SOLVING THE BM CAMELopardalis PUZZLE

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ABSTRACT

BM Camelopardalis (=12 Cam) is a chromospherically active binary star with a relatively large orbital eccentricity. Systems with large eccentricities usually rotate pseudosynchronously. However, BM Cam has been a puzzle since its observed rotation rate is virtually equal to its orbital period indicating synchronization. All available photometry data for BM Cam have been collected and analyzed. Two models of a modulated ellipticity effect are proposed, one based on equilibrium tidal deformation of the primary star and the other on a dynamical tidal effect. When the starspot variability is removed from the data, the dynamical tidal model was the better approximation to the real physical situation. The analysis indicates that BM Cam is not rotating pseudosynchronously but is rotating in virtual synchronism after all.
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BM Camelopardalis (= 12 Cam) is an SB1 in which a K1 giant orbits its unseen companion star in an eccentric 80-day orbit. The K1 giant shows strong emission in its H and K lines of Ca II, so it is chromospherically active (Abt, Duces, and Weaver 1969). The photometric variability, produced by longitudinally concentrated starspot regions, was discovered by Eaton et al. (1980). Hall and Osborn (1986) analyzed the photometric variability and found two periodicities, 0.3% faster than and 1.0% slower than the 80.174469 orbital period, presumably caused by two starspot regions at different latitudes. Hall (1986) calculated that, if the K1 giant is rotating pseudo-synchronously, the photometric period should be 45.2 ± 2.5. He noted that it was a disturbing coincidence to have the observed rotation rate virtually equal to the orbital period. If the rotation is not pseudosynchronous, then any period faster than or slower than 45 days would be possible, with no reason for 80 days to be preferred. This is the BM Cam puzzle.

To solve this puzzle we collected all available photometry. The 1979 and 1980 photometry discussed by Eaton et al. (1980) was not published but was available in our files. Fernandes (1983) published some 1983 photometry. We had in our files photometry from the years 1980 through 1985, obtained by 15 different observers and sent to us for analysis. Photometry obtained between 1983 and 1987 by the 10-inch automatic telescope in Arizona will be published by Boyd, Genet, Busby, Hall, and Strassmeier (1989). The preliminary analyses by Nelson et al. (1987) and by Strassmeier, Hall, Boyd and Genet (1989) were based on subsets of these data. We also had access to 1988 photometry obtained by the Vanderbilt 16-inch automatic telescope on Mt. Hopkins (Hall 1988). The analysis in this paper is restricted to the V-band data, which was the most extensive. All telescopes except that of Fernandes used the same comparison star, HR 1688. To compensate, we added −0.995 to his differential magnitudes.

One source of photometric variability which should be exactly in phase with the orbital period is the ellipticity effect (Morris 1985). That, however, should produce two maxima and two minima during each orbital cycle and thus show up in a periodogram at $P = 40$ days. A periodogram of the entire data set showed the most power around 80 days, very little around 40 days.

In the discussion which follows $m =$ magnitude, $e =$ orbital eccentricity, $w =$ angle of periastron measured from the ascending node of the spectroscopic primary in the direction of its orbital motion, $M =$ mean anomaly, $v =$ true anomaly, $a =$ orbital semi-major axis, $d =$ distance between star centers, and $R =$ stellar radius.

Then it occurred to us that the ellipticity effect should be modulated by the varying star-to-star separation in this eccentric orbit. The giant star should experience greater tidal deformation at periastron and less at apastron. This should be a strong effect because it depends on the cube of the ratio $R/d$ (Russell and Merrill 1952). If $w$ is near $90^\circ$ or $270^\circ$, then the light curve should show a maxima of equal height but minima of unequal depth. Abt, Dukes, and Weaver had found $w = 72.5^\circ$, not far from $90^\circ$, so we formed the tentative hypothesis that this modulated ellipticity effect could explain at least one of the strong 80-day periodicities.

To quantify this hypothesis, we considered two versions. The first assumes that the long axis of the tidally distorted star always points towards the other star and that the effect on the light curve, in magnitude units, is proportional to $(R/d)^3$. This would correspond to the theory of equilibrium tides. The second assumes that the long axis of the tidally distorted star rotates uniformly in time, i.e., directly
proportional to the mean anomaly, and that the effect on the light curve varies uniformly in time between the two extremes, i.e., between the maximum effect at \( d = a(1-e) \) and the minimum effect at \( d = a(1+e) \). This would correspond to the theory of dynamical tides. Both versions have the same \( \cos 2\phi \) dependence, where \( \phi \) is the angle between the line of sight and the long axis of the tidally elongated star. Thus, in both versions, the ellipticity effect vanishes at \( \phi = 45^\circ, 135^\circ, 215^\circ, \) and \( 225^\circ \).

For the first version, the change in magnitude produced by the ellipticity effect is given by

\[
\Delta m = k \left( \frac{d}{a} \right)^{-3} \cos 2\phi,
\]

(1)

where \( \phi = \nu + w - 90^\circ \) and the usual equations of the two-body problem can be used to compute \( d \) and \( v \) as functions of time.

