Consideration of Permanent Tidal Deformation in the Orbit Determination and Data Analysis for the Topex/Poseidon Mission

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Abstract

The effects of the permanent tidal effects of the Sun and Moon with specific applications to satellite altimeter data reduction are reviewed in the context of a consistent definition of geoid undulations. Three situations are distinguished: the tide free case, the "zero" case, and the mean case. These situations are applicable not only for altimeter reduction and geoid definition, but also for the second degree zonal harmonic of the geopotential and the equatorial radius. A recommendation is made that sea surface heights and geoid undulations placed on the Topex/Poseidon geophysical data record should be referred to the mean Earth case (i.e., with the permanent effects of the Sun and Moon included). Numerical constants for a number of parameters, including a flattening and geoid geopotential, are included.
Introduction

The tidal attraction of the Sun and Moon on the solid Earth can be represented in a series form that has constant and periodic terms. For numerous applications it is appropriate to remove the tidal effects from the measurements and parameters that may be affected. The manner in which this removal might be done has been discussed in the literature (Ekman, 1979; 1980; 1989; Groten, 1980; Heikkinen, 1979; Mather, 1978; Melbourne et al., 1983; Rapp, 1983; McCarthy et al., 1989).

Tidal attraction acts in a direct and indirect way. The direct effect on quantities such as potential, gravity (or gravitational attraction), shape of equipotential surfaces, etc. can be calculated knowing information about the masses and their positions in space. The direct attraction deforms the elastic Earth and thus causes an indirect change. The calculation of the indirect changes requires knowledge of parameters (primarily Love and Shida numbers) which depend on elastic properties of the Earth. To obtain observations and parameters for a tide-free Earth requires the removal of both the direct and indirect tidal effects. Both effects contain a permanent deformation at zero frequency. The removal of this portion of the indirect deformation requires knowledge of the Love and Shida numbers at zero frequency. It is considered that these numbers are distinct from those pertinent to the remainder of the time domain and cannot be distinguished from the static geopotential; thus the permanent part of the indirect deformation should not be removed from observations and parameters.

Resolution 16, adopted by the International Association of Geodesy at the 1983 IAG/IUGG meeting in Hamburg provides a formal statement on the current international convention dealing with tidal corrections. This resolution is as follows:

The International Association of Geodesy, recognizing the need for the uniform treatment of tidal corrections to various geodetic quantities such as gravity and station positions, and considering the reports of the Standard Earth Tide Committee and S.S.G. 2.55, Predictive Methods for Space Techniques, presented at XVIII General Assembly, recommends that:

1. the rigid Earth model be the Cartwright - Tayler - Edden model with additional constants specified by the International Centre for Earth Tides,
2. the elastic Earth model be that described by Wahr using the 1066 A model Earth of Gilbert and Dziewonski,
3. the indirect effect due to the permanent yielding of the Earth be not removed, and
4. ocean loading effects be calculated using the tidal charts and data produced by Schwiderski as working standards.

The key part of this resolution for the purpose of this paper is point 3. It is absolutely critical that different groups have the same understanding of the meaning of various parameters they use. The role of the permanent tide on potential coefficient models and station positions was also discussed in the report describing the Project Merit Standards (Melbourne et al., 1983, Appendix 5) and the IERS standards (McCarthy et al., 1989).

Increased interest in precisely defining the role of the permanent tide has recently arisen in the analysis of satellite altimeter data, where several groups are dynamically determining the geocentric location of the ocean surface. In addressing this problem it is essential to have consistency between geometric and potential effects. Comparisons between solutions are also greatly assisted if they are reported in a consistent way. Such agreement is needed because of the role of sea surface topography (SST) in ocean circulation studies. The
SST can be defined as the difference between the ocean surface and the geoid. Both the ocean surface and the geoid must be referred to the same tidal concepts of the permanent Earth tide. Discussions of permanent tidal considerations in the altimeter measurement reduction may be found in papers by Engelis (1985), Engelis and Knudsen (1989), Rapp (1989a), Marsh et al. (1990), and Nerem et al. (1990). Ekman (1988) has summarized the effects of the permanent Earth tide on a number of geophysical phenomena.

In addition to the consistent definitions necessary for the determination of the SST, it is important to have a consistent treatment of the dynamical perturbation of a satellite with respect to the permanent Earth tide effects. In lieu of identical software, it is important that all involved groups explicitly document the treatment of the permanent Earth tides in the orbit determination process.

