LARGE LIQUID ROCKET ENGINE
TRANSIENT PERFORMANCE SIMULATION SYSTEM

CONTRACT NO. NAS8-36994
SIX MONTH REPORT

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Large Liquid Rocket Engine Transient Performance Simulation, Contract NAS8-36994, is a program to design and develop a simulation system for rocket engine transient performance models. The system was designed and conceptually demonstrated during Phase I of the contract. The current Phase II of the contract will enhance the system, develop a detailed simulation of the Technology Test Bed Engine (TTBE), and deliver the model and system software to the National Aeronautics and Space Administration (NASA) George C. Marshall Space Flight Center (MSFC). The program is sponsored by NASA/MSFC with Mr. W.A. Adams acting as project manager. The Pratt & Whitney program manager is Mr. J.R. Mason.

The objective of the program is to apply state-of-the-art modeling and simulation technology to the rocket engine models. This application will be used to design, develop, and produce all liquid rocket transient simulation system compatible with MSFC control analysis software, simulation facilities, and computers. The simulation system will be referred to as the ROCETS (Rocket Engine Transient Simulation) System.

The report covers the third six months of ROCETS development.
SUMMARY

Phase 1 of the ROCETS program consists of seven technical tasks plus provision for reports and review; these tasks are:

1. Architecture
2. System Requirements
3. Component and Submodel Requirements
4. Submodel Implementation
5. Component Implementation
6. Submodel Testing and Verification
7. Subsystem Testing and Verification

During Phase I, the above tasks were completed and a Critical Design Review and Phase I completion review were conducted at MSFC. Phase II of Rocets consists of two technical tasks plus reports and reviews; these tasks are:

1. TTBE Model Data Generation
2. System Testing and Verification

During this period specific coding of the system processors was begun and the engineering representations of Phase I were expanded to produce a simple model of the Technology Test Bed Engine (TTBE). As code was completed, some minor modifications to the system architecture centering on the global variable common, GLOBVAR, were necessary to increase processor efficiency.

The engineering modules completed during Phase II are listed below:

1. INJT00 - Main Injector
2. MCHB00 - Main Chamber
3. NOZL00 - Nozzle Thrust Calculations
4. PBRN00 - Preburner
5. PIPE02 - Compressible flow without inertia
6. PUMP00 - Polytropic Pump
7. ROTR00 - Rotor Torque Balance/Speed Derivative
8. TURB00 - Turbine

Detailed documentation of these modules is in the appendix. In addition to the engineering modules, several submodules were also completed during the reporting period. These submodules include combustion properties, component performance characteristics (maps) and specific utilities. Documentation of the submodules will be included in updates to the System Design Specification (SDS).
Specific coding was begun on the system configuration processor. All functions necessary for multiple module operation have been completed but the SOLVER implementation is still under development. This portion of the processor has been incorporated into an in-house system for automated verification of the engineering modules. This system, the Verification Checkout Facility (VCF) allows interactive comparison of module results to store data as well as provides an intermediate checkout of the processor code.

After validation using the VCF, the engineering modules and submodules were used to build a simple Technology Test Bed Engine (TTBE). The simple TTBE model has 55 states and 2 algebraic loops. This simulation of main-stage TTBE operation demonstrates steady-state and transient operation of a full rocket simulation, used the ROCETS general property package, and provided additional information on system operation. The simple TTBE model provides a basis for completion of the detailed TTBE model which will be used to verify the ROCETS system.
SYSTEM REQUIREMENT CHANGES

The system requirements remain unchanged.

SYSTEM ARCHITECTURE CHANGES

Minor modifications to the global common structure are being implemented during coding. These changes center around the GLOBVAR common. Instead of breaking out all variables by type and storing them in separate arrays, all variables are being stored in the order encountered into the global common. The only variable types currently being used are real*4, character*4 and integer*4. The global variable names are still be stored in the GLOBNAM common as character*8. The changes should increase processor speed in the linked list search which verifies variable existence.

COMPONENT/SUBMODULE OVERVIEW

Modules

Thirteen component modules have been developed for the ROCETS system. Five modules were developed during Phase I of the contract and were used to model a simple engine sub-system. Eight more modules were developed under Phase II of the contract in order to model a complete rocket engine. The Phase I modules have been slightly revised under Phase II and have been incorporated into the rocket engine simulation (Simple Technology Test Bed Engine).

The component modules fall into two general categories -- state-derivative modules and non state-derivative modules. The state-derivative modules perform a function that can be described by differential equations. Associated with the module are "states" whose time derivatives are calculated by the module. Since the solver controls the value of the states, the modules require them as inputs and use them along with other inputs to calculate the state-derivatives which are module outputs. Non state-derivative modules have no states associated with them and are not directly controlled by the solver. The format for each module consists of an argument list, software identification number, history (revision dates), interface data, a description of inputs and outputs, a brief engineering description, a list of required subroutines/commons and finally the FORTRAN code.

Following is a list of the component modules that have been developed or updated, along with a brief description.

STATE-DERIVATIVE MODULES

- PBRN00 - PREBURNER MODULE. Combined combustion and volume dynamics module using overall density, temperature and oxidizer fraction as states.
**MCHB00** - MAIN COMBUSTION CHAMBER. Combustion and volume dynamics module using overall density, temperature and oxidizer fraction as states.

**INJT00** - MAIN FUEL INJECTOR. Perfect gas volume dynamics using overall density, temperature and oxidizer fraction as states.

**ROTR00** - ROTOR MODULE. Simple torque balance that handles two supply torques, two required torques, drag torque and extraction torque. Rotational speed is the state.

**PIPE00** - PIPE WITH INERTIA. Updated pipe flow module using flow as a state for incompressible flow.

**MIXR00** - FLOW MIXER. Updated flow mixer with reverse flow capability using density and internal energy as states.

**SPLT00** - FLOW SPLITTER. Updated flow splitter with reverse flow capability using density and internal energy as states.

**VOLM00** - LUMPED VOLUME. Updated volume module with reverse flow capability and heat transfer. Density and internal energy are the states.

**PUMP00** - PUMP MODULE. Polytropic pump with an internal iteration on density ratio.

**TURB00** - TURBINE MODULE. Isentropic turbine with heat transfer and compressibility effects.

**NOZL00** - NOZZLE MODULE. Isentropic nozzle with normal shock.

**PIPE01** - PIPE WITH LOSS. Updated pipe flow module without inertia. Pressure loss for incompressible flow.

**PIPE02** - FLOW THROUGH ORIFICE. Compressible flow through an orifice using a standard flow parameter relation.

---

**Submodules**

Twenty-one engineering submodules which are used to support the rocket engine component modules have been created or updated during this phase of the program. Basically, the sub-modules are divided into maps, utility and property submodules.

The map submodules, described in Table I, consist of component maps used to provide specific engine characteristics.
The utility submodules, described in Table II, consist of routines which perform specific functions or tasks.

The property submodules, described in Table III, consist of routines and maps used to calculate both combustion properties and real fluid properties. The combustion property submodules which were created during this phase consist of a system driver, model type interface routine and a series of combustion maps which are limited to reactions involving hydrogen and oxygen as the reactants. The real fluid property submodule was updated to meet the growing requirements of the ROCETS systems.

### TABLE I
**MAP SUBMODULES**

<table>
<thead>
<tr>
<th>SUBROUTINE</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>PMAP01</td>
<td>SSME ATD high pressure fuel pump map of both head and torque coefficients vs flow coefficient.</td>
</tr>
<tr>
<td>PMAP02</td>
<td>SSME ATD high pressure oxidizer (main stage) pump map of both head and torque coefficients vs flow coefficient.</td>
</tr>
<tr>
<td>PMAP03</td>
<td>SSME ATD high pressure oxidizer (preburner stage) pump map of both head and torque coefficients vs flow coefficient.</td>
</tr>
<tr>
<td>TBMP01</td>
<td>SSME ATD high pressure oxidizer turbine map of flow parameter vs PR with lines of speed parameter. Also, efficiency over velocity ratio vs velocity ratio.</td>
</tr>
<tr>
<td>TBMP02</td>
<td>SSME ATD high pressure fuel turbine map of flow parameter efficiency over velocity ratio vs velocity ratio vs PR with lines of speed parameter. Also, efficiency over velocity ratio vs velocity ratio.</td>
</tr>
<tr>
<td>RDMP20</td>
<td>Preburner/main chamber temperature rise curves.</td>
</tr>
</tbody>
</table>

### TABLE II
**UTILITY SUBMODULES**

<table>
<thead>
<tr>
<th>SUBROUTINE</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDNZ00</td>
<td>Calculates the performance of an isentropic nozzle with normal shock.</td>
</tr>
<tr>
<td>MACH03</td>
<td>Calculates Mach number</td>
</tr>
<tr>
<td>UNIT00</td>
<td>Determines all the SI/English units conversion constants</td>
</tr>
</tbody>
</table>
TABLE III
PROPERTY SUBMODULES

<table>
<thead>
<tr>
<th>SUBROUTINE</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>COMB00</td>
<td>The combustion property driver routine</td>
</tr>
<tr>
<td>PGAS00</td>
<td>The perfect gas model interface routine</td>
</tr>
<tr>
<td>CPGM00</td>
<td>Calculate CP and gamma for the products of combustion. The routine also allows for pressure ratio effects.</td>
</tr>
<tr>
<td>CPMP01</td>
<td>CP as a function of pressure &amp; temperature for hydrogen</td>
</tr>
<tr>
<td>RDM12</td>
<td>Molecular weight as a function of mixture ratio</td>
</tr>
<tr>
<td>RDMP13</td>
<td>Gamma as a function of oxidizer fraction</td>
</tr>
<tr>
<td>RDMP21</td>
<td>CP as a function of oxidizer fraction</td>
</tr>
<tr>
<td>ZGAS00</td>
<td>Calculates compressibility (Z) effects as a function of pressure, temperature and mixture ratio</td>
</tr>
<tr>
<td>ZZMP01</td>
<td>Compressibility as a function of pressure and temperature for hydrogen</td>
</tr>
</tbody>
</table>

REAL FLUID PROPERTIES - UPDATED

PROP00     | Provides thermodynamic properties for the five real fluid properties |

System Processors

The configuration processor is currently under full scale development. All functions necessary for a single module by module checkout have been completed. The system can currently interpret a single module/multiple node input and configure the necessary FORTRAN calls. Property capability has been added and currently state, state derivative and additional balance capabilities are being coded to allow multiple module checkouts to begin. Unit definition is also definable through configuration input. A common structure has been defined and is currently in place and is described in greater detail in the architecture section. A linked list system is used for determining variable existence and adding new variables to the list during configuration.

A system was created called VCF (Verification Checkout Facility) which allows interactive comparisons to be made between a ROCETS module and a section of...
code representing the same function in the DTM. Binary CAPA files are created using the DTM. This allows the engineering equations to be verified during the development phase.
The Configuration Preprocessor and the Verification Checkout Facility combine as illustrated above. A simple flag in the configuration input file either turns on or turns off the VCF system.
ROCETS Main System
Configuration PreProcessor
First level calls from main driver routine.

ROCONF
Main configuration driver

RCONF
Main reading routine

COMGEN
Common generation routine

CODEGEN
Code generation routine
RCONF
Main Reading Block

READMD
Module Reading Routine

READEQ
Equation Reading Routine

READPR
Property Reading Routine

READUT
Units Reading Routine

READEX
Externa ls Reading Routine

READRN
Run Reading Routine

READST
States Reading Routine
COMGEN
Common Generation Block

GENVAR

CHECKVAR
Variable Generation Routine

ADDVAR
Linked List Adding Routine
A working example for the module input and the generated code follows. For a two-module input, such as:

```
DEFINE SYSTEM

MODULE: 1 USING PIPE00
    NODE 1 IS IN
    NODE 2 IS VOL1;

MODULE: 2 USING PIPE01
    NODE 3 IS IN2
    NODE 4 IS VOL2;

END SYSTEM
```

The following common would have been generated by the configuration processor:

```
COMMON / GLOBVAR /
*x    IPRPL, IUPDAT, MODN1, NOD1IN, NOD2VOL1,
*x    AREA1, CF1, PTIN, PTVOL1, RHOIN,
*x    RHOVOL1, XLEN1, W1, DWDT1, MODN2,
*x    NODIN2, NOD2VOL2, CF2, PTIN2, PTVOL2,
*x    RHOIN2, W2
```

All of these variables were created using the standard naming convention described in the ROCETS standards.

The above input would also have resulted in the following calls to the engineering modules as listed below:

```
CALL PIPE00  ( IPRPL, IUPDAT, MODN1, NOD1IN, NOD2VOL1, NOD2VOL1, AREA1, CF1, PTIN, PTVOL1, RHOIN, RHOVOL1, XLEN1, W1, DWDT1 )
CALL PIPE01  ( IPRPL, IUPDAT, MODN2, NOD1IN2, NOD2VOL2, CF2, PTIN2, PTVOL2, RHOIN2, W2 )
```
The requirements for the property calls have recently been revised. Previously all properties were called as the following example shows:

\[
\text{PROPERTY: } \text{PTVOL1} = F(\text{RH0VOL1, UTVOL1 ) USING HYDROGEN;}
\]
\[
\text{PROPERTY: } \text{PTVOL2} = F(\text{RH0VOL2, UTVOL2 ) USING OXYGEN ;}
\]

This would have produced the following code:

```c
C
C BEGIN PROPERTY CALLS
C
CALL PROPO0 ('HYDROGEN',14,RH0VOL1,UTVOL1 ,PTVOL1 ,
* DUMYVOL1,DUMYVOL1,TEMP)
CALL PROPO0 ('OXYGEN ',14,RH0VOL2,UTVOL2 ,PTVOL2 ,
* DUMYVOL2,DUMYVOL2,TEMP)
```

Recently, the need for real combustion property calculations have altered the form of the property definition to the following:

\[
\text{PROPERTY}
\]
\[
\text{PROPO0}
\]
\[
\text{PTVOL1} = F(\text{RH0VOL1, UTVOL1 ) USING HYDROGEN}
\]
\[
\text{COMBO0}
\]
\[
<\text{combustion definition>};
\]

The combustion definition has not been resolved as of this printing. The type of calls generated would be exact like those listed above for PROPO0 and would be similar for COMBO0.
The VCF calls to the input panels from the program main for the above example look like:

```c
C
C CHECKOUT VERIFICATION FACILITY
C ENTRY CALL

    DO 100 I = 1, 2
    IRC = 0
    DO 50 J = 1, ISIZE(I)
  50 ARRAY(J) = ARGLIST(I,J)
    IF(I.EQ. 2) IRC = -1
100 CALL VCFIN ( MODNM(I), MMNEM(I), ARRAY, ISIZE(I), IRC )
```

The arrays MODNM and MMENM contain the character names of the module's actual subroutine name and the module's mnemonic, respectively. The array ARGLIST contains the argument lists as read from the data dictionary and ISIZE contains the actual size of each argument list. The variable IRC is used to verify that data was input to the simulation and as a single to the VCF routine that the last call has occurred.

