GRID3D-v2: An Updated Version of the GRID2D/3D Computer Program for Generating Grid Systems in Complex-Shaped Three-Dimensional Spatial Domains

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GRID3D-v2: An Updated Version of the GRID3D Computer Program for Generating Grid Systems in Complex-Shaped Three-Dimensional Spatial Domains

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1.0 INTRODUCTION

GRID2D/3D (refs. 1 and 2) is a powerful grid-generation package, capable of generating grid systems for complicated geometries in both two and three dimensions. This package, which employs algebraic grid-generation techniques, is computationally efficient and easy to use. Nonetheless, when the geometry is unusually complex (e.g., see fig. 1-1), the partitioning of the geometry into zones or blocks that are suitable for GRID2D/3D becomes very cumbersome. In order to make GRID2D/3D more versatile and readily applicable to geometries like the one shown in figure 1-1, a number of modifications were made to the package. These modifications are as follows:

(1) specification of boundary curves has been made more flexible;
(2) control over grid-point distribution has been increased;
(3) new interpolating functions based on tension splines have been added; and
(4) control over orthogonality at boundary surfaces has been increased.
GRID2D/3D is made up of two programs, GRID2D and GRID3D. GRID2D generates grid systems for two-dimensional (2-D) spatial domains, and GRID3D generates grid systems for three-dimensional (3-D) domains. The aforementioned modifications were made to GRID3D only. In this report, the original version of GRID3D as reported in references 1 and 2 will be referred to as GRID3D-v1. The new modified version will be referred to as GRID3D-v2.

In the remainder of this report, the theory and method behind the modifications are described, and the use of GRID3D-v2 is explained and illustrated by an example.
2.0 THEORY AND METHODOLOGY

In this section, the theory and methodology behind the modifications that were incorporated into GRID3D-v2 are described. First, specification of boundary curves is discussed, and a new approach for controlling distribution of grid points is explained. Next, the functional forms of new tension-spline based connecting curves are derived, and the properties of these functions examined. Finally, control of orthogonality of grid lines at boundaries is discussed.

Note that throughout this section, the reader is assumed to be familiar with the theory behind GRID3D-v1 which is described in reference 1.

2.1 Boundary Curves and Distribution of Grid Points

When generating 3-D grid systems with GRID3D-v1 or GRID3D-v2, we assume that the geometry of the spatial domain for which a grid is to be generated is completely described by the edge curves of the boundary surfaces (as used here, the edge curves are the four plane or twisted curves that define the boundary of a surface). The algorithms used in GRID3D-v1 and GRID3D-v2 were designed to construct grid systems from the edge curves; however, these algorithms differ in the way they specify edge curves and control grid-point distributions. This difference is described in this section.

In GRID3D-v1, the following approach is used:

(1) Each edge curve is given by specifying a set of node points that lie on the curve. From these node points, a parametric description of the curve is constructed by using tension-spline interpolation.

(2) The distribution of grid points within the domain (including edge curves) is controlled by specifying three one-dimensional stretching functions -- one for each family of grid lines (i.e., one for $\xi$-grid lines, one for $\eta$-grid lines, and one for $\zeta$-grid lines).

Although this approach provides flexibility in generating grid systems within complex-shaped spatial domains, it is inadequate in some cases. For example, it does not allow for edge curves to have derivative discontinuities such as those at cusps. Also, it does not provide adequate control over distribution of grid points in regions where geometry changes appreciably. As a specific example, the distribution of grid points on all constant-$\xi$ surfaces must be the same and cannot, even with partitioning, be made to vary from one section of the grid system to the next.
To overcome the shortcomings of GRID3D-v1, the following approach was used in GRID3D-v2:

1. Each edge curve is defined by specifying the location of the grid points on the curve. This can be done by either one of the following two procedures: (a) Specify a set of node points that lie on the edge curve for interpolation with a tension spline, and specify a stretching function that controls where on the edge curve the grid points lie. (b) Specify the grid points directly (in this case, a stretching function is not specified by the user, rather it is calculated based on arc length as will be shown herein).

2. The grid point distribution along grid lines of a given family is that obtained by the bilinear interpolation of the stretching functions used for the edge curves belonging to that family.

The specification of edge curves as described under (1(a)) is self explanatory, but the calculation of stretching functions as described under (1(b)) and (2) requires further explanation. For illustration, consider the $\zeta$-family of grid lines. Suppose all edge curves belonging to this family of grid lines are specified by using procedure (1(a)), and suppose the stretching functions $\xi_{00}(\zeta)$, $\xi_{10}(\zeta)$, $\xi_{01}(\zeta)$ and $\xi_{11}(\zeta)$ describe the distribution of the grid points on the edge curves located at $(\zeta = 0, \eta = 0)$, $(\zeta = 1, \eta = 0)$, $(\zeta = 0, \eta = 1)$ and $(\zeta = 1, \eta = 1)$, respectively. In GRID3D-v2, the stretching function for a $\zeta$-grid line at any $\zeta$-$\eta$ location is given by the bilinear interpolation of the stretching functions at the edge curves; namely

$$\hat{\xi}_{\zeta\eta}(\zeta) = [\xi_{00}(\zeta) (1-\zeta) + \xi_{10}(\zeta) \zeta] (1-\eta) + [\xi_{01}(\zeta) (1-\zeta) + \xi_{11}(\zeta) \zeta] \eta \quad (2.1)$$

Now, instead of all edge curves being defined by (1(a)), suppose that the edge curve at $\zeta = 0$ and $\eta = 0$ is defined by using procedure (1(b)); that is, by specifying the grid point coordinates directly. In order to use equation (2.1) for this case, a stretching function must be calculated for the edge curve. Since the stretching function is needed only at the grid point-locations along the edge curve, it can be calculated by using approximate arc length as follows:

$$\hat{\xi}_{00}(\zeta_k) = 0 \quad k=1 \quad (2.2a)$$

and

$$\hat{\xi}_{00}(\zeta_k) = \frac{d_k}{d_{KL}} \quad k=2,3,4,...,KL \quad (2.2b)$$

where

$$d_k = \sum_{n=2}^{k} [(x_n-x_{n-1})^2 + (y_n-y_{n-1})^2 + (z_n-z_{n-1})^2]^{1/2} \quad (2.2c)$$

and
\[ \zeta_k = (k-1) \Delta \zeta \quad \Delta \zeta = 1/(KL-1) \]  

In equation (2.2), \( k = 1,2,3,\ldots, KL \) denotes the grid points on the edge curve; \( KL \) is the total number of grid points on the curve; and \( x_n, y_n \) and \( z_n \) are the \( x-, y- \) and \( z \)-coordinates of the \( n \)-th grid point on the curve.

The same approach as that just described for the \( \zeta \)-family of grid lines is employed to determine the stretching functions for the \( \xi \)- and \( \eta \)-families of grid lines. This approach gives a smooth distribution of grid points throughout the domain.

Finally, note that all stretching functions available in GRID3D-v1 for controlling the distribution of grid points in the entire domain are available in GRID3D-v2 for controlling the distribution of grid points along edge curves defined by using procedure (1(a)) (i.e., by specifying a set of node points that lie on the edge curve and interpolating with a tension spline). In GRID3D-v2, a stretching function that allows asymmetric clustering of grid points along the edge curve was added. The new stretching function, which was developed by Vinokur (ref. 3), is given here.

Let \( t \in [0,1] \) be normalized distance or any monotonic parameter along a curve, and let \( \xi \in [0,1] \) be the computational coordinate along which grid points are equally spaced. Two user controlled parameters, \( s_0 \) and \( s_1 \), and two secondary parameters, \( A \) and \( B \), are defined as

\[ s_0 = \frac{d\xi(t = 0)}{dt} \quad \text{and} \quad s_1 = \frac{d\xi(t = 1)}{dt} \quad s_0, s_1 > 0 \]  

\[ A = \sqrt{s_0/s_1} \quad \text{and} \quad B = \sqrt{s_0 \cdot s_1} \]  

In terms of these parameters, the functional form of the stretching function can be written as

\[ t(\xi) = \frac{u(\xi)}{A + (1-A) u(\xi)} \]  

where the function \( u(\xi) \) depends on the value of the parameter \( B \), as shown in the following equations.

If \( B > 1.001 \), then

\[ u(\xi) = \frac{1}{2} \cdot \frac{\tanh(\Delta y (\xi - \frac{1}{2}))}{\tanh(\Delta y/2)} \]  

(2.6a)
where $\Delta y$ is obtained from the relation

$$B = \frac{\sinh(\Delta y)}{\Delta y} \quad (2.6b)$$

If $B < 0.999$, then

$$u(\xi) = \frac{1}{2} + \frac{\tan[\Delta x (\xi - \frac{1}{2})]}{2 \tan[\Delta x/2]} \quad (2.7a)$$

where $\Delta x$ is obtained from the relation

$$B = \frac{\sin(\Delta x)}{\Delta x} \quad (2.7b)$$

Finally, if $0.999 \leq B \leq 1.001$, then

$$u(\xi) = \xi \left[ 1 + 2 (B - 1) (\xi - \frac{1}{2}) (1 - \xi) \right] \quad (2.8)$$

The amount of clustering produced by the stretching function is controlled by the parameters $s_0$ and $s_1$ which are defined by equation (2.3). If $s_0$ and $s_1$ are greater than one, then grid points are clustered near the boundaries where $t = 0$ and $t = 1$. The greater $s_0$ and $s_1$, the greater is the clustering of grid points near the $t = 0$ and $t = 1$ boundaries, respectively. If $s_0$ and $s_1$ are less than one, then the grid spacing is larger near the boundaries than in the interior; the smaller $s_0$ and $s_1$, the greater the grid spacing near the boundaries.

Frequently, when using the stretching function given by equations (2.3) to (2.8), we must either solve equation (2.6(b)) for $\Delta y$, or solve equation (2.7(b)) for $\Delta x$. Vinokur (ref. 3) developed approximate analytical relations for both of these inversion problems as follows:

For equation (2.6(b)), which is used when $B > 1.001$, the approximate inverse when $1.001 < B < 2.7829681$ is

$$\Delta y = \sqrt{6\beta} \left( 1 - 0.15\beta + 0.057321429\beta^2 - 0.024907295\beta^3 \right.
+ 0.0077424461\beta^4 - 0.0010794123\beta^5 \left.) \quad (2.9a) \right)$$

where

$$\beta = B - 1 \quad (2.9b)$$
When $B > 2.7829681$,

$$\Delta y = v + (1 + 1/v) \ln(2v) - 0.02041793 + 0.24902722w + 1.9496443w^2 - 2.629447w^3 + 8.56795911w^4$$ \hspace{1cm} (2.10a)

where

$$v = \ln(B)$$ \hspace{1cm} (2.10b)

and

$$w = (1/B) - 0.028527431$$ \hspace{1cm} (2.10c)

For equation (2.7(b)), which is used when $B < 0.999$, the approximate inverse when $0 < B < 0.26938972$ is

$$\Delta x = \pi \left[1 - B + B^2 - (1 + \frac{\pi^2}{6}) B^3 + 6.794732B^4 - 13.205501B^5 + 11.726095B^6\right]$$ \hspace{1cm} (2.11)

When $0.26938972 < B < 0.999$,

$$\Delta x = \sqrt{6} \beta \left(1 + 0.15 \beta + 0.057321429\beta^2 + 0.048774238\beta^3 - 0.053337753\beta^4 + 0.075845134\beta^5\right)$$ \hspace{1cm} (2.12a)

where

$$\beta = 1 - B$$ \hspace{1cm} (2.12b)

### 2.2 Connecting Curves Based on Tension Spline Interpolation

Experience has shown that Hermite interpolation (cubic polynomials) as used in GRID3D-v1 sometimes results in connecting curves with too much curvature. In GRID3D-v2, the Hermite interpolation is replaced by tension-spline interpolation. The most attractive feature of tension-spline interpolation is that as the tension parameter is increased from zero to infinity, the interpolation function varies from being a cubic polynomial to being a linear polynomial. Thus, tension-spline interpolation offers increased control over the shape of the grid lines in the grid system. In this section, a derivation of the tension-spline interpolation function is given for the two- and four-
boundary methods. First, the interpolation for an arbitrary variable is derived. Then, the application
to algebraic grid generation is illustrated.

The tension-spline interpolation function is derived as follows: suppose the variable X is a
function of the parameter s on an interval [0,1], but only X(0), X(1), X'(0) and X'(1) (X' denotes
dX/ds) are known. A tension-spline interpolation of X(s) on the interval [0,1] is sought. A tension-
spline interpolation of X(s) is traditionally written in terms of X(0), X(1), X''(0) and X''(1), where
X'' = d^2X/ds^2, as follows (see, e.g., ref. 4):

\[ X(s) = \frac{X''(0) \sinh[\sigma(1-s)]}{\sigma^2 \sinh[\sigma]} \left( X(0) - \frac{X''(0)}{\sigma^2} \right)(1-s) \]
\[ + \frac{X''(1) \sinh[\sigma s]}{\sigma^2 \sinh[\sigma]} \left( X(1) - \frac{X''(1)}{\sigma^2} \right)s \]

where \( \sigma \) is the tension parameter. By differentiating equation (2.13) and evaluating the resulting
equation at the end points \( s = 0 \) and \( s = 1 \), we obtain

\[ X'(0) = \frac{X''(0) \cosh[\sigma]}{\sigma \sinh[\sigma]} - \left( X(0) - \frac{X''(0)}{\sigma^2} \right) + \frac{X''(1) \cosh[\sigma]}{\sigma \sinh[\sigma]} + \left( X(1) - \frac{X''(1)}{\sigma^2} \right) \]

and

\[ X'(1) = \frac{X''(0) \cosh[\sigma]}{\sigma \sinh[\sigma]} - \left( X(0) - \frac{X''(0)}{\sigma^2} \right) + \frac{X''(1) \cosh[\sigma]}{\sigma \sinh[\sigma]} + \left( X(1) - \frac{X''(1)}{\sigma^2} \right) \]

The above two simultaneous equations can be solved to give expressions for \( X''(0) \) and \( X''(1) \) in terms
of \( X(0), X(1), X'(0) \) and \( X'(1) \). Substituting the resulting expressions into equation (2.13) gives

\[ X(s) = X(0)h_1(s) + X(1)h_2(s) + X'(0)h_3(s) + X'(1)h_4(s) \]

\[ h_1(s) = c_1(1-s) + c_2s + c_2 \left( \frac{\sinh[\sigma(1-s)] - \sinh[\sigma s]}{\sinh[\sigma]} \right) \]

\[ h_2(s) = c_1s + c_2(1-s) - c_2 \left( \frac{\sinh[\sigma(1-s)] - \sinh[\sigma s]}{\sinh[\sigma]} \right) \]
\[ h_3(s) = c_3 \left( (1-s) - \frac{\sinh[\sigma(1-s)]}{\sinh[\sigma]} \right) + c_4 \left( s - \frac{\sinh[\sigma s]}{\sinh[\sigma]} \right) \]  

\[ h_4(s) = -c_4 \left( (1-s) - \frac{\sinh[\sigma(1-s)]}{\sinh[\sigma]} \right) - c_3 \left( s - \frac{\sinh[\sigma s]}{\sinh[\sigma]} \right) \]

where

\[ c_1 = 1 - c_2 \]  

\[ c_2 = \frac{\sinh[\sigma]}{2 \sinh[\sigma] - \sigma \cosh[\sigma] - \sigma} \]  

\[ c_3 = -\frac{\alpha}{(\beta^2 - \alpha^2)} \sinh[\sigma] \]  

\[ c_4 = -\frac{\beta}{(\beta^2 - \alpha^2)} \sinh[\sigma] \]

Equations (2.15) to (2.17) can be used to interpolate any function on an interval \([0,1]\), when the function's values and its first derivatives are known at the end points of the interval. The application of these equations to algebraic grid generation is straightforward. Consider, for example, the two-boundary technique (ref. 1). Suppose we want to generate a grid system between two constant-\(\eta\) boundary surfaces. The formulation in this case can be written as follows:

\[ r(\xi, \eta, \zeta) = r(\xi, \eta = 0, \zeta) \cdot h_1(\eta) + r(\xi, \eta = 1, \zeta) \cdot h_2(\eta) + \frac{\partial r(\xi, \eta = 0, \zeta)}{\partial \eta} \cdot h_3(\eta) + \frac{\partial r(\xi, \eta = 1, \zeta)}{\partial \eta} \cdot h_4(\eta) \]

where
The above equation has the same form as if Hermite interpolation were used, except for the definition of the functions \( h_1, h_2, h_3, \) and \( h_4 \), which in the case of Hermite interpolation are cubic polynomials (ref. 1). Note, however, that the functions used in tension-spline interpolation (eqs. (2.16) and (2.17)) have the property that as \( \sigma \to 0 \), \( h_1, h_2, h_3, \) and \( h_4 \) approach the cubic polynomials used in Hermite interpolation (see ref. 1). Also, as \( \sigma \to 0 \), \( h_1(s) \to (1-s), h_2(s) \to s, h_3(s) \to 0, \) and \( h_4(s) \to 0 \), giving rise to linear connecting functions (Lagrange interpolation; see ref. 1).

### 2.3 Specification of Derivatives at Boundaries

When the two- and four-boundary methods, as described in reference 1, are used to generate grid systems, the first-order derivatives involved (e.g., \( \partial r(\xi,\eta = 0,\zeta)/\partial \eta \) and \( \partial r(\xi,\eta = 1,\zeta)/\partial \eta \) in equation (2.18)) need to be specified. In GRID3D-v1 and GRID3D-v2, these derivatives are specified such that grid lines intersect boundary surfaces orthogonally. In this section, the methods used in GRID3D-v1 and GRID3D-v2 to calculate these derivatives will be explained. The two-boundary technique given by equation (2.18) will be used to illustrate the concepts.

In GRID3D-v1, the derivatives \( \partial r(\xi,\eta = 0,\zeta)/\partial \eta \) and \( \partial r(\xi,\eta = 1,\zeta)/\partial \eta \) (eq. (2.18)) are chosen as follows:

\[
\frac{\partial r(\xi,\eta = 0,\zeta)}{\partial \eta} = K_{\eta 1}(\xi,\zeta) \, t_{\eta 1} \quad \frac{\partial r(\xi,\eta = 1,\zeta)}{\partial \eta} = K_{\eta 2}(\xi,\zeta) \, t_{\eta 2} \tag{2.20}
\]

where

\[
t_{\eta 1} = - \frac{\partial r(\xi,\eta = 0,\zeta)}{\partial \xi} \times \frac{\partial r(\xi,\eta = 0,\zeta)}{\partial \zeta} \tag{2.21a}
\]

\[
t_{\eta 2} = - \frac{\partial r(\xi,\eta = 1,\zeta)}{\partial \xi} \times \frac{\partial r(\xi,\eta = 1,\zeta)}{\partial \zeta} \tag{2.21b}
\]

In equation (2.20), the terms \( K_{\eta 1}(\xi,\zeta) \) and \( K_{\eta 2}(\xi,\zeta) \) -- that is, the K-factors -- are specified by the user and are intended to control the magnitude of the derivatives. However, the magnitude of the
derivatives will also depend on the magnitude of the vectors \( t_{\eta 1} \) and \( t_{\eta 2} \), which in turn depend both on the geometry of the boundary surfaces at \( \eta = 0 \) and \( \eta = 1 \), respectively, and on the grid spacing on the surfaces. Thus, full control cannot be exerted over the magnitudes of the derivative terms \( \frac{\partial r(\xi, \eta = 0, \zeta)}{\partial \eta} \) and \( \frac{\partial r(\xi, \eta = 1, \zeta)}{\partial \eta} \), and for complex-shaped geometries this may pose a problem.

In order to overcome the aforementioned problems, in GRID3D-v2 the derivative terms \( \frac{\partial r(\xi, \eta = 0, \zeta)}{\partial \eta} \) and \( \frac{\partial r(\xi, \eta = 1, \zeta)}{\partial \eta} \) were defined as follows:

\[
\frac{\partial r(\xi, \eta = 0, \zeta)}{\partial \eta} = K_{\eta 1}(\xi, \zeta) \, e_{\eta 1} \quad \text{and} \quad \frac{\partial r(\xi, \eta = 1, \zeta)}{\partial \eta} = K_{\eta 2}(\xi, \zeta) \, e_{\eta 2}
\]  

(2.22)

where \( e_{\eta 1} \) and \( e_{\eta 2} \) are unit vectors that are normal to the boundary surfaces at \( \eta = 0 \) and \( \eta = 1 \), respectively; that is

\[
e_{\eta 1} = \frac{t_{\eta 1}}{|t_{\eta 1}|} \quad \text{and} \quad e_{\eta 2} = \frac{t_{\eta 2}}{|t_{\eta 2}|}
\]  

(2.23)

This approach allows total control over the magnitude of the derivatives \( \frac{\partial r(\xi, \eta = 0, \zeta)}{\partial \eta} \) and \( \frac{\partial r(\xi, \eta = 1, \zeta)}{\partial \eta} \) through the K-factors alone.

Finally, note that in GRID3D-v1, the K-factors were taken to be constants on each boundary surface, even though the authors realized that they could be allowed to vary. In GRID3D-v2, the K-factors are allowed to vary from point to point and can be controlled by the user.

In the next section, we show how GRID3D-v2 is used to generate grid systems. Several auxiliary programs were written to assist grid generation with GRID3D-v2. A description of these programs is given in appendix A. A complete listing of the GRID3D-v2 computer program is given in appendix B.
3.0 USING GRID3D-v2

In this section, the use of GRID3D-v2 is described. The section consists of two parts: first, an explanation of generating 3-D grid system with GRID3D-v2; and second, an example of such a grid system generated by using GRID3D-v2.

3.1 Generating a Three-Dimensional Grid System with GRID3D-v2

When using GRID3D-v2, the user must answer the following questions:

1) Should the two boundary technique or the four boundary technique be used?
2) Should any edge curve be defined by specifying the grid points directly?
3) How many grid points are desired in each of the \( \xi, \eta, \) and \( \zeta \) directions?
4) Is any clustering of grid points needed?
5) What K-factors should be used (see section 2.3 and pp. 18 to 20 in ref. 1) and are constant K-factors sufficient?

Once these questions have been answered, an input file for GRID3D-v2 must be constructed. The form of the input file is shown in figure 3-1. The various parameters in the input file are explained in table 3.1. The surface and edge curve numbering scheme embedded in GRID3D-v2 is shown in figure 3-2. Note that some of the edge curves are identical; that is, curves 3 and 9 are identical and so are curves 4 and 13, curves 7 and 10, and curves 8 and 14 (for further explanation of the edge curve numbering scheme, see p. 5 of ref. 2)

In the input file are values for K-factors at boundary surfaces -- a single value is assigned for all grid points on each surface. The user can modify the K-factor for any individual grid point or groups of grid points on each surface by adding FORTRAN statements into subroutine KFACTOR (see listing in Appendix B). Note that in addition to being used to generate grid points in the interior of the spatial domain, K-factors are also used to generate grid points on boundary surfaces themselves. For this latter case, the relevant K-factors are those at grid points that lie on the four edge curves bounding the surface. These K-factors can also be modified in subroutine KFACTOR.

Once an input file has been prepared (and an executable file created for GRID3D-v2 if subroutine KFACTOR was modified), GRID3D-v2 can be executed. Note, GRID3D-v2 reads the input file from unit 7 and writes output to unit 8.
Grid generation is an iterative process; that is, an acceptable grid system is generated by trial and error after generating a series of unacceptable grid systems. Some observations and rules of thumb that might be useful when generating grid systems with GRID3D are as follows:

1. If it is necessary to partition a spatial domain, then select partitioning surfaces that intersect boundary surfaces as orthogonally as possible. This approach minimizes skewness both at boundaries and in the interior of the domain.

2. A common boundary between two partitions should be specified in exactly the same way in the input files in order to guarantee a continuous grid across the common boundary.

3. For improved flexibility in modifying a grid system, the boundary curves should be defined by using node points for tension-spline interpolation and a stretching function (see Section 2.1), rather than specifying grid points directly.

4. When generating a grid system for a complicated geometry, first find stretching functions and K-factors for edge curves that give the desired distribution of grid points on the boundary surfaces. Afterwards, try to optimize grid-point distribution in the interior by modifying K-factors on boundary surfaces and/or the amount of tension in the connecting curves. If this two-step process does not yield an acceptable grid system, then try to modify grid-point distribution on boundary surfaces and/or re-partition the domain.

5. Start with low tension and low K-factors. Slowly increase K-factors to improve orthogonality at boundaries and eliminate overlapping grid lines. Increasing the amount of tension in the connecting curves also can eliminate overlapping grid lines by straightening grid lines.

6. Increased tension tends to straighten out grid lines whereas increased K-factors tend to increase the curvature of grid lines.

7. If K-factors are too high, then grid lines can overlap.

8. K-factors affect the grid spacing near the boundary surfaces. Increasing the K-factor at a boundary increases the grid spacing adjacent to that boundary. This effect can be beneficial in some instances but detrimental in others.

9. K-factors should vary smoothly from grid line to grid line. An exception to this general rule is when a grid line intersects a boundary surface at a cusp in the surface.

3.2 Example: A Spatial Domain With Irregular Boundaries

Figure 1-1 shows a cooling passage in a radial turbine blade. Figure 3-3 shows how this cooling passage was partitioned into blocks or zones for the purpose of grid generation. The partitioning that is shown was deemed necessary in order to get an acceptable grid system.
Figure 3-4 shows the grid system for partition number 18. The input file for this partition is given in table 3-2. The entire grid system generated by using GRID3D-v2 is shown in figures 3-5 and 3-6. The plotting in figures 3-4 to 3-6 were obtained by using 3DSURF -- the plotting package supplied with GRID2D/3D.

4.0 SUMMARY

A new version of the grid generation program GRID3D, which is a part of the grid generation package GRID2D/3D, has been developed. The new program is referred to as GRID3D-v2. This report describes GRID3D-v2 and how to use it. The capability of the program was demonstrated by generating a grid system for a very complicated geometry, namely a cooling passage inside a radial turbine blade.
Appendix A -- Support Programs

A.1 Description of Support Programs

To support the task of grid generation with GRID3D-v2, several auxiliary programs have been developed. Two of these programs, namely PRSURF and 3DSURF, were developed to allow the user to view grid systems generated with the package. Three more programs, namely 3DPREP, EDGE, and EDGPREP, were developed to aid in the preparation of input files for GRID3D-v2. Finally, one program, GRIDTST, was written to test the grid system for problems such as overlapping grid lines. Each of these programs and their use will be described in the following pages.

