
\end{align*}
\tag{37}
\]

which corresponds to a first order filter to reduce the errors in computing \(\dddot{x}_2 = M \dddot{x} + D \dddot{x}\). A second-order discrete filter for computing \(u\) and \(z\) can be obtained by differentiating \(u\) and \(z\) to obtain

\[
\begin{align*}
\dot{u} &= -\dddot{Z}_1 \ddot{x}_1 - \dddot{Z}_2 \ddot{x}_2 \\
\dot{z} &= \dddot{z} - \dddot{H}_1 \ddot{x}_1 - \dddot{H}_2 \ddot{x}_2
\end{align*}
\tag{38}
\]

and then substituting \(\dddot{x}_1\) and \(\dddot{x}_2\) from (17). Subsequently, (19) is applied to integrate the equations for \(u\) and \(z\) which yields

\[
\begin{align*}
\begin{bmatrix}
I + \delta \dddot{Z}_2 B + \delta^2 \dddot{Z}_1 M^{-1} B & \delta (\dddot{Z}_2 \dddot{L}_2 + \dddot{Z}_1 \dddot{L}_1 + \delta \dddot{Z}_1 M^{-1} \dddot{L}_2) \\
\delta (\dddot{H}_2 B + \delta \dddot{H}_1 M^{-1} B) & I + \delta \dddot{H}_1 (\dddot{L}_1 + \delta M^{-1} \dddot{L}_2) + \delta \dddot{H}_2 \dddot{L}_2
\end{bmatrix}
\begin{bmatrix}
\dot{u}^{n+1/2} \\
\dot{z}^{n+1/2}
\end{bmatrix}
= 
\begin{bmatrix}
\dot{u}^{n+1/2} \\
\dot{z}^{n+1/2}
\end{bmatrix}
\end{align*}
\tag{39}
\]
The net effects of this augmentation are to filter out the errors committed in estimating both \( \hat{x}_1 \) and \( \hat{x}_2 \). Solution of (39) for \( u^{n+1/2} \) and \( \bar{z}^{n+1/2} \) permits (25) and (26) to be solved in continuous time for \( \hat{x}_1^{n+1/2} \) and \( \hat{x}_2^{n+1/2} \). Subsequently, (29b) is used for \( \hat{x}_1^{n+1} \) and \( \hat{x}_2^{n+1} \).

The preceding augmentation (39) leads to an accurate estimate of the control force and observer error correction at the \((n+1/2)\) time step. Although (39) involves the solution of an additional algebraic equation, the equation size is relatively small (size = number of actuators \((m)\) plus the number of measurements \((r)\)). Thus, (39) is an efficient method for continuous time simulation of the Kalman filter equations provided the size of (39) is significantly lower than the first order form of (4). The next section discusses the relative efficiency of the present method and the conventional first order solution. More details on the equation augmentation procedure (39) may be found in Park and Belvin (1991).

Finally, it is noted that by following a similar time discretization procedure adopted for computing \( \hat{x}_1^{n+1/2} \) and \( \hat{x}_2^{n+1/2} \), the structural dynamics equation (1) can be solved by

\[
\begin{cases}
(M + \delta D + \delta^2 K)x_1^{n+1/2} = Mx_1^n + \delta x_2^n + \delta^2 Bu^{n+1/2} \\
x_2^{n+1/2} = x_2^n + \delta Bu^{n+1/2} - \delta Kx_1^{n+1/2}
\end{cases}
\]  

(40)

Thus, numerical solutions of the structural dynamics equation (1) and the observer equation (20) can be carried out within the second-order solution context, thus realizing substantial computational simplicity compared with the solution of first-order systems of equations (4) and the corresponding first-order observer equations (6).

It is emphasized that the solutions of both the structural displacement \( x \) and the reconstructed displacement \( \hat{x} \) employ the same solution matrix, \((M + \delta D + \delta^2 K)\). The computational stability of the present procedure can be examined as investigated in Park (1980) and Park and Felippa (1983, 1984). The result, when applied to the present case, can be stated as

\[
\delta^2 \lambda_{\text{max}} \leq 1
\]  

(41)

where \( \lambda_{\text{max}} \) is the maximum eigenvalue of

\[
(\lambda^2 I + \lambda \bar{z}_2 B + \bar{z}_1 M^{-1} B)y = 0
\]  

(42)

Experience has shown that \(|\lambda_{\text{max}}|\) is several orders of magnitude smaller than \( \mu_{\text{max}} \) of the structural dynamics eigenvalue problem:

\[
\mu My = Ky
\]  

