Late Stages of Accumulation and Early Evolution of the Planets

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ABSTRACT

This article briefly discusses recently developed solutions of problems that were traditionally considered fundamental in classical solar system cosmogony: determination of planetary orbit distribution patterns, values for mean eccentricity and orbital inclinations of the planets, and rotation periods and rotation axis inclinations of the planets. We will examine two important cosmochemical aspects of accumulation: the time scale for gas loss from the terrestrial planet zone, and the composition of the planets in terms of isotope data. We conclude that the early beginning of planet differentiation is a function of the heating of protoplanets during collisions with large (thousands of kilometers) bodies. This paper considers energetics, heat mass transfer processes, and characteristic time scales of these processes at the early stage of planet evolution.

INTRODUCTION

Using the theory of preplanetary cloud evolution and planet formation, which is based on the ideas of Schmidt, Gurevich, and Lebedinskiy, and which was developed in the works of Safronov (1969, 1982), we can estimate a number of significant parameters for the dynamics of bodies which accumulate in the planets. However, it long proved impossible to solve a number of problems in classical solar system cosmogony which were traditionally considered fundamental. Such problems include the theoretical derivation of patterns of planetary and satellite orbit distributions (the so-called Titius-Bode law), theoretical estimations of the value for
mean orbital eccentricities and inclinations, planet rotation periods and rotation axis inclinations, and other characteristics of the present structure of the Sun's planetary system. In addition, certain consequences of the theory and, most importantly, a conclusion on the relatively cold initial Earth and the late beginning of its evolution, clashed with data on geo- and cosmochemistry. These data are evidence of the existence of planet heating during the process of growth and very early differentiation. This paper briefly discusses a modified version of the theory which the authors developed in the 1970's and 1980's. Using this new version we were able to provide a fundamental solution to several key problems of planetary cosmogony and generate a number of new findings. The most significant of these appear to be an estimate of the composition of an accumulating Earth with data incorporated on oxygen isotopy and a conclusion on the early beginning of differentiation in growing planets.

FORMATION OF THE PLANETARY SYSTEM

Despite promising data on the existence of circumstellar disks, we have yet to discover an analogue to a circumsolar gas-dust disk. Nor have calculations (Ruzmaikina and Maeva 1986; Cassen and Summers 1983) produced a satisfactory picture of circumsolar disk formation, or a reliable estimate of its mass M and characteristic initial dimensions R. Nevertheless, a model of a low-mass disk (M < 0.1 M☉), with moderate turbulence, a hot circumsolar zone and cold periphery, has received the widest recognition in the works of a majority of authors.

STANDARD AND MK—DISK MODELS

A reconstruction of surface density distribution σ(R) according to Weidenschilling (1977) is shown in Figure 1. Mass of the disk, generated by adding on to produce the cosmic (solar) composition of present-day planet matter, is

\[(0.01 - 0.07)M_\odot, \text{ with } \sigma(R > R_\odot = 1AU) \propto (R/R_\odot)^{-3/2}.\]

It is usually supposed that by the time the Sun achieved main sequence, its luminosity L, did not greatly differ from the present L☉, and the temperature in the disk's central plane (z = 0) was on the order of a black body T ≈ 300(R_\odot/R)^{1/2}K. Estimations of the degree of ionization (N_e/N ≈ 10^{-11}) and gas conductivity (\lessapprox 10^3CGSE) in the central plane are insignificant, and the impact of the magnetic field is usually neglected in considering subsequent disk evolution. Using the hydrostatic equations
FIGURE 1 Surface density distribution $\sigma(R)$ in low-mass, circumstellar gaseous disks. The solid line indicates models of disks near F5 class stars: 1: $m/k = 1$; 2: $m/k = 1/2$; 3: $m/k = 1/4$; the dashed line indicates models of disks near G0 class stars: 4: $m/k = 1$; 5: $m/k = 1/2$; 6: $m/k = 1/4$; the dotted and dashed line shows disks near G5 class stars: 7: $m/k = 1$; 8: $m/k = 1/2$; 9: $m/k = 1/4$; the straight lines are critical density distributions for the corresponding classes of stars: 10: $\sigma_{cr}(F5)$; 11: $\sigma_{cr}(G0)$; 12: $\sigma_{cr}(G5)$. Disk mass in models 1-12 is equal to $M = 5 \times 10^{-2} M_\odot$. 13 is the standard model.
and the equation of the state of ideal gas at a temperature which is not dependent on \( z \), we yield a density distribution for \( z \):

\[
\rho(z) \approx \rho_0(R) \exp(-z^2/h^2), \quad h^2 = 2kTR^3/GM_\odot \mu,
\]

where \( \mu (\approx 2.3) \) is the mean molecular mass, and \( k \) is the Boltzmann constant. The model which has been termed standard is derived, by taking into account \( \sigma(R) \approx \sqrt{\pi} \rho_0(R) h(R) \):

\[
\rho_0(R) \propto R^{\alpha - \beta}, \quad P(R) \propto R^{-\beta}, \quad T(R) \propto R^{-\alpha}, \quad \alpha \lesssim 1, \beta \approx 3.
\]

