SOLAR NEUTRINOS AND THE MSW EFFECT FOR THREE-NEUTRINO MIXING

X. Shi
Department of Physics,
The University of Chicago
5640 S. Ellis Avenue, Chicago, IL 60637

David N. Schramm
Department of Physics,
Department of Astrophysics & Astronomy,
The University of Chicago,
5640 S. Ellis Avenue, Chicago, IL 60637
and
NASA/Fermiab Astrophysics Center,
Fermi National Accelerator Laboratory,
Box 500, Batavia, IL 60510-500

ABSTRACT

We consider three-neutrino MSW mixing, assuming $m_3 \gg m_2 > m_1$ as expected from theoretical consideration if neutrinos have mass, and calculate the corresponding mixing parameter space allowed by both the $^{37}\text{Cl}$ and Kamiokande II experiments. We also calculate the expected depletion for the $^{71}\text{Ga}$ experiment. Finally, we explore a range of theoretical uncertainty due to possible astrophysical effects by varying the $^8\text{B}$ neutrino flux and redoing the MSW mixing calculation.
I. Introduction:

The well known solar neutrino problem centers on the fact that the measured time averaged solar neutrino flux is lower than theoretical estimates by a factor of $2 \sim 3$ in two experiments, the Homestake $^{37}$Cl experiment and the Kamiokande neutrino-electron scattering experiment. In the $^{37}$Cl experiment, the observed neutrino flux $= 2.2 \pm 0.3$ SNU (Solar Neutrino Unit, $1$ SNU $= 10^{-36}$ capture events/target nucleus/sec$^1$), compared with the predicted $7.9$ SNU by the Standard Solar Model (SSM) of Bahcall $et$ $al.$ $^1$ or $5.8$ SNU of the French solar model.$^2$ Although many attempts have been made to assign an estimate of the uncertainty$^{3,4}$ in the theoretical numbers, the spread between the French and US models is probably as good as any estimate (it should be noted that the two models do approximately agree when the same choice of input parameters are made but differ primarily because of different choice as to the preferred input parameters). In the Kamiokande II experiment, the measured fluxes $= 0.45 \pm 0.06_{\text{stat}} \pm 0.06_{\text{syst}}$ relative to the predicted value of Bahcall $et$ $al.$ $^5$ The Kamiokande and Homestake experiments are mainly sensitive to the higher energy $^8$B neutrinos with the latter also have some admixture of $^7$Be neutrinos.$^1$ Furthermore, two new experiments using $^{71}$Ga (GALLEX and SAGE) which can see the main-line $pp$ neutrinos have recently begun operating, while preliminary results from SAGE also appear reduced relative to theory.$^6$ These reports remain to be established following subsequent calibration studies using $^{51}$Cr sources to verify that $^{71}$Ge produced by neutrino capture on $^{71}$Ga can indeed be quantitively detected.

Vacuum neutrino mixing was proposed to solve the problem, but it required large vacuum mixing angles relative to the Cabbibo angle ($\theta_c = 13^\circ$) to provide a maximal flux reduction of $1/N_\nu$, where $N_\nu$ is number of neutrino flavors involved in mixing.$^1$ The Mikheyev-Smirnov-Wolfenstein (MSW) matter mixing effect$^{7,8}$ enables neutrinos to flip flavor while propagating in matter even with a small vacuum mixing angle. Thus, it provides a more natural solution to the solar neutrino problem.$^9$ In the two-flavor mixing case, it has been found that in order to get the observed reduction with the MSW effect, only a triangular area in parameter space $\Delta = (m_3^2 - m_1^2) \sim \sin^2 2\theta_{12}/\cos 2\theta_{12}$ is allowed.$^1$ Full three-flavor MSW effects have also been investigated previously by Kuo and Pantaleone$^{10}$ and Barger $et$ $al.$$^{11}$ In this paper we reexamine the full three-flavor mixing in the light of the present experimental situation. Since we now know from LEP$^{12}$ and from cosmology$^{13}$ that there are indeed 3 neutrino families, it seems only reasonable that the formalism used should take into account this reality. However, we will see that the basic two-neutrino picture still gives a reasonable understanding of most of the action. We will also assume $m_3 \gg m_2 > m_1$, which, if neutrinos have mass, is a reasonable theoretical expectation. In section II, we derive the probability of solar neutrinos flipping their flavors on the way to the earth. In section III we calculate numerically the allowed parameter space from the data of the $^{37}$Cl and Kamiokande II experiments assuming three-flavor matter mixing.
Mixing between three orthogonal neutrino states can be generally described by a unitary operator $U$

$$|\nu_\alpha\rangle = U_{\alpha i} |\nu_i\rangle, \quad \alpha = e, \mu, \tau, \quad i = 1, 2, 3 \quad (1)$$

where $\nu_\alpha$ denotes flavor eigenstates and $\nu_i$ mass eigenstates.