For the second version, the change in magnitude produced by ellipticity is given by

\[
\Delta m = k (\alpha + \beta \cos M) \cos 2\phi,
\]

(2)

where now \( \phi = M + w - 90^\circ \). In this equation

\[
\alpha = v_2 (f_p + f_a)
\]

(3)

\[
\beta = v_2 (f_p - f_a)
\]

(4)

and

\[
f_p = (1 - e)^{-3}
\]

(5)

\[
f_a = (1 + e)^{-3}
\]

(6)

Note that, in this second version, there is no need to compute \( v \) or \( d \) as functions of time.

In both versions the coefficient \( k \) would correspond approximately to the coefficient \( A_2 \) as defined by Russell and Merrill (1952). In the case of a circular orbit, where \( d = a \) in equation (1) or \( e = 0 \) in equation (2), one would get \( k = A_2 \).

To try this hypothesis on BM Cam we proceeded by iteration. The observed magnitudes were plotted modulo; the known orbital period and means were taken in bins 0.02 phase units wide. There was considerable dispersion within each bin, of course, because of the other variability present in the system with supposedly different periodicities. We were encouraged that the resulting mean light curve had roughly the expected shape: two nearly equal maxima and two quite unequal minima. Then this mean light curve was subtracted from the observed magnitudes and the residuals examined.

The residuals showed something we recognized as variability produced by starspots. Within each observing season there was a roughly sinusoidal variation with a period similar to but significantly different from the orbital period. That period was about 82.5 up through 1984.5 and about 81.0 after that, with a half-cycle phase shift also around 1984.5. The amplitude of this roughly sinusoidal variation changed dramatically: up to a maximum of 0.15 at 1981.3, down to a minimum of 0.03 at 1984.0, up to another maximum of 0.15 at 1985.6, and possibly decreasing thereafter. In addition, the average light level changed significantly from year to year, covering a range of 0.06.
The next step of the iteration was to find analytical expressions to approximate this starspot variability. An assumed sinusoid required as its parameters the mean light level, the epoch of light minimum, the period, and the amplitude. Additionally, we allowed the mean light level to increase or decrease linearly with time.

The starspot variability was then removed from the original observed magnitudes with these analytical expressions. These residuals, which we presume contain only the ellipticity effect, were then fit in turn with equations (1) and with equation (2). Both fits yielded $P$, $k$, $e$, and $w$. The period $P$ enters as a parameter when we convert Julian date into mean anomaly.

Results with equation (2) were superior, in the sense that the sum of the squares of the residuals reached a much smaller minimum. We take this as indication that the theory in the second version is a better approximation to the real physical situation. The parameters are presented in Table I with their formal standard errors and compared to the corresponding elements of the Abt, Dukes, and Weaver (1969) solution to the spectroscopic orbit.

Table I

<table>
<thead>
<tr>
<th>Derived Elements</th>
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<tr>
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<tr>
<td>$P(\text{orb.})$</td>
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<tr>
<td>$e$</td>
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<td>$w$</td>
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The difference between the two determinations of the orbital period amounts to 5 standard errors and thus poses an apparent conflict. The spectroscopic determination was supposedly uncertain by only $\pm 0^d.00003$, but we can show that it could be significantly in error. The solution of Abt, Dukes, and Weaver (1969) was based on 13 radial velocities obtained in 1966, 1967, and 1968 plus three radial velocities obtained long before, in 1916 and 1917. That long 50-year baseline gave them their high
precision but depended critically on proper phasing. The three old velocities fell coincidentally at very nearly the same phase and had about the same velocity, namely \( V_r = -8 \) km/sec. If one places them on the falling slope of the radial velocity curve rather than the rising slope, one could have gotten orbital period of \( 80^d \ 04 \) or \( 80^d \ 79 \). The first of these would differ from our photometric determination by only 2 standard errors.

The difference between the two determinations of \( w \) amounts to 3 standard errors and thus poses another apparent conflict. There is, however, an easy explanation for this discrepancy, namely apsidal motion. The \( 22^\circ.5 \) increase in \( w \) between 1967 and 1984 would correspond to an apsidal motion period of 272 years. Theory shows that the rate of apsidal motion is proportional to the fifth power of the ratio \( R/a \). Since that ratio must be large in any binary with an observable ellipticity effect, it is expected that apsidal motion in BM Cam would be relatively rapid, i.e., measurable within a few decades.

The period of the photometric variation produced by the starspots is a measure of the rotation period of the K1 giant. The fact that it differs only a few percent from the orbital period indicates that BM Cam is not rotating pseudosynchronously but is rotating in virtual synchronism after all. From this we might conclude that the theory of pseudosynchronism does not apply when dynamical tides phase-locked with the orbital period are more important than the equilibrium tides.

This investigation is not quite finished. We need to improve the analytical representation of the starspot variability by allowing the amplitude of the assumed sinusoidal variation to increase or decrease linearly with time within each season. We need to repeat the iterative process a few more iterations, and we need to compute the theoretically expected apsidal motion period, which will require a determination or estimate of the orbital inclination and the mass ratio, neither of which is known directly for a single-lined spectroscopic binary nor for a non-eclipsing ellipsoidal variable.

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References


