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Optimal Control of Systems with Capacity-related noises N91-22310
Abstract

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In the ordinary theory of optimal control (LQR and Kalman Filter), the variances of the actuators and the sensors are assumed to be known(not related to the capacities of the devices). This assumption is not true in practice. Generally , a device with greater capacity to exert actuating forces and a sensor capable of sensing greater sensing range will generate noise of greater power spectral density.

When the ordinary theory of optimal control is used to estimate the errors of the outputs in such cases it will lead to faulty results, because the capacities of such devices are unknown before the system is designed. The performance of the system designed by the ordinary theory will not be optimal as the variances of the sensors and the actuators are neither known nor constant. The interaction between the control system and structure could be serious because the ordinary method will lead to greater feedback (Kalman gain) matrices.

The main purpose of this paper is to dvelop methods which can optimize the performance of systems when noises of the actuators and the sensors are related to their capacities. These methods will result in smaller feedback (Kalman gain) matrix. The smaller matrices will reduce the interaction between the control system and system structure and, thereby, reducing the requirements on the structures and consequently making the structure more flexible.

INTRODUCTION

In the optimal control of stochastic systems, we ordinarily assume that noises of the actuators and the sensors are not related to the capacity of the actutors and sensors[1,2,3] This assumption is not true in practice.Generally, the variances of actuators and the sensors, especially the actuators , are related to the capacities of the devices. Obviously, a fuel jet capable of generating a force of 100 lbs will have greater noises than the one capable of generating a force of 1 lb. It will be realistic and practical to assume that the noise variance of the actuators and the sensors be linear function of the variance of the controlling forces and the output of the sensors i.e., the observations. Under this assumption when a device is required to have greater capacity it will also introduce greater noise. The ordinary method of optimal control problems have at least three defects.

(a) It is hard to specify correctly the noise power spectral densities of the actuator and the sensor because the capacities of these devices are unknown

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before the system is designed

(b) The resultant feedback and Kalman gain matrices may not be optimal when the noises in these devices are not related to their capacities. Therefore, the performance of the control system may not be as good as it would be otherwise.

(c) When the noises are assumed to be not related to the capacities of the devices, the resultant feedback and Kalman gain matrices will be large, making the interaction between control system and the structure unsafe[4].

Because of these defects, it is hard for us to estimate the errors of the outputs. The errors of the outputs will be large, and the interaction between the control system and system structures will produce large errors.

In some control systems, such as communications satellite and on-orbit telescope, the precision of the control system is critical, and in the future missions their structures could be very flexible. The variances of these devices will be assumed to be linear functions of their capacities.

In this paper, we will develop methods which will optimize the performance of systems when noises of the actuators and the sensors are related to their capacities. The feedback (and Kalman gain) matrices are found by this methods will be automatically smaller than those found by ordinary methods. Therefore, the interaction between the control system and the strurcture will be reduced and thereby, permitting more flexible structures.

II PROBLEM STATEMENT

Let us first consider the optimal control of a first order system

$$\dot{x} = a x + u + w \tag{1.a}$$

$$u = - f x \tag{1.b}$$

$$J = E \left\{ x^2 + r u^2 \right\} \tag{1.c}$$

This is a steady-state optimal control problem with exact observation. f is the feedback coefficient to be determined. a and r are given parameters. E is the mean operator. w is a zero mean white Gaussian noise. Unlike the ordinary control problem, we assume that the variance of the noise w, can be described by

$$W = W_0 + \alpha \sigma_u^2$$

$$= W_0 + \alpha f^2 \sigma_x^2$$

where W_0, and α are non-negative constants. σ_u^2 can be considered as the nominal variance of the input, a good measure of the capacity of the actuator. From (1a) and (1b), we have

$$\dot{x} = (a - f)x + w \tag{3}$$

According to stochastic control theory, the variance of x, denoted by P can be determined by

$$2(a-f)P + W = 0 \tag{4}$$

Since $\sigma_x^2 = P$, eq.(4) reduces to

$$2(a - f) + W_0 + \alpha f^2 P = 0 \tag{5}$$

$$\text{or} \quad (2a - 2f + \alpha f^2) P + W_0 = 0 \tag{6}$$

Since P must be greater than or equal to zero, the following condition must hold

$$2a - 2f + \alpha f^2 < 0 \tag{7}$$

The cost functional can be written as

$$J = P + r f^2 P \tag{8}$$

Using eq.(6), we have

$$J = (1 + r f^2) \frac{W_0}{(2f - 2a - \alpha f^2)} \tag{9}$$

The inequality above indicate the stable region for the problem.

