A Scheme for Bandpass Filtering Magnetometer Measurements To Reconstruct Tethered Satellite Skiplfpe Motion

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A SCHEME FOR BANDPASS FILTERING MAGNETOMETER MEASUREMENTS
TO RECONSTRUCT TETHERED SATELLITE SKIPOPE MOTION

I. INTRODUCTION

The first flight of NASA's tethered satellite system is presently scheduled for 1992. It is to be carried onboard the space shuttle to a 300-km orbit with a 28.5° inclination [1]. There the satellite will be deployed outward along the orbit radius vector, to a tether length reaching 20 km, and later retrieved. It is well known that the tether may exhibit some very strange dynamics, one of which is the so-called skiprope motion [2]. This mode of vibration is of particular concern because it has very little damping and could pose a problem during satellite retrieval and docking. If this happens, the present plan is to have the shuttle flight crew execute an orbiter yaw maneuver that is properly phased relative to the rotating tether so as to cause the rotary motion to decay quickly [3]. However, to know when to maneuver, at what rate, and in what direction, requires knowing tether position as a function of time: and this means estimating it. The baseline approach is to do this on the ground, in near real-time, using a state observer. Originally, the observer used only satellite rate gyro measurements as inputs [4]. Afterwards, it was modified to use satellite magnetometer measurements also.

This paper describes a completely different approach to this problem. It uses the same magnetometer measurements, but filters them through bandpass filters tuned to the skiprope frequency. It is simple to implement, quite robust, and can be used in parallel with or as a backup to the observer. It is presented in this paper in the following manner. A theoretical development of its underlying equations is given in section II. These are utilized in the implementation scheme described in section III. Simulation results for it are given in section IV. Final comments are made in section V.

II. THEORETICAL DEVELOPMENT

Previous studies indicate a relationship between tether position caused by skiprope motion and satellite attitude, as shown in figure 1 [5]. Tether deflection at its midpoint, \( d \), is related to the list of the satellite \( \vec{\tau}_{s3} \) axis from the orbit radius vector, \( \alpha_{s3} \), by the relationship

\[
d = \frac{\alpha_{s3} L}{\pi}
\]

where \( L \) is tether length. As the tether skips, the satellite \( \vec{\tau}_{s3} \) axis cones about the orbit radius vector, \( \vec{r}_w \), at the skiprope frequency with the \( \vec{r}_{s3} \) axis lying in the plane of the tether, as shown in figure 1. Hence, if the coning motion can be reconstructed, the tether motion can be inferred. The scheme proposed here seeks to do that based on measurements of the Earth's magnetic field with an onboard magnetometer.
To develop this scheme, two coordinate frames are utilized. One is the satellite body-fixed frame $(i_1, i_2, i_3)$, and the other is the orbit-fixed frame $(i_L, i_V, i_W)$, shown in figure 2. As noted there, $i_L$ is aligned with the orbit velocity vector, $i_V$ with the orbit north, and $i_W$ with the orbit radius vector, directed away from the Earth. The $(i_1, i_2, i_3)$ frame is referenced to the $(i_L, i_V, i_W)$ frame by the Euler angles $(\alpha_{S1}, \alpha_{S2}, \alpha_{S3})$. The Earth’s magnetic field flux density vector $B$ is referenced to it by the parameters $(B, \beta_{EL}, \beta_{LV})$, as shown in figure 3, where $B$ is its magnitude in Gauss. It is assumed that the steady-state attitude motion of the satellite due to tether skiprope can be described by

\[
\begin{align*}
\alpha_{S2} &= \alpha_{S2M} \sin (\omega_{SK} t) \\
\alpha_{S1} &= \alpha_{S1M} \cos (\omega_{SK} t)
\end{align*}
\]

where $\omega_{SK}$ is the skiprope frequency and can be either positive or negative depending on the direction of coning. The angle $\alpha_{S3}$ is assumed constant and small enough, along with $\alpha_{S2}$ and $\alpha_{S1}$, that $\cos (\alpha_{S3}) \approx 1$ and $\sin (\alpha_{Si}) \approx \alpha_{Si}$ for $i = 1, 2, 3$. 

Figure 1. Tether skiprope motion.
Orbit Velocity

Vector $\vec{i}_w$ : Orbit Radius Vector

Orbit Velocity : $\vec{i}_U$

Vector $\vec{i}_{S1}$

$\alpha_{S1}$

$\alpha_{S2}$

$\alpha_{S3}$

Figure 2. Satellite axes referenced to orbital coordinates.