(43)

Considering that a typical explicit algorithm has its stability limit \( \mu_{\text{max}} \cdot h \leq 2 \), the maximum step size allowed by (42) is in fact several orders of magnitude larger than allowed by any explicit algorithm.
Computational Efficiency

Solution of the Kalman filtering equations in second-order form is prompted by the potential gain in computational efficiency due to the beneficial nature of matrix sparsity and symmetry in the solution matrix of the second-order observer equations. There is an overhead to be paid for the present second-order procedure, that is, the additional computations introduced to minimize the control force and observer error terms on the right-hand-side of the resulting discrete equations. The following paragraphs show the second-order solution is most advantageous for observer models with sparse coefficient matrices $M$, $D$ and $K$.

Solution of the first order Kalman filter equation (6) or the second-order form (25-26, 39) may be performed using a time discretization as given by (19). For linear time invariant (LTI) systems, the solution matrix is decomposed once and subsequently upper and lower triangular system solutions are performed to compute the observer state at each time step. Thus, the computations required at each time step result from calculation of the RHS and subsequent triangular system solutions. For the results that follow, the number of floating point operations per second (flops) are estimated for LTI systems of order $O(N)$. In addition, it is assumed that the mass, damping and stiffness matrices ($M$, $D$ and $K$) are symmetric and banded with bandwidth $\alpha N$, where $0 \leq \alpha \leq (0.5 - \frac{1}{2N})$.

The first-order Kalman filter equation (6) requires $(4N^2 + 2Nr + O(N))$ flops at each time step. The discrete time second-order Kalman filter solution (25-26, 33-34) require $(8\alpha^2N^2 + 2\alpha N^2 + 3Nm + 4Nr + O(N))$ flops and the continuous time second-order Kalman filter (25-26, 39) require $(8\alpha N^2 + 2\alpha N^2 + 5Nm + 6Nr + (r + m)^2 + O(N))$ flops at each time step. To examine the relative efficiency of the first-order and second-order forms, several cases are presented as follows.

First, a worst case condition is examined whereby $M$, $D$ and $K$ are fully populated ($\alpha = 0.5 - \frac{1}{2N}$) and $r = m = N$. For this condition, the number of flops are:

\[
\begin{align*}
\text{First Order} & \quad 6N^2 + O(N) \\
\text{Second Order Discrete} & \quad 10N^2 + O(N) \\
\text{Second Order Continuous} & \quad 18N^2 + O(N)
\end{align*}
\]

Thus, for non-sparse systems with large numbers of sensors and actuators relative to the system order, the first order Kalman filter is 300 percent more efficient than the second-order continuous Kalman filter solution presented herein.

For structural systems, $M$, and $K$ are almost always banded. In addition, the number of sensors and actuators is usually small compared to the system order $N$. Hence, the value of $\alpha$ for which the second-order form becomes more computationally attractive than the first order form must be determined. If the assumption is made that the number of
actuators \( (m) \) and the number of measurements \( (r) \) is proportional to the bandwidth \( r = m = \alpha N \), the value of \( \alpha \) which renders the second-order solution more efficient is readily obtained. For the second-order discrete Kalman filter, when \( \alpha \leq 0.394 \) the second-order form is more efficient. Similarly, the second-order continuous Kalman filter form is more efficient when \( \alpha \leq 0.279 \). Since \( \alpha \) obtains values approaching 0 when a modal based structural representation is used with few sensors and actuators, the second-order form can be substantially more efficient than the classical first-order form. A more detailed discussion can be found in Belvin (1989).

Implementation and Numerical Evaluations

The second-order discrete Kalman filtering equation derived in (25) and (26) have been implemented along with the stabilized form of the controller \( u \) and the filtered measurements \( \tilde{z} \) in such a way the observer computational module can be interfaced with the partitioned control-structure interaction simulation package developed previously (Belvin, 1989; Belvin Park, 1991; Alvin and Park, 1991). Table 1 contrasts the present CSI simulation procedure to conventional procedures. It is emphasized that the solution procedure of the present second-order discrete Kalman filtering equations (25) and (26) follows exactly the same steps as required in the solution of symmetric, sparse structural systems (or the plant dynamics in the jargon of control). It is this attribute that makes the present discrete observer attractive from the simulation viewpoint.