Flattening of the disk is high: \( \gamma = h/R \propto (RT)^{1/2}, \gamma \lesssim 0.1 \). Disk rotation is differential and differs little from Kepler’s:

\[
\omega = \omega_k (1 + \epsilon)^{1/2}, \quad \omega_k = V_k/R = \sqrt{GM_\odot/R^3},
\]

\[
\epsilon \approx (c_s^2/V_k^2)(\text{d} \ln P/\text{d} \ln R) \lesssim 0.1, \quad c_s^2 = kT/\mu.
\]

Here \( c_s \) is the speed of sound. In the standard model \( V_k^2 \gg c_s^2 \gg v_A^2 \), \( v_A \) is the Alfven velocity. Quasi-equilibrium disk models are constructed by Vityazev and Pechernikova (1982) which do not use the contemporary distribution \( \sigma(R) \). They were called MK-models since they are only defined by a mass \( M \) and a moment \( K \) of a disk which is rotating around a star with mass \( M_\ast \) and luminosity \( L_\ast \). Expressions for densities \( \rho(R,z) \) and \( \sigma(R) \) were obtained by resolving the system of hydrodynamic equations with additional conditions (low level of viscous impulse transport and minimum level of dissipative function). The distribution of surface density in disks was found to be:

\[
\sigma(R) \approx 1.55 \cdot 10^5 \tilde{m} \left( \frac{\tilde{m}}{k} \right)^7 \left( \frac{M_\ast}{M_\odot} \right)^{12.25} R^{1.75} \exp[-2.8 \left( \frac{\tilde{m}}{k} \right)^2 \left( \frac{M_\ast}{M_\odot} \right)^3 R] g/cm^2,
\]

where \( M \) is \( \tilde{m} \cdot 10^{-2} M_\odot \), \( K \) is \( k \cdot 10^{31} g \text{ cm}^2 \text{s}^{-1} \), and \( R \) is in AU. By varying \( \tilde{m} \) and \( k \), star mass \( M_\ast \), and luminosity \( L_\ast \), we can generate a set of models of quasi-equilibrium circumstellar disks (see Figure 1). We will note that the distributions of \( \sigma(R) \) in the standard and MK-models are qualitatively similar. However, there is a noticeable excess of matter in the remote zone in the first model in comparison to the latter ones. This excess may be related to diffusion spread of the planetesimal swarm during planet accumulation. Therefore, present planetary system dimensions may
primary stages of its evolution and of planet formation (Figure 2) within the framework of the aforementioned low-mass disk models.

**PLANETESIMAL MASS SPECTRUM N(M,T) AND MATTER REDISTRIBUTION**

After dust settles on the central plane and dust clusters are formed due to gravitational instability, there occurs the growth and compacting of some clusters, and the breakdown and absorption of others. This process is described in detail by Pechernikova and Vityazev (1988). We later briefly touch upon the specific features of the final stages of accumulation of sufficiently large bodies, when a stabilization effect develops for the orbits of the largest bodies (Vityazev et al. 1990). In the coagulation equation

\[
\frac{\partial n(m,t)}{\partial t} = \frac{1}{2} \int_0^m A(m', m - m')n(m')n(m - m')dm' - n(m,t) \int_0^m A_{max}(m, m')n(m')dm',
\]  

(5)
the subintegral kernel $A(m,m')$, describing collision efficiency, must take into account the less efficient diffusion of large bodies. Let us write $A(m,m') = A^* B / (A^* + B)$, where

$$A^* = \tau (r + r^1)^2 [1 + 2G(m + m^1)/(r + r^1)V^2]V = v_{ik} r_{ik},$$

is the usual coefficient which characterizes the collision frequency of gravitating bodies $m$ and $m'$ with good mixing,

$$B = [D(m) + D(m')] [\Delta R(m) + \Delta R(m')] = D_{ik} R_{ik},$$

is the coefficient accounting for diffusion, $D(m) = e^2(m) R^2 / \tau_E(m)$, $e$ is the eccentricity, $\tau_E$ is the characteristic Chandrasekhar relaxation time scale, $\Delta R = e R$. Estimates demonstrate that at the initial stages ($m \ll 0.1 m_\oplus$) $B \gg A^*$ and $A \rightarrow A^*$, while at the later stages ($m > 0.1 m_\oplus$) we have $B(i \sim k) < A^*(i \sim k)$ and in the limit $A(i \sim k) \rightarrow B(i \sim k)$. Pechernikova (1987) showed that if $A \propto (m^\alpha + m^{1\alpha}) \equiv m_\oplus^\alpha$ then the coagulation formula has an asymptotic solution that can be expressed as

$$n(m) \propto m^{-q}, q = 1 + \alpha / 2.$$

(6)