Figure 3. Earth's magnetic field referenced to orbital coordinates.
Given this model, it is straightforward to show that

\[
\begin{pmatrix}
\mathbf{i}_{s1} \\
\mathbf{i}_{s2} \\
\mathbf{i}_{s3}
\end{pmatrix} = \begin{bmatrix}
1 & \alpha_{s1} & -\alpha_{s2} \\
-\alpha_{s3} & 1 & \alpha_{s1} \\
\alpha_{s2} & -\alpha_{s1} & 1
\end{bmatrix}
\begin{pmatrix}
\mathbf{i}_t \\
\mathbf{i}_v \\
\mathbf{i}_w
\end{pmatrix}
\]

(3)

and

\[
\mathbf{\bar{B}} = -B \cos \beta_{FL} \sin \beta_{AZ} \mathbf{i}_t + B \cos \beta_{FL} \cos \beta_{AZ} \mathbf{i}_v + B \sin \beta_{FL} \mathbf{i}_w .
\]

(4)

From equations (3) and (4), the components of the Earth’s magnetic field in satellite axes are given by

\[
\begin{align*}
B_{s1} &= -B \cos \beta_{FL} \sin \beta_{AZ} + \alpha_{s3} B \cos \beta_{FL} \cos \beta_{AZ} - (B \sin \beta_{FL}) \alpha_{s2} \\
B_{s2} &= B \cos \beta_{FL} \cos \beta_{AZ} + \alpha_{s3} B \cos \beta_{FL} \sin \beta_{AZ} + (B \sin \beta_{FL}) \alpha_{s1} \\
B_{s3} &= B \sin \beta_{FL} - (B \cos \beta_{FL} \cos \beta_{AZ}) \alpha_{s1} - (B \cos \beta_{FL} \sin \beta_{AZ}) \alpha_{s2}.
\end{align*}
\]

(5)

By the same token, these are the satellite magnetometer outputs resolved into satellite axes. Substituting equation (2) into equation (5) and assuming for the remainder of the development that

\[
\alpha_{SM} = \alpha_{S1M} \pm \alpha_{S2M} .
\]

yields, after some manipulation,

\[
\begin{align*}
B_{s1C} &= \frac{B_{s1} + \alpha_{SM} \sin \beta_{FL} \cos (\omega_{SRt} + \pi/2)}{B} \\
B_{s2C} &= \frac{B_{s2} - \alpha_{SM} \cos \beta_{FL} \cos \beta_{AZ}}{B} \\
B_{s3C} &= \frac{B_{s3} - \alpha_{SM} \cos [\omega_{SRt} - (\beta_{AZ} + \pi)]}{B \cos \beta_{FL}}.
\end{align*}
\]

(6)
Hence, if the magnetometer outputs in satellite axes, \((B_{s1},B_{s2},B_{s3})\), are modified according to equation (6), the results are, in theory, three harmonics from which the following inferences can be made about satellite attitude motion, and hence tether motion, due to the skiprope phenomenon:

1. \(\alpha_{sM}\) is determined by the amplitude of \(B_{s3c}\). Tether deflection at the midpoint of the tether can then be determined using equation (1) and knowing the tether length \(L\).

2. The period of \(B_{s3c}\), \(T_{SR}\), determines the magnitude of \(\omega_{SR}\) by the relationship

\[
|\omega_{SR}| = 2\pi T_{SR}
\]

If \(B_{s1c}\) leads \(B_{s2c}\), then \(\omega_{SR} > 0\). If \(B_{s1c}\) lags \(B_{s2c}\), then \(\omega_{SR} < 0\).

3. When \(B_{s3c}\) is a maximum, then \(\omega_{SR} t = \beta_{AZ} + \pi\). At this point, \(i_{s3}\) lies in the plane formed by \(i_{W}\) and \(\vec{B}\), tilted toward \(\vec{B}\) from \(i_{W}\) by the angle \(\alpha_{sM}\). Then \(i_{s3}\) at any other point in time is readily determined, assuming \(\beta_{AZ}\) and \(\omega_{SR}\) are known. Figure 4 helps to clarify this. Here, \(\vec{B}_P\) and \(i_{s3P}\) are the projections of \(\vec{B}\) and \(i_{s3}\), respectively, onto the \(i_U - i_V\) plane. Tether position is \(\pi\) rad out of phase with \(i_{s3}\).

Hence, equation (6) and these associated inferences form the theoretical basis for the scheme proposed in this paper. A practical implementation of it is presented in section III.