3DSURF

The program 3DSURF was developed alongside the original version of GRID2D/3D to allow the user to plot grid systems on the computer screen. It was designed for IBM PC, XT and AT computer systems and compatibles. Two-dimensional grid output files from GRID2D/3D are already in the proper format for use with 3DSURF. Three-dimensional grid output files must, on the other hand, be processed using PRSURF (described next) to create input files for 3DSURF. A general description of 3DSURF and its use is given in Section 3.1 in reference 2.

PRSURF

The PRSURF program was written to process output files from GRID3D and create input files for 3DSURF. This program allows the user to select surfaces or parts of surfaces from the grid system (i.e., constant-ξ, constant-η, and constant-ζ surfaces) and to store them in a format compatible with 3DSURF. The program is interactive and self-explanatory.

PRSURF uses three files. The file containing the input to PRSURF (i.e., the output file from GRID3D) is read from unit 7. The output file from PRSURF (which becomes the input file for 3DSURF) is written to unit 9. Last, a file used for temporary storage of data is accessed as unit 8.
Finally, we mention that since 3DSURF is limited to handling surfaces that have 40 grid points per side or less, PRSURF automatically breaks any grid surface into sections that are 40 grid points by 40 grid points or smaller.

**3DPREP**

The user can apply 3DPREP to create input files for GRID3D-v2. The program prompts the user for all control parameters such as stretching functions and k-factors. It reads from files the coordinates of points defining the edge curves. These files must have the following format:

\[
\text{NP (number of points)}
\]

\[
\begin{align*}
  x_1, y_1, z_1 \\
  x_2, y_2, z_2 \\
  x_3, y_3, z_3 \\
  \vdots \\
  \vdots \\
  x_N, y_N, z_N
\end{align*}
\]

The program can read data points from these files in both forward and reverse order, and the user can let the program read data from several files (the whole file or only a part of the file) to put together a single edge curve. 3DPREP is useful, primarily, when many input files for GRID3D-v2 need to be created from the same set of data.

3DPREP is designed to run on the IBM PC, XT, AT, and compatibles, but it can easily be modified to run on other computer systems. The only modification that should be needed in such a case is the insertion of open statements into the program so that the user can interactively specify which files the program must access.

**EDGE**

The EDGE program was written to aid the user in generating grid points along an edge curve that cannot be represented by a single spline curve (e.g., edge curves possessing derivative discontinuities such as cusps) and which must, therefore, be defined in the input files for GRID3D-v2 by giving the grid point coordinates directly (see Section 2.1). The program was designed to
generate grid points on an edge curve that is composed of several sections, where each section is defined by a set of nodal points that are interpolated by a spline curve. The number of grid points on each section and their distribution within the section is controlled independently.

EDGE, which is designed to run on the IBM PC, XT, AT, and compatibles, reads input data from UNIT 1 and writes the output to UNIT 20. The input file must have the following format:

\[
\begin{align*}
\text{NS} & \quad \text{(number of sections)} \\
\text{Data for section} & \quad 1 \\
\text{Data for section} & \quad 2 \\
& \quad \vdots \\
\text{Data for section} & \quad \text{NS} \\
\end{align*}
\]

where the data for an arbitrary section number \( i \) are the following:

- \( IP_i \) (number of grid points on section \( i \ ))
- \( \sigma_i \) (tension parameter for the spline interpolation)
- \( NN_i \) (number of node points given on section \( i \ ))
- \( x_1, y_1, z_1 \)
- \( x_2, y_2, z_2 \)
- \( x_3, y_3, z_3 \)
- \( \ldots \)
- \( x_{NN_i}, y_{NN_i}, z_{NN_i} \)
- \( \text{StretchType}_i \)
- \( \text{Beta1}_i, \text{Beta2}_i \)

The meaning of the parameters \( \text{StretchType}_i, \text{Beta1}_i, \) and \( \text{Beta2}_i \) is explained in table 3-1. Note that in EDGE it is assumed that the curve being generated is continuous; that is, the last grid point on one
section must be the same as the first grid point on the next section. Thus the output from EDGE has the following format:

\[
\begin{align*}
&\text{IL} \quad \text{(number of grid points on the edge curve)} \\
&x_{1,1}, y_{1,1}, z_{1,1} \\
&x_{1,2}, y_{1,2}, z_{1,2} \\
&x_{1,3}, y_{1,3}, z_{1,3} \\
&\quad \ddots \quad \ddots \quad \ddots \\
&x_{1,IP_1-1}, y_{1,IP_1-1}, z_{1,IP_1-1} \\
&x_{2,1}, y_{2,1}, z_{2,1} \\
&x_{2,2}, y_{2,2}, z_{2,2} \\
&x_{2,3}, y_{2,3}, z_{2,3} \\
&\quad \ddots \quad \ddots \quad \ddots \\
&x_{2,IP_2-1}, y_{2,IP_2-1}, z_{2,IP_2-1} \\
&\quad \ddots \quad \ddots \quad \ddots \\
&x_{NS,1}, y_{NS,1}, z_{NS,1} \\
&x_{NS,2}, y_{NS,2}, z_{NS,2} \\
&x_{NS,3}, y_{NS,3}, z_{NS,3} \\
&\quad \ddots \quad \ddots \quad \ddots \\
&x_{NS,IP_NS-1}, y_{NS,IP_NS-1}, z_{NS,IP_NS-1} \\
&x_{NS,IP_NS}, y_{NS,IP_NS}, z_{NS,IP_NS}
\end{align*}
\]

where \( x_{i,j} \), \( y_{i,j} \), and \( z_{i,j} \) are the coordinates of grid point number \( j \) on section number \( i \). The total number of grid points is

\[
\text{IL} = 1 + \sum_{i=1}^{\text{NS}} (\text{IP}_i - 1)
\]
GRIDTST

The GRIDTST program is used to check grid systems of 3-D spatial domains for defects, such as overlapping grid lines, that result in a negative Jacobian, where the Jacobian is defined as follows (for further explanation of the Jacobian, see ref. 1):

\[ J = x_\xi(y_\eta z_\zeta - y_\zeta z_\eta) - x_\eta(y_\xi z_\zeta - y_\zeta z_\xi) + x_\zeta(y_\xi y_\eta - y_\eta y_\xi) \]

GRIDTST evaluates the Jacobian at every grid point, estimating the derivatives (i.e., \( x_\xi, y_\xi, z_\xi \), etc.) by using central difference approximations for grid points that do not lie on the boundary surfaces of the domain and by using second order accurate one-sided difference formulas where necessary on the boundary surfaces. If any negative Jacobians are found, GRIDTST prints a message on the computer screen, and the Jacobians and the grid point locations are written into a file.

The input into GRIDTST is the grid system generated by GRID3D-v2. The input is read from UNIT 1 whereas the output (if any) is written into UNIT 2. GRIDTST is written to run on the IBM PC, AT, XT, and compatibles, but it can be used on any computer system.
A.2 Listing of PRSURF

PROGRAM PRSURF

C This program writes out user picked surfaces from a 3-D grid

PARAMETER (IM=11, JM=51, KM=151)

INTEGER i, j, k, IL, JL, KL

REAL X(IM, JM, KM), Y(IM, JM, KM), Z(IM, JM, KM)

IERR=0

5 WRITE(*, 5001)
READ(*, *) IPICK

5001 FORMAT('2', 'Please enter:',
'S' 1 - if you want the boundary surfaces of the grid',
'S' to be saved',
'S' 2 - if you want to select surfaces to be saved',)

IF(IPICK.EQ.1) THEN
CALL PRGRID(X, Y, Z, IM, JM, KM)
ELSE IF(IPICK.EQ.2) THEN
CALL PRSRFS(X, Y, Z, IM, JM, KM)
ELSE IF(IERR.EQ.0) THEN
WRITE('You will get one more chance to make a'
WRITE('selection - Please enter any character'
WRITE('and then press RETURN')
READ(*,*)
IERR=1
GOTO 5
ELSE
STOP
ENDIF

STOP
END

SUBROUTINE PRGRID(XPnt, YPnt, ZPnt, IL2, JL2, KL2)

C This subroutine reads in grid point coordinates and writes out the
C coordinates of the grid points which lie along specified planes.

INTEGER i, j, k, IL, JL, KL, IL1, JL1, KL1

REAL XPnt(IL2, JL2, KL2),
YPnt(IL2, JL2, KL2),
ZPnt(IL2, JL2, KL2)
C Read in the grid size.

    READ(7,*) IL
    READ(7,*) JL
    READ(7,*) KL

C Read in the grid point locations.

    DO 7 i=1, IL
      DO 6 j=1, JL
        DO 5 k=1, KL
          READ(7,*) XPnt(i, j, k), YPnt(i, j, k), ZPnt(i, j, k)
        CONTINUE
      CONTINUE
    CONTINUE

C Calculate the number of sections the grid must be split into
C for plotting purposes. (Plotting routines can handle only a grid
C with a maximum dimension of 40. Here it is assumed that only the
C zeta-coordinate direction can involve more grid points than that)

    KS = (KL-1) / 39
    JS = (JL-1) / 39

C Print out the number of surfaces.

    IF((KS*39+1).LT.KL) THEN
      NSK=KS+1
    ELSE
      NSK=KS
    ENDIF
    IF((JS*39+1).LT.JL) THEN
      NSJ=JS+1
    ELSE
      NSJ=JS
    ENDIF
    NOSURF=2*NSJ+2*NSK+3*(NSK*NSJ)
    WRITE(9,*) NOSURF

C Print out the grid points.

    DO 21 m=1, NSJ
      j0=39*(m-1)+1
      jl=MIN(j0+39, JL)
      Jlm=jl-j0+1
      WRITE(9,*) IL
      WRITE(9,*) Jlm
      DO 20 i=1, IL
        DO 10 j=j0, jl
          WRITE(9,25) XPnt(i, j, KL), YPnt(i, j, KL), ZPnt(i, j, KL)
        CONTINUE
      CONTINUE
    CONTINUE

   25
CONTINUE

FORMAT(1X,F10.6,3X,F10.6,3X,F10.6)

DO 29 n=1,NSK

k0=39*(n-1)+1
k1=MIN(k0+39,KL)
KLn=k1-k0+1

WRITE(9,*) IL
WRITE(9,*) KLN

DO 28 i=1,IL
  DO 27 k=k0,k1
    WRITE(9,25) XPnt(i,JL,k),YPnt(i,JL,k),ZPnt(i,JL,k)
  CONTINUE
  CONTINUE
  CONTINUE

DO 44 m=1,NSJ

j0=39*(m-1)+1
j1=MIN(j0+39,JL)
JLm=j1-j0+1

DO 44 n=1,NSK

k0=39*(n-1)+1
k1=MIN(k0+39,KL)
KLn=k1-k0+1

WRITE(9,*) JLM
WRITE(9,*) KLN

DO 43 j=j0,j1
  DO 42 k=k0,k1
    WRITE(9,25) XPnt(IL,j,k),YPnt(IL,j,k),ZPnt(IL,j,k)
  CONTINUE
  CONTINUE
  CONTINUE

DO 110 m=1,NSJ

j0=39*(m-1)+1
j1=MIN(j0+39,JL)
JLm=j1-j0+1

DO 110 n=1,NSK

k0=39*(n-1)+1
k1=MIN(k0+39,KL)
KLn=k1-k0+1

WRITE(9,*) JLM
WRITE(9,*) KLN

CONTINUE
DO 109 j=j0,j1
   DO 108 k=k0,k1
       WRITE(9,25) XPnt(1,j,k),YPnt(1,j,k),ZPnt(1,j,k)
108    CONTINUE
109    CONTINUE
110    CONTINUE

DO 131 m=1,NSJ
   j0=39*(m-1)+1
   j1=MIN(j0+39,JL)
   JLm=j1-j0+1
   WRITE(9,*), JLm
   WRITE(9,*), IL
   DO 125 j=j0,j1
       WRITE(9,25) XPnt(i,j,1),YPnt(i,j,1),ZPnt(i,j,1)
125    CONTINUE
130    CONTINUE
131    CONTINUE

DO 134 n=1,NSK
   k0=39*(n-1)+1
   k1=MIN(k0+39,KL)
   KLn=k1-k0+1
   WRITE(9,*), KLn
   WRITE(9,*), IL
   DO 132 k=k0,k1
       WRITE(9,25) XPnt(i,1,k),YPnt(i,1,k),ZPnt(i,1,k)
132    CONTINUE
133    CONTINUE
134    CONTINUE

IH=(IL+1)/2
SUBROUTINE PRSRFS (X, Y, Z, IM, JM, KM)

C This subroutine reads in grid point coordinates and writes out the grid points which lie along planes specified by the user.

INTEGER i, j, k, IL, JL, KL
REAL X(IM, JM, KM), Y(IM, JM, KM), Z(IM, JM, KM)

WRITE(*, 5001)

C Read in the grid size.
READ(7, *) IL
READ(7, *) JL
READ(7, *) KL

C Read in the grid point locations.
DO 3 i=1, IL
   DO 2 j=1, JL
      DO 1 k=1, KL
         READ (7, *) X(i, j, k), Y(i, j, k), Z(i, j, k)
      CONTINUE
   CONTINUE
3 CONTINUE
NOSURF=0
WRITE(*, 5002) IL, JL, KL
READ(*,*) IPICK
IF(IPICK.EQ.1) THEN
   WRITE(*, 5003)
   READ(*,*) IPICK2
   IF(IPICK2.EQ.1) THEN
      WRITE(*,*)
      WRITE(*,*) 'Please enter the value of i'
      READ(*,*) I
      JFIRST=I
      JLAST=JL
      KFIRST=I
      KLAST=KL
   ELSE
      WRITE(*,*)
      WRITE(*,*) 'Please enter the value of i'
      READ(*,*) I
WRITE(*,*)
WRITE(*,*)' Please enter the lower and upper limit'
WRITE(*,*)' for the j coordinate (JFIRST, JLAST)'
READ(*,*) JFIRST, JLAST
WRITE(*,*)
WRITE(*,*)' Please enter the lower and upper limit'
WRITE(*,*)' for the k coordinate (KFIRST, KLAST)'
READ(*,*) KFIRST, KLAST
ENDIF
NSJ = (JLAST - JFIRST) / 39
IF (JFIRST + NSJ * 39. LT. JLAST) NSJ = NSJ + 1
NSK = (KLAST - KFIRST) / 39
IF (KFIRST + NSK * 39. LT. KLAST) NSK = NSK + 1
NOSURF = NOSURF + NSJ * NSK
DO 60 m = 1, NSJ
   j0 = 39 * (m - 1) + JFIRST
   jL = MIN(j0 + 39, JLAST)
   JLm = jL - j0 + 1
   DO 50 n = 1, NSK
      k0 = 39 * (n - 1) + KFIRST
      kL = MIN(k0 + 39, KLAST)
      KLn = kL - k0 + 1
      WRITE(8, *) JLm, KLn
      DO 40 j = j0, jL
         DO 30 k = k0, kL
            WRITE(8, 25) X(I, j, k), Y(I, j, k), Z(I, j, k)
      CONTINUE
      CONTINUE
      CONTINUE
   CONTINUE
IERR = 0
GOTO 5
ELSEIF (IPICK .EQ. 2) THEN
   WRITE(*, 5003)
   READ(*,*) IPICK2
   IF (IPICK2 .EQ. 1) THEN
      WRITE(*,*)
      WRITE(*,*)' Please enter the value of j'
      READ(*,*) J
      IFIRST = I
      ILAST = IL
      KFIRST = I
      KLAST = KL
   ELSE
      WRITE(*,*)
      WRITE(*,*)' Please enter the value of j'
      READ(*,*) J
      WRITE(*,*)
      WRITE(*,*)' Please enter the lower and upper limit'
      WRITE(*,*)' for the i coordinate (IFIRST, ILAST)'
      READ(*,*) IFIRST, ILAST
      WRITE(*,*)
      WRITE(*,*)' Please enter the lower and upper limit'
      WRITE(*,*)' for the k coordinate (KFIRST, KLAST)'
      READ(*,*) KFIRST, KLAST
   ENDIF
NSI=(ILAST-IFIRST)/39
IF(IFIRST+NSI*39.LT.ILAST)NSI=NSI+1
NSK=(KLAST-KFIRST)/39
IF(KFIRST+NSK*39.LT.KLAST)NSK=NSK+1
NOSURF=NOSURF+NSI*NSJ
DO 100 m=1,NSI
   i0=39*(m-1)+IFIRST
   il=MIN(j0+39,ILAST)
   ILM=il-i0+1
   DO 90 n=1,NSJ
      k0=39*(n-1)+KFIRST
      k1=MIN(k0+39,KLAST)
      KLN=k1-k0+1
      WRITE(8,*),ILM
      WRITE(8,*),KLN
      DO 80 i=i0,il
         DO 70 k=k0,k1
            WRITE(8,25)X(i,J,k),Y(i,J,k),Z(i,J,k)
      CONTINUE
      CONTINUE
      CONTINUE
      CONTINUE
      IERR=0
   GOTO 5

ELSEIF(IPICK.EQ.3)THEN
   WRITE(*,5003)
   READ(*,*)IPICK2
   IF(IPICK2.EQ.1)THEN
      WRITE(*,*)'Please enter the value of k'
      READ(*,*)K
      IFIRST=1
      ILAST=IL
      JFIRST=1
      JLAST=JL
   ELSE
      WRITE(*,*)'Please enter the value of k'
      READ(*,*)K
      WRITE(*,*)'Please enter the lower and upper limit for the i coordinate (IFIRST,ILAST)'
      WRITE(*,*)'for the j coordinate (JFIRST,JLAST)'
      READ(*,*)IFIRST,ILAST
      WRITE(*,*)'Please enter the lower and upper limit for the j coordinate (JFIRST,JLAST)'
      READ(*,*)JFIRST,JLAST
   ENDIF
   NSI=(ILAST-IFIRST)/39
   IF(IFIRST+NSI*39.LT.ILAST)NSI=NSI+1
   NSJ=(JLAST-JFIRST)/39
   IF(JFIRST+NSJ*39.LT.JLAST)NSJ=NSJ+1
   NOSURF=NOSURF+NSI*NSJ
   DO 140 m=1,NSI
      i0=39*(m-1)+IFIRST
C
C
RETURN

25 FORMAT(1X,F10.6,3X,F10.6,3X,F10.6)
5001 FORMAT(' ',///,                      Satisfaction
      $' 1 - if you want to save a constant-i surface',///,
      $' 2 - if you want to save a constant-j surface',///,
      $' 3 - if you want to save a constant-k surface',///,
      $' 0 - if you want to QUIT',///,
      $'(Recall: IL=',I3,',  JL=',I3,',  KL=',I3,')',///)
5003 FORMAT(' ',///, ' Please enter:',///,
      $' 1 - if you want the whole surface saved',///,
      $' 2 - if you want to specify a part of the',///,
      $' 0 - surface to be saved',///)
A.3 Listing of 3DPREP

PROGRAM PREPARE

C
DIMENSION TENSION(16),BETA1(16),BETA2(16),
X1(100),Y1(100),Z1(100),X(16,100),Y(16,100),Z(16,100)
INTEGER STRTYPE(16),N2B(4),TYPE(16),NODES(16)
REAL KXII,KXI2,KETA1,KETA2,KZETA1,KZETA2

WRITE(*,*)
WRITE(*,*)
WRITE(*,*)
WRITE(*,*)
WRITE(*,*)' This program prepares input files for GRID3D by'
WRITE(*,*)' reading the necessary information from the screen'
WRITE(*,*)' and from files.'
WRITE(*,*)
WRITE(*,*)

WRITE(*,*)'What technique is to be used.'
WRITE(*,*)'Enter 2 for the two-boundary technique'
WRITE(*,*)' or 4 for the four-boundary technique'
READ(*,*)ITECH

WRITE(*,*)'Enter IL, JL and KL'
READ(*,*)IL,JL,KL

WRITE(*,*)'Enter SigmaXi, SigmaEta, and SigmaZeta'
READ(*,*)SigXi,SigEt,SigZt

WRITE(*,*)'Enter kXII and kXI2'
READ(*,*)KXII,KXI2
WRITE(*,*)'Enter kETAI and kETA2'
READ(*,*)KETAI,KETA2
WRITE(*,*)'Enter kZETAI and kZETA2'
READ(*,*)KZETAI,KZETA2

WRITE(*,*)'The output file will be UNIT 20'
WRITE(20,'(I5)')ITECH,' Technique'
WRITE(20,'(I5,1X)')IL,' IL'
WRITE(20,'(I5,1X)')JL,' JL'
WRITE(20,'(I5,1X)')KL,' KL'
WRITE(20,'(F9.5,1X)')SigXi,' SigmaXi'
WRITE(20,'(F9.5,1X)')SigEt,' SigmaEta'
WRITE(20,'(F9.5,1X)')SigZt,' SigmaZeta'
WRITE(20,'(F9.5,1X)')KXII,' kXII'
WRITE(20,'(F9.5,1X)')KXI2,' kXI2'
WRITE(20,'(F9.5,1X)')KETAI,' kETAI'
WRITE(20,'(F9.5,1X)')KETA2,' kETA2'
WRITE(20,'(F9.5,1X)')KZETAI,' kZETAI'
WRITE(20,'(F9.5,1X)')KZETA2,' kZETA2'

DO 200 NSRF=1,2
   NE1=1+4*(NSRF-1)
   NE2=2+4*(NSRF-1)
   NSRF2=2+4*(NSRF-1)
200 CONTINUE
NE2 = NE1 + 1
NE3 = NE1 + 2
NE4 = NE1 + 3

C Get data for edge NE1:
C
WRITE(*, 2002) NE1
WRITE(*, 2001) NE1
READ(*, *) ITYPE
IF (ITYPE .NE. 1 .AND. ITYPE .NE. 2) GOTO 5

C
IF (ITYPE .EQ. 1) THEN
CALL GETGRP(X1, Y1, Z1, NOP)
IF (NOP .NE. KL) WRITE(*, *)
   'WARNING --- NUMBER OF GRID POINTS INCONSISTENT --'
   'EDGE', NE1
WRITE(20, 3001) ITYPE, NE1
DO 10 K = 1, KL
   WRITE(20, 3004) X1(K), Y1(K), Z1(K), K
10 CONTINUE
X11 = X1(1)
Y11 = Y1(1)
Z11 = Z1(1)
X1L = X1(KL)
Y1L = Y1(KL)
Z1L = Z1(KL)

C ELSEIF (ITYPE .EQ. 2) THEN
C
CALL GETNODES(X1, Y1, Z1, NOP)
C
WRITE(*, 2006) NE1
READ(*, *) TENSN
CALL GETSTR(NE1, ISTR1, BETA11, BETA21)
C
WRITE(20, 3001) ITYPE, NE1
WRITE(20, 3002) TENSN
WRITE(20, 3003) NOP
DO 20 I = 1, NOP
   WRITE(20, 3004) X1(I), Y1(I), Z1(I), I
20 CONTINUE
WRITE(20, 3005) ISTR1
IF (ISTR1 .NE. 4) WRITE(20, 3006) BETA11
IF (ISTR1 .EQ. 4) WRITE(20, 3007) BETA11, BETA21
X11 = X1(1)
Y11 = Y1(1)
Z11 = Z1(1)
X1L = X1(NOP)
Y1L = Y1(NOP)
Z1L = Z1(NOP)
C
ENDIF
C
C Get data for edge NE2:
C
WRITE(*, 2002) NE2
WRITE(*,2001)NE2
READ(*,*)ITYPE
IF(ITYPE.EQ.1 .AND. ITYPE.EQ.2)GOTO 25
C
IF(ITYPE.EQ.1)THEN
   CALL GETGRP(XI,YI,ZI,NOP)
   IF(NOP.NE.KL)WRITE(*,*);
   WRITE(20,3001)ITYPE,NE2
   DO 30 K=1,KL
       WRITE(20,3004)XI(K),Y1(K),ZI(K),K
   CONTINUE
   X21=XI(1)
   Y21=YL(1)
   Z21=ZI(1)
   X2L=XL(KL)
   Y2L=YL(KL)
   Z2L=ZI(KL)
C
ELSEIF(ITYPE.EQ.2)THEN
C
   CALL GETNODES(XI,YI,ZI,NOP)
C
   WRITE(*,2006)NE2
   READ(*,*)TENSN
   CALL GETSTR(NE1,ISTR2,BETAI2,BETA22)
C
   WRITE(20,3001)ITYPE,NE2
   WRITE(20,3002)TENSN
   WRITE(20,3003)NOP
   DO 40 I=1,NOP
       WRITE(20,3004)XI(I),YI(I),ZI(I),I
   CONTINUE
   WRITE(20,3005)ISTR2
   IF(ISTR2.NE.4)WRITE(20,3006)BETAI2
   IF(ISTR2.EQ.4)WRITE(20,3007)BETAI2,BETA22
   X21=XI(1)
   Y21=Y1(1)
   Z21=ZI(1)
   X2L=XI(NOP)
   Y2L=Y1(NOP)
   Z2L=ZI(NOP)
C
ENDIF
C
GET data for edge NE3:
C
WRITE(*,2002)NE3
WRITE(*,2003)NE3
READ(*,*)ITYPE
IF(ITYPE.EQ.1 .AND. ITYPE.EQ.2 .AND. ITYPE.EQ.3)GOTO 45
C
IF(ITYPE.EQ.1)THEN
   CALL GETGRP(XI,YI,ZI,NOP)
   IF(NOP.NE.IL)WRITE(*,*);
   WRITE(20,3001)ITYPE,NE2
   DO 30 K=1,KL
       WRITE(20,3004)XI(K),Y1(K),ZI(K),K
   CONTINUE
   X21=XI(1)
   Y21=YL(1)
   Z21=ZI(1)
   X2L=XL(KL)
   Y2L=YL(KL)
   Z2L=ZI(KL)
C
ENDIF
C
30
'EDGE',NE3
WRITE(20,3001) ITYPE,NE3
DO 50 I=1,IL
   WRITE(20,3004) XI(I),YI(I),ZI(I),I
CONTINUE

