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A CRITICAL COMPARISON OF TWO-EQUATION TURBULENCE MODELS

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ABSTRACT

Several two-equation models have been proposed and tested against benchmark flows by various researchers. For each study, different numerical methods or codes were used to obtain the results which were reported to be an improvement over other models. However, these comparisons may be overshadowed by the different numerical schemes used to obtain the results. With this in mind, several existing two-equation turbulence models, including $k-\epsilon$, $k-\tau$, $k-\omega$ and $q-\omega$ models, are implemented into a common flow solver code for near wall turbulent flows. Calculations were carried out for low Reynolds number, two-dimensional, fully developed channel and boundary layer flows. The quality of each model is based on several criterion including robustness and accuracy of predicting the turbulent quantities.

1. INTRODUCTION

The time averaged Navier-Stokes equations have more unknowns than the number of equations. In order to solve this closure problem, it is necessary to model the turbulent stress tensor, $\overline{u_i u_j}$, which appears in the time averaged momentum equation. Many semi-empirical models have been proposed, each with its own successes and flaws. The two-equation model is one of the more popular approaches. In this model, one equation related to the turbulent kinetic energy and one related to the turbulence length scale are solved along with the time averaged Navier-Stokes equations.

This paper summarizes two-equation turbulence models (including recently developed models) and compares the robustness and accuracy of different models which have appeared in the literature. For each model, calculations were carried out for two-dimensional, fully developed channel and flat plate boundary layer flows. These flows are appealing for model testing because they are simple and self-similar, yet demonstrate important features of wall bounded turbulent shear flows. In addition, we can compare the results from these calculations with Direct Numerical Simulations (DNS).

There were four types of two-equation models tested in this study:
1) $k-\epsilon$
2) $k-\omega$
3) $q-\omega$
4) $k-\tau$
where,

k = Turbulent Kinetic Energy,
q = $\sqrt{k}$,
ϵ = Dissipation Rate,
$\omega \propto \epsilon/k$ = Specific Dissipation Rate,
$\tau \propto k/\epsilon$ = Turbulent Time Scale

A list of the models which were tested are shown in the table below:

Ch	Chien[1] 1982	$k-\epsilon$
Sh	Shih[11] 1991	$k-\epsilon$
LB	Lam and Bremhorst[6] 1981	$k-\epsilon$
NH	Nagano and Hishida[9] 1987	$k-\epsilon$
NT	Nagano and Tagawa[10] 1990	$k-\epsilon$
LS	Launder and Sharma[7] 1974	$k-\epsilon$
JL	Jones and Launder[3] 1973	$k-\epsilon$
MK	Myong and Kasagi[8] 1988	$k-\epsilon$
YS	Yang and Shih[17] 1991	$k-\epsilon$
WI1	Wilcox[15] 1984	$k-\omega$
WI2	Wilcox[16] 1991	$k-\omega$
SAA	Speziale, Abid and Anderson[14] 1990	$k-\tau$
Co	Coakley[2] 1983	$q-\omega$

The time averaged momentum and continuity equations are written as:

$$\frac{\partial U_i}{\partial x_i} = 0 \tag{1}$$

$$\frac{\mathrm{D}U_i}{\mathrm{D}t} = \frac{\partial}{\partial x_i}(\nu \frac{\partial U_i}{\partial x_i}) - \frac{\partial \overline{u_i u_j}}{\partial x_j} - \frac{1}{\rho}\frac{\partial \overline{p}}{\partial x_i} \tag{2}$$

where the Reynolds stress is modeled as:

$$-\overline{u_i u_j} = \nu_T(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}) - \frac{2}{3}k\delta_{ij} \ . \tag{3}$$

From dimensional analysis, the eddy viscosity is:

$$\nu_T = cu'l'$$

where u' and l' are the turbulent velocity scale and turbulent length scale.

2. THE MODEL EQUATIONS

In a two-equation model, two turbulent quantities($k-\epsilon$, $k-\tau$, $k-\omega$ or $q-\omega$) are used to model the eddy viscosity. The turbulent transport equations of these quantities and the

eddy viscosity models are written below. The model constants and other model parameters may be found in the appendix.