---

**Figure 4.** Projection of \(i_{s3}\) and \(\vec{B}\) onto \(i_U - i_V\) plane.
III. IMPLEMENTATION SCHEME

In implementing equation (6), it is assumed that the satellite magnetometer outputs are telemetered to the ground and resolved into satellite axes, if necessary, to generate \((\hat{B}_{S1}, \hat{B}_{S2}, \hat{B}_{S3})\). The Earth’s magnetic field parameters \((B, \beta_{EL}, \beta_{AZ})\) are estimated on the ground via a math model of the Earth’s magnetic field and knowledge of the satellite orbit. Since \(\alpha_s\) is small, the terms in it are discarded to obviate the need to estimate it and thus simplify the scheme. Consequently, equation (6) is implemented as

\[
\begin{align*}
\hat{B}_{S1} &= \frac{B_{S1} + \hat{B} \cos \beta_{EL} \sin \beta_{AZ}}{\hat{B}} \pm \alpha_{SM} \sin \beta_{EL} \cos (\omega_{SR}t + \pi/2) \\
\hat{B}_{S2} &= \frac{B_{S2} - \hat{B} \cos \beta_{EL} \cos \beta_{AZ}}{\hat{B}} \pm \alpha_{SM} \sin \beta_{EL} \cos (\omega_{SR}t) \\
\hat{B}_{S3} &= \frac{B_{S3} - \hat{B} \sin \beta_{EL}}{\hat{B} \cos \beta_{EL}} \pm \alpha_{SM} \cos [\omega_{SR}t - (\beta_{AZ} + \pi)].
\end{align*}
\]

where \((\hat{\cdot})\) denotes an estimated variable. If the only motion of the satellite was that caused by tether skiprope, then equation (7) could be utilized directly. However, there will be satellite motion due to other effects that create unwanted signals in \((\hat{B}_{S1}, \hat{B}_{S2}, \hat{B}_{S3})\) and hence \((\hat{B}_{S1C}, \hat{B}_{S2C}, \hat{B}_{S3C})\). An example of this is pendulous motion, which is satellite oscillation about its center of mass. This is expected to have amplitudes on the order of several degrees at a frequency in the neighborhood of 0.03125 Hz. Motion due to tether skiprope is expected to be on the order of degrees or fractions of a degree, depending on the tether length [5]. Its frequency is around 0.005 Hz for tether lengths greater than 2 km. To eliminate unwanted signals like this, \((\hat{B}_{S1C}, \hat{B}_{S2C}, \hat{B}_{S3C})\) in equation (7) are each filtered by a bandpass filter whose transfer function has the form

\[
G(s) = \frac{2\xi (s/\omega_n)}{(s/\omega_n)^2 + 2\xi(s/\omega_n) + 1},
\]

where \(\xi \ll 1\) and \(\omega_n = \omega_{SR}\) ideally. At \(\omega = \omega_n = \omega_{SR}\), the filter passes the skiprope frequency with unity gain and zero phase shift while attenuating all other frequencies. Hence, it filters higher frequency signals like those due to pendulous motion, magnetometer electronic noise, and A/D converter quantization. In addition, it desensitizes the scheme to bias errors and low frequency errors, like those occurring at orbital frequency (0.00018 Hz). These are bound to exist in estimating the Earth’s magnetic field. Hence, the steady-state outputs of the bandpass filters should better reflect the right hand sides in equation (7) than the inputs. Inferences about satellite skiprope motion will be drawn from them.

Two final comments about implementation are appropriate. First, the smaller the \(\xi\) of the bandpass filters, the better they attenuate unwanted signals, but the longer it takes their outputs to reach steady state. The latter can be improved by choosing the initial states of each filter so that one equals the initial input to the filter and the other equals zero. Doing this, \(\xi = 0.1\) was found by simulation to give a
good compromise between filtering and speed. The other point to make is that the scheme will normally require two iterations, since the skiprope frequency can only be estimated a priori. For the first iteration, \( \omega_n \) should be set to the best guess for \( \omega_{SR} \). At steady state, the filtered output for \( \hat{B}_{53c} \) in equation (7) will reveal \( \omega_{SR} \) more precisely. Then, \( \omega_n \) can be tuned to it and the process repeated for the second iteration.