The first example is a truss beam shown in Fig. 1, consisting of 8 bays with nodes 1 and 2 fixed for cantilevered motions. The locations of actuator and sensor applications as well as their directions are given in Table 2. Figures 2, 3 and 4 are the vertical displacement histories at node 9 for open-loop, direct output feedback, and dynamically compensated feedback cases, respectively. Note the effectiveness of the dynamically compensated feedback case by the present second-order discrete Kalman filtering equations as compared with the direct output feedback cases. Figure 5 illustrates a testbed evolutionary model of an Earth-pointing satellite. Eighteen actuators and 18 sensors are applied to the system for vibration control and their locations are provided in Tables 3 and 4. Figures 6, 7, and 8 are a representative of the responses for open-loop, direct output feedback, and dynamically compensated cases, respectively. Note that \( u_x \) response by the dynamically compensated case does drift away initially even though the settling time is about the same as that by the direct output feedback case. However, the sensor output are assumed to be noise-free in these two numerical experiments. Although the objective of the present paper is to establish the computational effectiveness of the second-order discrete Kalman filtering equations, we conjecture that for noise-contaminated sensor output for which one would apply dynamic compensated strategies, the relative control performance may turn
out to be the opposite. Further simulations with the present procedure should shed light on the performance of dynamically compensated feedback systems for large-scale systems as they are computationally more feasible than heretofore possible.

Table 5 illustrates the computational overhead associated with the direct output feedback vs. the use of a dynamic compensation scheme by the output present Kalman filtering equations. In the numerical experiments reported herein, we have relied on Matlab software package (Wolfram, 1988) for the synthesis of both the control law gains and the discrete Kalman filter gain matrices. It is seen that the use of the present second-order discrete Kalman filtering equations for constructing dynamically compensated control laws adds computational overhead, only an equivalent of open-loop transient analysis of symmetric sparse systems of order N instead of $2N \times 2N$ dense systems.

Summary

The present paper has addressed the advantageous features of employing the same direct time integration algorithm for solving the structural dynamics equations also to integrate the associated continuous Kalman filtering equations. The time discretization of the resulting Kalman filtering equations is further facilitated by employing a canonical first-order form via a generalized momenta. When used in conjunction with the previously developed stabilized form of control laws (Park and Belvin, 1991), the present procedure offers a substantial computational advantage over the solution methods based on a first-order form when computing with large and sparse observer models.

Computational stability of the present solution method for the observer equation has been assessed based on the stability analysis result of partitioned solution procedures (Park, 1980). To obtain a sharper estimate of the stable step size, a more rigorous computational stability analysis is being carried out and will be reported in the future.

Acknowledgements

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References


Table 1a Partitioned Control-Structure Interaction Equations

Structure: a) \[ M\ddot{q} + D\dot{q} + Kq = f + Bu + Gw \]

Sensor Output: b) \[ z = Hx + v \]

Estimator: c) \[ M\ddot{\hat{q}} + D\dot{\hat{q}} + K\hat{q} = f + Bu + ML_2\gamma \]

Control Force: d) \[ \dot{u} + F_2M^{-1}Bu = F_2(M^{-1}\dot{\hat{p}} + L_2\gamma) + F_1\dot{\hat{q}} \]

Estimation Error: e) \[ \gamma + H_vL_2\gamma = \dot{z} - H_vM^{-1}(\dot{\hat{p}} - Bu) - H_d\dot{\hat{q}} \]

Table 1b Conventional Control-Structure Interaction Equations

Structure: a) \[ \dot{x} = Ax + Ef + Bu + Gw \]

Sensor Output: b) \[ z = Hx + v \]

Estimator: c) \[ \ddot{x} = A\ddot{x} + Ef + \dot{Bu} + L\gamma \]

Control Force: d) \[ u = -F\ddot{x} \]

Estimation Error: e) \[ \gamma = z - (H_d\ddot{q} + H_v\dot{\hat{q}}) \]

where \[ x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}, \quad \ddot{x} = \begin{bmatrix} \dddot{q} \\ \ddot{q} \end{bmatrix} \]

and

\[
H = \begin{bmatrix} H_d & H_v \end{bmatrix}, \quad L = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}, \quad E = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix} \]

\[
A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}D \end{bmatrix}, \quad \dot{B} = \begin{bmatrix} 0 \\ M^{-1}B \end{bmatrix}, \quad F = [F_1 \ F_2] \]

Table 1b Conventional Control-Structure Interaction Equations
TABLE 2a:
Actuator Placement for Truss Example Problem

