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DEVELOPMENT OF LOAD-DEPENDENT RITZ VECTOR METHOD FOR STRUCTURAL DYNAMIC ANALYSIS OF LARGE SPACE STRUCTURES

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ABSTRACT

In the analysis of large space structures, such as the Space Shuttle and Space Station Freedom, analytical models are dealt with which have a large number of degrees of freedom (DOF). It is important that the method used to analyze these structure be as accurate and efficient as possible in order that structural margins and areas requiring redesign can be identified and subsequent analysis be performed. This study involved the development and preliminary assessment of a method for dynamic structural analysis based on load-dependent Ritz vectors. The vector basis is orthogonalized with respect to the mass and structural stiffness in order that the equations of motion can be uncoupled and efficient analysis of large space structure performed. A series of computer programs was developed based on the algorithm for generating the orthogonal load-dependent Ritz vectors. Transient dynamic analysis performed on the Space Station Freedom using the software was found to provide solutions that required a smaller number of vectors than the modal analysis method. Error norms based on the participation of the mass distribution of the structure and spatial distribution of structural loading, respectively, were developed in order to provide an indication of vector truncation. These norms are computed before the transient analysis is performed. An assessment of these norms through a convergence study of the structural response was performed. The results from this assessment indicate that the error norms can provide a means of judging the quality of the vector basis and accuracy of the transient dynamic solution.

INTRODUCTION

The finite element method is the standard method for developing models of space structures for structural analysis. The finite element models of present and future space structures, such as the Space Shuttle and planned Space Station Freedom, possess a large number of degrees of freedom (DOF). To calculate the response of these structures for dynamic loading conditions involves using the finite element models in a transient dynamic or random structural vibration analysis. The standard procedure for doing this utilizes the modal analysis method. For structural models with a large number of DOF the modal vector basis is generally truncated because of the enormously large amount of computation effort and time required to calculate all eigenvectors and eigenvalues of the finite element model. The constraint of having to use a truncated modal basis and the fact that the computational effort to calculate vibration characteristics is costly compared to Ritz based methods [Bathe, 1982] gives motivation and justification for considering other procedures for generating an orthogonal vector basis.

This study is concerned with the development and preliminary assessment of a load-dependent Ritz vector method for use in structural dynamic analysis of large space structures. The scope of work in this study included developing an algorithm which was efficient in terms of numerical operations and has the ability to be used in a production environment. The algorithm was coded on the CRAY X-MP/EA 464 Supercomputer using the FORTRAN computer language. A transient dynamics analysis of the Space Station Freedom was performed using the computer program in order to assess the algorithm. This assessment involved comparing these results to those from a modal analysis. A summary of this assessment is presented in this report. Conclusions based on the finding of the analysis performed are noted and topics for future research are suggested. Also included in this report is a description of the series of computer programs developed which allow one to perform a transient dynamic load-dependent Ritz vector based analysis using NASTRAN [CSA/NASTRAN, 1988] finite models and to post process the results.

LOAD-DEPENDENT RITZ VECTOR ALGORITHM

Block Algorithm and Error Norms

The algorithm for generating an orthogonal load-dependent vector basis is based on using the static amplitudes of the dynamic loads as selected times. The vector basis generated aligns itself with respect to the loading, consequently the vector basis has the potential of a high participation with respect to the response to the dynamic loading. A summary of the algorithm is given in Fig. 1. The Ritz vectors are generated in blocks, each block having several vectors, in order to deal with closely spaced vibration frequencies and a spatial varying loading case that can often exist in space structures. The algorithm begins with the shifting of the structural stiffness matrix $\mathbf{K}$ using the shift constant σ, followed by the calculation of the set of displacements $\mathbf{U_0}$ reflecting the response to the block loading $\mathbf{F}(\mathbf{s})$. Shifting of $\mathbf{K}$ is necessary in order to remove the rigid body modes and permit $\mathbf{K}$ to be factorized. Any structural vibration frequencies near the shift point are well represented in the Ritz vector basis. The vectors $\mathbf{X}$ which are generated are orthogonalized with respect to the mass matrix $\mathbf{M}$ and contain a static residual to reduce the effects of truncation of higher frequencies. A final step of orthogonalizing the vectors $\mathbf{X}$ with respect to $\mathbf{K}$ is necessary to produce the vector basis ${}^{o}\mathbf{X}$ having the frequencies $\hat{\omega}$ in order to uncouple the equations of motion.

The concept of a block form was motivated by it use in previous work by the investigator [Ricles et. al., 1990] and that of other researchers [Nour-Omid and Clough, 1985].

