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STABILIZATION OF LARGE SPACE STRUCTURES BY LINEAR RELUCTANCE ACTUATORS

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Abstract

In this paper we consider application magnetic forces for stabilization of vibrations of flexible space structures. We investigate three electromagnetic phenomena, such as, a) magnetic body-force, b) reluctance torque, and c) magnetostriction, and analyse their application for stabilization of a beam. The magnetic body-force actuator utilizes the force that exists between poles of magnets. The reluctance actuator is configured in such a way that the reluctance of the magnetic circuit will be minimum when the beam is straight. Any bending of the beam increases the reluctance and hence generates a restoring torque that reduces bending. The gain of the actuator is controlled by varying the magnetizing current. Since the energy density of a magnetic device is much higher compared to piezo-electric or thermal actuators, it is expected that the reluctance actuator will be more effective in controlling the structural vibrations.

I. INTRODUCTION

The problems of modeling and control of flexible space structures have been a subject of considerable research interest in recent years. These future space vehicles will be large structures consisting of a rigid body and several flexible appendages, such as long beams, solar panels, large antennas etc. It is known that these space structures will possess low structural rigidity, high modal density and low damping. Consequently, in order for them to perform properly some active means of increasing the damping or the energy dissipation must be provided. There is a very large collection of research results available in the literature on the control and stabilization of flexible space structures. The references listed in this paper are only a small cross section of these results, and are not meant to be exhaustive.

Dynamic analysis and control system design of flexible structures are based on two different approaches: a) finite dimensional, and b) infinite dimensional. Although the finite dimensional approach [1 – 7] have been widely investigated in the past, the main objections are modal truncation, lack of *a priori* information of required mode numbers, and control spillover [15]. Because of these reasons, the infinite dimensional approach using partial differential equations appears to be more appropriate. Since large space structures are actually partly rigid and partly flexible, the complete mathematical model requires a combination of both ordinary differential equations and hyperbolic partial differential equations [8 - 16]. Stabilization of flexible space structures through active velocity feedback have been discussed in [8,9,13,14]. A more rigorous analysis of stabilization using semigroup theory is considered in [10,11]. Reference [16] describes the synthesis of optimal controls for this class of systems. Stabilization of flexible systems using thermal [17,18,19] and peizo-electric [20,21] actuators have been investigated in recent years. It has been shown both analytically and experimentally that thermo-elastic damping can be induced in materials by suitable application of thermal gradients. In [20,21] it has been shown that spatially distributed control actuators can be designed using piezoelectric polymers, and that feedback of beam tip angular velocity can be used for stabilization of vibrations of a beam.

In this paper, we investigate application of magnetic forces for stabilization of elastic structures. Magnetic forces and torques are developed in ferromagnetic systems in a variety of ways. Here we discuss three electromagnetic phenomena which have very good potential of stabilizing a vibrating structure; these are: a) magnetic body-force, b) reluctance torque, and c) magnetostriction. Magnetic body-force actuator relies on the force that exists between the poles of magnets. Reluctance torque is a consequence of the principle of conservation of energy, and arises due to the fact that the most stable configuration of a magnetic system is that of minimum reluctance. Magnetostriction causes generation of very high forces in ferromagnetic materials when subjected to applied magnetic fields. We show that a vibrating beam can be stabilized if the magnetizing current in the magnetic actuator is varied proportional to the rate of change of beam bending moment or the beam tip angular velocity. These magnetic actuators can be implemented using ferromagnetic or ferroplastic materials, and can be applied over the entire spatial domain of the elastic structure, thus emulating a distributed control actuator. Since the energy density of a magnetic device is much higher compared to piezo-electric or thermal actuators, it is expected that the magnetic actuator will be more effective in controlling the structural vibrations.

II. MAGNETIC ACTUATORS

A magnetomechanical transducer or actuator is a device that links a magnetic system and a mechanical system. The coupling between the two systems is through the magnetic field which acts as the energy storage device. A change in the stored energy leads to a energy conversion process to convert the magnetic energy to the mechanical energy, or vice-versa. There are several electromagnetic phenomena [22,23] that govern this energy conversion process among which the following are most important, and are commonly utilized in practical devices:

1. A mechanical force is exerted on a current carrying conductor in a magnetic field. Likewise, mechanical forces exist between two current carrying conductors because of their own magnetic fields.
2. A mechanical force is exerted on a movable ferromagnetic material tending to align it along the magnetic flux lines, or to reduce the reluctance of the flux path.
3. Most ferromagnetic materials show a small deformation in the presence of a magnetic field. This phenomenon is known as *magnetostriction*. Although the deformation is very small, the corresponding mechanical force may be very large.