Definitions

In discussing tidal effects on various quantities we start by distinguishing between tide free, zero value, and mean value. A tide-free value is the quantity from which all tidal effects have been removed. A mean value is the quantity from which the periodic tidal effects have been excluded, but the permanent deformations (both direct and indirect) are included. The mean value reflects a system in the presence of the constant effects of the Sun and the Moon. The zero value includes the indirect deformation effects associated with the permanent tidal deformation, but not the direct effects. The application of these terms to selected quantities is the subject of this paper.

Second Degree Harmonic of the Geopotential and the Flattening of the Reference Ellipsoid

The second degree zonal potential coefficient is well determined from the analysis of satellite tracking data. The tidal system in which \( J_2 \) has been reported varies. It is reasonable to remove the direct tidal influences of the Sun and the Moon as these can be directly computed from astronomical tidal theory. The use of frequency-independent time domain computations for the tidal deformation implicitly removes the indirect tidal contribution from \( J_2 \). This removal of the indirect effect on \( J_2 \) (using the adopted \( k_2 \) Love number) was done by Marsh et al. (1989) in the development of the GEM-T2 gravity model and therefore, the value of \( J_2 \) reported by Marsh et al. refers to a tide-free Earth. To be consistent with the IAG Resolution, the indirect tidal effect on \( J_2 \) should be added back to the GEM-T2 value. This effect is given as (Melbourne et al., 1983): \(-3.11080 \times 10^{-8} k_2\). The "zero-value \( J_2 \)" is then (with \( k_2 = 0.3 \)):

\[
J_{2z} = J_2^{\text{GEM-T2}} + 9.3324 \times 10^{-9} \tag{1}
\]

The definition of the flattening of a reference ellipsoid critically depends on the value of \( J_2 \). The flattening of the ellipsoid used in the IAG Geodetic Reference System 1967 and 1980 has been based on a "tide-free" value of \( J_2 \). To be consistent with Resolution 16, the appropriate \( J_2 \) value to use for flattening computations is the "zero-value" \( J_2 \) given by Eq. (1). As an example consider the \( J_2 \) value of the GEM-T2 (Marsh et al., 1989) model: we have (for the tide-free value):

\[
J_2 = 1082.626523 \times 10^{-6} \tag{2}
\]
The corresponding "zero value" would be:

\[ J_{2z} = 1082.635855 \times 10^{-6} \]  

(3)

We now temporarily adopt the constants used in GEM-T2 (*ibid*, 1989b):

\[ GM = 398600.436 \text{ km}^3/\text{s}^2 \]
\[ a_e = 6378137 \text{ m} \]
\[ \omega = 7.292115 \times 10^{-8} \text{ rad/sec} \]  

(4)

The flattening of the equipotential ellipsoid can be computed by the iterative evaluation of Eq. (2-92) in *Heiskanen and Moritz* (1967). Using the value of \( J_2 \) from Eqns. (2) and (3) we have:

\[ f = 1/298.257661944 \]  

(5)

\[ f_z = 1/298.256415307 \]  

(6)

The flattening of the Geodetic Reference System 1980, for comparison purposes, is:

\[ f = 1/298.257222101 \]  

(7)

The "zero-value" flattening is similar to that used by *Engelis* (1985) (i.e., \( f = 1/298.25657701 \)) for ocean circulation studies. The number of digits given reflects the definition of the constants as exact. The changing of \( a_e \) or \( GM \) by small amounts on the order of the accuracy of the value will have a substantial impact on the last 5 given digits of the inverse flattening. If the constants in Eq. (4) were used for the Topex/Poseidon standards and the zero value \( J_2 \), the flattening to be used would be given by Eq. (6). Alternatively, we compute \( f_z \) using the constants for \( GM \) and \( a_e \) given in *Wakker* (1990). Letting \( GM = 398600.4405 \text{ km}^3/\text{s}^2 \); \( a_e = 6378136.3 \text{ m} \), \( J_{2z} = 1082.636093 \times 10^6 \) (based on a rescaling of \( J_2 \) (GEM-T2) to the new value of \( a_e \)), we find \( f_z = 1/298.256435771 \).