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<td>2</td>
<td>18</td>
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<td>3</td>
<td>9</td>
<td>y</td>
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<tr>
<td>4</td>
<td>9</td>
<td>x</td>
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TABLE 2b:
Sensor Placement for Truss Example Problem

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<td>Position</td>
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<td>x</td>
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**TABLE 3:**

Actuator Placement for EPS Example Problem

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### TABLE 4:
Sensor Placement for EPS Example Problem

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<td>18</td>
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## TABLE 5:

CPU Results for ACSIS Sequential and Parallel Versions

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<td>Transient</td>
<td>4.5</td>
<td>5.6</td>
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<td>Truss</td>
<td>FSFB</td>
<td>9.4</td>
<td>10.2</td>
</tr>
<tr>
<td></td>
<td>K. Filter</td>
<td>13.0</td>
<td>10.7</td>
</tr>
<tr>
<td>582 DOF</td>
<td>Transient</td>
<td>98.6</td>
<td>100.3</td>
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<td>EPS7</td>
<td>FSFB</td>
<td>190.2</td>
<td>294.5</td>
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<tr>
<td></td>
<td>K. Filter</td>
<td>284.2</td>
<td>321.5</td>
</tr>
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</table>
Figure 1: Truss Beam Problem
Truss Model: Open Loop Transient Response

Figure 2: Truss Transient Response

Truss Model: Full State Feedback Response

Figure 3: Truss FSFB Response
Truss Model: Controlled Response with Kalman Filter

Figure 4: Truss Response with Filter
Figure 5: Evolutionary Earth-Pointing Satellite
EPS7 Model: Open Loop Transient Response

Figure 6: EPS Transient Response
EPS7 Model: Full State Feedback Response

![Graph showing deflection vs time for EPS FSFB Response]

Time, sec

-- -- -- Node 45, ux

-- -- -- Node 45, uz

-- -- -- Node 45, uy

**Figure 7: EPS FSFB Response**

---

Model: Controlled Response w/Kalman Filter

![Graph showing deflection vs time for EPS Response with Filter]

Time, sec

-- -- -- Node 45, ux

-- -- -- Node 45, uz

-- -- -- Node 45, uy

**Figure 8: EPS Response with Filter**
INTELLIGENT STRUCTURES

Edited by

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ELSEVIER APPLIED SCIENCE
LONDON and NEW YORK
PARALLEL COMPUTATIONS AND CONTROL
OF ADAPTIVE STRUCTURES

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ABSTRACT

The equations of motion for structures with adaptive elements for vibration control are presented for parallel computations to be used as a software package for real-time control of flexible space structures. A brief introduction of the state-of-the-art parallel computational capability is also presented. Time marching strategies are developed for an effective use of massive parallel mapping, partitioning and the necessary arithmetic operations. An example is offered for the simulation of control-structure interaction on a parallel computer and the impact of the approach presented herein for applications in other disciplines than aerospace industry is assessed.

1. Introduction

Active suppression of structural vibrations or active control of flexible structures has made considerable progress in recent years. As a result, it is now possible to actively suppress vibrations in mechanical systems emanating from machine foundations, in robotic manufacturing arms, truss-space structures and automobile suspension systems. A common characteristic to these applications of active control theory has been its discrete actuators and discrete sensors, ranging from proof mass actuators and gyro
dampers to strain gages and accelerometers. Because most available discrete actuators are inertia force-oriented devices, actuation often triggers coupling between the actuator dynamics and structural transients. A practical consequence of such coupling is a limitation of achievable final residual vibration level if both the actuator and structure possess insufficient passive damping level. It is noted that structures made of high stiffness composite materials have very low intrinsic damping, hence limiting the achievable residual vibration level for space maneuvering and space disturbance rejection purposes. This has been a motivating factor for the development of distributed actuators and sensors which are often embedded as an integral part of the structure so that control force can be effectively maintained by strain actuation, thus alleviating the undesirable actuator dynamics associated with inertia-force actuation.

Various activities that are being pursued by many investigators on the subject of adaptive structures may be categorized into three major thrusts: device developments, control laws synthesis and experimental demonstrations, and hardware/software implementation. The device developments effort has been the objective of many material scientists [1-3]. As the applications needs increase it is expected that functionally more reliable electrostrictive and magnetostrictive elements will be available for use in active control/strain damping with improved product quality.