The VCF output panel is also invoked from the main program with the following calls:

```c
C
C CALL ROUTINE TO WRITE OUTPUT DATA TO VCF FILE
C
    DO 1000 I = 1, 2
    DO 500 J = 1, ISIZE(I)
      IARRAY(J) = IPTLIST(I,J)
  500 ARRAY(J) = ARGLIST(I,J)
1000 CALL VCFHRT ( MODNM(I), MMNEM(I), ARRAY, IARRAY, ISIZE(I) )
C
C INVOKE VCF OUT PANEL SYSTEM
C
    CALL VCFOUT
```

The calls to VCFHRT simply place the output from the simulation into a location where the VCF output routine can have access to it. The VCFOUT routine is the actual output panel driver. The double assignment of ARRAY will be removed at next update since it is redundant.
An example of what the VCF input screen would look like for the first module configured in this example:

<table>
<thead>
<tr>
<th>ROCETS</th>
<th>I/O</th>
<th>DTM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>IPRPL</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>IUPDAT</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>MODN1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>NOD1IN</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>NOD2VOL1</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>AREA1</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>CF1</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>PTIN</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>PTVOL1</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>RHOIN</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>RHOVOL1</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>XLEN1</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>W1</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>DWDT1</td>
<td></td>
</tr>
</tbody>
</table>

An example of what the VCF output screen format would look like for the first module configured in this example (without values filled in):

<table>
<thead>
<tr>
<th>ROCETS_VAR</th>
<th>DTM_VAR</th>
<th>ROCETS_VAL</th>
<th>DTM_VAL</th>
<th>DELTA</th>
<th>%DELTA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 IPRPL</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 IUPDAT</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 MODN1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 NOD1IN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 NOD2VOL1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 AREA1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7 CF1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 PTIN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9 PTVOL1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 RHOIN</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11 RHOVOL1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12 XLEN1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13 W1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14 DWDT1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
This is an full and current example of a configuration input file:

```
DEFINE RUN
    VCF = ON
END RUN

DEFINE EXTERNALS
    PTIN , HTIN , PTEX ,
    CFLIN1, HTEX , VOLVOL ,
    CFLIN2 ;
END EXTERNALS

DEFINE SYSTEM

    MODULE: 1 USING PIPE00 ,
        NODE 1 IS IN ,
        NODE 2 IS VOL1 ;
    MODULE: 2 USING PIPE01 ,
        NODE 3 IS IN2 ,
        NODE 4 IS VOL2 ;

    PROPERTY: PTVOL1 = F(RHOVOL1, UTIVOL1 ) USING HYDROGEN ;
    PROPERTY: PTVOL2 = F(RHOVOL2, UTIVOL2 ) USING OXYGEN ;

    EQUATION: PT = PS * 1000/21.2 * TTIN *
        T ;
END SYSTEM
```
This is the FORTRAN that would be currently produced by the CONFIN file listed on the previous page:

```
PROGRAM MAIN
COMMON / ARGLIST / ARGLIST (2, 14)
COMMON / ARGVAR / IPTLIST (2, 14)
COMMON / MODULE / MMNEM (2), MODNM (2)
COMMON / COMSIZ / ISIZE (2)
CHARACTER*8 A, ARGLIST, ARRAY (50), MODNM, MMNEM
INTEGER IPTLIST, IARRY (50)

C CHECKOUT VERIFICATION FACILITY
C ENTRY CALL
DO 100 I = 1, 2
   IRC = 0
   DO 50 J = 1, ISIZE (I)
      50 ARRAY (J) = ARGLIST (I, J)
   IF (I .EQ. 2) IRC = -1
100 CALL VCFIN (MODNM (I), MMNEM (I), ARRAY, ISIZE (I), IRC)
C CALL READER ROUTINE TO BRING IN VALUES
C CALL VCFREAD
C CALL MAIN SIMULATION SUBROUTINE
C CALL ROCETS
C CALL ROUTINE TO WRITE OUTPUT DATA TO VCF FILE
C DO 1000 I = 1, 2
   DO 500 J = 1, ISIZE (I)
      IARRY (J) = IPTLIST (I, J)
   500 ARRAY (J) = ARGLIST (I, J)
1000 CALL VCFWRT (MODNM (I), MMNEM (I), ARRAY, IARRY, ISIZE (I))
C INVOKE VCF OUT PANEL SYSTEM
C CALL VCFOUT
END
SUBROUTINE ROCETS

COMMON / GLOBVAR /
  * IPRLP, IUPDAT, MODN1, NOD1IN, NOD2VOL1,
  * AREA1, CF1, PI1, PTVol1, RHOIN,
  * RHOVOL1, XLEN1, W1, DWD1, MODN2,
  * NOD1IN2, NOD2VOL2, CF2, PTIN2, PTVOL2,
  * RHOIN2, W2
  CHARACTER*4
  * MODN1, NOD1IN, NOD2VOL1, NOD2VOL2, CF2
  * , PTIN2
C CALL UNIT DEFINITION ROUTINE
C CALL UNIT00 ('ENGLISH ')
C BEGIN PROPERTY CALLS
C CALL PROP00 ('HYDROGEN', 14, RHOVOL1, UTVOL1, PTVOL1,
  * DUMYVOL1, DUMYVOL1, TEMP)
```

Pratt & Whitney
FR-20282-3
CALL PRPO0 ('OXGEN ',14,RHOVOL2,UTVOL2,PTVOL2,
  DUMYVOL2,DUMYVOL2,TEM)

BEGIN MODULE CALLS
  PT  = PS * 1000/21.2 * TTIN * 
  CALL PIPEO0 (IPRPL ,IUPDAT ,MODN1 ,NODIN , 
  NOD2VOL1,AREA1 ,CF1 ,PTIN ,PTVOL1 , 
  RHOIN ,RHOVOL1,XLEN1 ,W1 ,DWD1 )
  RETURN
END

SUBROUTINE VCFREAD

COMMON / GLOBVAR /
  * IPRPL , IUPDAT , MODN1 , NODIN , NOD2VOL1 ,
  * AREA1 , CF1 , PTIN , PTVOL1 , RHOIN ,
  * RHOVOL1 , XLEN1 , W1 , DWD1 , MODN2 ,
  * NODIN2 , NOD2VOL2 , CF2 , PTIN2 , PTVOL2 ,
  * RHOIN2 , W2
  CHARACTER*4
  * MODN1 ,NODIN ,NOD2VOL1 ,NOD2VOL2 ,CF2
  * ,PTIN2
  NAMELIST / ROCK /
  * IPRPL , IUPDAT , MODN1 , NODIN , NOD2VOL1 ,
  * AREA1 , CF1 , PTIN , PTVOL1 , RHOIN ,
  * RHOVOL1 , XLEN1 , W1 , DWD1 , MODN2 ,
  * NODIN2 , NOD2VOL2 , CF2 , PTIN2 , PTVOL2 ,
  * RHOIN2 , W2
  DO 100 I = 1, 2
  100 READ (15,ROCK)
  RETURN
END

SUBROUTINE VCFWRT (MODNM,MMNEM,ARRAY,IARRY,ISIZE)

COMMON / GLOBVAR /
  * IPRPL , IUPDAT , MODN1 , NODIN , NOD2VOL1 ,
  * AREA1 , CF1 , PTIN , PTVOL1 , RHOIN ,
  * RHOVOL1 , XLEN1 , W1 , DWD1 , MODN2 ,
  * NODIN2 , NOD2VOL2 , CF2 , PTIN2 , PTVOL2 ,
  * RHOIN2 , W2
  CHARACTER*4
  * MODN1 ,NODIN ,NOD2VOL1 ,NOD2VOL2 ,CF2
  * ,PTIN2

COMMON / VARTYP / TYPE( 22)

CHARACTER*8 MODNM,MMNEM,ARRAY(*)

REAL*4 VAL( 22)
INTEGER TYPE
INTEGER IARRY(*)

EQUIVALENCE (IPRPL ,VAL(1))
WRITE(16,10) MODNM,MMNEM
10 FORMAT('MODULE: ',A8,' MNEMONIC: ',A8)
DO 50 I = 1,ISIZE
  IF (TYPE(IARRY(I)).EQ.3) GOTO 20
  IF (TYPE(IARRY(I)).EQ.2) GOTO 30
  WRITE(16,100) ARRAY(I),VAL(IARRY(I))
  GOTO 50

Pratt & Whitney
FR-20282-3
C

RETURN
END

BLOCK DATA ONE
COMMON / GLOBNAM / A(22)
COMMON / VARTYP / TYPE(22)
COMMON / ARGLST / ARGLIST(2,14)
COMMON / ARGVAR / IPTLIST(2,14)
COMMON / MODULE / MMNEM(2),MODNM(2)
COMMON / COMSIZ / ISIZE(2)
CHARACTER*8 A, ARGLIST, MMNEM, MODNM
INTEGER IPTLIST

DATA A /
  'IPRPL ', 'IUPDAT ', 'MODN1 ', 'NOD1IN ', 'NOD2VOL1',
  'AREA1 ', 'CF1 ', 'PTIN ', 'PTVOL1 ', 'RHOIN ',
  'RHOVOL1 ', 'XLEN1 ', 'W1 ', 'DWDT1 ', 'MODN2 ',
  'NOD1IN2 ', 'NOD2VOL2', 'CF2 ', 'PTIN2 ', 'PTVOL2..',

DATA ARGLIST /
  'IPRPL ', 'IPRPL ', 'IUPDAT ', 'IUPDAT ', 'MODN1 ',
  'MODN2 ', 'NOD1IN ', 'NOD1IN ', 'NOD2VOL1', 'NOD2VOL2',
  'AREA1 ', 'CF2 ', 'CF1 ', 'PTIN2 ', 'PTIN ',
  'PTVOL2 ', 'PTVOL1 ', 'RHOIN2 ', 'RHOIN ', 'W2 ',
  'RHOVOL1 ', ' ', 'XLEN1 ', ' ', 'W1 ',

DATA IPTLIST /
  1, 1, 2, 2, 3,
  15, 4, 16, 5, 17,
  6, 18, 7, 19, 8,
  20, 9, 21, 10, 22,
  11, 0, 12, 0, 13,
  0, 14, 0/

DATA MMNEM /
  '1 ', '2 '

DATA MODNM /
  'PIPE00 ', 'PIPE01 '

DATA ISIZE /
  14, 10/

DATA TYPE /
  2, 2, 3, 3, 3,
  1, 1, 1, 1, 1,
  1, 1, 1, 1, 2,
  2, 3, 3, 3, 1,
  1, 1/
Simple TTBE Simulation

In order to complete system testing and module verification, a simple model of the Technology Test Bed Engine (TTBE) was completed. The simple TTBE demonstrates the ability to model a complete engine using the ROCETS methodology of combining generic engineering modules with an implicit solver. Figure 1 shows a schematic of the simple TTBE along with the 42 specific stations in the simulation. By using generic code, only the following 13 component modules are required by the simulation:

1. INJT00 - Main Injector
2. MCHB00 - Main Chamber
3. MIXR00 - Flow Mixer
4. NOZL00 - Nozzle Thrust Calculations
5. PBRN00 - Preburner
6. PIPE00 - Incompressible flow with inertia
7. PIPE01 - Incompressible flow without inertia
8. PIPE02 - Compressible flow without inertia
9. PUMP00 - Polytropic Pump
10. ROTR00 - Rotor Torque Balance/Speed Derivative
11. SPLT00 - Flow Splitter
12. TURB00 - Turbine
13. VOLM00 - Volume

The modules described above were hand processed into the simple TTBE simulation along with required property relationships and numerical utilities. There are 55 state variables and 2 algebraic loops required in the simulation as shown in Table IV. State derivatives and outputs are calculated from model inputs and states. The simulation employs the multivariable Newton-Raphson solver, SMITE, to simultaneously drive all state derivatives and algebraic loop errors to within a certain tolerance. SMITE operates on the matrix equation:

\[ \Delta Y = J \Delta X \]

Where \( \Delta Y \) is the amount that the errors need to change to be zero and \( \Delta X \) is the required change in the state or algebraic loop parameter. J, the solver Jacobian, is the matrix of partial derivatives for the states and algebraic loops. The J matrix is evaluated by perturbing each state and balance parameter. Then the J matrix is inverted and the new X values are calculated until the Y variables are within tolerance.

Using initial guesses from data of the Digital Transient Model (DTM) run at 100% RPL, SMITE successfully drives all TTBE model state derivatives and algebraic loop parameters to within a specific tolerance. This demonstrates the capability
of the ROCETS system to converge a rocket simulation to a steady-state point without running a transient.

Transient capability was demonstrated by running the simple TTBE simulation with small perturbations of valve areas about the 100% RPL point. Figures 2–3 show the results of stepping the oxidizer preburner valve open then closed. Figures 4–5 show the results for the same type of movement of the fuel preburner valve.

Additional algebraic loops (balances) were placed in the model to set preburner valve coefficients at points other than 100% RPL. The fuel preburner valve coefficient was iterated until chamber pressure (PTMCHB) was equal to the request:

\[ \text{PTMCHB}_{\text{Request}} = \text{PTMCHB}_{100\% \text{ RPL}} \times \text{RPL} \]

The LOX preburner valve coefficient was iterated until chamber oxidizer fraction (OFRMCHB) was equal to a constant value of 0.865. (This is equivalent to a mixture ratio of 6.407). Figure 6 shows main chamber pressure and oxidizer fraction as a function of RPL. A series of steady-state points between 60% and 119% RPL were then run with the solver iterating on valve coefficients until chamber pressure and LOX fraction were satisfied. This demonstrates the ability of the model to use the solver as a means to set a model parameter based on an input constraint. Output of the run gives a reference steady-state operating characteristic for the model and provides data for SMITE guess curves. Figure 7 shows fuel and LOX preburner valve coefficients and Figure 8 shows turbine speeds as a function of RPL.

The data shown in Figure 7 for valve coefficients as a function of RPL was tabularized and used to construct and open-loop control with RPL request input and valve data area requests calculated from the table. The valve requests were put through a first order lag to simulate actuator dynamics. Gross throttle transients were run by inputting an RPL request as a function of time and using the open-loop control to provide valve areas. Figures 9–13 show results of a run from 100% to 65% RPL decel and figures 14–18 show a run from 65% to 109% RPL.
**TABLE IV – Simple TTBE Simulation States and Algebraic Loops**

<table>
<thead>
<tr>
<th>STATE</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>OFRFPRB</td>
<td>Fuel Preburner Oxidizer Fraction</td>
</tr>
<tr>
<td>OFRMCHB</td>
<td>Main Chamber Oxidizer Fraction</td>
</tr>
<tr>
<td>OFRFMI</td>
<td>Main Fuel Injector Oxidizer Fraction</td>
</tr>
<tr>
<td>OFROPRB</td>
<td>Oxidizer Preburner Oxidizer Fraction</td>
</tr>
<tr>
<td>RHOFMCO</td>
<td>Volume FMCO Density</td>
</tr>
<tr>
<td>RHOFPBI</td>
<td>Fuel Preburner Injector Density</td>
</tr>
<tr>
<td>RHOFPRB</td>
<td>Oxidizer Preburner Injector Density</td>
</tr>
<tr>
<td>RHOF10S</td>
<td>Volume F10S Density</td>
</tr>
<tr>
<td>RHOMCHB</td>
<td>Main Chamber Density</td>
</tr>
<tr>
<td>RHOMFI</td>
<td>Main Fuel Injector Density</td>
</tr>
<tr>
<td>RHOMOIF</td>
<td>Main Oxidizer Injector Density</td>
</tr>
<tr>
<td>RHOOPBI</td>
<td>Oxidizer Preburner Injector Density</td>
</tr>
<tr>
<td>RHOOPRB</td>
<td>Oxidizer Preburner Density</td>
</tr>
<tr>
<td>RHOPBSF</td>
<td>Preburner Fuel Splitter Density</td>
</tr>
<tr>
<td>RHOPBSO</td>
<td>Preburner Oxidizer Splitter Density</td>
</tr>
<tr>
<td>RHO10</td>
<td>Volume 10 Density</td>
</tr>
<tr>
<td>RHO12</td>
<td>Volume 12 Density</td>
</tr>
<tr>
<td>RHO15B</td>
<td>Volume 15B Density</td>
</tr>
<tr>
<td>RHO8</td>
<td>Volume 8 Density</td>
</tr>
<tr>
<td>SNF2</td>
<td>Fuel Turbomachinery Speed</td>
</tr>
<tr>
<td>SN02</td>
<td>Oxidizer Turbomachinery Speed</td>
</tr>
<tr>
<td>TTFPRB</td>
<td>Fuel Preburner Temperature</td>
</tr>
<tr>
<td>TTMCBH</td>
<td>Main Chamber Temperature</td>
</tr>
<tr>
<td>TTMF1</td>
<td>Main Fuel Injector Temperature</td>
</tr>
<tr>
<td>TTOPRB</td>
<td>Oxidizer Preburner Temperature</td>
</tr>
<tr>
<td>UTFMCO</td>
<td>Volume FMCO Internal Energy</td>
</tr>
<tr>
<td>UTFPBI</td>
<td>Fuel Preburner Injector Internal Energy</td>
</tr>
<tr>
<td>UTF10S</td>
<td>Volume F10S Internal Energy</td>
</tr>
<tr>
<td>UTMO1</td>
<td>Main Oxidizer Injector Internal Energy</td>
</tr>
<tr>
<td>UTOPBI</td>
<td>Oxidizer Preburner Injector Internal Energy</td>
</tr>
<tr>
<td>UTPBSF</td>
<td>Preburner Fuel Splitter Internal Energy</td>
</tr>
<tr>
<td>UTPBSO</td>
<td>Preburner Oxidizer Splitter Internal Energy</td>
</tr>
<tr>
<td>UT10</td>
<td>Volume 10 Internal Energy</td>
</tr>
<tr>
<td>UT12</td>
<td>Volume 12 Internal Energy</td>
</tr>
<tr>
<td>UT15B</td>
<td>Volume 15B Internal Energy</td>
</tr>
<tr>
<td>UT7S</td>
<td>Volume 7S Internal Energy</td>
</tr>
<tr>
<td>UT8</td>
<td>Volume 8 Internal Energy</td>
</tr>
</tbody>
</table>
### STATE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>WCCV</td>
<td>Coolant Control Valve Flow</td>
</tr>
<tr>
<td>WFPHI</td>
<td>Fuel Preburner Fuel Flow</td>
</tr>
<tr>
<td>WFPOV</td>
<td>Fuel Preburner Oxidizer Flow</td>
</tr>
<tr>
<td>WFTI</td>
<td>Line FTI Flow</td>
</tr>
<tr>
<td>WFTID</td>
<td>Line FTID Flow</td>
</tr>
<tr>
<td>WF10</td>
<td>Line F10 Flow</td>
</tr>
<tr>
<td>WMFV</td>
<td>Main Fuel Valve Flow</td>
</tr>
<tr>
<td>WMOV</td>
<td>Main Oxidizer Valve Flow</td>
</tr>
<tr>
<td>WOPHI</td>
<td>Oxidizer Preburner Fuel Flow</td>
</tr>
<tr>
<td>WOPOV</td>
<td>Oxidizer Preburner Oxidizer Flow</td>
</tr>
<tr>
<td>W11</td>
<td>Line 11 Flow</td>
</tr>
<tr>
<td>W16</td>
<td>Line 16 Flow</td>
</tr>
<tr>
<td>W20</td>
<td>Line 20 Flow</td>
</tr>
<tr>
<td>W21</td>
<td>Line 21 Flow</td>
</tr>
<tr>
<td>W4</td>
<td>Line 4 Flow</td>
</tr>
<tr>
<td>W7</td>
<td>Line 7 Flow</td>
</tr>
<tr>
<td>W9</td>
<td>Line 9 Flow</td>
</tr>
</tbody>
</table>

