The Pulse-Pair Algorithm as a Robust Estimator of Turbulent Weather Spectral Parameters Using Airborne Pulse Doppler Radar

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Abstract

The pulse pair method for spectrum parameter estimation is commonly used in pulse Doppler weather radar signal processing since it is economical to implement and can be shown to be a maximum likelihood estimator. With the use of airborne weather radar for windshear detection, the turbulent weather and strong ground clutter return spectrum differs from that assumed in its derivation, so the performance robustness of the pulse pair technique must be understood. This paper analyzes the effect of radar system pulse to pulse phase jitter and signal spectrum skew on the pulse pair algorithm performance. Phase jitter effects may be significant when the weather return signal to clutter ratio is very low and clutter rejection filtering is attempted. The analysis can be used to develop design specifications for airborne radar system phase stability. The paper also shows that weather return spectrum skew can cause a significant bias in the pulse pair mean windspeed estimates, and that the poly pulse pair algorithm can reduce this bias. It is suggested that use of a spectrum mode estimator may be more appropriate in characterizing the windspeed within a radar range resolution cell for detection of hazardous windspeed gradients.
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I. Introduction

Ground based pulse Doppler radar is frequently used to provide information about weather conditions [1]. The coherent weather radar return typically includes a distribution of Doppler frequencies across the processing bandwidth, with a dominant Doppler frequency corresponding to a most probable windspeed value or in some sense an "average" within the range resolution cell [2]. The spread in the Doppler is influenced by the fact that the return is a scattering of incident electromagnetic energy from many distributed particles, by the spread in the antenna beam, and by the time domain windowing associated with the radar signal processing [2]. With adequate range resolution and with a scanning radar antenna, ground based systems have been shown to provide reliable estimates of windspeed spatial gradients associated with microbursts [3] and gust fronts which are considered hazardous to low altitude aircraft, e.g., in landing or take-off at a terminal area [1],[4].

Largely because of the computational efficiency, the pulse-pair algorithm [5]–[12] is commonly used to estimate average wind speed within each range resolution cell of the radar coverage sector. This algorithm can also provide a measure of wind turbulence by estimating the weather return Doppler spectrum spread. Airborne pulse Doppler radar is being considered to detect potentially hazardous windshear conditions [13],[14].

The pulse pair algorithm has been evaluated extensively in the literature [11]. It is implicitly assumed that the radar system is phase stable from pulse-to-pulse. In an airborne platform implementation, there are potential sources of system phase instability which may contribute to the decorrelation of pulse-to-pulse signal phase, e.g., the STALO (stable local oscillator) [15] which provides transmitted and reference carriers within the radar, quadrature processing at IF [16] which may contribute phase differences in the I and Q channels, and platform motion which may amplify other sources of phase instability. This system phase instability, or phase jitter, may be described as a
stochastic process. These effects may not be apparent when
observing the weather return Doppler spectrum, because of its
inherent spread. Earlier work [17] has considered this problem
assuming that the phase jitter is a white process. Results of an
analytical evaluation of signal to phase noise power compared
favorably to experimental results using a ground based radar with
an injection-locked magnetron. This report generalizes those
results by analyzing how phase jitter may affect the ability to
estimate Doppler spectrum parameters with the pulse pair algorithm
[18]. The results allow for the error in spectrum parameter
estimates to be computed from experimentally as well as
analytically described phase noise processes. It is shown that
the earlier white noise process results [17] are just a special
case.

Low altitude airborne weather radar returns are likely to be
dominated by ground clutter typically containing large spectral
power around the Doppler frequency corresponding to the aircraft
ground speed. Near a large urban airport there may be significant
return at other Doppler frequencies because of the antenna pattern
sidelobes and the structure of the surrounding area, including
highways, large buildings, multiple runways, moving vehicles, etc.
When the ground clutter spectrum is relatively narrow compared to
the Doppler processing bandwidth, it is anticipated that clutter
cancellation can be done very successfully without degrading the
weather spectrum [1], [19] using a high pass notch filter. The
notch bandwidth is generally limited by the loss in weather signal
sensitivity [20], [21]. As will be shown in this report, any radar
system phase noise will broaden the clutter spectrum causing
clutter power within the notch bandwidth to spill into the pass
band region, limiting clutter cancellation capability. This may
be a significant problem, particularly in a "dry microburst"
situation where the pre-filtered weather signal to ground clutter
power ratios are normally quite low (-30dB or less) [1].

Turbulent weather Doppler radar returns can have broad
spectra and may also be skewed, not meeting the symmetry
assumption in the pulse pair derivation. Any resulting errors in
the spectral parameter estimates may be further deteriorated by system phase instabilities. This report presents and develops a generalized statistical analysis approach for a quantitative assessment of pulse-to-pulse system phase jitter on the spectrum moment estimation quality and clutter cancellation capability, considering use of the pulse pair algorithm. Intrapulse phase uncertainty is not considered here.

