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# Acoustical Analysis of Gear Housing Vibration

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# ACOUSTICAL ANALYSIS OF GEAR HOUSING VIBRATION

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## Summary

This report describes the modal and acoustical analysis of the NASA gear-noise rig. Experimental modal analysis techniques were used to determine the modes of vibration of the transmission housing. The resulting modal data were then used in a boundary element method (BEM) analysis to calculate the sound pressure and sound intensity on the surface of the housing as well as the radiation efficiency of each mode.

In this report, the radiation efficiencies of the transmission-housing modes are compared with theoretical results for finite, baffled plates. The report also describes a method that uses the measured mode shapes and the BEM to predict the effect of simple structural changes on the sound radiation efficiency of the modes of vibration.

## Symbols

$C$	Helmholtz coefficient
$c$	speed of sound, m/sec
$I$	sound intensity, $W/m^2$
$i$	$\sqrt{-1}$
$k$	wave number
$P$	arbitrary field point on or around the housing
$p$	sound pressure, Pa
$Q$	arbitrary point on the housing
$S$	surface area, $m^2$
$v$	normal component of velocity, m/sec

$W$	sound power, W
$\rho_0$	density of acoustic medium, $kg/m^3$
$\sigma$	radiation efficiency
$\psi$	free space Green's function
$\psi'$	normal derivative of $\psi$
$\omega$	vibration frequency, rad/sec

## Introduction

Transmission noise is an important component of the total cabin noise in helicopters. Transmission noise reaches the cabin by two primary paths: a structural path by which vibrational energy is transmitted through transmission mounts and load-carrying members to the cabin walls, and a direct radiation path in which sound radiated from the transmission excites the cabin wall or passes into the cabin through openings (ref. 1). The relative importance of these paths depends on the specific design of the helicopter and the location of the transmission.

In this study, the direct radiation of sound from the transmission is of interest. Such sound is a function of the forces applied to the transmission, the structural properties of the transmission, and the radiation efficiency of the transmission. The transmission's vibrational response may be described by a superposition of its vibrational modes. It is, therefore, important to know the modes of vibration and the radiation efficiency of these modes. In this study, the vibrational modes of the transmission were measured by experimental modal analysis. The boundary element method (BEM) was used to

calculate (1) the acoustic field produced by each mode, (2) the sound pressure and sound intensity distributions on the surface of the transmission, (3) the far-field sound pressure directivity of each mode, and (4) the relative sound power and sound radiation efficiency of each of the transmission's vibrational modes.

## Modal Analysis Experiments

As a means of understanding the relationship between noise and gear design parameters, the NASA Lewis Research Center developed a gear-noise test rig (see fig. 1). In the present study, this rig was analyzed to determine the modes and natural frequencies of the gear housing. Eight modes were found in the 650- to 3000-Hz frequency range. Two of the gear-housing modes are shown in figures 2 and 3. Because the top of the gear housing is not as stiff as the sides, most of the modes resemble classic plate modes of the top surface. The mode in figure 2, for example, looks very much like a 1,1 plate mode, whereas the mode in figure 3 is similar to a 4,1 mode. All but one of the modes exhibited dominant plate modes of the top surface.

## Acoustical Analysis of Gear-Housing Modes

The boundary element method (BEM) was used to analyze the acoustical properties of the vibrational modes of the gear housing. This method has been used previously to analyze the vibrational modes of engines (refs. 2 and 3), to model the sound fields inside of automobiles and aircraft, and to predict the performance of mufflers and silencers (refs. 4 and 5).

### Background of the Boundary Element Method

The BEM is based on the Helmholtz integral equation

$$C(P)p(P) = \int_S [i\omega\rho_0 v(Q)\psi(P,Q) - p(Q)\psi'(P,Q)]dS(Q) \quad (1)$$

where

$$C(P) = 4(\pi) \text{ in } P \text{ (in the field around the housing)}$$

$$C(P) = 2(\pi) \text{ on } S \text{ (on the surface of the housing)}$$

Equation (1) states that the pressure  $p(P)$  at any point (see fig. 4) in the acoustic medium can be found by summing up (integrating) two terms over the entire surface  $S$  of the housings: the first term involves the vibration velocity  $v$  on the surface of the housing; and the second term involves the pressure  $p$  on the surface of the housing. However, the pressure on the surface of the housing is not known initially and must be determined numerically.