Before looking at the complication of three-flavor mixing, let us review the standard two-flavor formalism\textsuperscript{14,15,16} In the special case of two neutrino mixing between $\nu_e$ and $\nu_\tau$ ($\nu_\tau$ might be a linear combination of $\nu_\mu$ and $\nu_\tau$),

$$U = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \quad (2)$$

The Schrödinger Eq. of a neutrino travelling through matter in flavor basis is

$$i\frac{d}{dt} \begin{pmatrix} C_e(t) \\ C_\tau(t) \end{pmatrix} = M \begin{pmatrix} C_e(t) \\ C_\tau(t) \end{pmatrix} \quad (3)$$

Let $\Delta = m_2^2 - m_1^2$,

$$M = \frac{1}{4E} \begin{pmatrix} -\Delta \cos 2\theta + 2A & \Delta \sin 2\theta \\ \Delta \sin 2\theta & \Delta \cos 2\theta \end{pmatrix} = \frac{\Delta_M}{4E} \begin{pmatrix} -\cos 2\theta_M & \sin 2\theta_M \\ \sin 2\theta_M & \cos 2\theta_M \end{pmatrix} \quad (4)$$

where $A = 2\sqrt{2}\Gamma F n_e E$ is the induced mass of the electron neutrino due to the MSW effect. $F$ is the Fermi constant, $n_e$ is electron density of the matter and $E$ the neutrino energy. A term proportional to the identity matrix has been subtracted. In the second expression, $\Delta_M$ and $\theta_M$ are counterparts in matter of $\Delta$ and $\theta$, where

$$\Delta_M = [(\Delta \cos 2\theta - A)^2 + (\Delta \sin 2\theta)^2]^{1/2} \quad (5)$$

$$\tan 2\theta_M = \tan 2\theta /[1 - (A/\Delta) \sec 2\theta] \quad (6)$$

When $A = \Delta \cos 2\theta$, $\theta_M = \pi/4$, $\Delta_M$ reaches its minimum, the two mass eigenstates are almost degenerated and the neutrino encounters a resonance.

An interesting case occurs if neutrinos propagate through matter which has a slow and monotonically varying electron density. In that case the neutrinos can pass through the resonance adiabatically and flip their flavor (Fig.1). But generally, neutrinos evolve adiabatically in regions away from resonance. At resonance, they have some probability to jump between two mass eigenstates and thus don’t change their flavor completely. Such jump probability can be obtained by solving the Schrödinger eq.(3) assuming linear variation of electron density at the resonance region. The result is\textsuperscript{17}

$$P_{\text{jump}} = \exp\left[ -\frac{\pi \Delta \sin^2 2\theta}{4E \cos 2\theta} \frac{n_e}{[dn_e/dr]|_{\text{res}}} \right] \quad (7)$$
The probability for a $\nu_e$ produced at a high density region, passing through resonance, reaching the vacuum region, to maintain its flavor is:

\[ P = \frac{1}{2} + \left(\frac{1}{2} - P_{\text{jump}}\right)\cos 2\theta_M \cos 2\theta \]  

where $\theta_M$ is the mixing angle at which the $\nu_e$ is produced. If the density at which $\nu_e$ is produced is sufficiently large, $\theta_M \rightarrow \pi/2$ (see eq.(6)) and eq.(8) becomes

\[ P \approx \sin^2 \theta + P_{\text{jump}} \cos 2\theta. \]  

(8')