For the initial stages $\alpha = 2/3 - 4/3$ and $q_* = 4/3 - 5/3$. For the later stages $B(i \sim k) \propto m_\oplus^{-5/3}$ and $q^* \sim 1/6, q^* < q < q_*$, that is, the gently sloping power law spectrum for large bodies. With this finding we can understand the relative regularity of mass distribution in the planetary system. Unlike the findings of numerical experiments (Isaacman and Sagan 1977), only the low mass in the asteroid belt and a small Mars is an example of significant fluctuation. The authors, using numerical integration of equations, such as

$$\sigma_d = \frac{-1}{R} \frac{\partial}{\partial R} [R v_R \sigma_d] + \frac{3}{R} \frac{\partial}{\partial R} \left[ \left\{ \sqrt{R} \frac{\partial}{\partial R} \sqrt{R} D \sigma_d \right\} \right],$$

(7)

also examined the overall process of solid matter surface density redistribution $\sigma_d(R,t)$ ($\sigma_d \sim 10^{-2}\sigma$) resulting from planetesimal diffusion. The spread effect of a disk of preplanet bodies proved significant for the later stages and was primarily manifested for the outer areas (Vityazev et al. 1990). The effect is less appreciable for the zone of the terrestrial planet group, and we shall forego detailed discussion of it here.

**RELATIVE VELOCITY SPECTRUM**

Wetherill's numerical calculations for the terrestrial planet zone (1980) confirmed the order of value of relative velocities that had been estimated earlier by Safronov (1969). Similar calculations for the zone of outer planets
are preliminary. In particular, there is vagueness in the growth time scales for the outer planets. By simplifying the problem for an analytical approach, we can look separately at the problem of mean relative velocities \( \bar{v} \) (i.e., \( \bar{e}, \bar{i} \)) of the planetesimals and the problem of eccentricities \( e \) and inclination \( i \) of the orbits of accreting planets. The second problem is considered below. We will discuss here an effect which is important in the area of giant planets. As planetesimal masses \( (m) \) grow, their relative velocities \( (v) \) also increases. At a sufficiently high mean relative velocity \( (v \sim 1/3 \cdot \bar{V}_k) \) part of the bodies from the "high velocity Maxwell tail" may depart the system, carrying away a certain amount of energy and momentum. With this scenario, the formula for the mean relative velocity \( (Vityazev et al. 1990; Safronov 1969) \) must appear as follows:

\[
\frac{1}{v^2} \frac{dv^2}{dt} = \frac{\beta_1 - \nu_E}{\tau_E} - \frac{1}{\tau_g} - \frac{\beta_2}{\tau_s},
\]

where \( \tau_E, \tau_g \) and \( \tau_s \) are correspondingly the characteristic Chandrasekhar relaxation time scales, gas deceleration and the characteristic time scale between collisions, \( \beta_1 = 0.05 - 0.13 \) (Safronov 1969; Stewart and Kaula 1980), \( \beta_2 = 0.5 \) (Safronov 1969) and \( \nu_E \) is part of the amount of energy removed by the "rapid particles"

\[
\nu_E \approx \frac{1}{6} \int_{v_{cr}}^\infty v^2 n(v) dv / \int_0^\infty v^2 n(v) dv = \Gamma (5/2, b) / 6 \Gamma (5/2) \approx \frac{3 \nu^2_{cr}}{2 \bar{v}^2} \approx \frac{3}{2} \frac{\sqrt{2} - 1}{\bar{v}^2}.
\]

For now \( e \sim i \sim \bar{v} / \bar{V}_k \ll 1 \), the usual expressions for \( \bar{v} \) follow from (8), in particular, with \( \tau_g \gg \tau_E, \tau_s \) in a system of bodies of equal mass \( m \) we have

\[
\bar{v} = \sqrt{\frac{Gm}{\Theta r}}, \text{ where } \Theta \sim 1.
\]

Despite continuing growth of the mass, as \( e \) approaches \( e_{cr} \approx 0.3 - 0.4 \), \( \nu_E \) becomes comparable to \( \beta_1 \), the relative velocity \( \bar{v} \) in the system of remaining bodies discontinues growth. In other words, with \( e \approx e_{cr} \), the parameter \( \Theta \) in expression (10) grows with \( m \) proportionally to \( m^{2/3} \), reaching in the outer zone values \( \sim 10^3 \). This effect gives us acceptable time scales for outer planet growth. Because of low surface density, accreting planets in the terrestrial group zone cannot attain a mass sufficient with a build up of \( \bar{v} \) to \( e_{cr} \). Therefore the mean eccentricities do not exceed the values \( e_{max} = 0.2 - 0.25 \).
MASSES OF THE LARGEST BODIES IN A PLANET'S FEEDING ZONE