The surface of the housing is discretized into a mesh of surface elements of zero thickness. (In this study, the modal analysis grid shown in figs. 2 and 3 was used.) This discretization reduces equation (1) to numerical form (ref. 2) as

$$[A]\{p\} = [B]\{v\} \quad (2)$$

where  $\{v\}$  is a vector of known vibration velocity at the nodes (grid points) of the mesh,  $\{p\}$  is the vector of unknown pressures at the nodes, and  $[A]$  and  $[B]$  are square matrices that depend on the shape of both the surface  $S$  and the frequency  $\omega$ . For a given mode shape  $\{v\}$ , equation (2) may be solved to determine the pressure  $\{p\}$  at each node.

### Sound Intensity and Sound Power

Once both  $\{v\}$  and  $\{p\}$  are known at every node on the surface  $S$ , the pressure at any point in the near or far field can be determined from a discretized form of equation (1); further, the sound intensity  $I$  at every point  $Q$  on the surface may be calculated from

$$I(Q) = \Re_e[p(Q)v^*(Q)]/2 \quad (3)$$

where  $\Re_e$  denotes the real part of the expression in parentheses and  $*$  denotes the complex conjugate. The intensity is the sound power radiated per unit area of the housing; the total sound power is found by integrating the intensity over the surface  $S$ :

$$W = \int_S I(Q)dS(Q) \quad (4)$$

### Sound Radiation Efficiency

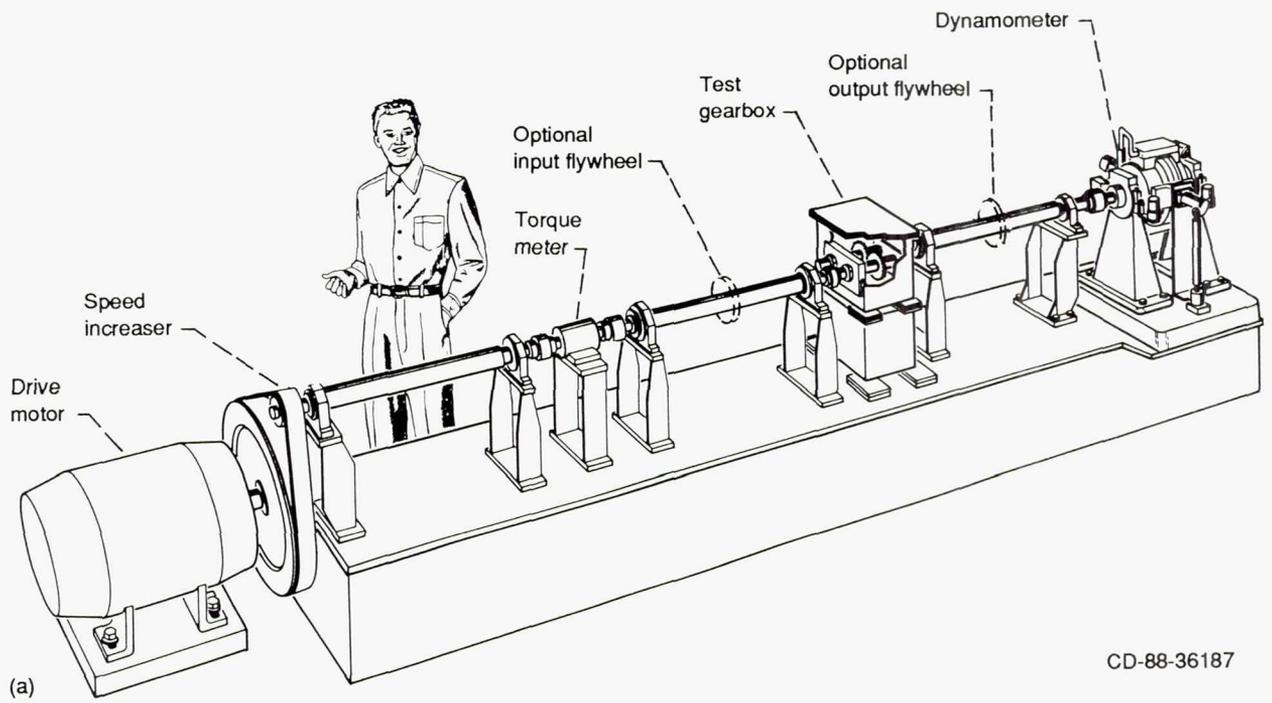
The radiation efficiency  $\sigma$  is the ratio of the sound power radiated by a vibrating structure, to the sound power that would be radiated by an equivalent flat piston vibrating in an infinite baffle:

$$\sigma = \frac{W}{\rho_0 c S \langle v^2 \rangle} \quad (5)$$

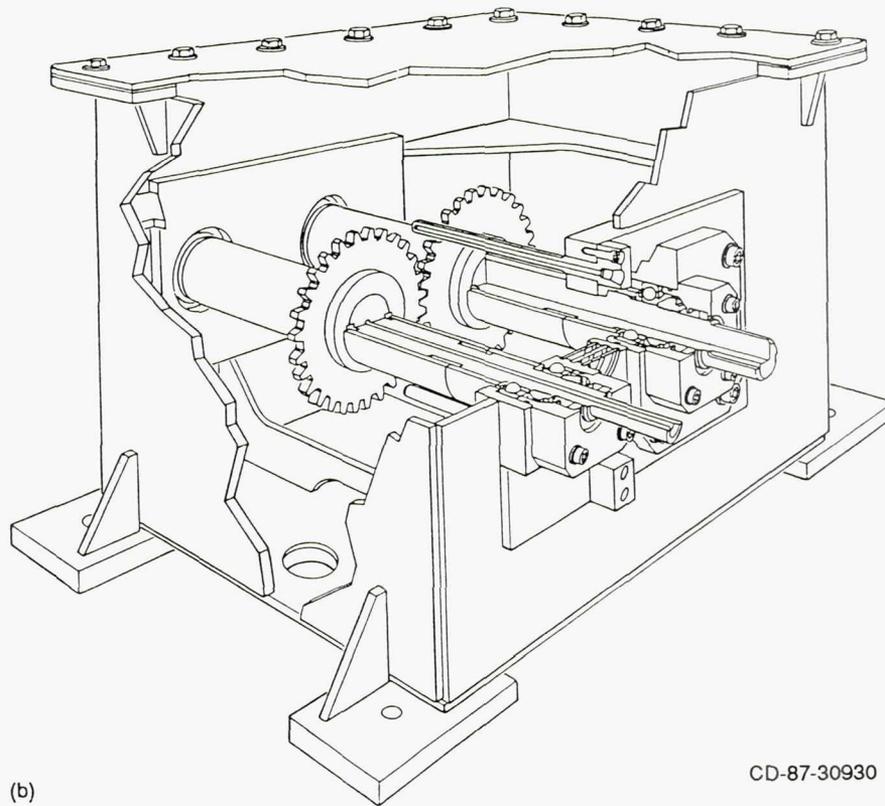
where  $c$  is the speed of sound and  $\langle v^2 \rangle$  is the mean square velocity of the surface  $S$ .

### The BEMAP Program

In this study, the BEMAP program (ref. 6) was used to perform the acoustical analysis just described. A flow chart of BEMAP is shown in figure 5. The input to BEMAP is the surface geometry of the structure (the transmission housing), the vibration of the structure, and the frequency of vibration. The vibration data may originate from modal analysis experiments, as in the present study, or from measurements on the surface, but analytical vibration data (e.g., from finite element analyses) may also be used. The vibration data may be normal-



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(b)

(a) Layout.  
 (b) Detail of gearbox.  
 Figure 1.—NASA gear-noise rig.