In Section II the pulse pair algorithm is briefly reviewed with the assumptions made in deriving this estimator clearly reemphasized. Results of the mean and width estimate quality which were previously derived (see [9],[22],[23]) are also restated, along with the corresponding assumptions made in those derivations. In Section III, the complex autocorrelation function of the radar return signal is described to incorporate the effects of system pulse-to-pulse phase jitter noise. Section IV then presents an analysis of the rederived pulse pair estimator quality using the expressions from Section II, modified to include the effects of the system phase error described in Section III. Section V discusses limitations on pulse pair estimation quality when ground clutter cancellation is used and radar system phase jitter noise is present. Section VI provides an analysis of the pulse pair estimate quality in the presence of a skewed weather return spectrum. The poly pulse pair algorithm [24] is discussed as a means of improving the pulse pair mean estimate bias brought about by spectrum skew. A summary and conclusions are given in Section VII.
II. Pulse Pair Spectrum Parameter Estimation

Consider the radar return Doppler spectrum $S(f)$ which is unambiguous over the processing bandwidth, i.e., the frequency interval $[-1/2T_s, 1/2T_s]$ where $T_s$ is the interpulse interval of a pulse sequence. If $f_d$ is the mean value of $S(f)$, then the autocorrelation of the pulse sequence for delay $T_s$ is the inverse Fourier transform of $S(f)$ which can be expressed as

$$R(T_s) = e^{j2\pi f_d T_s} \int_{-1/2T_s}^{1/2T_s} S(f)e^{j2\pi f f} df$$

(1)

If $S(f)$ is symmetric about $f_d$ and band-limited well within the interval $[-1/2T_s, 1/2T_s]$, the integral in (1) will be real and the autocorrelation at lag $T_s$ becomes

$$R(T_s) = |R(T_s)| e^{j2\pi f_d T_s}$$

(2)

It is apparent that if one can estimate the argument of the complex autocorrelation function at lag $T_s$, then the mean frequency $f_d$ can be estimated as

$$\hat{f}_d = \frac{1}{2\pi T_s} \text{arg}\{\hat{R}(T_s)\}$$

(3)

where

$$\hat{R}(T_s) = \frac{1}{M-1} \sum_{m=0}^{M-1} z(iT) z(iT+T_s)$$

(4)

Here $z(iT)$ represents the output complex video signal sequence indexed at pulse times $iT$ where $i=0,1,2,3,...,M-1$. The estimator (3) is unbiased if $S(f)$ is symmetric or narrow compared with the processing bandwidth [2] and is a maximum likelihood estimator if the pulse pairs are independent [25]. The estimator variance has been derived for a Gaussian shaped spectrum $S(f)$ [9] and is
\[
\text{VAR}(\hat{\ell}_d) = \left[8\pi^2 T_s \beta^2 (T_s) \right]^{-1} \left\{ M^2 \left[ 1 - \beta^2 (T_s) \right] \cdot \sum_{m=-\infty}^{M-1} \beta^2 (mT) (M - |m|) \right.
\]
\[
+ \frac{N^2}{MS^2} + 2N \cdot \left[ 1 - \beta (2T_s) \delta_{T_s,0} \delta_{T_s,0} + \frac{\beta (2T_s)}{M} \delta_{T_s,0} \right] \}
\]
\] (5)

where \(\beta (T_s) = \exp \{-2\pi^2 w^2 T_s^2 \}\) and for contiguous pulse pairs \(T = T_s\).

Here \(S\) is the signal power per sample
\[
S = \lim_{M \to \infty} \frac{1}{M} \sum_{i=0}^{M-1} |Z(iT)|^2
\] (6)

and \(N\) is the background (thermal) noise power per sample. The expression in (5) matches experimental results at very narrow spectrum widths or low SNR only if a very large number of samples is involved [10]. Also, for independent pulse pairs \((T \to \infty)\) the variance in (5) is the Cramer-Rao bound [25].

One form of the pulse pair width estimator which is independent of the mean frequency and independent of the spectrum shape when the width is sufficiently smaller than the processing bandwidth [10] is
\[
\hat{\ell}_d = \begin{cases} 
\frac{1}{\sqrt{2\pi T_s}} \left\{ \frac{\hat{R}(T_s)}{\hat{S}} \right\}^{1/2} & \text{when} \left| \hat{R}(T_s) \right| < \hat{S} \\
\delta & \text{when} \left| \hat{R}(T_s) \right| \geq \hat{S}
\end{cases}
\] (7)

where the signal power estimate
\[
\hat{S} = \frac{1}{L} \sum_{i=0}^{L-1} |Z(kT)|^2 \cdot N
\] (8)

is formed by subtraction of the known noise power from the total power estimate. Note that the estimate value \(\delta\) in (7) is simply a tag to indicate that the estimate of the magnitude of the first autocorrelation lag exceeds the estimate of the signal power, which is a computational anomaly. This form of the width estimator is asymptotically biased but the bias is inversely proportional to \(M\). It is an approximation to the maximum
likelihood estimator when the spectrum is Gaussian and pairs are independent [25]. The variance, which is the same as the ML estimate if the bias of (7) is removed, is given by [11, Eqn (5.4)]

$$\text{VAR} (\hat{\gamma}) = [32 M \pi^4 (WT_s)^2 \beta^2 (T_s) T_s^2]^{-1} \left\{ 2 \cdot [1 \cdot (1 + \delta_{T \cdot T_s, 0}) \beta^2 (T_s) \right.$$ 

$$+ \delta_{T \cdot T_s, 0} \beta^4 (T_s) \frac{N}{S} + [1 + (1 + \delta_{T \cdot T_s, 0}) \beta^2 (T_s)] \frac{N^2}{S^2} 

{\beta^2 (T_s)} \sum_{m = 1}^{M-1} \left[ 2 \beta^2 (mT) + \beta^2 (mT) \beta^2 (T_s) \right. 