All the above energy conversion processes are reversible in the sense that applications of mechanical forces or body deformations produce changes in the magnetic energy. In this research, we intend to utilize the magnetic-to-mechanical energy conversion processes for production of forces for stabilization of structural vibrations of elastic systems. In what follows, we present the fundamentals of three magnetic actuators which have very good potential of practical implementation for stabilization of large flexible space structures.

2.1 MAGNETIC BODY-FORCE ACTUATOR

The basic idea of this device is the magnetic body-force or stress acting between the magnetic poles. Consider the attraction of north and south poles of two magnets. The total force on one pole face is given by the integration of magnetic stress as

$$F_m = \int_A \frac{B_n^2}{2\mu_0}\, dA \qquad (1)$$

where B_n is the normal component of the field density to the surface, and μ_0 is the permeability of the air gap. Consider a magnetic system consisting of two ferromagnetic elements separated by a distance, and with I_1 and I_2 as the magnetizing currents as shown in Fig. 1.

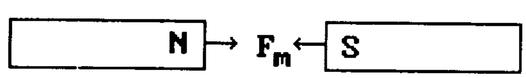

Fig. 1 Magnetic Body-Force

Then it can be shown that the resultant magnetic body-force is given by

$$F_m = k_1 I_1 I_2. \tag{2}$$

We may assume that one of the ferromagnetic materials is replaced by a permanent magnet, or an electro magnet with a constant exciting current. Then the force resulting from this magnetic system is of the form

$$F_m = k_2 I \tag{3}$$

where k_2 is suitable constant, and I is the magnetzing current. This analysis shows that this simple configuration of magnetic materials may be used for production of a force, and that this force could be made proportional to a control current. For the sake of simplicity, we assume that the magnetic force is distributed all over the spatial domain. In fact, for a single layer of ferromagnetic segments, this force may appear as a train of step functions. By using several layers of segments, one can obtain an average force that is distributed all over the spatial domain.

Now consider a flexible beam with a layer of ferromagentic segments rigidly attached to the upper surface of the beam, and another layer on the lower surface as shown in the Fig. 2.

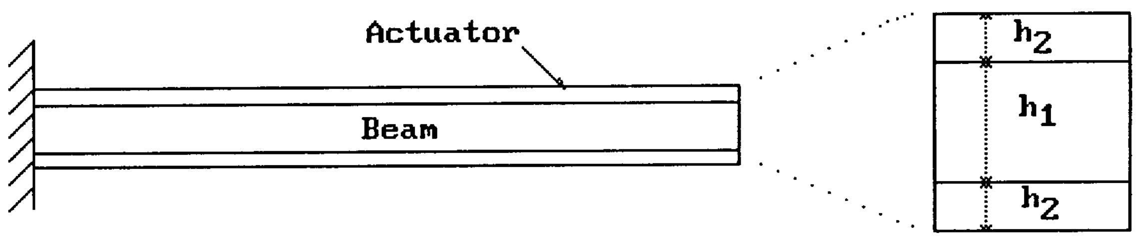

Fig. 2 Flexible Beam with Body-Force Actuator

We assume that the same magnetizing current is used for both the upper and the lower layers, and that the corresponding forces are same in magnitude but opposite in direction. This results in a bending moment given by

$$\begin{aligned} T(x,t) &= F_m(x,t)(h_1 + h_2) \\ &= c\, I(x,t) \end{aligned} \tag{4}$$

where c is a constant depending of the beam geometry and the properties of the magnetic material.

The dynamics of the transverse vibrations of a beam in the presence of this additional bending moment is given by

$$\rho \frac{\partial^2 y}{\partial t^2} + \frac{\partial^2}{\partial x^2}\left(Y \frac{\partial^2 y}{\partial x^2}\right) - \frac{\partial^2}{\partial x^2}\Big(c\, I(x,t)\Big) = 0, \quad x \in (0, L), \quad t \geq 0 \tag{5}$$

with the boundary conditions

$$\begin{aligned} &y(0,t) = 0 && Y \frac{\partial^2 y}{\partial x^2}(L,t) = c\, I(L,t) \\ &\frac{\partial y}{\partial x}(0,t) = 0 && \frac{\partial}{\partial x}\left(Y \frac{\partial^2 y}{\partial x^2}\right)(L,t) = c\, \frac{\partial I}{\partial x}(L,t) \end{aligned} \tag{6}$$

where y is the transverse deflection, Y is the flexural rigidity, and ρ is the mass density (per unit length) of the composite beam.