2.1 The k-ϵ Model

$$\nu_T = \text{(See Table 2)} \tag{4a}$$

$$\frac{Dk}{Dt} = \frac{\partial}{\partial x_i}[(\nu + \frac{\nu_T}{\sigma_k})\frac{\partial k}{\partial x_i}] + \Pi - \overline{u_i u_j}\frac{\partial U_i}{\partial x_j} - \epsilon + D \tag{5a}$$

$$\frac{D\epsilon}{Dt} = \frac{\partial}{\partial x_i}[(\nu + \frac{\nu_T}{\sigma_\epsilon})\frac{\partial \epsilon}{\partial x_i}] - C_1 f_1 \frac{1}{T_t}\overline{u_i u_j}\frac{\partial U_i}{\partial x_j} - C_2 f_2 \frac{\tilde{\epsilon}}{T_t} + E \tag{6a}$$

Wall BC: $k = U = 0, \quad \epsilon =$ see table 2

2.2 The k-τ Model

$$\tau = \frac{k}{\epsilon}$$

$$\nu_T = C_\mu f_\mu k \tau \tag{4b}$$

$$\frac{Dk}{Dt} = \frac{\partial}{\partial x_i}[(\nu + \frac{\nu_T}{\sigma_k})\frac{\partial k}{\partial x_i}] - \overline{u_i u_j}\frac{\partial U_i}{\partial x_j} - \frac{k}{\tau} \tag{5b}$$

$$\begin{aligned}\frac{D\tau}{Dt} = &\frac{\partial}{\partial x_i}[(\nu + \frac{\nu_T}{\sigma_{\tau 2}})\frac{\partial \tau}{\partial x_i}] - \frac{2}{\tau}(\nu + \frac{\nu_T}{\sigma_{\tau 2}})\frac{\partial \tau}{\partial x_i}\frac{\partial \tau}{\partial x_i} \\ &+ \frac{2}{k}(\nu + \frac{\nu_T}{\sigma_{\tau 1}})\frac{\partial k}{\partial x_i}\frac{\partial \tau}{\partial x_i} + (C_1 - 1)\frac{\tau}{k}\overline{u_i u_j}\frac{\partial U_i}{\partial x_j} + (C_2 f_2 - 1)\end{aligned} \tag{6b}$$

Wall BC: $k = U = \tau = 0$

2.3 The k-ω Model

$$\omega = \frac{\epsilon}{C_\mu k}$$

$$\nu_T = \frac{k}{\omega} \tag{4c}$$

$$\frac{Dk}{Dt} = \frac{\partial}{\partial x_i}[(\nu + \frac{\nu_T}{\sigma_k})\frac{\partial k}{\partial x_i}] - \overline{u_i u_j}\frac{\partial U_i}{\partial x_j} - C_\mu k\omega \tag{5c}$$

$$\frac{D\omega}{Dt} = \frac{\partial}{\partial x_i}[(\nu + \frac{\nu_T}{\sigma_\omega})\frac{\partial \omega}{\partial x_i}] - C_1 \frac{\omega}{k}\overline{u_i u_j}\frac{\partial U_i}{\partial x_j} - C_2\omega^2 - C_2 C_3 \omega (2\Omega_{i,j}\Omega_{j,i})^{\frac{1}{2}} \tag{6c}$$

$$\Omega_{i,j} = \frac{1}{2}(U_{i,j} - U_{j,i})$$

Wall BC: $k = U = 0, \quad \omega \to \frac{6\nu}{C_2 y^2}$

2.4 The q-ω Model

$$\omega = \frac{\epsilon}{k}, \quad q = \sqrt{k}$$

$$\nu_T = C_\mu f_\mu \frac{q^2}{\omega} \tag{4d}$$

$$\frac{Dq}{Dt} = \frac{\partial}{\partial x_i}[(\nu + \frac{\nu_T}{\sigma_q})\frac{\partial q}{\partial x_i}] - \frac{\overline{u_i u_j}}{2q}\frac{\partial U_i}{\partial x_j} - \frac{q\omega}{2} \tag{5d}$$

$$\frac{D\omega}{Dt} = \frac{\partial}{\partial x_i}[(\nu + \frac{\nu_T}{\sigma_\omega})\frac{\partial \omega}{\partial x_i}] - C_1 C_\mu(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i})\frac{\partial U_i}{\partial x_j} - C_2\omega^2 \tag{6d}$$