### ITERATION VARIABLE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>WFTRB</td>
<td>Fuel Turbine Flow – Iterated until equal to calculated value</td>
</tr>
<tr>
<td>WOTRB</td>
<td>LOX Turbine flow – Iterated until equal to calculated value</td>
</tr>
</tbody>
</table>
Figure 2 - Fuel Preburner Flow Coefficient and Flow vs Time
Figure 3 - Main Chamber Conditions as a Function of Time
FIGURE 4 - Lox Preburner Flow Coefficient and Flow vs Time
Figure 5 - Main Chamber Oxidizer Fraction as a Function of Time

MAIN CHAMBER OXIDIZER FRACTION

LOX PREBURNER OXIDIZER VALVE PERTURBATION

MAIN CHAMBER PRESSURE (PSI)
Figure 6 - Main Chamber Conditions as a Function of RPL
Figure 9 – RPL Request as a Function of Time
Figure 10 - Fuel Preburner Valve as a Function of Time
Figure 11 - Lox Preburner Valve as a Function of Time
Figure 13 - Main Chamber Conditions as a Function of Time
Figure 16 - Lox Preburner Valve as a Function of Time
Figure 17 - Turbine Speeds as a Function of Time
APPENDIX
3.4.2 ENGINEERING MANUAL

3.4.2.1 Module Cross Reference

3.4.2.2 Map Cross Reference

3.4.2.3 Map + Module Compatibility

3.4.2.4 Detailed Module Descriptions

1. Modules

A. Actuator
B. Auxiliary/Special
C. Control
D. Heat Exchanger
E. Injector
   1. INJT00
F. Line/Pipe
   1. PIPE00
   2. PIPE01
   3. PIPE02
G. Main Chamber
   1. MCHB00
H. Mixer
   1. MIXR00
I. Nozzle
   1. NOZL00
J. Pre-burner
   1. PBRN00
K. Pump General Description
   1. PUMP00
L. Rotor
   1. ROTR00
M. Secondary Flow
N. Sensor
O. Splitter
   1. SPLT00
P. Turbine
   1. TURB00
Q. Valve

45
R. Volume/Accumulator/Tank
  1. VOLM00

S. System Utility
  1. SSBL03
  2. TRAP03
  3. UNIT00

2. Sub-Modules
   A. Component Sub-modules
      1. CDNZ00
      2. MACH03
      3. FLPM02
   B. Property Sub-modules
      1. PROP00
      2. COMB00
   C. Map Sub-modules
      1. PMAPXX
      2. TBMPXX
   D. Utility Sub-modules
      1. SMIT01
      2. SUNB01
      3. PRPL00
      4. ITER01
### Module Cross Reference

<table>
<thead>
<tr>
<th>Code</th>
<th>Module</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>E-1</td>
<td>INJT00</td>
<td>Main fuel injector</td>
</tr>
<tr>
<td>F-1</td>
<td>PIPE00</td>
<td>Incompressible fluid flow in pipe with inertia</td>
</tr>
<tr>
<td>F-2</td>
<td>PIPE01</td>
<td>Incompressible fluid flow in pipe with loss</td>
</tr>
<tr>
<td>F-3</td>
<td>PIPE02</td>
<td>Compressible flow through an orifice</td>
</tr>
<tr>
<td>G-1</td>
<td>MCHB00</td>
<td>Main combustion chamber</td>
</tr>
<tr>
<td>H-1</td>
<td>MIXR00</td>
<td>Simple flow mixer</td>
</tr>
<tr>
<td>I-1</td>
<td>NOZL00</td>
<td>Isentropic nozzle</td>
</tr>
<tr>
<td>J-1</td>
<td>PRBN00</td>
<td>Preburner without purge</td>
</tr>
<tr>
<td>K-1</td>
<td>PUMP00</td>
<td>Pump module for polytropic process</td>
</tr>
<tr>
<td>L-1</td>
<td>ROTR00</td>
<td>Rotor</td>
</tr>
<tr>
<td>O-1</td>
<td>SPLT00</td>
<td>Simple flow splitter</td>
</tr>
<tr>
<td>P-1</td>
<td>TURB00</td>
<td>Turbine</td>
</tr>
<tr>
<td>R-1</td>
<td>VOLUM00</td>
<td>Volume/Accumulator/Tank</td>
</tr>
</tbody>
</table>
3.4.2.2 Map Cross Reference

1. PMAP01  -  Pratt and Whitney SSME High Pressure Fuel Pump
2. PMAP02  -  Pratt and Whitney SSME High Pressure Oxidizer Pump (main stage)
3. PMAP03  -  Pratt and Whitney SSME High Pressure Oxidizer Pump (Preburner stage)
4. TBMP01  -  Pratt and Whitney SSME High Pressure Oxidizer Turbine
5. TBMP02  -  Pratt and Whitney SSME High Pressure Fuel Turbine
3.4.2.3  Module + Map Compatibility

1. PUMP00  -  PMAP01, PMAP02, PMAP03
2. TURB00  -  TBMP01, TBMP02
**INPUTS:**

- NOD1 - Fuel Inlet Node
- NOD2 - Fuel Inlet Thermal Node
- NOD3 - Heat Flux Node - Fuel
- NOD4 - Oxidizer Inlet Node
- NOD5 - Oxidizer Inlet Thermal Node
- NOD6 - Heat Flux Node - Oxidizer
- NOD7 - Sleeve Inlet Node
- NOD8 - Sleeve Inlet Thermal Node
- NOD9 - Exit node
- NOD10 - Exit Thermal Node

**STATES:**

- OFR - Oxidizer Fraction
- $T_I$ - Injector Temperature
- $\rho$ - Overall Density

**Symbols:**

- $C_{PF}$ - Specific Heat - Fuel Turbine Discharge
- $C_{PO}$ - Specific Heat - Oxidizer Turbine Discharge
- $\gamma_F$ - Specific Heat Ratio - Fuel Turbine Discharge
- $\gamma_O$ - Specific Heat Ratio - Oxidizer Turbine Discharge
- $\gamma_S$ - Specific Heat Ratio - Sleeve
- $\gamma_I$ - Specific Heat Ratio - Main Fuel Injector
- $\gamma_E$ - Specific Heat Ratio - Exit
- $Q_F$ - Heat Flux - Main Chamber Cooling - Oxidizer
- $Q_O$ - Heat Flux - Main Chamber Cooling - Fuel
- $W_F$ - Fuel Flowrate
- $W_O$ - Oxidizer Flowrate
- $W_S$ - Sleeve Flowrate
- $W_E$ - Exit Flowrate
- $T_F$ - Fuel Temperature
- $T_O$ - Oxidizer Temperature
- OFR$_F$ - Oxidizer Fraction - Fuel Inlet
- OFR$_O$ - Oxidizer Fraction - Oxidizer Inlet
\( T_s \) - Sleeve Temperature  
\( T_e \) - Exit Temperature  
\( \text{Vol} \) - Main Fuel Injector Volume  

**OUTPUTS:**  
\[
\frac{d(OFR)}{dt} \quad \text{- Oxidizer Fraction Derivative}
\]
\[
\frac{dO}{dt} \quad \text{- Density Derivative}
\]
\[
\frac{dT_I}{dt} \quad \text{- Temperature Derivative}
\]

**CALCULATIONS:**

\[
WIN = W_s + W_F + W_O
\]

Total oxygen flow in:  
\[
WOIN = OFRF \times WF + OFRO \times WO
\]

**DERIVATIVE CALCULATIONS:**  
\[
m = O \times Vol
\]
\[
O = \frac{m}{Vol}
\]
\[
\frac{dO}{dt} = \frac{dm}{dt} \left( \frac{Vol}{(Vol)^2} \right)
\]

For a constant volume:  
\[
\frac{dO}{dt} = \frac{dm}{dt} \left( \frac{1}{Vol} \right)
\]
\[
\frac{dO}{dt} = \frac{WIN - W_{out}}{Vol}
\]
ENERGY ANALYSIS:

\[
\frac{dU}{dt} = \dot{Q}_o + \dot{Q}_F + W_F h_F + W_o h_o + W_s h_s - W_E h_E \tag{1}
\]

\[
\frac{dU}{dt} = \frac{d(mu)}{dt} = \frac{du}{dt} (m) + (u) \frac{dm}{dt}
\]

\[u = C_v T_l\]

\[
\frac{dU}{dt} = m C_v \frac{dT_l}{dt} + C_v T_l (W_F + W_O + W_S - W_E) \tag{2}
\]

Substitute (2) into (1) and \( H = C_p T \) and \( T_{EXIT} = T_I \).

\[m C_v \frac{dT_l}{dt} + C_v T_l (W_F + W_O + W_S - W_E) = \dot{Q}_F + \dot{Q}_o + W_FC_p T_F + W_SC_p T_S - W_E C_p T_l\]

Solving for \( \frac{dT_l}{dt} \):

\[
\frac{dT_l}{dt} = \left[ W_F (\gamma_F T_F - T_l) + W_o (\gamma_o T_o - T_l) + W_S (\gamma_S T_S - T_l) - W_E (\gamma_E - 1) T_l + \frac{(\dot{Q}_F + \dot{Q}_o)}{C_v} \right] \frac{1}{m}
\]
PIE 00 - Incompressible fluid flow in pipe with inertia and loss

INPUTS:

NOD1 - Inlet Thermal Node
NOD2 - Exit Thermal Node
A - Inlet Flow Area
CF - Flow Coefficient
Pin - Inlet Pressure
Pout - Exit Pressure
\( \rho \text{ in} \) - Inlet Fluid Density
L - Length of Pipe

OUTPUTS:

\( \frac{dW}{dt} \) - Flow derivative

STATES

W - Flow Rate

CALCULATIONS:

Definition of flow coefficient, CF:

\[
CF = Q \times \sqrt{\frac{GF}{\Delta P}}
\]

where \( GF = \frac{\rho}{\rho_{H_2O}} \) (specific gravity)

and \( Q \) = Volumetric flowrate (USGPM)

\[
Q = \frac{W}{\rho_{AV}}
\]

\[
\rho_{av} = \frac{(\rho_{in} + \rho_{out})}{2}
\]

\[
CF = \frac{W}{\rho_{av}} \sqrt{\frac{\rho_{av}}{\rho_{H_2O}}} \left( \frac{1}{\Delta P} \right)
\]

solving for \( \Delta P \):

\[
CF^2 = \frac{W^2}{\rho_{av}^2 \rho_{H_2O}} \left( \frac{1}{\Delta P} \right)
\]

\[
\Delta P = \frac{W^2}{\rho_{av}^2 \rho_{H_2O} CF^2}
\]

Define \( \alpha \):

\[
\alpha = \rho_{AV} \times CF^2
\]
Substitute into (1): \( \Delta P = \frac{W^2}{a} \times \text{const.} \) \hspace{1cm} (2)

Equation (2) represents the steady state relationship for \( W \) and \( \Delta P \).

Let's call this friction loss \( \Delta P_{\text{loss}} \).

**DERIVATIVE CALCULATIONS:**

Momentum analysis:

\[
F = \frac{Ma}{gc} \hspace{1cm} (1)
\]

\[
a = \frac{dV}{dt} \hspace{1cm} (2)
\]

\[
M = qAL \hspace{1cm} (3)
\]

\[
W = qAV \Rightarrow V = \frac{W}{qA} \hspace{1cm} (4)
\]

Substitute (2), (3) and (4) into (1):

\[
F = \frac{qAL}{gc} \left( \frac{dW}{dt} \right) \frac{1}{qA}
\]

\[
F = \frac{L}{gc} \left( \frac{dW}{dt} \right) \hspace{1cm} (5)
\]

Also:

\[
F = (\Delta P - \Delta P_{\text{loss}}) A \hspace{1cm} (6)
\]

Substitute (6) into (5) and solve for: \( \frac{dW}{dt} \)

\[
\frac{dW}{dt} = (\Delta P - \Delta P_{\text{loss}}) \left( \frac{Agc}{L} \right) \hspace{1cm} \text{(where \( \Delta P = P_{\text{in}} - P_{\text{out}} \))}
\]
PIPEØ1 - Incompressible fluid flow in pipe with loss.

**INPUTS:**

- NOD1 - Inlet Thermal Node
- NOD2 - Exit Thermal Node
- CF - Flow Coefficient
- Pin - Inlet Pressure
- Pout - Exit Pressure
- \( \rho_{\text{in}} \) - Inlet Fluid Density

**OUTPUT:**

- W - Flow Rate

**CALCULATIONS:**

Definition of flow coefficient, CF:

\[
CF = Q \times \sqrt{\frac{GF}{\Delta P}}
\]

where \( GF = \frac{Q}{Q_{H_2O}} \) (specific gravity)

and \( Q \) = Volumetric Flowrate (USGPM)

\[
Q = \frac{W}{\rho_{\text{in}}}
\]

\[
CF = \frac{W}{\rho_{\text{in}}} \sqrt{\frac{\rho_{\text{in}}}{Q_{H_2O}} \left( \frac{1}{\Delta P} \right)}
\]

Solving for W:

\[
CF^2 = \frac{W^2}{\rho_{\text{in}}^2} \frac{\rho_{\text{in}}}{Q_{H_2O}} \frac{1}{\Delta P}
\]

\[
W = \sqrt{\Delta P \, \rho_{\text{in}} \, CF^2 \, Q_{H_2O}} \quad (1)
\]

Define \( \alpha \):

\[
\alpha = \rho_{\text{in}} \times CF^2
\]

Substitute into (1)

\[
W = \sqrt{\alpha \, \Delta P \times \text{const.}}
\]
PIPEØ2 – Compressible flow through an orifice

INPUTS:

NOD1 – Inlet Thermal Node
NOD2 – Exit Thermal Node
K – Number of Head Losses
A – Throat Area
Pin – Inlet Pressure
Pout – Exit Pressure
R – Gas Constant
Tin – Inlet Temperature
γin – Inlet Specific Heat Ratio

OUTPUTS:

W – Flowrate

CALCULATIONS:

\[ \text{PR} = \frac{\text{Pin}}{\text{Pout}} \]

Submodule FLPMEØ2 gives a value for FP from PR, K, γin.

