Balancing Reliability and Cost to Choose the Best Power Subsystem

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ABSTRACT

How does a design engineer or manager choose between a power subsystem with .990 reliability and a more costly power subsystem with .995 reliability? When is the increased cost of a more reliable power subsystem justified?

A mathematical model is presented for computing total (spacecraft) subsystem cost including both the basic subsystem cost and the expected cost due to the failure of the subsystem. This model is then used to determine power subsystem cost as a function of reliability and redundancy. Minimum cost and maximum reliability and/or redundancy are not generally equivalent. Two example cases are presented. One is a small satellite, and the other is an interplanetary spacecraft.

INTRODUCTION

The methods described here can be applied to power subsystems in launch vehicles, satellites, and on earth. In addition, they can be utilized in many other types of applications which do not necessarily involve power.

High reliability is not necessarily an end in itself. High reliability may be desirable in order to reduce the statistically expected cost due to a subsystem failure. However, this may not be the wisest use of funds since the expected cost due to subsystem failure is not the only cost involved. The subsystem itself may be very costly. We cannot consider either the cost of the subsystem or the expected cost due to subsystem failure separately. We therefore minimize the total of the two costs, i.e., the total of the cost of the subsystem plus the expected cost due to subsystem failure.

We will be looking at subsystems which are part of a larger main system, such as a power subsystem which is a part of a main satellite system. In development of our analyses, we will talk about subsystems and main systems. In the examples, we will bring out power subsystems and larger aerospace-related systems.

Expected value is an important ingredient in our quest for finding the best power subsystem. Therefore, we'll first consider the expected cost due to subsystem failure, which is written as $E\{\text{cost due to subsystem failure}\}$. As with all expected values, this number depends upon both the dollar cost and the probability of its occurrence. Let $c_1$ be the dollar cost due to failure of the subsystem, including all costs incurred by subsystem failure (but not the cost of the subsystem itself). This number could be the entire cost of the main system (even greater in some circumstances) if failure of the subsystem resulted in complete failure of the main system. In other instances $c_1$ would be less than the cost of the main system, e.g., failure of the subsystem resulted in only a partial failure of the main system.

Now the expected cost due to subsystem failure is $c_1$ times the probability that this cost will be experienced. Subsystem failure for us can only occur when the main system is good. If the main system fails (for other than failure of the subsystem), we'll not experience cost due to subsystem failure. So, we discount the $E\{\text{cost due to subsystem failure}\}$ by multiplying by the reliability of the main system. For example, let's consider a power subsystem in a rocket. The rocket may explode on the launch pad due to a fuel problem. Even if the power subsystem would have failed in flight, we would not experience this failure. Let $r_M$ be the reliability of the main system (for other than failure of the subsystem), and let $r_S$ be the reliability of the subsystem.
Then \( E\{\text{cost due to subsystem failure}\} \)
\[
= c_1 \Pr\{\text{subsystem failure} | \text{mainsystem good}\} \\
\times \Pr\{\text{main system good}\} \\
= c_1 (1-r_S) r_M = r_M c_1 (1-r_S).
\]

We can minimize this expected cost by building a subsystem with an extremely low probability of failure, i.e., a subsystem with extremely high reliability. In this situation it is not clear that we should build the most reliable subsystem possible since this will minimize only the expected cost due to subsystem failure but does not consider the cost of building the subsystem itself. To make this decision, we should not consider the two costs separately. We therefore minimize the total of the two costs, i.e., the total of the cost of the subsystem plus the expected cost due to subsystem failure. The total cost to be minimized is given by

\[ C = \text{cost of the subsystem} + E\{\text{cost due to subsystem failure}\} \]
\[ = \text{cost of subsystem} + r_M c_1 (1-r_S). \]

In minimizing cost \( C \) we see that we are balancing the cost of the subsystem against the expected cost due to subsystem failure.

**NON-REDUNDANT POWER SUBSYSTEMS:**
**SELECTING THE BETTER OF TWO ALTERNATIVES**

Let's look at a microsatellite example. Suppose that we have two possible power subsystems under consideration for the microsatellite. Power subsystem 1, which costs $200,000, has a .97 reliability. Power subsystem 2, with a cost of $100,000, has a .94 reliability. Without further information and analysis, there is no clear "best" power subsystem, and the choice is often based upon the amount budgeted for the power subsystem.

For further analysis, let us say that the main microsatellite system has a reliability (exclusive of the subsystem under consideration) of \( r_M = .96 \). We'll further assume that failure of the power subsystem will result in a cost of \( c_1 = $10,000,000 \). Let us first compare the \( E\{\text{cost due to subsystem failure}\} \) for each of the two power subsystems. For power subsystem 1,

\[ E\{\text{cost due to subsystem failure}\} \]
\[ = r_M c_1 \Pr\{\text{subsystem failure} \} \\
\times \Pr\{\text{main system good} \} \\
= r_M c_1 (1-r_S) = .96 \times $10,000,000 \times .04 = $288,000. \]

For power subsystem 2,

\[ E\{\text{cost due to subsystem failure}\} \]
\[ = r_M c_1 (1-r_S) = .96 \times $10,000,000 \times .06 = $576,000. \]

Since power subsystem 2 is less reliable than power subsystem 1, it has a higher expected cost of failure. However, since 2 is also less expensive to build, we need to compare the overall cost, \( C \), for 1 and for 2.

For power subsystem 1,

\[ C_{S1} = $200,000 + $288,000 = $488,000. \]

For power subsystem 2,

\[ C_{S2} = $100,000 + $576,000 = $676,000. \]

Since \( C_{S1} < C_{S2} \), we select power subsystem 1 over power subsystem 2.