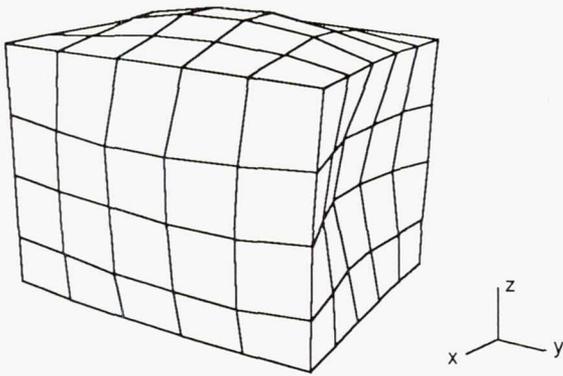


Figure 2.—A 658-Hz mode of the transmission housing.

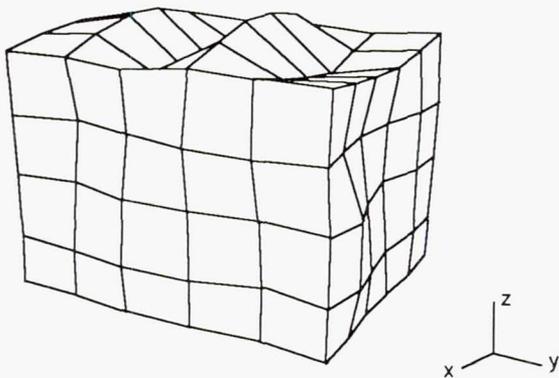


Figure 3.—A 2722-Hz mode of the transmission housing.

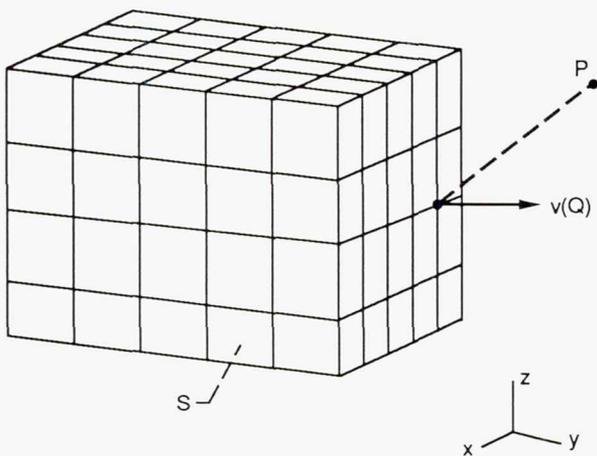


Figure 4.—Gearbox with mesh showing BEM parameters.

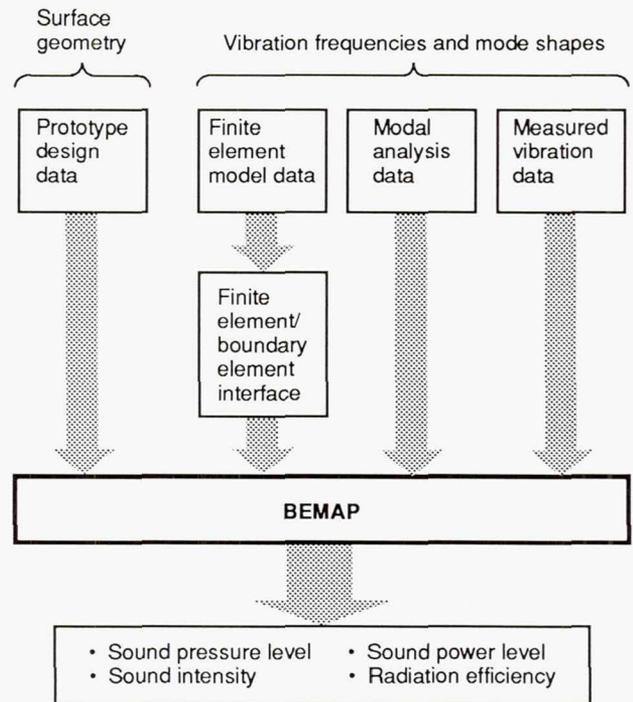


Figure 5.—Flow chart for BEMAP input and output data.

ized, as in the case of mode shapes, or absolute, as when the data are obtained from actual measurements or from finite element, forced vibration models. Regardless of the source of the vibration, the magnitude and phase of the vibration at a number of discrete surface points (nodes) must be provided. The spacing of the nodes depends on (1) the variation of the surface vibration data, (2) the shape of the surface, and (3) the frequency.