IV. SIMULATION RESULTS

A computer simulation was developed to test the scheme described in section III, subject to the following conditions. (See the appendix for a listing of the program.) The tethered satellite was assumed to be in a 300-km orbit with a 28.5° inclination. Its motion relative to the orbital coordinate frame described in section II was given by

\[
\begin{align*}
\alpha_{S3} &= \alpha_{S3B} \\
\alpha_{S2} &= \alpha_{S2B} + \alpha_{SR2M} \sin (\omega_{SR} t) + \alpha_{P2M} \sin (\omega_p t + \pi/4) \\
\alpha_{S1} &= \alpha_{S1B} + \alpha_{SR1M} \cos (\omega_{SR} t) + \alpha_{P1M} \sin (\omega_p t + \pi/4)
\end{align*}
\]

where

\begin{align*}
\alpha_{S3B} &= 5^\circ \\
\alpha_{S2B} &= 1^\circ \\
\alpha_{S1B} &= 1^\circ \\
\alpha_{SR2M} &= 0.5^\circ \\
\alpha_{SR1M} &= 0.75^\circ \\
\alpha_{P2M} &= 2^\circ \\
\alpha_{P1M} &= 2^\circ \\
\omega_{SR} &= 2\pi(0.005) \text{ rad/s} \\
\omega_p &= 2\pi(0.03125) \text{ rad/s}
\end{align*}

Hence, the satellite showed skiprope and pendulous motion, plus bias errors. The bandpass filters were set so that
\[ \omega_{\nu} = \omega_{\nu s} = 2\pi(0.005\text{Hz}) \]
\[ \xi = 0.1 \]

which implies that the filters were already tuned to the skiprope frequency. The Earth’s magnetic field was modeled by a six-displaced-dipole model. The estimated field was given by

\[ \hat{B} = 1.1B \]
\[ \hat{\beta}_{\nu} = \beta_{\nu} + 5^\circ \]
\[ \hat{\beta}_{\nu s} = \beta_{\nu s} + 5^\circ \]

where \((B, \beta_{\nu}, \beta_{\nu s})\) are its true values. The magnetometer axes were assumed to be aligned with the satellite axes with measurements made 16 times per second and quantized to an LSB = 0.02 milli-Gauss [6]. This corresponds to a 16-bit A/D converter scaled to a range of \(\pm 0.65536\) Gauss.

For these conditions, the simulation results are shown in figures 5, 6, and 7. Using the rules outlined in section II for interpreting the plots in figure 7, the following conclusions are drawn about the skiprope motion. In figure 7, observe that \(\hat{B}_{s1c}\) leads \(\hat{B}_{s2c}\) by \(\pi/2\) rad; hence, \(\omega_{\nu s} > 0\). From \(\hat{B}_{s3c}\), it can be seen that \(T_{\nu s} = 200s\) and so \(\omega_{\nu s} = 2\pi(0.005)\text{ rad/s}\). The amplitude of \(\hat{B}_{s3c}\) shows that \(\hat{\omega}_{s} = 0.0125\) rad = 0.72°. Hence, the deflection of the tether at its midpoint due to skiprope is \(d = 0.0125 \ell \pi / 2\) by virtue of equation (1). At \(t = 100, 200, 300\ldots\) s, the \(\hat{\beta}_{s1}\) axis lies in the plane of \(\hat{\beta}_{w}\) and \(\hat{B}\), tilted toward \(\hat{B}\). At \(t = 100, 200, 300\ldots\) s, it remains in the same plane, but tilted away from \(\hat{B}\). The tether is \(\pi\) rad out of phase with the projection of \(\hat{\beta}_{s1}\) onto the \(\hat{\beta}_{w}\) plane.

In all aspects, the estimated skiprope motion matches the actual reasonably well, in spite of the demanding test conditions. This verifies the scheme and demonstrates its robustness.

V. FINAL COMMENTS

This paper has presented a unique scheme for reconstructing tethered satellite skiprope motion by bandpass filtering satellite magnetometer outputs telemetered to the ground. It was tested in a computer simulation with a demanding set of test conditions. This showed it to be quite robust.

As a final remark, this scheme is not just limited to the tethered satellite skiprope problem posed here. Indeed, it has potential application wherever:

1. A body cones about a known axis while measuring a known vector in body-fixed axes.

2. There is a need to know the time history, or perhaps just the fundamental characteristics of, the coning motion.
Figure 5. Earth’s magnetic field versus time.

Figure 6. Magnetometer outputs versus time.
Figure 7. Bandpass filter outputs versus time.
REFERENCES


C THIS PROGRAM IS NAMED TSSMAG.FORT(CRAY6). IT TESTS SCHEME TO
C RECONSTRUCT TETHERED SATELLITE ATTITUDE MOTION DUE TO TETHER
C SKIPROPE USING MAGNETOMETER OUTPUTS. BANDPASS FILTERS ARE USED TO
C RECONSTRUCT SKIPROPE FREQUENCY COMPONENTS. LOWPASS FILTER IS USED TO
C RECONSTRUCT AS3. FOURTH ORDER RUNGA-KUTTA IS USED TO SOLVE FILTER
C DIFF. EQS. THIS PROGRAM WAS LAST REVISED ON FEB 19, 1991.