The stable region for ordinary problem is defined by

$$a - f < 0 \qquad (10)$$

The stable regions describe by equations (7) and (10) are plotted in Fig. 1. Obviously, the stable region of the present problem is only a subset of the region of the ordinary problem. The stable region becomes smaller when α becomes greater and this region is not directly related to the constant term. For certain values of $a > 0$ and α, it is possible that there is no f which lie in the stable region, i.e., such a system can't be stabilized.

The optimal feedback control can be found by differentiating equation (9) with respect to f and equating the derivative of J with respect to f to zero. The derivative of J with respect to f after simplification can be written in the form

$$\frac{dJ}{df} = \frac{2 W_0}{(2f - 2a - \alpha f^2)^2} \left(r f^2 - 2 r a f + \alpha f - 1 \right)$$

Equating the derivative of J to zero and solving the quadratic equation in f and neglecting the extraneous solution, we obtain the optimal feedback control as follows.

$$f = \frac{-(\alpha - 2ra) + \sqrt{(\alpha - 2ra)^2 + 4r}}{2r} \qquad (11a)$$

Figs 2,4 and 6 show the ratio of optimal feedback (the value of f given by equation (11a)) to the feedback found by ordinary method vs. r for various values of a and α. We can see that the value of f/fo is less than 1, i.e., when the noise of the actuator is capacity related, the optimal feedback tends to decrease.The reason for this is that a greater feedback corresponds to a greater actuator signal, and increased capacity of the device and increased noise power spectral density of the noise.Therefore, a smaller feedback matrix will be preferred. When r becomes smaller, α becomes bigger and a becomes greater. The difference between f and f_0 will becomes greater. The reason is not hard to imagine. When r becomes smaller, the feedback by the ordinary method becomes greater even it is out of the stable region, while the feedback by the present method although becomes greater but the increment will not be significant because it has not taken the increase in noise power into consideration, and the feedback will never be out of the stable region. When α (alpha) becomes greater, noise is more related to the capacity and the system will more seriously depend on the feedback. Figs 3,5 and 6 show the ratios of optimal cost found by the propsed method to the case when f is found by the ordinary method. Fig.5 does not have a plot for $\alpha = 1$, because the feedback found by the ordinary method is out of stable region, and the ratios, Jo/J is infinite.

III GENERRAL CASE

In the above section, we have solved a simple problem by theoretical approach. In general system contains multiple states with multiple inputs and multiple outputs, and the meassurements are corrupted by noises. Then , the problem can be stated as follows.

$$\min_{F,K} \quad J = E\{ y^T Q y + u^T R u \} \tag{11b}$$

with constraints

$$y = H x$$

$$\dot{x} = A x + B u + G w \tag{12.a}$$

$$u = - F \hat{x} \tag{12.b}$$

$$\dot{\hat{x}} = A \hat{x} + B u + K (z - M \hat{x}) \tag{12.c}$$

where

y= Output vector
x= State vector
u= Control Vector
z= Measurement Vector
x= Estimated State Vector
w= input noise vector (zero-mean white Gaussian noise)
v= Measurement noise vector(zero mean white Gaussian noise)

F = Feedback matrix
K = Kalman Gain Matrix
and the matrices are of appropriate dimension

The noise covariance matrices are given by

$$E\{ w \; w^T \} = W$$

$$E\{ v \; v^T \} = V$$

The only difference between the ordinary problem and proposed problem is that the model of the covariance matrices w and v. Ordinarily W and V are assumed to be constant matrices which are not related to the capacities of the actuators and sensors. In this paper W and V are assumed to be matrices whose covariance matrices are functions of the capacities of actuators and sensors.