Wall BC: $k = U = 0, \quad \frac{\partial \omega}{\partial y} = 0$

3. MODEL TESTING

The momentum, continuity, Reynolds stress, eddy viscosity and turbulent transport equations (Equations 1-6) are solved simultaneously in a numerical code. The numerical scheme is based on GENMIX, a parabolic code developed by Patanker and Spaulding[13]. The turbulent transport equations and momentum equation are solved by a space marching finite difference method obtained by integrating over control volumes.

Two dimensional channel flow calculations were made at $Re_\tau = 180$ and $Re_\tau = 395$. These cases were compared with DNS data of Kim et al[4]. Calculations for the two- dimensional flat plate boundary layer flow at $Re_\theta = 1410$ were compared with DNS data of Spalart[12]. Some flat plate boundary layer comparisons were made between experimental data of Klebanoff[5] at $Re_\theta = 7700$ and solutions of various models.

Results from channel flow at $Re_\tau = 180$ and $Re_\tau = 395$ appear in Figures 1-6 and Figures 7-12, respectively. Results from flat plate boundary layer flow appear in Figures 13-24.

An important criterion for two-equation model comparisons is not only how well the model predicts mean velocity and shear stress, but also the turbulent kinetic energy and dissipation (or specific dissipation) rate. These predictions should provide appropriate turbulent velocity and length scales so that the model can be applied to more complex flows for which a simple mixing length model often fails. Some researchers maintain that it is not critical that the turbulent kinetic energy and the turbulent length scale are predicted with great accuracy. However, one may imagine that a two-equation model making unreasonable turbulent velocity and length scale predictions would be very questionable when applied to more general flows. A model which accurately predicts the shear stress and mean velocity does not imply that it has correctly modeled the turbulent kinetic energy and length scale. In fact, if the turbulent kinetic energy is incorrect, then the length scale must also be incorrect to compensate for the error in the turbulent kinetic energy. For this case, two wrongs are making a right. This warrants some caution when computing flows for other geometries.

The comparisons made in this study are only for rather simple flows. However, We think they are important. Because if a model does not correctly predict a simple flow, it cannot, in general, be expected to correctly model a more complicated flow. Although the comparisons may be highly subjective, it is clear that the JL, LS, WI1 and WI2 models underpredict the near wall turbulent kinetic energy compared to the other models.

The standard $k-\epsilon$ model has been proven to provide good results in the high Reynolds number range. It is therefore attractive for a near wall $k-\epsilon$ turbulence model to approach the standard $k-\epsilon$ model away from the wall. The LB, LS and YS models are the only $k-\epsilon$ models in this study which possess this characteristic.

Because the boundary layer and channel flows are self-similar, the solutions should be independent of the initial conditions. However, some of the models (SAA, Co, and LB) have difficulty when the initial conditions contain large gradients. The Co Model is particularly dependent on the initial conditions. Even slight perturbations to the initial conditions will yield noticeably different solutions with this model.

JL, LS, WI1 and WI2 are the only models which do not contain y^+. Damping functions which contain y^+ are not desirable because y^+ is erroneous near flow separations and not well defined near complicated geometries. Unfortunately, these are the same models which poorly predict the near wall turbulent quantities.