To obtain the observed neutrino flux reduction, a large fraction of the $\nu_e$'s produced at the center of the Sun have to change their flavor. This requires the following: (a) part of the solar neutrinos have to pass through a resonance; (b) from eq.(8) and (8'), $P_{\text{jump}}$ must be sufficiently small; (c) also from eq.(8) and (8') $\sin^2 \theta$ has to be sufficiently small. These conditions yield constraints on neutrino masses and mixing angles and limit them to a triangle in parameter space $\Delta$ vs $\sin^2 2\theta / \cos 2\theta$. Each region of the triangle produces a different spectrum for the resultant neutrino flux to be observed. On the horizontal region, only high energy neutrinos go through resonance and change flavor; thus, the high energy part of neutrino spectrum gets "cut off". This region is effectively ruled out because the Kamiokande II experiment which is only sensitive to high energy neutrinos ($E_{\text{threshold}}$ of recoil electron=7.5MeV) detected a larger fraction of its theoretical flux than the $^{37}$Cl experiment ($E_{\text{threshold}}=0.814$MeV) did. On the diagonal region, the lower energy neutrinos go through resonance with a smaller $P_{\text{jump}}$, they are thereby suppressed more than the higher energy ones. This is consistent with experiments. On the vertical part of the triangle the entire spectrum is uniformly suppressed. This is also not contradictory to the observation at the 3$\sigma$ level.

II. Matter mixing for three-neutrino mixing:

The three-neutrino matter mixing is more complicated. It involves three mixing angles. The unitary operator

\[ U = e^{i\psi \lambda_7} e^{-i\phi \lambda_5} e^{i\omega \lambda_2} \]  

(9)

where $\lambda_2, \lambda_5, \lambda_7$ are the generators of SU(3),

\[ \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}. \]  

(10)

In our problem, we are only interested in how much the $\nu_e$ flux is reduced without caring about which flavor it is converted to. We can rotate the flavor basis by $\exp(-i\psi \lambda_7)$ to get rid of $\psi$. The new basis is $\nu_e, \nu_\mu, \nu_\tau$ and

\[ U = e^{-i\phi \lambda_5} e^{i\omega \lambda_2} = \begin{pmatrix} \cos \phi \cos \omega & \cos \phi \sin \omega & -\sin \phi \\ -\sin \omega & \cos \omega & 0 \\ \sin \phi \cos \omega & \sin \phi \sin \omega & \cos \phi \end{pmatrix}. \]  

(11)
In general there are two resonances. These resonances can be coupled so it is hard to see when they happen and how neutrinos behave while crossing these resonances. However, if \( m_3 \gg m_2 > m_1 \), the resonance between two lower mass eigenstates can be well decoupled from the higher one. The assumption that \( m_3 \gg m_2 > m_1 \) is reasonable from seesaw model consideration. This also would be anticipated if \( \nu_\tau \), the heaviest neutrino, is associated with Hot Dark Matter where \( m_{\nu_\tau} \sim 25 \text{eV} \). Kuo and Pantaleone found that under such assumption the lower resonance happens when

\[
A \cos^2 \phi = \Delta \cos 2\omega
\]  

(12)

and the higher resonance occurs when \( A = m_3^2 \cos 2\phi \). We will follow the above assumption to solve the Schrödinger equation for three-neutrino mixing. We also assume that the higher resonance requires a much higher density than that of the Sun and becomes irrelevant. To find \( P_{\text{jump}} \) for the lower resonance, we further rotate the basis by \( \exp(i\phi \lambda_3) \). The new basis becomes \( \nu_e \), \( \nu_\mu \) (both linear combinations of \( \nu_e \) and \( \nu_\mu \)) and \( \nu_\tau \). The associated Schrödinger equation is

\[
i \frac{d}{dt} \begin{pmatrix} C_{e'}(t) \\ C_{\mu'}(t) \\ C_{\tau'}(t) \end{pmatrix} = M \begin{pmatrix} C_{e'}(t) \\ C_{\mu'}(t) \\ C_{\tau'}(t) \end{pmatrix}
\]  

(13)

with

\[
M = \frac{1}{4E} \begin{pmatrix} -\Delta \cos 2\omega + 2A \cos^2 \phi & \Delta \sin 2\omega & -A \sin 2\phi \\ \Delta \sin 2\omega & \Delta \cos 2\omega & 0 \\ -A \sin 2\phi & 0 & 2m_3^2 - m_2^2 - m_1^2 + 2A \sin^2 \phi \end{pmatrix}
\]  

(14)

Under the assumption \( m_3^2 \gg m_2^2 \) and \( m_1^2 \), we can see that \( C_{\tau'} \) oscillates so fast that it averages out (if we take the see-saw model, \( m_3/m_2 \sim m_2^2/m_\mu^2 \sim 10^2 \) using leptons (or \( \sim m_\mu^2/m_e^2 \sim 10^4 \) using quarks), \( C_{\tau'} \) would oscillate \( \sim 10^4 (10^6) \) times faster than the other two components). Then the transition probability of the lower resonance \( P_{\text{jump}} \) is given by eq.(7).