REAL NU2,LA2,LA3,GA2
DIMENSION RVG(3),BMAG(3),AA(38)

C SPECIFY AND COMPUTE CONSTANTS-----------------------------

REAL PI,TWOPI,PIO2,RFD,DFR,GME,ALT,REA,RV,
OMO,OMA,OMG,LA3,XJ,PSI,ASB3,ASB2,ASB1,WSR,
ASR2M,ASR1M,WP,AP2M,AP1M,PHIP,WNF,ZETAB,
ZETAF,DT,NPMAX,TMAX

C STARTING CONDITIONS
IDUM=1
READ (4,2) IDUM

2 FORMAT (14)
PI=3.1415926
TWOPI=2.0*PI
PIO2=PI/2.0
RFD=0.017453292
DFR=1.0/RFD

C ALT=ORBIT ALT IN KM
C REA=RADIUS OF EARTH IN KM
GME=398601.2
ALT=300.0
REA=6371.2
RV=REA+ALT

C OMO=ORBIT RATE IN RAD/SEC
OMO=SGRT(GME/(RV**3))
TO=TWOPI/OMO

C LA3=ORBIT ICLINATION IN RAD

C OMA=ORBIT REGRESSION RATE IN RAD/SEC (SEE THOMPSON'S "INTRO TO SPACE
DYNAMICS, P. 99 FOR EQUATIONS GIVEN BELOW)
LA3=RFD*28.5
XJ=1.637E-03
PSI=TWOPI*XJ*COS(LA3)*(REA/RV)**2
OMA=-PSI/TO

C OMG=EARTH RATE IN RAD/SEC
OMG=RFD*360.0/(24.0*3600.0)
ASB3=RFD*5.0
ASB2=RFD*1.0
ASB1=RFD*1.0
WSR=TWOPI*0.0050
ASR2M=RFD*0.50
ASR1M=RFD*0.75
WP=TWOPI*0.03125
AP2M=RFD*2.0
AP1M=RFD*2.0
PHIP=RFD*45.0
WNF=TWOPI*0.0001
ZETAB=0.707
ZETAF=0.100
DT=1.0/16.0
NPMAX=16
TMAX=6000.0
PRINT*,OMO,OMA,OMG

C COMPUTE GAIN AND PHASE OF BANDPASS FILTER TUNED FOR
C SKIPROPE FREQUENCY
RATIO=WSR/WNF

14
XR=1.0-RATIO*RATIO
XI=2.0*ZETA*F*RATIO
GM=RATIO/SQRT(XR*XR+XI*XI)
GP=PI/2-ATAN2(XI,XR)
PRINT*, ZETA, WNF, WSR, GM, GP

C SPECIFY ERRORS IN ESTIMATING EARTH’S MAGNETIC FIELD
BHEF=0.10
BELHE=RFD*5.0
BAZHE=RFD*5.0

C MAGNETOMETER OUTPUT QUANTIZATION PARAMETERS
NBITS=16
BSMAX=0.65536
IQ=1
IQFST1=1
IQFST2=1
IQFST3=1

C DUMMY VARIABLE FOR PLOT VARIABLES NOT USED
DUM=0.0

C SPECIFY INITIAL CONDITIONS-----------------------------------------------
T=0.0
NP=NPMAX
START=1.0

C GA2=ANGLE FROM VERNAL EQUINOX TO PRIME MERIDIAN, NORTH IS POSITIVE,
C IN RAD
C LA2=LONGITUDE OF ASCENDING NODE IN RAD
C NU2=POSITION OF VEHICLE IN ORBIT WRT ASCENDING NODE IN RAD
GA2=0.0
LA2=0.0
NU2=0.0

C INITIAL STATES OF FILTERS:
BS1PF=0.0
BS1PFD=0.0
BS2PF=0.0
BS2PFD=0.0
BS3PF=0.0
BS3PFD=0.0

C IFLGMF=0 GIVES CONSTANT EARTH MAG FIELD WRT LOCAL VERTICAL AXES
C IFLGMF=1 GIVES 6-DISPLACED-DIPOLE MODEL OF EARTH MAG FIELD WITH
C REALISTIC TETHERED SATELLITES ORBIT MECHANICS---------------------
IFLGMF=1
10 CONTINUE