During the generation of vectors $\mathbf{X_i}$ the norm ϵ_{u_i} is computed, and is intended to provide an indication of the participation of the mass distribution. This norm is computed using the displacements $\mathbf{U_i}$ at each cycle and the initial displacement set $\mathbf{U_0}$, where at cycle i:

$$\epsilon_{u_i} = \frac{||\mathbf{MU_i}||_\infty}{||\mathbf{MU_0}||_\infty} \tag{1}$$

For the analysis performed the behavior of ϵ_{u_i} was studied to determine whether it could be used as criteria to judge when to terminate the generation of vectors. A second norm ϵ_L, intended to represent the degree of participation of the spatial load distribution, was also calculated and studied to determine whether it was reliable for use in judging the quality of the vector basis ${}^{o}\mathbf{X}$ in responding with high participation to the loading. This norm is computed by the following formula, where for k vectors in ${}^{o}\mathbf{X}$:

$$\epsilon_L = \frac{|\,\mathbf{P(t)e_L}\,|}{\mathbf{P^T(t)P(t)}} \tag{2}$$

1. Dynamic Equilibrium Equations:

$$\mathbf{M\ddot{Z}(t) + C\dot{Z}(t) + KZ(t) = F(s)\alpha(t)}$$

2. Initial Calculations:

(a) Shift and Factorize Stiffness Matrix

$$\mathbf{K^* = K + \sigma M}$$
$$\mathbf{K^* = LDL^T}$$

(b) Solve for Static Response to Block Loading

$$\mathbf{K^* U_0 = F(s)}$$

3. Calculate Rigid Body Modes:

(a) Use Geometric Description and DOF Relationship to Describe Rigid Body Motions $\mathbf{X_1}$

(b) Generate First Block(s) of Orthonormalized Ritz Vectors

$$\mathbf{X_1} = \beta \mathbf{\overline{X}_1}$$
$$\beta = \left(\mathbf{\overline{X}_1^T M \overline{X}_1}\right)^{-1}$$

(c) Remove Rigid Body Modes From Static Block $\mathbf{U_0}$ (Gram-Schmidt Orthogonalization)

$$\mathbf{U_1 = U_0 - X_1\left(X_1^T M U_0\right)}$$

4. Generate Additional Ritz Vectors $\mathbf{X_i}, i = 2, \ldots, n-1$:

(a) Solve for $\mathbf{\overline{X}_i}$

$$\mathbf{K^* \overline{X}_i = M U_{i-1}}$$

(b) M-Orthogonalize $\mathbf{\overline{X}_i}$ against previous blocks (Gram-Schmidt)

$$\mathbf{X_i^* = \overline{X}_i - \sum_{j=m}^{i-1} X_j X_j^T M \overline{X}_i} \qquad \mathbf{1 \le m \le i-2}$$

(c) M-Orthogonalize Vectors in Block $\mathbf{X_i^*}$ by Modified Gram-Schmidt to obtain $\mathbf{X_i}$

(d) Remove New Ritz Block $\mathbf{X_i}$ From Static Block $\mathbf{U_{i-1}}$ (Gram-Schmidt)

$$\mathbf{U_i = U_{i-1} - X_i\left(X_i^T M U_{i-1}\right)}$$

5. Add Static Block Residual $\mathbf{U_{n-1}}$ as Static Correction Terms $\mathbf{X_n}$.

6. Make Ritz Vectors $\mathbf{X}$ Stiffness Orthogonal (Optional - Uncouples Equations of Motion):

(a) Form and Solve the Reduced $n \times n$ eigenvalue Problem

$$\mathbf{K^{**} = X^T K^* X}$$

$$\left[\mathbf{K^{**}} - \hat{\omega}^2 \mathbf{I}\right] \mathbf{\Psi} = 0$$

(b) Compute Final Ritz Vectors $\mathbf{{}^{o}X = X\Psi}$

Fig. 1.- Block Form Load-Dependent Ritz Vector Algorithm for Semipositive Definite Systems.

where $\mathbf{P(t)}$ is the load vector at time t and $\mathbf{e_L}$ the error in the representation of the load in Ritz coordinates, with

$$\mathbf{e_L} = \mathbf{P(t)} - \sum_{\mathbf{j=1}}^{\mathbf{k}} {}^{\mathbf{o}}\mathbf{X_j}\mathbf{P^T(t)M}{}^{\mathbf{o}}\mathbf{X_j} \tag{3}$$

Software Development

The computer program named RITZ was developed to generate ${}^{\mathbf{o}}\mathbf{X}$, $\hat{\omega}$, and ϵ_u. RITZ interfaces with the bulk NASTRAN data set and NASTRAN OUTPUT4 [CSA/NASTRAN, 1988] files containing the structural mass and stiffness matrices to define the structural model. To maximize the efficiency, the symmetry and any sparseness of these matrices are exploited in their incore storage and factorization. A second computer program named NAS-DYN was developed which gives the user the choice of calculating either the norm ϵ_L or the transient dynamic response of the structure. Either Ritz vectors or NASTRAN modal vectors can be used. The results of the transient analysis can be post processed by a third newly developed program POSTRN for producing plot files which are compatible with the JSC Structures and Mechanics Division PLOT2D graphics software.