If we let

\[
\Phi_{1,2,3}(t_1, t_2) = \exp \left[ -i \int_{t_1}^{t_2} E_{1,2,3}(t) dt \right],
\]  

(15)

\( E_1, E_2, E_3 \) are energy eigenvalues of three mass eigenstates, the time evolution of a \( \nu_e \) produced at time \( t_i \), passing through lower resonance, detected at time \( t_f \) is

\[
U_{e1}(t_i)\Phi_1(t_i, t_r)|\nu_1(t_i)\rangle + U_{e2}(t_i)\Phi_2(t_i, t_r)|\nu_2(t_i)\rangle + U_{e3}(t_i)\Phi_3(t_i, t_r)|\nu_3(t_i)\rangle
\]  

(before resonance)

(16)

and

\[
\Phi_1(t_r, t_f)|a_{1} U_{e1}(t_i)\Phi_1(t_i, t_r) - a_{2} U_{e2}(t_i)\Phi_2(t_i, t_r)||\nu_1(t_f)\rangle
\]
\[+\Phi_2(t_r, t_f)[a_1^* U_{e2}(t_i) \Phi_2(t_i, t_r) + a_2 U_{e1}(t_i) \Phi_1(t_i, t_r)]|\nu_2(t_f)\]
\[+\Phi_3(t_r, t_f)a_3 U_{e3}(t_i) \Phi_3(t_i, t_r)|\nu_3(t_f)\]
(after resonance) \(^{(16')}\)

where \(|a_2|^2 = P_{\text{jump}} \cos^2 \phi\) (remember the \(P_{\text{jump}}\) we calculated earlier was in the basis \(\nu_e', \nu_{\mu}', \nu_{\tau}'\), a rotation of basis \(\nu_e, \nu_\mu, \nu_\tau\) by angle \(\phi\) , \(|a_1|^2 = 1 - P_{\text{jump}} \cos^2 \phi\), and \(|a_3|^2\), from the above decoupling approximation, is 1. The amplitude of detecting a \(\nu_e\) at \(t_f\) is

\[A_{e,e} = [a_1 U_{e1}(t_i) \Phi_1(t_i, t_f) - a_2^* U_{e2}(t_i) \Phi_2(t_i, t_f)] U_{e1}(t_f)\]
\[+ [a_1^* U_{e2}(t_i) \Phi_2(t_i, t_f) + a_2 U_{e1}(t_i) \Phi_1(t_i, t_r)] U_{e2}(t_f)\]
\[+ a_3 U_{e3}(t_i) U_{e3}(t_f) \Phi_3(t_i, t_f)\]. \(^{(17)}\)

After averaging over the position of production, i.e. \(t_i\), and the position of detection \(t_f\), the probability \(P\) of survival of \(\nu_e\) is

\[P = |a_1 U_{e1}(t_i) U_{e1}(t_f)|^2 + |a_2^* U_{e2}(t_i) U_{e1}(t_f)|^2 + |a_3 U_{e2}(t_i) U_{e2}(t_f)|^2\]
\[+ |a_2 U_{e1}(t_i) U_{e2}(t_f)|^2 + |a_3 U_{e3}(t_i) U_{e3}(t_f)|^2\]. \(^{(18)}\)

Taking expression of \(U_{ei}\) from eq.(11)

\[U_{e1} = \cos \phi \cos \omega, \quad U_{e2} = \cos \phi \sin \omega, \quad U_{e3} = -\sin \phi\] \(^{(19)}\)

we get

\[P = \left[\frac{1}{2} + \left(\frac{1}{2} - P_{\text{jump}} \cos^2 \phi\right) \cos 2\omega_M \cos 2\omega \cos^2 \phi_M \cos^2 \phi + \sin^2 \phi_M \sin^2 \phi\right]\]
\[\approx \left[\frac{1}{2} + \left(\frac{1}{2} - P_{\text{jump}} \cos^2 \phi\right) \cos 2\omega_M \cos 2\omega \cos^4 \phi + \sin^4 \phi\right]\] \(^{(20)}\)

where \(\phi_M\) and \(\omega_M\) are \(\phi(t_i)\) and \(\omega(t_i)\), the mixing angles at the position of production; \(\phi(t_f)\) and \(\omega(t_f)\) are \(\phi\) and \(\omega\) if we detect in vacuum. We can see that the \(\nu_e\) flux will suffer an additional overall suppression when \(\phi \neq 0\).