The capacity of an actuator can be reasonably be represented by the nominal varianace of the actuator signal,

$$\sigma_{u_i}^2 = E_\infty \{u_i^2\} = f_i E_\infty \{\hat{x}\hat{x}^T\} f_i^T \triangleq f_i P_{\hat{x}} f_i^T$$

where f_i is the ith row of F.

And , we will assume the variance of an actuator to be a linear function of its capacities , i.e.,

$$W = \mathrm{diag}\{ w_1, w_2, \quad w_m \}$$

$$w_i = w_{i_0} + \alpha_i \sigma_{u_i}^2$$

where w_{i_0} and α_i are non-negative constants

Similarly,

$$V = \mathrm{diag}\{ V_1 , V_2 , \cdots \cdots V_l \}$$

$$V_i = V_{i_0} + \beta_i \sigma_{z_i}^2$$

where V_{i_0} and β_i are non - negative constants

$$\sigma_{z_i}^2 = E\{ z_i^2 \} = m_i E \{x x^T\} m_i \triangleq m_i P_x m_i^T$$

where m_i is the ith row of M.

Clearly the covarince matrices become functions of the feedback and Kalman gain matrices.

Equation (12.a) ,(12.b) and (12.c) can be written as

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \begin{bmatrix} A & -BF \\ KM & A - BF - KM \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \begin{bmatrix} G & 0 \\ 0 & K \end{bmatrix} \begin{bmatrix} w \\ v \end{bmatrix}$$

According to the Stochastic Control theory the covariance matrices satisfy the following equations

$$\begin{bmatrix} P_x & P_{x\hat{x}} \\ P_{x\hat{x}} & P_{\hat{x}} \end{bmatrix} \begin{bmatrix} A & -BF \\ KM & A - BF - KM \end{bmatrix}^T +$$

$$\begin{bmatrix} A & -BF \\ KM & A - BF - KM \end{bmatrix} \begin{bmatrix} P_x & P_{x\hat{x}} \\ P_{x\hat{x}} & P_{\hat{x}} \end{bmatrix}$$

$$+ \begin{bmatrix} G & 0 \\ 0 & K \end{bmatrix} \begin{bmatrix} W & 0 \\ 0 & V \end{bmatrix} \begin{bmatrix} G & 0 \\ 0 & K \end{bmatrix}^T = 0$$

where W and V are functions of P_x , $P_{\hat{x}}$, F and K .To solve the above optimization problem, we probably have to use numerical approach

IV . DIRECT APPROACH

The simplest way to solve the problem is to use direct approach. In the direct approach, we assume that all the elements of F and K are parameters. The cost J can be found by solving equation (11b) iteratively when F and K are given. Various techniques of optimization theory can be used to find the optimum value of F and K.

However, this method can solve only problems of smaller dimension.

For relatively large problems, the number of parameters will be large and the computational efforts to find the cost for given F and K will also be large;therefore, the total computational load will be large

It seeems that the challenging problem here is development of computationally efficient fast algorithm to sove the feedback gain and the Kalman gain