SPACE STATION ANALYSIS

Program of Investigation

The Space Station Freedom was analyzed for a simulated docking with the Space Shuttle. A view of the Station is given in Fig. 2. In the analyses performed the NASTRAN MB15YZ model of the Station with 2803 DOF was used along with docking load case 915L, consisting of the set of transient forces shown in Fig. 3 applied to the end of the docking arm. The position of the PV arrays in this model had an orientation in the Y-Z plane as opposed to the X-Y plane as shown in Fig. 2. RITZ was used to generate vector bases with 30, 60, and 90 vectors and DYNTRN used to compute the elastic displacements and total accelerations of selected DOF based on a constant damping ratio of 0.01. The vector bases were generated using a block size of 6 vectors, with $\mathbf{U_0}$ consisting of six displacement vectors corresponding to the response of the six individual docking loads acting on the structure at time=1.93 seconds. The response at the tip of an outboard PV array (node 8022 in Fig. 2) is discussed in this report. Neither a complete solution

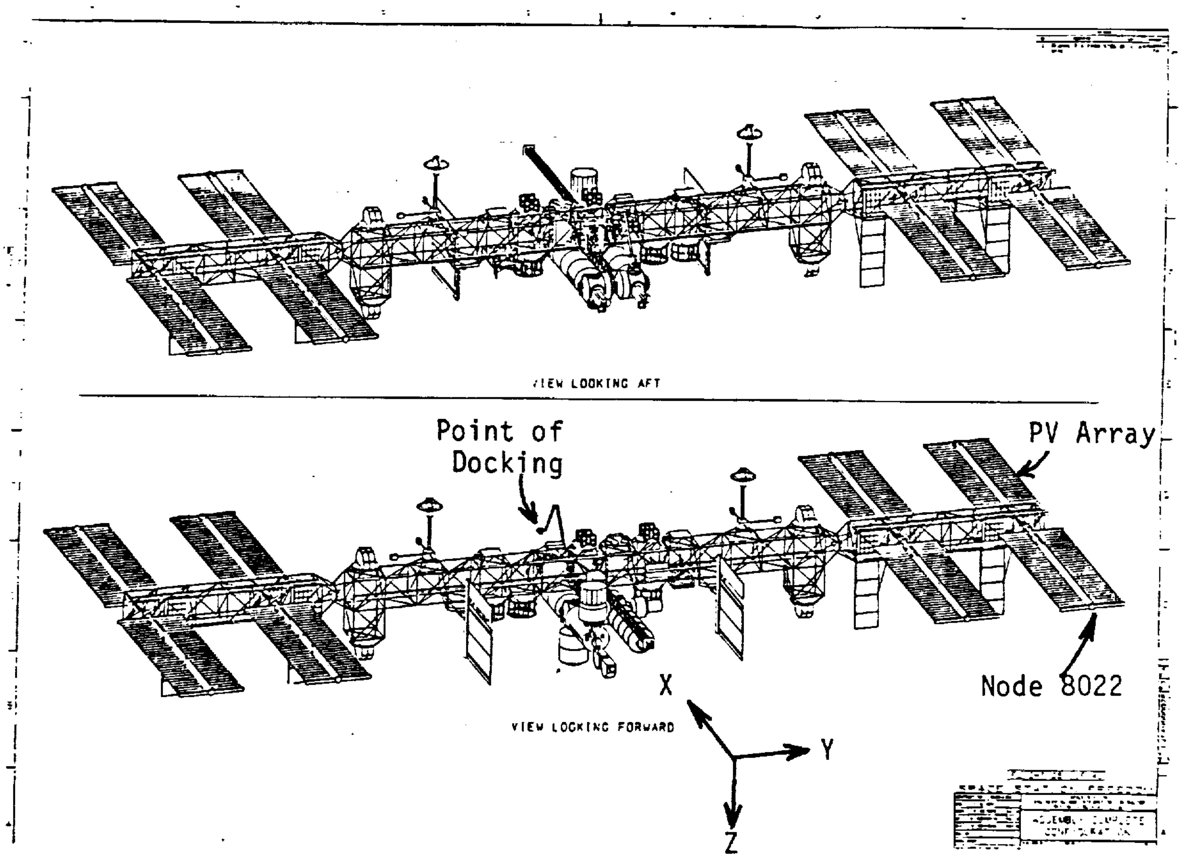
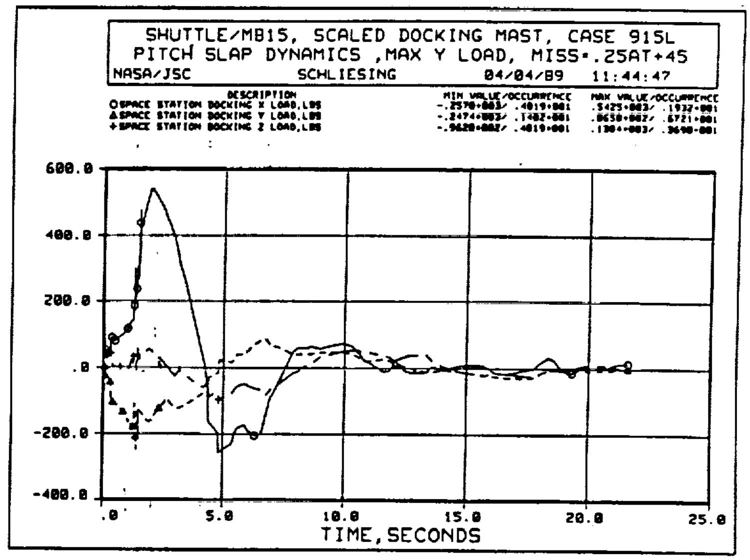


Fig. 2-Space Station Freedom with PV Arrays in The X-Y Plane.