From eq.(20) we can calculate the experimental constraints on parameter space \(\Delta\) vs. \(\sin^2 2\omega / \cos 2\omega\), which reduces to \(\Delta\) vs. \(\sin^2 2\theta / \cos 2\theta\) when \(\phi = 0\).

III. Numerical Result:

Figures 2–4 show the allowed parameter space for different \(\phi\) with 1\(\sigma\) and 3\(\sigma\) errors. from the results of both \(^{37}\)Cl and the Kamiokande II experiments (by convention, we replace \(\omega\) by \(\theta\). Fig.2(b) is basically consistent with Chen and Cherry’s result.\(^{20}\) We can see that the horizontal region is still ruled out, in agreement with our previous discussion of the two-flavor mixing case. It is interesting to note that at the 1\(\sigma\) level (Fig.2(a) to
4(a)) there is no overlap for the vertical region either. This is understandable: for the 1σ
error, the lower bound of the Kamiokande II is $0.45 - \sqrt{0.06^2 + 0.06^2} = 0.365$, which is
higher than the upper bound from $^{37}$Cl experiment $2.5/7.9 \approx 0.32$; besides, on the vertical
region, the neutrino spectrum is uniformly suppressed. Thus, there can't be any overlap.
So at the 1σ level the only possible parameter space is the diagonal region.

From Fig.2(b) to Fig.4(b) we find at the 3σ level that the vertical region is then
allowed. However, part of this region will show day-night effect because of the regeneration
of electron-neutrino flux in the earth.\(^{21}\) So, after considering the Kamiokande II's limit on
the absence of a day-night effect, region $\sin^22\theta/\cos2\theta > 0.02$ and $\Delta = 2 \times 10^{-6} - 10^{-5}$eV\(^2\) is
also excluded at 90% C. L. when $\phi = 0$.\(^{22}\) At large $\phi$, the overall suppression factors in
eq.(20) which are not affected by neutrinos travelling through the earth (because of its
low density) will smooth out the regeneration. Therefore we expect the excluded region
to shrink.

When $\phi = 0.1$rad (Fig.3), we find no significant change in parameter space. This case
would be interesting if the reported 1% mixing of 17keV neutrino in $\beta$ decay turn out to
be true (then the 17keV neutrino would serve as the heaviest neutrino species).\(^{23}\) At large
$\phi$, Fig.4 shows that the allowed parameter space is shifted, but we still find the same
situation with respect to the vertical and diagonal regions as in $\phi = 0$ case.

The ongoing $^{71}$Ga experiments have a very low threshold 0.233MeV. Thus, they will
bring more severe constraints on the parameter space. As mentioned earlier, at present
SAGE gives a preliminary low result.\(^6\) This would suggest that the central diagonal region
would be the final answer if the MSW effect is the solution to the solar neutrino problem.

In standard Solar Models, the $^8$B neutrino flux is very sensitive to the temperature
at the center of the Sun ($\propto T_c^{18}$) whereas the other neutrino sources are less sensitive.\(^1,24\)
Therefore the theoretical expected fluxes calculated for current $^8$B dominated solar neutrino experiments depend drastically on what $T_c$ is calculated in the Solar Model. This
gives rise to greater astrophysical uncertainty for the determination of neutrino parameter
space. To try to estimate the sensitivity to the uncertainty we have calculated the allowed
parameter spaces for different central temperatures. If we assume the central temperature
is only slightly varied, the predicted solar $^8$B neutrino flux would be changed by some factor
while other neutrino fluxes are almost unaffected. Fig.5(a) and (b) show the overlap of
two experimental results under different solar $^8$B fluxes at 1σ error. We see that a change
of the solar $^8$B flux by a full factor of 2 will only shift the overlaped region back and forth
without inducing any new allowed parameter space (after the completion of this work, we
notice that Smirnov also calculated the allowed parameter space for different $T_c$, but at 2σ
level\(^{25}\)). We also find that at 1σ error, in order for the neutrino to have any allowed MSW
parameter space, the predicted $^8$B neutrino flux for the $^{37}$Cl experiment must be higher
than $\sim 2.5$ SNU when $\phi = 0$ and $\sim 5$ SNU when $\phi = 0.5$rad. For the $^{71}$Ga experiments,
such changes in the the solar $^8$B neutrino flux produces negligible effect because Gallium
experiments are dominated by the $pp$ neutrino flux.

