Transmission Overhaul Estimates for Partial and Full Replacement at Repair

M. Savage
University of Akron
Akron, Ohio

and

D.G. Lewicki
Propulsion Directorate
U.S. Army Aviation Systems Command
Lewis Research Center
Cleveland, Ohio

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Timely overhauls produce in-flight service reliability greater than the calculated design reliabilities of the transmission components. Although necessary for aircraft safety, transmission overhauls contribute to aircraft expense. Predictions of the transmission's maintenance needs at the design stage should enable the development of more cost-effective and reliable transmissions in the future.

This work estimates the frequency of overhaul and the number of transmissions or components needed to support the overhaul schedule. Two methods based on the two-parameter Weibull statistical distribution for component life are used to estimate the time between overhauls. These methods predict transmission lives for maintenance schedules which (1) repair the transmission with a complete system replacement or (2) repair only failed components of the transmission. An example illustrates the methods.

INTRODUCTION

Aircraft transmissions include bearings and gears which have finite fatigue lives with detectable failure warnings. The two-parameter Weibull distribution statistically describes the drive system bearing and gear life [1-4]. The in-flight service reliability of aircraft transmissions is much higher than the design reliability of their components. Transmission overhauls provide the difference. By monitoring the onset of potential fatigue failures, one can use just-in-time overhauls to maintain the transmission economically and reliably [5].

A two-parameter Weibull distribution provides the transmission system life model for repairing a transmission with full-system replacement [6,7]. The sum of the component failure rates predicts the repair frequency for maintenance with partial-replacement repair [8,9].

Renewal theory is a secondary statistical model that describes the maintenance process. It estimates the number of replacements needed to maintain transmission reliability with a specified maintenance schedule. The theory considers the ongoing sequence of use, failure onset, repair, and return to use. For this sequence, renewal theory predicts the frequency of component replacement and the number of replacements needed to support the service maintenance schedule [10-12].

Confidence theory complements these statistics with estimates of the likelihood of the predictions. Higher confidence levels require more spare parts to cover a greater range of possible situations [11,12].

This work presents these theories and applies them to a simple transmission (Fig. 1) to show their use. Estimates of drive system component failure onset rate and replacement needs are essential in design. They allow one to compare the worth of different designs from a maintenance cost perspective, and they help assess the cost of operating a proposed drive system design.

COMPONENT RELIABILITY

The two-parameter Weibull distribution is a statistical function commonly used to describe fatigue life data. It can describe a variety of life patterns in which the reliability of a component is the complement of its probability of failure.
In statistics, reliability is a double negative. Reliability, or the act of surviving, is the state of not having failed. Statistics count single events such as the act of failing. A part can fail only once, whereas it survives for its entire life. The probability of failure for the two-parameter Weibull distribution, which is a direct statistic, is

\[ F = 1.0 - e^{-\left(\frac{t}{\theta}\right)^b} = 1.0 - R \]  

(1)

The derivative of Eq. (1) with respect to life is the probability density function \( f \):

\[ f = b \left(\frac{t}{\theta}\right)^{b-1} e^{-\left(\frac{t}{\theta}\right)^b} \]  

(2)

The probability density function is a histogram of life failures for a unit population. The Weibull reliability function can be expressed as a log reciprocal:

\[ \ln \left(\frac{1}{R} \right) = \left(\frac{t}{\theta}\right)^b \]  

(3)

In working with the high-reliability range, the \( t_{10} \) life often replaces the characteristic life \( \theta \) as the scaling parameter. In terms of \( t_{10} \) life, Eq. (3) is

\[ \ln \left(\frac{1}{R} \right) = \ln \left(\frac{1}{0.9}\right) \left(\frac{t}{t_{10}}\right)^b \]  

(4)

Even though it is cumbersome, manufacturers use Eq. (4) as the two-parameter Weibull distribution of bearings to place 90-percent reliability lives in the catalogs [13].

In both Eqs. (3) and (4), the log of the reliability reciprocal is proportional to the life raised to the Weibull slope. Taking the log of either equation generates a straight-line plot as shown in Fig. 2. The plot is a probability graph for the two-parameter Weibull distribution.