The study of control laws synthesis and demonstration employing adaptive elements has been one of the predominant activities in recent years. As scientists accumulate experience in the characterization of the coupling between the structure and the adaptive element, the applications will then be expanded from the current beam-like structures to the truss long beams, plates and shells. In order to effectively utilize as many adaptive elements as necessary for actively controlling the vibration of such large-scale structures in real-time operations, it will be imperative that the software/hardware components in the real-time control loop must be able to process data fast enough so that control commands and the measurements can be carried without saturating and/or jamming the control system.

With the advent of new technology in distributed actuators and sensors [4-9], it appears that a combination of decentralized/distributed and hierarchical control strategies can be a viable alternative to conventional centralized control strategies. The real-time computer control of such systems as well as design of such control systems through iterating on simulations and hardware realizations thus will require the processing of a vast amount of data from and to the distributed actuators and sensors. A significant part of such data processing for the decentralized actuators and sensors is planned to be self-managed, viz., there will be embedded microprocessors for each actuator and sensor pair or for each group of them. However, the necessary links between the decentralized control systems and the global control system as well as the necessary global control strategy will still require computational power far in excess of presently available real-time data processing capability. In addition, if one contemplates the performance of neural-network control or adaptive control for onboard real-time control of large-scale space structures, the computational need will dramatically increase beyond the current capability. As a case in point, even for the control of 20-bay truss beam vibrations by
three proof mass actuators and six sensors, NASA/Langley is relying on CRAY-XMP for adequate real-time data processing requirements.

The objective of this paper is thus to present a computational framework by which one can bring the two emerging new technologies together, namely, the distributed actuators and sensors and the parallel computing capability, toward the real-time control of vibrations in large structural systems such as space stations, space cranes and in-space construction facilities. We will then discuss the potential for applying such a space technology to mitigate and/or minimize the earthquake damage of ground structures such as high-rise buildings, bridges and lifeline equipment.

2. Models for Structures with Embedded Actuators and Sensors

The coupling between the structural behavior and an adaptive electrostrictive element, whether it is embedded or surface-mounted, is primarily due to the following constitutive relation [3,10-12]:

\[
\begin{align}
\{ e \} &= \begin{bmatrix} \varepsilon & g \\ -g^T & c \end{bmatrix} \{ v \} \\
\{ \sigma \} &= \begin{bmatrix} e \end{bmatrix}
\end{align}
\]  

(1)

where \( e \) and \( v \) are the electrical displacement (charges/unit area) and the electric field (volt/unit area), \( \sigma \) and \( \varepsilon \) are the stress and strain, and \( \varepsilon, g \) and \( c \) are the constitutive coefficient matrices, respectively. For magnetostrictive elements, one needs to replace \( e \) and \( v \) by the magnetic field (H) and the magnetic induction (B), respectively, and the subsequent derivations will hold without any loss of generality.

The coupled equations of motion for the structure and the adaptive elements can proceed by augmenting the standard procedure for the structure with the electric transient equations plus the appropriate modification of the structural equilibrium equations that reflect the coupled constitutive equations (1). The resulting coupled structural-piezoelectric equations of motion take the following form [13-15]:

\[
\begin{align}
\text{Structure:} & \quad a) \quad M\ddot{q} + D\dot{q} + (K_s + K_a)q = f + Sa \\
& \quad q(0) = q_0, \quad \dot{q}(0) = \dot{q}_0 \\
\text{Sensor Output:} & \quad b) \quad y = H_p q + H_r \dot{q} + H_s a \\
\text{Actuator:} & \quad c) \quad \dot{a} + \Theta a = B_a u - \tilde{S}^T \begin{bmatrix} q \\ \dot{q} \end{bmatrix} \\
\text{Controller:} & \quad d) \quad \ddot{u} + Gu = Ly
\end{align}
\]

(2)

where

\[ a = \begin{bmatrix} e \\ v \end{bmatrix}, \quad u = \begin{bmatrix} I_0 \\ V_0 \end{bmatrix} \]
In the preceding equations, $M$ is the mass matrix, $D$ is the damping matrix, $K$ is the stiffness matrix due to structural strain-displacement relations and $K_a$ is the stiffness matrix due to the strain actuation. $f(t)$ is the applied force. $S$ is the actuator projection matrix. $H_p$, $H_r$ and $H_0$ are the sensor calibration gain matrices. $\Theta$ is the actuator dynamic characteristics, $B_a$ is the gain matrix that translates the applied current/charge and voltage into the corresponding strain and strain rate where $\tilde{S}$ is the transducer conversion gain. $q$ is the generalized displacement vector and and the superscript dot denotes time differentiation, and $u$ is the control law that consists of the applied current (or charge), $I_0$, and voltage across the electrostrictive devices, $V_0$. $G$ is the electric circuit characteristics, and $L$ is the optimum direct feedback gain matrix. The case of dynamic compensations can be augmented to (2) by introducing an observer. But in subsequent discussions we limit ourselves to direct feedback cases only.