**Sea Surface Heights**

A sea surface height is the distance along an ellipsoid normal between the sea surface and the reference ellipsoid. A sea surface height can be computed from a satellite altimeter measurement after numerous corrections are made to the original measurement and tidal effects are taken into account. The tidal effects are associated with ocean tides and solid Earth tides. Ocean tide corrections are computed from a model that defines the tidal surface relative to a mean surface associated with the deformed solid Earth. The solid Earth tide correction reflects the vertical displacement of the crust of the Earth with respect to the ellipsoid due to the attraction of the Sun and Moon. This displacement is:

\[ \Delta h = h_2 \frac{W_2}{g} \]  

(8)

where \( h_2 \) is the second degree Love number, \( W_2 \) is the second degree tidal potential, and \( g \) is the average acceleration of gravity. We restrict this discussion to the second degree terms of the tidal potential. The value of \( \Delta h_i \) for mass \( M_i \) as used in the Seasat data corrections is (*Parke et al.*, 1980):

\[ \Delta h_i = h_2 \frac{M_i}{M_e} \frac{a_e^2}{d_i^3} \left[ \frac{2}{3} \cos^2 \theta_i - \frac{1}{2} \right] \]  

(9)

where \( M_i \) is the mass of the body (Sun or Moon), \( a_e \) is the Earth's equatorial radius, \( M_e \) is the mass of the Earth, \( d_i \) is the distance from the center of mass of the Earth to the body (Sun or Moon), and \( \theta_i \) is the angle between the vectors from the center of the Earth to the subsatellite point and from the center of the Earth to the center of the mass of the tide generating third body.

Let \( S_{\text{ot}} \) be the sea surface height after the ocean tide correction has been made and \( S_{\text{rf}} \) be the sea
surface height after the full, solid Earth tide correction has been made (i.e., a "tide-free" sea surface height). We have:

\[ S_{TF} = S_{OT} - (\Delta h_s + \Delta h_m) \]

where \( \Delta h_s \) and \( \Delta h_m \) are the evaluations of (9) for the Sun and Moon, respectively. A constant (time independent) part (\( \Delta h_c \)) of the Earth tide correction can be computed so that the following holds:

\[ \overline{\Delta h_s} + \overline{\Delta h_m} + \overline{\Delta h_c} = 0 \]

where the overbars indicate the constant or zero frequency term in the correction. Using nominal constants one has (Parke et al., 1980):

\[ \Delta h_c = 0.198 h_2 \left( \frac{3}{2} \sin^2 \phi - \frac{1}{2} \right) \text{ meters} \]

where 0.198 is the numerical constant implied by the constant tidal potential of the Cartwright and Edden (1973) model. This factor also holds for the epoch 1990-2000 based on the constant term given by Cartwright (1990).

A mean sea surface height (\( S_m \)) is now computed as:

\[ S_m = S_{TF} - (\Delta h_s + \Delta h_m + \Delta h_c) \]

This mean sea surface height includes the permanent deformation of the crust of the Earth. The sum of the three corrections was given in the Seasat and Geosat geophysical data records. The definition of \( S_m \) to include the permanent tidal deformation is an important step in refining our definitions of sea surface topography.

The Geoid and Geoid Undulations

The geoid is an equipotential surface of the Earth’s gravity field. It is defined in such a way to approximate the mean sea surface in ocean areas. Concerns about sea surface topography and permanent tidal effects creates a need for a precise definition. As discussed by Heikkinen (1979), Ekman (1988) and others, one can consider three types of geoids: the tide-free geoid, the zero geoid, and the mean geoid.

The mean geoid is the equipotential surface that would exist in the presence of the constant or permanent effects of the Sun and Moon. The zero geoid is the surface after the removal of the direct tidal potential effects from the mean geoid and the tide free geoid is that surface if the complete (direct and indirect) effects of the Sun and Moon are removed. The latter surface requires an assumption on the zero frequency Love number. To be consistent with the IAG resolution, the geoid surface of primary interest for geodetic purposes would be the zero geoid. The potential on this geoid can be defined through the usual spherical harmonic expansions (Heiskanen and Moritz, 1967, Chapter 2; Rapp, 1971):

\[ W_o = \frac{GM}{r_o} \left[ 1 + \sum_{n=2}^{\infty} \left( \frac{4\pi}{r_o} \right)^n \sum_{m=0}^{n} \left( \overline{c}_{nm} \cos m\lambda_o \right) + \overline{s}_{nm} \sin m\lambda_o \right] + CFP \]

where \( GM \) is the product of the gravitational constant and the mass of the Earth; \( r_o, \theta_o, \lambda_o \) are the spherical coordinates of a point on the geoid; \( \overline{c}_{nm}, \overline{s}_{nm} \), are the fully normalized potential coefficients; \( \overline{P}_{nm} (\cos \theta_o) \), fully normalized associated Legendre functions; and CFP is the centrifugal force potential.