STABILIZATION

The beam dynamics described above contains a controllable parameter $I(x,t)$ which may be appropriately regulated in order to achieve a stabilizing action. For this purpose we follow the Lyapunov type analysis. Consider the total energy of beam vibrations given by

$$V(t) = \frac{1}{2}\int_0^L \left\{ \rho\left|\frac{\partial y}{\partial t}\right|^2 + Y\left|\frac{\partial^2 y}{\partial x^2}\right|^2 \right\} dx. \tag{7}$$

Then using the dynamics (5) along with the boundary conditions, we obtain

$$\frac{dV}{dt} = \int_0^L c\, I(x,t)\, \frac{\partial^3 y}{\partial x^2\, \partial t}\, dx. \tag{8}$$

This clearly shows that for asymptotic decay of vibration energy the magnetizing control current may be chosen as

$$I(x,t) = -k\, \frac{\partial^3 y}{\partial x^2\, \partial t}(x,t) \tag{9}$$

where k is a suitable gain, in other words, the control current should be proportional to the rate of change of bending moment of the beam.

It is interesting to note that this ferromagnetic actuator essentially introduces in the system a type of damping commonly known as "*structural damping*" in the literature. Indeed, substituting the equation (9) into the dynamics (5), the beam equation can be rewritten as

$$\rho\frac{\partial^2 y}{\partial t^2} + \frac{\partial^2}{\partial x^2}\left(Y\frac{\partial^2 y}{\partial x^2}\right) + k\frac{\partial^5 y}{\partial x^4\, \partial t} = 0, \tag{10}$$

in which the last term represents the structural damping. Note that the damping parameter k is very small for naturally occurring structural damping of elastic materials. In this case the control current can be suitably regulated so as to obtain the desired damping.

In case the feedback current is assumed to be uniform all over the length of the beam, equation (8) reduces to

$$\frac{dV}{dt} = c\, I(t)\, \frac{\partial^2 y}{\partial x\, \partial t}. \tag{11}$$

Hence considering a feedback current proportional to the tip angular velocity of the beam, i.e.,

$$I(t) = -k\frac{\partial^2 y}{\partial x\, \partial t}(L,t) \tag{12}$$

we obtain asymptotic stability of the system.

The control laws discussed above require regulation of the control current proportional to the angular velocity of the tip of the beam, or the rate of change of the (distributed) bending moment. For practical applications it may be relatively easier to measure the tip angular velocity only. Proportional variations of the control current can be done by suitable electronic circuits. One can also consider on-off or deadzone type of controls derived [9] from (8).

2.2 RELUCTANCE ACTUATOR

A property of a conservative system is that its energy is a function of only its state, and given sufficient time, the system always attains its rest state at which the energy is minimum. Consider a magnetic circuit containing a movable member. The energy stored in the magnetic field is minimum when the movable member attains a position for which the magnetic reluctance is minimum. Any perturbation of this position would imply a higher energy state of the system, and hence would lead to the production of a restoring force or torque that will realign the movable member to the minimum reluctance position. This is the fundamental principle of the Reluctance Actuator.

Consider a magnetic circuit consisting of two ferromagnetic segments as shown before; but in this case we assume that these segments can undergo an angular displacement relative to each other.

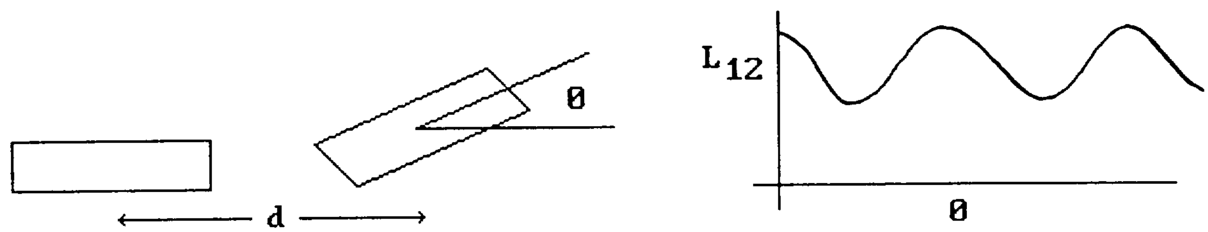

Fig. 3 Variation of Mutual Inductance with Angular Position

The magnetic potential energy stored in the air gap depends on the mutual inductance and the magnetizing currents, and is given by

