Development of reliability and risk assessment of structural components and structures is a major activity at Lewis Research Center. It consists of five program elements: (1) probabilistic loads, (2) probabilistic finite element analysis, and (3) probabilistic material behavior, (4) assessment of reliability and risk, and (5) probabilistic structural performance evaluation. Recent progress includes: (1) the evaluation of the various uncertainties in terms of cumulative distribution functions for various structural response variables based on known or assumed uncertainties in primitive structural variables, (2) evaluation of the failure probability, (3) reliability and risk-cost assessment, and (4) an outline of an emerging approach for eventual certification of man-rated structures by computational methods. Collectively, the results demonstrate that the structural durability/reliability of man-rated structural components and structures can be effectively evaluated by using formal probabilistic methods.

INTRODUCTION

It is becoming increasingly evident that deterministic structural analysis methods will not be sufficient to properly design critical components in future structures in general and aerospace structures in particular. These structural components are subjected to a variety of complex, and severe cyclic loading conditions, including high temperatures and high temperature gradients. Most of these are quantifiable only as best engineering estimates. These complex loading conditions subject the material to complex and coupled nonlinear behavior which depends on stress, temperature, and time. Complex and coupled nonlinear material behavior is nonuniform, is very difficult to determine experimentally, and perhaps impossible to describe deterministically. In addition, hot rotating structural components for aerospace propulsion (engines) are relatively small. Fabrication tolerances on these components, which in essence are small thickness variations, can have significant effects on the component structural response. Fabrication tolerances by their very nature are statistical. Furthermore, the attachment of components in the vehicles integrated structural system generally differs by some unknown amount from that assumed for designing the component. In summary, the fundamental aspects -- (1) loading conditions, (2) materials behavior, (3) geometric configuration, and (4) supports (attachments) to integrated structures inherently include a variety of uncertainties of unknown magnitude.

There are generally two approaches to handle this wide variety of uncertainties: (1) current practice, and (2) probabilistic evaluation. Current practice is adequate where the new engine structure is not very different from an existing one. However, this approach is costly and requires long time-schedules for future structures which will be entirely different from any existing ones. The second approach is to develop probabilistic structural analysis methods where the uncertainties in all the parameters of the four fundamental aspects are described by appropriate probability functions.

Development of the probabilistic structural analysis methodology (PSAM) is an on-going activity at NASA Lewis and is a joint program of in-house and sponsored research (ref. 1). Theoretical considerations, computer codes, and other relevant applications are described in papers presented in conferences (refs. 2 to 6). Activities and progress up to June 1989 are
summarized in reference 7. The objectives of this brief paper are (1) to summarize the fundamental aspects of PSAM and (2) to demonstrate the application of this methodology to a specific example (the reliability/risk of turbine blade components of rocket propulsion systems). The specific example includes the four fundamental aspects (key elements) required in probabilistic structural analysis of future structures, namely: (1) probabilistic loads, (2) probabilistic finite element analysis, (3) probabilistic description of complex coupled nonlinear material behavior, and (4) evaluation of reliability and risk. Throughout the specific example discussion, appropriate comments are included to illustrate the generality of the method and its application to aerospace and other structures in general.

**FUNDAMENTAL CONSIDERATION**

Central to the probabilistic structural analysis is the fundamental consideration that: Uncertainties observed in the structural performance (displacements, frequencies, buckling, global fracture toughness, stresses/strains) of structures can be quantified in terms of corresponding uncertainties in basic parameters (primitive variables). The primitive variables are those which are used to describe the structure and its respective environment. For example: (1) structural configuration, (2) boundary conditions (attachments), (3) loading conditions, and (4) material thermomechanical nonlinear behavior.

The uncertainties in these primitive variables are then integrated through structural mechanics to quantify the uncertainties in the global structural responses (displacements) and are decomposed to quantify the uncertainties in local responses (stresses/strains). The concept is schematically illustrated in figure 1. The structural component is the blade which is modeled for finite element analysis. The input uncertainties are the blade loads (centrifugal, pressure, and temperature), geometry and material variables. The output is quantification of uncertainties in structured responses or in local stresses for probable fracture initiation. Brief descriptions are given in subsequent sections on each of these as it is applied to the specific example.

**PROBABILISTIC SIMULATION OF LOADS**

The fundamental assumption for the probabilistic simulation of loads is that each individual load condition can be probabilistically synthesized from four primitive parts: (1) steady state, (2) periodic, (3) random, and (4) spike. Each of these parts, except random, is described by a nominal or deterministic portion and a probabilistic perturbation about this nominal portion. The resulting distribution is similar to the schematic, in figure 1 upper left and as further described in reference 7. One justification for synthesizing each loading condition in terms of primitive parts is that experts, over the years, have developed good judgment of the ranges of perturbations about nominal or deterministic conditions. A computer code (Composite Load Spectra) has been developed to synthesize the four parts of each load condition by using (1) available data from various sources of past experience, (2) probability theory, and (3) a dedicated expert system, which includes the information supplied by the experts.