C DETERMINE EARTH’S MAG FIELD FLUX DENSITY VECTOR IN GAUSS IN LOCAL
C VERTICAL AXES----------------------------------------------------------
OE11=COS(NU2)*COS(LA3)*COS(LA2)-SIN(NU2)*SIN(LA2)
OE12=COS(NU2)*SIN(LA3)
OE13=-COS(NU2)*COS(LA3)*SIN(LA2)-SIN(NU2)*COS(LA2)
OE21=-SIN(LA3)*COS(LA2)
OE22=COS(LA3)
OE23=SIN(LA3)*SIN(LA2)
OE31=SIN(NU2)*COS(LA3)*COS(LA2)+COS(NU2)*SIN(LA2)
OE32=SIN(NU2)*SIN(LA3)
OE33=-SIN(NU2)*COS(LA3)*SIN(LA2)+COS(NU2)*COS(LA2)
SGA2=SIN(-GA2)
CGA2=COS(-GA2)
AEG11=CGA2
AEG12=0.
AEG13=-SGA2
AEG21=0.
AEG22=1.0
AEG23=0.
AEG31=SGA2
AEG32=0.
AEG33=CGA2

C SUBROUTINE CALL STATMENTS

TEMF=GA2-LA2
CTEM=COS(TEMF)
CLA3=COS(-LA3)
SNU2=SIN(-NU2)
STEM=SIN(TEMF)
CNU2=COS(-NU2)
SLA3=SIN(-LA3)

RVG(1) = (-SNU2*CTEM*CLA3-STEM*CNU2)*RV
RVG(2) = (SNU2*SLA3)*RV
RVG(3) = (-SNU2*STEM*CLA3+CTEM*CNU2)*RV

CALL BFIELD(RVG, START, BMAG)

BMAGT=SQRT(BMAG(1)**2+BMAG(2)**2+BMAG(3)**2)

BEG1=AEG11*BEG1+AEG12*BEG2+AEG13*BEG3
BEG2=AEG21*BEG1+AEG22*BEG2+AEG23*BEG3
BEG3=AEG31*BEG1+AEG32*BEG2+AEG33*BEG3

BU=OE11*BEG1+OE12*BEG2+OE13*BEG3
BV=OE21*BEG1+OE22*BEG2+OE23*BEG3
BW=OE31*BEG1+OE32*BEG2+OE33*BEG3

C IFLGMG=0 GIVES CONSTANT EARTH MAG FIELD IN LOCAL VERTICAL AXES
C IFLGMG=1 GIVES VARIABLE EARTH MAG FIELD IN LOCAL VERTICAL AXES------

IF(IFLGMG.EQ.1)GO TO 15

B=0.3
BEL=RFD*30.0
BAZ=RFD*30.0
BU=-B*COS(BEL)*SIN(BAZ)
BV=B*COS(BEL)*COS(BAZ)
BW=B*SIN(BEL)

CONTINUE

B=SQRT(BU**2+BV**2+BW**2)

BP=SQRT(BU**2+BV**2)

BUN=-BU
BEL=ATAN2(BW,BP)
BAZ=ATAN2(BUN,BV)

C DETERMINE EULER ANGLES RELATING SATELLITE AXES TO LOCAL VERTICAL AXES-----------------------

THSR=WSR*T
THP=WP*T+PHIP
A33=A3B3
ASR2=ASR2M*SIN(THSR)
ASR1=ASR1M*COS(THSR)
AP2=AP2M*SIN(THP)
AP1=AP1M*SIN(THP)
\[ AS_2 = ASB_2 + ASR_2 + AP_2 \]
\[ AS_1 = ASB_1 + ASR_1 + AP_1 \]

**C DETERMINE MAGNETOMETER OUTPUTS IN SATELLITE AXES**

\[ S_1 = \sin(AS_1) \]
\[ C_1 = \cos(AS_1) \]
\[ S_2 = \sin(AS_2) \]
\[ C_2 = \cos(AS_2) \]
\[ S_3 = \sin(AS_3) \]
\[ C_3 = \cos(AS_3) \]
\[ T_{11} = C_2 \cdot C_3 \]
\[ T_{12} = -S_2 \]
\[ T_{13} = S_1 \]
\[ T_{21} = S_1 \cdot S_2 \cdot C_3 - C_1 \cdot S_3 \]
\[ T_{22} = S_1 \cdot S_2 \cdot S_3 + C_1 \cdot C_3 \]
\[ T_{23} = S_1 \cdot C_2 \]
\[ T_{31} = C_1 \cdot S_2 \cdot C_3 + S_1 \cdot S_3 \]
\[ T_{32} = C_1 \cdot S_2 \cdot S_3 - S_1 \cdot C_3 \]
\[ T_{33} = C_1 \cdot C_2 \]

\[ B_{S1} = T_{11} \cdot BU + T_{12} \cdot BV + T_{13} \cdot BW \]
\[ B_{S2} = T_{21} \cdot BU + T_{22} \cdot BV + T_{23} \cdot BW \]
\[ B_{S3} = T_{31} \cdot BU + T_{32} \cdot BV + T_{33} \cdot BW \]