To simplify the transfer of vibration data and surface shape into BEMAP, a number of software interfaces have been written. An interface from an experimental modal analysis software package was used in the present study to import the grid point coordinates and modal data (e.g., figs. 2 and 3) into BEMAP. Similar interfaces have been developed for importing vibration data from several popular finite element programs.

The output quantities from BEMAP are the sound pressure and sound intensity on the surface of the structure, the sound power radiated by the structure, the radiation efficiency of the structure, and the sound pressure at any point in the acoustic field surrounding the structure. Note that because the vibration of the surface  $\{v\}$  is normalized (i.e., a mode shape), the sound pressure and sound intensity determined in equations (2) and (3) are relative values. In this case, the relative sound pressure and sound intensity distributions on the surface demonstrate how a given mode radiates sound energy (i.e., which regions of the surface are responsible for radiation).

The radiation efficiency  $\sigma$  (eq. (5)) is a function of the mode shape and the frequency of vibration. Even though a mode shape is a normalized vibration,  $\sigma$  is an absolute value; it is a property of the mode. Therefore, the radiation efficiency can be used to compare the sound radiating characteristics of different structural modes.

## Results

### Transmission Housing Modes

To illustrate how the acoustical analysis described herein may be applied to structural vibration, consider the two transmission housing modes in figures 2 and 3. In figure 2, a 658-Hz mode, the top, front, and rear surfaces of the transmission housing are vibrating in phase (i.e., outward) with one another, whereas the two end surfaces are vibrating in phase with each other (i.e., inward), but out of phase with the other three surfaces. (Since the bottom of the housing is much thicker than the other sides and was rigidly attached to the frame supporting the transmission, it had negligible motion. Therefore, throughout this paper, the vibration of the bottom surface and the support frame are neglected.)

In figure 3, a 2722-Hz mode, most of the deflection is in the top surface of the housing with a small amount of deflection on the ends and virtually no motion on the front and rear surfaces. Considered as a plate, the top surface is vibrating in a 4,1 bending mode.

### Sound Pressure and Sound Intensity Distributions

The relative sound pressure distribution on the surface of the transmission housing for the 658-Hz mode (fig. 2) is shown in figure 6. The relative sound pressure varies from a maximum where the deflection is the highest (near the center of each surface of the housing) to a minimum where the deflection is the lowest (at the vertical edges of the housing), a range of 30 dB. Each of the color bands in figure 6 represents a 3.5-dB change in sound pressure.

The relative sound intensity distribution of the 658-Hz mode is shown in figure 7. The sound intensity distribution shows "hot spots" where sound energy is radiated from the housing. Because the sound intensity is the sound power per unit area, it is a better indicator of the sound energy radiated by the mode than is the sound pressure level. For example, the sound pressure level distribution in figure 6 shows high sound levels at the center of all three visible surfaces of the housing. However, the sound intensity distribution in figure 7 shows the right end does not contribute much to the total sound power. (The discrepancy may be due to the velocity being out of phase with the pressure on this surface, which would reduce the sound intensity and, therefore, the sound power from the right side of the housing.)

The relative sound intensity distribution for the 2722-Hz mode (fig. 3) is shown in figure 8. (Recall that this distribution cannot be compared quantitatively to the intensity distribution in fig. 7 because each mode shape is normalized by the maximum deflection of that mode.) It is clear from figure 8 that there is a broad region on the top surface of the housing where appreciable sound energy is radiated. From the appearance of the mode (fig. 3), one might expect sound cancellation (hence weak sound intensity) near the top surface. However, for this relatively high frequency (2722 Hz), the structural wavelength is comparable to the acoustical wavelength, and the cancellation is minimal.