(a)



(b)

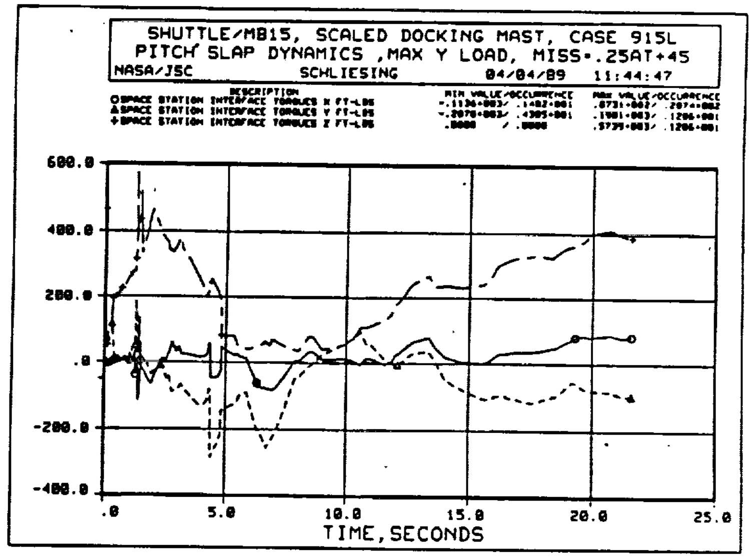


Fig. 3 Docking Load Case 915L (a) Translational, and (b) Rotational Transient Forces.

based on a numerical integration of the coupled equations of motion nor a complete modal vector basis was available to compare with the Ritz solution. For purposes of this preliminary study a comparison was made with solutions obtained from a truncated modal basis consisting of 210 modes. The frequency range of the modes ranged from 0 to 6.0 Hz. Fig. 4 shows a comparison of the frequency range of the modal vectors and selected Ritz vector bases. The 60 Ritz vector basis with a static residual (identified as SR in the legend of Fig. 4) has approximately the same frequency range as the modal basis. It can be seen that a larger Ritz vector basis has a greater span and that this span is extended for those basis containing the static residual as opposed to not having it (identified as NSR in the figure's legend). All Ritz vector basis presented in this report were generated with a shift constant of $\sigma = 1.0$, except for the 60 vector basis with $\sigma = 200.0$ identified in Fig. 4 as SHI. The effect of using a larger value for σ is that the basis has more distantly spaced frequencies and a greater span.

Presentation of Transient Response

A time history plot of the translational displacements and accelerations in the x-direction are shown in Figs. 5 and 6 for the 90 Ritz vector and modal solution. The Ritz based displacements are shown to agree closely with the modal solution, whereas there exists a greater discrepancy between the accelerations computed by the two methods. The maximum discrepancy in the transient translational displacement and acceleration along the X-, Y-, and Z-axes based on the results from the modal vectors and Ritz vectors with a 90, 60, 30 vector basis are shown in Fig. 7. These results correspond to the node at the end of the outboard PV array. It is apparent in Fig. 7 that the discrepancy between the 210 modal vector and Ritz solutions decreases as the number of vectors in the Ritz basis is increased from 30 to 90 vectors. The Ritz solution with a 60 vector basis is able to achieve the same result as the 210 modal vector solution for displacements. The Ritz based solution for accelerations shows greater discrepancies with the modal solution, and requires a larger number of vectors in the basis to achieve the same level as accuracy as that found in the displacements at 60 Ritz vectors.