{\left. + \beta (mT + T_s (1 - \delta_{T \cdot T_s, 0})) \cdot 4 \beta (mT + T_s) \cdot \beta (mT) \beta^{-1} (T_s) \right] 

{\cdot (1 - \frac{m}{M})} \right\} $$

(9)

where $\beta(.)$, $S$, and $N$ are as defined above. This result is not reliable for very narrow widths or when the SNR and the number of pairs are small.
III. The Radar Return Autocorrelation Considering System Phase Jitter Effects

The transmitted radar signal at each pulse time is a burst of a sinusoid defined by

\[ V(t) = V_0 e^{j(2\pi f_c t + \alpha)} \]

where \( \alpha \) is a random phase considered constant throughout the pulse duration and \( f_c \) is the transmitted carrier frequency. The complex demodulated video signal at the receiver output can be represented as

\[ Z(t) = Z_0(t)e^{j(2\pi f_d t + \phi(t))} \]

where the envelope function \( Z_0(t) \) is a narrowband random process determined primarily by the nature of the source of the radar return, \( f_d \) represents the mean Doppler frequency of the return, and \( \phi(t) \) is a random process associated with the phase of the return. The process \( \phi(t) \) is considered to have stationary increments where statistical changes are very slow as compared to the interpulse period \( T_s \). The phase process modelled in terms of \( \phi(t) \) might include STALO oscillator phase drift, phase instabilities within the modulator or demodulator, or platform motion, i.e., anything that might contribute to uncertainty in the phase of the return signal from pulse to pulse. Any short term intrapulse phase fluctuations associated with the source of the return, e.g., the weather, are considered to be a part of the process \( Z_0(t) \). The interpulse phase variations modelled in terms of \( \phi(t) \) will be referred to as phase jitter and are isolated for further study.

Assuming that the envelope process is statistically independent of the phase jitter process, the autocorrelation of the return can be characterized as

\[ R'(T_s) = E\{Z(t)Z(t+T_s)\} = R(T_s)E\{e^{-j\phi(t)}e^{j\phi(t+T_s)}\} \]

where the first factor incorporates characteristics of the weather return and the last factor is associated with the phase jitter.
Excluding other effects the autocorrelation associated with the weather return is

\[ R(T_s) = E\{Z_0(t)Z_0(t+T_s)\} = R_w e^{j2\pi f_0 T_s} \] (13)

where, for the Gaussian shaped weather return spectrum,

\[ R_w = R_0 e^{-\frac{2\pi^2 W^2 T_s^2}{2}} \] (14)

and is just \( \beta(T_s) \) used in (5) and (9). If the phase \( \phi(t) \) and \( \phi(t+T_s) \) can be considered jointly Gaussian, then the last expectation in (12) can be evaluated [26] as

\[ E\{e^{j[\phi(t+T_s) - \phi(t)]}\} = \exp{-\int_{-\infty}^{\infty} S_\phi(f) [1 - \cos(2\pi f T_s)] df} \] (15)

where \( S_\phi(f) \) is the phase jitter spectrum. If \( S_\phi(f) \) is an even function, then this can be further reduced to

\[ E\{e^{j[\phi(t+T_s) - \phi(t)]}\} = \exp{R_\phi(T_s) \int_{-\infty}^{\infty} S_\phi(f) df} \] (16)

where \( R_\phi(T_s) \) is the autocorrelation of the phase process at lag \( T_s \), i.e., the inverse Fourier transform of \( S_\phi(f) \) evaluated at \( T_s \).

For this case the exponent in (15) is the structure function of the random phase process [2],[27], a basic characteristic of the process describing the intensity of the fluctuations with periods smaller than or comparable to \( T_s \). A structure function approaching zero characterizes a phase process which is highly correlated yielding a value for the exponential in (16) near unity. For this case the autocorrelation function \( R'(T_s) \) from (12) would simply be \( R(T_s) \) and the phase process \( \phi(t) \) would have no effect on the received signal autocorrelation function. The effects of phase jitter on pulse pair estimates can be evaluated by considering the autocorrelation of the return signal as

\[ R'(T_s) = R(T_s) \exp{R_\phi(T_s) \int_{-\infty}^{\infty} S_\phi(f) df} \] (17)
The function given in (17) can be determined in closed form only when \( R_\phi(\tau) \) is known or can be determined in closed form from \( S_\phi(f) \).

It can be noted that this result generalizes an earlier published derivation [17] for the special case when \( S_\phi(f) \) is constant, characteristic of a white process. For the white process, the autocorrelation function \( R_\phi(T_s)=0 \) since \( R_\phi(\tau) \) an impulse having a non-zero value \( R_\phi(0)=\sigma^2 \) only, the process variance. Since the integral of the spectrum in (17) yields \( \sigma^2 \), then (17) becomes

\[
R'(T_s) = R(T_s) \exp \left[ -\sigma^2 \right] \tag{18}
\]

which is precisely the result obtained in [17].