### Radiation Efficiency

The radiation efficiencies of the eight transmission-housing modes are shown in figure 9. As seen in figure 9, all of the modes are very efficient radiators of sound. These results can be checked by using Wallace's plate radiation theory (ref. 7) to make an approximate analysis. This theory is strictly valid only for simply supported plates in an infinite baffle, and is based on  $k/k_b$ , the ratio of the acoustic-to-bending wave numbers. The acoustic wave number  $k = \omega/c$ ; the bending wave number is given by

$$k_b = \left[ (m\pi/a)^2 + (n\pi/b)^2 \right]^{1/2} \quad (6)$$

where  $a$  and  $b$  are the dimensions of the plate and  $m, n$  are mode numbers. When  $k/k_b \ll 1$ , the radiation efficiency is much less than unity and highly dependent on the mode numbers  $m, n$ . When  $k/k_b \approx 1$ , the radiation efficiency is less strongly affected by the mode numbers and approaches unity. Above  $k/k_b = 1$ , the radiation efficiency of each mode reaches a maximum value between one and two; it then decreases monotonically to unity for  $k/k_b \gg 1$ .

In figure 10 the radiation efficiencies of seven of the eight transmission-housing modes are compared to the radiation efficiencies of a rectangular plate having the same dimensions as the top surface of the transmission housing, as determined by the method of reference 7. (The mode at 2000 Hz was omitted since it did not exhibit a dominant plate mode on the top surface of the transmission housing.) Figure 10 shows the plate mode number for each mode, and it shows that the wave number ratio for all of the modes lies between 0.75 and 1.35. This is the region where the radiation efficiency is high.

The two sets of data in figure 10 follow the same trend, an indication that the transmission housing is dominated by modes which resemble those of a flat panel. However, because the top surface of the housing is not baffled and because there is radiation from the other sides of the transmission, there are some discrepancies between the theoretical data and the BEMAP data.

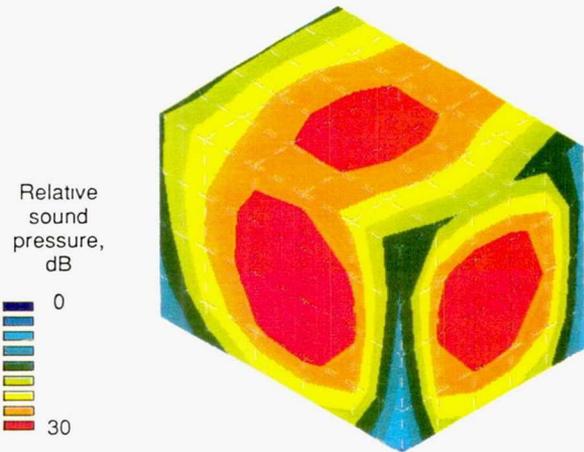


Figure 6.—Relative sound pressure distribution on surface of transmission housing for the 658-Hz mode. Each color band represents a 3.5-dB change.

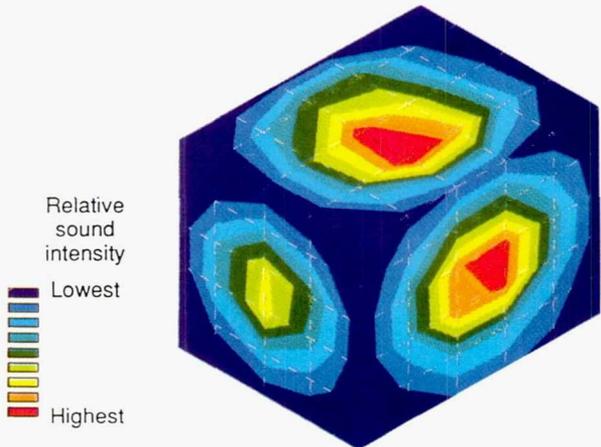


Figure 7.—Relative sound intensity distribution on surface of transmission housing for the 658-Hz mode. Each color band represents a  $1.4\text{-W/m}^2$  change.