The average life is the mean time to failure (MTTF), which is the sum of all times to failure divided by the total number of failures. For a continuous distribution, the total number of failures is unity, and the sum of all lives to failure is the integral of time or life times the probability density function. Integrating from zero to infinity gives the mean life:

\[ t_{av} = MTTF = \int_0^\infty f(t) \, dt \]  

(5)

Substituting the probability density function of Eq. (2) for the two-parameter Weibull distribution and integrating yields the well-known gamma function \( \Gamma \) multiplied by the characteristic life \( \theta \):

\[ t_{av} = MTTF = \theta \Gamma \left(1 + \frac{1}{b}\right) \]  

(6)

The solid curve in Fig. 3 is a plot of the ratio of the two-parameter Weibull mean life to the characteristic life versus Weibull slope. The mean life equals the characteristic life at \( b = 1.0 \), drops below the characteristic life to a minimum relative value at \( b = 2.15 \), and then increases back to the characteristic life as \( b \) approaches infinity. When \( b \) is infinite, the distribution is an impulse with all lives equal to the characteristic life.

By a similar integration, one can find the standard deviation of the two-parameter Weibull distribution. The standard deviation is the square root of the second moment of the component life distribution about the mean.

\[ \sigma_f = \sqrt{\int_0^\infty (t - t_{av})^2 f(t) \, dt} \]  

(7)

![Fig. 2. Two-parameter Weibull probability plot.](image)

![Fig. 3. Average life and standard deviation of life ratios to characteristic life for a Weibull distribution as a function of the Weibull slope.](image)
In terms of the gamma function, the standard deviation of the two-parameter Weibull distribution is

\[ \sigma_i = \theta \sqrt{ \Gamma\left(1 + \frac{2}{b_i}\right) - \Gamma^2\left(1 + \frac{1}{b_i}\right)} \]  

(8)

The standard deviation of a distribution is a measure of the scatter of the distribution. It is valuable in estimating a confidence limit for the average life.

The broken curve in Fig. 3 is a plot of the ratio of the standard deviation of the two-parameter Weibull distribution to its characteristic life versus the Weibull slope. At a slope of one, the distribution is the exponential distribution, which has a large scatter. As the slope increases to two, the scatter decreases rapidly and continues to decrease with increasing slope.

SYSTEM LIFE WITH FULL REPLACEMENT

To model the transmission life based on full replacement, one must have a model for the system as a complete system which treats the system as a single component. The life of a drive system can be considered to be a strict series probability model of the lives of its components [7]. In this model, the reliability of the system \( R_s \) is the product of the reliabilities of all the components:

\[ R_s = \prod_{i=1}^{n} R_i \]  

(9)

The high speed of drive system components and the spray of loose debris warrant the strict series probability model. If any component fails, debris may be present which could accelerate the fatigue damage in other components. Therefore, the drive system will need an overhaul to return it to a high state of reliability when any element fails.

The log of the reciprocal of Eq. (9) is:

\[ \ln \left( \frac{1}{R_s} \right) = \sum_{i=1}^{n} \ln \left( \frac{1}{R_i} \right) \]  

(10)

and substitution of Eq. (4) into Eq. (10) for each component yields

\[ \ln \left( \frac{1}{R_s} \right) = \ln \left( \frac{1}{R_{10}} \right) + \sum_{i=1}^{n} \frac{t_{10,i} - t}{t_{10,i}} b_i \]  

(11)

In Eq. (11), \( t_{10,i} \) is the life of the entire drive system for the system reliability \( R_{10} \). It is also the life of each component at the same drive system reliability \( R_{10} \). For consistency in Eq. (11), all the component lives must have the same counting base of hours.

Equation (11) is a two-parameter Weibull distribution only when all the Weibull exponents \( b_i \) are equal. However, a two-parameter Weibull distribution can approximate Eq. (11) quite well.

Equation (12) is the drive system two-parameter Weibull relationship. It includes the system reliability parameters \( b_s \) and \( t_{10,s} \):

\[ \ln \left( \frac{1}{R_s} \right) = \ln \left( \frac{1}{R_{10}} \right) + \frac{t_{10,s} - t}{t_{10,s}^2} b_s \]  

(12)

The straight-line reliability relationship of Eq. (12) can be fit numerically to the more exact relationship of Eq. (11) with a linear regression. The slope of the fitted straight line is the drive system Weibull slope \( b_s \), and \( t_{10,s} \) is the life at which the drive system reliability \( R_s \) equals 90 percent on the straight line.