R E F E R E N C E S

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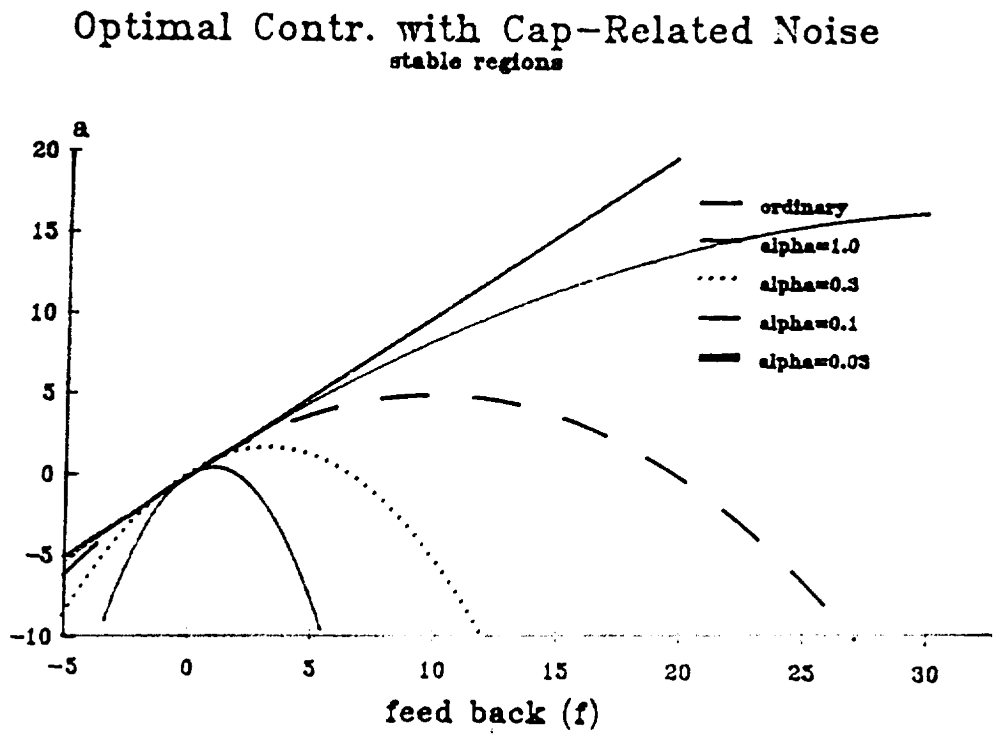


Optimal Contr. with Cap-Related Noise
stable regions
a
20
15
10
5
0
-5
-10
-5
0
5
10
15
20
25
30
feed back (f)
ordinary
alpha=1.0
alpha=0.3
alpha=0.1
alpha=0.03