Definition of flow parameter: \( \text{FP} = \frac{W \sqrt{R \times T_{IN}}}{P_{IN} \times A} \)

Solve for \( W \):

\[ W = \frac{FP \times P_{IN} \times A}{\sqrt{R \times T_{IN}}} \]
INPUTS:
- NOD1 - Inlet Thermal Node
- NOD2 - Inlet Node
- NOD3 - Exit Node
- NOD4 - Exit Thermal Node
- NOD5 - Exit Node
- NOD6 - Exit Thermal Node
- NOD7 - Main Chamber Exit Node
- NOD8 - Exit Thermal Node
- OFR_{MFI} - Oxidizer Fraction, Main Fuel Inj.
- W_{out} - Exit Flowrate
- W_{FIG} - Fuel Igniter Flowrate
- W_{MFI} - Main Fuel Injector Flowrate
- W_{MOI} - Main Oxid. Injector Flowrate
- W_{OIG} - Oxid. Igniter Flowrate
- W_{OTH} - Other Fuel Flowrate
- VOL - Main Chamber Volume
- T_F - Fuel Temperature
- \gamma - Specific Heat Ratio (Chamber)

STATES:
- OFR - Oxidizer Fraction
- T_c - Main Comb. Ch. Temperature
- \epsilon - Overall Density

OUTPUTS:
\frac{d(OFR)}{dt} - Oxidizer fraction derivative
\( \frac{d \rho}{dt} \) - Density derivative

\( \frac{dT_c}{dt} \) - Temperature derivative

**CALCULATIONS:**

\[ m = \rho \times \text{Vol} \]

Total oxidizer & Fuel Flows:

\[ W_{OIN} = W_{MOI} + W_{OIG} + W_{MFI} \times OFR_{MFI} \]

\[ W_{FIN} = W_{FIG} + W_{OTH} + W_{MFI} \times (1 - OFR_{MFI}) \]

Flow-Based Mixture Ratio:

\[ OF_w = \frac{W_{OIN}}{W_{FIN}} \]

Mass-based mixture ratio:

\[ OF_m = \frac{OFR}{1 - OFR} \]

Temperature Rise \( \Delta T \)

Where \( \Delta T = f(OFR) \)

\[ T^*_c = T_F + \Delta T \]

**DERIVATIVE CALCULATIONS:**

\[ \rho = \frac{m}{\text{Vol}} \]

\[ \frac{d \rho}{dt} = \frac{\frac{dm}{dt} \text{(Vol)} - \frac{d\text{Vol}}{dt} \rho \text{(m)}}{\text{(Vol)}^2} \]

For a constant volume:

\[ \frac{d \rho}{dt} = \frac{\frac{dm}{dt} \text{(Vol)}}{\text{(Vol)}^2} = \frac{dm}{dt} \left( \frac{1}{\text{Vol}} \right) \]

\[ \frac{d \rho}{dt} = \frac{W_{FIN} + W_{OIN} - W_{out}}{\text{Vol}} \]

Energy analysis:

\[ \frac{dU}{dt} = (W_{FIN} + W_{OIN}) h_c - (W_{out}) h_c \quad (1) \]

\[ \frac{dU}{dt} = m \frac{du}{dt} + u \frac{dm}{dt} \quad (2) \]
Substituting (2) into (1) and solving for \( \frac{du}{dt} \) yields:

\[
\frac{du}{dt} = \left[ (W_{F_{IN}} + W_{O_{IN}})h^* - (W_{out}) h_c - u \frac{dm}{dt} \right] / m \quad (3)
\]

For a perfect gas:

\[
h = C_p T \Rightarrow \frac{dh}{dT} = C_p dT
\]

\[
u = C_v T \Rightarrow \frac{du}{dT} = C_v dT
\]

From continuity:

\[
\frac{dm}{dt} = (W_{F_{IN}} + W_{O_{IN}}) - W_{out}
\]

Substituting perfect gas relationships and \( \frac{dm}{dt} \) into (3) yields:

\[
\frac{du}{dt} = \left[ (W_{F_{IN}} + W_{O_{IN}}) C_p T^* - W_{out} C_p T_c - C_v T_c (W_{F_{IN}} + W_{O_{IN}} - W_{out}) \right] \left( \frac{1}{m} \right)
\]

Simplifying yields:

\[
\frac{dT_c}{dt} = \left[ (W_{F_{IN}} + W_{O_{IN}})(\gamma_c T_c^* - T_c) - W_{out}(\gamma_c - 1) T_c \right] \left( \frac{1}{m} \right)
\]

OFR derivative:

\[
OFR = \frac{m_o}{m_T} = \frac{m_o}{m_o + m_F}
\]

Assuming homogeneous volume: \((OFR) W_{T_{EX}} = W_{O_{EX}}\)

\((1 - OFR) W_{T_{EX}} = W_{F_{EX}}\)

\[
\frac{d(OFR)}{dt} = \left( \frac{dm_o}{dt} m_T - \frac{dm_T}{dt} m_o \right) / m_T^2
\]

\[
= \frac{1}{m_T} \left[ \frac{dm_o}{dt} - \frac{dm_T}{dt} \frac{m_o}{m_T} \right]
\]
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\[
\frac{d(OFR)}{dt} = \frac{1}{m_T} \left[ \frac{dm_o}{dt} - (OFR) \frac{dm_T}{dt} \right]
\]  \hspace{1cm} (1)

\[\frac{dm_o}{dt} = W_{oIN} - W_{oEX} = W_{oIN} - (OFR)W_{TEX} \]  \hspace{1cm} (2)

\[\frac{dm_T}{dt} = W_{TIN} - W_{TEX} \]  \hspace{1cm} (3)

Substituting (2) and (3) into (1) yields:

\[
\frac{d(OFR)}{dt} = \frac{1}{m_T} \left[ W_{oIN} - (OFR)W_{TEX} - (OFR) (W_{TIN} - W_{TEX}) \right]
\]

Simplifying:

\[
\frac{d(OFR)}{dt} = \frac{1}{m_T} \left[ W_{oIN} - (OFR)W_{TIN} \right]
\]
**MIXRØØ** - Simple Flow Mixer

**INPUTS:**
- NOD1 - Inlet 1 Thermal Node
- NOD2 - Inlet 1 Node
- NOD3 - Inlet 2 Thermal Node
- NOD4 - Inlet 2 Node
- NOD5 - Exit Node
- NOD6 - Exit Thermal node
- $h_1$ - Inlet Stream 1 Enthalpy
- $h_2$ - Inlet Stream 2 Enthalpy
- $h_3$ - Exit Enthalpy
- $h_{mix}$ - Mixer Enthalpy
- $W_1$ - Inlet 1 Flowrate
- $W_2$ - Inlet 2 Flowrate
- $W_3$ - Exit Flowrate
- VOL - Mixer Volume

**OUTPUTS:**
- $\frac{dq}{dt}$ - Density Derivative
- $\frac{du}{dt}$ - Internal Energy Derivative

**STATES:**
- $\rho$ - Mixer Density
- $u$ - Mixer Internal Energy
DERIVATIVE CALCULATIONS:

\[ m = e \times \text{vol} \]

\[ \frac{dm}{dt} = W_1 + W_2 - W_3 \]

\[ e = \frac{m}{\text{Vol}} \]

\[ \frac{d\text{Vol}}{dt} = \frac{\left(\frac{dm}{dt}\right)(\text{Vol}) - \left(\frac{d\text{Vol}}{dt}\right)(m)}{\text{Vol}^2} \]

For a constant Volume:

\[ \frac{d\text{Vol}}{dt} = \frac{dm}{dt} \left(\frac{1}{\text{Vol}}\right) \]

\[ \frac{d\text{Vol}}{dt} = \frac{(W_1 + W_2 - W_3)}{\text{Vol}} \]

Energy:

\[ \frac{dU}{dt} = W_1 h_1 + W_2 h_2 - W_3 h_{\text{mix}} \quad \text{(assuming no heat transfer)} \] (1)

\[ \frac{dU}{dt} = \frac{d(mu)}{dt} = m \left( \frac{du}{dt} \right) + u \left( \frac{dm}{dt} \right) \] (2)

Substituting (2) into (1):

\[ m \left( \frac{du}{dt} \right) + u \left( \frac{dm}{dt} \right) = W_1 h_1 + W_2 h_2 - W_3 h_{\text{mix}} \]

Solving for \( \frac{du}{dt} \):

\[ \frac{du}{dt} = \left[ W_1 h_1 + W_2 h_2 - W_3 h_{\text{mix}} - u \left( \frac{dm}{dt} \right) \right] \left( \frac{1}{m} \right) \]
NOZLOØ - Isentropic Nozzle

INPUTS:
- NOD1 - Inlet Thermal Node
- NOD2 - Exit Thermal Node
- $P_{IN}$ - Inlet Pressure Main Chamber
- $P_{OUT}$ - Exit Pressure (Ambient)
- $T_{IN}$ - Inlet Temperature
- AR - Area Ratio
- AREA - Throat Area
- $\gamma$ - Specific Heat Ratio
- R - Gas Constant
- CS - Loss Coefficient

OUTPUTS:
- $F_g$ - Gross Thrust
- $SI$ - Specific Impulse
- $M_e$ - Mach Number At Exit
- $W_T$ - Flowrate At Throat

CALCULATIONS:
- $PR = \frac{P_{IN}}{P_{OUT}}$
- Submodule CDNZØØ gives values for FP and CF.
- Definition of flow parameter: $FP = \frac{W_T \sqrt{R x T_{IN}}}{AREA x P_{IN}}$
- Solve for $W_T$: $W_T = \frac{(FP)(P_{IN})(AREA)}{\sqrt{(R)(T_{IN})}}$
Thrust: \[ F_g = P_{TN} \times \text{AREA} \times CF \] where CF is a thrust coefficient

Specific impulse: \[ SI = \frac{F_g}{W_i} \]
PBRO00 - Preburner without purge

**INPUTS:**
- NOD1 - Oxidizer Inlet Flow Node
- NOD2 - Fuel Inlet Flow Node
- NOD3 - Fuel Inlet Thermal node
- NOD4 - Exit Flow Node
- OFLT - Mixture Ratio (Light Level)
- $T_F$ - Fuel Temperature
- VOL - Preburner Volume
- $W_F$ - Fuel Flowrate
- $W_O$ - Oxidizer Flowrate
- $W_{OUT}$ - Exit Flowrate
- $\gamma$ - Specific Heat Ratio

**OUTPUTS:**
\[
\frac{d(OFR)}{dt} \quad \text{Oxidizer fraction derivative}
\]
\[
\frac{dT_c}{dt} \quad \text{Temperature derivative}
\]
\[
\frac{d\rho}{dt} \quad \text{Density derivative}
\]

**STATES:**
- OFR - Oxidizer Fraction
- $T_c$ - Preburner Temperature
- $\rho$ - Overall Density

**CALCULATIONS:**
Flow - Based
Mixture Ratio : $OF_{w} = \frac{W_O}{W_F}$
Mass – Based Mixture Ratio: \( OF_m = OFR / (1 - OFR) \)

Temperature Rise (\( \Delta T \)): \( T_c^* = T_F + \Delta T \)

Where \( \Delta T = f(OFR) \) if preburner is lit
or \( \Delta T = \text{const.} \) if preburner is not lit

**DERIVATIVE CALCULATIONS:**

\[
\frac{dQ}{dt} = \frac{dm}{dt} \left( \frac{VOL}{dt} \right) - \frac{dv_{vol}}{dt} \left( m \right)
\]

For a constant volume:

\[
\frac{dQ}{dt} = \frac{dm}{dt} \left( \frac{Vol}{dt} \right) = \frac{dm}{dt} \left( \frac{1}{Vol} \right)
\]

\[
\frac{dQ}{dt} = \frac{W_F + W_o - W_{out}}{Vol}
\]

Energy analysis:

\[
\frac{dU}{dt} = (W_F + W_o) h^* - (W_{out}) h_c
\]

(1)

\[
\frac{dU}{dt} = m \frac{du}{dt} + u \frac{dm}{dt}
\]

(2)

Substituting (2) into (1) and solving for \( \frac{du}{dt} \) yields.

\[
\frac{du}{dt} = \left[(W_F + W_o)h^* - (W_{out}) h_c - u \frac{dm}{dt}\right]/m
\]

(3)

For a perfect gas:

\[
h = C_P T \Rightarrow dh = C_P dT
\]

\[
u = C_V T \Rightarrow du = C_V dT
\]

From continuity:

\[
m = m_F + m_o
\]

\[
\frac{dm}{dt} = (W_F + W_o) - W_{out}
\]

Substituting perfect gas relationships and \( \frac{dm}{dt} \) into equation (3) yields:

\[
\frac{du}{dt} = \left[(W_F + W_o) C_P T_c - W_{out} C_P T_c - C_V T_c (W_F + W_o - W_{out})\right] \left(\frac{1}{m}\right)
\]

\[
\frac{du}{dt} = \frac{d(T_c)}{dt} = C_V \left(\frac{dT_c}{dt}\right)
\]
Simplifying yields:

\[
\frac{dT_c}{dt} = \left[ (W_F + W_o)(\gamma_c T_c - T_c) - W_{out}(\gamma_o - 1)T_c \right] \left( \frac{1}{m} \right)
\]

OFR derivative:

\[
OFR = \frac{m_o}{m_T} = \frac{m_o}{m_o + m_F}
\]

Assuming homogeneous volume: 

\[(OFR) \quad W_{T_{EX}} = W_{o_{EX}}
\]

\[(1 - OFR) \quad W_{T_{EX}} = W_{F_{EX}}
\]

\[
\frac{d(OFR)}{dt} = \left( \frac{dm_o}{dt} m_T - \frac{dm_T}{dt} m_o \right) / m_T^2
\]

\[
= \frac{1}{m_T} \left[ \frac{dm_o}{dt} - (OFR) \frac{dm_T}{dt} \right]
\]

\[
= \frac{1}{m_T} \left[ \frac{dm_o}{dt} - (OFR) \frac{dm_T}{dt} \right]
\]

\[
= \frac{1}{m_T} \left[ \frac{dm_o}{dt} - (OFR) \frac{dm_T}{dt} \right]
\]

\[
\frac{dm_o}{dt} = W_o - W_{o_{EX}} = W_o - (OFR)W_{T_{EX}}
\]

\[
\frac{dm_T}{dt} = W_{T_{IN}} - W_{T_{EX}}
\]

Substituting (2) and (3) into (1) yields:

\[
\frac{d(OFR)}{dt} = \frac{1}{m_T} \left[ W_o - (OFR)W_{T_{EX}} - (OFR)(W_{T_{IN}} - W_{T_{EX}}) \right]
\]

Simplifying:

\[
\frac{d(OFR)}{dt} = \frac{1}{m_T} \left[ W_o - (OFR)W_{T_{IN}} \right]
\]
TEMPERATURE RISE VS. OXIDIZER FRACTION

- The graph shows the relationship between temperature rise (in degrees Celsius) and oxidizer fraction.
- The x-axis represents the oxidizer fraction ranging from 0.0 to 1.0.
- The y-axis represents the temperature rise ranging from 0 to 7000.
- The graph indicates a non-linear increase in temperature rise as the oxidizer fraction increases.

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PUMPØØ - Pump Module For Polytropic Process

INPUTS:

- NOD1 - Flow Node
- NOD2 - Inlet Thermal Node
- NOD3 - Exit Thermal Node
- NOD4 - Shaft Work Node
- W - Flowrate
- \( P_{IN} \) - Inlet Pressure
- \( \rho_{IN} \) - Inlet Density
- \( h_{IN} \) - Inlet Enthalpy
- \( N \) - Rotational Speed

OUTPUTS:

- \( P_{OUT} \) - Discharge Pressure
- \( h_{OUT} \) - Discharge Enthalpy
- \( \rho_{OUT} \) - Discharge Density
- \( T_{ORQ} \) - Torque Required

This module iterates on \( \frac{Q_{OUT}}{Q_{IN}} \) until the exit pressure that results from the property call using \( h_{out} \) and \( \rho_{OUT} \) equals the exit pressure calculated from the polytropic equation (iterate until \( P_{OUT} = P_{OUTc} \)).