Studies on accumulation theory previously assumed that in terms of mass a significant runaway of planet embryos from the remaining bodies in the future planet's feeding zone occurred at an extremely early stage. According to Safronov's well-known estimates (1969), the mass of the largest body (after the embryo) \( m_1 \) was \( 10^{-2} - 10^{-3} \) of the mass \( m \) of the accreting planet. Pechernikova and Vityazev (1979) proposed a model for expanding and overlapping feeding zones, and they considered growth of the largest bodies. The half-width of a body's feeding zone is determined by mean eccentricity \( \bar{e} \) of orbits of the bulk of bodies at a given distance from the Sun:

\[
\Delta R(t) \simeq e(t)R \simeq \bar{v}(t)R/V_k, \tag{11}
\]

The characteristic mixing time scale for \( R \) in this zone virtually coincides with the characteristic time scale for the transfer of regular energy motion to chaotic energy motion, and the characteristic time scale for energy exchange between bodies. It is clear from (10) and (11) that

\[
\bar{v}(t) \propto r(t), \bar{e}(t) \propto r(t), \Delta R(t) \propto r(t). \tag{12}
\]

The mass of matter in the expanding feeding zone

\[
Q(R,t) = 2\pi \int_{R-\Delta R}^{R+\Delta R} \sigma_d R \, dR, \quad \tag{13}
\]

will also grow with a time scale \( \propto r(t) \), while disregarding the difference in matter diffusion fluxes across zone boundaries. When the mass of a larger body \( m(t) \) begins to equal an appreciable amount \( Q(t) \) with the flow of time (see Figure 3), the growth of the feeding zone decelerates. At this stage the larger bodies of bordering zones begin to leave behind, in terms of their mass, the remaining bodies in their zones. However, (as seen in Figure 3), this runaway by mass is much less than was supposed in earlier studies.

MASSES, RELATIVE DISTANCES, AND THE NUMBER OF PLANETS

Mass increase of the largest body in a feeding zone is represented by the well-known formula:

\[
dm/dt = \pi r^2 (1 + 2\Theta) \rho_d \bar{v} = 2(1 + 2\Theta) r^2 W_k \sigma_d, \tag{14}
\]

where the surface density of condensed matter \( \sigma_d \) can be considered a sufficiently smooth function \( R \) and it can be assumed that \( \sigma_d(t=0) = \)
FIGURE 3 The region of determination and model distributions for $m_1/m$ as the ratio of the mass, $m_1$, of the largest body in the feeding zone of a growing planet to the mass of a planet $m$: 1: $m_1/m = 1 - 0.62 (m/Q)^{0.3}$, which corresponds to the growth of bodies in the expanding feeding zone; 2: $m_2/m$; 3: $m_3/m$; 4: $m_4/m = 1 - m/Q$; 5: $m_5/m = (1 - m/Q)^2$. The circles and dots indicate the results of numerical simulation of the process of terrestrial planet accumulation (Ipatov 1987; Wetherill 1985).
with its values $\sigma_0$ and $v$ in each zone. As bodies precipitate to the largest body in a given zone $m$, a growing portion of matter is concentrated in $m$ and the corresponding decrease in surface density is written as:

$$\sigma_\infty(t) = \sigma_\infty(t = 0)[1 - m(t)/Q(t)]. \quad (15)$$

From (10) through (15) for the growth rate of a planet’s radius we have (with $v \neq 2$):

$$\frac{dr}{dt} = \frac{(1 + 2\Theta)\sigma_\infty W_k \left(\frac{R_0}{R}\right)^v}{2\pi \delta} \left\{1 - \frac{2\delta(2 + v)\tau(t)(R/R_\infty)^v}{3\sigma_\infty \tau^2[1 + v]^3 + (1 - v)^2 + 1}\right\}. \quad (16)$$

It follows from (16) that the largest body which is not absorbed by the other bodies (a planet) ceases growing when it reaches a certain maximum radius (mass). The value of this radius is only determined by the parameters of the preplanetary disk. If we put a zero value in the bracket in (16), in the first approximation for $\tau$, we can yield $\max r$, $\max m$, $\max \tau$, and $\max \Delta R$. In particular,

$$\max r = \left(\frac{5\pi}{\Theta \delta M_\odot}\right)^{1/4} \cdot \sqrt{2\sigma_\infty(R_\odot)}r_\infty^{5/4}, \text{cm};$$

$$\max \tau = \frac{2}{3} \left(\frac{5\pi \delta R}{\Theta M_\odot}\right)^{1/2} \cdot \max r. \quad (17)$$

The growth time scale is an integration (16). It is close to the one generated by Safronov (1969) and Wetherill (1980) and is on the order of 65-90 million years for accreting 80-90% of the mass.