Generally, for any real system, the phase noise spectrum would be specified by measured values. Even if an analytical expression could be fit to these data for any particular case, the likelihood that (17) would yield a closed form expression is small, unless, for example the spectrum has a Gaussian form. Thus (17) will generally involve some numerical procedure. Since the assumption of a Gaussian class phase noise spectrum will yield a closed form expression for (17), analysis results are included in Section IV based on this assumption. For the Gaussian case the phase noise spectrum is modelled as

\[
S_\phi(f) = A_0 \exp \left( -\frac{f^2}{2\Delta f_c^2} \right) \tag{19}
\]

where the relationship between the phase jitter spectrum width parameter \( \Delta f_c \) and the total phase jitter power \( \sigma^2 \) is represented by

\[
\sigma^2 = \int_{-\infty}^{\infty} S_\phi(f) df = \sqrt{\pi \Delta f_c A_0} \tag{20}
\]

With the Gaussian jitter spectrum, the autocorrelation of the received signal from (17) can be obtained analytically as [27]

\[
R'(T_s) = R(T_s) \exp \left[ -\sigma^2 \left( 1 - e^{-\omega^2 \Delta f_c^2 T_s^2} \right) \right] \tag{21}
\]
This modified autocorrelation at lag $T_s$ is explicitly a function of the spectrum width parameter $\Delta f_c$ and the total power $\sigma^2$ so that, if $\Delta f_c=0$ or if $\sigma^2=0$, this reduces to the expression of $R(T_s)$ without phase jitter noise included. As will be shown in the following section, analysis results based on the assumption of a Gaussian jitter spectrum are close to those obtained assuming an inverse power law type spectrum, i.e., $S_\phi(f)=Kf^{-n}$, which is considered more representative of oscillator phase noise [15]. In fact, as it turns out with the pulse pair algorithm analysis, the Gaussian spectrum assumption yields a result which may be interpreted as an upper bound spectrum parameter estimation error.
IV. Analysis of Pulse Pair Spectral Estimation Error Considering Phase Jitter Noise

A modern Doppler radar system with good phase stability is expected to have a narrow phase jitter spectrum, and from (17) the phase jitter should have little effect on the weather spectrum. Figure 1 shows a Gaussian weather return Doppler spectrum with and without phase jitter. The weather spectrum has zero mean and width of 15% of the processing bandwidth with the power normalized to unity. Assuming statistically independent phase jitter and a Gaussian spectrum, the total power level is 9% of the signal power. The phase jitter spectrum width is 28% of the processing bandwidth, which is less than that of an inverse power law spectrum with the same total power. There is little apparent difference between the two spectra in Figure 1.

![Figure 1. Example of Two Simulated Return Doppler Spectra Considering Phase Jitter Effect.](image)
Figure 2 shows the pulse pair mean estimate standard deviation for this same phase jitter and varying weather spectrum widths. The phase jitter noise causes more than a 15% increase at $\omega T_s=0.30$ and can cause up to 50% increase. Considering the rms error of the pulse pair width estimate in Figure 3, the phase jitter again causes about a 15% increase. These results have prompted further analysis of the phase jitter noise effect.

The pulse pair estimate errors discussed in Section II can be easily modified using the results of Section III. From (17) only the magnitude of the complex autocorrelation is affected by phase jitter, so the mean estimate in (3), involving only the phase of the autocorrelation, should not be biased. Conversely, the variance of the mean estimate is found by replacing $\beta(T_s)$ in (5) with $\beta'(T_s)$ which, using (17) for the arbitrary $S_\phi(f)$ is

$$\beta'(T_s) = \beta(T_s) \exp \left\{ R_\phi(T_s) \cdot \int_{-\infty}^{\infty} S_\phi(f) df \right\}$$

and using (21) for the Gaussian $S_\phi(f)$ is

$$\beta'(T_s) = \beta(T_s) \exp \left\{ -\sigma^2 \left[ 1 - e^{-\pi^2 \Delta f_c^2 T_s^2} \right] \right\}$$

Figure 4 shows the mean error standard deviation considering 1) no phase jitter error and a 0dB Doppler to thermal noise power ratio (SNR) ([9], Figure 2), 2) phase jitter described by a set of published oscillator phase noise spectrum data [28], and 3) a Gaussian phase jitter spectrum model with 75 Hz width ($\Delta f_c=75$) and the same total jitter power ($\sigma^2=0.16$). These plots use the expression in (5) and the modifications given in (22) and (23), respectively. The numerical example was computed using real data and an adaptive quadrature algorithm [29]. It appears that the Gaussian phase jitter spectrum model can provide a result close to that which might be obtained from an inverse power law model and, more importantly, a useful upper bound. Thus, the pulse pair estimators were analyzed further using the Gaussian spectrum model, considering a Doppler to thermal noise power ratio of 0dB.
\[ \sqrt{M SD(fT_s)} \]

Figure 2. Error Standard Deviation of the Pulse Pair Spectrum Mean Estimate for the Example in Figure 1.

\[ \sqrt{(\sqrt{M SD(wT_s)})^2 + M(T_B)^2} \]