Fig.1

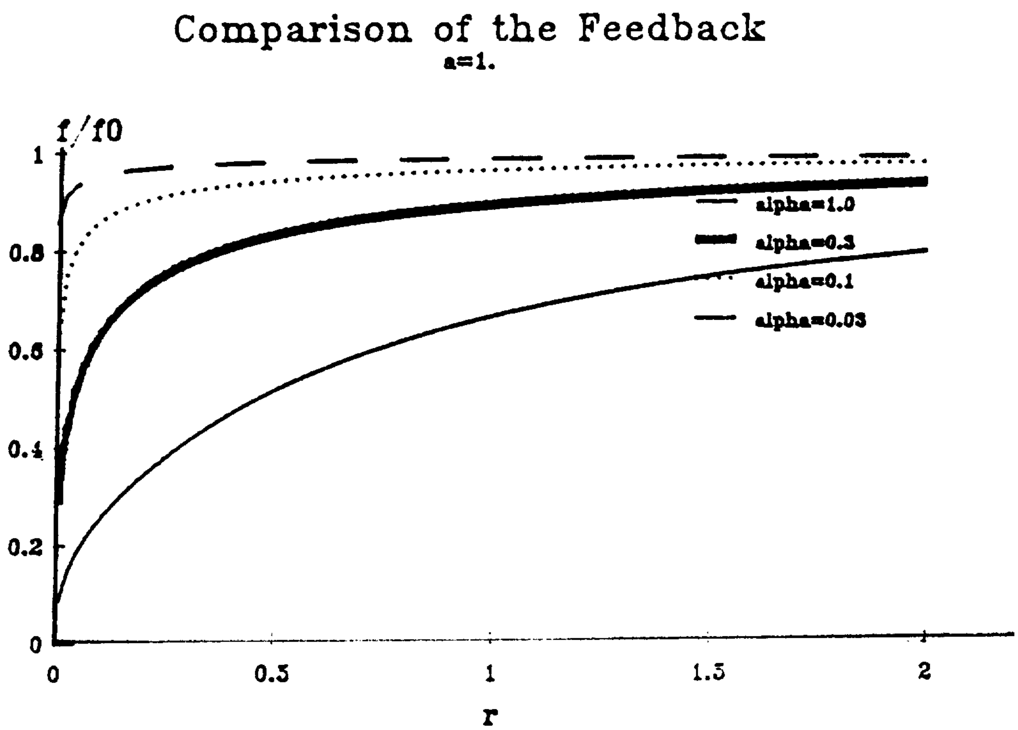

Comparison of the Feedback
a=1.
f/f0
alpha=1.0
alpha=0.3
alpha=0.1
alpha=0.03
r

Fig.4

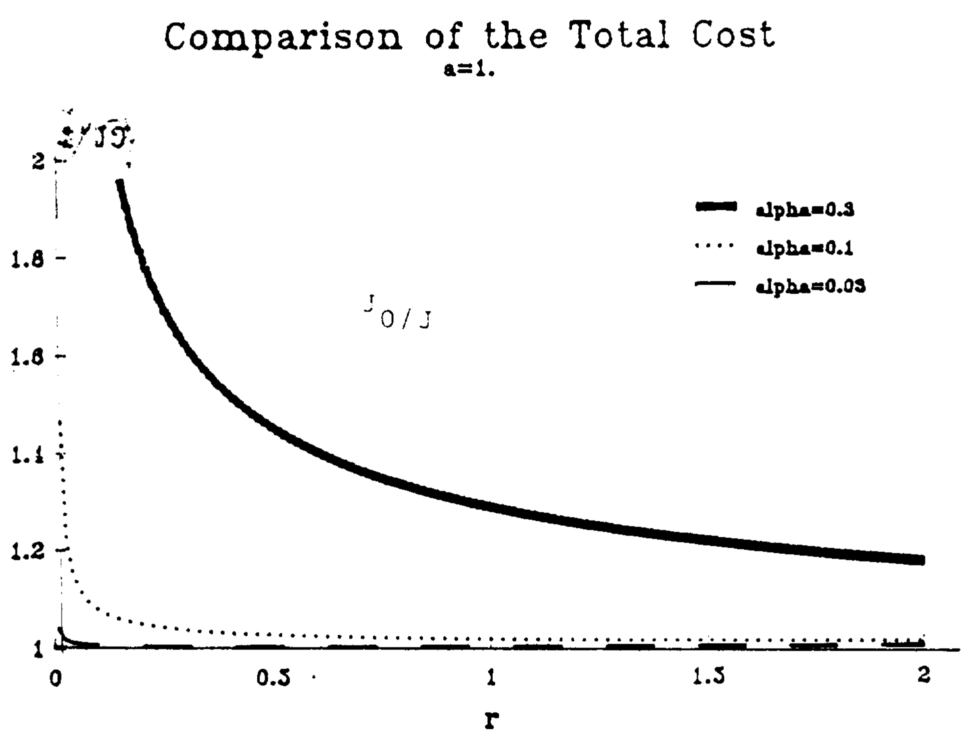

Comparison of the Total Cost
a=1.
alpha=0.3
alpha=0.1
alpha=0.03
J0/J
r

Fig.5

Comparison of the Feedback

a=0.