C ELSEIF (ITYPE.EQ.2) THEN

   CALL GETNODES(X1,Y1,Z1,NOP)

   WRITE(*,2006)NE3
   READ(*,*)TENSN
   CALL GETSTR(NE3,ISTR3,BETA13,BETA23)

   WRITE(20,3001)ITYPE,NE3
   WRITE(20,3002)TENSN
   WRITE(20,3003)NOP
   DO 60 I=1,NOP
      WRITE(20,3004) XI(I),YI(I),ZI(I),I
   CONTINUE
   WRITE(20,3005)ISTR3
   IF (ISTR3.NE.4) WRITE(20,3006)BETA13
   IF (ISTR3.EQ.4) WRITE(20,3007)BETA13,BETA23

C ELSEIF (ITYPE.EQ.3) THEN

   ITYPE=2
   NOP=2
   I1=1
   I2=2
   WRITE(*,2006)NE3
   READ(*,*)TENSN
   CALL GETSTR(NE3,ISTR3,BETA13,BETA23)

   WRITE(20,3001)ITYPE,NE3
   WRITE(20,3002)TENSN
   WRITE(20,3003)NOP
   WRITE(20,3004) XI1,Y11,Z11,I1
   WRITE(20,3004) XI2,Y21,Z21,I2
   XI(1)=XI1
   XI(2)=XI2
   Y1(1)=Y11
   Y1(2)=Y21
   Z1(1)=Z11
   Z1(2)=Z21
   WRITE(20,3005)ISTR3
   IF (ISTR3.NE.4) WRITE(20,3006)BETA13
   IF (ISTR3.EQ.4) WRITE(20,3007)BETA13,BETA23

C ENDIF

C TYPE(8+NSRF)=ITYPE
C NODES(8+NSRF)=NOP
C TENSION(8+NSRF)=TENSN3
C STRTYPE(8+NSRF)=ISTR3
C BETA1(8+NSRF)=BETA13
C BETA2(8+NSRF)=BETA23
C IF (ITYPE.EQ.1) IMAX=IL
C IF (ITYPE.EQ.2) IMAX=NOP
C DO 70 I=1,IMAX
X(8+NSRF,I)=XI(I)
Y(8+NSRF,I)=YI(I)
Z(8+NSRF,I)=ZI(I)

CONTINUE

Get data for edge NE4:

WRITE(*,2002)NE4
WRITE(*,2003)NE4
READ(*,*)ITYPE
IF(ITYPE.NE.1 .AND. ITYPE.NE.2 .AND. ITYPE.NE.3) GO TO 75

IF(ITYPE.EQ.1) THEN
   CALL GETGRP(XI,YI,ZI,NOP)
   IF(NOP.NE.IL)WRITE(*,*)
   / 'WARNING --- NUMBER OF GRID POINTS INCONSISTENT -'
   / 'EDGE',NE4
   WRITE(20,3001) ITYPE, NE4
   DO 80 I=I,IL
      WRITE(20,3004) XI(K), YI(K), ZI(K), K
   CONTINUE
ELSEIF (ITYPE.EQ.2) THEN
   CALL GETNODES(XI,YI,ZI,NOP)
   WRITE(*,2006)NE4
   READ(*,*)TENSN
   CALL GETSTR(NE4,ISTR4,BETA14,BETA24)
   WRITE(20,3001) ITYPE,NE4
   WRITE(20,3002) TENSN
   WRITE(20,3003) NOP
   WRITE(20,3004) XI(I), YI(I), ZI(I), I
   CONTINUE
   WRITE(20,3005) ISTR4
   IF(ISTR4.NE.4) WRITE(20,3006) BETA14
   IF(ISTR4.EQ.4) WRITE(20,3007) BETA14, BETA24
ELSEIF (ITYPE.EQ.3) THEN
   ITYPE=2
   NOP=2
   I1=1
   I2=2
   WRITE(*,2006)NE4
   READ(*,*)TENSN
   CALL GETSTR(NE4,ISTR4,BETA14,BETA24)
   WRITE(20,3001) ITYPE,NE4
   WRITE(20,3002) TENSN
   WRITE(20,3003) NOP
   WRITE(20,3004) XI(I), YI(I), ZI(I), I
   WRITE(20,3004) XI(1), YI(1), ZI(1), I1
   WRITE(20,3004) XI(2), YI(2), ZI(2), I2
   XI(1)=XI1
   XI(2)=XI2
   YI(1)=YI1
   YI(2)=YI2

32
C
C
Z1(1)=Z1L
Z1(2)=Z2L
WRITE (20,3005) ISTR4
IF (ISTR4 .NE. 4) WRITE (20,3006) BETA14
IF (ISTR4 .EQ. 4) WRITE (20,3007) BETA14, BETA24
C
ENDIF
C
TYPE (12+NSRF) = ITYPE
NODES (12+NSRF) = NOP
TENSION (12+NSRF) = TENSN4
STRTYPE (12+NSRF) = ISTR4
BETAI (12+NSRF) = BETA14
BETA2 (12+NSRF) = BETA24
IF (ITYPE .EQ. 1) IMAX=IL
IF (ITYPE .EQ. 2) IMAX=NOP
DO 100 I=1,IMAX
    X (16+NSRF, I) = X1 (I)
    Y (16+NSRF, I) = Y1 (I)
    Z (16+NSRF, I) = Z1 (I)
100
CONTINUE
C
C
ELSEIF (ITECH .EQ. 4) THEN
C
DO 500 NSRF=3,4
    NE1=1+4*(NSRF-1)
    NE2=NE1+1
    NE3=NE1+2
    NE4=NE1+3
C
Write data for edge NE1:
C
WRITE (20,3001) TYPE(NE1), NE1
IF (TYPE(NE1) .EQ. 1) THEN
    DO 210 I=1, IL
        WRITE (20,3004) X(NE1,I), Y(NE1,I), Z(NE1,I), I
210 CONTINUE

33
ELSEIF (TYPE (NE1) .EQ. 2) THEN
WRITE (20, 3002) TENSION (NE1)
WRITE (20, 3003) NODES (NE1)
DO 220 I = 1, NODES (NE1)
   WRITE (20, 3004) X (NE1, I), Y (NE1, I), Z (NE1, I), I
CONTINUE
WRITE (20, 3005) STRTYPE (NE1)
IF (STRTYPE (NE1) .NE. 4) WRITE (20, 3006) BETA1 (NE1)
IF (STRTYPE (NE1) .EQ. 4)
   WRITE (20, 3007) BETA1 (NE1), BETA2 (NE1)
ENDIF

Write data for edge NE2:

WRITE (20, 3001) TYPE (NE2), NE2
IF (TYPE (NE2) .EQ. 1) THEN
   DO 230 I = 1, IL
      WRITE (20, 3004) X (NE2, I), Y (NE2, I), Z (NE2, I), I
   CONTINUE
ELSEIF (TYPE (NE2) .EQ. 2) THEN
   WRITE (20, 3002) TENSION (NE2)
   WRITE (20, 3003) NODES (NE2)
   DO 240 I = 1, NODES (NE2)
      WRITE (20, 3004) X (NE2, I), Y (NE2, I), Z (NE2, I), I
   CONTINUE
   WRITE (20, 3005) STRTYPE (NE2)
   IF (STRTYPE (NE2) .NE. 4) WRITE (20, 3006) BETA1 (NE2)
   IF (STRTYPE (NE2) .EQ. 4)
      WRITE (20, 3007) BETA1 (NE2), BETA2 (NE2)
   ENDIF
ENDIF

Get data for edge NE3:

WRITE (*, 2002) NE3

WRITE (*, *) 'Enter the following:
WRITE (*, *) 1 if grid points are specified'
WRITE (*, *) 2 if nodes for splining are specified'
WRITE (*, *) 3 if you want to let the end points of'
WRITE (*, *) edges already entered define the curve'
WRITE (*, *) 4 if you want to use the end points of'
WRITE (*, *) edges already defined but add (by '
WRITE (*, *) typing in directly) some points in '
WRITE (*, *) between'
WRITE (*, *)
WRITE (*, *)
WRITE (*, *)
WRITE (*, *)
WRITE (*, *) 'Enter your choice'
READ (*, *) ITYPE

34
IF (ITYPE .NE. 1 .AND. ITYPE .NE. 2 .AND. ITYPE .NE. 3 .AND. ITYPE .NE. 4) GOTO 305

IF (ITYPE .EQ. 1) THEN
   CALL GETGRP(X1,Y1,Z1,NOP)
   IF (NOP .NE. JL) WRITE(*,*)
   ' WARNING ---- NUMBER OF GRID POINTS INCONSISTENT - ',
   ' EDGE', NE3
   WRITE(20,3001) ITYPE, NE3
   DO 310 J=I,JL
      WRITE(20,3004) X1(J), Y1(J), Z1(J), J
   CONTINUE
ELSEIF (ITYPE .EQ. 2) THEN
   CALL GETNODES(X1,Y1,Z1,NOP)
   WRITE(*,2006) NE3
   READ(*,*), TENS
   CALL GETSTR(NE3, ISTR3, BETA13, BETA23)
   WRITE(20,3001) ITYPE, NE3
   WRITE(20,3002) TENS
   WRITE(20,3003) NOP
   WRITE(20,3005) ISTR3
   IF (ISTR3 .NE. 4) WRITE(20,3006) BETA13
   IF (ISTR3 .EQ. 4) WRITE(20,3007) BETA13, BETA23
ELSEIF (ITYPE .EQ. 3) THEN
   ITYPE=2
   NOP=2
   I1=1
   I2=2
   WRITE(*,2006) NE3
   READ(*,*), TENS
   CALL GETSTR(NE3, ISTR3, BETA13, BETA23)
   WRITE(20,3001) ITYPE, NE3
   WRITE(20,3002) TENS
   WRITE(20,3003) NOP
   WRITE(20,3004) X(NE1,1), Y(NE1,1), Z(NE1,1), I1
   WRITE(20,3004) X(NE2,1), Y(NE2,1), Z(NE2,1), I2
   WRITE(20,3005) ISTR3
   IF (ISTR3 .NE. 4) WRITE(20,3006) BETA13
   IF (ISTR3 .EQ. 4) WRITE(20,3007) BETA13, BETA23
ELSEIF (ITYPE .EQ. 4) THEN
   WRITE(_,*) ' How many nodes do you want to add?'
   READ(*,*), NOP
   DO 330 I=1, NOP
      WRITE(20,3004) X(NE1, I), Y(NE1, I), Z(NE1, I), I
   CONTINUE
330
9  WRITE(20,3005) ISTR3
   IF (ISTR3 .NE. 4) WRITE(20,3006) BETA13
   IF (ISTR3 .EQ. 4) WRITE(20,3007) BETA13, BETA23
CALL GETNEWND(X1,Y1,Z1,I)
330 CONTINUE
ITYPE=2
NOP=NOP+2
II=1
C
WRITE(*,2006)NE3
READ(*,*)TENSN
CALL GETSTR(NE3,ISTR3,BETA13,BETA23)
C
WRITE(20,3001)ITYPE,NE3
WRITE(20,3002)TENSN
WRITE(20,3003)NOP
WRITE(20,3004)X(NE1,1),Y(NE1,1),Z(NE1,1),II
DO 340 I=2,NOP-1
  WRITE(20,3004)X(I-1),Y(I-1),Z(I-1),I
CONTINUE
WRITE(20,3004)X(NE2,1),Y(NE2,1),Z(NE2,1),NOP
WRITE(20,3005)ISTR3
IF(ISTR3.NE.4)WRITE(20,3006)BETA13
IF(ISTR3.EQ.4)WRITE(20,3007)BETA13,BETA23
ENDIF
C
C
GET data for edge NE4:

C
405 WRITE(*,2002)NE4
C
WRITE(*,*)'Enter the following:'
WRITE(*,*)' 1 if grid points are specified'
WRITE(*,*)' 2 if nodes for splining are specified'
WRITE(*,*)' 3 if you want to let the end points of'
WRITE(*,*)' edges already entered define the curve'
WRITE(*,*)' 4 if you want to use the end points of'
WRITE(*,*)' edges already defined but add (by '
WRITE(*,*)' typing in directly) some points in '
WRITE(*,*)' between'
WRITE(*,*)
WRITE(*,*)
WRITE(*,*)
WRITE(*,*)
WRITE(*,*)'Enter your choice'
READ(*,*)ITYPE
C
IF(ITYPE.NE.1 .AND. ITYPE.NE.2 .AND. ITYPE.NE.3
   .AND. ITYPE.NE.4)GOTO 405
C
IF(ITYPE.EQ.1)THEN
  CALL GETGRP(X1,Y1,Z1,NOP)
  IF(NOP.NE.JL)WRITE(*,*)
     'WARNING --- NUMBER OF GRID POINTS INCONSISTENT -'
     ' EDGE',NE4
  WRITE(20,3001)ITYPE,NE4
  DO 410 J=1,JL
    WRITE(20,3004)X1(J),Y1(J),Z1(J),J
  CONTINUE
C
410
ELSEIF (ITYPE.EQ.2) THEN
   CALL GETNODES(X1,Y1,Z1,NOP)
   WRITE(*,2006)NE4
   READ(*,*)TENSN
   CALL GETSTR(NE4,ISTR4,BETA14,BETA24)

WRITE (20,3001) ITYPE,NE4
WRITE (20,3002) TENSN
WRITE (20,3003) NOP
DO 420 I=1,NOP
   WRITE (20,3004) X1(I),Y1(I),Z1(I),I
CONTINUE

WRITE (20,3005) ISTR4
IF (ISTR4 .NE. 4) WRITE (20, 3006) BETA14
IF (ISTR4 .EQ. 4) WRITE (20,3007) BETA14,BETA24

ELSEIF (ITYPE.EQ.3) THEN
   ITYPE=2
   NOP=2
   I1=1
   I2=2
   WRITE(*,2006)NE4
   READ(*,*)TENSN
   CALL GETSTR(NE4,ISTR4,BETA14,BETA24)

WRITE (20,3001) ITYPE,NE4
WRITE (20,3002) TENSN
WRITE (20, 3003) NOP
IMAX=IL
IF (TYPE(NE1) .EQ. 2) IMAX=NODES(NE1)
WRITE (20,3004) X(NE1,IMAX),Y(NE1,IMAX),Z(NE1,IMAX),I1
IMAX=IL
IF (TYPE(NE2) .EQ. 2) IMAX=NODES(NE2)
WRITE (20,3004) X(NE2,IMAX),Y(NE2,IMAX),Z(NE2,IMAX),I2
WRITE (20, 3005) ISTR4
IF (ISTR4 .NE. 4) WRITE (20,3006) BETA14
IF (ISTR4 .EQ. 4) WRITE (20,3007) BETA14,BETA24

ELSEIF (ITYPE.EQ.4) THEN
   WRITE(*,*)' How many nodes do you want to add?'
   READ(*,*)NOP
   DO 430 I=1,NOP
      CALL GETNEWND(X1,Y1,Z1,I)
   CONTINUE
   ITYPE=2
   NOP=NOP+2
   I1=1
   WRITE(*,2006)NE4
   READ(*,*)TENSN
   CALL GETSTR(NE4,ISTR4,BETA14,BETA24)

WRITE (20,3001) ITYPE,NE4

37
WRITE (20, 3002) TENSN
WRITE (20, 3003) NOP
IMAX=IL
IF (TYPE (NE1) .EQ. 2) IMAX=NODES (NE1)
WRITE (20, 3004) X (NE1, IMAX), Y (NE1, IMAX), Z (NE1, IMAX), IL
DO 440 I=2, NOP-1
  WRITE (20, 3004) X1(I-1), Y1(I-1), Z1(I-1), I
440  CONTINUE
IMAX=IL
IF (TYPE (NE2) .EQ. 2) IMAX=NODES (NE2)
WRITE (20, 3004) X (NE2, IMAX), Y (NE2, IMAX), Z (NE2, IMAX), I2
WRITE (20, 3005) ISTR4
IF (ISTR4 .NE. 4) WRITE (20, 3006) BETA14
IF (ISTR4 .EQ. 4) WRITE (20, 3007) BETA14, BETA24

C
ENDIF
CONTINUE
ENDIF
C
2001  FORMAT('////', ' Enter the TYPE for edge ', I2,
      $'/',' Enter:/',//,' 1 if grid points along the edge are to be ',
      $'specified/',//,' 2 if nodes for splining are to be',
      $'specified/',//)
2002  FORMAT('////////////',' SPECIFYING EDGE ', I2,///)
2003  FORMAT('////', ' Enter the TYPE for edge ', I2,
      $'/',' Enter:/',//,' 1 if grid points along the edge are to be ',
      $'specified/',//,' 2 if nodes for splining are to be',
      $'specified/',//,' 3 if end nodes of edges already ',
      $'entered ',/ ' define the curve completely',//)
2006  FORMAT('////////////', ' Enter the TENSION parameter for curve ', I2)

3001  FORMAT(' ', I3, ' Type - EDGE NO: ', I2,
      $'----------------------------------------')
3002  FORMAT(' ', F6.2, ' Tension parameter')
3003  FORMAT(' ', I3, ' Number of nodes')
3004  FORMAT(3X, 3(F8.5, 3X), '//', I2)
3005  FORMAT(' ', I3, ' StretchType')
3006  FORMAT(' ', F8.4, ' Stretching parameter BETA')
3007  FORMAT(' ', 2(F8.4, 3X), ' Stretching parameters BETA1 and',
      $' BETA2')
3010  FORMAT(' ', I3, ' StretchType ', I2, ' ----------------')

STOP
END

C
SUBROUTINE GETNODES (X1, Y1, Z1, NOP)
C
This subroutine reads in and arranges nodal points to define
C an edge.
C
DIMENSION X1(100), Y1(100), Z1(100)
C
INUMO=10
NOP=0

WRITE(*,*)'How many sections is the edge composed of?'
READ(*,*)NOSECT

DO 100 ISECT=1,NOSECT

INNUM=INNUM0+ISECT
WRITE(*,200)ISECT,INNUM
READ(INNUM,*)NP
WRITE(*,201)NP
READ(*,*)N1
WRITE(*,202)
READ(*,*)N2

N12=IABS(N2-N1)+1
NOP=NOP+N12

IF(N2.GE.N1)THEN
  DO 10 J=1,N1-1
  READ(INNUM,*)
  CONTINUE
10
  I=NOP-N12
  DO 20 J=N1,N2
  I=I+1
  READ(INNUM,*)XI(I),Y1(I),Z1(I)
  CONTINUE
20
ELSE
  DO 30 J=1,N2-1
  READ(INNUM,*)
  CONTINUE
30
  I=NOP+1
  DO 40 J=N2,N1
  I=I-1
  READ(INNUM,*)XI(I),Y1(I),Z1(I)
  CONTINUE
40
ENDIF
CLOSE(INNUM)
CONTINUE

FO_/AT(' Section ',I2,' will be read in from UNIT',I3)
FO_/AT(' There are ',I2,' points in the file.
FO_/AT(' Enter the number of the point that is to be the',I2,' the first on the current section."
FO_/AT(' Enter the number of the point that is to be the',I2,' the last on the current section."
RETURN
END

C==============================================#
C
C SUBROUTINE GETGRP(X1,Y1,Z1,NOP)
This subroutine reads in grid point coordinates for an edge and stores in either forward or reversed order.

```
DIMENSION XI(100), YI(100), ZI(100)

INNUM0 = 10
NOP = 0

WRITE(*,*) 'How many sections is the edge composed of?'
READ(*,*) NOSECT

DO 500 ISECT = 1, NOSECT
   INNUM = INNUM0 + ISECT
   WRITE(*, 200) ISECT, INNUM
   READ(INNUM, *) NP
   WRITE(*, 201) NP
   READ(*, *) NI
   WRITE(*, 202)
   READ(*, *) N2

   N12 = IABS(N2 - NI) + 1
   NOP = NOP + N12

   IF (N2 .GE. NI) THEN
      DO 10 J = I, NI - 1
         READ(INNUM, *)
      CONTINUE
      I = NOP - N12
      DO 20 J = NI, N2
         I = I + 1
         READ(INNUM, *) XI(I), YI(I), ZI(I)
      CONTINUE
   ELSE
      DO 60 J = I, N2 - 1
         READ(INNUM, *)
      CONTINUE
      I = NOP + 1
      DO 70 J = N2, NI
         I = I - 1
         READ(INNUM, *) XI(I), YI(I), ZI(I)
      CONTINUE
   ENDIF
   CLOSE(INNUM)
   CONTINUE
```

500 FORMAT(' Section ', I2, ' will be read in from UNIT', I3)
200 FORMAT(' There are ', I2, ' grid points in the file.', /
         ' Enter the number of the grid point that is to be the', /
         ' the first or the current section.')
SUBROUTINE GETNEWND(X1, Y1, Z1, I)

This subroutine reads in from the screen new points that are
to be included on the edge.

DIMENSION X1(100), Y1(100), Z1(100)

WRITE(*,2001) I
READ(*,*)X1(I), Y1(I), Z1(I)

RETURN
END

SUBROUTINE GETSTR(NEI, ISTR, BETA1, BETA2)

This subroutine reads information regarding stretching function
along edge NEI.

WRITE(*,2011) NEI
WRITE(*,2020)
100 READ(*,*)ISTR
IF(ISTR.LT.4 .AND. ISTR.GT.0)THEN
    WRITE(*,2031) NEI
    READ(*,* )BETA1
ELSEIF (ISTR.EQ.4) THEN
    WRITE(*,2036) NEI
    READ(*,* )BETA1, BETA2
ELSEIF (ISTR.EQ.0) THEN
    BETA1=1.1
ELSE
    WRITE('*',' Please enter a number from 0 to 4'
        GOTO 100
ENDIF

2011 FORMAT( ' Enter the STRETCH TYPE for edge ',I2)
2020 FORMAT( ', Enter: 0 for no stretching',/,
            0.001 for concentration near lower boundary',/,
            0.002 for concentration near upper boundary',/,
            0.003 for concentration near both boundaries',/,
            0.004 for concentration near both boundaries',/,
            (one parameter stretching function)',/,
            (two-parameter stretching function)'
2031 FORMAT( ' Enter the stretching parameter (BETA) for edge ',
            /I2)
2036 FORMAT( ' Enter the stretching parameters (BETA1 and BETA2) ',
            /for edge ',I2)

RETURN
END
A.4 Listing of EDGE

PROGRAM EDGE
C This program generates grid points along an edge which is given by a set of discrete nodes. The edge can be made up of up to several sections. A parametric tension spline is fit through each section. Each section has its own control parameters such as number of grid points, tension, and stretching functions. NOTE: Subroutines have been adopted from GRID3D (and modified slightly) to generate the grid points on the curves.

PARAMETERS:
MxBPts - Maximum number of nodes per section
MxGSiz - Maximum number of grid points per section
MxSect - Maximum number of sections
MxBCvs - Should not be modified

PARAMETER (MxBPts=31, MxGSiz=101, MxSect=5, MxBCvs=1)

DIMENSION x(MxSect,MxBPts), y(MxSect,MxBPts), z(MxSect,MxBPts), zx(MxSect,MxBPts), zy(MxSect,MxBPts), zz(MxSect,MxBPts), s(MxSect,MxBPts), Tensn(MxSect)

DIMENSION Diag(MxBPts), OfDiag(MxBPts), Right(MxBPts)

DIMENSION XB(MxGSiz,MxSect), YB(MxGSiz,MxSect), ZB(MxGSiz,MxSect), StrB(MxGSiz,MxBCvs)

INTEGER NDPts(MxSect), ILS(MxSect), StrTp

WRITE(*,*) ' Running PROGRAM EDGE'
WRITE(*,*)
WRITE(*,*)
WRITE(*,*)' The input data will be read in from UNIT 1'
WRITE(*,*)' The output will be written into UNIT 20'
WRITE(*,*)
WRITE(*,*)
READ(1,*)NOSECT

IL=0
DO 200 iS=1,NOSECT
    CALL RdSctIn(x,y,z,NDPts,is,Tensn,1,MxBPts,MxSect,
      StrTp,Betal,Beta2,ILS(is))
    CALL PTSpln(x,y,z,s,zx,zy,zz,Diag,OfDiag,Right,NDPts,
      Tensn(is),is,MxBPts,MxSect)
    CALL CalcStr2(1,ILS(is),StrTp,Betal,Beta2,
      StrB,MxBCvs,MxGSiz)

200

42
CALL EdgGPts(is,1,ILS(is),XB,YB,ZB,StrB,x,y,z,s,
  $  
  zx,zy,zz,NDPts,Tensn(is),
  $  
  MxBCvs,MxBPts,MxGSiz,MxSect)

IL=IL+ILS(is)
CONTINUE

IL=IL-(NOSECT-I)

WRITE(20,*)IL,' Number of grid points'

IP=0

DO 301 is=1,NOSECT
  DO 300 i=1,ILS(is)-I
    IP=IP+I
    WRITE(20,3001)XB(i,is),YB(i,is),ZB(i,is),IP
  CONTINUE
300 CONTINUE
301 CONTINUE
is=NOSECT
i=ILS(NOSECT)

WRITE(20,3001)XB(i,is),YB(i,is),ZB(i,is),IL

FORMAT
  (',3X,3(F9.6,3X),' ',I3)
STOP
END

C
C   SUBROUTINE RdSctIn (x,y,z,NDPts,CrvNum, Tensn, InNum, MxBPts,MxSect,
C                      $  StrTp, Betal, Beta2, IL)
C
C This SUBROUTINE reads in the information concerning discrete points on
C the boundaries. This information is used for generating spline-fitted
C boundary approximation curves.
C
INTEGER CrvNum, i, NDPts(MxSect), InNum, StrTp, IL

REAL  x(MxSect,MxBPts), y(MxSect,MxBPts),
      $  z(MxSect,MxBPts), Tensn(MxSect)

READ(InNum,* ) IL
READ(InNum,* ) Tensn(CrvNum)
READ(InNum,* ) NDPts(CrvNum)

DO 10 i=1,NDPts(CrvNum)
  READ(InNum,* ) x(CrvNum,i), y(CrvNum,i), z(CrvNum,i)
10 CONTINUE

READ(InNum,* )StrTp
IF (StrTp.NE.4) THEN
  READ(InNum,* )Betal
ELSE
  READ(InNum,* )Betal,Beta2
ENDIF

RETURN
END

C
C   SUBROUTINE CalcS (x,y,z,NDPts,CrvNum,MxBPts,MxSect)
C
43
This SUBROUTINE calculates the spline parameter, s, as an approximate arc length.

