Probability of Failure and Risk Assessment of Propulsion Structural Components

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Prepared for the
1989 JANNAF Propulsion Meeting
Cleveland, Ohio, May 23–25, 1989
Because of the increasing need to account for the uncertainties in material properties, loading conditions, geometry, etc., a methodology has been developed to determine structural reliability and to assess the associated risk. The methodology consists of a probabilistic structural analysis by a probabilistic finite element computer code NESSUS (Numerical Evaluation of Stochastic Structures Under Stress) and a generic probabilistic material property model. The methodology is versatile and is equally applicable to structures operating at high- and cryogenic-temperature environments. Results obtained demonstrate that the issues of structural reliability and risk can be formally evaluated by using the methodology developed which is inclusive of the uncertainties in material properties, structural parameters, and loading conditions. The methodology is described in some detail with illustrative examples.

INTRODUCTION

The probabilistic structural analysis methods (PSAM) (ref. 1) have been developed to analyze the effects of fluctuating loads, variable material properties, uncertainties in analytical models and geometry, etc., especially for high-performance structures such as space shuttle main engine (SSME) turbopump blades. In the deterministic approach, uncertainties in the responses are not quantified, and the actual safety margin remains unknown. Risk is calculated after extensive service experience. However, probabilistic structural analysis provides a rational alternative method to quantify uncertainties in structural performance and durability. NESSUS (Numerical Evaluation of Stochastic Structures Under Stress) is a probabilistic structural analysis computer code developed under the PSAM project which integrates finite element methods and reliability algorithms (refs. 2 and 3). This code is capable of predicting the scatters of structural response variables (such as stress, displacement, natural frequencies, and buckling loads) subjected to all kinds of uncertainties. These are subsequently compared with their probable failure modes to assess the risk of component fracture. Probable failure modes are defined for different structures and their respective service environments. For example, failure events (such as stress greater than strength, displacement exceeds...
maximum allowables, or avoidance of resonance) are often used for the reliability assessment. Probability of occurrence of those failure events can be determined once the probability distributions of the requisite structural response variables are calculated by NESSUS.

In undertaking a reliability and risk analysis, all suspected sources of uncertainties must be taken into account in order to control the probability of failure in service environments within an acceptable range. Structural reliability and risk obtained by a formal probabilistic methodology can be useful in evaluating the traditional design and in setting quality control requirements, inspection criteria, and retirement for cause. This methodology can also be used to identify candidate materials and design concepts in the absence of a technology base. In this report, the methodology developed to assess the structural reliability and risk/cost is described.

CONCEPT OF PROBABILISTIC STRUCTURAL ANALYSIS

In a probabilistic structural analysis, the primitive variables which affect the structural behaviors have to be identified. These include temperature, material properties, structural geometry, loading conditions, etc. and should be described by their respective probability distributions. A structural analysis performed by NESSUS with the predetermined probability distributions of all the primitive (random) variables will produce corresponding scatter (uncertainties) in the structural responses such as displacement, stress, natural frequencies, etc. The concept is illustrated by figure 1.

STRUCTURE OF NESSUS

NESSUS consists of three major modules: NESSUS/PRE, NESSUS/FEM, and NESSUS/FPI. NESSUS/PRE is a preprocessor used for the preparation of the statistical data needed to perform the probabilistic finite element analysis. It allows the user to describe the uncertainties in the structural parameters (random variables) at nodal points of a finite element mesh. The uncertainties in these parameters are specified over this mesh by defining the mean value and standard deviation of the random variable at each point, together with an appropriate form of correlation. Correlated random variables are then decomposed into a set of uncorrelated vectors by a modal analysis. For a strongly correlated random field, the number of dominant random variables in the set of uncorrelated vectors will be much less than the number in a weakly correlated random field. Thus, the computational time required for the analysis will also be reduced significantly.