The results from the application of the Composite Load Spectra computer code to probabilistically simulate loads for two blades are summarized in table 1. The comparisons with the measured data are in very good agreement considering the large number of primitive variables (47) required to synthesize these loads. The conclusion is that methods can be developed and are available to probabilistically synthesize complex load conditions for hot aerospace structures and structures in general.
PROBABILISTIC FINITE ELEMENT STRUCTURAL ANALYSIS

The fundamental assumption for developing probabilistic finite element methods (PFEM) for structural analysis is that the uncertainties in each primitive structural variable can be represented by an assumed probabilistic distribution. Primitive structural variables are those which are used to describe a structure such as: (1) stiffness, (2) strength, (3) thickness and tolerance, (4) spatial location, (5) attachment, and (6) various nonlinear material dependencies (temperature, stress, time, etc) as is schematically illustrated in figure 1 upper right. See also reference 7. Subsequently, the uncertainties in the load conditions (synthesized by the composite load spectra) and the uncertainties in the primitive structural variables are computationally synthesized by performing probabilistic finite element structural analysis to simulate uncertainties in the structural response of a specific structural component or structure. The structural response is generally described in terms of usual quantities such as displacement, frequencies, buckling loads, and structural fracture toughness as was already mentioned.

PFEM has been formalized and integrated into a computer code identified as NESSUS (numerical Evaluation of Stochastic Structures Under Stress). NESSUS contains a library of finite elements and is driven by an expert system. It can be used to probabilistically evaluate all types of structures. Representative results obtained using NESSUS on various structures are given in references 1 to 7 and for the specific example will be discussed in a later section. The combined effects of the primitive variable uncertainties on structural response are generally shown as probability distributions, figure 1 bottom right (stress from NESSUS). The information generated for these probability distributions can also be used to evaluate the sensitivities which influence these distributions. The significant point is the PFEM yields a wealth of information which can be used to evaluate: (1) the uncertainties in the structural response, and (2) the sensitivities which can be used to adjust the design for enhanced probability of success. The important conclusion is that probabilistic finite element methods can be developed and are available to quantify uncertainties in the structural performance of a variety of structures. In addition the sensitivities that influence this performance can be evaluated and ranked.

PROBABILISTIC SIMULATION OF MATERIAL BEHAVIOR

The fundamental assumptions to probabilistically simulate complex nonlinear material behavior are: (1) a relationship for material behavior can be developed in terms of primitive variables affecting this behavior and (2) the uncertainties in the primitive variables can be described by assumed distributions (ref. 7). A multifactor interaction model (MFIM) for this relationship has been developed and incorporated in the NESSUS computer code. The MFIM is used to develop resistance probability functions as shown in figure 1, bottom right (strength).

This MFIM is applied to specific structures to probabilistically determine: (1) the resistance curve for damage (crack) initiation and (2) damage propagation and its effects on global structural response. The results for the most probable point for damage initiation, the most probable path, and the degradation in structural integrity can then be determined for specified probabilities. The important observation is that the uncertainties in damage initiation, propagation and subsequent effects on structural performance can be probabilistically simulated by the methodology described herein. It is worthy of note that this methodology, in general, is applicable to a variety of structures, and can readily be incorporated to monitor the in-service health of structures in general and aerospace structures in particular.
RISK-COST ASSESSMENT

The methodology described previously has been extended to perform reliability and risk-cost assessments. In order to accomplish this, (1) the cost for component/structure service readiness needs to be quantified and (2) the cost as a consequence of component/structure failure must be established. Both of these have been integrated into the probabilistic structural analysis methodology (ref. 8). The results from the application of this methodology to the specific example blade are summarized in figure 3 in terms of fatigue cycles to failure.

The important observation from the aforementioned discussion is that the reliability and risk-cost of structures in general and man-rated structures in particular can be assessed using probabilistic methods of the type described herein. The implications are far-reaching because these methods are primarily computational and can be applied to existing structures to evaluate their risk for continuing service as well as those on the design board and those still in the conceptual phase.