Figure 3. R.M.S. Error of the Pulse Pair Spectrum Width Estimate for the Example in Figure 1.
Figure 5 shows the standard deviation of the mean estimate error versus phase jitter spectrum widths, with the total phase noise power held to 9% of the signal power (σ=0.3). The maximum phase width of 0.4 corresponds to that shown earlier in Figure 1. The curve for Δf_c=0 matches the result of Zrnic' for this case [9]. It can be noted that phase jitter has very little effect on this error. Figure 6 is a plot of the bias in the spectrum width estimate formed using (7) with R'(T_s) from (21) replacing R(T_s) and M=128. Note that phase jitter can bias the width estimates for very narrow weather spectra. Figure 7 shows the rms spectrum width estimate error for this same case. Here rms error is the square root of the sum of the variance and the square of the error bias. The variance is found from (9) with β'(Ts) from (23) replacing β(T_s). The phase jitter noise power is again 9% of the total signal power in the return. Note that phase jitter has very little effect on the width estimate error. In Figures 6 and 7 the curves for Δf_c=0 match those published by Zrnic' [9] for 0 dB SNR.

![Figure 4](image)

**Figure 4.** Comparison of Error Standard Deviation of the Pulse Pair Spectrum Mean Estimate.
This analysis verifies that phase jitter should have little effect upon the pulse pair spectrum estimates, except for a bias in the mean when the weather spectrum is narrow. In the next section the effect of phase jitter in the presence of strong ground clutter return is shown to be a more formidable problem.

Figure 5. Error Standard Deviation of the Pulse Pair Spectrum Mean Estimate Considering Phase Jitter.
Figure 6. Pulse Pair Spectrum Width Estimate Bias With Phase Jitter

\[
\sqrt{\left(\sqrt{\text{SD}(\hat{\text{WT}_s})} \right)^2 + \text{M}(\text{T}_s \text{B})^2}
\]

Figure 7. R.M.S. Error of the Pulse Pair Spectrum Width Estimate Considering Phase Jitter.
V. Phase Jitter Noise Limitations of Clutter Cancellation

With an aircraft on final approach and the radar antenna in a look-ahead orientation, the airborne Doppler weather radar ground clutter returns are typically dominated by a very strong spectral power around the Doppler frequency corresponding to the aircraft ground speed, treated here as the zero frequency reference. At least a 10 dB signal-to-clutter ratio (SCR) is needed for accurate mean velocity estimation with pulse pair, while width estimation may require 15 dB SCR [30]. Typically, a high pass filter with a notch width selected to attenuate the clutter will be chosen. When the clutter spectrum is narrow and separated from the weather spectrum, clutter filtering can be done very successfully without degrading the weather spectrum [1], [20], [21].

Assuming the clutter and weather return spectra to be Gaussian, the return spectrum can be modelled as the sum of two normal functions

\[ S(f) = \frac{C}{\sqrt{2\pi W_c}} \exp \left\{ -\frac{f^2}{2W_c^2} \right\} + \frac{1}{\sqrt{2\pi W_s}} \exp \left\{ -\frac{(f-f_d)^2}{2W_s^2} \right\} \]  

(24)

where \( C = 10^{(\text{SCR}/10)} \), \( f_d \) is the mean frequency of a weather return spectrum, the clutter is assumed to be zero mean, and \( W_c \) and \( W_s \) are the spectrum width of clutter and weather return, respectively.

Considering phase jitter noise with a Gaussian spectrum (24) can be rewritten as [26]

\[ S(f) = Ce^{-2\sigma^2} \sum_{k=0}^{\infty} \frac{2^k \sigma^{2k}}{\sqrt{2\pi(W_c^2+k\Delta f_c^2)^{1/2}}} \exp \left\{ -\frac{f^2}{2(W_c^2+k\Delta f_c^2)} \right\} \]

\[ +e^{-2\sigma^2} \sum_{k=0}^{\infty} \frac{2^k \sigma^{2k}}{\sqrt{2\pi(W_s^2+k\Delta f_c^2)^{1/2}}} \exp \left\{ -\frac{(f-f_d)^2}{2(W_s^2+k\Delta f_c^2)} \right\} \]

(25)

where only the first four terms are needed for small \( \sigma \).
Consider the use of an ideal high pass filter having a stopband width of $2B$ and a stop attenuation of $ATT$ (in dB). The ideal filter has a constant stopband gain and a passband gain of 0 dB, with no transition band. Then the SCR of the filtered return spectrum is represented by

$$SCR = 10 \log_{10} \left( \frac{SR}{CR} \right)$$

where

$$CR = Ce^{-2\sigma^{2}} \left[ Q \left( \frac{B}{W_{c}} \right) + 2\sigma^{2}Q \left( \frac{B}{W_{1}} \right) + 2\sigma^{4}Q \left( \frac{B}{W_{2}} \right) + \frac{4}{3} \sigma^{6}Q \left( \frac{B}{W_{3}} \right) \right]$$

$$+ 10^{-ATT/10}Ce^{-2\sigma^{2}} \left[ \left\{ 1-Q \left( \frac{B}{W_{c}} \right) \right\} + 2\sigma^{2} \left\{ 1-Q \left( \frac{B}{W_{1}} \right) \right\} + 2\sigma^{4} \left\{ 1-Q \left( \frac{B}{W_{2}} \right) \right\} + \frac{4}{3} \sigma^{6} \left\{ 1-Q \left( \frac{B}{W_{3}} \right) \right\} \right]$$