**SUBROUTINE SplMat (Diag, OfDiag, Right, w, s, NDPts, T, CrvNum, MxBPts, MxSect)**

This SUBROUTINE forms the parametric tension spline matrix for a particular boundary curve data set.

**SUBROUTINE SplSlv (Diag, OfDiag, Right, Deriv2, NDPts, CrvNum, MxBPts, MxSect)**

This SUBROUTINE solves the diagonally dominant parametric tension
C spline matrix for a given data set using the Gauss-Seidel iteration.
C Convergence is assumed after 20 iterations.
C
INTEGER i, j, NDPts(MxSect), CrvNum

REAL Diag(MaxBPts), OfDiag(MaxBPts), Right(MaxBPts),
  Derv2(MaxSect, MaxBPts)

C Initialize the second derivative matrix to all zeroes.
C
DO 10 i=1,NDPts(CrvNum)
  Derv2(CrvNum, i)=0.0
10 CONTINUE

C Calculate the second derivative values using 20 iterations of
C the Gauss-Seidel method.
C
DO 30 j=1,20
  DO 20 i=2,NDPts(CrvNum)-i
    Derv2(CrvNum, i)= (Right(i) -OfDiag(i)*Derv2(CrvNum, i+1)
       -OfDiag(i-1)*Derv2(CrvNum, i-1) )
       /Diag(i)
  20 CONTINUE
30 CONTINUE

FUNCTION SplVal (s,w,Derv2,sval,T,n,CrvNum, MxBPts, MxSect)
C
This real function finds the w-value (x-value or y-value) corresponding
C to a specified s-value using the parametric tension spline curve
C generated for a particular boundary curve data set.
C
INTEGER n, CrvNum
C
REAL s(MaxSect, MxBPts), w(MaxSect, MxBPts), Derv2(MaxSect, MxBPts),
  sval, T, h, Interim, Temp1, Temp2
C
Templ=sval-s(CrvNum, n)
  h=s(CrvNum, n+1)-s(CrvNum, n)
  Temp2=s(CrvNum, n+1)-sval
  Interim=Derv2(CrvNum, n)/T**2*SINH(T*Temp2)/SINH(T*h)
  + (w(CrvNum, n)-Derv2(CrvNum, n)/T**2)*Temp2/h
  SplVal=Interim+Derv2(CrvNum, n+1)/T**2*SINH(T*Temp1)
  /SINH(T*h)+(w(CrvNum, n+1)
  -Derv2(CrvNum, n+1)/T**2)*Temp1/h

RETURN
END

SUBROUTINE PTSpln(x,y,z,s,XDerv2,YDerv2,ZDerv2,Diag,OfDiag,
  Right,NDPts,Tensn,CrvNum,MxBPts,MxSect)
C
This SUBROUTINE forms the main routine for the parametric tension
C spline process.

INTEGER NDPts(MxSect), CrvNum

REAL Diag(MxBPts), OfDiag(MxBPts), Right(MxBPts),
 $ XDerv2(MxSect,MxBPts), YDerv2(MxSect,MxBPts),
 $ ZDerv2(MxSect,MxBPts), Tensn,
 $ x(MxSect,MxBPts), y(MxSect,MxBPts),
 $ z(MxSect,MxBPts), s(MxSect,MxBPts)

CALL CalcS(x,y,z,s,NDPts,CrvNum,MxBPts,MxSect)
CALL SplMat(Diag,OfDiag,Right,x,s,NDPts,Tensn,CrvNum,
 $ - MxBPts,MxSect)
CALL SplSlv(Diag,OfDiag,Right,XDerv2,NDPts,CrvNum,MxBPts,MxSect)
CALL SplMat(Diag,OfDiag,Right,y,s,NDPts,Tensn,CrvNum,
 $ - MxBPts,MxSect)
CALL SplSlv(Diag,OfDiag,Right,YDerv2,NDPts,CrvNum,MxBPts,MxSect)
CALL SplMat(Diag,OfDiag,Right,z,s,NDPts,Tensn,CrvNum,
 $ - MxBPts,MxSect)
CALL SplSlv(Diag,OfDiag,Right,ZDerv2,NDPts,CrvNum,MxBPts,MxSect)

RETURN
END

C---------------------------------------------------------------------

SUBROUTINE SplInt(n,s,SValue,NDPts,CurCrv,MxBPts,MxSect)

C This SUBROUTINE finds the proper interval in which a point on a specified
C boundary lies. The interval indicates which initial data points the
C point in question lies between and thus which spline coefficients to
C use.

INTEGER i, n, CurCrv, NDPts(MxSect)

REAL Temp, SValue, s(MxSect,MxBPts)

n=1
i=NDPts(CurCrv)

10 IF ((n.EQ.1).AND.(i.GT.1)) THEN
   I=I-1
   Temp=SValue-s(CurCrv,i)

   IF (Temp.GT.0.0) THEN
      n=i
   ENDIF
   GOTO 10
ENDIF
RETURN
END

C---------------------------------------------------------------------

SUBROUTINE FAInew(AlNew,Alpha,B,Str)
This SUBROUTINE computes the new Alpha value after stretching as 
AlNew. Alpha is a dummy variable representing either Xi, Eta or Zeta.

C

INTEGER Str

REAL Alpha, Templ, Temp2, B2, AlNew, B

AlNew=Alpha
Temp1=(B+1)/(B-1)

IF (Str.EQ.1) THEN
  Temp2=Templ**(1-Alpha)
  AlNew=((B+1)-(B-1)*Temp2)/(Temp2+1)*1
ENDIF

IF (Str.EQ.2) THEN
  B2=0
  Temp2=Templ**((Alpha-B2)/(1-B2))
ENDIF

IF (Str.EQ.3) THEN
  B2=0.5
  Temp2=Templ**((Alpha-B2)/(1-B2))
ENDIF

RETURN
END

============================================

SUBROUTINE CalcStr2(EdgNum, NGPts, StrTp, Betal, Beta2,
$ StrB, MxBCvs, MxGSiz)

This subroutine calculates the distribution function base on the 
stretching parameters 'StrTp' and 'Beta'

INTEGER NGPts, StrTp, EdgNum, i

REAL StrB(MxGSiz,MxBCvs), Betal, Beta2, A, B, DZ

StrB(1,EdgNum)=0.
IF(StrTp.LE.3) THEN
  DO 10 i=1,NGPts-1
    Alpha=(i-1.)/(NGPts-1.)
    CALL FAiNew(AiNew,Alpha,Betal,StrTp)
    StrB(i,EdgNum)=AiNew
  CONTINUE
ELSEIF(StrTp.EQ.4) THEN
  CALL Str4Prm(Betal,Beta2,A,B,DZ)
  DO 20 i=2,NGPts-1
    Alpha=(i-1.)/(NGPts-1.)
    CALL Str4(AiNew,Alpha,A,B,DZ)
    StrB(i,EdgNum)=AiNew
  CONTINUE
ENDIF

StrB(NGPts,EdgNum)=1.
SUBROUTINE EdgGPts (CrvNum, EdgNum, NGPts, XB, YB, ZB, StrB, x, y, z, s, zx, zy, zz, NDPts, Tensn, MxBCvs, MBPts, MxGSiz, MxSect)

This subroutine calculates the grid point location along an edge based on a spline curve fitted through specified nodal points and a given distribution function.

INTEGER CrvNum, EdgNum, NGPts, NDPts(MxSect), i, n
REAL $XB(MxGSiz,MxSect), YB(MxGSiz,MxSect), ZB(MxGSiz,MxSect), $StrB(MxGSiz,MxBCvs), x(MxSect,MBPts), y(MxSect,MBPts), $z(MxSect,MBPts), zx(MxSect,MBPts), zy(MxSect,MBPts), $zz(MxSect,MBPts), s(MxSect,MBPts), Tensn

SRa=S(CrvNum,NDPts(CrvNum))

DO 10 i=1,NGPts
SB=SRa*StrB(i,EdgNum)
CALL SplInt(n,s,SB,NDPts,CrvNum,MBPts,MxSect)
XB(i,CrvNum)=SplVal(s,x,zx,SB,Tensn,n,CrvNum,MBPts,MxSect)
YB(i,CrvNum)=SplVal(s,y,zy,SB,Tensn,n,CrvNum,MBPts,MxSect)
ZB(i,CrvNum)=SplVal(s,z,zz,SB,Tensn,n,CrvNum,MBPts,MxSect)
10 CONTINUE

RETURN
END

SUBROUTINE Str4Prm(S0,SI,A,B, DZ)
REAL S0, SI, A, B, DZ, Y, PI

PI=ACOS(-1.)
A=SQR(S0/SI)
B=SQR(S0*SI)

IF(B.GT.1.001)THEN
IF(B.LE.2.7829681)THEN
  Y=B-1
  DZ=SQR(6.*Y)*(1.-0.15*Y+0.057321429*(Y**2)
$+0.024907295*(Y**3)+0.0077424461*(Y**4)$
$-0.0010794123*(Y**5))
ELSEIF(B.GT.2.7829681)THEN
  V=LOG(B)
  W=1./B - 0.028527431
  DZ=V+(1.+1./V)*LOG(2.*V)-0.02041793+0.24902722*W
$+1.9496443*W**2-2.6294547*W**3+8.56795911*W**4
ENDIF
ELSEIF(B.LT.0.999)THEN
IF (B .LE. 0.26938972) THEN
   DZ = PI * (1. - B + B**2 - (1. + (PI**2) / 6.) * (B**3) + 6.794732 * (B**4)
   - 13.205501 * (B**5) + 11.726095 * (B**6))
$ ELS E
   Y = B - 1
   DZ = SQRT (6. * Y) * (1. + 0.15 * Y + 0.057321429 * (Y**2)
$   + 0.048774238 * (Y**3) - 0.053337753 * (Y**4)
$   + 0.075845134 * (Y**5))
ENDIF
ENDIF
RETURN
END

SUBROUTINE Str4 (AiNew, Alpha, A, B, DZ)
REAL AiNew, Alpha, A, B, DZ, U, T

This subroutine calculates the value of the two-sided Vinokur stretching function based on the value of the parameters A, B, and DZ, and on the value of the "computational" coordinate Alpha.

IF (B .GT. 1.001) THEN
   U = 0.5 + TANH (DZ * (Alpha - 0.5)) / (2. * TANH (DZ / 2.))
ELSEIF (B .LT. 0.999) THEN
   U = 0.5 + TAN (DZ * (Alpha - 0.5)) / (2. * TAN (DZ / 2.))
ELSE
   U = Alpha * (1. + 2. * (B - 1) * (Alpha - 0.5) * (1 - Alpha))
ENDIF
T = U / (A + (1. - A) * U)
AiNew = T

RETURN
END
A.5 Listing of EDGPREP

PROGRAM EDGPREP
C
PARAMETER (MxNode=100)
C
INTEGER ISTR, ILS, NOP
C
REAL TENSION, BETA1, BETA2,
     / Xl(MxNode), Yl(MxNode), Zl(MxNode)
C
C This program prepares input files for EDGE by reading the
C necessary information from the screen and from files.
C
WRITE(*,*)
WRITE(*,*)
WRITE(*,*)
WRITE(*,*) 'This program prepares an input file for the program'
WRITE(*,*) 'EDGE. EDGE generates the grid point coordinates on'
WRITE(*,*) 'a curve given by a set of nodes through which'
WRITE(*,*) 'a spline curve can be fitted. The spline curve can'
WRITE(*,*) 'be made up from several sections.'
WRITE(*,*) 'Enter the number of sections'
WRITE(*,*)
READ(*,*) NOSECT
C
WRITE(*,*)
WRITE(*,*)
WRITE(*,*) 'The data will be written into UNIT 20'
WRITE(20,*) NOSECT, ' Number of sections'
C
DO 100 is=1, NOSECT
WRITE(*,*)
WRITE(*,*)
WRITE(*,*)
WRITE(*,*)
WRITE(*,*) 'Now enter data for section number', is
WRITE(*,*)
WRITE(*,*) 'Enter number of grid points to be on the section'
WRITE(*,*)
READ(*,*) ILS
CALL GETNODES(Xl, Yl, Zl, NOP, MxNode)
C
WRITE(*, 2001) is
READ(*,*) TENSION
CALL GETSTR(is, ISTR, BETA1, BETA2)
C
WRITE(20, 3001) ILS, is
WRITE(20, 3002) TENSION
WRITE(20, 3003) NOP
DO 20 I=1, NOP
   WRITE(20, 3004) Xl(I), Yl(I), Zl(I), I
20 CONTINUE
WRITE(20, 3005) ISTR
IF(ISTR.NE.4)WRITE(20,3006)BETA1
IF(ISTR.EQ.4)WRITE(20,3007)BETA1,BETA2

CONTINUE

FORMAT(//,' Enter the TENSION parameter for section ',I2)
FORMAT(',I3,' No of gridpts: Section ',I2,
$'-------------------')
FORMAT(',F6.2,' Tension parameter')
FORMAT(',I3,' Number of nodes')
FORMAT(3X,3(F8.5,3X),' ---',I2)
FORMAT(' ',I3, ' StretchType')
FORMAT(' ',F8.4,' Stretching parameter BETA')
FORMAT(' ',2(F8.4,3X),' Stretching parameters BETA1 and',
$' BETA2')

STOP

END

SUBROUTINE GETNODES(X1,Y1,Z1,NOP,MxNode)

This subroutine reads in and arranges nodal points to define two parallel curve sections (one on each blade surface).

DIMENSION X1(MxNode),Y1(MxNode),Z1(MxNode)

INNUM0=10
NOP=0
PI=ACOS(-1.)

WRITE(*,'The data for the section can be read in several'
WRITE(*,'parts, where each part is a single node point or a'
WRITE(*,'series of node points. Note, the node points can'
WRITE(*,'be read in both forward and reverse order from'
WRITE(*,'the input files.')
WRITE(*,'How many parts is the section composed of?'
READ(*,NOSECT)

DO 100 ISECT=1,NOSECT

INNUM=INNUM0+ISECT
WRITE(*,200)ISECT,INNUM
READ(INNUM,*NP
WRITE(*,201)NP
READ(*,N1
WRITE(*,202)
READ(*,N2

N12=IBS(N2-N1)+1
NOP=NOP+N12

IF(N2.GE.N1)THEN
DO 10 J=1,N1-1
   READ(INNUM,*)
CONTINUE
C
I=NOP-N12
DO 20 J=N1,N2
   I=I+1
   READ(INNUM,*)X1(I),Y1(I),Z1(I)
CONTINUE
C
ELSE
C
DO 30 J=1,N2-1
   READ(INNUM,*)
CONTINUE
C
I=NOP+1
DO 40 J=N2,N1
   I=I-1
   READ(INNUM,*)X1(I),Y1(I),Z1(I)
CONTINUE
ENDIF
C
CLOSE(INNUM)
CONTINUE
C
FORMAT(' Part ',I2, ' will be read in from UNIT ',I3)
FORMAT(' There are ',I2, ' node points in the file.',/
   ' Enter the number of the node point that is to be the',/
   ' the first on the current part.' )
FORMAT(' Enter the number of the node point that is to be the',/
   ' the last on the current part.' )
RETURN
END
C
C ........................................................................
C
SUBROUTINE GETSTR(NEI,ISTR,BETA1,BETA2)
C
C This subroutine reads information regarding the stretching function
C along curve section NEI.
C
WRITE(*,2011)NEI
WRITE(*,2020)
I00 READ(*,*)ISTR
IF(ISTR.LT.4 .AND. ISTR.GT.0)THEN
   WRITE(*,2031)NEI
   READ(*,*)BETA1
ELSEIF(ISTR.EQ.4)THEN
   WRITE(*,2036)NEI
   READ(*,*)BETA1,BETA2
ELSEIF(ISTR.EQ.0)THEN
   BETA1=1.1
ELSE
   WRITE(*,'*') ' Please enter a number from 0 to 4'
   GOTO I00
ENDIF
C
2011 FORMAT(' Enter the STRETCH TYPE for section',I2)
2020 FORMAT(/,' Enter: 0 for no stretching',/,
      '1 for concentration near lower boundary',/,
      '2 for concentration near upper boundary',/,
      '3 for concentration near both boundaries',/,
      '4 for two-parameter stretching function')
2031 FORMAT(' Enter the stretching parameter (BETA) for section ',
      '/I2)
2036 FORMAT(' Enter the stretching parameters (BETA1 and BETA2) ',
      '/for section ',I2)
C
RETURN
END
A.6 Listing of GRIDTST

PROGRAM GRIDTST
PARAMETER (IM=11, JM=51, KM=151)

C---------------------------------------------------------------------
C This program is used to test whether all Jacobians for a grid system are positive.
C---------------------------------------------------------------------

DIMENSION X(IM, JM, KM), Y(IM, JM, KM), Z(IM, JM, KM)

DIMENSION DXDXI(IM), DXDET(IM), DXDZET(IM), DYDXI(IM), DYDET(IM)
/, DYDZET(IM), DZDXI(IM), DZDET(IM), DZDZET(IM)

Read in the grid point coordinates.
READ(1,*) IL
READ(1,*) JL
READ(1,*) KL
DO 5 I=1, IL
DO 5 J=1, JL
DO 5 K=1, KL
READ(1,*) X(I, J, K), Y(I, J, K), Z(I, J, K)
CONTINUE

DXI=I./FLOAT(IL)
DET=J./FLOAT(JL)
DZET=K./FLOAT(KL)

Calculate the metric coefficients at the regular grid points.
INEG=0
DO 30 K=1, KL
DO 20 J=1, JL
CALL DDXIJK(DXI, IL, JL, KL, X, Y, Z, J, K
/, DXDXI, DYDXI, DZDXI, IM, JM, KM)
CALL DDETKJ(DET, IL, JL, KL, X, Y, Z, J, K
/, DXDET, DYDET, DZDET, IM, JM, KM)
CALL DDZETJK(DZET, IL, JL, KL, X, Y, Z, J, K
/, DZDXI, DZDET, DZDZET, IM, JM, KM)
DO 10 I=1, IL
RJACB=/
/(DXDXI(I)*DYDET(I)*DZDZET(I)-DZDET(I)*DYDZET(I))
+/DYDZET(I)*DXDET(I)*DZDZET(I)-DXDET(I)*DZDZET(I))
+/DZDXI(I)*DXDET(I)*DYDZET(I)-DYDET(I)*DZDZET(I))
IF (RJACB.LE.0) WRITE (2, 50) I, J, K, RJACB
IF (RJACB.LE.0) INEG=INEG+1
CONTINUE
CONTINUE
CONTINUE
IF (INEG.GT.0) WRITE (*,*) ' NEGATIVE JACOBIANS FOUND'
SUBROUTINE DDXIJK (DXI, IL, JL, KL, X, Y, Z, J, K, DXDXI
/ ,DYDXI, DZDXI, IM, JM, KM)

C This subroutine calculates the derivatives of X, Y, and Z
(i.e., the coordinates of the Cartesian coordinate system)
with respect to the coordinate XI of the transformed coordinates.

DIMENSION X(IM, JM, KM), Y(IM, JM, KM), Z(IM, JM, KM)
/ ,DXDXI(IM), DYDXI(IM), DZDXI(IM)

DXI2=2.*DXI
DXDXI(1) = (-X(3, J, K)+4.*X(2, J, K)-3.*X(1, J, K))/DXI2
DYDXI(1) = (-Y(3, J, K)+4.*Y(2, J, K)-3.*Y(1, J, K))/DXI2
DZDXI(1) = (-Z(3, J, K)+4.*Z(2, J, K)-3.*Z(1, J, K))/DXI2
DO 10 I=2,1L-I
   DXDXI(I) = (X(I+1, J, K)-X(I-1, J, K))/DXI2
   DYDXI(I) = (Y(I+1, J, K)-Y(I-1, J, K))/DXI2
   DZDXI(I) = (Z(I+1, J, K)-Z(I-1, J, K))/DXI2
10 CONTINUE
DXDXI(IL) = (3.*X(IL, J, K)-4.*X(IL-I, J, K)+X(IL-2, J, K))/DXI2
DYDXI(IL) = (3.*Y(IL, J, K)-4.*Y(IL-I, J, K)+Y(IL-2, J, K))/DXI2
DZDXI(IL) = (3.*Z(IL, J, K)-4.*Z(IL-I, J, K)+Z(IL-2, J, K))/DXI2

RETURN
END

SUBROUTINE DDETJK (DET, IL, JL, KL, X, Y, Z, J, K, DXDET
/ ,DYDET, DZDET, IM, JM, KM)

C This subroutine calculates the derivatives of X, Y, and Z
(i.e., the coordinates of the Cartesian coordinate system)
with respect to the coordinate ETA of the transformed coordinates.

DIMENSION X(IM, JM, KM), Y(IM, JM, KM), Z(IM, JM, KM)
/ ,DXDET(IM), DYDET(IM), DZDET(IM)
DET2 = 2. * DET

IF (J.EQ.1) THEN
   DO 10 I = 1, IL
      DXDET(I) = (-X(I, J+2, K) + 4. * X(I, J+1, K)
                  / -3. * X(I, J, K)) / DET2
      DYDET(I) = (-Y(I, J+2, K) + 4. * Y(I, J+1, K)
                  / -3. * Y(I, J, K)) / DET2
      DZDET(I) = (-Z(I, J+2, K) + 4. * Z(I, J+1, K)
                  / -3. * Z(I, J, K)) / DET2
   CONTINUE
10 ELSE IF (J.EQ.JL) THEN
   DO 20 I = 1, IL
      DXDET(I) = (3. * X(I, J, K) - 4. * X(I, J-1, K)
                  / + X(I, J-2, K)) / DET2
      DYDET(I) = (3. * Y(I, J, K) - 4. * Y(I, J-1, K)
                  / + Y(I, J-2, K)) / DET2
                  / + Z(I, J-2, K)) / DET2
20 CONTINUE
ELSE
   DO 30 I = 1, IL
      DXDET(I) = (X(I, J+1, K) - X(I, J-1, K)
                  / - DZET2)
      DYDET(I) = (Y(I, J+1, K) - Y(I, J-1, K)
                  / - DZET2)
      DZDET(I) = (Z(I, J+1, K) - Z(I, J-1, K)
                  / - DZET2)
30 CONTINUE
ENDIF

RETURN
END

SUBROUTINE DDZETJK(DZET, IL, JL, KL, X, Y, Z, J, K, DXDET, DYDET, DZDET, IM, JM, KM)

This subroutine calculates the derivatives of X, Y, and Z (i.e., the coordinates of the Cartesian coordinate system) with respect to the coordinate ZETA of the transformed coordinates.

DIMENSION X(IM, JM, KM), Y(IM, JM, KM), Z(IM, JM, KM)

DZET2 = 2. * DZET

IF (K.EQ.1) THEN
   DO 10 I = 1, IL
      DXDET(I) = (-X(I, J, K+2) + 4. * X(I, J, K+1)
                  / -3. * X(I, J, K)) / DZET2
      DYDET(I) = (-Y(I, J, K+2) + 4. * Y(I, J, K+1)
                  / -3. * Y(I, J, K)) / DZET2
10 END
DZDZET(I) = (-Z(I, J, K+2) + 4 .* Z(I, J, K+1) - 3 .* Z(I, J, K)) / DZET2

CONTINUE

ELSE IF (K.EQ.KL) THEN
    DO 20 I=I, IL
        DXDZET(I) = (3 .* X(I, J, K) - 4 .* X(I, J, K-1) + X(I, J, K-2)) / DZET2
        DYDZET(I) = (3 .* Y(I, J, K) - 4 .* Y(I, J, K-1) + Y(I, J, K-2)) / DZET2
    CONTINUE

ENDIF

RETURN
END
Appendix B -- LISTING OF PROGRAM GRID3D-v2

PROGRAM GRID3D

PARAMETER (MxSrfs=4, MxBCvs=16, MxBPts=21, MxGSiz=31)

This SUBROUTINE generates a three-dimensional grid system using the
"two-boundary" or "four-boundary" algebraic grid generation techniques.
Boundary surface edge curves are formed from sets of nodal points by
using parametric tension splines. Boundary surfaces are formed by
using the "bi-directional 3-D Hermite interpolation" technique.