The \( \overline{C}_{2,0} \) (\( \overline{C}_{2,0} = -J_2/\sqrt{5} \)) to be used in Eq. (14) would be the zero value as would be given by Eq. (3). A procedure to calculate geoid undulations through (14) is described in Rapp (1971) and Shum (1983). Basically, \( W_o \) and related constants are defined and then the \( r_o \) is found that will yield \( W_o \).
with the given set of potential coefficients. The $W_o$ should be computed from $GM$, $a_e$, $\omega$ and the zero-tide $J_2$.

Another procedure to calculate a geoid undulation is through the definition of a disturbing potential, $T$, and the use of Bruns' formula (Heiskanen and Moritz, 1967, Eq. (2-144)). The disturbing potential is the difference between the "true" potential at a point, and the normal potential $U$, usually defined by an equipotential reference ellipsoid. One has:

$$T(r, \theta, \lambda) = W(r, \theta, \lambda) - U(r, \theta, \lambda) \quad (15)$$

The calculation of $U$ requires the definition of the four fundamental constants: $GM$, $\omega$, $a_e$, and $J_2$. The $J_2$ may be the nontidal value (as used in GRS67 and GRS80) or the preferred "zero value" as discussed earlier. For these discussions, we assume that $J_2$ is referred to the "zero" case, i.e., the indirect deformation is retained in the value. Conceptually, the $W_o$ value is calculated with the $C_{2,0}$ in the same system as used in the normal potential definition. Setting the zero and first degree harmonics of $T$ to zero we have:

$$T(r, \theta, \lambda) = \frac{GM}{r} \sum_{n=2}^{\infty} \sum_{m=0}^{n} \frac{(2n+1)}{4d_i^2} \left( \begin{array}{c} \sin^2 \varepsilon - 1 \sin^2 \phi - 1 \end{array} \right)$$

The geoid undulation is found from the Bruns' equation:

$$N(r, \theta, \lambda) = \frac{T(r, \theta, \lambda)}{\gamma} \quad (17)$$

where $r, \theta, \lambda$ is formally a point on the geoid but in practice, is a point on the reference ellipsoid. The value of $\gamma$ is a normal value of gravity computed at $r, \theta$.

The geoid undulation computed through Eq. (16) and Eq. (17), with the "zero-value" $C_{2,0}$ (and zero value $C_{2,0}$ (or flattening)) in the reference potential) would be considered the zero geoid height or undulation.

The potential of the mean geoid would be found by adding the direct tidal potential. For a mass $M_i$ we have for the permanent potential tidal effect (Ekman, 1988, Eq. (5)):

$$\bar{W}_i = \frac{GM_i a^2}{4d_i^2} \left( \frac{3}{2} \sin^2 \varepsilon - 1 \right) \left( 3 \sin^2 \phi - 1 \right) \quad (18)$$

where $\varepsilon$ is the inclination of the ecliptic to the Equator for the Sun, and is the orbit plane to the Equator for the Moon; and $d_i$ is the same as used in Eq. (9). One can add this potential (evaluated for both the Sun and Moon) to Eq. (14) to obtain the mean geoid potential. An alternative procedure is to calculate the separation between the mean geoid and the zero geoid. This is given as (Heikkinen, 1979; Ekman, 1988; Rapp, 1989a; Marsh et al., 1990; Nerem et al., 1990):

$$N_M = N_Z - 0.198 \left( \frac{3}{2} \sin^2 \phi - 1 \right) \text{ meters} \quad (19)$$

The value of $N_M$ computed from Eq. (16) and (17) with the "zero-value" $J_2$ and the use of Eq. (19) would be consistent with the mean sea surface height defined by Eq. (13). The difference between these two values defines sea surface topography in a consistent system:

$$SST = S_M - N_M = S_M - N_Z + 0.198 \left( \frac{3}{2} \sin^2 \phi - 1 \right) \text{ meters} \quad (20)$$

The value of $SST$ computed from Eq. (20) should be consistent with oceanographic estimates. If the zero geoid ($N_Z$) is used to compute the $SST$, the
reported SST height should be corrected by adding the last term in Eq. (20).