Analysis of Results

The norm ϵ_u is plotted against the number of blocks of Ritz vectors in

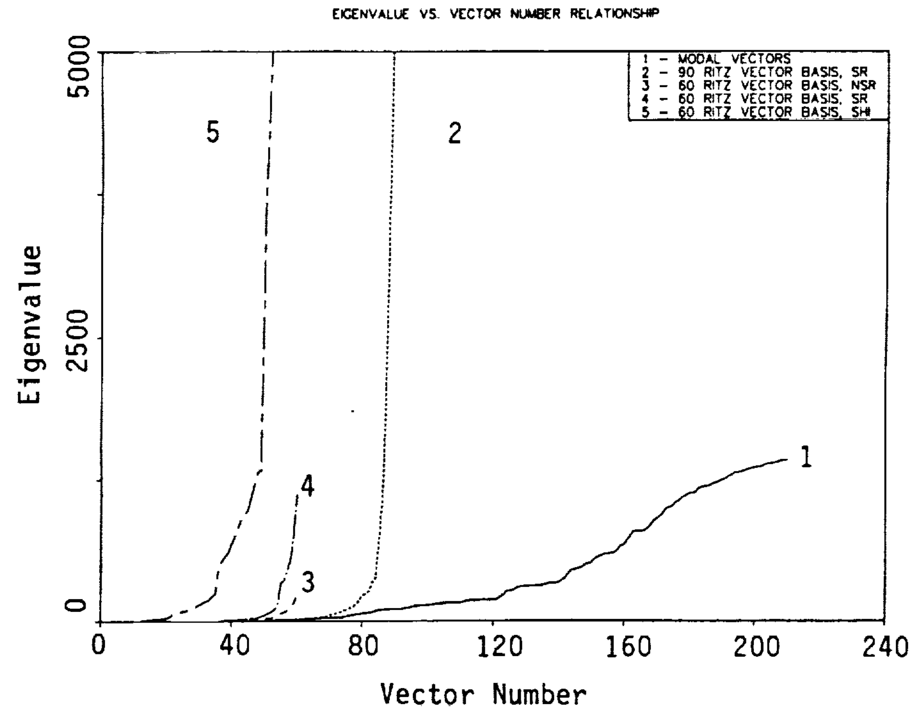


Fig. 4.- Span of Eigenvalue (Frequecy) Range of Modal and Ritz Vector Bases.

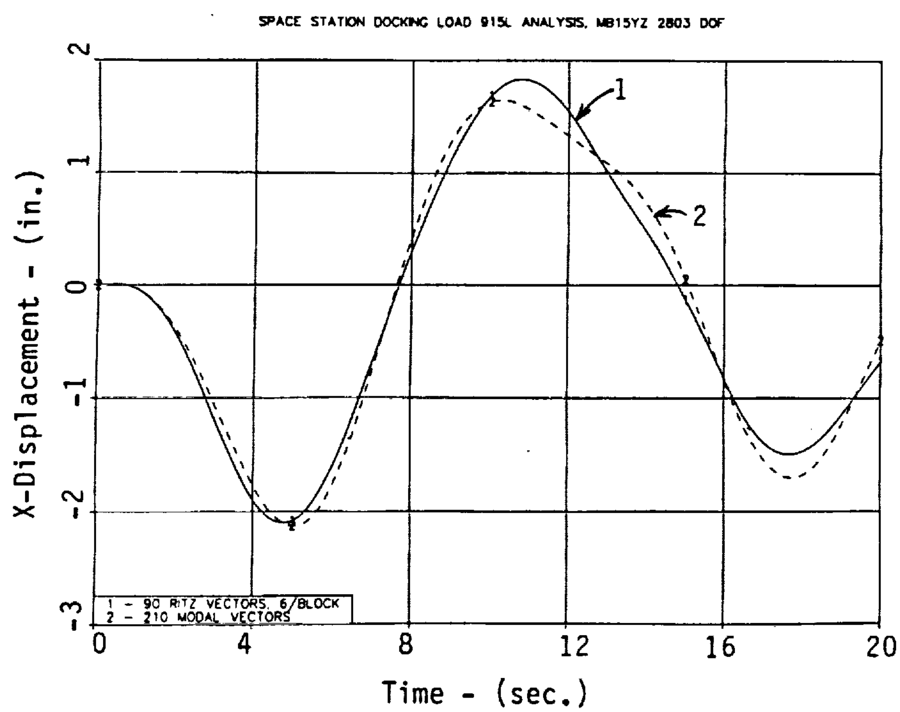


Fig. 5.- Displacement Response at End of Outboard Pv Array.

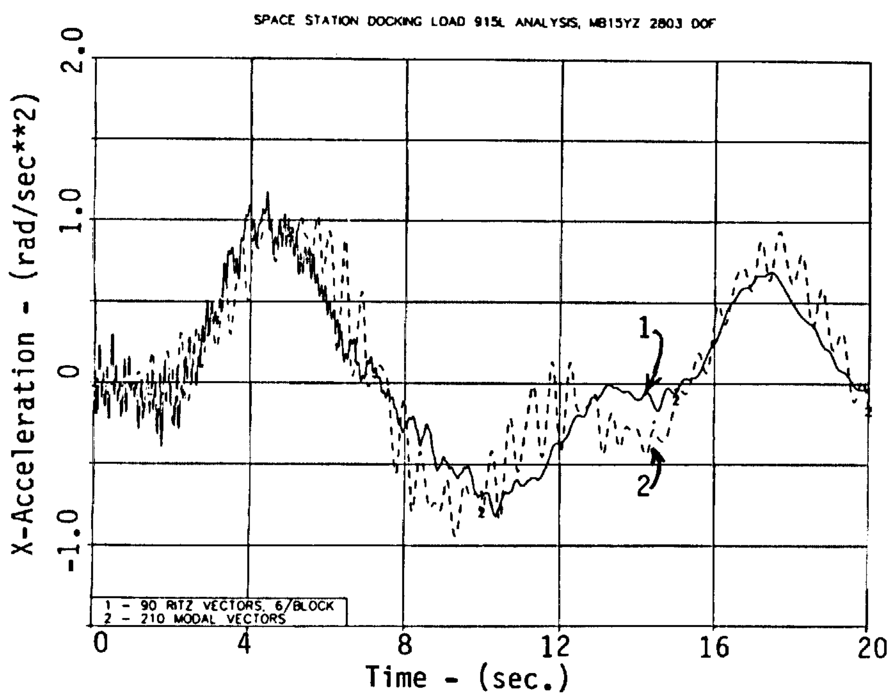


Fig. 6.- Acceleration Response at End of Outboard PV Array.