The Wilcox models (WI1 and WI2) suffer from a numerically awkward boundary condition for ω at the wall:

$$\omega \rightarrow \frac{6\nu}{C_2 y^2} \text{ as } y^+ \rightarrow 0$$

We cannot define ω at $y^+ = 0$. We have tried two ways to approximate ω as y^+ approaches 0. First, we chose a large number for ω_{wall} and, second, we used an asymptotic $\omega_{wall} = \frac{6\nu}{C_2 y^2}$. Test cases showed that the solution does not converge as $\omega_{wall} \rightarrow \infty$ or $y^+ \rightarrow 0$ for either model. In addition, both Wilcox models underpredict the turbulent kinetic energy peak value for both boundary layer and channel flows.

4. CONCLUSION

In our calculations, $k-\epsilon$ models such as Ch, NT, Sh and YS were robust and also gave the best predictions of overall turbulent quantities. However they all contain an undesirable y^+ in their damping function.

To explore the capabilities as well as the deficiencies of these models, further testing of these models in more complex flows, such as, flows with adverse pressure gradients is needed.

APPENDIX: Model Parameters and Damping Functions

Table 1 : $k-\epsilon$ Model Parameters

Model	Π	D	E
Ch	0	$-\frac{2\nu k}{y^2}$	$-\frac{2\nu\epsilon}{y^2}exp(-.5y^+)$
Sh	$[\frac{.05}{f_\mu[1-exp(-y^+)]}\frac{\nu_T}{\sigma_k}k_{,j}]_{,j}$	0	$\nu\nu_T(\frac{\partial^2 U}{\partial y^2})^2$
LB	0	0	0
NH	0	$-2\nu(\frac{\partial\sqrt{k}}{\partial y})^2$	$\nu\nu_T(1-f_\mu)(\frac{\partial^2 U}{\partial y^2})^2$
NT	0	0	0
JL	0	$-2\nu(\frac{\partial\sqrt{k}}{\partial y})^2$	$2\nu\nu_T(\frac{\partial^2 U}{\partial y^2})^2$
LS	0	$-2\nu(\frac{\partial\sqrt{k}}{\partial y})$	$2\nu\nu_T(\frac{\partial^2 U}{\partial y^2})^2$
MK	0	0	0
YS	0	0	$\nu\nu_T(\frac{\partial^2 U}{\partial y^2})^2$

Table 2 : $k-\epsilon$ Model Parameters

Model	$\tilde{\epsilon}$	T_t	ν_T	$BC\epsilon_w$
Ch	ϵ	$\frac{k}{\epsilon}$	$C_\mu f_\mu\frac{k^2}{\tilde{\epsilon}}$	0
Sh	$\epsilon-\nu\frac{k_{,i}k_{,i}}{2k}$	$\frac{k}{\epsilon}$	$C_\mu f_\mu\frac{k^2}{\tilde{\epsilon}}$	$\nu\frac{\partial^2 k}{\partial y^2}$
LB	ϵ	$\frac{k}{\epsilon}$	$C_\mu f_\mu\frac{k^2}{\tilde{\epsilon}}$	$\nu\frac{\partial^2 k}{\partial y^2}$
NH	ϵ	$\frac{k}{\epsilon}$	$C_\mu f_\mu\frac{k^2}{\tilde{\epsilon}}$	0
NT	ϵ	$\frac{k}{\epsilon}$	$C_\mu f_\mu\frac{k^2}{\tilde{\epsilon}}$	$\nu\frac{\partial^2 k}{\partial y^2}$
JL	ϵ	$\frac{k}{\epsilon}$	$C_\mu f_\mu\frac{k^2}{\tilde{\epsilon}}$	0
LS	ϵ	$\frac{k}{\epsilon}$	$C_\mu f_\mu\frac{k^2}{\tilde{\epsilon}}$	0
MK	ϵ	$\frac{k}{\epsilon}$	$C_\mu f_\mu\frac{k^2}{\tilde{\epsilon}}$	$\nu\frac{\partial^2 k}{\partial y^2}$
YS	ϵ	$\frac{k}{\epsilon}+(\frac{\nu}{\epsilon})^{1/2}$	$C_\mu f_\mu k T_t$	$2\nu(\frac{\partial\sqrt{k}}{\partial y})^2$