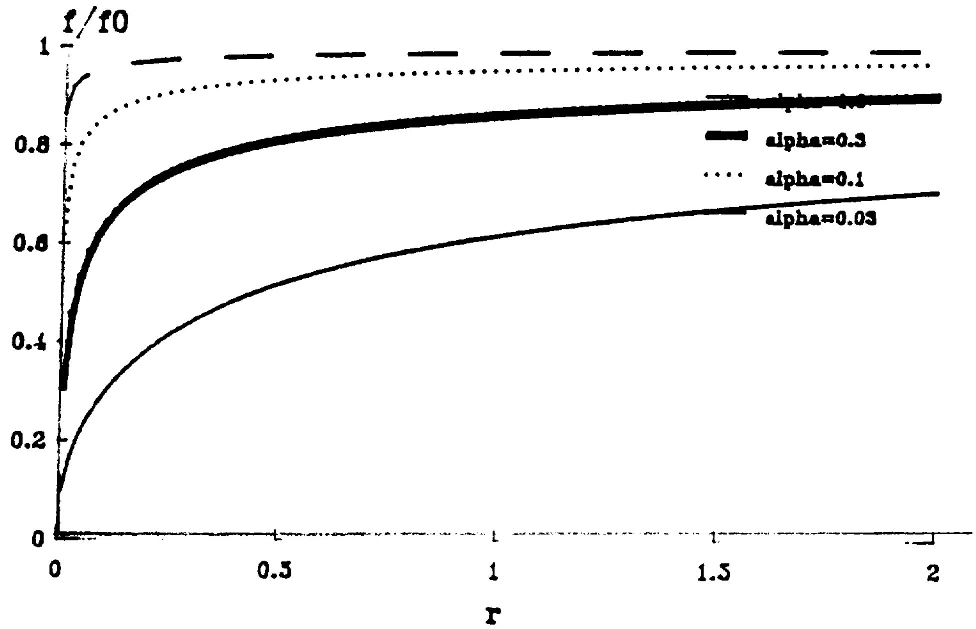


Fig.2

Comparison of the Total Cost

a=0.