CALCULATIONS:

\[
N_{rad} = N \times \left( \frac{2\pi}{60} \right)
\]

Submodule MAP gives values for H (head) and Torque from \( W, \rho_{OUT} \) and \( N \).

\[
\eta = \frac{\text{work out}}{\text{work in}} = \frac{(W)(H)(gr)}{(N)(Torq)(gc)}
\]

where \( gr \) = acceleration due to gravity

\[
\Delta h = \frac{(H)(gr)}{(\eta)(gc)}
\]
\( h_{\text{OUT}} = h_{\text{IN}} + \Delta h \)

A property call using \( e_{\text{OUT}} \) and \( h_{\text{OUT}} \) gives a value for \( P_{\text{OUT}} \). To get \( P_{\text{OUT}} \) let's look at a polytropic process:

**Polytropic Headrise**

For a Polytropic Process

\[
P \cdot v^n = C \tag{1}
\]

\[
P \cdot v^{1/n} = C^{1/n} \tag{2}
\]

\[
v = C^{1/n} P^{-1/n} \tag{3}
\]

\[
H = \text{headrise} = \int_1^2 v dP
\]

\[
H = \int_1^2 v dP
\]

Substitute (3) into (4)

\[
H = \int_1^2 C^{1/n} P^{-1/n} dP
\]

\[
H = C^{1/n} \int_1^2 P^{-1/n} dP
\]

\[
H = \left( C^{1/n} P_2^{1-1/n} - C^{1/n} P_1^{1-1/n} \right) \left( \frac{n}{n-1} \right)
\]

\[
H = \left( \frac{n}{n-1} \right) \left( C^{1/n} P_2^{1-1/n} - C^{1/n} P_1^{1-1/n} \right) \tag{5}
\]

Substitute (2) into (5)

\[
H = \left( \frac{n}{n-1} \right) \left[ v_2 P_2^{1/n} P_2^{1-1/n} - v_1 P_1^{1/n} P_1^{1-1/n} \right]
\]

\[
H = \left( \frac{n}{n-1} \right) \left\{ v_2 P_2 - v_1 P_1 \right\}
\]
For a polytropic process

For a polytropic process

\[ P \cdot v^n = k \]

or

\[ P_1 \cdot v_1^n = P_2 \cdot v_2^n \]  

(1)

(2)

\[ \frac{P_2}{P_1} = \left( \frac{v_1}{v_2} \right) \]

(3)

\[ \ln \left( \frac{P_2}{P_1} \right) = n \ln \left( \frac{v_1}{v_2} \right) \]

(4)

\[ n = \frac{\ln \left( \frac{P_2}{P_1} \right)}{\ln \left( \frac{v_1}{v_2} \right)} \]

also from 3:

\[ \frac{P_2}{P_1} \cdot \frac{v_2}{v_1} = \left( \frac{v_1}{v_2} \right)^n \cdot \frac{v_2}{v_1} \]

(5)

\[ \frac{P_2}{P_1} \cdot \frac{v_2}{v_1} = \left( \frac{v_2}{v_1} \right)^n \cdot \left( \frac{v_2}{v_1} \right) \]

\[ \frac{P_2}{P_1} \cdot \frac{v_2}{v_1} = \left( \frac{v_2}{v_1} \right) \]

\[ \ln \left( \frac{P_2}{P_1} \frac{v_2}{v_1} \right) = (1-n) \ln \left( \frac{v_2}{v_1} \right) \]

\[ n = 1 - \frac{\ln \left( \frac{P_2}{P_1} \frac{v_2}{v_1} \right)}{\ln \left( \frac{v_2}{v_1} \right)} \]
let $\beta = \frac{n}{n-1}$

\[
\frac{n}{n-1} = \frac{\ln \left( \frac{P_2}{P_1} \right) \left( \frac{\nu_2}{\nu_1} \right)}{1 - \frac{\ln \left( \frac{\nu_2}{\nu_1} \right)}{\ln \left( \frac{P_2}{P_1} \right) \left( \frac{\nu_2}{\nu_1} \right)}}
\]

\[
\frac{n}{n-1} = \frac{\ln \left( \frac{\nu_2}{\nu_1} \right) - \ln \left( \frac{P_2}{P_1} \right) \left( \frac{\nu_2}{\nu_1} \right)}{\ln \left( \frac{P_2}{P_1} \right) \left( \frac{\nu_2}{\nu_1} \right)}
\]

\[
\frac{n}{n-1} = \frac{\ln \left( \frac{P_2}{P_1} \right) \left( \frac{\nu_2}{\nu_1} \right) - \ln \left( \frac{\nu_2}{\nu_1} \right)}{\ln \left( \frac{P_2}{P_1} \right) \left( \frac{\nu_2}{\nu_1} \right)}
\]

\[
\ln \left( \frac{P_2}{P_1} \right) \left( \frac{\nu_2}{\nu_1} \right) - \ln \left( \frac{\nu_2}{\nu_1} \right) = \ln \frac{\frac{P_2}{P_1} \left( \frac{\nu_2}{\nu_1} \right)}{\frac{P_2}{P_1}} = \ln \left( \frac{P_2}{P_1} \right)
\]

\[
\frac{n}{n-1} = \frac{\ln \left( \frac{P_2}{P_1} \right)}{\ln \left( \frac{P_2}{P_1} \right) \left( \frac{\nu_2}{\nu_1} \right)}
\]

\[
H = \left( \frac{n}{n-1} \right) \left[ P_2 \nu_2 - P_1 \nu_1 \right]
\]

\[
H = \left( \frac{n}{n-1} \right) \left[ \frac{P_2}{\nu_2} - \frac{P_1}{\nu_1} \right]
\]

\[
H \left( \frac{n-1}{n} \right) = \frac{P_2}{\nu_2} - \frac{P_1}{\nu_1}
\]

\[
H \left( \frac{n-1}{n} \right) + \frac{P_1}{\nu_1} = \frac{P_2}{\nu_2}
\]

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\[ P_2 = \varrho_2 \left[ H \left( \frac{n-1}{n} \right) + \frac{P_1}{\varrho_1} \right] \]

If \[ \beta = \frac{n}{n-1} \Rightarrow \frac{1}{\beta} = \frac{n-1}{n} \]

\[ P_1 = \varrho_2 \left[ \frac{H}{\beta} + \frac{P_1}{\varrho_1} \right] \]

Module implementation with \( v \) and \( P \) instead of \( v \) and \( P \)

\[
POUT_C = q_{OUT} \left[ \frac{H}{\beta} + \frac{PIN}{\varrho_{IN}} \right]
\]

FORTRAN implementation of \( \beta \) calculation

\[
\beta = \frac{\ln \left( \frac{P_2}{P_1} \right)}{\ln \left( \frac{\varrho_2}{\varrho_1} \right)}
\]

\[
\frac{v_2}{v_1} = \frac{\varrho_1}{\varrho_2}
\]

then

\[
\beta = \frac{\ln \left( \frac{P_2}{P_1} \right)}{\ln \left( \frac{\varrho_2}{\varrho_1} \right)} = \frac{\ln \left( \frac{P_2}{P_1} \right)}{\ln \left( \frac{\varrho_2}{\varrho_1} \right)}
\]

\[
\beta = \frac{\ln \left( \frac{P_2}{P_1} \right)}{\ln \left( \frac{P_2}{P_1} \right) - \ln \left( \frac{\varrho_2}{\varrho_1} \right)}
\]

\[
\beta = \frac{1}{1 - \left[ \frac{\ln \left( \frac{\varrho_2}{\varrho_1} \right)}{\ln \left( \frac{P_2}{P_1} \right)} \right]}
\]
**INPUTS:**

- NOD1 - High Pressure Turbine Node
- NOD2 - Low Pressure Turbine Node
- NOD3 - High Pressure Pump Node
- NOD4 - Low Pressure Pump Node
- \( T_{T1} \) - High Pressure Turbine Torque
- \( T_{T2} \) - Low Pressure Turbine Torque
- \( T_{P1} \) - High Pressure Pump Torque
- \( T_{P2} \) - Low Pressure Pump Torque
- \( T_D \) - Drag Torque
- \( T_{XTR} \) - Extraction Torque
- \( n_{T1} \) - High Pressure Turbine Gear Ratio
- \( n_{T2} \) - Low Pressure Turbine Gear Ratio
- \( n_{P1} \) - High Pressure Pump Gear Ratio
- \( n_{P2} \) - Low Pressure Pump Gear Ratio
- \( n_{XTR} \) - Extraction Gear Ratio
- \( I_P \) - Polar Moment of Inertia

**OUTPUTS:**

- \( \frac{dN_{NOD}}{dt} \) - Rotor speed derivative

**CALCULATIONS:**

\[
\Sigma Tn = I_P \alpha \quad (1)
\]
\[
\alpha = \frac{dN_{NOD}}{dt} \quad (2)
\]

Substituting (2) into (1)

\[
\frac{dN_{NOD}}{dt} = \frac{\Sigma Tn}{I_P}
\]

\[
\frac{dN_{NOD}}{dt} = \frac{(T_{T1} n_{T1} + T_{T2} n_{T2} - T_{P1} n_{P1} - T_{P2} n_{P2} - T_D - T_{XTR} n_{XTR})}{I_P}
\]
SPLTØØ - Simple Flow Splitter

**INPUTS:**
- NOD1 - Inlet Thermal Node
- NOD2 - Inlet Node
- NOD3 - Exit Node
- NOD4 - Exit Thermal Node
- NOD5 - Exit Node
- NOD6 - Exit Thermal Node
- \( h_1 \) - Inlet Enthalpy
- \( h_2 \) - Exit Enthalpy
- \( h_3 \) - Exit Enthalpy
- \( h_{\text{SPLT}} \) - Splitter Enthalpy
- VOL - Splitter Volume
- \( W_1 \) - Inlet Flowrate
- \( W_2 \) - Exit Flowrate
- \( W_3 \) - Exit Flowrate

**OUTPUTS:**
- \( \frac{d\rho}{dt} \) - Density derivative
- \( \frac{du}{dt} \) - Internal energy derivative

**CALCULATIONS:**
- \( m = \rho \times \text{vol} \)
- \( \frac{dm}{dt} = W_1 - W_2 - W_3 \)
- \( \dot{\rho} = \frac{m}{Vol} \)
- \( \frac{d\rho}{dt} = \frac{(\frac{dm}{dt})(Vol) - (\frac{dVol}{dt})(m)}{Vol^2} \)
For a constant Volume:

$$\frac{dQ}{dt} = \frac{dm}{dt} \left( \frac{1}{\text{Vol}} \right)$$

$$\frac{dQ}{dt} = \frac{(W_1 + W_2 - W_3)}{\text{Vol}}$$

$$\frac{dU}{dt} = W_1 h_1 - W_2 h_{SPLT} - W_3 h_{SPLT}$$ (assuming no heat transfer) (1)

$$\frac{dU}{dt} = \frac{d(mu)}{dt} = m\left( \frac{du}{dt} \right) + u\left( \frac{dm}{dt} \right)$$ (2)

Substituting (2) into (1):

$$m\left( \frac{du}{dt} \right) + u\left( \frac{dm}{dt} \right) = W_1 h_1 - W_2 h_{SPLT} - W_3 h_{SPLT}$$

Solving for \( \frac{du}{dt} \):

$$\frac{du}{dt} = \left[ W_1 h_1 - W_2 h_{SPLT} - W_3 h_{SPLT} - u\left( \frac{dm}{dt} \right) \right] \left( \frac{1}{m} \right)$$
TURBØØ - Turbine

INPUTS:
NOD1 - Inlet Thermal Node
NOD2 - Discharge Thermal Node
NOD3 - Heat Transfer Node
NOD4 - Shaft Work Node
R - Inlet Gas Constant
γ - Inlet Specific Heat Ratio
Z - Inlet Compressibility Factor
PIN - Inlet Pressure
TIN - Inlet Temperature
POUT - Exit Pressure
N - Rotational Speed
QDOT - Heat Transfer Rate into Turbine
QFRC - Fraction of Heat That Goes to Turbine Work
MAP - Turbine Characteristic Map

OUTPUTS:
W - Turbine Flowrate
TORQ - Turbine Torque
TOUT - Discharge Temperature

CALCULATIONS:
PR = PIN / POUT

N_rad = N \left( \frac{2\pi}{60} \right)

Submodule MAP gives values for FP, \Delta h' and \eta

Definition of Flow Parameter: FP = \frac{W\sqrt{R \times TIN}}{P}

Solve for W:
W = \frac{FP \times P}{\sqrt{R \times TIN}}
Real $\Delta h$: \[ \Delta h = \Delta h' \times \eta + (Q_{\text{DOT}} \times Q_{\text{FRC}}) \]

TORQ = \( \frac{W \Delta h}{N_{\text{rad}}} \)

Exit Temperature: \[ \Delta h = C_p \Delta T \]

\[ C_p = R \left( \frac{\gamma}{\gamma - 1} \right) \]

\[ T_{IN} - T_{OUT} = \frac{(\Delta h - Q_{\text{DOT}})}{C_p} \]

\[ T_{OUT} = T_{IN} - \frac{(\Delta h - Q_{\text{DOT}})(\gamma - 1)}{\gamma} \]
VOLM00 - Lumped Volume With Energy and Continuity

**INPUTS:**

- NOD1 - Inlet Thermal Node
- NOD2 - Inlet Node
- NOD3 - Exit Node
- NOD4 - Exit Thermal Node
- NOD5 - Heat Transfer Node
- Vol - Volume
- hin - Inlet Enthalpy
- hout - Exit Enthalpy
- hvol - Volume Enthalpy
- \( \dot{Q} \) - Heat Flux into the Volume
- Win - Inlet Flowrate
- Wout - Exit Flowrate

**OUTPUTS:**

- \( \frac{d\rho}{dt} \) - Density Derivative
- \( \frac{du}{dt} \) - Internal Energy Derivative

**DERIVATIVE CALCULATIONS:**

\[ m = \rho \times \text{vol} \]

\[ \frac{dm}{dt} = W_{in} - W_{out} \]

\[ \rho = \frac{m}{\text{Vol}} \]

\[ \frac{d\dot{Q}}{dt} = \frac{\left(\frac{dm}{dt}\right)(\text{Vol}) - \left(\frac{d\text{Vol}}{dt}\right)(m)}{\text{Vol}^2} \]
For a constant volume:

\[
\frac{d\rho}{dt} = \frac{dm}{dt} \left( \frac{1}{\text{Vol}} \right)
\]

\[
\frac{d\rho}{dt} = \frac{W_{in} - W_{out}}{\text{Vol}}
\]

Energy:

\[
\frac{dU}{dt} = (W_{in}) (hin) - (W_{out}) (h_{vol}) + \dot{Q}
\]  \hspace{1cm} (1)

\[
\frac{dU}{dt} = \frac{d(mu)}{dt} = (m) \left( \frac{du}{dt} \right) + (u) \left( \frac{dm}{dt} \right)
\]  \hspace{1cm} (2)

substituting (2) into (1)

\[
m \left( \frac{du}{dt} \right) + u \left( \frac{dm}{dt} \right) = (W_{in})(hin) - (W_{out})(h_{vol}) + \dot{Q}
\]

solving for \( \frac{du}{dt} \):

\[
\frac{du}{dt} = \left( (W_{in})(hin) - (W_{out})(h_{vol}) - u \frac{dm}{dt} + \dot{Q} \right) \left( \frac{1}{m} \right)
\]
SSBL03 - Steady State Balance Routine

SSBL03 is an interface to SMIT01 which is designed to perform a steady-state balance on a given model. The call list for SSBL03 is defined as follows:

ISSPNT = Print option flag
  0 = No steady-state print
  1 = Steady-state print to unit 6

NSTATE = Number of States
STATE = Array of State Variables
DSTATE = Array of State Variable Derivatives
SNORM = Array of normalizing values for State Variables
SNAME = Array of State Variable Names
ISON = Array of on/off codes for State Variables
  0 = State not iterated
  1 = State iterated

NVAR = Number of algebraic loops (balances)
XVAR = Array of balance independent variables
YVAR = Array of balance dependent variables
XNORM = Array of normalizing values for balances
XNAME = Array of names for balances
IXON = Array of on/off codes for balances
  0 = Balance not iterated
  1 = Balance iterated

MXPASS = Maximum number of convergence passes
ISAMAT = Same Jacobian Flag
  0 = Do no use previous matrix
  1 = Use previous Jacobian
ISIG = Control flag
  0 = On first pass to set initialization
  1 = Evaluate derivatives and errors
  2 = Converged

SSBL03 provides input to SMIT01 so that both states and algebraic loop (balance) parameters are iterated until their respective errors are within tolerance. The error terms for states are calculated as the derivative divided by the state and the error terms for the balance parameters are the input error terms (YVAR).

The states and balance parameters are normalized before going into SMIT01 by the input normalization values, SNORM, and XNORM. If the input normalizing value is zero, the initial value of the state or balance parameter will be used.
Values for the SMIT01 input DELTAX and DXALOW are input based on the normalization values.

Specific states and balances may be turned on or off based on the input switching arrays ISON and IXON. SSBL03 builds the dependent and independent variable arrays for SMIT01 based on these inputs and the normalization discussed above. If the switch value is 1, the variable will be included in the SMIT01 array and therefore iterated, if the switch value is zero, the variable will not be operated on by SMIT01.

If requested via the flag ISSPNT, SSBL03 will provide a detailed print of the steady-state balance. The steady-state print includes the following:

1. The base point dependent and independent variables sent to SMIT01
2. The dependent and independent SMIT01 variables during the generation of the J matrix.
3. The dependent and independent SMIT01 variables during each convergence attempt along with the changes in each variable and the percent reduction in the sum of the squared errors.
4. A message providing SMIT01 statistics upon completion.
TRAP03 - Transient Integration Routine

TRAP03 is the transient counterpart of SSBL03 and has the same inputs as SSBL03 with the following additions:

1. DT = Transient time increment
2. TIME = Transient time

TRAP03 is an implicit integration routine which can simultaneously solve algebraic loops. The predicted STATE values at TIME = TIME + DT are the previous time values and MSIT01 is used to solve a trapezoidal corrector. This is performed by interfacing to MSIT01 similarly to SSBL03 except that the state error terms become:

\[ \varepsilon_{STATE} = \frac{STATE_{predicted} - STATE_{calculated}}{SNORM} \]

STATE\text{calculated} is determined from the previous time STATE and DSTATE, the current time DSTATE and the time increment DT as follows:

\[ STATE_{calculated} = STATE_{previous} + (DSTATE + DSTATE_{previous}) \times DT/2. \]

Because the current derivative is used in the calculation of \( STATE_{calculated} \), SMIT01 is required.

Additionally, TRAP03 can solve multiple algebraic loops at the same time as solving for the states. This is accomplished by constructing the SMIT01 input arrays in the same manner as SSBL03, except that the error terms for the STATES are as \( \varepsilon_{STATE} \) rather than DSTATE divided by STATE as in SSBL03. The error terms for the algebraic loops are calculated in the same way as in SSBL03.