SYSTEM LIFE WITH PARTIAL REPLACEMENT

To model the transmission life based on partial replacement, one can treat the full system as a collection of independent components. Separate analysis of each component will predict the number of replacements needed. If no two components are repaired at the same overhaul, the maximum number of overhauls is equal to the sum of all individual component replacements. One can estimate the mean time between overhauls as the total service time divided by the number of replacements for this component sum repair calculation and for the full-system repair.

RENEWAL THEORY

Renewal theory estimates the number of replacements as a function of the component failure distribution and its life [10-12]. It assumes that failed components will be replaced just before they fail, which models an unending sequence of use and repair. Aircraft drive system maintenance follows this pattern closely.

The mean number of failures is the infinite sum of the probabilities of at least \( i \) failures in the life period \( t \). This function, \( M(t) \), is the renewal function. It is expressed as

\[ M(t) = f(t) + \int_0^t M(t-x)f(x) \, dx \]  

(13)

The derivative of the renewal function with respect to life is the renewal density function:

\[ m(t) = f(t) + \int_0^t m(t-x)f(x) \, dx \]  

(14)

These equations give the number of replacements needed to support a maintenance schedule. Their solution involves a series of convolution integrals that can be performed on any failure distribution. However, the solution, which is an oscillation of replacement numbers about a straight line, is not easily obtained. The solid curve of Fig. 4 shows the renewal function for a component with a two-parameter Weibull reliability, \( \theta = 5000 \) hr and \( b = 1.5 \). Tabulated solutions to the renewal function for the two-parameter Weibull distribution are available [12].

An approximation for the renewal function [11] is

\[ M_b(t) = \frac{t}{t_{av}} - \frac{t_{av} - \theta^2}{2t_{av}^2} \]  

(15)

The accuracy of this approximation increases as \( t \) increases. Equation (15) is an asymptote.
for the true renewal function of low-scatter distributions. For high-scatter distributions, it approximates the true renewal function closely.

The renewal function is the probability of replacement for a single component. Its value goes above one because multiple replacements can occur. For a set of Q identical components, the total number of replacements is the product:

\[ N_r = Q M(t) \]

Estimates of replacement inventory need a margin for variations from the mean, as do repair frequency estimates. Confidence statistics based on the renewal standard deviation provide one means for determining this margin. The broken curve of Fig. 4 is a plot of the renewal function standard deviation versus life for the component with a characteristic life of 5000 hr and a Weibull slope of 1.5 for which the solid curve of Fig. 4 plots the renewal mean.

The approximation for the standard deviation of the renewal function uses the third moment of the life distribution. For the two-parameter Weibull distribution, the third moment is

\[ \mu_3 = \int_0^\infty t^3 f(t) \, dt = \theta^3 \Gamma\left(1 + \frac{3}{\beta}\right) \]

Figure 5 shows the third moment of the two-parameter Weibull distribution divided by the cube of the characteristic life versus the Weibull slope.

The approximation for the standard deviation of the renewal function is [11]

\[ \sigma_{s_3}(t) = \sqrt{\frac{\mu_3}{4\bar{t}_3} + \left(\frac{\bar{t}_3^2 + \sigma_t^2}{4\bar{t}_3}\right)^3} \]

The standard deviation of the renewal function gives a measure of the scatter in replacement needs from one sample to the next. Estimates of replacement inventory need a margin for variations from the mean, as repair frequency estimates do. Confidence statistics provide one means for determining this margin.

CONFIDENCE STATISTICS

In predicting replacement rates and maintenance inventories, direct theory provides mean or "average" estimates. These estimates come from the statistics of a universal population. With enough cases, they will be the true average values.

In any real situation, the number of drive systems under service is a limited sample. Confidence statistics estimate how differently a small sample may behave from its universal population. It uses the standard deviation of the universal failure distribution and the sample size to estimate the mean of the sample. Confidence intervals are shown by the broken lines in Fig. 2.

For many samples of the same size, the mean of the samples has a normal distribution about the overall mean. The standard deviation of the means is

\[ \sigma_{\bar{z}} = \frac{\sigma_f}{\sqrt{Q}} \]

where \( Q \) is the size of the sample.

In reliability predictions, the lower confidence bound is valuable in aircraft applications. This confidence distribution estimates the life at which a large percentage of the samples of a given set will survive. For a high confidence, this life is less than the mean life for the entire population. For a 90-percent confidence,

\[ \bar{t}_{av,90} = \bar{t}_{av} - Z_{0.05} \sigma_{av} \]

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Fig. 4. Renewal function and renewal function standard deviation for a two-parameter Weibull distribution with \( \theta = 5000 \) hr, and \( \beta = 1.5 \).