$$SR = e^{-2\sigma^{2}} \left[ \left\{ 1-P \left( \frac{B-f_{d}}{W_{s}} \right) \right\} + 2\sigma^{2} \left\{ 1-P \left( \frac{B-f_{d}}{Y_{1}} \right) \right\} + 2\sigma^{4} \left\{ 1-P \left( \frac{B-f_{d}}{Y_{2}} \right) \right\} \right]$$

$$+ \frac{4}{3} \sigma^{6} \left\{ 1-P \left( \frac{B-f_{d}}{Y_{3}} \right) \right\} + 10^{-ATT/10}e^{-2\sigma^{2}} \left[ P \left( \frac{B-f_{d}}{W_{s}} \right) + 2\sigma^{2}P \left( \frac{B-f_{d}}{Y_{1}} \right) + 2\sigma^{4}P \left( \frac{B-f_{d}}{Y_{2}} \right) + \frac{4}{3} \sigma^{6}P \left( \frac{B-f_{d}}{Y_{3}} \right) \right]$$

with

$$w_{k} = \left( \frac{w_{c}^{2}+k^{2}}{2} \right)^{1/2}; \quad Q(x) = 2 \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-y^{2}/2} dy$$

and

$$y_{k} = \left( \frac{y_{c}^{2}+k^{2}}{2} \right)^{1/2}; \quad P \left( \frac{B-f_{d}}{z} \right) = \int_{z}^{\infty} \frac{1}{\sqrt{2\pi}z} e^{-y^{2}/2} dy.$$
A continuous frequency analysis, to avoid aliasing the spectra, is used to show the effect of phase jitter noise on the post-filtered SCR. With a weather signal power of 1.0, the pre-filtered clutter power is set to give an SCR of either 0 dB or -30 dB, to represent a wet microburst or a dry microburst situation, respectively. The clutter spectrum width is 30 Hz and the weather spectrum width is 60 Hz with the clutter rejection filter stopband width B=150 Hz. The weather spectrum mean value is varied from 100 to 250 Hz. Figure 8 shows how the post-filtered SCR varies as the phase noise power is varied from 0 to 0.25 for different weather location parameters, a notch attenuation of 50 dB, and a pre-filtered SCR of 0 dB. There is adequate SCR even with large phase jitter power. Figure 9 shows a similar result with a -30 dB pre-filtered SCR and a 70 dB notch attenuation. With no phase jitter, there should be adequate post-filtered SCR for reliable pulse pair spectrum estimates, but if the phase jitter power exceeds 0.05, a post-filtered SCR greater than 10 dB can not be achieved.

Even though the presence of phase jitter may have little effect upon the signal parameter estimation quality using pulse pair, as discussed in Section IV, the results here indicate that even small amounts of phase noise can be a serious problem in the presence of strong ground clutter returns.
stopband width (\(B\)) = 150 Hz, ATT = 50 dB

Figure 8. Phase Noise Effects on Clutter Filtering of 50dB Stopband Attenuation for a Doppler Weather Return With 0dB SCR.

stopband width (\(B\)) = 150 Hz, ATT = 70 dB

Figure 9. Phase Noise Effects on Clutter Filtering of 70dB Stopband Attenuation for a Doppler Weather Return With -30dB SCR.
VI. Estimating Parameters of a Skewed Weather Spectrum

A. Pulse Pair Parameter Estimates of a Skewed Spectrum

The pulse pair algorithm uses estimates of the complex autocorrelation function at zero lag and at lag $T_s$, the pulse separation time. Estimate quality is in the context of symmetric spectra. One earlier study [31] considered narrow spectra and showed that spectrum skew should not be a handicap for covariance estimators. However, particularly with turbulent weather returns, the Doppler spectrum may be broad and not symmetric. This section evaluates the resulting bias in terms of the spectrum skewness considering widths up to about 25% of the Doppler bandwidth. A standard model for a skewed statistical distribution is used.

Consider a piecewise Gaussian skewed spectrum modelled as

\[
S_n(f) = \begin{cases} 
\frac{2}{1+p} \frac{1}{\sqrt{2\pi w_1}} e^{-\frac{f^2}{2w_1^2}} & ; \ f < 0 \\
\frac{2p}{1+p} \frac{1}{\sqrt{2\pi w_2}} e^{-\frac{f^2}{2w_2^2}} & ; \ f > 0 
\end{cases}
\]

where the standard deviation ratio $p=w_1/w_2$ defines the degree of skewness $g$, i.e. [32],

\[
g = \frac{4\sqrt{2}}{\sqrt{\pi}} \left( \frac{3}{2} \cdot \frac{3}{(p^2+1)^{\frac{3}{2}}} \cdot (p^2+1)^{\frac{3}{2}} \right)
\]

This skewness parameter varies proportionally to skew from $g=0$ for no skew ($p=1$) to larger values, e.g., $g=3.14$ for a case which may be considered large skew ($p=10$) as may be seen in Figure 10. For a narrow symmetric Gaussian spectrum ($w_1=w_2=w$), the integral in (1) can be reduced to one simple term, $\exp(-2\pi^2 w^2 T_s^2)$, as stated in (14). For the skewed spectrum model in (31) the integral in (1)
Figure 10. Relationship Between the Parameter $p$ and the Degree of Skewness $g$.