INTEGER CrvNum, SrfNum, NSurfs, InNum, OutNum,
  i, j, k, NDPts(4), AAL(MxSrfs), BBL(MxBCvs),
  NGPts(MxBCvs), Type, StrTp, EdgNum, ZoneNo

REAL EtStep, XiStep, ZtStep, AAStep, BBStep,
  SigmaXi, SigmaEt, SigmaZt,
  KXi1, KXi2, KEta1, KEta2, KZeta1, KZeta2,
  BetaXi, BetaEt, BetaZt, BetaAA, BetaBB,
  h1(MxGSiz), h2(MxGSiz), h3(MxGSiz), h4(MxGSiz),
  h5(MxGSiz), h6(MxGSiz), h7(MxGSiz), h8(MxGSiz),
  kS(MxSrfs,MxBCvs,MxGSiz), kl(MxSrfs,MxGSiz),
  k2(MxBCvs,MxGSiz), k3(MxBCvs,MxGSiz), k4(MxBCvs,MxBCvs),
  SigmaAA(MxBCvs), SigmaBB(MxBCvs),
  XB(MxGSiz,4), YB(MxGSiz,4), ZB(MxBCvs,4),
  X1(MxBAVs), X2(MxBAVs), X3(MxBAVs), X4(MxBAVs),
  Y1(MxBAVs), Y2(MxBAVs), Y3(MxBAVs), Y4(MxBAVs),
  Z1(MxBAVs), Z2(MxBAVs), Z3(MxBAVs), Z4(MxBAVs),
  StrB(MxBCvs,4),
  EtStl1(MxBAVs), EtStl2(MxBAVs), EtStl3(MxBAVs),
  EtStl4(MxBAVs), EtStl5(MxBAVs),
  EtStl6(MxBAVs), ZtSt1(MxBAVs), ZtSt2(MxBAVs),
  ZtSt3(MxBAVs), ZtSt4(MxBAVs),
  ZtSt5(MxBAVs), ZtSt6(MxBAVs),

REAL
  PXSIPE(MxBAVs,MxBAVs), PXS2PE(MxBAVs,MxBAVs),
  PYS1PE(MxBAVs,MxBAVs), PYS2PE(MxBAVs,MxBAVs),
  PZS1PE(MxBAVs,MxBAVs), PZS2PE(MxBAVs,MxBAVs),
  PXS3Zt(MxBAVs,MxBAVs), PXS4Zt(MxBAVs,MxBAVs),
  PYS3Zt(MxBAVs,MxBAVs), PYS4Zt(MxBAVs,MxBAVs),
  PZS3Zt(MxBAVs,MxBAVs), PZS4Zt(MxBAVs,MxBAVs),

REAL
  Diag(MxBAVs), OfDiag(MxBAVs), Right(MxBAVs),
  XDerv2(4,MxBAVs), YDerv2(4,MxBAVs),
  ZDerv2(4,MxBAVs),
  x(4,MxBAVs), y(4,MxBAVs),
  z(4,MxBAVs), s(4,MxBAVs),
  zx(4,MxBAVs), zy(4,MxBAVs),
  zz(4,MxBAVs),

REAL PX1PBB(MxBAVs), PX2PBB(MxBAVs),
  PY1PBB(MxBAVs), PY2PBB(MxBAVs),
  PZ1PBB(MxBAVs), PZ2PBB(MxBAVs),
  PX1PAA(MxBAVs), PX2PAA(MxBAVs),
  PY1PAA(MxBAVs), PY2PAA(MxBAVs),
  PZ1PAA(MxBAVs), PZ2PAA(MxBAVs),

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C Specify input and output device unit numbers for Region i. This is
C convenient for running the program on a PC. For a mainframe, you will
C need to use the FORTRAN OPEN and CLOSE statements or alter the input
C to use a namelist.

INNum=7
OUTNum=8

C Read in the grid control information.

CALL RdGrIn(II,JJ,KK,NSurfs,SigmaXi,SigmaEt,SigmaZt,
    $     kXi1,kXi2,kEta1,kEta2,kZeta1,kZeta2,INNum)

C Set various parameters for the grid generation routines.

CALL KFctrs(1,kS,k1,k2,k3,k4,kXi1,kXi2,kEta1,kEta2,
    $     kZeta1,kZeta2,MxSrfs,MxGSiz)
Calculate the grid point spacings in the transformed domain.

XiStep = 1.0 / (II - 1)
EtStep = 1.0 / (JJ - 1)
ZtStep = 1.0 / (KK - 1)

Calculate the boundary surface grid point locations for each surface.

DO 40 SrfNum = 1, NSurfs

Read in the edge curve nodal points and form the boundary surface
edge curves for surface SrfNum by splining.

DO 30 CrvNum = 1, 4
    EdgNum = (SrfNum - 1) * 4 + CrvNum
    READ (InNum, *) Type
    IF (Type.EQ.1) THEN
        CALL RdGrPIn(NGPts(EdgNum), XB, YB, ZB, CrvNum, MxGSiz, InNum)
        CALL CalSt1(NGPts(EdgNum), XB, YB, ZB, CrvNum, EdgNum, $ StrB, MxBCvs, MxGSiz)
    ELSE
        CALL RdCvIn(x, y, z, NDPts, CrvNum, Tensn, InNum, MxBPts, $ StrTp, Betal, Beta2)
        CALL PTSpln(x, y, z, s, zx, zy, zz, Diag, OfDiag, Right, NDPts, $ Tensn(CrvNum), CrvNum, MxBPts)
        CALL CalSt2(EdgNum, NGPts(EdgNum), StrTp, Betal, Beta2, $ StrB, MxBCvs, MxGSiz)
        CALL EdgGPts(CrvNum, EdgNum, NGPts(EdgNum), XB, YB, ZB, StrE, $ x, y, z, s, zx, zy, zz, NDPts, Tensn(CrvNum), $ MxBCvs, MxBPts, MxGSiz)
    ENDIF
30 CONTINUE

Calculate the boundary surface edge derivative values for surface SrfNum.

CALL EdgDer(PX1PAA, PX2PAA, PY1PAA, PY2PAA, P21PAA, P22PAA,
Calculate the boundary surface grid point locations for surface SrfNum.

CALL TwoBnd(XS,YS,ZS,SrfNum,AAL(SrfNum),BBL(SrfNum),
SigmaBB(SrfNum),k1,k2,StrB,
h1,h2,h3,h4,X1,X2,X3,X4,
Y1,Y2,Y3,Y4,Z1,Z2,Z3,Z4,PX1PBB,PX2PBB,
PY1PBB,PY2PBB,PZ1PBB,PZ2PBB,PX1PAA,PX2PAA,
PY1PAA,PY2PAA,PZ1PAA,PZ2PAA,PX3PBB,PX4PBB,
PY3PBB,PY4PBB,PZ3PBB,PZ4PBB,
MxBCvs,MxGSiz,MxSrfs)

CALL ForBnd(XS,YS,ZS,SrfNum,AAL(SrfNum),BBL(SrfNum),
SigmaAA(SrfNum),SigmaBB(SrfNum),
k3,k4,StrB,
h1,h2,h3,h4,h5,h6,h7,h8,
X1,X2,X3,X4,Y1,Y2,Y3,Y4,Z1,Z2,Z3,Z4,
PX1PBB,PX2PBB,PY1PBB,PY2PBB,PZ1PBB,PZ2PBB,
PX1PAA,PX2PAA,PY1PAA,PY2PAA,PZ1PAA,PZ2PAA,
PX3PBB,PX4PBB,PY3PBB,PY4PBB,PZ3PBB,PZ4PBB,
PX3PAA,PX4PAA,PY3PAA,PY4PAA,PZ3PAA,PZ4PAA,
MxBCvs,MxGSiz,MxSrfs)

CONTINUE

IF(NSurfs.EQ.2)THEN
  DO 60 SrfNum=3,4
    DO 50 CrvNum=3,4
      EdgNum=(SrfNum-1)*4 + CrvNum
      READ(InNum,*)StrTp
      IF(StrTp.NE.4)THEN
        READ(InNum,*)Betal
      ELSE
        READ(InNum,*)Betal,Beta2
      ENDIF
      CALL CalSt2(EdgNum,NGPts(EdgNum),StrTp,Betal,Beta2,
       StrB,MxBCvs,MxGSiz)
  CONTINUE
  CONTINUE
ENDIF

Calculate the interior grid point locations.

CALL TwoSrf(XPnt,YPnt,ZPnt,II,JJ,KK,SigmaEt,kS,
EtStl1,EtStl2,EtStl5,EtStl6,
XiStep,EtStep,ZtStep,XS,YS,ZS,h1,h2,h3,h4,
PX1PE,PX2PE,PYS1PE,PYS2PE,PZS1PE,
PZS2PE,MxGSiz,MxSrfs)

IF (NSurfs.EQ.4) THEN
  CALL ForSrf(XPnt,YPnt,ZPnt,II,JJ,KK,
  SigmaEt,SigmaZt,kS,
  EtStl1,EtStl2,EtStl5,EtStl6,
This SUBROUTINE calculates the grid point locations between two specified surfaces using the "two-boundary technique".

INTEGER i, j, k, StrXi, StrEt, StrZt, II, JJ, KK
REAL Xi, Eta, Zeta, XiNew, EtaNew, ZetaNew, LL1, LL2,
$PXSIxi, PXSIxi, PXSIxi, PXSIxi, PXSIxi, PXSIxi, PXSIxi, PXSIxi,$
$PXSIxi, PXSIxi, PXSIxi, PXSIxi, PXSIxi, PXSIxi, PXSIxi, PXSIxi,$
BetaXi, BetaEt, BetaZt, XiStep, EtStep, ZtStep,
EtSt11(MxGSiz), EtSt12(MxGSiz),
EtSt15(MxGSiz), EtSt16(MxGSiz),
kS(MxSrfs, MxGSiz, MxGSiz),
h1(MxGSiz), h2(MxGSiz), h3(MxGSiz), h4(MxGSiz),
PXSIPE(MxGSiz, MxGsz), PXSIPE(MxGSiz, MxGsz),
PXSIPE(MxGSiz, MxGsz), PXSIPE(MxGSiz, MxGsz),
PXSIPE(MxGSiz, MxGsz), PXSIPE(MxGSiz, MxGsz),
PXSIPE(MxGSiz, MxGsz), PXSIPE(MxGSiz, MxGsz),
PXSIPE(MxGSiz, MxGsz), PXSIPE(MxGSiz, MxGsz),
PXSIPE(MxGSiz, MxGsz), PXSIPE(MxGSiz, MxGsz),
$XS(MxSrfs, MxGsiz, MxGsiz),
YS(MxSrfs, MxGsiz, MxGsiz),
ZS(MxSrfs, MxGsiz, MxGsiz),
XPnt(MxGSiz, MxGsiz, MxGsiz),
YPnt(MxGSiz, MxGsiz, MxGsiz),
ZPnt(MxGSiz, MxGsiz, MxGsiz)

Calculate the derivative values along the constant Xi/Zeta boundaries.

FXSIxi=($X$S(1,1,2)-$X$S(1,1,1))/XiStep
FXSIxi=($X$S(2,1,2)-$X$S(2,1,1))/XiStep
FXSIxi=($Y$S(1,1,2)-$Y$S(1,1,1))/XiStep
FXSIxi=($Y$S(2,1,2)-$Y$S(2,1,1))/XiStep
FXSIxi=($Z$S(1,1,2)-$Z$S(1,1,1))/XiStep
FXSIxi=($Z$S(2,1,2)-$Z$S(2,1,1))/XiStep
FXSIxi=($X$S(1,2,1)-$X$S(1,1,1))/ZtStep
FXSIxi=($X$S(2,2,1)-$X$S(2,1,1))/ZtStep
FXSIxi=($Y$S(1,2,1)-$Y$S(1,1,1))/ZtStep
FXSIxi=($Y$S(2,2,1)-$Y$S(2,1,1))/ZtStep
FXSIxi=($Z$S(1,2,1)-$Z$S(1,1,1))/ZtStep
FXSIxi=($Z$S(2,2,1)-$Z$S(2,1,1))/ZtStep
PZS1Zt = (ZS(1, 1, i) - ZS(1, 1, I))/ZtStep
PZS2Zt = (ZS(2, 1, i) - ZS(2, 1, I))/ZtStep
LL1 = ((PYS1Xi*PZS1Zt - PZS1Xi*PYS1Zt)**2
+ (PXS1Xi*PZS1Zt - PZS1Xi*PXS1Zt)**2)**0.5
+ (PXS1Xi*PYS1Zt - PYS1Xi*PXS1Zt)**2)
LL2 = ((PYS2Xi*PZS2Zt - PZS2Xi*PYS2Zt)**2
+ (PXS2Xi*PZS2Zt - PZS2Xi*PXS2Zt)**2)**0.5

PXSIPE(I, I) = -kS(I, I, I)*(PYSIXi*PZSIZt - PZSIXi*PYSIZt)/LL1
PXS2PE(I, I) = -kS(2, I, I)*(PYS2Xi*PZS2Zt - PZS2Xi*PXS2Zt)/LL2
PYS1PE(I, I) = kS(I, I, I) * (PXSIXi*PZSIZt - PZSIXi*PXSIZt)/LL1
PYS2PE(I, I) = kS(2, I, I)*(PXS2Xi*PZS2Zt - PZS2Xi*PXS2Zt)/LL2
PZSIPE(I, I) = -kS(I, I, I)*(PYSIXi*PZSIZt - PZSIXi*PYSIZt)/LL1
PZS2PE(I, I) = -kS(2, I, I) * (PYS2Xi*PZS2Zt - PZS2Xi*PYS2Zt)/LL2

DO 55 i = 2, II - 1
PXSIPE(I, I) = -kS(I, I, I)* (PYSIXi*PZSIZt - PZSIXi*PYSIZt)/LL1
PXS2PE(I, I) = -kS(2, I, I) * (PYS2Xi*PZS2Zt - PZS2Xi*PXS2Zt)/LL2
PYS1PE(I, I) = kS(I, I, I) *(PXSIXi*PZSIZt - PZSIXi*PXSIZt)/LL1
PYS2PE(I, I) = kS(2, I, I) * (PXS2Xi*PZS2Zt - PZS2Xi*PXS2Zt)/LL2
PZSIPE(I, I) = -kS(I, I, I) * (PYSIXi*PZSIZt - PZSIXi*PYSIZt)/LL1
PZS2PE(I, I) = -kS(2, I, I) * (PYS2Xi*PZS2Zt - PZS2Xi*PYS2Zt)/LL2

DO 55 i = 2, II - 1
PXSIPE(I, I) = -kS(I, I, I)* (PYSIXi*PZSIZt - PZSIXi*PYSIZt)/LL1
PXS2PE(I, I) = -kS(2, I, I) * (PYS2Xi*PZS2Zt - PZS2Xi*PXS2Zt)/LL2
PYS1PE(I, I) = kS(I, I, I) *(PXSIXi*PZSIZt - PZSIXi*PXSIZt)/LL1
PYS2PE(I, I) = kS(2, I, I) * (PXS2Xi*PZS2Zt - PZS2Xi*PXS2Zt)/LL2
PZSIPE(I, I) = -kS(I, I, I) * (PYSIXi*PZSIZt - PZSIXi*PYSIZt)/LL1
PZS2PE(I, I) = -kS(2, I, I) * (PYS2Xi*PZS2Zt - PZS2Xi*PYS2Zt)/LL2

DO 55 i = 2, II - 1
PXSIPE(I, I) = -kS(I, I, I)* (PYSIXi*PZSIZt - PZSIXi*PYSIZt)/LL1
PXS2PE(I, I) = -kS(2, I, I) * (PYS2Xi*PZS2Zt - PZS2Xi*PXS2Zt)/LL2
PYS1PE(I, I) = kS(I, I, I) *(PXSIXi*PZSIZt - PZSIXi*PXSIZt)/LL1
PYS2PE(I, I) = kS(2, I, I) * (PXS2Xi*PZS2Zt - PZS2Xi*PXS2Zt)/LL2
PZSIPE(I, I) = -kS(I, I, I) * (PYSIXi*PZSIZt - PZSIXi*PYSIZt)/LL1
PZS2PE(I, I) = -kS(2, I, I) * (PYS2Xi*PZS2Zt - PZS2Xi*PYS2Zt)/LL2
\[ C \]
\[
L_{1i} = \left( (P_{YSi}X_{i}P_{ZSi}Z_{i} - P_{ZSi}X_{i}P_{YSi}Z_{i})^2 + (P_{XSi}X_{i}P_{ZSi}Z_{i} - P_{ZSi}X_{i}P_{XSi}Z_{i})^2 + (P_{XSi}X_{i}P_{YSi}Z_{i} - P_{YSi}X_{i}P_{XSi}Z_{i})^2 \right)^{0.5}
\]
\[
P_{XSi}PE(i, I) = -k_{S}(i, i, I) \frac{(P_{YSi}X_{i}P_{ZSi}Z_{i} - P_{ZSi}X_{i}P_{YSi}Z_{i})}{L_{1i}}
\]
\[
P_{XS2i}PE(i, I) = -k_{S}(2, i, I) \frac{(P_{YSi}X_{i}P_{ZSi}Z_{i} - P_{ZSi}X_{i}P_{YSi}Z_{i})}{L_{2i}}
\]
\[
P_{YSi}PE(i, I) = k_{S}(I, i, I) \frac{(P_{XSi}X_{i}P_{ZSi}Z_{i} - P_{ZSi}X_{i}P_{YSi}Z_{i})}{L_{1i}}
\]
\[
P_{YS2i}PE(i, I) = k_{S}(2, i, I) \frac{(P_{XSi}X_{i}P_{ZSi}Z_{i} - P_{ZSi}X_{i}P_{YSi}Z_{i})}{L_{2i}}
\]
\[
P_{YSi}PE(i, I) = -k_{S}(I, i, I) \frac{(P_{XSi}X_{i}P_{YSi}Z_{i} - P_{YSi}X_{i}P_{XSi}Z_{i})}{L_{1i}}
\]
\[
P_{YS2i}PE(i, I) = -k_{S}(2, i, I) \frac{(P_{XSi}X_{i}P_{YSi}Z_{i} - P_{YSi}X_{i}P_{XSi}Z_{i})}{L_{2i}}
\]
\[ C \]
\[
DO 70 k = 2, KK-1
\]
\[
P_{XS1i} = (X_{1}(k, 1, 2) - X_{1}(k, 1, 1))/X_{i}Step
\]
\[
P_{XS2i} = (X_{2}(k, 1, 2) - X_{2}(k, 1, 1))/X_{i}Step
\]
\[
P_{YS1i} = (Y_{1}(k, 1, 2) - Y_{1}(k, 1, 1))/Y_{i}Step
\]
\[
P_{YS2i} = (Y_{2}(k, 1, 2) - Y_{2}(k, 1, 1))/Y_{i}Step
\]
\[
P_{ZS1i} = (Z_{1}(k, 1, 2) - Z_{1}(k, 1, 1))/Z_{i}Step
\]
\[
P_{ZS2i} = (Z_{2}(k, 1, 2) - Z_{2}(k, 1, 1))/Z_{i}Step
\]
\[
P_{XS1t} = (X_{1}(k+1, 1, 1) - X_{1}(k-1, 1, 1))/2/Z_{t}Step
\]
\[
P_{XS2t} = (X_{2}(k+1, 1, 1) - X_{2}(k-1, 1, 1))/2/Z_{t}Step
\]
\[
P_{YS1t} = (Y_{1}(k+1, 1, 1) - Y_{1}(k-1, 1, 1))/2/Z_{t}Step
\]
\[
P_{YS2t} = (Y_{2}(k+1, 1, 1) - Y_{2}(k-1, 1, 1))/2/Z_{t}Step
\]
\[
P_{ZS1t} = (Z_{1}(k+1, 1, 1) - Z_{1}(k-1, 1, 1))/2/Z_{t}Step
\]
\[
P_{ZS2t} = (Z_{2}(k+1, 1, 1) - Z_{2}(k-1, 1, 1))/2/Z_{t}Step
\]
\[
L_{1i} = \left( (P_{YSi}X_{i}P_{ZSi}Z_{i} - P_{ZSi}X_{i}P_{YSi}Z_{i})^2 + (P_{XSi}X_{i}P_{ZSi}Z_{i} - P_{ZSi}X_{i}P_{XSi}Z_{i})^2 + (P_{XSi}X_{i}P_{YSi}Z_{i} - P_{YSi}X_{i}P_{XSi}Z_{i})^2 \right)^{0.5}
\]
\[
L_{2i} = \left( (P_{YS2i}X_{i}P_{ZS2i}Z_{i} - P_{ZS2i}X_{i}P_{YS2i}Z_{i})^2 + (P_{XS2i}X_{i}P_{ZS2i}Z_{i} - P_{ZS2i}X_{i}P_{XS2i}Z_{i})^2 + (P_{XS2i}X_{i}P_{YS2i}Z_{i} - P_{YS2i}X_{i}P_{XS2i}Z_{i})^2 \right)^{0.5}
\]
\[ C \]
\[
P_{XS1PE}(i, 1) = -k_{S}(1, i, 1) \frac{(P_{YSi}X_{i}P_{ZSi}Z_{i} - P_{ZSi}X_{i}P_{YSi}Z_{i})}{L_{1i}}
\]
\[
P_{XS2PE}(i, 1) = -k_{S}(2, i, 1) \frac{(P_{YSi}X_{i}P_{ZSi}Z_{i} - P_{ZSi}X_{i}P_{YSi}Z_{i})}{L_{2i}}
\]
\[
P_{YS1PE}(i, 1) = k_{S}(1, i, 1) \frac{(P_{XSi}X_{i}P_{ZSi}Z_{i} - P_{ZSi}X_{i}P_{YSi}Z_{i})}{L_{1i}}
\]
\[
P_{YS2PE}(i, 1) = k_{S}(2, i, 1) \frac{(P_{XSi}X_{i}P_{ZSi}Z_{i} - P_{ZSi}X_{i}P_{YSi}Z_{i})}{L_{2i}}
\]
\[
P_{ZS1PE}(i, 1) = -k_{S}(1, i, 1) \frac{(P_{XSi}X_{i}P_{YSi}Z_{i} - P_{YSi}X_{i}P_{XSi}Z_{i})}{L_{1i}}
\]
\[
P_{ZS2PE}(i, 1) = -k_{S}(2, i, 1) \frac{(P_{XSi}X_{i}P_{YSi}Z_{i} - P_{YSi}X_{i}P_{XSi}Z_{i})}{L_{2i}}
\]
PXSIPE(I, k) =-kS(I, II, k) * (PYSIXi*PZSIZt-PZSIXi*PYSIZt)/LLI
PXSIPE(I, k) = kS(I, II, k) * (PXSIXi*PZSIZt-PZSIXi*PXSIZt)/LLI
PXSIPE(I, k) = kS(2, II, k) * (PXS2Xi*PZS2Zt-PZS2Xi*PXS2Zt)/LL2
PXSIPE(I, k) = kS(I, II, k) * (PYSIXi*PYSIZt-PYSIXi*PYStZt)/LL1
PXSIPE(I, k) = kS(2, II, k) * (PXS2Xi*PYS2Zt-PXS2Xi*PXS2Zt)/LL2
PXSIPE(I, k) = kS(I, II, k) * (PXSIXi*PYSIZt-PYSIXi*PXStZt)/LL1
PXSIPE(I, k) = kS(2, II, k) * (PXS2Xi*PYS2Zt-PXS2Xi*PXS2Zt)/LL2
PXSIPE(I, k) = kS(I, II, k) * (PYSIXi*PYSIZt-PYSIXi*PYStZt)/LL1
PXSIPE(I, k) = kS(2, II, k) * (PXS2Xi*PYS2Zt-PXS2Xi*PXS2Zt)/LL2
PXSIPE(I, k) = kS(I, II, k) * (PYSIXi*PYSIZt-PYSIXi*PYStZt)/LL1
PXSIPE(I, k) = kS(2, II, k) * (PXS2Xi*PYS2Zt-PXS2Xi*PXS2Zt)/LL2
PXSIPE(I, k) = kS(I, II, k) * (PYSIXi*PYSIZt-PYSIXi*PYStZt)/LL1
PXSIPE(I, k) = kS(2, II, k) * (PXS2Xi*PYS2Zt-PXS2Xi*PXS2Zt)/LL2
PXSIPE(I, k) = kS(I, II, k) * (PYSIXi*PYSIZt-PYSIXi*PYStZt)/LL1
PXSIPE(I, k) = kS(2, II, k) * (PXS2Xi*PYS2Zt-PXS2Xi*PXS2Zt)/LL2

DO 60 i=2, II-1

PXSI2Xi= (XS(I, k, i+1) -XS(I, k, i-1))/2/XiStep
PXSI2Xi= (XS(2, k, i+1) -XS(2, k, i-1))/2/XiStep
PXSI2Xi= (YS(I, k, i+1) -YS(I, k, i-1))/2/XiStep
PXSI2Xi= (YS(2, k, i+1) -YS(2, k, i-1))/2/XiStep
PXSI2Zt= (XS(I, k, i+1) -XS(I, k, i-1))/2/ZtStep
PXSI2Zt= (XS(2, k, i+1) -XS(2, k, i-1))/2/ZtStep
PXSI2Zt= (YS(I, k, i+1) -YS(I, k, i-1))/2/ZtStep
PXSI2Zt= (YS(2, k, i+1) -YS(2, k, i-1))/2/ZtStep

LLI= ((PYSIXi*PZSIZt-PZSIXi*PYSIZt)**2
        + (PXSIXi*PZSIZt-PZSIXi*PXSIZt)**2
        + (PYSIXi*PYSIZt-PYSIXi*PXSIZt)**2)**0.5
LL2= ((PYS2Xi*PZS2Zt-PZS2Xi*PYS2Zt)**2
        + (PXS2Xi*PZS2Zt-PZS2Xi*PXS2Zt)**2
        + (PYS2Xi*PYS2Zt-PYS2Xi*PXS2Zt)**2)**0.5

CONTINUE

C

PXSIPE(I, K) =-kS(1, I, K) * (PYS1X1*PZS1Zt-PZS1X1*PYS1Zt)/LL1
PXSIPE(1, K) = kS(2, I, K) * (PXS2Xi*PZS2Zt-PZS2Xi*PYS2Zt)/LL2
PXSIPE(1, K) = kS(I, I, K) * (PXS1Xi*PZS1Zt-PZS1Xi*PYS1Zt)/LL1
PXSIPE(2, K) = kS(2, I, K) * (PXS2Xi*PZS2Zt-PZS2Xi*PYS2Zt)/LL2
PXSIPE(1, K) = kS(1, I, K) * (PXS1Xi*PYS1Zt-PYS1Xi*PXStZt)/LL1
PXSIPE(2, K) = kS(2, I, K) * (PXS2Xi*PYS2Zt-PXS2Xi*PXS2Zt)/LL2

CONTINUE

C

PXSIPE(I, K) =-kS(1, I, K) * (PYS1X1*PZS1Zt-PZS1X1*PYS1Zt)/LL1
PXSIPE(1, K) = kS(2, I, K) * (PXS2Xi*PZS2Zt-PZS2Xi*PYS2Zt)/LL2
PXSIPE(1, K) = kS(I, I, K) * (PXS1Xi*PZS1Zt-PZS1Xi*PYS1Zt)/LL1
PXSIPE(2, K) = kS(2, I, K) * (PXS2Xi*PYS2Zt-PXS2Xi*PXS2Zt)/LL2
PXSIPE(1, K) = kS(1, I, K) * (PXS1Xi*PYS1Zt-PYS1Xi*PXStZt)/LL1
PXSIPE(2, K) = kS(2, I, K) * (PXS2Xi*PYS2Zt-PXS2Xi*PXS2Zt)/LL2