Internal checking is performed so that the J matrix used by SMIT01 is consistent. If a state or algebraic loop is switched on or off during a run a new J matrix is immediately evaluated. If convergence fails the SMIT01 variables are reset to the initial values (previous converged point) and DT is reduced by a factor of DTRDCN and convergence is attempted again. DT may be reduced a total number MXDTRD times on any given integration step. The current values of DTRDCN and MXDTRD are both 3 therefore allowing a factor of 27 reduction in DT. If the point still fails, they are set equal to the initial failed values. TRAP03 will fail when the number of failures above exceeds the value of MXFAIL. MXFAIL is currently set at 5. DTRDCN, MXDTRD and MXFAIL are all internal variables to TRAP03.
SUBMODULE CDNZ0Ø : Isentropic Nozzle Analysis

Basically, all nozzles can be analyzed as an isentropic process which allows the use of the isentropic flow relationships. Hence, the expansion process of nozzles can be represented on an H–S diagram as drawn below:

A nozzle analysis requires the following input as known conditions:

They are: PR, AR, γ, Po, To, Ath

The input parameters defined above represent pressure ratio (PR), area ratio (AR), ratio of specific heats (γ), chamber pressure (Po), chamber temperature (To) and throat area (Ath).

For a first test, we need to check and see if the PR is choke. The isentropic total to static pressure can be defined as:

\[ \frac{P_o}{P_s} = \left(1 + \frac{\gamma - 1}{2} \frac{m^2}{r-1}\right)^{\frac{r}{r-1}} \]  

At the throat, the choked PR occurs at Mn = 1

\[ \left(\frac{P_o}{P_s}\right)_{\text{CHOKED}} = \left(1 + \frac{\gamma - 1}{2} m^2\right)^{\frac{r}{r-1}} = \left(\frac{\gamma + 1}{2}\right)^{\frac{r}{r-1}} \]

\[ \therefore \left(\frac{P_o}{P_s}\right)_{\text{CHOKED}} = \left(\frac{\gamma + 1}{2}\right)^{\frac{r}{r-1}} \]  

If PR > \( \left(\frac{P_o}{P_s}\right)_{\text{CHOKED}} \) then the nozzle is choked
For a second test, we need to check and see if the flow parameter at the throat (FP\textsubscript{THROAT}) is choked. The flow parameter is defined as:

\[ FP = \frac{W \sqrt{R T_o}}{P_o A} = \sqrt{\gamma} \frac{mn}{\sqrt{1 + \frac{\gamma - 1}{2} mn^2}} \tag{3} \]

The flow parameter defined by equation (2) is a function of \( \gamma \) and Mn. This relationship can also be defined as a function of \( \gamma \) and PR by defining Mn as a function of PR.

Solving equation (1) for Mn as a F(PR).  

\[ 1 + \frac{\gamma - 1}{2} mn^2 = \left( \frac{P_o}{P_s} \right)^{\frac{\gamma - 1}{\gamma}} \]

\[ mn^2 = \frac{2}{\gamma - 1} \left[ \left( \frac{P_o}{P_s} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right] \]

\[ mn = \left\{ \left( \frac{2}{\gamma - 1} \right) \left[ \left( \frac{P_o}{P_s} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right] \right\}^{\frac{1}{2}} \tag{4} \]

Now that a relationship for Mn as a F(PR) has been determined; it can be used to define the flow parameter as function of \( \gamma \) and PR. Let's substitute equation (4) into (3)

\[ FP = \frac{\sqrt{\frac{2 \gamma}{\gamma - 1} \left[ \left( \frac{P_o}{P_s} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right]}}{\sqrt{1 + \frac{\gamma - 1}{2} \left( \frac{P_o}{P_s} \right)^{\frac{\gamma - 1}{\gamma}}}} \frac{\gamma - 1}{2(\gamma - 1)} \]

Simplifying:

\[ FP = \frac{\sqrt{\frac{2 \gamma}{\gamma - 1} \left[ \left( \frac{P_o}{P_s} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right]}}{\left( \frac{P_o}{P_s} \right)^{\frac{\gamma - 1}{\gamma}} \left\{ \left( \frac{P_o}{P_s} \right)^{\frac{\gamma - 1}{\gamma}} \right\}^{\frac{1}{2}}} \tag{5} \]
The choked flow parameter can now be defined by substituting the choked pressure ratio defined by equation (2) into equation (5).

\[
FP_{\text{CHOKED}} = \frac{\left(\frac{2\gamma}{\gamma - 1}\right)^{\frac{1}{2}} \left(\frac{r_{1}^{1/2}}{\gamma - 1} - 1\right)^{1/2}}{\left[\left(\frac{\gamma - 1}{2}\right)^{\frac{3}{2}}\right]^{1/2}}
\]

Let the checked pressure term be defined as CHOKED

\[
CHOKED = \left\{\left[\frac{\gamma - 1}{2}\right]^{\frac{1}{2}} \left[\frac{\gamma + 1}{2}\right]^{\frac{1}{2}}\right\}^{1/2}
\]

\[ FP_{\text{CHOKED}} = \left(\frac{2\gamma}{\gamma - 1}\right)^{1/2} \cdot CHOKED \]

Let the nozzle pressure term be defined as EXIT:

\[
EXIT = \left\{PR^{r_{1}^{-1}} - 1\right\}^{1/2}
\]

\[ FP_{\text{EXIT}} = \left(\frac{2\gamma}{\gamma - 1}\right)^{1/2} \cdot EXIT \]

Finally,

\[ FP_{\text{THROAT}} = FP_{\text{EXIT}} \cdot AR \]

If \( FP_{\text{THROAT}} > FP_{\text{CHOKED}} \) then the nozzle is choked.
UN-CHOKED nozzle

Let's calculate the exit Mach number using subroutine Mach03 which guess \( M_{\text{EXIT}} \) as a function of \( F_P_{\text{EXIT}} \).

\[ m_{\text{EXIT}} = F(F_P_{\text{EXIT}}) \]

Vary \( M_{\text{EXIT}} \) until \( (F_P_{\text{EXIT}})_{\text{CALC}} - F_P_{\text{EXIT}} \leq \) tolerance

The above balance uses subroutine ITER01 and calculates \( (F_P_{\text{EXIT}})_{\text{CALC}} \).

\[ (F_P_{\text{EXIT}})_{\text{CALC}} = \frac{\sqrt{\gamma} \ m_{\text{EXIT}}}{\left(1 + \frac{\gamma-1}{2} m_{\text{EXIT}}^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}} \]

\[ F_P_{\text{THROAT}} = F_P_{\text{EXIT}} \times AR \]

\[ \left(\frac{P_o}{P_1}\right)_{\text{EXIT}} = PR \]

Go calculate the thrust coefficient

CHOKED nozzle

If \( AR < 1 \) then

CONVERGENT AREA

\[ m_{\text{EXIT}} = 1.0 \]

\[ \left(\frac{P_o}{P_1}\right)_{\text{EXIT}} = P_{R\text{CHOKED}} \]

\[ F_P_{\text{THROAT}} = F_P_{\text{CHOKED}} \times AR \]

Go calculate the thrust coefficient

Else

DIVERGENT AREA

\[ F_P_{\text{THROAT}} = F_P_{\text{CHOKED}} \]

\[ \therefore F_P_{\text{EXIT}} = F_P_{\text{CHOKED}} / AR \]
Since this is a divergent nozzle, the analysis needs to check for internal shocks.
Let's calculate the exit Mach number using subroutine Mach03 which guess Mn_{EXIT} as a function of FP_{EXIT}.

\[ m_{n_{EXIT}} = F(FP_{EXIT}) \]

Vary Mn_{EXIT} until \((FP_{EXIT})_{CALC} - FP_{EXIT} \leq \text{tolerance}\)

\[ \left( \frac{p_0}{p_t} \right)_{EXIT} = \left( 1 + \frac{\gamma - 1}{2} \right) m_{n_{EXIT}}^{\frac{2}{\gamma - 1}} \]

1st test: 
\( P_{EXIT} > P_{AMB} \)

\[ RPR = \frac{PR}{\left( \frac{p_0}{p_t} \right)_{EXIT}} \]

If \( RPR \geq 1 \) then

The exit pressure \((P_{EXIT})\) is greater than ambient pressure \((P_{AMB})\) which indicates that there is no shock in the nozzle. Go calculate the thrust coefficient.

\[ Ps^2 = \frac{2\gamma m_{n_{EXIT}}^2}{\gamma + 1} - \frac{\gamma - 1}{\gamma + 1} \]

2nd test \( P_{EXIT} \) (after shock > \( P_{AMB} \))

\[ RPRS = \left( \frac{Ps^2}{Ps^{\gamma}} \right) \cdot RPR \]

If \( RPRS > 1 \), then

The exit static pressure \((P_{EXIT})\) is greater than ambient pressure are the check. Hence the nozzle does not have a shock wave. Go calculate the thrust coefficient.

A normal shock exist in the nozzle. Let's guess Mn, (Mach number upstream of the shock).

\[ mn_1 = 1 + \frac{(mn_{EXIT} - 1)}{2} \]

This allow the total to total pressure ratio \((P_{T1} / P_{T2})\) across the shock to calculate.
Calculating exit Mach number downstream of the shock.

\[
\frac{P_{T2}}{P_{T1}} = \left[ \frac{\gamma + \frac{1}{\gamma}}{1 + \frac{\gamma + 1}{\gamma} \frac{\gamma}{\gamma}} \right] \left/ \left[ \frac{2\gamma}{\gamma + 1} \frac{\gamma}{\gamma + 1} \right] \right.
\]

\[
\text{Mn}_{\text{EXIT}} = F \left( \text{FP}_{\text{EXIT}} \right) \text{ using subroutine Mach03}
\]

\[
\left( \frac{P_o}{P_s} \right)_{\text{EXIT}} = \left( 1 + \frac{\gamma - 1}{2} \frac{\text{Mn}_{\text{EXIT}}^2}{\text{Mn}_{\text{EXIT}}^2} \right)^{\frac{1}{\gamma - 1}}
\]

Vary Mn_1 until \( \left[ \frac{(P_o/P_s)_{\text{EXIT}}}{(PR) \left( P_{T2} \setminus P_{T1} \right)} \right] \leq \text{tolerance} \)

\[
\left( \frac{P_o}{P_s} \right)_{\text{EXIT}} = PR
\]

Go calculate the thrust coefficient.

**Calculation of The Thrust Coefficient**

\[
\frac{P_{\text{EXIT}}}{P_o} = \frac{1}{\left( \frac{P_o}{P_s} \right)_{\text{EXIT}}}
\]

The ideal throat coefficient \( \text{CF}_{\text{IDL}} \) with \( P_{\text{EXIT}} = P_{\text{AMB}} \) is defined as:

\[
\text{CF}_{\text{IDL}} = \gamma \left\{ \left( \frac{2}{\gamma - 1} \right) \left( \frac{2}{\gamma + 1} \right) \right\}^\frac{r+1}{2} \left[ 1 - \left( \frac{1}{PR} \right)^\frac{r+1}{r} \right]\]

The ideal thrust coefficient (no real losses, but includes expansion losses) is defined as:
Velocity coefficient is defined as:

\[ CV = \frac{(CF' - LOSS_{REAL})}{CF_{IDL}} \]
SUBMODULE FLPM02 – Flow Calculation Through A Restriction

It is often necessary to include the effects of a flow path restriction in a simulation. This involves developing a relationship between flow and total pressure loss.

FLOW CALCULATION:

The general problem of calculating the mass flow rate through a restriction can be simplified considerably if the restriction is considered to be an orifice. This reduces the scope of the problem from developing a general theory applicable to all possible geometries to deriving a calculational procedure for only one geometry. The general situation is thus reduced to calculation of flow through an orifice and proper sizing of an orifice.

As flow moves through an orifice it is first accelerated until it attains a maximum velocity at the throat. The flow is then diffused as it is decelerated to the downstream conditions. During this process, the total and static pressures change as depicted in Figure I. There is little loss in total pressure as the flow nears the throat, although static pressure changes considerably due to the increased fluid velocity. Most of the total pressure loss occurs as the fluid is diffused downstream of the throat. The static pressure is increasing, however, as a portion of the velocity head available at the throat is converted to pressure. Total temperature does not change throughout the process.

Typically, the upstream and downstream total pressures and temperatures are known, from which flow is to be determined. Since the total pressure and temperature at the throat is nearly the same as upstream, flow could be calculated using compressible flow relations if static pressure at the throat were known.

Static pressure at the throat can be calculated if it is assumed that the total pressure loss across the orifice is related to the velocity head at the throat. Numerous experiments have been conducted which justify this assumption, although most of the empirical data is presented in terms of discharge and flow coefficients. Discharge and flow coefficients are useful for control valve work but are generally restricted in their range of applicability. It is more convenient, therefore, to define a loss coefficient from which static pressure at the throat can be determined.

Referring to Figure II, the loss coefficient is defined as:

\[ L = \frac{\text{Total pressure loss}}{\text{Velocity head at throat}} = \frac{P_{T,\text{up}} - P_{T,\text{down}}}{P_{T,\text{up}} - P_{s,\text{throat}}} \]
Rearranging, the total to static pressure ratio at the throat is given by

\[ \frac{P_{T\text{up}}}{P_{S\text{throat}}} = \frac{PR(L)}{1 + PR(L-1)} \]

Figure I. Total and Static Pressure Variation in an Orifice

where \( PR = \frac{P_{T\text{up}}}{P_{T\text{down}}} \) and it is assumed the orifice is not choked. If the calculated total to static pressure ratio is greater than the choke value, then the orifice is choked, in which case, the actual total to static pressure ratio at the throat is equal to the choke value.

With the total to static pressure ratio known, the flow parameter \((W \sqrt{RT_i/P_tA})\) can be determined at the throat based on compressible flow relations:

\[ W = \frac{\sqrt{2857 \left( PR_{\frac{P_{T\text{down}}}}{P_tA}^{\frac{1}{n}} - 1 \right)}}{(\gamma - 1) \left( PR_{\frac{P_{T\text{down}}}}{P_tA}^{\frac{1}{n}} \right)} \]
Since total temperature remains constant, and total pressure at the throat is assumed equal to the upstream total pressure, the mass flow rate can be determined,

\[ W = F P \frac{(P_{TuP})(A_{eff})}{R(T_{TuP})} \]

![Diagram showing pressure drop and velocity head at throat](image)

Figure II. Loss coefficient relates total pressure lost to velocity head at the orifice throat.
ROCETS REAL FLUID AND COMBUSTION PROPERTY PACKAGES

The ROCETS property package consists of five real fluid properties and one "perfect gas" combustion property. Both property packages consist of a driver subroutine and a property/model interface subroutine for each of the five fluids and the combustion method. The driver routine is the decision maker and then routes all information to the correct property or model interface which the accesses the property maps for the real fluid properties or the subroutines for the combustion properties to look up the desired results. The program logic of the ROCETS real fluid properties and the combustion property package is shown Figures 3.4.2.4.1 and 3.4.2.4.2 respectively. The five basic properties are para-hydrogen, oxygen, methane, nitrogen and helium. The combustion properties provide a perfect gas combustion analysis using hydrogen and oxygen as the reactants. The driver has been set up such that it can be expanded to allow real gas combustion and Brinkly combustion at some future date. The combustion properties are from the digital transient model (DTM), but have been upgraded to meet ROCETS software standards.

In the combustion property package, the driver receives basic inputs such as model type (MTYPE), property option (IOPT), the independent variables (XVAR1, XVAR2, XVAR3 and XVAR4) and the map indices IHOLD1 and IHOLD2. This information is then passed to the model interface which accesses the required information from the combustion property subroutines and maps. The call statement for the driver routine has the following form:

```
CALL COMBO0 (MTYPE, IOPT, XVAR1, XVAR2, XVAR3, XVAR4, OVAR1, OVAR2, IHOLD1, IHOLD2)
```

It should be noted that MTYPE is a character*8 variable which defines the method to be used. For example, if the user wishes to access combustion properties using a perfect gas model, they would define MTYPE = 'PGAS'.