One can state the following for distances between two accreting planets:

$$R_{n+1} - R_n \approx \Delta_n R + \Delta_{n+1} R = \varepsilon_n R_n + \varepsilon_{n+1} R_{n+1}, \quad (18)$$

hence

$$R_{n+1}/R_n \approx (1 + \varepsilon_n)/(1 - \varepsilon_{n+1}) = b. \quad (19)$$

In view of (17) the following theoretical estimate can be made for terrestrial planets: $b(\max \varepsilon = 0.2-0.25) = 1.5-1.67$. For the zone of outer planets $b(\max \varepsilon = \varepsilon_{cr} = 0.32-0.35) = 1.85-2.3$. The real values $b$ in the present solar system are cited in Table 1. The theory that was developed not only explains the physical meaning of the Titius-Bode law, but also provides a satisfactory estimate of parameter $b$. The partial overlapping of zones,
embryo drift, and the effects of radial redistribution of matter (Vityazev et al. 1990) complicates the formulae, but this does not greatly influence the numerical values max m, max \( \varepsilon \), and b.

An estimate of the number of forming planets can easily be generated from (19) for a preplanetary disk with moderate mass and distribution \( a(R) \) according to the standard or MK-models with pre-assigned outer and inner boundaries:

\[
N = \frac{1}{1 + \max \varepsilon} \approx \frac{1}{1 + \max \varepsilon} 
\]

With low values of max \( \varepsilon \) from (20) we have \( N \approx 1/(1 + \max \varepsilon) \max \varepsilon \) and yield a natural explanation of the results of the numerical experiment (Isaacman and Sagan 1977): \( N \approx 1/e \). For a circumsolar disk with initial mass \( \lesssim 0.1M_\odot \), the theory offers a satisfactory estimate of the number of planets which formed:

\[
N(R^* \lesssim 10^2 \text{AU, } R_* \gtrsim 10^{-1} \text{AU, } \max \varepsilon \approx e_{cr} = 1/3) \lesssim 10. 
\]

**DYNAMIC CHARACTERISTICS OF FORMING PLANETS**

Workers were long unsuccessful in developing estimates of planet eccentricities, orbital inclinations, and mean periods of axis rotation, which were formed during the planets' growth process. In other words, estimates that fit well with observational data. Perchernikova and Vityazev (1980, 1981) demonstrated that by taking into account the input of large bodies, the existing discrepancy between theory and observations could be resolved. When accreting planets approach and collide with large bodies at the earliest stages \( \varepsilon \) increases. A rounding off of orbit takes place at the final stage: by the time accumulation is completed estimated values for \( \varepsilon \) are close to present "mean" values (Laska 1988). The same can be said for orbital inclinations. Vityazev and Pechernikova (1981) and Vityazev et al. (1990) developed a theory for determining mean axial spin periods and axis
inclinations. The angular momentum vector for the axial spin of a planet $K$, inclined at an angle $\epsilon$ to axis $z$ (where $z$ is perpendicular to the orbital plane), is equal to the sum of the regular component $K_1$, which is directed along axis $z$ and the random component $K_2$, which is inclined at an angle $\gamma$ to $z$. For $K_1$ in a modified Giuli-Harris approximation (for terrestrial planets) the following was generated:

$$K_1 = \frac{48}{\pi} \sqrt{\frac{2G}{5}} \left( \frac{2M_\odot}{R^3} \right)^{1/4} \left( \frac{3}{4\pi\rho} \right)^{5/12}$$

$$m_p^{5/3} F_1(m/Q), \quad F_1(m/Q = 1) = 9.6 \cdot 10^{-2}.$$ (21)

Dispersion $K_2$ is ($\varphi = 11/6$):

$$DK_2 \approx 4.18 \cdot 10^{-2} \varphi^{-1/3} Gm_p^{10/3} \cdot F_2(m/Q), \quad F_2(m/Q = 1) = 0.123.$$ (22)

The authors demonstrated that as planets accumulate what takes place is essentially direct rotation ($\tau \lesssim \gamma < 90^\circ$). Large axis rotation inclinations and reverse rotation of individual planets are a natural outcome of the accumulation of bodies of comparable size. It is clear from (21) and (22) that the theoretical dependence of the specific axis rotation momentum ($\propto m^{2/3}$) approaches what has been observed. It is worth recalling that this theory does not allow us to determine the direction and velocity at which a planet, forming at a given distance, will rotate. It only gives us the corresponding probability (Figure 4).