For further information on expected values or on selecting the best subsystem in simple situations as above, you may refer to [2]. We also note that the methods contained in this paper do not consider time-related functions, such as the cost of failure as a function of mission time. Time-related functions are covered in considerable depth in [2].

**THE EFFECT OF REDUNDANCY:**
**K OUT-OF-N:G SUBSYSTEMS**

In this article we'll direct our attention to a specific type of subsystem, called a \( k \)-out-of-\( n:G \) subsystem. Such a subsystem has \( n \) modules, of which \( k \) are required to be good for the subsystem to be good. As an example consider the situation where the engineer has a certain power requirement. He may meet this requirement by having one large power module, two smaller modules, etc. The number of modules required is called \( k \). For example, the engineer may decide that \( k = 4 \). This means that each module is \( 1/4 \) of the full required power. Therefore, the subsystem must have 4 or more modules for the full required power. The number of modules used in the subsystem is called \( n \). For example, an \( n = 6 \) and \( k = 4 \) subsystem would have 6 modules each of 1/4 th power and thus would have the output capability of 1.5 times the required power. The engineer is free to choose \( n \) and \( k \). Selection of the different values of \( n \) and \( k \) results in different subsystems, each with different costs and reliabilities. Since each \( n \) and \( k \) yields a different subsystem with different costs, we can therefore choose the subsystem, i.e., the \( n \) and \( k \), which will minimize cost \( C \).
Here we'll assume perfect switching devices (if needed) of negligible cost and independence of the modules of the subsystem.

Although there are many variations of the k-out-of-n: G model, we'll present a few of these possibilities to give some idea of its uses and potentialities.

**MODEL 1** (k fixed and n variable)

The simplest k-out-of-n: G model is one where the modules are independent and all have common probability \( p \) of being good and common probability of failure \( q = 1 - p \). Let \( X \) count the number of good modules. Now \( E \{ \text{cost due to subsystem failure} \} \)

\[
= r_MC_1 \Pr \{ \text{subsystem failure} \} \\
= r_MC_1 \sum_{x=0}^{k-1} \binom{n}{x} p^x q^{n-x} \quad (1)
\]

Recall that \( C = \text{cost of subsystem} + E \{ \text{cost due to subsystem failure} \} \). We therefore need also to consider the cost of the subsystem. First consider a simple situation where \( k \) is fixed. Here we are free to choose \( n \). Then \( n-k \) will be the redundancy or number of spares in the subsystem. If each module costs \( c_4 \) then the cost of subsystem = \( nc_4 \). Using this with (1) we obtain

\[
C = \text{cost of subsystem} + E \{ \text{cost due to subsystem failure} \} \\
= nc_4 + r_MC_1 \sum_{x=0}^{k-1} \binom{n}{x} p^x q^{n-x}.
\]

We wish to find the \( n \) which minimizes cost \( C \).

As an example, let's look at an interplanetary spacecraft. Consider the situation where \( k = 1 \), i.e., only one power module is required to be operational for the power subsystem to be operational. Suppose that the reliability of this single module is .95, i.e., \( p = .95 \). Let the reliability of the main spacecraft system for other than failure of the subsystem be .9, i.e., \( r_M = .9 \). Suppose that the cost of one power module is 1 (hundred million dollars, for example) i.e., \( c_4 = 1 \), and that the cost due to failure of the power subsystem is 10 (hundred million), i.e., \( c_1 = 10 \).
subsystem. A

Consider the example of an optimal

rM,

For in

g(3)

values

subsystem costs

as a single

subsystem would

smaller

g(k)

be the cost of a subsystem of k modules.

The n = 2 power subsystem is the best. If p falls below .83, then no redundancy is required. Figure 2 reveals that if .83 < p < .98, then the n = 2 power subsystem is the best. If p > .98, then a subsystem consisting of exactly k modules is the best. If p < .83, then more redundancy is required (n = 3). If p = .83, then the reliability of the satellite (for other failure of the subsystem) be .99, i.e., r_M = .99. Furthermore, the cost of building a single module capable of full power is .5 million, i.e., c_3 = .5. And last, let's say that each power module has a reliability of .95.

Suppose for a small satellite that the cost due to power subsystem failure, c_1, is 100 (million dollars). Let the reliability of the satellite (for other failure of the subsystem) be .99, i.e., r_M = .99. Furthermore, the cost of building a single module capable of full power is .5 million, i.e., c_3 = .5. And last, let's say that each power module has a reliability of .95.

From Figure 3 we see, at p = .95, that the n = 2, k = 1 power subsystem is the best (has the lowest value of C). Note however, if p < .948, that the n = 4, k = 2 subsystem is the best. Additionally, note that this is a much flatter curve. Unless we're fairly sure that p is close to .95, we might choose the n = 4, k = 2 subsystem, since it gives us a relatively low value for C over a wide range of p.

Figure 3

Example for Model 2.

BASIC PROGRAMS

The authors will provide, upon request, copies of BASIC programs (Quickbasic 4.5) to both evaluate C and also to search for an n and k which minimize C. These programs are also appropriate for models other than the two we've covered here (see [1] or [2] for more detailed information on other models). If you wish a copy on disk, please send a formatted disk floppy with your request. We also note that all models may be used when k is fixed by replacing nc_3 g(k)/k by nc_4.
Additionally, the cost of launching the subsystem may easily be considered merely by including this cost in C for the various models.

CONCLUSIONS

The methods brought forward in this paper can be used to make very definitive decisions in choosing the best power (or other) subsystem from a number of alternatives. Essentially we minimize the total of two costs, i.e., the total of the cost of the (power) subsystem itself plus the expected cost due to the failure of the (power) subsystem. A computer program which utilizes the methods is available. Its output plots can yield very clear, obvious, and straightforward decisions.

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