NESSUS/FEM is a finite element code used for the structural analysis and parameter sensitivity evaluation. It generates a database containing all the response information corresponding to a small variation of each independent random variable. The algorithm used in NESSUS/FPI requires an explicit response function in order to perform a reliability analysis. In complicated structural analysis problems, response can only be available implicitly through a finite element model. To overcome this difficulty, the response function is expressed parametrically with the database.
NESSUS/FPI (Fast Probability Integrator) is an advance reliability module (ref. 3). This module extracts the database generated by NESSUS/FEM to develop a response or a performance model in terms of uncorrelated random variables. The probabilistic structural response is calculated from the performance model. For a given response value, the probability of exceeding this value is estimated by a reliability method, which treats the problem as a constrained minimization. This step is called a point probability estimation. The cumulative distribution function is generated by running FPI at several response values. One alternative for generating the distribution function for any given response is to conduct a direct Monte Carlo simulation study. However, in general, it is very costly. NESSUS/FPI provides a method which not only produces an accurate probability distribution, but also requires less computing time than that required for Monte Carlo simulation, especially in low-probability regions.

PROBABILITY MATERIAL PROPERTIES MODEL

A generic material behavior model (ref. 4) is used to evaluate the scatter of material properties for the structures subjected to high-temperature environments and high-cycle loading conditions. The fundamental assumption for this model is that the material properties behavior can be simulated by primitive (random) variables. The general form of this model is shown in equation (1).

\[ M_p = M_{po} \left( \frac{T_F - T}{T_F - T_0} \right)^n \left( \frac{S_F - \sigma}{S_F - \sigma_0} \right)^p \left( \frac{\log N_{MF} - \log N_{MO}}{\log N_{MF} - \log N_{MO}} \right)^q \]  

where \( M_p \) is the material property at current temperature after the application of fatigue cycles; \( M_{po} \) is the reference material property at reference temperature \( T_0 \) (usually room temperature), reference stress \( \sigma_0 \), and reference fatigue cycles \( N_{MO} \); \( T_F \), \( S_F \), and \( N_{MF} \) are the final temperature, final material strength, and final mechanical cycle, respectively; \( T \) is current blade temperature; \( \sigma \) is the existing blade stress; and \( N_M \) is the number of fatigue cycles to be considered in the failure analysis. The exponents \( n \), \( p \), and \( q \) are determined from available experiment data, or they are estimated from the anticipated material behavior due to the particular primitive random variable.

APPLICATION TO A SPACE SHUTTLE MAIN ENGINE (SSME) TURBINE BLADE

The methodology described previously was applied to a SSME turbine blade (fig. 2) which was subjected to complex mechanical and thermal loads. (Probabilistic Structural Analysis for Select Space Propulsion System Components. First Annual Report by Southwest Research Institute, Rocketdyne, University of Arizona, Marc Analysis Corp., and Columbia University for NASA Lewis Research Center under Contract NAS3-24389, March 1986.) Mechanical loads consist of centrifugal force and the differential pressure across the airfoil. Centrifugal force is induced by rotational speed. Since it is difficult to maintain a constant rotational speed, the centrifugal force has to be considered as a random variable. Differential pressure is also random because of pressure fluctuation. Random thermal loads are due to combustion irregularities which cause a random temperature distribution in the blade. Uncertainties in the blade geometry arise during the manufacturing process.
The scatter in material properties is caused by nonuniformities in the material.

The turbine blade is modelled by 40 4-node shell elements with 55 nodal points. In this study, seven random fields were considered as listed in table I. In previous analysis (ref. 5), the probability density function of material properties such as Young's modulus, thermal expansion coefficient, and material strength are assumed. In the present study, they are simulated by the probabilistic material property model as defined by equation (1). The material property is a function of its reference value, temperature, and fatigue cycles. The statistics of the primitive random variables are listed in table II. Since the material is a function of stress, and stress is a function of material property in an implicit way, an iterative procedure is necessary to obtain a convergent solution for stress and material properties.

To start the iteration procedure, the probability distributions of the material properties were predicted by equation (1) with its reference value and the blade temperature only, since the variations of the blade stresses were unknown. A probabilistic structural analysis using NESSUS was performed, and the probability distributions of stresses were determined. The generic probabilistic material property model was applied again at the given fatigue cycle with these newly estimated stress variations. The probability distributions of material properties were updated, and NESSUS was rerun. The procedure was repeated until the probability distributions of the material properties and stress converged. During the iteration process, the joint cumulative distribution functions of nodal stresses were also calculated to apply the generic probabilistic material property model on the entire blade. It was found that only a few iterations were needed for a convergence.