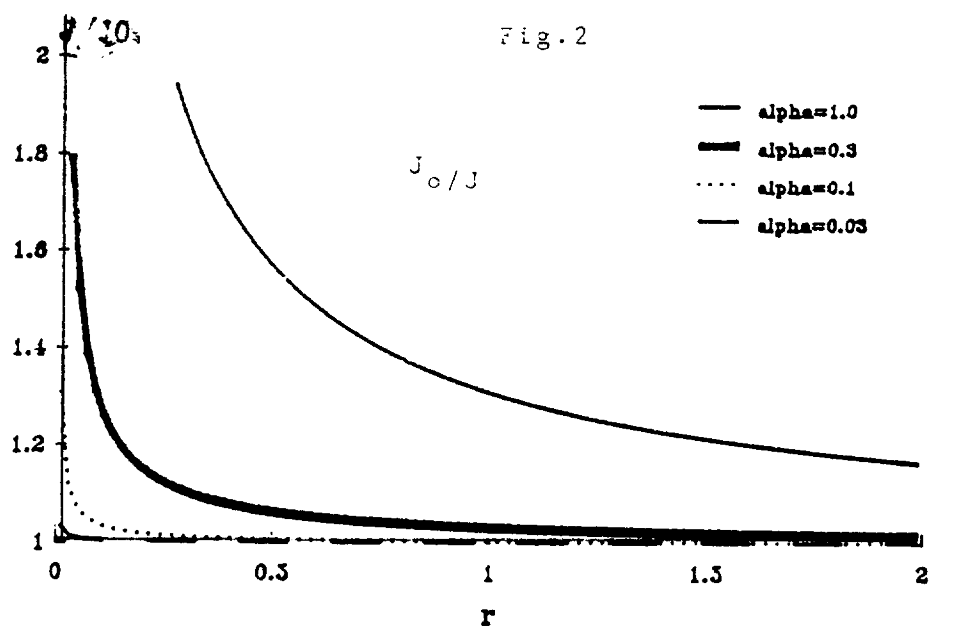
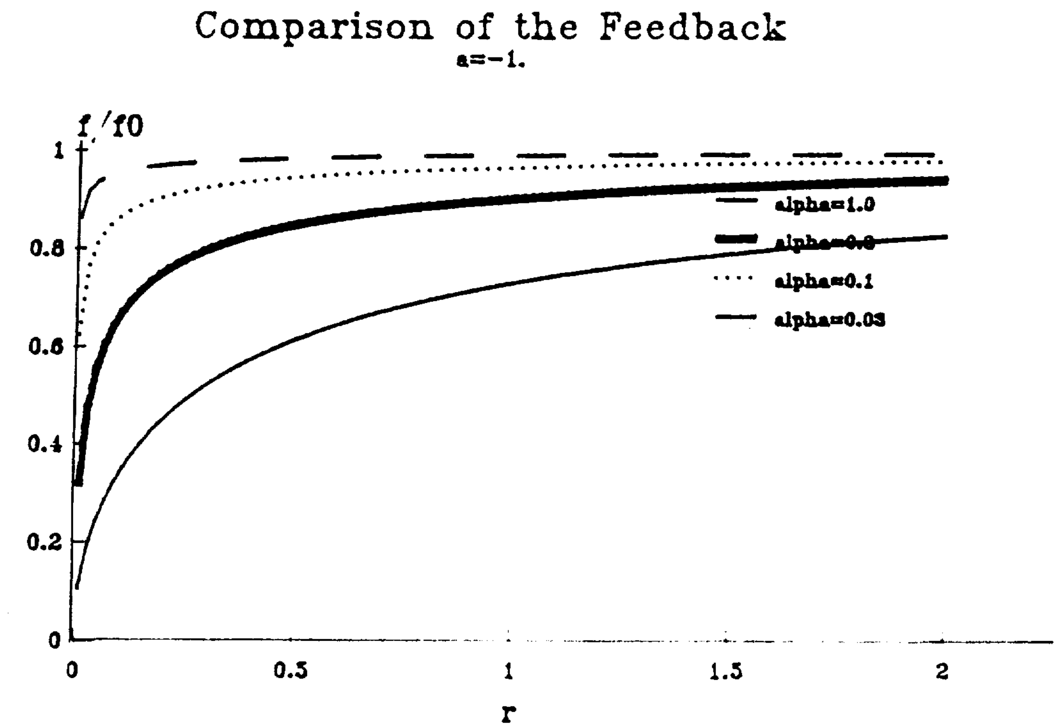
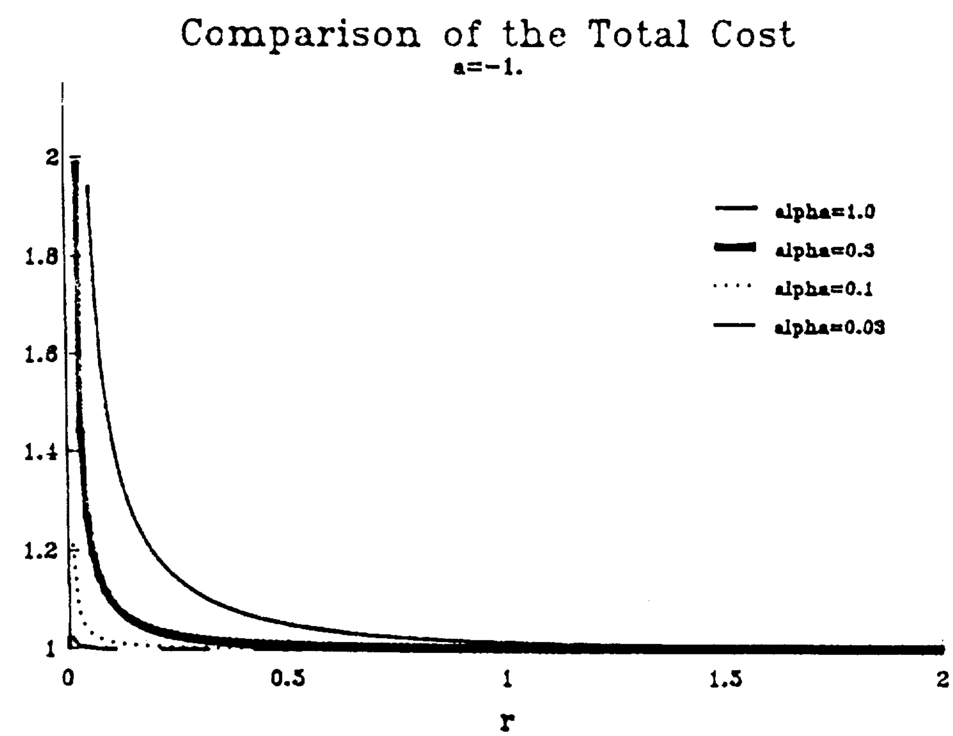


Fig.3



Comparison of the Feedback
a=-1.
f/f0
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
r
alpha=1.0
alpha=0.3
alpha=0.1
alpha=0.03

Fig.6



Comparison of the Total Cost
a=-1.
2
1.8
1.6
1.4
1.2
1
0
0.5
1
1.5
2
r
alpha=1.0
alpha=0.3
alpha=0.1
alpha=0.03

Fig.7