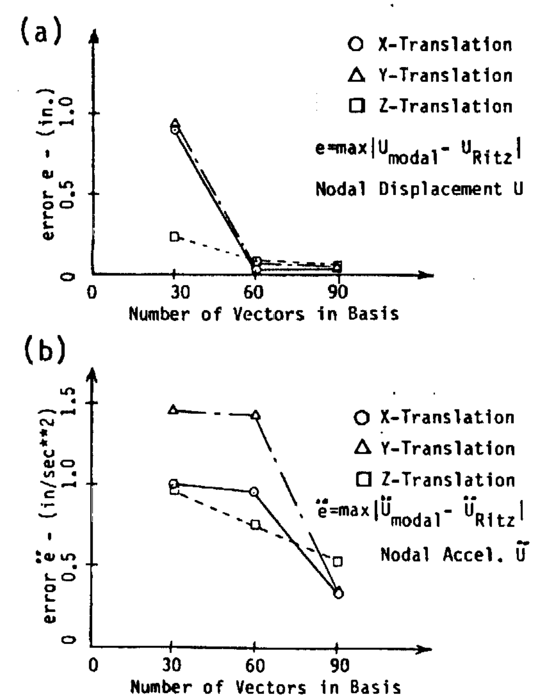


Fig. 7.- Error Between Modal and Ritz solutions for (a) Displacements, and (b) Accelerations at end of PV Array.

Fig. 8 to establish whether it can give a good indication of when to terminate the generation of vectors. An examination the data reveals that for 30 Ritz vectors (5 blocks x 6 vectors per block = 30) the value of ϵ_u is approximately 50 times greater than that for 60 Ritz vectors (10 blocks x 6 vectors per block = 60), at which the displacements were in very close agreement with the modal solution. The fact that ϵ_u has become quite small at 60 Ritz vectors suggest that there is not much to be gained by 30 additional vectors to achieve a 90 vector basis. As discussed above this was found to be true only for the displacements. Thus, ϵ_u could be considered more reliable for displacements than accelerations .

The behavior of ϵ_u in Fig. 8 shows that a fluctuation in the curve can exist where a local maximum develops. This phenomenon is quite noticeable for the case of 5 blocks with 6 vectors per block. This behavior is associated the structure responding more to the forces related to the displacements $\mathbf{U_i}$ as opposed to $\mathbf{U_0}$. Numerical experimentation indicated that apparently this occurs when the block loads used to calculate $\mathbf{U_0}$ are not uniformly displacing the mass, leading to displacement patterns $\mathbf{U_i}$ in some of the later cycles which 'accelerate' more the mass. This same experimentation indicated that the fluctuation tends to occur in the initial cycles of vector generation and that as the number of vectors is increased the phenomenon appears to decay in a similar manner as shown in Fig. 8. This phenomenon is not as pronounced in the vector basis generated using fewer vectors per block, as indicated by the results shown in Fig. 8 for a block size of 1 vector. Asymptotic convergence of ϵ_u to zero can thereby be expected as the number of vectors is increased. A threshold value for ϵ_u to terminate the process of generation of vectors can not be assigned at this time, for its sensitivity to models with a more uniformly distributed mass is not known as well as the fact that the results presented are from only one loading condition.

A presentation of the norm ϵ_L associated with the translational components of the docking loading at time=1.93 seconds is given in Figs. 9 to 11. These results include the 60 and 90 Ritz vector basis (cases already discussed above, which contain the static residual identified as SR in the legends of Figs. 9 to 11) in addition to the 210 modal basis and a 60 Ritz vector basis without a static residual (identified as NSR in the legends). It can be observed that the Ritz vectors' basis can better represent the load than the modal vectors. This representation is improved as a static residual is added and further improved as the number of Ritz vectors in the basis is