It is noted that the control laws, unlike conventional control-structure interaction systems, are not directly fed back into the structural equations. Instead, the controller is simply a regulator controlling the electric charge, the voltage or the current. These regulated electric quantities are then fed into the piezoelectric sensors and actuators. Hence, it is the piezoelectric actuation that triggers feedback into the structures.

3. Parallel Computations for the Dynamics of Adaptive Structures

The earliest recorded computational results in mechanics were the parabolic trajectory calculations of a falling body by Galileo [16]. Since then, most scientific computations have been carried out by anthropomorphic algorithms, viz. step-by-step binary and/or decimal arithmetics. To set the stage properly for the present objective, parallel computations of the dynamic response of structures with distributed adaptive elements, we recall a passage by Kepler to John Napier, the inventor of a logarithmic table:

Newton was essentially dependent upon the results of Kepler's calculations, and these calculations might not have been completed but for the aid of that logarithms afforded. Without the logarithms, ..., the development of modern science might have been very different [17].

In terms of the present day data processing requirement, Napier's logarithmic table in 1614 contained about 100 kilobytes, which was perhaps the most important computational aid to Kepler and Newton. Three and one-half centuries later we are witnessing gigabytes of tables being stored and retrieved at our disposal [18]. But these tables complement the weakness of the human mind and computational speed: long term memory and human arithmetic speed. In addition, for problems requiring a sequential nature of computations, i.e., ballistic trajectories which deal only with the position and velocity of a single shell or quasi-static equilibrium equations of a building structure, the computing activities do not interact with "time" and the computing efficiency affects only the humanpower efficiency for completing the computational task.
There are many important scientific and engineering endeavors whose computations must be fast enough for real-time delivery of the computed results. A classical example was Richardson's lattice model for weather prediction by numerical process in 1922 [19]. The motivation for adopting such a lattice concept was due to the fact that the equation state at each lattice node takes on a different value set in time and an efficient way of interchanging and transmitting the nodal values at each time step was mandatory if the computations were to be carried out in real-time to predict the weather. Indeed, this was the dawn of the parallel computing era, even though the basic idea had to wait for its validity for 60 years. Today, many controls engineering activities have been implemented by using computers so that their intended functions can be monitored and controlled in real-time. These include chemical processing, autopiloting and vibration control of simple structures. It is important to note that the computational framework employed for such applications is based on sequential architecture. Hence, we believe that future improvements that can deal with large parameter models and large parameter controls must adopt a parallel computational framework. One such area is the dynamics and control of large structures with distributed/embedded adaptive elements.

In order to carry out the necessary parallel computations, there are three distinct steps that must be addressed: discretizing the structure into appropriate partitions, mapping the physical partitions onto the processors, and step advancing of the equation states. These will be discussed below.

3.1 Partitioning and Mapping of Adaptive Structures

Ideally, if the sensor and actuator leads fall on the discrete nodes, no spatial interpolation would be necessary. However, such a situation is either difficult to realize or may prohibit the use of spatially convolving sensors [20] that are known to filter certain harmonic signals for minimizing phase lag in the feedback loop. Hence, we will assume that the sensor and actuator characteristics can be interpolated to the discrete nodes; in this way the partition boundaries can be chosen arbitrarily regardless of the physical locations of the sensor and actuator leads. In addition, this approach can lead to a natural embedding of the sensor and actuator characteristics into the finite element or boundary integral structural models. Once the partitioning is accomplished, the next step is to map the discrete partitions for adaptive elements onto the corresponding multiprocessors.

Consider an adaptive structure that has been modeled as a set of discrete elements as shown in Fig. 1. In a sequential computing environment, in order to advance the necessary computations for the present states, the arithmetic operations are carried out step-by-step for each node at a time. Hence, each nodal-state computations is performed in a manner similar to one courier delivering and picking up all the mails throughout the entire routes. In a parallel computational environment, in contrast, there can be as many couriers as necessary who comb through the routes concurrently in order to pick up and deliver all the mail at once. One of the most popular concepts in executing such tasks is the hypercube architecture (see Fig. 2) whose every node
Fig. 1  Discrete Model of Adoptive Structures

Fig. 2  Hypercube Interconnection Network of a 32-Processor
(each node represents a processor)
is associated with a processor. Thus, to process the necessary computations for an adaptive structure with 19 partitions, one can assign the 19 adaptive elements to 19 processors as shown in Fig. 3. The procedure for assigning the physical domain (elements) to the parallel processors with minimal interprocessor communications is called mapping.