$$W_m = L_{12} I_1 I_2. \tag{13}$$

The mutual inductance L_{12} varies with the angular orientation of the two segments relative to each other. It is clear from Fig. 3 that when θ is 0° or 360°, reluctance is minimum so that inductance is at the maximum value. Similarly, when θ is 180°, reluctance is maximum with the correspondingly small inductance. Hence the mutual inductance can be expressed as

$$L_{12} = L_0 + L_\sigma \cos\theta. \tag{14}$$

Any rotation of the movable member would tend to increase the air gap, and hence would increase the reluctance, or decrease the inductance. Then according to the principle of conservation of energy, a restoring torque is produced that would realign the movable member with the stationary member. This restoring torque is given by

$$\begin{aligned} T &= \frac{\partial W(I_1, I_2, \theta)}{\partial \theta} \\ &= -L_\sigma I_1 I_2 \sin\theta. \end{aligned} \tag{15}$$

Clearly, the torque reduces to zero when there is no angular deflection, i.e., when the two segments are aligned. In what follows, we show that this torque can be utilized to stabilize a vibrating beam.

Consider a cantilever beam with a string of ferromagnetic segments interlaced by air gaps as shown in the Fig. 2. Consider two typical segments located at the axial distances $x - \frac{d}{2}$ and $x + \frac{d}{2}$ respectively, where d is the distance between the two segments. The angular orientation of these segments on a perturbed beam will be given by $\frac{\partial y}{\partial x}(x - \frac{d}{2})$ and $\frac{\partial y}{\partial x}(x + \frac{d}{2})$ respectively. Hence the relative angle between the two segments is

$$\begin{aligned} \theta(x,t) &= \frac{\partial y}{\partial x}(x + \frac{d}{2}) - \frac{\partial y}{\partial x}(x - \frac{d}{2}) \\ &\simeq d\,\frac{\partial^2 y}{\partial x^2}(x,t) \end{aligned} \tag{16}$$

Using this equation in (15) and assuming small angle perturbations of the beam, the restoring torque becomes

$$T(x,t) = -L_\sigma I_1 I_2 \sin\left(d \frac{\partial^2 y}{\partial x^2}\right)$$
$$\simeq -d\, L_\sigma I_1 I_2 \frac{\partial^2 y}{\partial x^2}(x,t) \qquad (17)$$

For simplicity we assume that the magnetizing currents are equal, and $I_1 = I_2 = I$. Then the dynamics of transverse vibration of the beam is given by

$$\rho \frac{\partial^2 y}{\partial t^2} + Y \frac{\partial^4 y}{\partial x^4} + kI^2 \frac{\partial^4 y}{\partial x^4} = 0 \qquad (18)$$

with appropriate boundary conditions. This shows that the reluctance torque essentially increases the flexural rigidity of the elastic material, and this stiffening action is independent of the direction of the magnetizing current. Thus reluctance torque can be used to introduce artificial flexural rigidity in elastic members. Alternatively, feedback control schemes can be designed to stabilize the system. Indeed, after some analysis using the energy function (7), it can be shown that a feedback current proportional to the rate of change of bending moment of the elastic member,i.e.,

$$I_1 I_2(t) = -q \frac{d}{dt} \left\| \frac{\partial^2 y}{\partial x^2} \right\|^2_{L_2} \qquad (19)$$

can be used to stabilize the system. Here q is the gain of the controller.

2.3 MAGNETOSTRICTION

Magnetostriction is the elastic deformation of a magnetic material due to the change in the magnetic field. If a ferromagnetic bar such as nickel, cobalt, is subjected to an applied magnetic field, it shrinks in length. If the bar is restrained from contracting, a mechanical force is developed and mechanical energy can be extracted. For some magnetic materials the action is to elongate rather than contract while in some others first to elongate and then contract. The change in length is usually very small and of the order of 0.01%, but the resulting force may be very large of the order of 200 N/cm^2 or 300 psi. It is important to note that the stress due to magnetostriction is independent of the direction of the applied magnetic field. As such the mechanical force obtainable from a magnetostrictive device will be bounded between zero and some upper limit depending on the strength of the applied field.

III. CONCLUSIONS

We consider stabilization of flexible structures using three types of magnetomotive forces: a) magnetic body-force, b) reluctance torque, and c) magnetostriction. We prove stabilization of the system using the first two types of forces. This requires feedback of rate of bending moment of the structure in the form of a magnetizing current. It is important to note that magnetic body-force and the reluctance torque are complementary and occur simultaneously, in other words, the same hardware will produce two types of stabilizing action in the vibrating system. Although magnetostriction produces a mechanical force that can be extracted, at this time it is not clear whether this can be utilized to produce any stabilizing action.

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