Table 3 : $k-\epsilon$ Model Parameters

Model	C_μ	C_1	C_2	σ_k	σ_ϵ
Ch	.09	1.35	1.8	1.0	1.3
Sh	.09	1.45	2.0	1.3	1.3
LB	.09	1.44	1.92	1.0	1.3
NH	.09	1.45	1.9	1.0	1.3
NT	.09	1.45	1.9	1.4	1.3
JL	.09	1.45	2.0	1.0	1.3
LS	.09	1.44	1.92	1.0	1.3
MK	.09	1.4	1.8	1.4	1.3
YS	.09	1.44	1.92	1.0	1.3

Table 4 : $k-\epsilon$ Damping Functions

Model	f_μ	f_1	f_2
Ch	$1-exp(-.0115y^+)$	1	$1-.22exp(-\frac{R_t^2}{36})$
Sh	$1-exp(-.006y^+-4e^{-4}y^{+^2}+2.5e^{-6}y^{+^3}-4e^{-9}y^{+4})$	1	$1-.22exp(-\frac{R_t^2}{36})$
LB	$(1-e^{-.0165R_k})^2(1+\frac{20.5}{R_t})$	$1+(\frac{.05}{f_\mu})^3$	$1-exp(-R_t^2)$
NH	$[1-exp(-\frac{y^+}{26.5})]^2$	1	$1-.3exp(-R_t^2)$
NT	$[1-exp(-\frac{y^+}{26})]^2(1+\frac{4.1}{R_t^{3/4}})$	1	$(1-.3exp(-(\frac{R_t}{6.5})^2))$ $(1-exp(\frac{-y^+}{6}))^2$
JL	$exp(\frac{-2.5}{1+R_t/50})$	1	$1-.3exp(-R_t^2)$
LS	$exp(\frac{-3.4}{(1+R_t/50)^2})$	1	$1-.3exp(-R_t^2)$
MK	$(1+\frac{3.45}{\sqrt{R_t}})(1-exp(-\frac{y^+}{70}))$	1	$(1-\frac{2}{9}exp(-\frac{R_t^2}{36}))$ $(1-exp(\frac{y^+}{5}))^2$
YS	$1-exp(-.004y^+-5e^{-5}y^{+^2}+2e^{-6}y^{+^3}-8e^{-8}y^{+^4})$	1	$1-.22exp(-\frac{R_t^2}{36})$

Table 5 : $k-\omega$ Model Parameters

Model	C_μ	C_1	C_2	C_3	σ_k	σ_ω
WI1	.09	$\frac{5}{9}$	$\frac{3}{40}$	0	2.0	2.0
WI2	.09	$\frac{1}{C_\mu}[C_2(1+\sqrt{C_\mu})-\frac{1}{\sigma_\omega}\kappa^2\sqrt{C_\mu}]$	$\frac{3}{40}$	1	2.0	2.0

Table 6 : $k-\tau$ Model Parameters

Model	σ_k	$\sigma_{\tau 1}$	$\sigma_{\tau 2}$	C_1	C_2	C_μ
SAA	1.36	1.36	1.36	1.44	$1.83[1-\frac{2}{9}exp(\frac{-Re_t^2}{36})]$	.09

Table 7 : $k-\tau$ Model Damping Functions

Model	f_2	f_μ
SAA	$1-exp(\frac{-y^+}{4.9})$	$(1+\frac{3.45}{\sqrt{R_t}})[1-exp(\frac{-y^+}{70})]$

Table 8 : $q-\omega$ Model Parameters

Model	σ_q	σ_ω	C_μ	C_1	C_2	f_μ
Co	1.0	1.3	.09	$.405f_\mu + .045$	.92	$1-exp(-.0065R_k)$

$R_k = \sqrt{k}\frac{y}{\nu}$, $y^+ = \frac{u_\tau y}{\nu}$ $R_t = \frac{k^2}{\nu\epsilon}$