Based on Figures 2-5, we can conclude that if $^{71}$Ga does give a very low flux, $\Delta = m_2^2 - m_1^2$ will lie in $10^{-7}$ to $2 \times 10^{-6}$eV$^2$, and $\sin^2 2\theta$ will be 0.01-0.3, which is naturally in
agreement with Bahcall and Bethe's prediction$^{26}$ (using only two neutrino families and a
similar analysis of the $^{37}$Cl and Kamiokande implications) and consistent with the Cabibbo
angle $\theta_c$ ($\sin^2 2\theta_c = 0.2$). This corresponds to a $\nu_\mu$ mass of $\sim 10^{-3}$eV, which is more
than $10^3$ times larger than that of some SUSY see-saw models, and $\sim 5$ times smaller
than the lowest $m_{\nu_\mu}$ value in the SO(10) see-saw model, both of which are discussed by
Bludman et al.$^{27}$ Thus our $m_{\nu_\mu}$ is within the admittedly large theoretically predicted range.
Generally, if we take the see-saw inspired relationship $m_{\nu_\tau}/m_{\nu_\mu} = m^2_t/m^2_e \times C(C \sim O(1)$
but larger than 1)$^{27}$, using $m_e = 1.55$GeV, $m_t = 124 \pm 34$GeV,$^{28}$ we obtain $m_{\nu_\tau} \sim 10$eV.
This would make $\nu_\tau$ a natural candidate for the Hot Dark Matter (see also discussion in
ref. 29). Similarly, we obtain from the see-saw model $m_{\nu_e} \sim 10^{-8}$eV. These numbers are
consistent with the current experimental bounds on the neutrino masses and the neutrino
oscillation parameters (Fig.6).$^{30,31}$

IV. Conclusion:

After taking into account the three-neutrino matter mixing under the assumption
that $m_3 \gg m_2 > m_1$, we find that for various values of the second mixing angle $\phi$,
the neutrino mixing parameters most probably lie in the diagonal region of parameter
space $\Delta vs. \sin^2 2\theta/cos2\theta$. The vertical region is less probable but hasn't been ruled out at
$3\sigma$ level (except those parts ruled out by the lack of a day-night effect). The horizontal
region of the parameter space is excluded by current experiments. The Gallium experiment
may give us a final resolution between the diagonal area and vertical area. Changing the
high energy $^8$B neutrino flux will shift the allowed parameter space, but will not change
the shape of allowed space significantly. The basic robustness of two-flavor MSW mixing
results is not weakened by taking into account the full three neutrino flavors.

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NASA/Fermilab Astrophysics Center.
Reference:

6 John Wilkerson, talk at PASCOS, Boston 1991.


Figure Captions:

Fig.2-4
The doted region is allowed by the $^{37}$Cl experiment; the crossed region is allowed by the Kamiokande II. Percentages at the lower left show expected flux for the $^{71}$Ga experiment relative to SSM.
2(a) $\phi=0$, 1$\sigma$ error. 2(b) $\phi=0$, 3$\sigma$ error.
3(a) $\phi=0.1$, 1$\sigma$ error. 3(b) $\phi=0.1$, 3$\sigma$ error.
4(a) $\phi=0.5$, 1$\sigma$ error. 4(b) $\phi=0.5$, 3$\sigma$ error.

Fig.5
This figure shows the parameter space allowed by both the $^{37}$Cl experiment and Kamiokande II with different predicted solar $^8$B neutrino fluxes for the $^{37}$Cl experiment assumed.
5(a) $\phi = 0$, 1$\sigma$ error. 5(b) $\phi = 0.5$, 1$\sigma$ error.

Fig.6
The current experimental constraints on neutrino oscillation parameters. [1] $\nu_{\mu} \rightarrow \nu_e$; [2] $\nu_{\mu} \leftrightarrow \nu_{\tau}$; [3] $\nu_{e} \leftrightarrow \nu_{\tau}$. [4] $\bar{\nu}_e \rightarrow \nu_{\tau}$; [1], [2] and [3] are from accelerator experiments, [4] is from reactor experiments. The left side of the lines is excluded at 90% C. L..
Fig. 2(a)
Fig. 2(b)
Fig. 3(a)
Fig. 3(b)
Fig. 4(b)
Fig. 5(a)
Fig. 5(b)
Fig. 6

\[ \log(\Delta m^2 / eV^2) \]

\[ \log(\sin^2 2\theta) \]