Fig. 5. Third life moment to characteristic life cubed ratio for a Weibull distribution as a function of the Weibull slope.
Since the behavior of samples differs from the behavior of the "ideal" distribution, confidence estimates help one to see the effects of sample size on the life and replacement estimates.

EXAMPLE

For the single-mesh transmission shown in Fig. 1, the 90-percent reliability lives for the bearings and gears are shown in Table 1. The Weibull slope for the bearings is 1.2, and for the gears is 2.5. For a fleet of \( Q = 50 \) aircraft, we would like to estimate the number of overhauls in the first 10 000 hr of service and the number of replacement components needed to support these overhauls. Two types of overhaul are treated - full replacement and failed-component replacement only. All estimates will be with 90-percent confidence for the 50 aircraft sample size. From Eq. (6), the average lives were determined for each component. From Eq. (8), the standard deviations for each component were determined. The results are shown in Table 1.

Full Replacement

To treat the transmission as a complete system undergoing full-replacement repair, one can use the two-parameter Weibull system model of Eq. (12). The parameters \( b_1 \) and \( \eta_{10} \) for Eq. (12) come from a least squares fit to Eq. (11). The two-parameter Weibull slope is \( b_1 = 1.57 \) for the transmission, and the system 90-percent reliability life is \( \eta_{10,1} = 1050 \text{ hr} \). From Eq. (3), the transmission characteristic life is \( \eta = 4440 \text{ hr} \). For these data, the transmission average life is \( \eta_{av} = 3990 \text{ hr} \) with a standard deviation of \( \sigma_f = 90 \text{ hr} \).

The renewal function can estimate the number of transmissions needed for full replacement in a continual sequence of failure warning, repair, and return to service for the 50 aircraft. For an average life of 3990 hr and a standard deviation life of 2600 hr, Eqs. (15) and (16) give the total number of replacements in the period from 0 to 4

From Eq. (17), the third moment of the transmission life distribution is \( \mu_3 = 1.62 \times 10^{11} \text{ hr}^3 \), and Eq. (18) gives the standard deviation of the renewal function for the transmission.

Equations (18) and (19) give the standard deviation of the number of replacements in the period from 0 to 4. Finally, a relationship similar to Eq. (20) gives the replacement estimate for complete transmissions with a 90-percent confidence that the replacements will be less.

For the first 10 000 hr of operation, this procedure estimates an average number of 111 replacements for the 50 aircraft. A confidence limit of 90 percent boosts this estimate to 121 transmission replacements for 500 000 fleet service hours. This represents a mean time between overhauls of 4130 hr and a total spare parts requirement of 726 parts.

Partial Replacement

When only the failing components are replaced, the renewal function can estimate the number of replacements needed, also. Applying the calculations of this procedure for each of the six components in the transmission estimates the number of components needed to support a partial-repair maintenance schedule with a 90-percent confidence.

Table 2 summarizes these calculations for the four bearings and two gears in the transmission. Adding the total number of components that renewal theory estimates will need to be replaced yields 72 bearings and 135 gears, for a total of 207 components. This total of 207 spare parts is significantly less than the 726 parts required by the 121 full-transmission replacements required of the other service procedure.

If each component failure required its own overhaul, then 207 overhauls would be required with the same 90-percent confidence as used for the full-replacement calculations. Dividing the 500 000 fleet service hours by the maximum number of 207 overhauls yields an estimate for the mean time between overhauls equal to 2420 hr. This is 1710 hr less than the mean time between overhauls for full-transmission replacement because it does not consider repair of components near failure in a maintenance session.

By only replacing the failed components, one would need 86 more overhauls, but 519 less parts.