RELIABILITY/CERTIFICATION -- AN EMERGING APPROACH

The collective observations from the previous discussion led to an emerging approach to computationally simulate structural reliability, risk components qualification, and eventually vehicle structure certification. The general steps for this emerging approach are outlined as follows:

1. Develop a coarse structure or structural component/vehicle (global) analysis model.
2. Conduct probabilistic structural analysis (PSA) of the types described herein.
3. Identify the critical component/structure areas from the results of PSA.
4. Perform global/local PSA’s to evaluate nonlinear effects and to locate probable sites of damage initiation.
5. Determine the most probable damage propagation path.
6. Evaluate probable structural degradation along this path.
7. Establish probable path extent for violation of specified structural performance criteria (for example, 10-percent increase in displacement or 5 percent reduction in the frequency of the first vibration mode).
8. Assess the corresponding reliability and risk and decide on their acceptability.
9. Schedule inspection intervals and retirement for cause criteria based on the results of items 5, 6, and 7.
10. Verify with probabilistically selected (using respective sensitivities) critical structural components and prototype structure tests.
11. Design a suitable in-service health monitoring system using the results from items 8 and 9 above in order to ascertain that the component/structure will meet the acceptable reliability and risk requirements.
CONCLUSIONS

A methodology has been developed for the formal probabilistic quantification of uncertainties in the structural performance and subsequent reliability and risk of man-rated structures. The key elements in this methodology are: (1) probabilistic load simulation, (2) probabilistic finite element analysis, (3) probabilistic simulation of complex nonlinear material behavior, and (4) risk-cost assessment. This methodology is described in terms of fundamental aspects and application to a specific structural component which is a turbopump blade of the Space Shuttle Main Engine (SSME) and which was selected for its complexity in order to demonstrate what can be done by using this methodology. The specific example illustrates how the uncertainties in all the basic parameters (primitive variables) for loads, structure and material behavior are incorporated in order to probabilistically simulate the uncertainties in the structural response (global and local). Also, the example illustrates how the reliability and risk-cost can be assessed. Collectively, the summary of the fundamental considerations and the results from the specific example demonstrate that a formal methodology is available to evaluate the reliability and risk-cost of man-rated structures in aerospace environments as well as structures in general. In addition, an emerging approach is outlined which can be used to computationally qualify and eventually certify future structures.

REFERENCES

### TABLE 1

**HPOTP & HPFTP Parameters**  
Phase II Engine Calculated vs Measured

<table>
<thead>
<tr>
<th>Condition</th>
<th>HPOTP</th>
<th></th>
<th>HPFTP</th>
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<tbody>
<tr>
<td></td>
<td>Speed</td>
<td>Turbine Temp</td>
<td>Speed</td>
<td>Turbine Temp</td>
</tr>
<tr>
<td></td>
<td>Calc</td>
<td>Measured</td>
<td>Calc</td>
<td>Measured</td>
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<tr>
<td></td>
<td>(rpm)</td>
<td>(rpm)</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hardware - 2σ random</td>
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<td>-- 53</td>
<td>288</td>
<td>-- 65</td>
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<tr>
<td>Test - 2σ random</td>
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<td>-- 157</td>
<td>298</td>
<td>-- 20</td>
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<tr>
<td>Total random</td>
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<td>-- 165</td>
<td>554</td>
<td>-- 70</td>
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<td>Low NPSP - det</td>
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<td>-- 225</td>
<td>55</td>
<td>-- 52</td>
</tr>
<tr>
<td>High NPSP - det</td>
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<td>-- -219</td>
<td>-04</td>
<td>-- -62</td>
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<tr>
<td>Range = random + det</td>
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<td>1500</td>
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<tr>
<td>Max</td>
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<tr>
<td>Min</td>
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<td>27500</td>
<td>34482</td>
<td>34750</td>
</tr>
</tbody>
</table>

**Meanings:**  
- **Calc:** Calculated value  
- **Measured:** Measured value  
- **°R:** Degree Rank  
- **Measured variation for phase II test set**  
- **Variations in engine hardware**  
- **Initial test conditions — Inlet temperatures & mixture ratio**  
- **Duty cycle effects of Inlet pressures plus correlated 2σ variations of cavitation**
FIGURE 1
Component Response Analysis Using CLS Coupled With PSAM

Turbine Blade Loading

Nessus Turbine Blade Coarse Model

Input Variables

Centrifugal

Pressure & Temperature

Operating Stress

Probability of Occurrence

STRESS-β (FROM NESSUS)

STRENGTH-δ (FROM GENERIC PROBABILISTIC MATERIAL PROPERTY MODEL)

RESPONSE SCATTER

PROBABLE DAMAGE INITIATION

RESISTANCE SCATTER

LeRC Contracts

CLS - Composite Loads Spectra
PSAM - Probabilistic Structural Analysis Methods - SWRI
FIGURE 2

PROBABILISTIC RISK-COST ASSESSMENT

PROBABILITY OF DAMAGE INITIATION

TOTAL COST

= COST FOR COMPONENTS
SERVICE READINESS

+ PROBABILITY OF DAMAGE INITIATION

* CONSEQUENTIAL COST DUE TO
DAMAGE INITIATION

TOTAL COST

1000

100

1

10^4 10^5 10^6

FATIGUE CYCLES

1.0

10^{-2}

10^{-4}

10

10^4 10^5 10^6

FATIGUE CYCLES