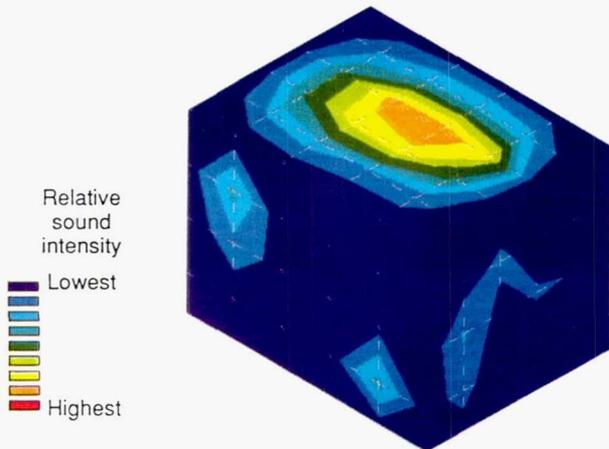


Figure 8.—Relative sound intensity distribution on surface of transmission housing for the 2722-Hz mode. Each color band represents a  $3.2\text{-W/m}^2$  change.

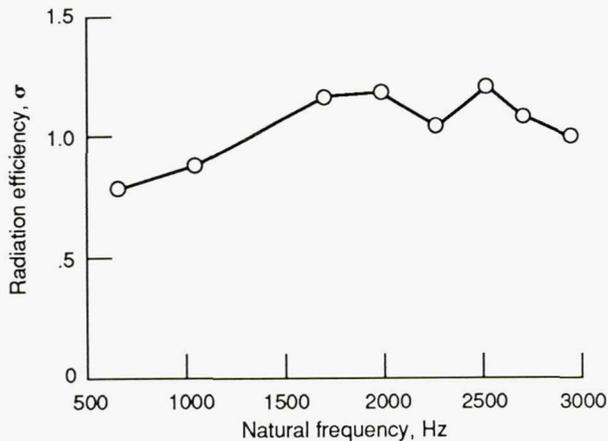


Figure 9.—Radiation efficiencies of the eight transmission housing modes.

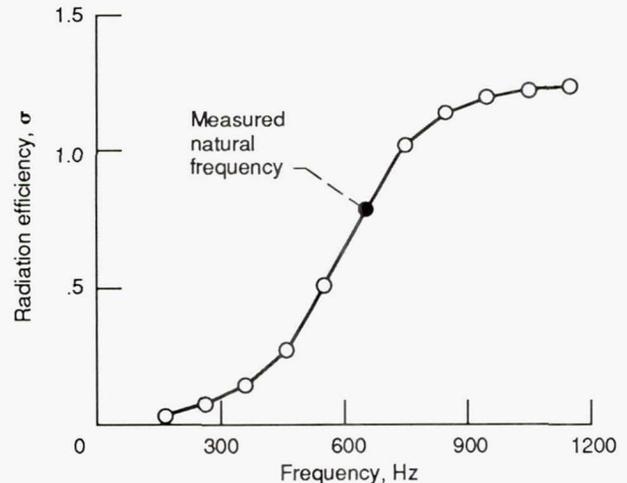


Figure 11.—Radiation efficiency of the 658-Hz vibration mode as a function of natural frequency.

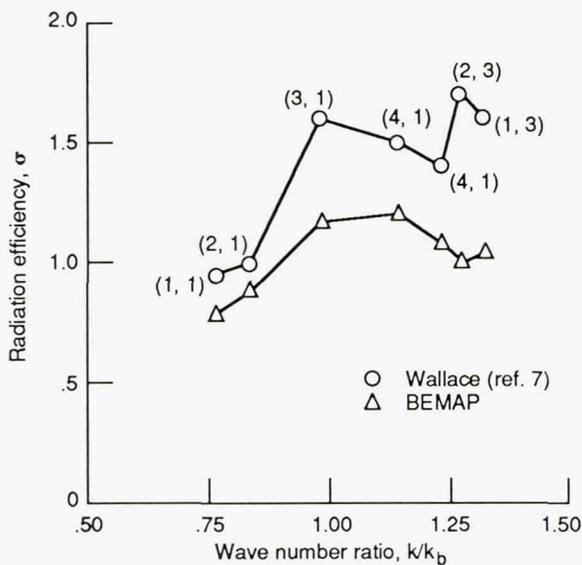


Figure 10.—Radiation efficiencies calculated by using BEMAP and plate theory.