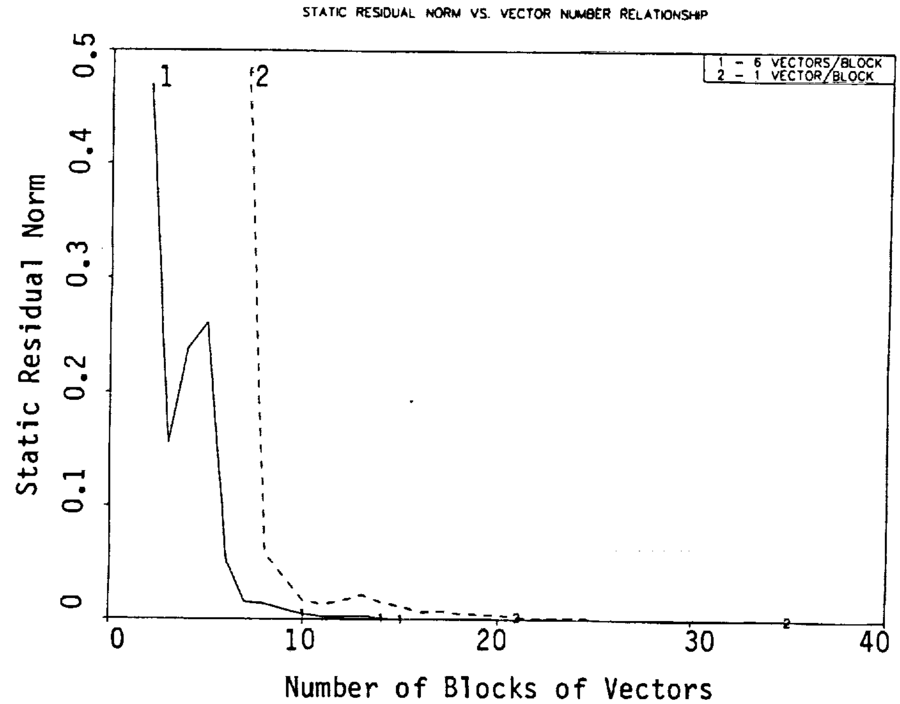


Fig. 8.- ε_u-Block Number Relationship, Block Sizes 1 and 6 Vectors.

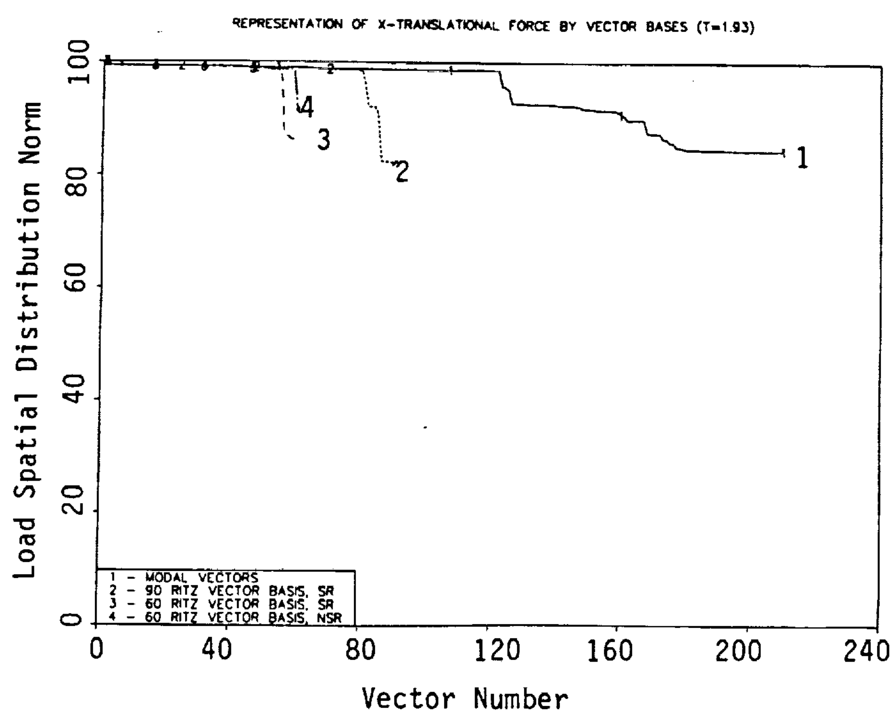


Fig. 9.- ε_L Norm for X-Translational Docking Force at Time 1.93 sec.

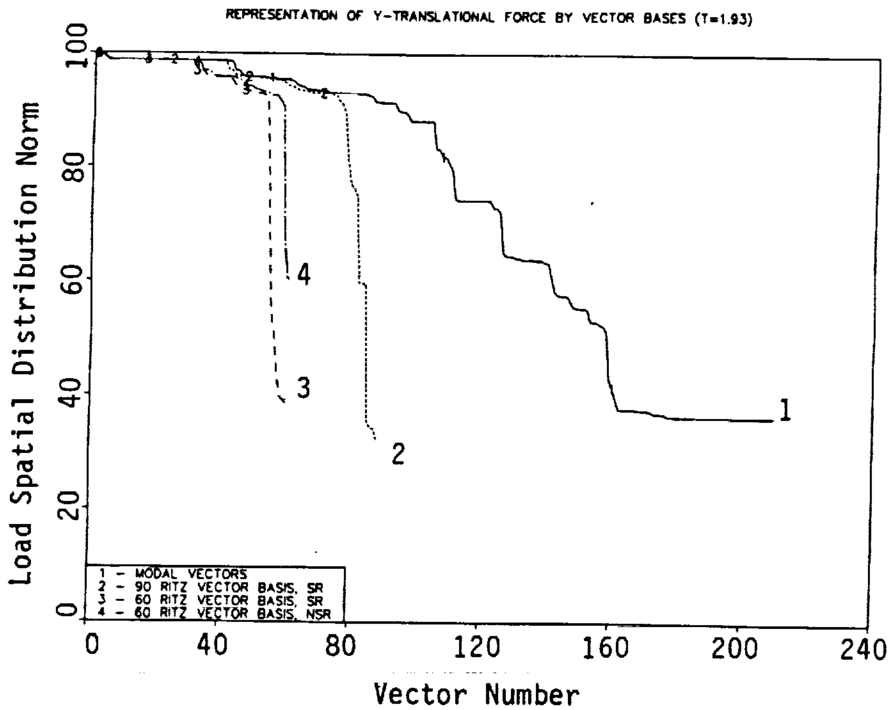


Fig. 10.- ε_L Norm for Y-Translational Docking Force at Time 1.93 sec.