will include both a real term

$$a = \int_{1}^{2T_s} S_n(f) \cos(2\pi f T_s) \, df = \frac{2}{1 + p} \left( \frac{1}{2} e^{-2\pi^2 w_1^2 T_s^2} + \frac{w_2}{2w_1} e^{-2\pi^2 w_2^2 T_s^2} \right)$$

and an imaginary term

$$b = \frac{2}{1 + p} \int_{0}^{\frac{1}{2T_s}} \left[ \frac{1}{2\pi w_1} \cdot e^{-\frac{f^2}{2w_1^2}} \cdot e^{-\frac{f^2}{2w_2^2}} \right] \sin(2\pi f T_s) \, df$$

Using (33) and (34), the bias in the pulse pair mean and width estimates can be represented by

$$\text{mean bias} = \left| \frac{1}{2\pi T_s} \tan^{-1} \left( \frac{b}{a} \right) - f_m \right|$$

(35)
width bias = \left| \frac{1}{\sqrt{2\pi T_s}} \left| \ln\left( \frac{1}{\sqrt{a^2 + b^2}} \right) \right|^{1/2} - W \right| \quad (36)

where the true mean \( f_m \) and the true width W in (31) are

\[
f_m = \int_{-T_s/2}^{T_s/2} f S_n(f) \, df = \frac{2}{1+p} \frac{1}{\sqrt{2\pi}} \left( p w_2 - w_1 \right)
\]

\[
W^2 = \int_{-T_s/2}^{T_s/2} (f-f_m)^2 S_n(f) \, df = \frac{1}{1+p} \left( w_1^2 + p w_2^2 - f_m^2 \right)
\]  

Estimate biases as given in (35) and (36) are plotted as functions of true width W and the skewness parameter g in Figures 11 and 12. In Figure 11, if there is no skew (g=0), the pulse pair estimator is unbiased. As skew is increased there is a sharp increase in the bias. With the skewness parameter greater than zero, the percentage bias error is essentially independent of the specific value of skew, but is strongly related to the spectrum width W. As can be seen, the bias error due to skewness is not negligible if the spectrum is broad. Figure 12 shows that a broad spectrum with a large degree of skewness can degrade the quality of the pulse pair width estimate, but it does not seem to be as serious as the mean estimate bias shown in Figure 11. Figure 13 is included to compare the mean estimate rms error for the case of a skewed spectrum (p=2) with that of a symmetric Gaussian spectrum having an equivalent overall normalized width WT_s. Equations (35) and (36) was used to determine the rms error for the unbiased Gaussian spectrum. The error caused by the skewness may seriously degrade the pulse pair estimation quality if the return Doppler spectrum width is 40% or more of the processing bandwidth. It is difficult to see by comparing Figures 11 and 13, but the effect of skewness on the variance of the mean error is not significant.
Figure 11. Pulse Pair Mean Estimate Bias Error Versus Skewness.

Figure 12. Pulse Pair Width Estimate Bias Error Versus Skewness.
\[ \sqrt{\left( \text{MSD}(WT_s) \right)^2 + M(T_s B)^2} \]

Figure 13. Mean Estimate R.M.S. Error With Skewed Gaussian Spectrum for Degree of Skewness \( g = 1.99 \).

B. Mean Estimate Bias Reduction with Poly-Pulse Pair

As noted in Section II, the pulse pair mean estimator is based upon a linear approximation to the derivative of the phase function of the autocorrelation estimate, i.e., differentiating (2), the mean estimate in (3) is

\[ \hat{\theta} = \frac{1}{2\pi} \frac{d\theta(\tau)}{d\tau} \bigg|_{\tau=0} = \frac{1}{2\pi} \frac{\hat{\theta}(T_s)}{T_s} \]  

(39)

where \( \Theta(T_s) \) is the phase function (argument of \( R(T_s) \)). There is no approximation error in (39) for a symmetric spectrum, but a large error can occur in a skewed spectrum since \( \Theta(T_s) \) is no longer a linear function of \( T_s \). An alternative is to approximate \( \Theta(\tau) \) as a low order polynomial (order \( n \geq 1 \)), i.e.,

\[ \Theta(\tau) = \sum_{i=1, \text{odd}}^{n} a_i \tau^i \]  

(40)
where $\Theta(\tau)$ must be an odd function of $\tau$ [25] since the spectrum is always real valued.

Now (39) may be rewritten as

$$
\hat{f}' = \frac{1}{2\pi} \frac{d\Theta(\tau)}{d\tau} \bigg|_{\tau=0} = \frac{1}{2\pi} \hat{\alpha}_1
$$

(41)

where $\alpha_1$ can be computed from estimates of the complex autocorrelation function for lags $\tau = T_s, 2T_s, 3T_s,$ etc. using the poly-pulse pair algorithm [24],[33]. For example, using (40), with a third order model $(n=3)$

$$
\Theta(\tau) = a_3\tau^3 + a_1\tau
$$

(42)

The argument of $R(T_s)$, $\Theta(\tau)|_{\tau=T_s}$, and the argument of $R(2T_s)$, $\Theta(\tau)|_{\tau=2T_s}$ can be estimated by pairing pulses spaced at $T_s$ and also pairing pulses at spacing $2T_s$. From (42), solving a pair of simultaneous equations for $a_1$, will yield a mean frequency estimate according to (41), termed the third order poly-pulse pair mean frequency estimate. Figure 14 shows the mean estimate bias, considering the same situation as depicted in Figure 13, for the pulse pair estimator, along with the poly-pulse pair mean bias for both a third order $(n=3)$ and a fifth order $(n=5)$ polynomial in (40). The poly-pulse pair method was used to estimate the argument of the complex autocorrelation at 2 lags and at 3 lags respectively in determining the mean frequency estimate from (41).