Table 1. - Single-mesh transmission properties

<table>
<thead>
<tr>
<th>Component</th>
<th>Load, kN</th>
<th>10-percent life, ( \eta_{10} ), ( 10^6 ) cycles</th>
<th>10-percent life, ( \eta_{10} ), hr</th>
<th>Average life, ( \eta_{av} ), hr</th>
<th>Standard deviation of life, ( \sigma_f ), hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bearing 1</td>
<td>4.1</td>
<td>317</td>
<td>2640</td>
<td>16 187</td>
<td>13 570</td>
</tr>
<tr>
<td>Bearing 2</td>
<td>10.3</td>
<td>578</td>
<td>4820</td>
<td>29 554</td>
<td>24 776</td>
</tr>
<tr>
<td>Pinion</td>
<td>5.8</td>
<td>290</td>
<td>2486</td>
<td>5 426</td>
<td>2 324</td>
</tr>
<tr>
<td>Bearing 3</td>
<td>10.3</td>
<td>868</td>
<td>7230</td>
<td>44 330</td>
<td>24 460</td>
</tr>
<tr>
<td>Bearing 4</td>
<td>4.1</td>
<td>475</td>
<td>3960</td>
<td>24 280</td>
<td>20 097</td>
</tr>
<tr>
<td>Gear</td>
<td>5.8</td>
<td>380</td>
<td>3170</td>
<td>6 920</td>
<td>2 963</td>
</tr>
<tr>
<td>Transmission</td>
<td>5.8</td>
<td>127</td>
<td>1060</td>
<td>3 990</td>
<td>2 600</td>
</tr>
</tbody>
</table>

Table 2. - 10 000-hr repair estimates for 50 transmissions

<table>
<thead>
<tr>
<th>Transmission component</th>
<th>Replacements required</th>
<th>Mean</th>
<th>Standard deviation</th>
<th>90-percent confidence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bearing 1</td>
<td>23</td>
<td>5.2</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>Bearing 2</td>
<td>9</td>
<td>4.1</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>Pinion</td>
<td>72</td>
<td>4.7</td>
<td>78</td>
<td></td>
</tr>
<tr>
<td>Bearing 3</td>
<td>4</td>
<td>3.6</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>Bearing 4</td>
<td>13</td>
<td>4.4</td>
<td>19</td>
<td></td>
</tr>
<tr>
<td>Gear</td>
<td>52</td>
<td>4.3</td>
<td>57</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>173</td>
<td>207</td>
<td></td>
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</table>
The same high reliability would be present for both maintenance procedures because of the on-board failure monitoring system. These estimates are for cost and scheduling purposes only.

SUMMARY OF RESULTS

Two methods of estimating the time between transmission overhauls and the number of replacement components needed are presented. The first treats full replacement of failed transmissions, whereas the second treats replacement of failed components only. Confidence statistics are applied to both methods to improve the statistical estimate of sample behavior.

The method to predict overhaul timing with full replacement is based on a two-parameter Weibull system life model. The relationship between the system life model and the component life models is presented. In addition, formulas for the mean and standard deviation of the two-parameter Weibull distribution are given.

Renewal theory is presented as a tool to estimate the number of replacements in a transmission undergoing a consistent maintenance procedure. The theory is useful for estimating replacements for both full and partial transmission-replacement procedures. Approximation formulas are given for the mean and standard deviations of the renewal function. These approximations are valid for two-parameter Weibull distribution lives amongst others. Formulas for sample replacement numbers are given in terms of the renewal function.

Single-sided confidence theory is presented for the replacement number and overhaul timing estimates. A transmission example is presented to illustrate the methods. Comparisons of overhaul timing and spare-part requirements are made in the example between full-transmission replacement and partial component-replacement overhauls. High reliability is assured for the transmissions in both cases by the on-board monitoring system.

NOMENCLATURE

- $b$: Weibull slope
- $e$: base of the natural log
- $F$: probability distribution function (probability of failure)
- $f$: probability density function
- $\text{Ln}$: natural log
- $t$: life, hr
- $M$: renewal function
- $M_e$: approximate renewal function
- $\text{MTTF}$: mean time to failure
- $m$: renewal density function
- $N_r$: number of replacements
- $Q$: sample size
- $R$: reliability (probability of survival)
- $x$: integration time variable, hr
- $z_{10}$: number of standard deviations from the mean which cuts off a 10-percent population tail
- $\Gamma$: gamma function
- $\theta$: characteristic life, hr
- $\mu_3$: third moment of a probability density function
- $\sigma$: standard deviation
- $\sigma_f$: standard deviation of Weibull function
- $\sigma_{me}$: standard deviation of renewal function

Subscripts:

- $\text{av}$: average or mean
- $i$: index
- $n$: number of components
- $s$: system
- $1$: index value
- $10$: 10-percent failure, 90-percent reliability
- $90$: 90-percent confidence

REFERENCES

<table>
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| 17. Key Words (Suggested by Author(s)) | Transmissions (Machine elements)  
Weibull density functions  
Life (Durability)  
MTBF |