The Equatorial Radius of the Reference Ellipsoid

The determination of the parameters of the reference ellipsoid has been a historical goal of geodesy. Numerous techniques used historically and in current terms are described in Rapp (1989b). Numerous definitions exist. For example: 1) the size of the ellipsoid should be such that the average geoid undulation over the whole Earth should be zero, and 2) the ellipsoid should be a best fit to the mean ocean surface. The latter definition is of specific interest to us for oceanographic purposes. The definition is further complicated by the existence of sea surface topography and permanent tidal deformation. A discussion of some of these factors may be found in Rizos (1980) and Engelis (1985). For ocean studies, a meaningful definition is one where the mean (over the oceans) difference between sea surface heights and geoid undulations should be zero. Specifically, we seek a reference ellipsoid where (Engelis, 1985):

$$M (S - N) = 0$$  \hspace{1cm} (21)

where the $S$ values (sea surface heights) are referred to the ideal ellipsoid and $N$ is the geoid undulation. The sea surface heights and geoid undulations must be given in a consistent (zero or mean) system. The global average undulation (zero or mean) will be zero with respect to a consistently defined (zero or mean) ellipsoid.

Recent analysis of altimeter data has simultaneously solved for potential coefficient models, sea surface topography representations and other parameters such as an altimeter bias. The bias can be interpreted as a correction to the equatorial radius adopted for reference purposes if the "true" altimeter instrumental bias has been determined. Engelis and Knudsen (1989) report an average bias for 17 days of Seasat data of 86 cm which implies an equatorial radius of 6378136.14 m. This number is uncertain by about 10 cm because of altimeter instrumental bias and treatment of the permanent tidal effects. Nerem et al. (1990) determined an identical value of the equatorial radius using 80 days of Geosat data.

Denker and Rapp (1990) processed 1 year of Geosat data starting from GEM-T1 orbits. The procedure followed by Engelis and Knudsen (1989) was used in a modified way for the Geosat analysis. The average bias found by Denker and Rapp (ibid) was 59 cm implying an equatorial radius of 6378136.41 m. This value depends on altimeter calibration, the precision of the determination of the center of mass of the Earth (and therefore the origin of the coordinate system), etc. This equatorial radius would be interpreted as a fit to a reference ellipsoid where the average value of the sea surface topography is zero. As the original sea surface heights refer to the "mean" case this equatorial radius should also refer to a mean value. The corresponding equatorial radius for the zero case would be reduced by .099 m (based on Eq. (19) and Figure 1 of Heikkinen, (1979)). Therefore the "zero-value" equatorial radius, based on this Geosat analysis, is 6378136.31 m.

An alternative procedure to calculate the equatorial radius is based on comparisons of geoid undulations derived geometrically from Doppler-derived satellite positions, and geoid undulations implied by a set of potential coefficients. Results reported by Rapp (1987) implied an equatorial radius of 6378136.2 m. Improved gravity models (and transformation parameters) imply an equatorial radius between 6378136.20 and 6378136.33 m. The consistency with the altimeter-implied result is
remarkable although it should be noted that the treatment of the permanent tidal effects in the Doppler orbital analysis has not been researched.

**Station Positions**

The treatment of solid Earth tides plays an important role in the definition of the positions of points fixed to the crust of the Earth. Satellite orbital studies will incorporate solid Earth tide models in their analysis. Equations that can be used to calculate the solid Earth tide effects are given in Melbourne et al. (1983, Appendix 5) and McCarthy et al. (1989, p. 27). There is a zero frequency component of this correction. In association with the "zero-value" concept discussed earlier (for \( J_2 \) and geoid undulation), the indirect permanent deformation is included in the station position values when they are computed. This procedure is that adopted for use in the definition of the Conventional Terrestrial Reference System (CTRS) by the International Earth Rotation Service (Boucher and Altamimi, 1989). Specifically, let \( X_o \) be the station positions with the full solid Earth tide removed and \( X_\sigma \) be the coordinates with the constant (zero frequency) part included. Then:

\[
\overline{X}_o = X_o + \Delta X_{\text{perm}} \tag{22}
\]

with

\[
\Delta X_{\text{perm}} = \frac{X_o}{X_\sigma} \Delta h_{\text{perm}} \tag{23}
\]

with

\[
\Delta h_{\text{perm}} = -0.121 \left( \frac{3}{2} \sin^2 \phi - \frac{1}{2} \right) \text{ meters} \tag{24}
\]

The numerical constant was computed with a nominal Love number \( (h_2) \) of 0.6090 (Melbourne et al., 1983, p. A5-7; McCarthy et al., 1989, p. 28).