Once the combustion property model type has been defined, the correct model interface is accessed. The model interface routines have the following form:

```
CALL XGASAA (IOPT, XVAR1, XVAR2, XVAR3, XVAR4, OVAR1, OVAR2, IHOLD1, IHOLD2)
```

The single character prefix X defines the model to be used and two character suffix defines the version number. The following models are available or are planned: P-perfect gas model (available), R-Real gas model (not available) and B-Brinkly model (not available). A complete list of all options and parameters used in the argument list are defined in Table 3.4.2.4.4 and the units are defined in Table 3.4.2.4.5.
The perfect gas model interface defined above obtains property information in both subroutines and property maps from the Digital Transient Model (DTM). However, the subroutines found in the DTM have been upgraded to ROCETS software standards and the maps which are from the DTM input file were put into subroutine format. Basically, the subroutines which are called from the model interface access results from tabular data which are in uni-variant or bi-variant form. The univariant maps are set up in x-axis and y-axis coordinate system and use a linear map reader (SUNBOO) to interpolate. The bi-variant maps are set up in an x-axis, y-axis and z-axis coordinate system and use a corresponding point map reader (CPMRØ2) to interpolate. The map indices IHOLD1 and IHOLD2 are used to track the corresponding map locations and are responsible for a significant reduction in the cost of accessing the maps.

At this stage of development, the real gas and Brinkly gas combustion models are not available. However, as they are implemented into the ROCETS system, these models will be discussed in greater detail.

The ROCETS combustion property package has 7 property maps that are used to drive the perfect gas model interface. The real gas and Brinkly gas interface have not yet been implemented. The following tables define the subroutines and maps which are used.

**COMBUSTION PROPERTY ROUTINES**

<table>
<thead>
<tr>
<th>PERFECT GAS</th>
<th>REAL GAS</th>
<th>BRINKLY GAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPGMØØ</td>
<td>NONE</td>
<td>NONE</td>
</tr>
<tr>
<td>CPMPØ1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RDMP13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RDMP21</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RDMP12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ZGASØØ</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ZZMPØ1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

These maps are presented in figures 3.4.2.4.27 - 3.4.2.4.31. The perfect gas combustion property package allows the calculation of basic thermodynamic properties which are corrected for compressibility effects for the hot gas system components such as the preburner, turbine, injectors and the main chamber.

In the basic fluid property package, the driver routine receives basic property inputs such as fluid type (FLUID), property option (IOPT), the independent
variables (XVAR1 and XVAR2) and the station number (KSTA). The user supplied information is then passed to the particular property interface which accesses the required information from the property maps. The call statement for the driver routine has the following from:

CALL PROP00 (FLUID, IOPT, XVAR1, XVAR2, PVAR, OVAR1, OVAR2, KSTA)

It should be noted that FLUID is a character*8 variable which defines the fluid by name. For example, if the user wishes to access hydrogen property data, he would define FLUID = 'HYDROGEN'.

Once the fluid property has been defined, the correct property interface can be accessed. The property interface routines for the five basic properties have the following form:

CALL XXPROP (IOPT, XVAR1, XVAR2, DVAR, OVAR1, OVAR2)

The two character prefix XX defines the fluid to be used. The following fluids are available: H2 – para-hydrogen, O2 – oxygen, N2 – nitrogen, HE – helium and ME – methane. A complete list of all options and parameters used in the argument list are defined in Table 3.4.2.4.1 and units are defined in Table 3.4.2.4.2.

Each property interface defined above obtains property data from a series of real fluid property maps. The call statements to the maps follow a basic format which uses a one letter prefix to define the fluid, a three character map variable name and two character version number. The map variables define how a particular map is set up. Let's consider an example for a hydrogen map. The fluid prefix is H. Hence, the call for a hydrogen map would have the following form: HXYZAA. A hydrogen map call would have the following form:

CALL HXYZAA (XX, YY, ZZ, IOPT, IHLD1, IHLD2)

All property maps are set up in an x-axis, y-axis and z-axis coordinate system and uses a corresponding point map reader (CPMR02) to interpolate. The advantage of a corresponding point reader is that a family of discontinuous curves such as a property map can be adequately handled. The map reader has two options. Option one determines zz as a function of xx and yy. Option two determines xx as a function of zz and yy. The integer variables IHLD1 and IHLD2 are map indicies which are stored internally for each call to a particular property interface routine by specifying KSTA.

These indicies significantly reduces the amount of CPU time required to access information from the maps.
The ROCETS property package has 19 property data maps that are used to drive the interface routines. The following table defines the maps which are available:

### REAL FLUID PROPERTY ROUTINES

<table>
<thead>
<tr>
<th>PARA—HYDROGEN</th>
<th>OXYGEN</th>
<th>METHANE</th>
<th>NITROGEN</th>
<th>HELIUM</th>
</tr>
</thead>
<tbody>
<tr>
<td>HRHP01</td>
<td>ORHP01</td>
<td>MRPH01</td>
<td>NRHP01</td>
<td>ERHP01</td>
</tr>
<tr>
<td>HRUP01</td>
<td>ORUP01</td>
<td>MTPH01</td>
<td>NPHT01</td>
<td>EPHT01</td>
</tr>
<tr>
<td>HPUT01</td>
<td>OPUT01</td>
<td>MHPS01</td>
<td>NHPS01</td>
<td>EHPS01</td>
</tr>
<tr>
<td>HHPS01</td>
<td>OHPS01</td>
<td>MCPT01</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>MCVT01</td>
</tr>
</tbody>
</table>

These maps are presented in plot form in Figures 3.4.2.4.3 – 3.4.2.4.26 and are restricted to the pressure and temperature ranges defined in Table 3.4.2.4.3. It should be noted that MCPT and MCVT define $C_p$ and $C_v$ as a function of temperature with lines of constant pressure.

This particular property package has also been optimized to minimize cost. This was accomplished by tracking the indices of the property maps for up to 100 calls for each fluid by storing unique integer values assigned to KSTA for each call to a particular fluid. A speed trial was performed which ran 100,000 calls to PROP00 for para–hydrogen on the IBM 3083 for both an iterative and noniterative solution. The iterative solution required 510 seconds of real time and 1.14 CRU’s of execution time. The noniterative solution required 207 seconds of real time and 0.46 CRU’s of execution time.
### TABLE 3.4.2.4.1

PROPERTY INTERFACE PARAMETERS OPTIONS

<table>
<thead>
<tr>
<th>IOPT</th>
<th>XVAR1</th>
<th>XVAR2</th>
<th>DVAR</th>
<th>QVAR1</th>
<th>QVAR2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>PRES</td>
<td>TEMP</td>
<td>ENTH</td>
<td>DENS</td>
<td>TEMP</td>
</tr>
<tr>
<td>2</td>
<td>PRES</td>
<td>TEMP</td>
<td>ENTR</td>
<td>DENS</td>
<td>TEMP</td>
</tr>
<tr>
<td>4</td>
<td>PRES</td>
<td>ENTH</td>
<td>ENTR</td>
<td>DENS</td>
<td>TEMP</td>
</tr>
<tr>
<td>5</td>
<td>PRES</td>
<td>ENTR</td>
<td>TEMP</td>
<td>DENS</td>
<td>TEMP</td>
</tr>
<tr>
<td>6</td>
<td>PRES</td>
<td>TEMP</td>
<td>DENS</td>
<td>DENS</td>
<td>TEMP</td>
</tr>
<tr>
<td>8</td>
<td>PRES</td>
<td>TEMP</td>
<td>CP</td>
<td>GAMMA</td>
<td>DENS</td>
</tr>
<tr>
<td>9</td>
<td>PRES</td>
<td>TEMP</td>
<td>TH</td>
<td>CONDVISC</td>
<td>DENS</td>
</tr>
<tr>
<td>10</td>
<td>TEMP</td>
<td>DENS</td>
<td>PRES</td>
<td>ENTR</td>
<td>ENTH</td>
</tr>
<tr>
<td>11</td>
<td>PRES</td>
<td>DENS</td>
<td>ENTH</td>
<td>TEMP</td>
<td>ENTR</td>
</tr>
<tr>
<td>12</td>
<td>DENS</td>
<td>ENTH</td>
<td>PRES</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>13</td>
<td>PRES</td>
<td>ENTH</td>
<td>DENS</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>14</td>
<td>DENS</td>
<td>INT.ENERGY</td>
<td>PRES</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>15</td>
<td>PRES</td>
<td>INT.ENERGY</td>
<td>DENS</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

NOTE: Of the eleven options above, only (9) for helium and methane are not available.
<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>ENGLISH</th>
<th>SI</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRESSURE</td>
<td>Lbf/IN²</td>
<td>N/m²</td>
</tr>
<tr>
<td>TEMPERATURE</td>
<td>DEG R</td>
<td>DEG K</td>
</tr>
<tr>
<td>DENSITY</td>
<td>LBM/IN³*</td>
<td>KG/m³</td>
</tr>
<tr>
<td>ENTHALPY</td>
<td>BTU/LBM</td>
<td>J/Kg</td>
</tr>
<tr>
<td>INTERNAL ENERGY</td>
<td>BTU/LBM</td>
<td>J/Kg</td>
</tr>
<tr>
<td>CP</td>
<td>BTU/IN-SEC-DEG R</td>
<td>J/Kg – DEG K</td>
</tr>
<tr>
<td>THERM. COND.</td>
<td>BTU/FT-HR-R</td>
<td>J/m – SEC – DEG K</td>
</tr>
<tr>
<td>VISCOSITY</td>
<td>LBM/FT-SEC</td>
<td>KG/m – SEC</td>
</tr>
<tr>
<td>ENTROPY</td>
<td>BTU/LBM-DEG R</td>
<td>J/Kg – DEG K</td>
</tr>
</tbody>
</table>
TABLE 3.4.2.4.3

NBS PROPERTY PRESSURE AND TEMPERATURE LIMITS

<table>
<thead>
<tr>
<th>FLUID PROPERTY</th>
<th>PRESSURE RANGE (Lbf/IN²)</th>
<th>TEMPERATURE RANGE (DEG R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Para-hydrogen</td>
<td>0.0 – 17,404</td>
<td>24.8 – 720.0</td>
</tr>
<tr>
<td>Oxygen</td>
<td>0.0 – 17,404</td>
<td>97.8 – 720.0</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>0.0 – 20,000</td>
<td>113.7 – 3420.0</td>
</tr>
<tr>
<td>Helium</td>
<td>0.0 – 14,500</td>
<td>5.4 – 2,700.0</td>
</tr>
<tr>
<td>Methane</td>
<td>1.7 – 10,000</td>
<td>163.2 – 900.0</td>
</tr>
</tbody>
</table>
### TABLE 3.4.2.4.4

**COMBUSTION PROPERTY INTERFACE PARAMETER OPTIONS**

<table>
<thead>
<tr>
<th>IOPT</th>
<th>XVAR1</th>
<th>XVAR2</th>
<th>XVAR3</th>
<th>XVAR4</th>
<th>OVAR1</th>
<th>OVAR2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>PT</td>
<td>TT</td>
<td>OFR</td>
<td>PR</td>
<td>CP</td>
<td>GAMA</td>
</tr>
<tr>
<td>2</td>
<td>OFR</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>RGAS</td>
<td>XMW</td>
</tr>
<tr>
<td>3</td>
<td>PT</td>
<td>TT</td>
<td>OFR</td>
<td>NA</td>
<td>Z</td>
<td>NA</td>
</tr>
<tr>
<td>4</td>
<td>RHO</td>
<td>TT</td>
<td>OFR</td>
<td>NA</td>
<td>PT</td>
<td>RGAS</td>
</tr>
</tbody>
</table>
### TABLE 3.4.2.4.5
PARAMETERS AND UNITS USED IN XGASAA

<table>
<thead>
<tr>
<th>PARAMETER (ARG)</th>
<th>UNITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRESSURE (PT)</td>
<td>ENGLISH</td>
</tr>
<tr>
<td>TEMPERATURE (TT)</td>
<td>Lbf/in²</td>
</tr>
<tr>
<td>DENSITY (RHO)</td>
<td>°R</td>
</tr>
<tr>
<td>GAS CONSTANT (RGAS)</td>
<td>Lbm/in³</td>
</tr>
<tr>
<td>SPECIFIC HEAT RATIO (GAMA)</td>
<td>Lbf in/Lbm °R</td>
</tr>
<tr>
<td>SPECIFIC HEAT (CP)</td>
<td>D'LESS</td>
</tr>
<tr>
<td>COMPRESSIBILITY (Z)</td>
<td>Btu/Lbm°R</td>
</tr>
<tr>
<td>MOLECULAR WEIGHT (XMW)</td>
<td>D'LESS</td>
</tr>
<tr>
<td>PRESSURE RATIO (PR)</td>
<td>Lbm/Lbm-MOLE</td>
</tr>
<tr>
<td>OXIDIZER FRACTION (OFR)</td>
<td>D'LESS</td>
</tr>
</tbody>
</table>
### TABLE 3.4.2.4.6

**SUBROUTINE PARAMETERS AND RANGES**

<table>
<thead>
<tr>
<th>SUBROUTINE</th>
<th>PARAMETER</th>
<th>RANGE</th>
<th>UNITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZZMPØ1</td>
<td>PT</td>
<td>14.7 - 2000</td>
<td>Lbf/IN²</td>
</tr>
<tr>
<td></td>
<td>TT</td>
<td>50 - 400</td>
<td>°R</td>
</tr>
<tr>
<td></td>
<td>ZZ</td>
<td>0.45 - 1.55</td>
<td>D'LESS</td>
</tr>
<tr>
<td>RDMP12</td>
<td>OFR</td>
<td>0 - 9999</td>
<td>D'LESS</td>
</tr>
<tr>
<td></td>
<td>XMW</td>
<td>2 - 32</td>
<td>Lbm/Lbm-mole</td>
</tr>
<tr>
<td>CPMPØ1 (1)</td>
<td>PT</td>
<td>50 - 5000</td>
<td>Lbg/IN²</td>
</tr>
<tr>
<td></td>
<td>TT</td>
<td>100 - 2000</td>
<td>°R</td>
</tr>
<tr>
<td></td>
<td>PR</td>
<td>1 - 4</td>
<td>D'LESS</td>
</tr>
<tr>
<td></td>
<td>GAMA</td>
<td>1.4</td>
<td>D'LESS</td>
</tr>
<tr>
<td>RDMP21</td>
<td>OFR</td>
<td>0 - 1.0</td>
<td>D'LESS</td>
</tr>
<tr>
<td></td>
<td>CP</td>
<td>0.22 - 3.6</td>
<td>Btu/(Lbm°R)</td>
</tr>
<tr>
<td>RDMP13</td>
<td>OFR</td>
<td>0 - 1.0</td>
<td>D'LESS</td>
</tr>
<tr>
<td></td>
<td>GAMA</td>
<td>1.2 - 1.4</td>
<td>D'LESS</td>
</tr>
</tbody>
</table>

**NOTE:** (1) $C_p$ agrees with $H_2$ (NBS) within +/- 2% when inlet pressure (for turbines) is less than 2000 PSIA and within +/- 60% when inlet pressure is less than 5000 PSIA.
ROCETS
Combustion Property Package

CALL COMBOO (I MYPE,ldpt...