The third order poly-pulse pair mean estimate variance can be easily derived using (41) and the set of equations from (42) to yield

$$
\text{var}(\hat{f}') = \text{var}(\hat{f}) - \frac{T_s^2}{2\pi} E\{\hat{a}_2a_1\hat{a}_1\} - \frac{T_s^4}{4\pi} E\{(a_3-a_3)^2\}
$$

(43)

where $\text{var}(f)$ is the variance of the conventional pulse pair method. The first term may be positive or negative and the second term will actually serve to reduce the pulse pair estimate variance. In any case, since the pulse interval $T_s$ is generally very small, the higher order terms may be ignored to yield
\[ \text{var}(\hat{f}') = \text{var}(\hat{f}) \]  

These results suggest that the poly-pulse pair method can improve the quality of mean estimates in the presence of skewed spectra.

\% mean bias error by skewed spectrum (g=1.99)

![Graph of mean bias error by skewed spectrum](image)

Figure 14. Performance Comparison Between Poly-Pulse Pair and Conventional Method.

C. Mode Versus Mean Estimation

With the possibility of Doppler spectrum skew, there is a question as to whether the estimated "average" windspeed Doppler within each range cell should be the mean value or the mode (most probable value). The mode may better characterize windshear. With skewness, the difference between the mean and the mode can be quite large, as seen in Figure 15 showing the normalized difference DT_s between the mode and the true mean of a skewed spectrum as width and skew are varied. Figure 16 shows the
difference between the mode and the pulse pair mean with variation in skew and spectrum width. For larger widths, due to increased sensitivity to spectrum skew, the pulse pair mean is not a good mode estimator.

This suggests that mode estimation techniques should be considered for pulse Doppler radars operating in turbulent weather environments. This has led to an investigation of spectral decomposition techniques for mode location [26],[34]. It may be possible to operate without clutter rejection pre-filtering in locating a weather spectrum mode, thus avoiding deleterious effects of radar system phase instabilities compounded by clutter rejection filtering, as was discussed in Section V.

Figure 15. Normalized Difference Value Between True Mean and Mode of Skewed Spectrum.
Figure 16. Normalized Difference Value Between Pulse Pair Mean Estimate and Mode of Skewed Spectrum.
VII. Summary

The widely used pulse pair spectrum parameter estimator is being considered for a high resolution airborne weather radar system for low altitude windshear detection. The robustness of pulse pair estimator performance in the presence of turbulent weather, high ground clutter environments, and coherent radar system phase instabilities is considered. Originally proposed for independent pulse pairs with Gaussian weather return spectra, it has been widely used for contiguous pairs and is being considered for weather echoes which have non-Gaussian spectra.

Previous analysis of radar system pulse-to-pulse phase jitter effects has been generalized and used to evaluate its effect on pulse pair estimate quality. As shown here, the effect is largely insignificant, except in the presence of very low signal to clutter ratios when clutter rejection filtering is applied. These results can help determine appropriate radar system phase stability design specifications and also suggest that spectrum parameter estimation without clutter rejection pre-filtering may be necessary in the presence of strong clutter environments.

With turbulent weather returns, and the potential for non-Gaussian spectra, it has been shown that lack of spectrum symmetry can contribute to a significant bias in the pulse pair mean estimates. The previously defined poly-pulse pair method is demonstrated as a potential solution. With skewed weather spectra and with potentially large clutter returns, the whole issue of best characterizing the "average" windspeed within a range resolution cell is raised. The pulse pair algorithm is simply a maximum entropy estimator assuming a first order autoregressive model of the return spectrum. A higher order mode estimator may be more useful if spectral modes can be identified and classified.
References


The pulse-pair method for spectrum parameter estimation is commonly used in pulse Doppler weather radar signal processing since it is economical to implement and can be shown to be a maximum likelihood estimator. With the use of airborne weather radar for wind shear detection, the turbulent weather and strong ground clutter return spectrum differs from that assumed in its derivation, so the performance robustness of the pulse-pair technique must be understood. This paper analyzes the effect of radar system pulse-to-pulse phase jitter and signal spectrum skew on the pulse-pair algorithm performance. Phase jitter effects may be significant when the weather return signal-to-clutter ratio is very low and clutter rejection filtering is attempted. The analysis can be used to develop design specifications for airborne radar system phase stability. The paper also shows that weather return spectrum skew can cause a significant bias in the pulse-pair mean wind speed estimates, and that the poly-pulse-pair algorithm can reduce this bias. It is suggested that use of a spectrum mode estimator may be more appropriate in characterizing the wind speed within a radar range resolution cell for detection of hazardous wind speed gradients.