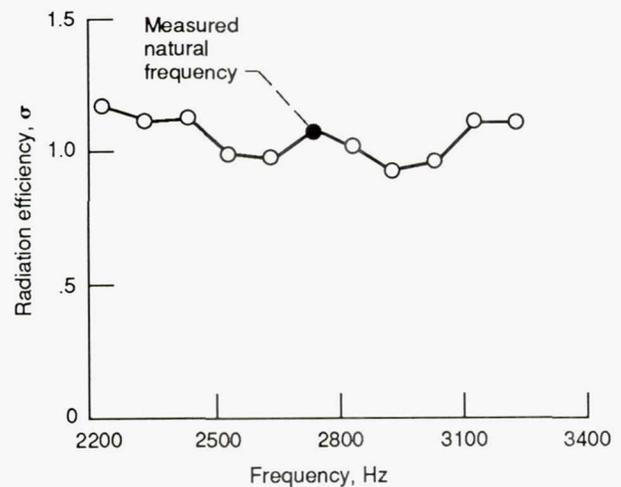


Figure 12.—Radiation efficiency of the 2722-Hz vibration mode as a function of natural frequency.

## Structural Modification and Noise Control

Structural modification of the transmission housing is a logical method of noise control. In general, the spectrum of the forces which excite a structure is dominated by relatively low frequency components. Modifications which stiffen a structure tend to push the vibrational modes to higher frequencies where these forces are weaker—an effect that reduces vibration. However, this simplistic approach ignores the effect of radiation efficiency on the radiation of sound. Equation (5) shows that the sound power is directly proportional to the radiation efficiency. Thus, while modification of the structure may reduce the mean-square vibration, it may also increase the radiation efficiency.

Consider the effect of a straightforward modification to the thickness of the transmission housing. This alteration would change the natural frequencies of the modes without materially altering the mode shapes. (A similar effect can be obtained by changing the composition of the transmission-housing material.) To see the effect of such a modification, the radiation efficiency of each mode was determined at frequencies above and below the measured value. These data are plotted in figures 11 and 12 for the 658- and 2722-Hz modes, respectively. (The actual measured natural frequency of each mode is represented by the filled data symbol.)

The radiation efficiency of the 658-Hz mode increases with frequency, as shown in figure 11. Thus, a thicker transmission housing will radiate more noise if the reduction in vibration

does not offset the increase in radiation efficiency. It is also possible that making the transmission housing thicker may cause a mode with a high radiation efficiency to move from a frequency where it is not excited to a new frequency (e.g., a gear mesh frequency) where it radiates considerably more sound energy.

By contrast, the radiation efficiency of the 2722-Hz mode is almost independent of frequency, as see in figure 12. Thickening of the housing will not increase the radiation efficiency of this mode, and unless the new frequency coincides with a gear mesh frequency, the sound energy radiated by this mode will decrease.

## Concluding Remarks

This report shows that the vibrational modes of a transmission housing may be analyzed to determine their impact on the radiated sound energy. The boundary element method (BEM) can be used to determine from vibration velocity data the sound intensity on the surface of the transmission housing. The BEM can find the radiation efficiency of each vibration mode in order to assess acoustically the importance of each mode. The BEM can also be used to examine "what if" strategies for possible structural modifications such as altering the thickness or changing the material composition of the housing.

In this report, the BEM was used to analyze the radiation characteristics of the vibration modes of the NASA gear-noise rig. The radiation efficiency of each mode was determined and compared to approximate values calculated by Wallace's method, which is based on plate radiation theory.

Because the transmission-housing modes are similar in appearance to those of simple rectangular plates, plate radiation theory was partially successful in estimating the radiation efficiency of the transmission-housing modes. The present approach of calculating the radiation efficiency of each mode by using the BEM is superior because, unlike plate radiation theory, the BEM makes no assumptions about boundary conditions, baffling, and uniformity of the mode shapes.

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