CONTINUE

C
\[ PYS2PE(1, KK) = kS(2, 1, KK) \times (PXS2Xi*PZS2Zt-PZS2Xi*PXS2Zt)/LL2 \]
\[ PZS1PE(1, KK) = -kS(1, 2, KK) \times (PXS1Xi*PYS1Zt-PYS1Xi*PXS1Zt)/LL1 \]
\[ PZS2PE(1, KK) = -kS(2, 1, KK) \times (PXS2Xi*PYS2Zt-PYS2Xi*PXS2Zt)/LL1 \]
\[ PXS1PE(i, KK) = (XS(1, KK, ii) - XS(1, KK, iii))/XiStep \]
\[ PXS2PE(i, KK) = (XS(2, KK, ii) - XS(2, KK, iii))/XiStep \]
\[ PYS1PE(i, KK) = (YS(1, KK, ii) - YS(1, KK, iii))/XiStep \]
\[ PYS2PE(i, KK) = (YS(2, KK, ii) - YS(2, KK, iii))/XiStep \]
\[ PZS1PE(i, KK) = (ZS(1, KK, ii) - ZS(1, KK, iii))/XiStep \]
\[ PZS2PE(i, KK) = (ZS(2, KK, ii) - ZS(2, KK, iii))/XiStep \]
\[ LL1 = ((PYS1Xi*PZS1Zt-PZS1Xi*PYS1Zt)**2 + (PXS1Xi*PZS1Zt-PZS1Xi*PXS1Zt)**2 + (PXS1Xi*PYS1Zt-PYS1Xi*PXS1Zt)**2)**0.5 \]
\[ LL2 = ((PYS2Xi*PZS2Zt-PZS2Xi*PYS2Zt)**2 + (PXS2Xi*PZS2Zt-PZS2Xi*PXS2Zt)**2 + (PXS2Xi*PYS2Zt-PYS2Xi*PXS2Zt)**2)**0.5 \]
\[ CONT INUE \]

DO 75 \( i = 2, II-1 \)
\[ PXS1Xi = (XS(1, KK, i) - XS(1, KK, i)/2)/XiStep \]
\[ PXS2Xi = (XS(2, KK, i) - XS(2, KK, i)/2)/XiStep \]
\[ PYS1Xi = (YS(1, KK, i) - YS(1, KK, i)/2)/XiStep \]
\[ PYS2Xi = (YS(2, KK, i) - YS(2, KK, i)/2)/XiStep \]
\[ PZS1Xi = (ZS(1, KK, i) - ZS(1, KK, i)/2)/XiStep \]
\[ PZS2Xi = (ZS(2, KK, i) - ZS(2, KK, i)/2)/XiStep \]
\[ PXS1Zt = (XS(1, KK, i) - XS(1, KK, i)/2)/ZtStep \]
\[ PXS2Zt = (XS(2, KK, i) - XS(2, KK, i)/2)/ZtStep \]
\[ PYS1Zt = (YS(1, KK, i) - YS(1, KK, i)/2)/ZtStep \]
\[ PYS2Zt = (YS(2, KK, i) - YS(2, KK, i)/2)/ZtStep \]
\[ PZS1Zt = (ZS(1, KK, i) - ZS(1, KK, i)/2)/ZtStep \]
\[ PZS2Zt = (ZS(2, KK, i) - ZS(2, KK, i)/2)/ZtStep \]
\[ LL1 = ((PYS1Xi*PZS1Zt-PZS1Xi*PYS1Zt)**2 + (PXS1Xi*PZS1Zt-PZS1Xi*PXS1Zt)**2 + (PXS1Xi*PYS1Zt-PYS1Xi*PXS1Zt)**2)**0.5 \]
\[ LL2 = ((PYS2Xi*PZS2Zt-PZS2Xi*PYS2Zt)**2 + (PXS2Xi*PZS2Zt-PZS2Xi*PXS2Zt)**2 + (PXS2Xi*PYS2Zt-PYS2Xi*PXS2Zt)**2)**0.5 \]

CONT INUE
C Calculate the interior grid point locations.

DO 100 k=1,KK
    Zeta=(k-1.)/(KK-1.)
DO 90 i=1,II
    Xi=(i-1.)/(II-1.)
DO 80 j=1,JJ
    EtaNew=(EtSt11(j)*(1.-Xi)+EtSt12(j)*Xi)*(1.-Zeta)
        +(EtSt15(j)*(1.-Xi)+EtSt16(j)*Xi)*Zeta
    CALL FindHs(h1(j),h2(j),h3(j),h4(j),EtaNew,SigmaEt)
    XPnt(i,j,k)=h1(j)
        *XS(1,k,i)+h2(j)*XS(2,k,i)
        +h3(j)*PXS1PE(i,k)
        +h4(j)*PXS2PE(i,k)
    YPnt(i,j,k)=h1(j)
        *YS(1,k,i)+h2(j)*YS(2,k,i)
        +h3(j)*PYS1PE(i,k)
        +h4(j)*PYS2PE(i,k)
    ZPnt(i,j,k)=h1(j)
        *ZS(1,k,i)+h2(j)*ZS(2,k,i)
        +h3(j)*PZS1PE(i,k)
        +h4(j)*PZS2PE(i,k)
80 CONTINUE
90 CONTINUE
100 CONTINUE
C
RETURN
END

C=========================================================================
C
SUBROUTINE ForSrf(XPnt,YPnt,ZPnt,II,JJ,KK,SigmaEt,SigmaZt,kS,
    EtSt11,EtSt12,EtSt15,EtSt16,
    ZtSt1,ZtSt2,ZtSt5,ZtSt6,
    XS,YS,ZS,XiStep,EtStep,ZtStep,
    h1,h2,h3,h4,h5,h6,h7,h8,
    PXS1PE,PXS2PE,PYS1PE,PYS2PE,PZS1PE,PZS2PE,
    PXS3PE,PXS4PE,PYS3PE,PYS4PE,PZS3PE,PZS4PE,
    P2X00,P2X01,P2X10,P2X11,P2Y00,P2Y01,P2Y10,P2Y11,
    P2Z00,P2Z01,P2Z10,P2Z11
    BetaXi,BetaEt,BetaZt,XiStep,EtStep,ZtStep

This SUBROUTINE adjusts the grid so that the other two surfaces of the
region are mapped correctly using the "four-boundary technique".

INTEGER i, j, k, StrXi, StrEt, StrZt, II, JJ, KK
REAL Xi, Eta, Zeta, XiNew, EtaNew, ZetaNew, LL3, LL4,
    h1(MxGSiz), h2(MxGSiz), h3(MxGSiz), h4(MxGSiz),
    h5(MxGSiz), h6(MxGSiz), h7(MxGSiz), h8(MxGSiz),
    PXS3Xi, PXS4Xi, PYs3Xi, PYs4Xi, PZS3Xi, PZS4Xi,
    PXS3PE, PXS4PE, PYS3PE, PYS4PE, PZS3PE, PZS4PE,
    P2X00, P2X01, P2X10, P2X11, P2Y00, P2Y01, P2Y10, P2Y11,
    P2Z00, P2Z01, P2Z10, P2Z11
REAL BetaXi, BetaEt, BetaZt, XiStep, EtStep, ZtStep,
    EtSt11(MxGSiz),EtSt12(MxGSiz),
    EtSt15(MxGSiz),EtSt16(MxGSiz),
    ZtSt1(MxGSiz),ZtSt2(MxGSiz),
    ZtSt5(MxGSiz),ZtSt6(MxGSiz),
Calculate the derivative values along the constant Xi/Eta boundaries.

\[
\begin{align*}
PXS3Xi &= (XS(3, 2, 1) - XS(3, 1, 1))/XiStep \\
PXS4Xi &= (XS(4, 2, 1) - XS(4, 1, 1))/XiStep \\
PY3Xi &= (YS(3, 2, 1) - YS(3, 1, 1))/XiStep \\
PYS4Xi &= (YS(4, 2, 1) - YS(4, 1, 1))/XiStep \\
PZ3Xi &= (ZS(3, 2, 1) - ZS(3, 1, 1))/XiStep \\
PZ4Xi &= (ZS(4, 2, 1) - ZS(4, 1, 1))/XiStep \\
PXS3PE &= (XS(3, 1, 2) - XS(3, 1, 1))/EtStep \\
PXS4PE &= (XS(4, 1, 2) - XS(4, 1, 1))/EtStep \\
PY3PE &= (YS(3, 1, 2) - YS(3, 1, 1))/EtStep \\
PYS4PE &= (YS(4, 1, 2) - YS(4, 1, 1))/EtStep \\
PZ3PE &= (ZS(3, 1, 2) - ZS(3, 1, 1))/EtStep \\
PZ4PE &= (ZS(4, 1, 2) - ZS(4, 1, 1))/EtStep \\
LL3 &= ((PYS3Xi*PZS3PE - PZS3Xi*PYS3PE)**2 + (PXS3Xi*PZS3PE - PZS3Xi*PXS3PE)**2 + (PXS3Xi*PYS3PE - PYS3Xi*PXS3PE)**2)**0.5 \\
LL4 &= ((PYS4Xi*PZS4PE - PZS4Xi*PYS4PE)**2 + (PXS4Xi*PZS4PE - PZS4Xi*PXS4PE)**2 + (PXS4Xi*PYS4PE - PYS4Xi*PXS4PE)**2)**0.5 \\
PXS3Xi &= (XS(3, II, 1) - XS(3, II-1, 1))/XiStep \\
PXS4Xi &= (XS(4, II, 1) - XS(4, II-1, 1))/XiStep \\
PY3Xi &= (YS(3, II, 1) - YS(3, II-1, 1))/XiStep \\
PYS4Xi &= (YS(4, II, 1) - YS(4, II-1, 1))/XiStep \\
PZ3Xi &= (ZS(3, II, 1) - ZS(3, II-1, 1))/XiStep \\
PZ4Xi &= (ZS(4, II, 1) - ZS(4, II-1, 1))/XiStep \\
PXS3PE &= (XS(3, II, 2) - XS(3, II, 1))/EtStep \\
PXS4PE &= (XS(4, II, 2) - XS(4, II, 1))/EtStep \\
PY3PE &= (YS(3, II, 2) - YS(3, II, 1))/EtStep \\
PYS4PE &= (YS(4, II, 2) - YS(4, II, 1))/EtStep \\
PZ3PE &= (ZS(3, II, 2) - ZS(3, II, 1))/EtStep \\
PZ4PE &= (ZS(4, II, 2) - ZS(4, II, 1))/EtStep \\
LL3 &= ((PYS3Xi*PZS3PE - PZS3Xi*PYS3PE)**2 + (PXS3Xi*PZS3PE - PZS3Xi*PXS3PE)**2 + (PXS3Xi*PYS3PE - PYS3Xi*PXS3PE)**2)**0.5
\end{align*}
\]
DO 45 i=2,II-1
  PXS3Xi = (XS (3, i+1, 1) - XS (3, i-1, 1)) / 2 / XiStep
  PXS4Xi = (XS (4, i+1, 1) - XS (4, i-1, 1)) / 2 / XiStep
  PYS3Xi = (YS (3, i+1, 1) - YS (3, i-1, 1)) / 2 / XiStep
  PYS4Xi = (YS (4, i+1, 1) - YS (4, i-1, 1)) / 2 / XiStep
  PZS3Xi = (ZS (3, i+1, 1) - ZS (3, i-1, 1)) / 2 / XiStep
  PZS4Xi = (ZS (4, i+1, 1) - ZS (4, i-1, 1)) / 2 / XiStep
  PXS3PE = (XS (3, i, 2) - XS (3, i, 1)) / EtStep
  PXS4PE = (XS (4, i, 2) - XS (4, i, 1)) / EtStep
  PYS3PE = (YS (3, i, 2) - YS (3, i, 1)) / EtStep
  PYS4PE = (YS (4, i, 2) - YS (4, i, 1)) / EtStep
  PZS3PE = (ZS (3, i, 2) - ZS (3, i, 1)) / EtStep
  PZS4PE = (ZS (4, i, 2) - ZS (4, i, 1)) / EtStep
  LL3 = ((PYS3Xi*PZS3PE-PZS3Xi*PYS3PE)**2)
  + (PXS3Xi*PYS3PE-PYS3Xi*PXS3PE)**2
  + (PXS3Xi*PYS3PE-PYS3Xi*PXS3PE)**2)*0.5
  CONTINUE

DO 60 j=2, JJ-I
  PXS3Xi = (XS (3, 2, j) - XS (3, 1, j)) / XiStep
  PXS4Xi = (XS (4, 2, j) - XS (4, 1, j)) / XiStep
  PYS3Xi = (YS (3, 2, j) - YS (3, 1, j)) / XiStep
  PYS4Xi = (YS (4, 2, j) - YS (4, 1, j)) / XiStep
  PZS3Xi = (ZS (3, 2, j) - ZS (3, 1, j)) / XiStep
  PZS4Xi = (ZS (4, 2, j) - ZS (4, 1, j)) / XiStep
  PXS3PE = (XS (3, 1, j+1) - XS (3, 1, j-1)) / 2 / EtStep
  PXS4PE = (XS (4, 1, j+1) - XS (4, 1, j-1)) / 2 / EtStep
  PYS3PE = (YS (3, 1, j+1) - YS (3, 1, j-1)) / 2 / EtStep
  PYS4PE = (YS (4, 1, j+1) - YS (4, 1, j-1)) / 2 / EtStep
  PZS3PE = (ZS (3, 1, j+1) - ZS (3, 1, j-1)) / 2 / EtStep
  PZS4PE = (ZS (4, 1, j+1) - ZS (4, 1, j-1)) / 2 / EtStep
  LL3 = ((PYS3Xi*PZS3PE-PZS3Xi*PYS3PE)**2)
  + (PXS3Xi*PYS3PE-PYS3Xi*PXS3PE)**2
  + (PXS3Xi*PYS3PE-PYS3Xi*PXS3PE)**2)*0.5
  CONTINUE
\[
\begin{align*}
\text{PXS3Zt}(1, j) &= -k_S(3, 1, j) \cdot (PYS3Xi \cdot PZS3PE - PZS3Xi \cdot PYS3PE) / LL3 \\
\text{PXS4Zt}(1, j) &= -k_S(4, 1, j) \cdot (PYS4Xi \cdot PZS4PE - PZS4Xi \cdot PYS4PE) / LL4 \\
\text{PYS3Zt}(1, j) &= k_S(3, 1, j) \cdot (PXS3Xi \cdot PZS3PE - PZS3Xi \cdot PXS3PE) / LL3 \\
\text{PYS4Zt}(1, j) &= k_S(4, 1, j) \cdot (PXS4Xi \cdot PZS4PE - PZS4Xi \cdot PXS4PE) / LL4 \\
\text{PZS3Zt}(1, j) &= -k_S(3, 1, j) \cdot (PYS3Xi \cdot PYS3PE - PYS3Xi \cdot PXS3PE) / LL3 \\
\text{PZS4Zt}(1, j) &= -k_S(4, 1, j) \cdot (PXS4Xi \cdot PYS4PE - PYS4Xi \cdot PXS4PE) / LL4 \\
\end{align*}
\]

DO 50 i=2, II-1

\[
\begin{align*}
\text{PXS3Xi} &= (XS(3, i+1, j) - XS(3, i-1, j)) / 2 / XiStep \\
\text{PXS4Xi} &= (XS(4, i+1, j) - XS(4, i-1, j)) / 2 / XiStep \\
\text{PYS3Xi} &= (YS(3, i+1, j) - YS(3, i-1, j)) / 2 / XiStep \\
\text{PYS4Xi} &= (YS(4, i+1, j) - YS(4, i-1, j)) / 2 / XiStep \\
\text{PZS3Xi} &= (ZS(3, i+1, j) - ZS(3, i-1, j)) / 2 / XiStep \\
\text{PZS4Xi} &= (ZS(4, i+1, j) - ZS(4, i-1, j)) / 2 / XiStep \\
\text{PXS3PE} &= (XS(3, i+1, j+1) - XS(3, i, j-1)) / 2 / EtStep \\
\text{PXS4PE} &= (XS(4, i+1, j+1) - XS(4, i, j-1)) / 2 / EtStep \\
\text{PYS3PE} &= (YS(3, i+1, j+1) - YS(3, i, j-1)) / 2 / EtStep \\
\text{PYS4PE} &= (YS(4, i+1, j+1) - YS(4, i, j-1)) / 2 / EtStep \\
\text{PZS3PE} &= (ZS(3, i+1, j+1) - ZS(3, i, j-1)) / 2 / EtStep \\
\text{PZS4PE} &= (ZS(4, i+1, j+1) - ZS(4, i, j-1)) / 2 / EtStep \\
\end{align*}
\]

LL3 = ((PYS3Xi \cdot PZS3PE - PZS3Xi \cdot PYS3PE) ** 2)

\[
\begin{align*}
\text{LL4} &= ((PYS4Xi \cdot PZS4PE - PZS4Xi \cdot PYS4PE) ** 2) \\
\text{PXS3Zt}(II, j) &= -k_S(3, II, j) \cdot (PYS3Xi \cdot PZS3PE - PZS3Xi \cdot PYS3PE) / LL3 \\
\text{PXS4Zt}(II, j) &= -k_S(4, II, j) \cdot (PYS4Xi \cdot PZS4PE - PZS4Xi \cdot PYS4PE) / LL4 \\
\text{PYS3Zt}(II, j) &= k_S(3, II, j) \cdot (PXS3Xi \cdot PZS3PE - PZS3Xi \cdot PXS3PE) / LL3 \\
\text{PYS4Zt}(II, j) &= k_S(4, II, j) \cdot (PXS4Xi \cdot PZS4PE - PZS4Xi \cdot PXS4PE) / LL4 \\
\text{PZS3Zt}(II, j) &= -k_S(3, II, j) \cdot (PYS3Xi \cdot PYS3PE - PYS3Xi \cdot PXS3PE) / LL3 \\
\text{PZS4Zt}(II, j) &= -k_S(4, II, j) \cdot (PXS4Xi \cdot PYS4PE - PYS4Xi \cdot PXS4PE) / LL4 \\
\end{align*}
\]
\[ PYS4Zt(i, j) = kS(4, i, j) \times (PXS4Xi \times PYS4PE - PYS4Xi \times PXS4PE) / LL4 \]
\[ PZS3Zt(i, j) = -kS(3, i, j) \times (PXS3Xi \times PYS3PE - PYS3Xi \times PXS3PE) / LL3 \]
\[ PZS4Zt(i, j) = -kS(4, i, j) \times (PXS4Xi \times PYS4PE - PYS4Xi \times PXS4PE) / LL4 \]

CONTINUE

C

\begin{align*}
PXS3Xi &= (XS(3, 2, JJ) - XS(3, 1, JJ)) / XiStep \\
PXS4Xi &= (XS(4, 2, JJ) - XS(4, 1, JJ)) / XiStep \\
PY33Xi &= (YS(3, 2, JJ) - YS(3, 1, JJ)) / XiStep \\
PYS4Xi &= (YS(4, 2, JJ) - YS(4, 1, JJ)) / XiStep \\
PZ33Xi &= (ZS(3, 2, JJ) - ZS(3, 1, JJ)) / XiStep \\
PZ4Xi &= (ZS(4, 2, JJ) - ZS(4, 1, JJ)) / XiStep \\
PXS3PE &= (XS(3, 1, JJ) - XS(3, 1, JJ-1)) / EtStep \\
PXS4PE &= (XS(4, 1, JJ) - XS(4, 1, JJ-1)) / EtStep \\
PY33PE &= (YS(3, 1, JJ) - YS(3, 1, JJ-1)) / EtStep \\
PYS4PE &= (YS(4, 1, JJ) - YS(4, 1, JJ-1)) / EtStep \\
PZ33PE &= (ZS(3, 1, JJ) - ZS(3, 1, JJ-1)) / EtStep \\
PZ4PE &= (ZS(4, 1, JJ) - ZS(4, 1, JJ-1)) / EtStep \\
LL3 &= \left( \frac{PYS3Xi \times PZS3PE - PZS3Xi \times PYS3PE}{LL3} \right) \times 0.5 \\
LL4 &= \left( \frac{PYS4Xi \times PZS4PE - PZS4Xi \times PYS4PE}{LL4} \right) \times 0.5 \\
\end{align*}

C

\begin{align*}
PXS3Xi &= (XS(3, 2, JJ) - XS(3, 1, JJ)) / XiStep \\
PXS4Xi &= (XS(4, 2, JJ) - XS(4, 1, JJ)) / XiStep \\
PY33Xi &= (YS(3, 2, JJ) - YS(3, 1, JJ)) / XiStep \\
PYS4Xi &= (YS(4, 2, JJ) - YS(4, 1, JJ)) / XiStep \\
PZ33Xi &= (ZS(3, 2, JJ) - ZS(3, 1, JJ)) / XiStep \\
PZ4Xi &= (ZS(4, 2, JJ) - ZS(4, 1, JJ)) / XiStep \\
PXS3PE &= (XS(3, 1, JJ) - XS(3, 1, JJ-1)) / EtStep \\
PXS4PE &= (XS(4, 1, JJ) - XS(4, 1, JJ-1)) / EtStep \\
PY33PE &= (YS(3, 1, JJ) - YS(3, 1, JJ-1)) / EtStep \\
PYS4PE &= (YS(4, 1, JJ) - YS(4, 1, JJ-1)) / EtStep \\
PZ33PE &= (ZS(3, 1, JJ) - ZS(3, 1, JJ-1)) / EtStep \\
PZ4PE &= (ZS(4, 1, JJ) - ZS(4, 1, JJ-1)) / EtStep \\
LL3 &= \left( \frac{PYS3Xi \times PZS3PE - PZS3Xi \times PYS3PE}{LL3} \right) \times 0.5 \\
LL4 &= \left( \frac{PYS4Xi \times PZS4PE - PZS4Xi \times PYS4PE}{LL4} \right) \times 0.5 \\
\end{align*}

C

\begin{align*}
PXS3Zt(II, JJ) &= -kS(3, II, JJ) \times (PXS3Xi \times PYS3PE - PYS3Xi \times PXS3PE) / LL3 \\
PXS4Zt(II, JJ) &= -kS(4, II, JJ) \times (PXS4Xi \times PYS4PE - PYS4Xi \times PXS4PE) / LL4 \\
PYS3Zt(II, JJ) &= -kS(3, II, JJ) \times (PYS3Xi \times PYS3PE - PYS3Xi \times PXS3PE) / LL3 \\
PYS4Zt(II, JJ) &= -kS(4, II, JJ) \times (PYS4Xi \times PYS4PE - PYS4Xi \times PXS4PE) / LL4 \\
PZ3Zt(II, JJ) &= -kS(3, II, JJ) \times (PZ3Xi \times PYS3PE - PZ3Xi \times PXS3PE) / LL3 \\
PZ4Zt(II, JJ) &= -kS(4, II, JJ) \times (PZ4Xi \times PYS4PE - PZ4Xi \times PXS4PE) / LL4 \\
\end{align*}
DO 65 i=2,II-1
   PXS3Xi=(XS(3,i+1,JJ)-XS(3,i-1,JJ))/2/XiStep
   PXS4Xi=(XS(4,i+1,JJ)-XS(4,i-1,JJ))/2/XiStep
   PYS3Xi=(YS(3,i+1,JJ)-YS(3,i-1,JJ))/2/XiStep
   PYS4Xi=(YS(4,i+1,JJ)-YS(4,i-1,JJ))/2/XiStep
   PZS3Xi=(ZS(3,i+1,JJ)-ZS(3,i-1,JJ))/2/XiStep
   PZS4Xi=(ZS(4,i+1,JJ)-ZS(4,i-1,JJ))/2/XiStep
   PXS3PE=(XS(3,i,JJ)-XS(3,i,JJ-1))/EtStep
   PXS4PE=(XS(4,i,JJ)-XS(4,i,JJ-1))/EtStep
   PYS3PE=(YS(3,i,JJ)-YS(3,i,JJ-1))/EtStep
   PYS4PE=(YS(4,i,JJ)-YS(4,i,JJ-1))/EtStep
   PZS3PE=(ZS(3,i,JJ)-ZS(3,i,JJ-1))/EtStep
   PZS4PE=(ZS(4,i,JJ)-ZS(4,i,JJ-1))/EtStep
       LL3=((PYS3Xi*PZS3PE-PZS3Xi*PYS3PE)**2
            +(PXS3Xi*PZS3PE-PZS3Xi*PXS3PE)**2
            +(PYS3Xi*PXS3PE-PXS3Xi*PYS3PE)**2)**0.5
       LL4=((PYS4Xi*PZS4PE-PZS4Xi*PYS4PE)**2
            +(PXS4Xi*PZS4PE-PZS4Xi*PXS4PE)**2
            +(PYS4Xi*PXS4PE-PXS4Xi*PYS4PE)**2)**0.5
C
   PXS3Zt(i,JJ)=-kS(3,i,JJ)*(PYS3Xi*PZS3PE-PZS3Xi*PYS3PE)/LL3
   PXS4Zt(i,JJ)=-kS(4,i,JJ)*(PYS4Xi*PZS4PE-PZS4Xi*PYS4PE)/LL4
   PYS3Zt(i,JJ)= kS(3,i,JJ)*(PXS3Xi*PZS3PE-PZS3Xi*PXS3PE)/LL3
   PYS4Zt(i,JJ)= kS(4,i,JJ)*(PXS4Xi*PZS4PE-PZS4Xi*PXS4PE)/LL4
   PZS3Zt(i,JJ)=-kS(3,i,JJ)*(PXS3Xi*PYS3PE-PYS3Xi*PXS3PE)/LL3
   PZS4Zt(i,JJ)= kS(4,i,JJ)*(PXS4Xi*PYS4PE-PYS4Xi*PXS4PE)/LL4

CONTINUE

Calculate the grid point locations everywhere.