The blade is assumed to be subjected to 100,000 constant amplitude load cycles which degrade the modulus and strength. The critical points of the large displacement and high stress are depicted in figure 3. As shown in figure 4, at the root of the leading edge, the probability distribution of modulus is reduced significantly after the application of cyclic loads, where the initial distribution is that without the fatigue load, and the final distribution is the one after the application of the fatigue load. Changes in the probability density function of tip displacement with and without the application of cyclic loads are also shown in figure 5. As expected, the tip displacements are increased after the application of cyclic loads because the blade becomes softer. Material strength degradation after cyclic loads is calculated with the converged stress by the probabilistic material property model as shown in figure 6. Once the stress-strength relationships are determined at a given fatigue cycle, the probability that the stress is greater than the corresponding material strength will be determined. The probability is calculated by

\[ P_f = \int_{-\infty}^{+\infty} \left[ \int_{-\infty}^{+\infty} f_S(s)ds \right] f_\sigma(x)dx \]  

(2)

where \( f_\sigma \) is the probability density function (pdf) of effective stress calculated by the probabilistic structural analysis using NESSUS, and \( f_S \) is the pdf of material strength simulated by the probabilistic material property model. By varying the number of cycles and repeating the procedure described previously, a Risk-Fatigue cycle curve is developed for critical locations as shown in figure 7. This curve is useful for assessing the risk of structural
fracture. For instance, at a given acceptable risk level, the number of fatigue cycles to initial local failure can be determined. With this information available, criteria can be set for quality control, inspection intervals, and retirement for cause.

The risk-cost assessment is the evaluation of the relationship between structural reliability and total cost associated with the structure. Total cost is the sum of initial cost and a fraction of consequential cost as defined in equation (3). The fraction is weighted by the probability of failure. The initial cost is the cost for component service readiness. The consequential cost is the cost incurred due to failure.

\[ C_t = C_i + P_f \times C_f \]  

where \( C_t \) represented total cost, \( C_i \) represented initial cost, \( P_f \) is the probability of failure, and \( C_f \) is the consequential cost. Since the lower initial cost is often associated with higher risk for the structural failure, and higher initial cost will normally reduce the risk, the total cost can be minimized for an acceptable structural reliability. For example, the initial cost is a function of the mean strength of the material as shown in figure 8. The greater the strength, the more reliable the structure. However, high initial cost is needed to improve the mean strength. Total cost is shown in figure 9. There exists a point of diminishing return. This can be identified as the lowest total cost with no loss of structural reliability. A similar result was also obtained for the case of strength quality improvement as shown in figures 10 and 11.

**SUMMARY OF RESULTS**

In summary, a reliability and risk cost methodology has been developed. It consists of a probabilistic structural analysis by NESSUS and a generic probabilistic material model. The methodology is versatile and equally applicable to hot and cold structures where data is difficult to obtain. The methodology is demonstrated by using it to assess the risk associated with fatigue cycles to initiate local failure in SSME blades.

**REFERENCES**


TABLE I. - RANDOM INPUT DATA

<table>
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<tr>
<th>Random fields</th>
<th>Number of dependent random variables</th>
<th>Mean</th>
<th>Standard deviation (or coefficient of variation)</th>
<th>Correlation length</th>
<th>Number of independent random variables</th>
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<tbody>
<tr>
<td>X coordinate</td>
<td>55</td>
<td>Deterministic coordinate</td>
<td>0.01 in.</td>
<td>5.0</td>
<td>13</td>
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<tr>
<td>Y coordinate</td>
<td>55</td>
<td>Deterministic coordinate</td>
<td>0.01 in.</td>
<td>5.0</td>
<td>13</td>
</tr>
<tr>
<td>Z coordinate</td>
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<td>Deterministic coordinate</td>
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<td>13</td>
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<tr>
<td>Temperature</td>
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<td>Steady-state temperature</td>
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<tr>
<td>Modulus</td>
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<td>23 000 000 psi</td>
<td>0.10 (coefficient of variation)</td>
<td>3.0</td>
<td>16</td>
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<tr>
<td>Pressure</td>
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<td>Rotation speed</td>
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<td>0.01 (coefficient of variation)</td>
<td>N/A</td>
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TABLE II. - PRIMARY VARIABLE PROBABILITY DISTRIBUTIONS FOR PROBABILISTIC MATERIAL PROPERTY MODEL