COSMOCHEMICAL ASPECTS OF EVOLUTION

Vityazev and Pechernikova (1985) and Vityazev et al. (1989) have repeatedly discussed the problem of fusing physico-mechanical and physico-chemical approaches in planetary cosmogony. We will only mention two important findings here. Vityazev and Pechernikova (1985, 1987) proposed a method for estimating the time scale for gas removal from the terrestrial planet zone. They compared the theory of accumulation and data on ancient irradiation by solar cosmic rays of the olivine grains and chondrules of meteorite matter with an absolute age of 4.5 to 4.6 billion years.

TIME SCALE FOR THE REMOVAL OF GAS FROM THE TERRESTRIAL PLANET ZONE

It is easy to demonstrate that if gas with a density of at least $10^{-2}$ of the original amount remains during the formation of meteorite parent bodies in the preplanetary disk between the asteroid belt and the Sun, then
Vector $K$ of the observed rotation is shown. An initial rotational period of 10 hours was assumed for Earth. A rotational period equal to 15 hours was assumed for Uranus. The lined area corresponds to reverse rotation.
solar cosmic rays (high-energy nuclei of iron and other elements) could not have irradiated meteorite matter grains. Vityazev and Pechernikova (1985, 1987) showed that irradiation occurred when 100- to 1000-kilometer bodies appeared. The reasoning was that prior to that time nontransparency of the swarm of bodies was still sufficiently high, while less matter would have been irradiated at a later stage than the 5-10% that has been discovered experimentally. The conclusion then follows that there was virtually no gas as early as the primary stage of terrestrial planet accumulation. This conclusion is important for Earth science and comparative planetology because it is evidence in favor of the view of gas-free accumulation of the terrestrial planets at the later stages and repudiates the hypothesis of an accretion-induced atmosphere (see, for example the works of the Hayashi school).

ON THE COMPOSITION OF THE TERRESTRIAL PLANETS

According to current thinking, the Earth (and other planets) was formed from bodies of differing mass and composition. It is supposed that the composition of these bodies is, at least partially, similar to meteorites. Several constraints on the possible model composition of primordial Earth (generated by the conventional mixing procedure) can be obtained from a comparison of data on the location of meteorite groups and mafic bedrock on Earth on the diagram $\sigma^{17}O - \sigma^{18}O$ and density data. Pechernikova and Vityazev (1989) found constraints from above on Earth's initial composition with various combinations of different meteorite groups: the portion of carbonaceous chondrite-type matter for Earth was < 10%, chondrite (H, L, LL, EH, EL) < 70% and achondrite (Euc) + iron < 80%. They proposed a method which can be used to determine multicomponent mixtures of Earth's model composition. The composition of Mars can also be determined from the hypothesis of the Martian origin of shergottites. Confirmation of the authors' hypothesis on the removal of the silicate shell from proto-Mercury (Vityazev and Pechernikova 1985; Pechernikova and Vityazev 1987) would mean that there is an approximately homogeneous composition for primary rock-forming elements in the entire zone 0.5-1.5 AU.

EARLY EVOLUTION OF THE PLANETS

Vityazev (1982) and Safronov and Vityazev (1983) showed that by the stage where 1000-kilometer bodies are formed, there commences a moderate, and subsequently, increasingly intensive process of impact processing, heating metamorphism, melting, and degassing of the matter of colliding bodies. It has been concluded that > 90% of the matter of bodies which
went into forming the terrestrial planets had already passed the swarm through repeated metamorphism and melting, both at the surface and in the cores of bodies analogous to parent bodies of meteorites.

**INITIAL EARTH TEMPERATURE AND ENERGY SOURCES**

If we take into account the collisions of accreting planets with 1000-kilometer bodies, we conclude that there were extremely heated interiors, beginning with protoplanetary masses $M > 10^{-2}M_\oplus$. The ratio of the earliest and current estimates for primordial Earth are given in Figure 5. The latest temperature estimates indicate the possibility that differentiation began in the planet cores long before they attained their current dimensions.