C
DO 90 k=1,KK
   Zeta=(k-1.)/(KK-1.)
DO 80 i=1,II
   Xi=(i-1.)/(II-1.)
DO 70 j=1,JJ
   Eta=(j-1.)/(JJ-1.)
   EtaNew=(EtSt11(j)*(1.-Xi)+EtSt12(j)*Xi)*(1.-Zeta)
   $ + (EtSt15(j)*(1.-Xi)+EtSt16(j)*Xi)*Zeta
   $ ZetaNew=(ZtSt1(k)*(1.-Xi)+ZtSt2(k)*Xi)*(1.-Eta)
   $ + (ZtSt5(k)*(1.-Xi)+ZtSt6(k)*Xi)*Eta
   CALL FindHs(h1(j),h2(j),h3(j),h4(j),EtaNew,SigmaEt)
   CALL FindHs(h5(k),h6(k),h7(k),h8(k),ZetaNew,SigmaZt)
   XFnt(i,j,k)=XFnt(i,j,k)
   $ + (XS(3,i,j)-h1(j)*XS(1,1,i)
   $ - h2(j)*XS(2,1,i)

72
-h3(j)*PXS1PE(i,1)
-h4(j)*PXS2PE(i,1)) *h5(k)
+(XS(4,i,j)-h1(j)*XS(1,1,i)
-h2(j)*XS(2,1,i)
-h3(j)*XS1PE(1,1)
-h4(j)*XS2PE(1,1)) *h5(k)
+(PXS3Zt(i,j)-(h1(j)*PXS3Zt(1,i)
+h2(j)*PXS3Zt(1,JJ)
+h3(j)*P2X00+h4(j)*P2X01)) *h7(k)
+(PXS4Zt(i,j)-(h1(j)*PXS4Zt(1,i)
+h2(j)*PXS4Zt(1,JJ)
+h3(j)*P2X10+h4(j)*P2X11)) *h8(k)

YPnt(i,j,k)=YPnt(i,j,k)
+(YS(3,i,j)-h1(j)*YS(1,1,i)
-h2(j)*YS(2,1,i)
-h3(j)*YS1PE(1,1)
-h4(j)*YS2PE(1,1)) *h5(k)
+(YS(4,i,j)-h1(j)*YS(1,1,i)
-h2(j)*YS(2,1,i)
-h3(j)*YS1PE(1,1)
-h4(j)*YS2PE(1,1)) *h5(k)
+(PYS3Zt(i,j)-(h1(j)*PYS3Zt(1,i)
+h2(j)*PYS3Zt(1,JJ)
+h3(j)*P2Y00+h4(j)*P2Y01)) *h7(k)
+(PYS4Zt(i,j)-(h1(j)*PYS4Zt(1,i)
+h2(j)*PYS4Zt(1,JJ)
+h3(j)*P2Y10+h4(j)*P2Y11)) *h8(k)

ZPnt(i,j,k)=ZPnt(i,j,k)
+(ZS(3,i,j)-h1(j)*ZS(1,1,i)
-h2(j)*ZS(2,1,i)
-h3(j)*ZS1PE(1,1)
-h4(j)*ZS2PE(1,1)) *h5(k)
+(ZS(4,i,j)-h1(j)*ZS(1,1,i)
-h2(j)*ZS(2,1,i)
-h3(j)*ZS1PE(1,1)
-h4(j)*ZS2PE(1,1)) *h5(k)
+(PZS3Zt(i,j)-(h1(j)*PZS3Zt(1,i)
+h2(j)*PZS3Zt(1,JJ)
+h3(j)*P2Z00+h4(j)*P2Z01)) *h7(k)
+(PZS4Zt(i,j)-(h1(j)*PZS4Zt(1,i)
+h2(j)*PZS4Zt(1,JJ)
+h3(j)*P2Z10+h4(j)*P2Z11)) *h8(k)

70 CONTINUE
80 CONTINUE
90 CONTINUE
C
RETURN
END

C-----------------------------------------------C
C
SUBROUTINE PrGrid (XPnt,YPnt,ZPnt,II, JJ, KK, OutNum,MxGSiz)
C
This SUBROUTINE prints (to output) the grid point x, y, and z coordinates.
C
INTEGER i, j, k, II, JJ, KK, OutNum
C
REAL XPnt(MxGSiz,MxGSiz,MxGSiz),
$ \text{YPnt (MxGSiz, MxGsz, MxGSiz)}$

$ \text{ZPnt (MxGSiz, MxGsz, MxGSiz)}$

C

WRITE (OutNum, *) II
WRITE (OutNum, *) JJ
WRITE (OutNum, *) KK

C

DO 30 i = 1, II
   DO 20 j = 1, JJ
      DO 10 k = 1, KK
         WRITE (OutNum, 35) XPnt(i, j, k), YPnt(i, j, k), ZPnt(i, j, k)
      CONTINUE
   CONTINUE
30 CONTINUE

C

FORMAT (1X, F10.6, 3X, F10.6, 3X, F10.6)

RETURN

END

C=================================================================================================C

SUBROUTINE RdGrIn (II, JJ, KK, NSurfs, SigmaXi, SigmaEt, SigmaZt, $ kXil, kXi2, kEtal, kEta2, kZetal, kZeta2, InNum)

This SUBROUTINE reads in the desired grid information for grid control.

INTEGER StrXi, StrEt, StrZt, InNum, II, JJ, KK
REAL kXil, kXi2, kEtal, kEta2, kZetal, kZeta2,
$ BetaXi, BetaEt, BetaZt, SigmaXi, SigmaEt, SigmaZt

READ(InNum, *) NSurfs
READ(InNum, *) II
READ(InNum, *) JJ
READ(InNum, *) KK

READ(InNum, *) SigmaXi
READ(InNum, *) SigmaEt
READ(InNum, *) SigmaZt

READ(InNum, *) kXil
READ(InNum, *) kXi2
READ(InNum, *) kEtal
READ(InNum, *) kEta2
READ(InNum, *) kZetal
READ(InNum, *) kZeta2

RETURN

END

C=================================================================================================C

SUBROUTINE RdCvIn (x, y, z, NDPts, CrvNum, Tensn, InNum, MxBPts, $ StrTp, Beta1, Beta2)

This SUBROUTINE reads in the information concerning discrete points on
This information is used for generating spline-fitted boundary approximation curves.

```
INTEGER CrvNum, i, NDPts(4), InNum, StrTp
REAL x(4,MxBPts), y(4,MxBPts),
     z(4,MxBPts), Tensn(4)

READ(InNum,*) Tensn(CrvNum)
READ(InNum,*) NDPts(CrvNum)

DO 10 i=1,NDPts(CrvNum)
     READ(InNum,*) x(CrvNum,i), y(CrvNum,i), z(CrvNum,i)
10 CONTINUE

READ(InNum,*) StrTp
IF (StrTp.NE.4) THEN
     READ(InNum,*) Betal
ELSE
     READ(InNum,*) Betal, Beta2
ENDIF
RETURN
END
```

```
SUBROUTINE CalcS (x,y,z,s,NDPts,CrvNum,MxBPts)
This SUBROUTINE calculates the spline parameter, s, as an approximate arc length.

INTEGER NDPts(4), CrvNum, i
REAL x(4,MxBPts), y(4,MxBPts),
     z(4,MxBPts), s(4,MxBPts)

s(CrvNum,1)=0.0

DO 10 i=2,NDPts(CrvNum)
     s(CrvNum,i)=s(CrvNum,i-1)
          +SQRT((x(CrvNum,i)-x(CrvNum,i-1))**2
                + (y(CrvNum,i)-y(CrvNum,i-1))**2
                + (z(CrvNum,i)-z(CrvNum,i-1))**2)
10 CONTINUE
RETURN
END
```

```
SUBROUTINE SplMat (Diag,OfDiag,Right,w,s,NDPts,T,CrvNum,MxBPts)
This SUBROUTINE forms the parametric tension spline matrix for a particular boundary curve data set.

INTEGER i, NDPts(4), CrvNum
```
REAL Diag(MxBPts), OfDiag(MxBPts), Right(MxBPts),
$ w(4,MxBPts), s(4,MxBPts), T, h, hm$

 Diag(1)=1.0
 OfDiag(1)=0.0
 Right(1)=0.0

DO 10 i=2,NDPts(CrvNum)-1
   h=s(CrvNum,i+1)-s(CrvNum,i)
   hm=s(CrvNum,i)-s(CrvNum,i-1)
   Diag(i)=(T*COSH(T*hm)/SINH(T*hm)-1/hm+T*COSH(T*h))/SINH(T*h)
   $ -1/h)/T**2$
   OfDiag(i)=(1/h-T/SINH(T*h))/T**2
   Right(i)=(w(CrvNum,i+1)-w(CrvNum,i))/h
   $ -(w(CrvNum,i)-w(CrvNum,i-1))/hm$
 10 CONTINUE

Diag(NDPts(CrvNum))=1.0
 OfDiag(NDPts(CrvNum)-1)=0.0
 Right(NDPts(CrvNum))=0.0

RETURN
END

C SUBROUTINE SplSlv (Diag,OfDiag,Right,Derv2,NDPts,CrvNum,MxBPts)
C
C This SUBROUTINE solves the diagonally dominant parametric tension
C spline matrix for a given data set using the Gauss-Seidel iteration.
C Convergence is assumed after 20 iterations.
C
INTEGER i, j, NDPts(4), CrvNum

REAL Diag(MxBPts), OfDiag(MxBPts), Right(MxBPts),
$ Derv2(4,MxBPts)$

C Initialize the second derivative matrix to all zeroes.

DO 10 i=1,NDPts(CrvNum)
   Derv2(CrvNum,i)=0.0
10 CONTINUE

C Calculate the second derivative values using 20 iterations of
C the Gauss-Seidel method.

DO 30 j=1,20
   DO 20 i=2,NDPts(CrvNum)-1
      Derv2(CrvNum,i)=(Right(i)-OfDiag(i)*Derv2(CrvNum,i+1)
   $ -OfDiag(i-1)*Derv2(CrvNum,i-1))/Diag(i)$
   20 CONTINUE
30 CONTINUE

RETURN
END

C-------------------------------------------------------------------------------------------------
FUNCTION SplVal (s, w, Derv2, sval, T, n, CrvNum, MxBPts)

This real function finds the w-value (x-value or y-value) corresponding
to a specified s-value using the parametric tension spline curve
generated for a particular boundary curve data set.

INTEGER n, CrvNum

REAL s(4, MxBPts), w(4, MxBPts), Derv2(4, MxBPts),
$ sval, T, h, Interim, Temp1, Temp2

Temp1 = sval - s(CrvNum, n)

h = s(CrvNum, n + 1) - s(CrvNum, n)

Interim = Derv2(CrvNum, n) / T**2 * SINH(T * Temp2) / SINH(T * h)
$ + . . .

SplVal = Interim + Derv2(CrvNum, n + 1) / T**2 * SINH(T * Temp1)
$ . . .

RETURN
END

SUBROUTINE PTSpln(x, y, z, s, XDerv2, YDerv2, ZDerv2, Diag, OfDiag,

This SUBROUTINE forms the main routine for the parametric tension
spline process.

INTEGER NDPts(4), CrvNum

REAL Diag(MxBPts), OfDiag(MxBPts), Right(MxBPts),
$ XDerv2(4, MxBPts), YDerv2(4, MxBPts),
$ ZDerv2(4, MxBPts), Tensn,
$ x(4, MxBPts), y(4, MxBPts),
$ z(4, MxBPts), s(4, MxBPts)

CALL CalcS(x, y, z, s, NDPts, CrvNum, MxBPts)
CALL SplMat(Diag, OfDiag, Right, x, s, NDPts, Tensn, CrvNum, MxBPts)
CALL SplSlv(Diag, OfDiag, Right, XDerv2, NDPts, CrvNum, MxBPts)
CALL SplMat(Diag, OfDiag, Right, y, s, NDPts, Tensn, CrvNum, MxBPts)
CALL SplSlv(Diag, OfDiag, Right, YDerv2, NDPts, CrvNum, MxBPts)
CALL SplMat(Diag, OfDiag, Right, z, s, NDPts, Tensn, CrvNum, MxBPts)
CALL SplSlv(Diag, OfDiag, Right, ZDerv2, NDPts, CrvNum, MxBPts)

RETURN
END

SUBROUTINE FindHs(h1, h2, h3, h4, n, s)

This SUBROUTINE computes the h factors used in Hermite interpolation.
REAL  h1, h2, h3, h4, n, s
    al, a2, a3, a4, a, b, bbaa, sh, ch, shsn, shsnl

IF(s.NE.0)THEN
    sh=sinh(s)
    ch=cosh(s)
    a2=sh/(2.*sh-s*ch-s)
    al=1-a2
    a=s*ch-sh
    b=sh-s
    bbaa=b*b-a*a
    a3=-a*sh/bbaa
    a4=b*sh/bbaa
    shsn=sinh(s*n)/sh
    shsnl=sinh(s*(1.-n))/sh
    h1=a2*(shsnl-shsn)+al*(1.-n)+a2*n
    h2=a2*(shsn-shsnl)+a2*(1.-n)+al*n
    h3=a3*((1.-n)-shsnl)+a4*(n-shsn)
    h4=a4*(shsnl-(1.-n))+a3*(shsn-n)
ELSE
    h1= 2*n**3-3*n**2+1
    h2=-2*n**3+3*n**2
    h3= n**3-2*n**2+n
    h4= n**3-n**2
ENDIF

RETURN
END

SUBROUTINE SplInt(n,s,SValue,NDPts,CurCrv,MxBPts)

This SUBROUTINE finds the proper interval in which a point on a specified
boundary lies. The interval indicates which initial data points the
point in question lies between and thus which spline coefficients to
use.

INTEGER  i, n, CurCrv, NDPts(4)

REAL  Temp, SValue, s(4,MxBPts)

n=1
i=NDPts(CurCrv)
10  IF ((n.EQ.1).AND.(i.GT.1)) THEN
    i=i-1
    Temp=SValue-s(CurCrv,i)
    IF (Temp.GT.0.0) THEN
        n=i
    ENDIF
    GOTO 10
ENDIF
SUBROUTINE FAiNew(AiNew, Alpha, B, Str)
C
This SUBROUTINE computes the new Alpha value after stretching as
AiNew. Alpha is a dummy variable representing either Xi, Eta or Zeta.
C
INTEGER Str
C
REAL Alpha, Templ, Temp2, B2, AINew, B
C
AiNew=Alpha
Temp1=(B+1)/(B-1)
C
IF (Str.EQ.1) THEN
Temp2=Temp1**(1-Alpha)
AiNew=((B+1)-(B-1)*Temp2)/(Temp2+1)*1
ENDIF
C
IF (Str.EQ.2) THEN
B2=0
Temp2=Temp1**((Alpha-B2)/(1-B2))
ENDIF
C
IF (Str.EQ.3) THEN
B2=0.5
Temp2=Temp1**((Alpha-B2)/(1-B2))
ENDIF
C
RETURN
END
C
SUBROUTINE EdgPts(X1,X2,X3,X4,Y1,Y2,Y3,Y4,Z1,Z2,Z3,Z4,AL,BL,$ AAStep, BBStep, x, y, z, s, zx, zy, zz, NDPts, Tensn,$ StrAA, StrBB, BetaAA, BetaBB, MxBPts, MxGSiz)
C
This SUBROUTINE calculates the grid point locations along the surface
edges.
C
INTEGER Act, Bct, n1, n2, n3, n4,$ AL, BL, StrAA, StrBB, NDPts(4)
C
REAL AA, BB, AAiNew, BBiNew, S1, S2, S3, S4, BBStep, AAStep,$ S1AA, S2AA, S3BB, S4BB,$ X1(MxGSiz), X2(MxGSiz), X3(MxGSiz), X4(MxGSiz),$ Y1(MxGSiz), Y2(MxGSiz), Y3(MxGSiz), Y4(MxGSiz),$ Z1(MxGSiz), Z2(MxGSiz), Z3(MxGSiz), Z4(MxGSiz),$ x(4,MxBPts), y(4,MxBPts), z(4,MxBPts),$ s(4,MxBPts), zx(4,MxBPts), zy(4,MxBPts),$ zz(4,MxBPts), Tensn(4), BetaAA, BetaBB
C
C Initialize the grid point locations along boundaries 1 and 2.

AA=0.0

DO 10 ACt=I,AL
    CALL FAI-New(AANew, AA, BetaAA, StrAA)
    S1=AANew*S1AAR
    S2=AANew*S2AAR
    CALL SplInt(n1, s, S1, NDPts, 1, MxBPts)
    X1(ACt)=SplVal(s, x, zx, S1, Tensn(1), n1, 1, MxBPts)
    Y1(ACt)=SplVal(s, y, zy, S1, Tensn(1), n1, 1, MxBPts)
    Z1(ACt)=SplVal(s, z, zz, S1, Tensn(1), n1, 1, MxBPts)
    AA=AA+AAStep
10   CONTINUE

C Calculate the grid point locations along boundaries 3 and 4.

BB=0.0

DO 20 BCt=I,BL
    CALL FAINew(BBNew, BB, BetaBB, StrBB)
    S3=BBNew*S3BBR
    S4=BBNew*S4BBR
    CALL SplInt(n3, s, S3, NDPts, 3, MxBPts)
    X3(BCt)=SplVal(s, x, zx, S3, Tensn(3), n3, 3, MxBPts)
    Y3(BCt)=SplVal(s, y, zy, S3, Tensn(3), n3, 3, MxBPts)
    Z3(BCt)=SplVal(s, z, zz, S3, Tensn(3), n3, 3, MxBPts)
    BB=BB+BBStep
20   CONTINUE

RETURN

END

SUBROUTINE EdgDer(PX1PAA, PX2PAA, PY1PAA, PY2PAA, PZ1PAA, PZ2PAA,
                   PX3PBB, PX4PBB, PY3PBB, PY4PBB, PZ3PBB, PZ4PBB,
                   X1, X2, X3, X4, Y1, Y2, Y3, Y4, Z1, Z2, Z3, Z4,
                   AL, BL, MxGSiz)

INTEGER ACt, BCt, AL, BL

REAL AAStep, BBStep, PX3PBB(MxGSiz), PX4PBB(MxGSiz),
     PY3PBB(MxGSiz), PY4PBB(MxGSiz), PZ3PBB(MxGSiz),
     PZ4PBB(MxGSiz), PX1PAA(MxGSiz), PX2PAA(MxGSiz),

80
Calculate step size in the AA and BB directions.

\[ AA\text{Step} = 1. / (AL - 1) \]
\[ BB\text{Step} = 1. / (BL - 1) \]

Calculate the derivative values along the constant AA boundaries.

\[ PX1\text{PAA}(1) = (X1(2) - X1(1)) / AA\text{Step} \]
\[ PX2\text{PAA}(1) = (X2(2) - X2(1)) / AA\text{Step} \]
\[ PY1\text{PAA}(1) = (Y1(2) - Y1(1)) / AA\text{Step} \]
\[ PY2\text{PAA}(1) = (Y2(2) - Y2(1)) / AA\text{Step} \]
\[ PZ1\text{PAA}(1) = (Z1(2) - Z1(1)) / AA\text{Step} \]
\[ PZ2\text{PAA}(1) = (Z2(2) - Z2(1)) / AA\text{Step} \]

DO 10 ACT = 2, AL - 1

\[ PX1\text{PAA}(ACT) = (X1(ACT + 1) - X1(ACT - 1)) / 2 / AA\text{Step} \]
\[ PX2\text{PAA}(ACT) = (X2(ACT + 1) - X2(ACT - 1)) / 2 / AA\text{Step} \]
\[ PY1\text{PAA}(ACT) = (Y1(ACT + 1) - Y1(ACT - 1)) / 2 / AA\text{Step} \]
\[ PY2\text{PAA}(ACT) = (Y2(ACT + 1) - Y2(ACT - 1)) / 2 / AA\text{Step} \]
\[ PZ1\text{PAA}(ACT) = (Z1(ACT + 1) - Z1(ACT - 1)) / 2 / AA\text{Step} \]
\[ PZ2\text{PAA}(ACT) = (Z2(ACT + 1) - Z2(ACT - 1)) / 2 / AA\text{Step} \]

CONTINUE

Calculate the derivative values along the constant BB boundaries.

\[ PX3\text{PBB}(1) = (X3(2) - X3(1)) / BB\text{Step} \]
\[ PX4\text{PBB}(1) = (X4(2) - X4(1)) / BB\text{Step} \]
\[ PY3\text{PBB}(1) = (Y3(2) - Y3(1)) / BB\text{Step} \]
\[ PY4\text{PBB}(1) = (Y4(2) - Y4(1)) / BB\text{Step} \]
\[ PZ3\text{PBB}(1) = (Z3(2) - Z3(1)) / BB\text{Step} \]
\[ PZ4\text{PBB}(1) = (Z4(2) - Z4(1)) / BB\text{Step} \]

DO 20 BCt = 2, BL - 1

\[ PX3\text{PBB}(BCt) = (X3(BCt + 1) - X3(BCt - 1)) / 2 / BB\text{Step} \]
\[ PX4\text{PBB}(BCt) = (X4(BCt + 1) - X4(BCt - 1)) / 2 / BB\text{Step} \]
\[ PY3\text{PBB}(BCt) = (Y3(BCt + 1) - Y3(BCt - 1)) / 2 / BB\text{Step} \]
\[ PY4\text{PBB}(BCt) = (Y4(BCt + 1) - Y4(BCt - 1)) / 2 / BB\text{Step} \]
\[ PZ3\text{PBB}(BCt) = (Z3(BCt + 1) - Z3(BCt - 1)) / 2 / BB\text{Step} \]

CONTINUE
SUBROUTINE TwoBnd(XS, YS, ZS, SrfNum, AL, BL, SigmaBB, k1, k2, " $ StrB,h1,h2,h3,h4,X1,X2,X3,X4, $
$ Y1,Y2,Y3,Y4,Z1,Z2,Z3,Z4,PX1PBB,PX2PBB, $
$ PY1PBB, PY2PBB, PX1PBB, PX2PBB, PX1PAA, PX2PAA, $
$ PY1PAA, PY2PAA, PX1PAA, PX2PAA, PX3PBB, PX4PBB, $
$ PY3PBB, PY4PBB, PX3PBB, PX4PBB, $
$ $ MxBCvs, MxGSiz, MxSrfs)$

This SUBROUTINE calculates the interior grid point locations between two specified boundaries (1 and 2) by using transfinite Hermite interpolation.

INTEGER ACT, BCT, AL, BL, StrAA, StrBB, SrfNum, Edgl, Edg2, $
Edg3, Edg4$

REAL AA, BB, AANew, BBNew, LL1, LL2, $
Box1i, Box1j, Box1k, Box2i, Box2j, Box2k, $
Temp1i, Temp1j, Temp1k, Temp2i, Temp2j, Temp2k, $
k1(MxSrfs,MxGSiz), k2(MxSrfs,MxGSiz), $
BetaAA, BetaBB, BBStep, AAStep, $
h1(MxGSiz), h2(MxGSiz), h3(MxGSiz), h4(MxGSiz), $
X1(MxGSiz), X2(MxGSiz), X3(MxGSiz), X4(MxGSiz), $
y1(MxGSiz), y2(MxGSiz), y3(MxGSiz), y4(MxGSiz), $
z1(MxGSiz), z2(MxGSiz), z3(MxGSiz), z4(MxGSiz)$

REAL PX1PBB(MxGSiz), PX2PBB(MxGSiz), $
PY1PBB(MxGSiz), PY2PBB(MxGSiz), $
PX1PBB(MxGSiz), PX2PBB(MxGSiz), $
PY1PAA(MxGSiz), PX2PAA(MxGSiz), $
PX1PAA(MxGSiz), PX2PAA(MxGSiz), $
PX3PBB(MxGSiz), PX4PBB(MxGSiz), $
PY3PBB(MxGSiz), PY4PBB(MxGSiz), $
PX3PBB(MxGSiz), PX4PBB(MxGSiz), $
XS(MxSrfs,MxGSiz,MxGSiz), $
YS(MxSrfs,MxGSiz,MxGSiz), $
ZS(MxSrfs,MxGSiz,MxGSiz), $
StrB(MxGSiz,MxBCvs)$

Calculate the step size in the AA and BB directions.

AAStep=1./(AL-1.)
BBStep=1./(BL-1.)

Calculate the edge numbers for the surface 'SrfNum'.

Edgl=(SrfNum-1)*4 + 1
Edg2=(SrfNum-1)*4 + 2
Edg3=(SrfNum-1)*4 + 3
Edg4=(SrfNum-1)*4 + 4
Calculate the derivative values for grid line orthogonality.