**C MAGNETOMETER OUTPUT QUANTIZATION**

CALL QUANT(IQ, IQFST1, NBITS, BSMAX, BS1, BS1Q)
CALL QUANT(IQ, IQFST2, NBITS, BSMAX, BS2, BS2Q)
CALL QUANT(IQ, IQFST3, NBITS, BSMAX, BS3, BS3Q)

\[ B_{SI} = BS1Q - BS1 \]
\[ B_{S2} = BS2Q - BS2 \]
\[ B_{S3} = BS3Q - BS3 \]

**C COMPUTE INPUTS TO FILTERS**

\[ BH = (1.0 + BHEF) \cdot B \]
\[ BELH = BEL + BELHE \]
\[ BAZH = BAZ + BAZHE \]
\[ SBELH = \sin(BELH) \]
\[ CBELH = \cos(BELH) \]
\[ SBAZH = \sin(BAZH) \]
\[ CBAZH = \cos(BAZH) \]
\[ BSOP = (BS1Q \cdot CBAZH + BS2 \cdot SBAZH) / (BH \cdot CBELH) \]
\[ BS1P = (BS1Q \cdot BH \cdot CBELH \cdot SBAZH) / BH \]
\[ BS2P = (BS2Q - BH \cdot CBELH \cdot CBAZH) / BH \]
\[ BS3P = (BS3Q - BH \cdot SBELH) / (BH \cdot CBELH) \]

**C SPECIFY BETTER INITIAL CONDITIONS FOR FILTER OUTPUTS AT T=0.0----------**

IF(T.GT.0.0) GO TO 40

\[ BSOPF = BSOP \]
\[ BS1PF = BS1P \]
\[ BS2PF = BS2P \]
\[ BS3PF = BS3P \]

40 CONTINUE

**C TEST FOR TIME TO STOP OR STORE DATA IN ARRAYS FOR PLOTTING LATER------**

IF(T.GT.TMAX) GO TO 100

20 IF(NP.LT.NPMAX) GO TO 30

BELU = DFR*BEL
BAZU = DFR*BAZ
BELHU = DFR*BELH
BAZHU = DFR*BAZH
AS3H = BSOPF
BS1CH = (BS1PFD/WNF)/GM
BS2CH = (BS2PFD/WNF)/GM
BS3CH = (BS3PFD/WNF)/GM
AA(01) = T
AA(02) = B
AA(03) = BELU
AA(04) = BAZU
AA(05) = BH
AA(06) = BELHU
AA(07) = BAZHU
AA(08) = BS1
AA(09) = BS2
AA(10) = BS3
AA(11) = BS1Q
AA(12) = BS2Q
AA(13) = BS3Q
AA(14) = BS1QE
AA(15) = BS2QE
AA(16) = BS3QE
AA(17) = BSOP
AA(18) = BS1P
AA(19) = BS2P
AA(20) = BS3P
AA(21) = BSOPF
AA(22) = BS1PF
AA(23) = BS2PF
AA(24) = BS3PF
AA(25) = BSOPFD
AA(26) = BS1PF
AA(27) = BS2PF
AA(28) = BS3PF
AA(29) = BS1CH
AA(30) = BS2CH
AA(31) = BS3CH
AA(32) = AS3H
AA(33) = ASR1
AA(34) = ASR2
AA(35) = AP1
AA(36) = AP2
AA(37) = AS1
AA(38) = AS2

WRITE (7) AA
NP = 0
30 CONTINUE
C UPDATE FILTER OUTPUTS, ORBIT/EARTH ANGLES, TIME, AND COUNTER FOR
C STORING DATA TO PLOT-----------------------------------------------------
CALL LPF2 (DT, WNB, ZETAB, BSOP, BSOPF, BSOPFD)
CALL LPF2 (DT, WNF, ZETA, BS1P, BS1PF, BS1PF)
CALL LPF2 (DT, WNF, ZETA, BS2P, BS2PF, BS2PF)
CALL LPF2 (DT, WNF, ZETA, BS3P, BS3PF, BS3PF)
GA2 = GA2 + OMG*DT
LA2 = LA2 + OMA*DT
NU2 = NU2 + OMO*DT
IF (NU2.GT.TWOPI) NU2 = NU2 - TWOPI
T = T + DT
NP=NP+1
GO TO 10
100 CONTINUE
STOP
END