IF MYPE NE. B04S

CALL BGA500 (I DPT.XVAR1)

IF MYPE NE. B04S

CALL ZGA500 (I DPT.XVAR1)

IF MYPE NE. CR04S

CALL ZGA500 (I DPT.XVAR1)

WRITE TINESY G4S. NO COMBUSTION PR0PERTIES NOT AVAILABLE

STOP

RETURN

ORIGINAL PAGE IS OF POOR QUALITY
PRATT & WHITNEY
HYDROGEN PROPERTY MAP
DENSITY VS PRESSURE AT CONSTANT $U_0$
PRATT & WHITNEY
HYDROGEN PROPERTY MAP
DENSITY VS PRESSURE AT CONSTANT ENTHALPY

FIGURE 3.4.2.4.4
HYDROGEN PROPERTY MAP
PRESSURE VS TEMPERATURE AT CONSTANT U

FIGURE 3.4.2.4.6
PRATT & WHITNEY
HYDROGEN PROPERTY MAP
ENTROPY VS ENTHALPY AT CONSTANT PRESSURE

FIGURE 3.4.2.4.7
OXYGEN PROPERTY MAP

DENSITY VS PRESSURE AT CONSTANT U
PRATT & WHITNEY
OXYGEN PROPERTY MAP
DENSITY VS PRESSURE AT CONSTANT ENTHALPY

FIGURE 3.4.2.4.9
PRATT & WHITNEY
OXYGEN PROPERTY MAP
PRESSURE VS TEMPERATURE AT CONSTANT U

FIGURE 3.4.2.4.10
OXYGEN PROPERTY MAP
PRESSURE VS TEMPERATURE AT CONSTANT U
PRATT & WHITNEY
OXYGEN PROPERTY MAP
ENTROPY VS ENTHALPY AT CONSTANT PRESSURE

FIGURE 3.4.2.4.12
PRATT & WHITNEY

METHANE PROPERTY MAP

DENSITY VS PRESSURE AT CONSTANT ENTHALPY

FIGURE 3.4.2.4.13
PRATT & WHITNEY
METHANE PROPERTY MAP
ENTHALPY VS TEMPERATURE AT CONSTANT PRESSURE

FIGURE 3.4.2.4.14
PRATT & WHITNEY
METHANE PROPERTY MAP
ENTHALPY VS TEMPERATURE AT CONSTANT PRESSURE

FIGURE 3.4.2.4.15
METHANE PROPERTY MAP

ENTROPY VS ENTHALPY AT CONSTANT PRESSURE

FIGURE 3.4.2.4.16
PRATT & WHITNEY
METHANE PROPERTY MAP
TEMPERATURE VS CV

FIGURE 3.4.2.4.17
DENSITY VS PRESSURE AT CONSTANT ENTHALPY
PRATT & WHITNEY
NITROGEN PROPERTY MAP
PRESSURE VS TEMPERATURE AT CONSTANT ENTHALPY
PRATT & WHITNEY
NITROGEN PROPERTY MAP
PRESSURE VS TEMPERATURE AT CONSTANT ENTHALPY

FIGURE 3.4.2.4.21
PRATT & WHITNEY
NITROGEN PROPERTY MAP
ENTROPY VS ENTHALPY AT CONSTANT PRESSURE

FIGURE 3.4.2.4.22
PRATT & WHITNEY
HELIUM PROPERTY MAP
DENSITY VS PRESSURE AT CONSTANT ENTHALPY

FIGURE 3.4.2.4.23
HELIUM PROPERTY MAP
PRESSURE VS TEMPERATURE AT CONSTANT ENTHALPY

FIGURE 3.4.2.4.24
PRATT & WHITNEY
HELIUM PROPERTY MAP
PRESSURE VS TEMPERATURE AT CONSTANT ENTHALPY

FIGURE 3.4.2.4.25
PRATT & WHITNEY
HELUM PROPERTY MAP
ENTROPY VS ENTHALPY AT CONSTANT PRESSURE

FIGURE 3.4.2.4.26
PRATT & WHITNEY
COMBUSTION PROPERTY MAP
MOLECULAR WEIGHT VS MIXTURE RATIO

Figure 3.4.2.4.28
Figure 3.4.2.4.29
Figure 3.4.2.4.30
PRATT & WHITNEY
COMBUSTION PROPERTY MAP
RATIO OF SPECIFIC HEATS VS LOX FRACTION

LOX FRACTION - DLESS
0.0 0.2 0.4 0.6 0.8 1.0

RATIO OF SPECIFIC HEATS - DLESS
1.40 1.35 1.30 1.25 1.20

Figure 3.4.2.4.31
PMAPXX - Pump Characteristic MAP Format 1

INPUTS:

IOPT - MAP Option
ε - MAP Density
N - Pump Speed
W - Flow
SFSP - Speed Scale Factor
SFW - Flow Scale Factor
SFHD - Head Scale Factor
SFTQ - Torque Scale Factor

OUTPUTS:

HD - Pump Head
TORQ - Pump Torque

DESCRIPTION OF MAP OPTION:

(I) (IOPT = -1) : Values returned in call list are design values (100% RPL).

(II) (IOPT=Ø) : Scale factors are calculated from the input values. (HD and TORQ are inputs when this option is chosen).

SFSP = NDES/N
SFW = WDES/W
SFHD = HDDES/HD
SFTQ = TORQDES/TORQ

(III) (IOPT = 1) : MAP is read (curve fit) by using input values and given scale factors from call list.

NMAP = N/SFSP
WMAP = W/SFW

CALCULATIONS:

Speed (in radians):

NR = NMAP \frac{2\pi}{60}

Tip speed is defined as:

U = \frac{D}{2} \left( \frac{2\pi}{60} \right)^{N_{MAP}}

Flow coefficient (Φ) is defined as:

Φ = \frac{Q}{AU} (shown below)
Dimensional analysis:

By relating the speed of rotation, pump diameter and volumetric flow rate the following equation results:

\[
\frac{Q}{ND^3} = CONSTANT
\]

\[
= \left( \frac{Q}{D^2} \right) \left( \frac{1}{ND} \right)
\]

We know that: \( U \propto ND \) and \( A \propto D^2 \)

Substituting yields:

\[
\frac{Q}{A} \left( \frac{1}{U} \right) = CONSTANT = \Phi
\]

\[
\frac{Q}{AU} = \Phi
\]

Head rise coefficient can now be read from the MAP using flow coefficient and speed as inputs:

Torque coefficient can be read from the MAP using flow coefficient as an input:

NOTE: For option III (IOPT=1) scale factors are applied to the inputs before reading the MAPS as defined under Option II above.
PUMP HEAD:

Dimensional analysis for pump volumetric flow rate, pressure loss, density and diameter yields:

\[ \frac{Q \sqrt{\Delta P}}{D^2} = \text{CONSTANT} \]

Squaring yields \( \frac{Q^2 \Delta P}{D^2} = \text{CONSTANT} \) \hspace{1cm} (1)

Pump head is defined as: \( H_p = \frac{\Delta P}{g r} \) \hspace{1cm} (2)

We know that: \( Q \propto UD^2 \) \hspace{1cm} (3)

Substituting (2) and (3) into (1):

\[ \frac{U^2 D^4}{D^4 H_p g r} = \text{CONSTANT} \]

\[ H_{\text{MAP}} = \frac{U^2 \psi}{g r} \]

PUMP TORQUE:

Dimensional analysis of pump area, diameter, tip speed and density related to pump torque yields the following:

\[ T_{\text{MAP}} = \frac{g ADU^2}{g c} x CTRQ \]

For Option III (IOPT = 1) scale factors are applied to the head and torque.

\[ H_p = H_{\text{MAP}} \times SFHD \] \hspace{1cm} (pump head)

\[ T_P = T_{\text{MAP}} \times SFTQ \] \hspace{1cm} (pump torque)
PMAP01 - Pratt & Whitney SSME High Pressure Fuel Pump
Pump Map Format 1
PMAP02 – Pratt & Whitney SSME High Pressure LOX Pump (Main Stage) Pump Map Format 1
PMAP03  -  Pratt & Whitney SSME High Pressure LOX Pump
(Preburner Stage) Pump Map Format 1
PREBURNER STAGE TORQUE COEFFICIENT

FLOW COEFFICIENT

0.00 0.01 0.02 0.03 0.04 0.05

0.00 0.05 0.10 0.15 0.20 0.25 0.30

LOX PREBURNER STAGE CHARACTERISTIC

10000
15000
20000
25000

SPEED
SPEED
SPEED
SPEED
SUBMODULE TBMPXX - Turbine Characteristic MAP Format 1

INPUTS:

IOPT - MAP Option
N - Rotational Speed
TIN - Inlet Temperature
PIN - Inlet Pressure
PEX - Exit Pressure
γ - Specific Heat Ratio
R - Gas Constant
SFSPD - Speed Scale Factor
SFPR - Pressure Ratio Scale Factor
SFPSI - PSI Scale Factor
SFETA - Design Point Efficiency Adder

OUTPUTS:

FP - Flow Parameter
η - Efficiency
Δh' - Ideal Delta h

OPTION ONE (IOPT = -1)
This option will return design conditions (100% RPL) for all arguments in the call list.

OPTION TWO (IOPT = 0)
This option will calculate the scale factors based on the input conditions and the design conditions as follows:

\[ SFSPD = \frac{N_{des}}{N} \]

\[ SFPR = \frac{(PR_{des} - 1)/PR_{des}}{(PR - 1)/PR} \]

\[ SFPSI = \frac{PSI}{PSI_{des}} \]

\[ SFETA = (\eta - \eta_{des})(SFSPD)^2 \]

OPTION THREE (IOPT = 1)
This option reads the MAP using the input parameters and input scale factors to calculate the MAP parameters (actual numbers used when reading the maps).
CALCULATIONS:

Speed parameter: \( N_{pm} = \frac{N}{\sqrt{R \times T_{IN}}}. \)

IDEAL \( \Delta h \):

\[
\Delta h' = C_p(T_{IN} - T_{OUT})
\]

\[
\Delta h' = -C_p T_{IN} \left( \frac{T_{OUT}}{T_{IN}} - 1 \right)
\]

\[
\frac{T_{OUT}}{T_{IN}} = \left( \frac{P_{OUT}}{P_{IN}} \right)^{\gamma-1}
\]

\[
C_p = \frac{\gamma R}{\gamma - 1}
\]

Substitute (2) and (3) into (1):

\[
\Delta h' = -\frac{T_{IN} \times R}{\gamma - 1} \left[ \left( \frac{1}{PR} \right)^{\gamma-1} - 1 \right]
\]

Tip Speed:

\[
U = \frac{D}{2} \quad N_r = \frac{D}{2} \times \left( \frac{2\pi}{60} \right)(N)
\]

\[
U = \left( \frac{D}{2} \right) \left( \frac{2\pi}{60} \right)(N)
\]

Axial velocity: Axial velocity, \( C \) is obtained from the kinetic energy term in the energy equation such that:

\[
\Delta h' = \frac{C^2}{2gc}
\]

\[
C = \sqrt{2gc \Delta h'}
\]

Define PSI: \( PSI = \) velocity ratio

\[
PSI = \frac{U}{C}
\]
For option 3 (IOPT = 1) MAP parameters are calculated:

\[
PRMAP = \frac{PR}{[PR(1 - SFPR) + SFPR]}
\]

\[
SNMAP = N_{pm} \times SFSPD
\]

\[
PSIMAP = \Psi \times SFPSI
\]

Flow parameter can now be read from the MAP using pressure ratio and speed parameter as inputs:

\[
\frac{\eta \text{MAP}}{\text{PSIMAP}} = \frac{\text{TIMAP}}{\text{PSIMAP}} = \frac{\text{T}_{MAP}}{t} \times SFETA \times \left(\frac{\text{SNMAP}}{\text{SNDES}}\right)^2
\]

Efficiency can be determined by reading \(\eta \text{MAP}/\text{PSIMAP}\) as a function of \(\text{PSIMAP}\):

\[
\eta = \eta \text{MAP} + SFETA \times (\text{SNMAP} / \text{SNDES})^2
\]
TBMP01 – Pratt & Whitney SSME High Pressure Oxidizer Turbine
Turbine Map Format 1.
EFFICIENCY / VELOCITY RATIO

HIGH PRESSURE LOX TURBINE

Pratt & Whitney
FR-20282-3

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TBMP02 - Pratt & Whitney SSME High Pressure Fuel Turbine
Turbine Map Format 1.
SUBMODULE SMIT01 – Multivariable Iteration Routine

Utility submodule SMIT01 is a multi-variable iteration routine using a modified Newton-Raphson technique. The matrix equation used in SMIT01 is as follows:

\[ \Delta Y = J \Delta X \]

Where \( \Delta Y \) is the amount that the errors, or dependent variables, need to change to be zero; and \( \Delta X \) is the associated change in the independent variable. \( J \), the solver Jacobian, is a matrix of partial derivative generated with the model. SMIT01 makes a number of passes equal to the number of iteration variables plus 1 through the model to generate the Jacobian. The first pass is kept as a base point and the each of the next \( n \) variable passes generates a column of the \( J \) matrix by perturbing one of the independent variables and observing the changes in the dependent variables from the base point. The perturbation size for each of the independent variables is set through the SMIT01 inpt array \( \text{DELTAX} \). Each element of \( \text{DELTAX} \) is the perturbation magnitude of the corresponding independent variable. The \( J \) matrix is then inverted, and changes in the independent variable values, \( \Delta X \), are calculated as follows:

\[ \Delta X = J^{-1} \Delta Y \]

The SMIT01 input array \( \text{DXALOW} \) is used to limit the change in the independent variables along the same direction as the \( \Delta X \) vector. This is done to prevent excessive changes in independent variables on a single pass. This process is iterated until the dependent variables are within a specific tolerance. The maximum number of iterations allowed is input to SMIT01 as the variable \( \text{MXPASS} \). The tolerance for each variable is set in the SMIT01 input array \( \text{TOL} \).

The arrays \( \text{XVAR} \) and \( \text{YVAR} \) contain the independent and dependent variables respectively. \( \text{XVAR} \) is input to SMIT01 and changed as above until each \( \text{YVAR} \) is within the specified tolerance. The number of variables is input as \( \text{NVAR} \).

To improve the efficiency of SMIT01, Broyden's algorithm for updating the inverse \( J \) matrix \( J^{-1} \), is incorporated. Broyden's method updates the inverse \( J \) matrix without evaluating or inverting a new matrix providing a large savings in number of total passes through the model. The matrix is updated with a secant method and can be performed during a convergence attempt. Broyden's algorithm is applied whenever the reduction in the sum of the squared errors is less than the SMIT01 input \( \text{SSEBRD} \). It may still be necessary to evaluate a new \( J \) matrix if SMIT01 fails to converge at an acceptable rate. If the improvement in the sum of the squared errors is less than \( \text{SSEJAC} \) for greater than \( \text{MXFAIL} \).
consecutive passes, a new J matrix is evaluated. SSEJAC and MXFAIL are SMIT01 inputs.

Other SMIT01 inputs are defined as follows:

1. IPNT = Print flag
   0 = No print
   1 = SMIT01 statistics output at completion

2. KFILE = Logical unit number for print output

3. ILOOP = Loop number for nesting SMIT01. Two SMIT01 loops can be nested; ILOOP is used to distinguish internal variable for each loop.

4. ISAMAT = Flag to use previous J matrix
   0 = Do not use stored J matrix
   1 = Used stored J matrix

5. IDTYPE = Pertubation type for J matrix calculation
   1 = Backward difference
   0 = Central difference
   +1 = Forward difference

The flag ISIG is used to control the program flow. It is set as 0 on the first call to SMIT01 and is set by SMIT01 on subsequent passes as follows:

   1 = New X's returned in an attempt to converge
   2 = Perturbation pass to create Jacobian matrix
   3 = Iteration has converged
   4 = Jacobian is singular
   5 = Maximum iterations failed to converge

Additional parameters are output as statistics on SMIT01 operation as follows:

PCTRDN = Percent reduction in the sum of squares error for convergence attempt

KCOUNT = Convergence attempt counter.

KTOTAL = Total pass counter (including J matrix passes).

KJACOB = Number of J matrices generated

KBROYD = Number of Boyden updates
SUBMODULE SUNB01  - Univariant or bi-variant sunbar-type map reader with option to read map in any direction and either extrapolate or return corner values.

INPUTS
A = Array containing map
X = Abscissa value. Normally first independent variable
Y = Family value. Normally second independent variable
(Not used for univariant curve)
T = Ordinate value. Normally dependent variable

IOPT = OPTION CODE
1 = X and Y are input, T is returned
   (X only for univariant)
2 = T and Y are input, X is returned
   (T only for univariant)
3 = X and T are input, Y is returned
   (Not applicable for univariant)

OUTPUTS:
IERRX = Error code describing the abscissa
0 = Normal return
1 = X value too small. Lowest X used.
2 = X value too small. Extrapolated value used.
3 = X value too large. Largest X used.
4 = X value too large. Extrapolated value used.

IERRY = Error code describing the family
0 = Normal return
1 = Y value too small. Lowest Y used.
2 = Y value too small. Extrapolated value used.
3 = Y value too large. Largest Y used.
4 = Y value too large. Extrapolated value used.

Method of setting up the A array
Abscissa = X
Ordinate = T
Family = Y

A (1) = Storage space for X position indicator -- initially zero
A (2) = Storage space for Y position indicator -- initially zero, or curve identification number
A (3) = Number of X values defining each line
A (4) = Number of Y values (must be zero for univariant curve) then, beginning with A(5), the map is input as:

A (5) - 4(4+NX) = X values in ascending order
A (5+NX) - A(4+NX+NY) = Y values in ascending order
A (5+NX+NY) = T for X1,Y1
A (6+NX+NY) = T for X1,Y2

A(2*N¥+NX+4) = T for X1,YNY
A(2*N¥+NX+5) = T for X2,Y1
A(2*N¥+NX+6) = T for X2, Y2
A(3*N¥+NX+4) = T for X2,YNY
A(NX+N¥*NX+N¥+4) = T for XNX,YNY

NOTE: In the above setup, NX=number of X's, NY=number of Y's for bivariant curve. Curve for univariant, A(5) - A(4+NX) contains the X values, A(5+NX) - A(2*N¥+4) contains the T values.

COMMENTS:

1. For normal reading (IOPT=1), X is bracketed first, followed by Y bracketing. Since X and Y are bracketed independently, care must be exercised when setting up maps to ensure correct values will result. If maps are not fairly flat, a corresponding point map reader should be used.

2. For reading X as a function of T and DY (IOPT=2), Y is bracketed first, followed by T bracketing. When corner values are returned, they look as if X and T were swapped.

3. When reading Y as a function of T and X (IOPT=3), X is bracketed first, followed by T bracketing. This option is invalid for univariant curves.

4. Univariant and Bivariant curves are calculated similarly but there are no Y-calculations.

5. Multiple valued functions (more than 1 T for given X and Y) should not be read with Option 2.