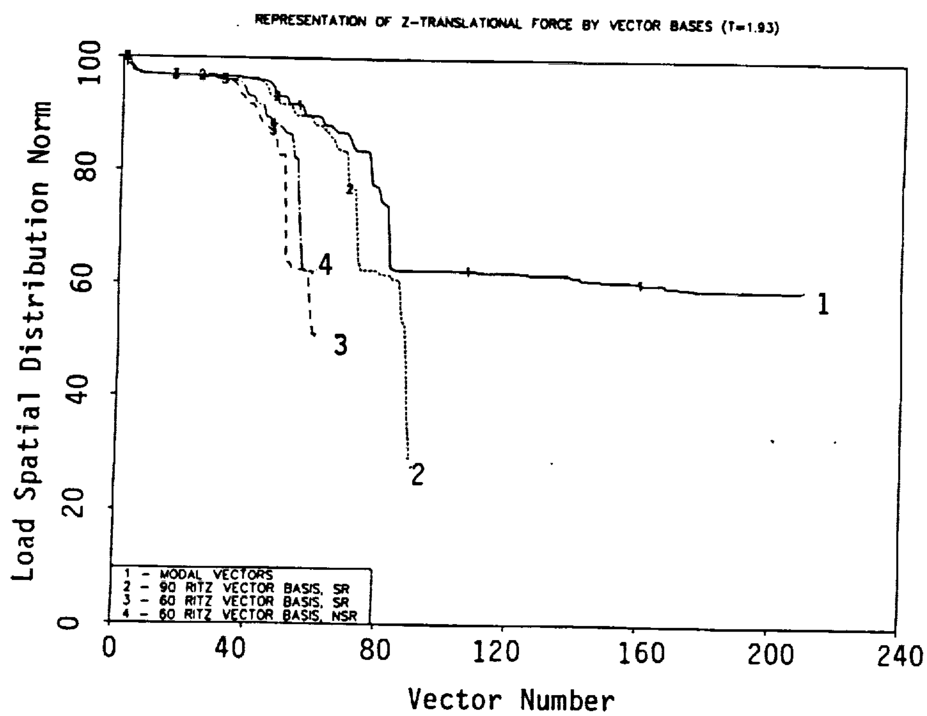


Fig. 11.- ε_L Norm for Z-Translational Docking Force at Time 1.93 sec.

increased. It can also be observed that representation of the loading by the 90 Ritz vector basis is as good (X- and Y-translation loadings) or better (Z-translational loading) compared to that of the 210 modal vector basis. The fact that fewer Ritz vectors were found to be required than modal vectors to achieve comparable results for ϵ_L gives an indication that the Ritz vector bases align more with the loading than the modal vector basis, such that the product of the Ritz vectors with the load vector is larger than the product using modal and the load vector. The participation of the Ritz modes $^o\mathbf{X}$ in the dynamic response can therefore be expected to occur, as opposed to the possibility of having the vectors orthogonal to the loading as is sometimes the case in modal analysis. A close examination of Figs. 9 to 11 shows that most of the modal vectors' contribution to the load representation by its basis occurs in the mid frequency range (the vector number in Figs. 9 to 11 correspond to the ordering of the frequencies). Also, the modal vectors which are orthogonal to the loading are spread throughout the spectrum. This orthogonality is identified by the parts of the curve joining the values of ϵ_L where the slope is zero.

A comparison of the behavior of ϵ_u with that of ϵ_L indicates that although ϵ_u may become small, ϵ_L may not. Since convergence for accelerations was observed to require more Ritz vectors, more emphasis should be placed on ϵ_L for analysis in which the acceleration response is of prime interest when attempting to judge the quality of the vector basis for the selected loading conditions. This could perhaps be extended to situations where the Ritz vectors were generated based on a particular load case and there is the desire to use them for another load condition.

CONCLUSIONS

Based on the results of the preliminary analysis of the Space Station Freedom using the load-dependent Ritz vector algorithm presented herein, the following conclusions are noted:

1. Load-dependent Ritz vectors appear to provide accurate displacement solutions using fewer number of vectors than modal based vectors. The use of ϵ_u for criteria to stop the generation of the vector process appears to be promising.

2. More load-dependent Ritz vectors are required when computing accelerations. Criteria for judging the quality of the vector basis using ϵ_L needs to be more strict when acceleration response is of interest as opposed to displacements.

3. Load-dependent Ritz vectors have a broader frequency range than the equivalent number of modal vectors and are able to better represent the loading function. These features of the Ritz vectors are enhanced by including the static residual in the basis.

Areas which are suggested for future study include:

1. A study of the basic behavior and calibration of ϵ_u and ϵ_L for different load cases and structures, respectively.

2. Assessments of the reliability of the vector basis generated from a different load case than that used in the transient dynamic analysis and whether ϵ_L can provide a means of judging the vector basis quality in such cases.

3. The optimization of the algorithm to take advantage of the vectorization procedure by the CRAY X-MP/EA 464.

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