<table>
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<th>Variable</th>
<th>Distribution type</th>
<th>Mean</th>
<th>Standard deviation</th>
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<tr>
<td>$T_F$, °F</td>
<td>Normal</td>
<td>2750</td>
<td>51.4</td>
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<tr>
<td>$T_0$, °F</td>
<td>Normal</td>
<td>68</td>
<td>2.04</td>
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<td>$S_F$, ksi</td>
<td>Normal</td>
<td>212.0</td>
<td>10.6</td>
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<tr>
<td>$\sigma_0$</td>
<td>Constant</td>
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<td>0</td>
</tr>
<tr>
<td>$N_{MF}$</td>
<td>Lognormal</td>
<td>$10^8$</td>
<td>5x10^8</td>
</tr>
<tr>
<td>$N_{MO}$</td>
<td>Lognormal</td>
<td>$10^3$</td>
<td>50</td>
</tr>
<tr>
<td>$n$</td>
<td>Normal</td>
<td>0.25</td>
<td>0.0075</td>
</tr>
<tr>
<td>$p$</td>
<td>Normal</td>
<td>0.25</td>
<td>0.0075</td>
</tr>
<tr>
<td>$q$</td>
<td>Normal</td>
<td>0.25</td>
<td>0.0075</td>
</tr>
</tbody>
</table>
INPUT PDF
(LOAD, MATERIAL, GEOMETRY, BOUNDARY CONDITIONS)

\[
P_X(x) = \int_{-\infty}^{\infty} P_Y(y) dy
\]

OUTPUT PDF
(DISPLACEMENT, STRAIN, STRESS)

FIGURE 1. - CONCEPT OF PROBABILISTIC STRUCTURAL ANALYSIS.

STOCHASTIC THERMOMECHANICAL LOADS
STRESS CONCENTRATION
UNCERTAIN BOUNDARY CONDITIONS

FIGURE 7. - UNCERTAINTIES IN PROBABILISTIC STRUCTURE ANALYSIS.

GLOBAL DISPLACEMENTS

LOCAL STRESSES

FIGURE 3. - SOME BLADE SHOWING LOCATIONS WHERE PROBABILISTIC STRUCTURAL RESPONSE WAS EVALUATED (RANDOM LOADINGS: PRESSURE, TEMPERATURE, AND ROTATIONAL SPEED; RANDOM GEOMETRY; RANDOM MATERIAL PROPERTIES; MECHANICAL CYCLE = 1000 CAD).

PROBABILITY DISTRIBUTION

FIGURE 4. - PROBABILISTIC MODULUS SIMULATED BY USING THE GENERIC PROBABILISTIC MATERIAL PROPERTY MODEL (MPA).

(A) NODE A, (MEAN STRESS STRENGTH RATIO = 0.85). (B) NODE B, (MEAN STRESS STRENGTH RATIO = 0.82).

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Figure 5. - Probabilistic displacements calculated by Nevisus ([10^{-3}] in.).

Figure 6. - Probabilistic fatigue stress/strength simulated by using the generic probabilistic material property model, ksi.

Figure 7. - Probability of local failure due to fatigue cycles at node A.

Figure 8. - Initial cost and structural reliability quantified in terms of mean strength (given quality).

Figure 9. - Total cost quantified in terms of mean strength (given quality).

Figure 10. - Initial cost and structural reliability quantified in terms of quality control (given mean strength).

Figure 11. - Total cost quantified in terms of quality control (given mean strength).
**Probability of Failure and Risk Assessment of Propulsion Structural Components**

**Abstract**

Because of the increasing need to account for the uncertainties in material properties, loading conditions, geometry, etc., a methodology has been developed to determine structural reliability and to assess the associated risk. This methodology consists of a probabilistic structural analysis by a probabilistic finite element computer code NESSUS (Numerical Evaluation of Stochastic Structures Under Stress) and a generic probabilistic material property model. The methodology is versatile and is equally applicable to structures operating at high- and cryogenic-temperature environments. Results obtained demonstrate that the issues of structural reliability and risk can be formally evaluated by using the methodology developed which is inclusive of the uncertainties in material properties, structural parameters, and loading conditions. The methodology is described in some detail with illustrative examples.