The primary energy sources in the planets are known to us. In addition to energy released during the impact processes of accumulation, the most significant sources for Earth are: energy from gravitational differentiation released during stratification into the core and mantle ($\sim 1.5 \times 10^{38}$ergs) and energy from radioactive decay ($\lesssim 1 \times 10^{38}$ergs). It is important for specific zones to account for the energy of rotation released during tidal evolution of the Earth-Moon system ($\sim 10^{37}$ergs) and the energy of chemical transformations ($\lesssim 10^{37}$ergs). However, these factors are not usually taken into account.
account in global models. The energy from radioactive decay at the initial stages plays a subordinate role. However, the power of this source may locally exceed by several times the mean value $\epsilon r(U,\text{Th},K) \approx 10^{-6}\text{erg/cm}^3\text{s}$ (in the case of early differentiation and concentration of U, Th, and K in near-surface shells). The power of the shock mechanism is significantly greater even on the average: $\epsilon_{\text{imp}} \sim 10^{-4}-10^{-5}\text{erg/cm}^3\text{s}$. Intermediate values are generated for the energy of gravitational differentiation $\epsilon_{G,D} \sim 10^{-5}-10^{-6}\text{erg/cm}^3\text{s}$ (the time scale for core formation is 0.1 to one billion years) and the source of equivalent adiabatic heating during collapse of $\sim 10^{-6}\text{erg/cm}^3\text{s}$.

### HEAT MASS TRANSFER PROCESSES

Heat-mass transfer calculations in planetary evolution models are currently made using various procedures to parameterize the entire system of viscous liquid hydrodynamic equations for a binary or even single-component medium. This issue was explored (Safronov and Vityazev 1986; Vityazev et al. 1990) in relation to primordial Earth. We will merely note here the order of values for effective temperature conductivities, which were used for thermal computations in spherically symmetrical models with a heat conductivity equations such as:

$$\frac{\partial T}{\partial t} = \frac{1}{R^2} \frac{\partial}{\partial R} \left( R^2 \Sigma K \frac{\partial T}{\partial R} \right) + \frac{\Sigma \epsilon_i}{\rho \epsilon_p}. \quad (23)$$

The value which is normally assumed for aggregate temperature conductivity is a function of molecular mechanisms $K_m = 10^{-2}\text{cm}^2/\text{s}$. For shock mixing during crater formation, the effective mean is $K_{\text{imp}} \sim 1-10\text{ cm}^2/\text{s}$. Similar values have effective values for thermal convection ($K_c \sim 10^{-1}-10\text{ cm}^2/\text{s}$) and gravitational differentiation ($K_{G,D} \sim 1-10\text{ cm}^2/\text{s}$). It is clear from these estimates that the energy processes in primordial Earth exceeded by two orders and more contemporary values in terms of intensity.

### INITIAL COMPOSITION INHOMOGENEITIES

The planets were formed from bodies with slightly differing compositions, and which accumulated at various distances. Variations in their composition and density were, on the average, on the same order as the adjacent planets:

$$\frac{|\delta c_o|}{\bar{c}} \approx \frac{|\delta \rho_o|}{\bar{\rho}} \lesssim 0.1 \quad (24)$$

These fluctuations were smoothed out during shock mixing with crater formation, but remained on the order of
where $\xi (\sim 10^2 - 10^3)$ is the ratio of mass removed from the crater to the mass of the fallen body. Using the distribution of bodies by mass (6), we can estimate the distribution of composition and density fluctuations both for a value ($\delta \rho$) and for linear scales (l). For a fixed $\delta \rho$, spaces occupied by small-scale and large-scale fluctuations are comparable. It can be demonstrated that during planet growth, relaxation of inhomogeneities already begins (floating of light objects and sinking of heavy ones). In the order of magnitude, velocity $v$, relaxation time scale $\tau$, effective temperature for one scale inhomogeneities occupying a part of the volume $c$, are obtained from the expressions:

$$|v| = |\delta \rho| g l / 5 n, \tau \sim R^2_\infty / v l$$ (26)

$$K \propto c v l, \dot{\epsilon} \sim |\delta \rho| g c l v l / R_\infty,$$

where $g$ is the acceleration of gravity, and $\eta$ is the viscosity coefficient. Where $g \approx 10^3$ cm/s, $\eta \approx 10^{20}$ poise, $\delta \rho / \rho \sim 10^{-3} - 10^{-4}$, $c \approx 0.1$, we have $\tau \approx 10^2$ years, $\dot{\epsilon} \approx \dot{\epsilon}_r (U, Th, K)$. Estimates show that, owing to intensive heat transfer during relaxation of such inhomogeneities, the accreting planet establishes a positive temperature gradient (central areas are hotter than the external). This runs contrary to previous assessments (Safronov, 1959, 1969, 1982; Kaula, 1980). The second important conclusion is that heavy component enrichment may occur towards the center, which is sufficient for closing off large-scale thermal convection. This is due to relaxation of the composition fluctuations during planet growth. Both of these conclusions require further verification in more detailed computations.