Of several techniques available for the processor mapping of the computational domains [22], we will adopt a heuristic mapper developed by Farhat [23] since it can accommodate both the synchronous and asynchronous cases with robust and acceptable complexities. An application of this mapping technique for modeling a bulkhead substructure for massively parallel computing is shown in Fig. 4. A similar mapping can be used for parallel computations of adaptive structures.

3.2 Parallel Data Structure and Algorithms

We will assume that each processor is assigned to carry out all the necessary computations for at least one set of a sensor, an actuator, and a controller or a group of them. Therefore, the word partition does not necessarily imply a finite element; it can be a substructure, an element or even a sublayer within the composite layer that includes a sensor or an actuator. In carrying out the step-advancing in time, one may invoke an implicit or explicit direct time integration algorithm. When an implicit algorithm is employed, one needs to communicate not only the state variable vectors but also the associated matrices, i.e., the stiffness matrix, among the processors. Although we will show our results using implicit algorithms, we will, for illustrative purposes, restrict ourselves to an explicit direct time integration algorithm as it is intrinsically parallel and the data structure aspects can be explained more succinctly via an explicit algorithm. It should, however, be mentioned that the choice of the solution algorithm can greatly influence the design and implementation of the necessary mapping and data structure.

Consider the explicit integration of the equations of motion for the structure (2a) as recalled here:

\[ -M\ddot{q} + f_{int} = f + f_{cont} \]  

where \( f_{int} \) and \( f_{cont} \) are the internal and applied control forces, respectively, given by

\[ f_{int} = Dq + (K_s + K_a)q \]

\[ f_{cont} = Sa \]

The use of the central difference algorithm to integrate (3) leads to the following difference equations in time

\[
\begin{align*}
\dot{q}^{n+\frac{1}{2}} &= q^{n-\frac{1}{2}} + hM^{-1}(f^n + f_{cont}^n - f_{int}^n) \\
q^{n+1} &= q^n + h\dot{q}^{n+\frac{1}{2}}
\end{align*}
\]  

(4)
Fig. 3  Physical Domain and Its Mapping Onto Hypercube Processors
Fig. 4  Decomposition of the Structure with
"Finite Element Chips"
Fig. 5  Partitioning and Communication Requirement
4. Implementation and Illustrative Example

The mapping, partitioning and data structures above discussed have been implemented based on a shared-memory concurrent machine (Alliant FX/8) by modifying the software framework developed for finite element computations [26] and the control-structure interaction simulation and design software developed in [27, 28]. At present the following specialized systems of equations are implemented:

\[
\begin{align*}
\text{Structure:} & \quad a) \quad M\ddot{q} + D\dot{q} + Kq = f + Bu + Gw \\
& \quad q(0) = q_0, \quad \dot{q}(0) = \dot{q}_0 \\
\text{Sensor Output:} & \quad b) \quad z = Hx + m \\
\text{Estimator:} & \quad c) \quad \dot{x} = Ax + Ef - fBu + L(z - H\dot{x}) \\
& \quad \dot{x}(0) = 0 \\
\text{Control Force:} & \quad d) \quad u = -Fx
\end{align*}
\]

where

\[
x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}, \quad \dot{x} = \begin{bmatrix} \ddot{q} \\ \dot{q} \end{bmatrix}
\]

and

\[
H = [H_d \ H_v], \quad L = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}, \quad F = [F_1 \ F_2]
\]

It is noted that in the above implemented equations, we have merged the actuator and the control law equations into one by neglecting the actuator and control law dynamics. Instead, we have introduced an estimator equation as we do not have all the measurements needed for complete feedback. In the above equations, B and \( \tilde{B} \) represent the input influence matrix for actuator locations whereas G and \( \tilde{G} \) represent the disturbance locations. The vector q is the generalized displacement, w is a disturbance vector and the vector m is measurement noise. In Eq. (6b), z is the measured sensor output. The matrix \( H_d \) is the matrix of displacement sensor locations and \( H_v \) is the matrix of velocity sensor locations. The state estimator in Eq. (6c) may or may not be model based. The superscript * and ' denote the estimated states and time differentiation respectively. The input command, u, is a function of the state estimator variables, \( \ddot{q} \) and \( \dot{q} \), and \( F_1 \) and \( F_2 \) are control gains. The observer is governed by A, the state matrix representing the plant dynamics, and L, the filter gain matrix.