**Conclusions**

The permanent tides created by the attraction of the Sun and Moon must be precisely considered in satellite and terrestrial analysis. This paper draws attention to specific areas of interest in the area of satellite altimetry, sea surface topography and geoid undulation. A guiding theme was Resolution 16 of the 1983 IAG meeting in which it was recommended that "the indirect effect due to the permanent yielding of the Earth be not removed."

For applications to be dealt with in the Topex/Poseidon mission, the following recommendations are made for quantities that are influenced by the permanent tidal effects:

**A. \( J_2 \) and the Flattening**

The preferred \( J_2 \) is one that includes the indirect permanent deformation. This value is to be used in computing the flattening of the reference ellipsoid. Using the \( GM = 398600.4405 \ \text{km}^3\text{s}^{-2}, a_e = 6378136.3 \ \text{m} \) as given in Wakker (1990), \( \omega = 7.292115 \times 10^{-8} \ \text{rad sec}^{-1} \), and with the rescaled GEM-T2 \( J_2 \), the "zero" case flattening is 1/298.256435771.

**B. Sea Surface Heights**

Sea surface heights should have the permanent tidal effects included when the values are reported. The total Earth tide effect is usually removed in data reductions with the constant (or zero frequency) part added back in. This procedure is consistent with what has been done with the Seasat and Geosat Geophysical Data Records. The resultant values refer to the mean sea surface.
C. Geoid Undulations

A clear distinction must be maintained between the nontidal, zero, and mean geoid and the undulation of these surfaces. For most geodetic purposes the "zero geoid undulations" are preferred. For sea surface topography determinations using the sea surface heights defined in part B, the mean geoid is to be used. The correction between the zero and mean geoid is given by Eq. (19). To avoid confusion the undulations of the "mean geoid" should be given on the Topex/Poseidon Geophysical Data Record.

D. Equatorial Radius

For geodetic purposes, the equatorial radius of the ellipsoid fitting the zero geoid is appropriate. For oceanographic purposes, one might argue that the equatorial radius associated with the mean geoid is most appropriate. Based on the previous discussions in this paper, a suitable "zero" equatorial radius is 6378136.3 m.

E. Geoid Potential

The value of \( W_0 \) is defined once \( GM, a_e, f \) (or \( J_2 \)) are defined. Using \( GM = 398600.4405 \) km\(^3\)s\(^{-2}\), \( a_e = 6378136.3 \) m, \( f = 1/298.25643771 \), and \( \omega = 7.292115 \times 10^{-8} \) rad/sec, and the value of \( W_0 \) computed using Eq. (2-61) of Heiskanen and Moritz (1967) is: 62636858.546 m\(^2\) s\(^{-2}\). This value is subject to change if \( J_2, GM \) or \( a_e \) are changed.

F. Station Positions

The station positions to be used in the Topex/Poseidon mission should be such that the permanent deformation is included in the station definition. This statement is consistent with the standards of the International Earth Rotation Service in the definition of the Conventional Terrestrial Reference System.

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References


Rapp, R.H., Geometric Geodesy, Part II, Dept. of Geodetic Science and Surveying, The Ohio State University, Columbus, 1989b.

Rizos, C., The role of the gravity field in sea surface topography studies, dissertation, School of Surveying, University of New South Wales, Sydney Australia, 1980.

Shum, C.K., Altimeter methods for satellite geodesy, Center for Space Research Report, CSR-82-2, The University of Texas at Austin, 1983.

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<th>15. Supplementary Notes</th>
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<tr>
<td>Richard H. Rapp: The Ohio State University, Columbus, OH; R. Steven Nerem: NASA-GSFC, Greenbelt, MD; C.K. Shum: The University of Texas at Austin, Austin, TX; Steven M. Klosko and Ronald G. Williamson: ST Systems Corporation, Lanham, MD.</td>
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<th>16. Abstract</th>
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<td>The effects of the permanent tidal effects of the Sun and Moon with specific applications to satellite altimeter data reduction are reviewed in the context of a consistent definition of geoid undulations. Three situations are applicable not only for altimeter reduction and geoid definition, but also for the second degree zonal harmonic of the geopotential and the equatorial radius. A recommendation is made that sea surface heights and geoid undulations placed on the Topex/Poseidon geophysical data record should be referred to the mean Earth case (i.e., with the permanent effects of the Sun and Moon included). Numerical constants for a number of parameters, including a flattening and geoid geopotential, are included.</td>
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<td>Permanent Tide, Geopotential, Geoid, Topex/Poseidon</td>
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