**"THERMAL EXPLOSIONS" IN PRIMORDIAL EARTH**

Peak heat releases, as these relatively large bodies fall, exceed by many orders the values for $\dot{\epsilon}_{imp}$ which are listed above. Entombed melt sites seek to cool, giving off heat to the enclosing medium. However, density-based differentiation, triggering a separation of the heavy (Fe-rich) component from silicates, may deliver enough energy for the melt area to expand. Foregoing the details (Vityazev et al., 1990), let us determine the critical dimensions of such an area. Let us write (23) in nondimensional form:

$$\frac{\partial \Theta}{\partial \tau} = \frac{\partial}{\partial \xi} (1 + Pe e^{\Theta / \nu}) \frac{\partial \Theta}{\partial \xi} + \Gamma \cdot e^\Theta + \Gamma_r,$$ (27)

where
\[ \Theta = \frac{(T - T_m)E}{RT_m^2}, \Gamma = \frac{\Delta \rho c g h^2 v_o E}{4\lambda RT_m^2}, \Gamma_r = \frac{\dot{\epsilon}_r h^2 E}{4\lambda RT_m^2}, Pe_o = \left( \frac{\nu_o \epsilon}{K} \right)^{1/n}, \]

is the difference between heavy and light component densities, \( c \) is the heavy component portion in terms of volume, \( h \) is the characteristic size of the area (layer here), \( E \) is the energy for activation in the expression for the viscosity coefficient, \( \lambda \) is the heat conductivity coefficient, \( v_o(\Theta = 0) \) is the Stokes' velocity, or filtration rate, whose numerical value is found from the condition \( \epsilon_r = \epsilon_{GD} \). It can be demonstrated that for \( \Gamma > \Gamma_{cr} \) (\( \Gamma_{cr} = 0.88 \) for the layer and \( \Gamma_{cr} = 3.22 \) for the sphere) and \( \Theta > \Theta_{cr} \) (\( \Theta_{cr} = 1.2 \) for the layer and \( \Theta_{cr} = 1.6 \) for the sphere) with \( \Theta = 0 \) at the area boundaries, conditions are maintained which promote a "thermal explosion." The emerging differentiation process can release enough energy to develop the process in space and accelerate it in time. For \( \Delta \rho = 4.5 \text{ g/cm}^3, \rho_c = 0.2, \rho_{c_p} = 10^6 \text{erg/cm}^3 \text{ K}, RT_m^2/E = 50 \text{K}, \alpha = 10^{-2}\text{cm}^2/\text{s}, \) and a Peclet number < 1, critical dimensions of the area (\( h_{cr} \)) are on the order of several hundred kilometers.

**CHARACTERISTIC TIME SCALES FOR THE EARLY DIFFERENTIATION OF INTERIORS**

Experimental data on meteorite material melt is too meager to make reliable judgments as to the composition of phases which are seeking to divide in the field of gravitational pull. Classical views, hypothesizing that the heavy (Fe-rich) component separates from silicates and sinks, via the filtration mechanism or as a large diapiric structures in the convecting shell, have only recently been expressed as hydrodynamic models. Complications with an estimate of the characteristic time scales for separation are, first of all, related to highly ambiguous data on numerical viscosity values for matter in the interiors. Variations in the temperature and content of fluids on the order of several percent, close to liquidus-solidus curves, alter viscosity numerical values by orders of magnitude. It is clear from this that even for Stokes’ (slowed) flows, velocity \( v \propto n^{-1} \) and characteristic time scales \( \tau \propto n \) are uncertain. Secondly, existing laboratory data point to the coexistence of several phases (components) with sharply varied rheology. This further complicates the separation picture. Nevertheless, a certain overall mechanism which is weakly dependent on concrete viscosity values and density differences, was clearly functioning with interiors differentiation. A number of indirect signs are evidence of this; they indicate the very early and concurrent differentiation of all terrestrial planets, including the Moon.

The following are time scale estimates of the formation of the Earth’s core:
1) From paleomagnetic data, the core existed 2.8 to 3.5 billion years ago;
2) Based on uranium-lead data, it was formed in the first 100 to 300 million years.

(The following do not clash with items 1 and 2 listed above).
3) Data on the formation of a protective, ionosphere or magnetosphere screen for $^{26}\text{Ne}$ and $^{36}\text{Ar}$ prior to the first 700 million years, and
4) Data on intensive degassing in the first 100 million years for I-Xe.

CONCLUSION

The solution (generated in the 1970's-80's at least in principle) to the primary problems of classical planetary cosmogony has made it possible to move towards a synthesis of the dynamic and cosmochemical approaches. The initial findings appear to be promising. However, they require confirmation. Clearly, there is no longer any doubt that intensive processes which promoted the formation of their initial shells occurred at the later stages of planet formation. At the same time, if we are to make significant progress in this area, we will need to conduct experimental studies on localized matter separation and labor-intensive numerical modeling to simulate large-scale processes of fractioning and differentiation of the matter of planetary cores.

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