The software thus implemented was used to test its applicability to solve the control-structure interaction design of a model Earth Pointing Satellite (EPS), shown in Fig. 6, which is a derivative of a geostationary platform proposed for the study of Earth Observation Sciences. Two flexible antennas are attached to a truss bus. Typical missions involve pointing one antenna to earth, while tracking or scanning with
Fig. 6 Earth Pointing Satellite Structure
Table 1. EPS Vibration Frequencies (Hz.)

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1-6)</td>
<td>0.000</td>
</tr>
<tr>
<td>(7)</td>
<td>0.242</td>
</tr>
<tr>
<td>(8)</td>
<td>0.406</td>
</tr>
<tr>
<td>(9)</td>
<td>0.565</td>
</tr>
<tr>
<td>(10)</td>
<td>0.656</td>
</tr>
<tr>
<td>(11,12)</td>
<td>0.888</td>
</tr>
<tr>
<td>(13)</td>
<td>1.438</td>
</tr>
<tr>
<td>(14)</td>
<td>1.536</td>
</tr>
<tr>
<td>(15,16)</td>
<td>1.776</td>
</tr>
<tr>
<td>(17,18)</td>
<td>3.026</td>
</tr>
<tr>
<td>(19)</td>
<td>3.513</td>
</tr>
<tr>
<td>(20)</td>
<td>3.531</td>
</tr>
</tbody>
</table>

A small disturbance force was applied to the nominal EPS system in the form of a reboost maneuver. The force acted at the center of gravity in the Y-axis direction for 0.1 seconds at a 10 N force level and from 0.1 to 0.2 seconds the force level was -10 N. The disturbance was removed after 0.2 seconds. Figure 7 shows the open-loop angular response about the X-axis of the 15 m antenna. A small amount of passive damping was assumed (D = 0.0002 K). The vibrational response produced more than 4.5 μ radians of RMS pointing error due to this small reboost disturbance. Although many modes participate in the flexible body response, this particular reboost maneuver strongly excites modes near 4 Hz. The following paragraphs present an integrated control and structure design which seeks to lower the vibrational response of the EPS subject to some additional constraints. Figure 8 shows the closed-loop angular response about the X-axis of the 15 m antenna after design optimization. The pointing error is significantly reduced from that of the open-loop system shown.

5. Future Work and Discussions

The example problem analyzed in the previous section used a set of lumped actuators and localized sensors instead of distributed adaptive actuators and spatially integrated sensors. While such a model at best capture the adaptive elements used by Anderson et al. [29], Matsunaga [30], and Takahara [31], it can not simulate on a large scale the distributed usage of piezoelectric actuators and sensors proposed by de Luis [32], Rogers et al. [33], and Burk and Hubbard [34]. Our immediate future work will concentrate on the implementation of distributed adaptive elements and assess their practical applicability beyond the currently reported beam-like structural components. In this regard, we are exploring an adaptation of neural-network concepts [35] in the modeling and parallel computations of controlled structures with adaptive elements. Specifically, the limits of the applicability of distributed parameter modeling and control theory and discrete structures with discrete actuators and sensors, and their cross-over...
Fig. 7 Open-Loop Response of EPS Structure

Fig. 8 Closed-Loop Response of Structure EPS
performance must be investigated. Design, modeling, simulation and testing criteria from such studies will provide greater insight into the eventual adoptions of adaptive structures as viable choice for future space systems design alternatives.

The real-time simulation procedures presented herein may be applicable to the vibration control of lifeline equipment, and secondarily in minimizing the damage of buildings during earthquakes. In this applications, the sensor measurements used herein can be directly applicable to the vibration and earthquake-causing forces on the structures. An idea that may prove to be crucial in this case is the use of earthquake-generated natural force as vibration minimization actuators' forces. In other words, instead of trying to mitigate the earthquake-generating forces, exploit the natural forces instantly to activate certain vibration minimizing devices! Research along this line may in the end lead to the design of actuators attachable to the columns and floors, if properly triggered during earthquakes, can minimize damages based on the natural forces.

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