SUBROUTINE LPF2(DT,WN,ZETA,U,X1,X2)
C THIS SUBROUTINE UPDATES STATES FOR 2ND ORDER LOW PASS FILTER USING
C 4TH ORDER RUNGA-KUTTA-----------------------------------------------
X1P=X1
X2P=X2
SUM1=0.0
SUM2=0.0
DO 5 I=1,4
WFACT=1.0
DTP=DT/2.0
IF (I.EQ.2.OR.I.EQ.3) WFACT=2.0
IF (I.EQ.3) DTP=DT
X1D=X2P
X2D=WN*(WN*(U-X1P)-2.0*ZETA*X2P)
SUM1=SUM1+WFACT*X1D
SUM2=SUM2+WFACT*X2D
X1P=X1+DTP*X1D
X2P=X2+DTP*X2D
5 CONTINUE
X1=X1+SUM1*DT/6.0
X2=X2+SUM2*DT/6.0
RETURN
END

SUBROUTINE BFIELD(RV,START,BMAG)
C THIS IS 6-DISPLACED-DIPOLE MODEL OF THE EARTH'S MAG FIELD
C FLUX DENSITY IS GIVEN IN GAUSS-------------------------------------
DIMENSION RV(3),RO(12,3),R(3),BMAG(3),EM(12,3)
IF (START.NE.1.) GO TO 4
R0(1,1)=500.805
R0(2,1)=-292.9318
R0(3,1)=455.075
R0(4,1)=414.636
R0(5,1)=-1611.292
R0(6,1)=14.958
C
R0(1,2)=146.800
R0(2,2)=-818.6409
R0(3,2)=-902.802
R0(4,2)=1540.2948
R0(5,2)=1095.094
R0(6,2)=1243.2935
C
R0(1,3)=718.994
R0(2,3)=583.55
R0(3,3)=791.483
R0(4,3)=-65.878
R0(5,3)=-282.390
R0(6,3)=-849.084
C
EM(1,1)=1.423489E6
EM (2,1) = 4.558039E6
EM (3,1) = -7.045997E6
EM (4,1) = -5.55610E6
EM (5,1) = 1.54356E6
EM (6,1) = 2.920314E6

EM (1,2) = -18.247848E6
EM (2,2) = 3.575713E6
EM (3,2) = 3.744393E6
EM (4,2) = -6.96502E6
EM (5,2) = -5.93411E6
EM (6,2) = 4.470056E6

EM (1,3) = 2.104301E6
EM (2,3) = -3.079872E6
EM (3,3) = -4.175109E6
EM (4,3) = 6.801980E6
EM (5,3) = -8.56652E6
EM (6,3) = -1.303223E6

START = 0.0

CONVERT FLUX DENSITY FROM WEBERS/(METERS**2) TO GAUSS

END

SUBROUTINE QUANT(IQ, IQFST,NBITS,XMAX,X,XQ)

IF (IQFST.NE.1) GO TO 10
XLSB = (2.0 * XMAX) / (2.0 ** NBITS)
XMAXP = XMAX - XLSB
XMIN = -XMAX
PRINT *, NBITS, XMAX, XLSB, XMAXP, XMIN
IQFST = 0
10 CONTINUE
XQ = X
IF (IQ.EQ.0) GO TO 20
XDUM = XQ
IF (XDUM.LT.XMIN) XDUM = XMIN
IF (XDUM.GT.XMAXP) XDUM = XMAXP
XDUM = XDUM + XMAXP + XLSB
XDUM = XDUM/XLSB
1 XDUM = XDUM
XDUM = XDUM/XLSB
XDUM = XDUM * XLSB
XQ=XDUM-XMAX
20 CONTINUE
RETURN
END
A Scheme for Bandpass Filtering Magnetometer Measurements To Reconstruct Tethered Satellite Skiprope Motion

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This paper presents a unique scheme for reconstructing tethered satellite skiprope motion by ground processing of satellite magnetometer measurements. The measurements are modified based on ground knowledge of the Earth's magnetic field and passed through bandpass filters tuned to the skiprope frequency. Simulation results are presented which verify the scheme and show it to be quite robust. The concept is not just limited to tethered satellites. Indeed, it can be applied wherever there is a need to reconstruct the coning motion of a body about a known axis, given measurements of a known vector in body-fixed axes.