AA = 0.0

```
DO 20 ACT=1, AL
   Box1i = AA*(PY1PAA(AL)*P4PBB(1) -P2PAA(AL)*PY4PBB(1))
   + (1-AA)*(PY1PAA(AL)*P3PBB(1) -P2PAA(AL)*PY3PBB(1))
   Box1j = AA*(PX1PAA(AL)*P4PBB(1) -P2PAA(AL)*PX4PBB(1))
   + (1-AA)*(PX1PAA(AL)*P3PBB(1) -P2PAA(AL)*PX3PBB(1))
   Box1k = AA*(PX1PAA(AL)*PY4PBB(1) -PY1PAA(AL)*PX4PBB(1))
   + (1-AA)*(PX1PAA(AL)*PY3PBB(1) -PY1PAA(AL)*PX3PBB(1))
   Box2i = AA*(PY2PAA(AL)*P4PBB(BL) -P2PAA(AL)*PY4PBB(BL))
   + (1-AA)*(PY2PAA(AL)*P3PBB(BL) -P2PAA(AL)*PY3PBB(BL))
   Box2j = AA*(PX2PAA(AL)*P4PBB(BL) -P2PAA(AL)*PX4PBB(BL))
   + (1-AA)*(PX2PAA(AL)*P3PBB(BL) -P2PAA(AL)*PX3PBB(BL))
   Box2k = AA*(PX2PAA(AL)*PY4PBB(BL) -PY2PAA(AL)*PX4PBB(BL))
   + (1-AA)*(PX2PAA(AL)*PY3PBB(BL) -PY2PAA(AL)*PX3PBB(BL))
   Templi = PY1PAA(ACt)*Box1i + P2PAA(ACt)*Box1j
   Templj = PX1PAA(ACt)*Box1i + P2PAA(ACt)*Box1j
   Templk = PX1PAA(ACt)*Box1j + PY1PAA(ACt)*Box1i
   Temp2i = PY2PAA(ACt)*Box2i + P2PAA(ACt)*Box2j
   Temp2j = PX2PAA(ACt)*Box2i + P2PAA(ACt)*Box2j
   Temp2k = PX2PAA(ACt)*Box2j + PY2PAA(ACt)*Box2i
   LL1 = (Templi**2 + Templj**2 + Templk**2)**0.5
   LL2 = (Temp2i**2 + Temp2j**2 + Temp2k**2)**0.5
PX1PBB(ACt) = k1(SrfNum, ACt)* Templi/LL1
PX2PBB(ACt) = k2(SrfNum, ACt)* Temp2i/LL2
PY1PBB(ACt) = k1(SrfNum, ACt)* Templj/LL1
PY2PBB(ACt) = k2(SrfNum, ACt)* Temp2j/LL2
PZ1PBB(ACt) = k1(SrfNum, ACt)* Templk/LL1
PZ2PBB(ACt) = k2(SrfNum, ACt)* Temp2k/LL2
```

AA = AA + AAStep
```
20 CONTINUE
```
Calculate the interior grid point locations.

```
DO 40 ACT=1, AL
   DO 30 BCt=1, BL
   AA = (ACT-1.)/(AL-1.)
   BBNew = StrB(BCt, Edg3) * (1. - AA) + StrB(BCt, Edg4) * AA
   CALL FindHs(hl(BCt), h2(BCt), h3(BCt), h4(BCt), BBNew, SigmaBB)
   XS(SrfNum, ACT, BCt) = hl(BCt)*X1(ACt) + h2(BCt)*X2(ACt)
   + h3(BCt)*PX1PBB(ACt) + h4(BCt)*PX2PBB(ACt)
```
```
YS(SrfNum,Act,BCt) = h1(BCt) * Y1(Act) + h2(BCt) * Y2(Act) + h3(BCt) * PY1PBB(Act) + h4(BCt) * PY2PBB(Act)
ZS(SrfNum,Act,BCt) = h1(BCt) * Z1(Act) + h2(BCt) * Z2(Act) + h3(BCt) * PZ1PBB(Act) + h4(BCt) * PZ2PBB(Act)

CONTINUE
CONTINUE
RETURN
END

SUBROUTINE ForBnd(XS, YS, ZS, SrfNum, AL, BL, SigmaAA, SigmaBB, k3, k4, StrB, hl, h2, h3, h4, h5, h6, h7, h8, Xl, X2, X3, X4, YI, Y2, Y3, Y4, ZI, Z2, Z3, Z4, PX1PBB, PX2PBB, PY1PBB, PY2PBB, PZ1PBB, PZ2PBB, PX1PAA, PX2PAA, PY1PAA, PY2PAA, PZ1PAA, PZ2PAA, PX3PBB, PX4PBB, PY3PBB, PY4PBB, PZ3PBB, PZ4PBB, PX3PAA, PX4PAA, PY3PAA, PY4PAA, PZ3PAA, PZ4PAA, MxBCvs, MxGSiz, MxSrfs)

This SUBROUTINE adjusts the grid so that the other two boundaries (3 and 4) of the surface are mapped correctly using transfinite Hermite interpolation.

INTEGER Act, BCt, AL, BL, StrAA, StrBB, i, j, SrfNum, Edgl, Edg2, Edg3, Edg4
REAL AA, BB, AANew, BBNew, LL3, LL4, Box3i, Box3j, Box3k, Box4i, Box4j, Box4k, Temp3i, Temp3j, Temp3k, Temp4i, Temp4j, Temp4k, P2Y00, P2Y01, P2Y10, P2Y11, P2X00, P2X01, P2X10, P2X11, P2Z00, P2Z01, P2Z10, P2Z11, k3(MxSrfs,MxGSiz), k4(MxSrfs,MxGSiz), BetaAA, BetaBB, BBStep, AAStep, h1(MxGSiz), h2(MxGSiz), h3(MxGSiz), h4(MxGSiz), h5(MxGSiz), h6(MxGSiz), h7(MxGSiz), h8(MxGSiz), X1(MxGSiz), X2(MxGSiz), X3(MxGSiz), X4(MxGSiz), Y1(MxGSiz), Y2(MxGSiz), Y3(MxGSiz), Y4(MxGSiz), Z1(MxGSiz), Z2(MxGSiz), Z3(MxGSiz), Z4(MxGSiz)
REAL PX1PBB(MxGSiz), PX2PBB(MxGSiz), PY1PBB(MxGSiz), PY2PBB(MxGSiz), PX1PAA(MxGSiz), PX2PAA(MxGSiz), PY1PAA(MxGSiz), PY2PAA(MxGSiz), PX3PBB(MxGSiz), PX4PBB(MxGSiz), PY3PBB(MxGSiz), PY4PBB(MxGSiz), PX3PAA(MxGSiz), PX4PAA(MxGSiz), PY3PAA(MxGSiz), PY4PAA(MxGSiz), P2XII, XS(MxSrfs,MxGSiz, MxGSiz), YS(MxSrfs,MxGSiz, MxGSiz), ZS(MxSrfs,MxGSiz, MxGSiz), StrB(MxGSiz, MxBCvs)

C Calculate the step size for directions AA and BB.
AAStep=1./(AL-1.)
BBStep=1./(BL-1.)

Calculate edge numbers for the surface 'SrfNum'
Edgl=(SrfNum-1)*4 + 1
Edg2=(SrfNum-1)*4 + 2
Edg3=(SrfNum-1)*4 + 3
Edg4=(SrfNum-1)*4 + 4

Calculate the derivative values for grid line orthogonality.

BB=0.0

DO 20 BCt=1,BL
  Box3i = BB* (PY2PAA(1)*PZ3PBB(BL)
  $  -PZ2PAA(1)*PY3PBB(BL))
  $  +(1-BB)* (FY1PAA(1)*PZ3PBB(1)
  $  -PZ1PAA(1)*PY3PBB(1))
  Box3j = BB* (PX2PAA(1)*PZ3PBB(BL)
  $  -PZ2PAA(1)*PX3PBB(BL))
  $  +(1-BB)* (PX1PAA(1)*PZ3PBB(1)
  $  -PZ1PAA(1)*PX3PBB(1))
  Box3k = BB* (PX2PAA(1)*PY3PBB(BL)
  $  -PY2PAA(1)*PX3PBB(BL))
  $  +(1-BB)* (PY1PAA(1)*PY3PBB(1)
  $  -PY1PAA(1)*PX3PBB(1))
  Box4i = BB* (PY2PAA(1)*PZ4PBB(BL)
  $  -PZ2PAA(1)*PY4PBB(BL))
  $  +(1-BB)* (FY1PAA(1)*PZ4PBB(1)
  $  -PZ1PAA(1)*PY4PBB(1))
  Box4j = BB* (PX2PAA(1)*PZ4PBB(BL)
  $  -PZ2PAA(1)*PX4PBB(BL))
  $  +(1-BB)* (PX1PAA(1)*PZ4PBB(1)
  $  -PZ1PAA(1)*PX4PBB(1))
  Box4k = BB* (PX2PAA(1)*PY4PBB(BL)
  $  -PY2PAA(1)*PX4PBB(BL))
  $  +(1-BB)* (PY1PAA(1)*PY4PBB(1)
  $  -PY1PAA(1)*PX4PBB(1))
  Temp3i = PZ3PBB(BCt)*Box3j+PY3PBB(BCt)*Box3k
  Temp3j = PZ3PBB(BCt)*Box3i-PX3PBB(BCt)*Box3k
  Temp3k = PY3PBB(BCt)*Box3i+PX3PBB(BCt)*Box3j
  Temp4i = PZ4PBB(BCt)*Box4j+PY4PBB(BCt)*Box4k
  Temp4j = PZ4PBB(BCt)*Box4i-PX4PBB(BCt)*Box4k
  Temp4k = PY4PBB(BCt)*Box4i+PX4PBB(BCt)*Box4j
  LL3 = (Temp3i**2+Temp3j**2+Temp3k**2)**0.5
  LL4 = (Temp4i**2+Temp4j**2+Temp4k**2)**0.5

PX3PAA(BCt) = k3(SrfNum, BCt)*Temp3i/LL3
PX4PAA(BCt) = k4(SrfNum, BCt)*Temp4i/LL4
PY3PAA(BCt) = k3(SrfNum, BCt)*Temp3j/LL3
PY4PAA(BCt) = k4(SrfNum, BCt)*Temp4j/LL4
PZ3PAA(BCt) = -k3(SrfNum, BCt)*Temp3k/LL3
PZ4PAA(BCt) = -k4(SrfNum, BCt)*Temp4k/LL4

BB = BB + BBStep
CONTINUE

Set the cross-derivative terms equal to zero.

```
P2X00=0.0
P2X10=0.0
P2X01=0.0
P2X11=0.0
P2Y00=0.0
P2Y10=0.0
P2Y01=0.0
P2Y11=0.0
P2Z00=0.0
P2Z10=0.0
P2Z01=0.0
P2Z11=0.0
```

Calculate the grid point locations everywhere.

```
DO 40 i=1,AL
  DO 30 j=1,BL
    AA=(i-1.)/(AL-1.)
    BB=(j-1.)/(BL-1.)
    AANew=StrB(i,Edg1)*(1.-BB)+StrB(i,Edg2)*BB
    BBNew=StrB(j,Edg3)*(1.-AA)+StrB(j,Edg4)*AA
    CALL FindHs(h1(j),h2(j),h3(j),h4(j),BBNew,SigmaBB)
    CALL FindHs(h5(i),h6(i),h7(i),h8(i),AANew,SigmaAA)
    XS(SrfNum,i,j)=XS(SrfNum,i,j)
      + (X3(j)-h1(j)*X1(1)
          -h2(j)*X2(1)
          -h3(j)*PX1PBB(1)
          -h4(j)*PX2PBB(1)))*h5(i)
      + (X4(j)-h1(j)*X1(AL)
          -h2(j)*X2(AL)
          -h3(j)*PX1PBB(AL)
          -h4(j)*PX2PBB(AL)))*h6(i)
      + (PX3PAA(j)-(h1(j)*PX3PAA(1)
          +h2(j)*PX3PAA(BL)
          +h3(j)*P2X00+h4(j)*P2X01))*h7(i)
      + (PX4PAA(j)-(h1(j)*PX4PAA(1)
          +h2(j)*PX4PAA(BL)
          +h3(j)*P2X10+h4(j)*P2X11))*h8(i)
    YS(SrfNum,i,j)=YS(SrfNum,i,j)
      + (Y3(j)-h1(j)*Y1(1)
          -h2(j)*Y2(1)
          -h3(j)*PY1PBB(1)
          -h4(j)*PY2PBB(1)))*h5(i)
      + (Y4(j)-h1(j)*Y1(AL)
          -h2(j)*Y2(AL)
          -h3(j)*PY1PBB(AL)
          -h4(j)*PY2PBB(AL)))*h6(i)
      + (PY3PAA(j)-(h1(j)*PY3PAA(1)
          +h2(j)*PY3PAA(BL)
          +h3(j)*P2Y00+h4(j)*P2Y01))*h7(i)
      + (PY4PAA(j)-(h1(j)*PY4PAA(1)
          +h2(j)*PY4PAA(BL)
          +h3(j)*P2Y10+h4(j)*P2Y11))*h8(i)
    ZS(SrfNum,i,j)=ZS(SrfNum,i,j)
```

SUBROUTINE XiEtFl (XPnt, YPnt, ZPnt, II, JJ, KK, JI, J2, NKCsps, KCusp, $ NoIts, MxGSiz)
C This SUBROUTINE smooths 3D, constant Zeta grid planes which have been
C disturbed by a constant Zeta cusp in a Xi-Zeta boundary surface. The
C process produces smoother grid lines in the Zeta direction.
C
INTEGER kc, i, j, k, l, II, JJ, KK, NICsps, NoIts,
$ KCusp(MxGSiz), Jl, J2
C
REAL XPnt(MxGSiz,MxGSiz,MxGSiz), YPnt(MxGSiz,MxGSiz,MxGSiz),
$ ZPnt(MxGSiz,MxGSiz,MxGSiz),
REAL XPnt(II, JJ, KK), YPnt(II, JJ, KK), ZPnt(II, JJ, KK)
C
DO 30 k=1,NKCps
kc=KCusp(k)
DO 20 i=2,II-1
DO 10 j=Jl+1, J2-1
DO 5 i=1,NoIts
$ XPnt(i, j, kc)=0.5*(XPnt(i, j, kc+1)-2*XPnt(i, j, kc)
$ +XPnt(i, j, kc-1)) +XPnt(i, j, kc)
$ YPnt(i, j, kc)=0.5*(YPnt(i, j, kc+1)-2*YPnt(i, j, kc)
$ +YPnt(i, j, kc-1)) +YPnt(i, j, kc)
$ ZPnt(i, j, kc)=0.5*(ZPnt(i, j, kc+1)-2*ZPnt(i, j, kc)
$ +ZPnt(i, j, kc-1)) +ZPnt(i, j, kc)
$ XPnt(i, j, kc-1)=0.25*(XPnt(i, j, kc)-2*XPnt(i, j, kc-2)
$ +XPnt(i, j, kc-1)) +XPnt(i, j, kc-1)
$ YPnt(i, j, kc-1)=0.25*(YPnt(i, j, kc)-2*YPnt(i, j, kc-2)
$ +YPnt(i, j, kc-1)) +YPnt(i, j, kc-1)
$ ZPnt(i, j, kc-1)=0.25*(ZPnt(i, j, kc)-2*ZPnt(i, j, kc-2)
$ +ZPnt(i, j, kc-1)) +ZPnt(i, j, kc-1)
$ XPnt(i, j, kc+1)=0.25*(XPnt(i, j, kc+2)-2*XPnt(i, j, kc+1)
$ +XPnt(i, j, kc))+XPnt(i, j, kc)
$ YPnt(i, j, kc+1)=0.25*(YPnt(i, j, kc+2)-2*YPnt(i, j, kc+1)
$ +YPnt(i, j, kc))+YPnt(i, j, kc)
$ ZPnt(i, j, kc+1)=0.25*(ZPnt(i, j, kc+2)-2*ZPnt(i, j, kc+1)
$ +ZPnt(i, j, kc))+ZPnt(i, j, kc)
SUBROUTINE RdGrPin(NGPts, XB, YB, ZB, CrvNum, MxGSiz, InNum)
INTEGER NGPts, CrvNum
REAL XB(MxGSiz, 4), YB(MxGSiz, 4), ZB(MxGSiz, 4)
This subroutine reads in the coordinates of the grid points on one edge.
DO 10 i=1,NGPts
READ(InNum,*) XB(i, CrvNum), YB(i, CrvNum), ZB(i, CrvNum)
10 CONTINUE
RETURN
END

SUBROUTINE CalStI(NGPts, XB, YB, ZB, CrvNum, EdgNum, StrB, MxBCvs, MxGSiz)
INTEGER NGPts, CrvNum, EdgNum, i
REAL XB(MxGSiz, 4), YB(MxGSiz, 4), ZB(MxGSiz, 4),
$ StrB(MxGSiz, MxBCvs)
StrB(1, EdgNum) = 0.
DO 10 i=2,NGPts
   StrB(i, EdgNum) = StrB(i-1, EdgNum) +
$   SQRT((XB(i, CrvNum) -XB(i-1, CrvNum))**2 +
$   (YB(i, CrvNum) -YB(i-1, CrvNum))**2 +
$   (ZB(i, CrvNum) -ZB(i-1, CrvNum))**2)
10 CONTINUE
SMax = StrB(NGPts, EdgNum)
DO 20 i=2,NGPts
   StrB(i, EdgNum) = StrB(i, EdgNum) / SMax
20 CONTINUE
RETURN
END
SUBROUTINE CalSt2(EdgNum, NGPts, StrTp, Beta1, Beta2,
$ StrB, MxBCvs, MxGSiz)
C
C This subroutine calculates the distribution function based on the
C stretching parameters 'StrTp' and 'Beta'
C
INTEGER NGPts, StrTp, EdgNum, i
C
REAL StrB(MxGSiz, MxBCvs), Beta1, Beta2, A, B, DZ
C
StrB(1, EdgNum) = 0.
IF (StrTp .LE. 3) THEN
  DO 10 i = 1, NGPts - 1
    Alpha = (i - 1.) / (NGPts - 1.)
    CALL FAiNew(AiNew, Alpha, Beta1, StrTp)
    StrB(i, EdgNum) = AiNew
  CONTINUE
ELSEIF (StrTp .EQ. 4) THEN
  CALL Str4Prm(Betal, Beta2, A, B, DZ)
  DO 20 i = 2, NGPts - 1
    Alpha = (i - 1.) / (NGPts - 1.)
    CALL Str4(AiNew, Alpha, A, B, DZ)
    StrB(i, EdgNum) = AiNew
  CONTINUE
ENDIF
StrB(NGPts, EdgNum) = 1.
C
RETURN
END
C
C---------------------------------------
C
SUBROUTINE EdgGPts(CrvNum, EdgNum, NGPts, XB, YB, ZB, StrB,
$ x, y, z, s, zx, zy, zz, NDPts, Tensn,
$ MxBCvs, MxBPts, MxGSiz)
C
C This subroutine calculates the grid point location along an edge
C based on a spline curve fitted through specified nodal points and a
given distribution function.
C
INTEGER CrvNum, EdgNum, NGPts, NDPts(4), i, n
C
REAL XB(MxGSiz, 4), YB(MxGSiz, 4), ZB(MxGSiz, 4),
$ StrB(MxGSiz, MxBCvs), x(4, MxBPts), y(4, MxBPts),
$ z(4, MxBPts), s(4, MxBPts), Tensn
C
SRa = S(CrvNum, NDPts(CrvNum))
C
DO 10 i = 1, NGPts
  SB = SRa * StrB(i, EdgNum)
  CALL SplInt(n, s, SB, NDPts, CrvNum, MxBPts)
  XB(i, CrvNum) = SplVal(s, x, zx, SB, Tensn, n, CrvNum, MxBPts)
  YB(i, CrvNum) = SplVal(s, y, zy, SB, Tensn, n, CrvNum, MxBPts)
  ZB(i, CrvNum) = SplVal(s, z, zz, SB, Tensn, n, CrvNum, MxBPts)
10 CONTINUE
SUBROUTINE Str4Prm(S0, S1, A, B, DZ)

REAL S0, S1, A, B, DZ, Y, PI

This subroutine calculates the parameters A, B, and DZ for the two-sided Vinokur stretching function.

PI = ACOS(-1.)

A = SQRT(S0/S1)
B = SQRT(S0*S1)

IF (B .GT. 1.001) THEN
IF (B .LE. 2.7829681) THEN
Y = B - 1
DZ = SQRT(6.*Y) *(1. - 0.15*Y + 0.057321429*(Y**2)
- 0.024907295*(Y**3) + 0.0077424461*(Y**4)
- 0.0010794123*(Y**5))
ELSEIF (B .GT. 2.7829681) THEN
V = LOG(B)
W = 1./B - 0.028527431
DZ = V + (1. + 1./V)*LOG(2.*V) - 0.02041793 + 0.24902722*W
+ 1.9496443*(W**2) - 2.6294547*(W**3) + 8.56795911*(W**4)
ENDIF
ELSEIF (B .LT. 0.999) THEN
IF (B .LE. 0.26938972) THEN
DZ = PI*(1. - B + B**2 - (1. + (PI**2)/6.)*B**3)
+ 6.794732*(B**4) - 13.205501*(B**5) + 11.726095*(B**6)
ELSE
Y = B - 1
DZ = SQRT(6.*Y) *(1. + 0.15*Y + 0.057321429*(Y**2)
+ 0.048774238*(Y**3) - 0.053337753*(Y**4)
+ 0.075845134*(Y**5))
ENDIF
ENDIF

RETURN
END

SUBROUTINE Str4(AiNew, Alpha, A, B, DZ)

REAL AiNew, Alpha, A, B, DZ, U, T

This subroutine calculates the value of the two-sided Vinokur stretching function based on the value of the parameters A, B, and DZ, and on the value of the "computational" coordinate Alpha.

IF (B .GT. 1.001) THEN
U = 0.5 + TANH(DZ*(Alpha - 0.5))/(2.*TANH(DZ/2.))

RETURN
END
ELSEIF (B.LT.0.999) THEN
  \[ U = 0.5 + \frac{\tan(DZ \cdot (\text{Alpha} - 0.5))}{2 \cdot \tan(DZ/2)} \]
ELSE
  \[ U = \text{Alpha} \cdot (1 + 2 \cdot (B - 1) \cdot (\text{Alpha} - 0.5) \cdot (1 - \text{Alpha})) \]
ENDIF
T=U/(A+(1-A)*U)
A1New=T

RETURN
END

SUBROUTINE KFctrs(ZoneNo,kS,k1,k2,k3,k4,kXi1,kXi2,kEtal,kEta2,kZetal,kZeta2,MxSrfs,MxGSiz)

This subroutine is used to set the k-factors that are to be used. The value of the k-factors is first set equal to the user specified values of kXi1, kXi2, kEtal, kEta2, kZetal and kZeta2. After that the user can modify the k-factors for individual grid lines as desired.

INTEGER ZoneNo
REAL kS(MxSrfs,MxGSiz,MxGSiz), k1(MxSrfs,MxGsiz),
$ k2(MxSrfs,MxGSiz), k3(MxSrfs,MxGSiz), k4(MxSrfs,MxGSiz),
$ kXi1, kXi2, kEtal, kEta2, kZetal, kZeta2

Set the starting values of the k-factors:
first, k-factors used in generating interior grid points

DO 100 il=1,MxGSiz
  DO 100 i2=1,MxGSiz
    kS(1,il,i2)=kEtal
    kS(2,il,i2)=kEta2
    kS(3,il,i2)=kZetal
    kS(4,il,i2)=kZeta2
  CONTINUE
DO 200 il=1,MxGSiz
  k1(1,il)=kXi1
  k2(1,il)=kXi2
  k3(1,il)=kZetal
  k4(1,il)=kZeta2
  k1(2,il)=kXi1
  k2(2,il)=kXi2
  k3(2,il)=kZetal
  k4(2,il)=kZeta2
  k1(3,il)=kEtal
  k2(3,il)=kEta2
  k3(3,il)=kXi1
  k4(3,il)=kXi2
  k1(4,il)=kEtal

100 CONTINUE

then, k-factors used to generate boundary surfaces.

DO 200 il=1,MxGSiz
  k1(1,il)=kXi1
  k2(1,il)=kXi2
  k3(1,il)=kZetal
  k4(1,il)=kZeta2
  k1(2,il)=kXi1
  k2(2,il)=kXi2
  k3(2,il)=kZetal
  k4(2,il)=kZeta2
  k1(3,il)=kEtal
  k2(3,il)=kEta2
  k3(3,il)=kXi1
  k4(3,il)=kXi2
  k1(4,il)=kEtal

200 CONTINUE
Here, the user can make any desired modification of the K-factors to improve the grid that he/she is generating. This part of the subroutine will be case dependent.

RETURN
END
REFERENCES


### TABLE 3.1 — Guide To Grid Control Parameters

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TABLE 3.2 — Listing of Input File For Generation of Grid System
For Zone 18 of Radial Turbine Coolant Passage

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3 StretchType
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Figure 1.1 — Radial turbine coolant passage.
$n$ (Technique: $n = 2$ or $n = 4$)

IL
JL
KL
$\sigma_\xi$
$\sigma_\eta$
$\sigma_\zeta$
k$\xi_1$
k$\xi_2$
k$\eta_1$
k$\eta_2$
k$\zeta_1$
k$\zeta_2$

Type-1

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Type-2

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Type-3

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Type-m

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<th>Information for Edge m</th>
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$m = 8$ if $n = 2$ (two boundary technique)
$m = 16$ if $n = 4$ (four boundary technique)

Figure 3.1 — Input-file format for GRID3D-v2.
Information for Edge i

if Type-i = 1:

\[
\begin{align*}
  x_1 & \ y_1 & \ z_1 \\
  x_2 & \ y_2 & \ z_2 \\
  \vdots \\
  x_{NL} & \ y_{NL} & \ z_{NL}
\end{align*}
\]

NL=IL for i=3,4,9,10,13 and 14
NL=JL for i=11,12,15 and 16
NL=KL for i=1,2,3,4

if Type-i = 2:

\[
\begin{align*}
  \sigma \\
  NP \\
  x_1 & \ y_1 & \ z_1 \\
  x_2 & \ y_2 & \ z_2 \\
  \vdots \\
  x_{NP} & \ y_{NP} & \ z_{NP}
\end{align*}
\]

\(\sigma = \text{tension for spline}\)
NP = number of node points

Figure 3.1 (concluded)
Figure 3.2 — Edge curve and boundary surface numbering scheme for GRID3D-v2.
Figure 3.3 — Partitioning of the spatial domain of radial turbine coolant passage into zones for grid generation.
Figure 3.4 — Grid system for zone 18 of radial turbine coolant passage.
Figure 3.5 — Grid system for the whole radial turbine coolant passage (2-D view).
Figure 3.6 — Grid system for the whole radial turbine coolant passage (3-D view).
GRID3D-v2: An Updated Version of the GRID2D/3D Computer Program for Generating Grid Systems in Complex-Shaped Three-Dimensional Spatial Domains

E. Steinthorsson, T. I-P. Shih, and R. J. Roelke

In order to generate good quality grid systems for complicated three-dimensional spatial domains, the grid-generation method used must be able to exert rather precise controls over grid-point distributions. In this report, several techniques are presented that enhance control of grid-point distribution for a class of algebraic grid-generation methods known as the two-, four-, and six-boundary methods. These techniques include variable stretching functions from bilinear interpolation, interpolating functions based on tension splines, and normalized "K-factors." The techniques developed in this study were incorporated into a new version of GRID3D called GRID3D-v2. The usefulness of GRID3D-v2 was demonstrated by using it to generate a three-dimensional grid system in the coolant passage of a radial turbine blade with serpentine channels and pin fins.