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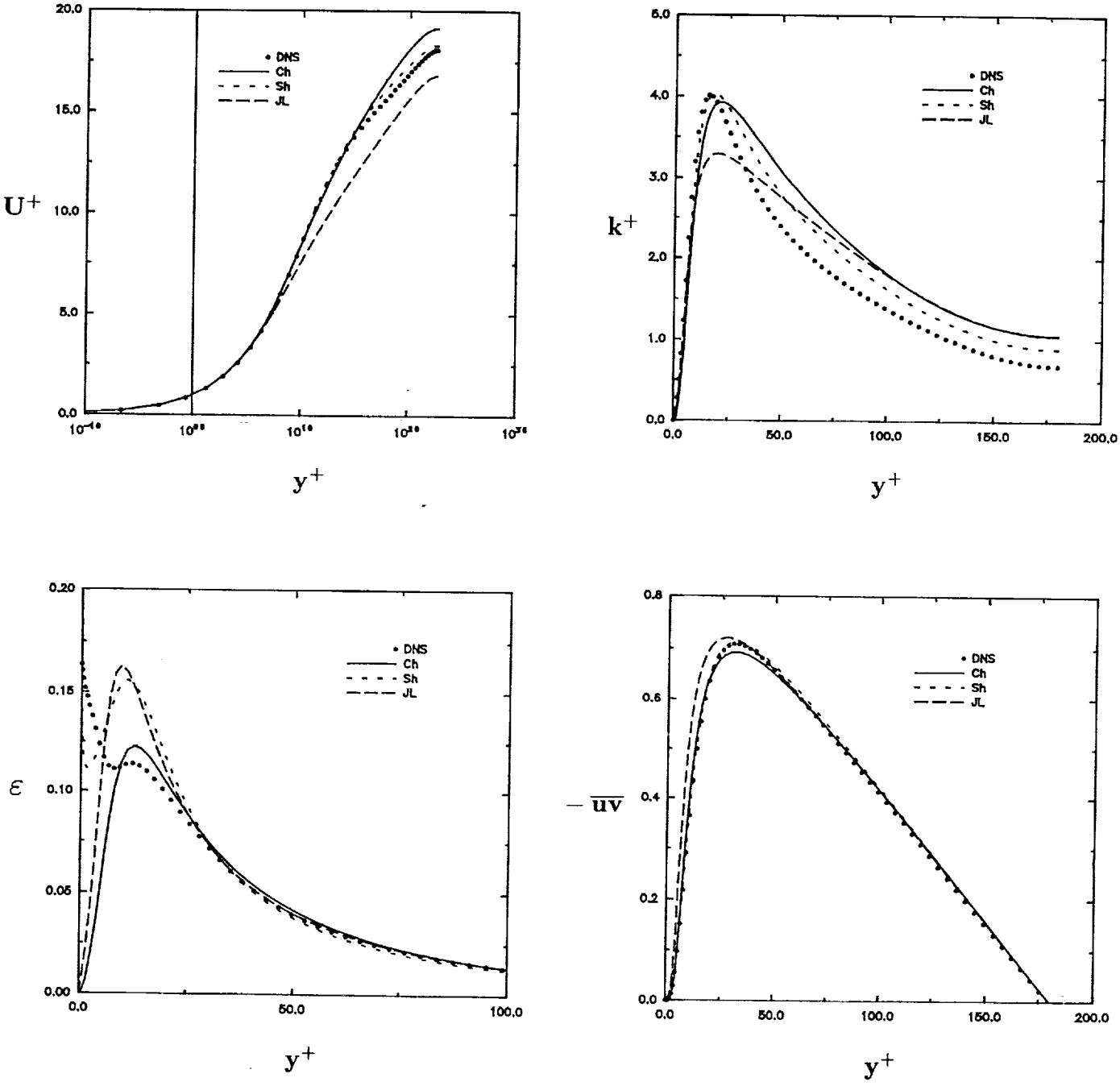


FIGURE 1
2-D Channel Flow $Re_\tau = 180$

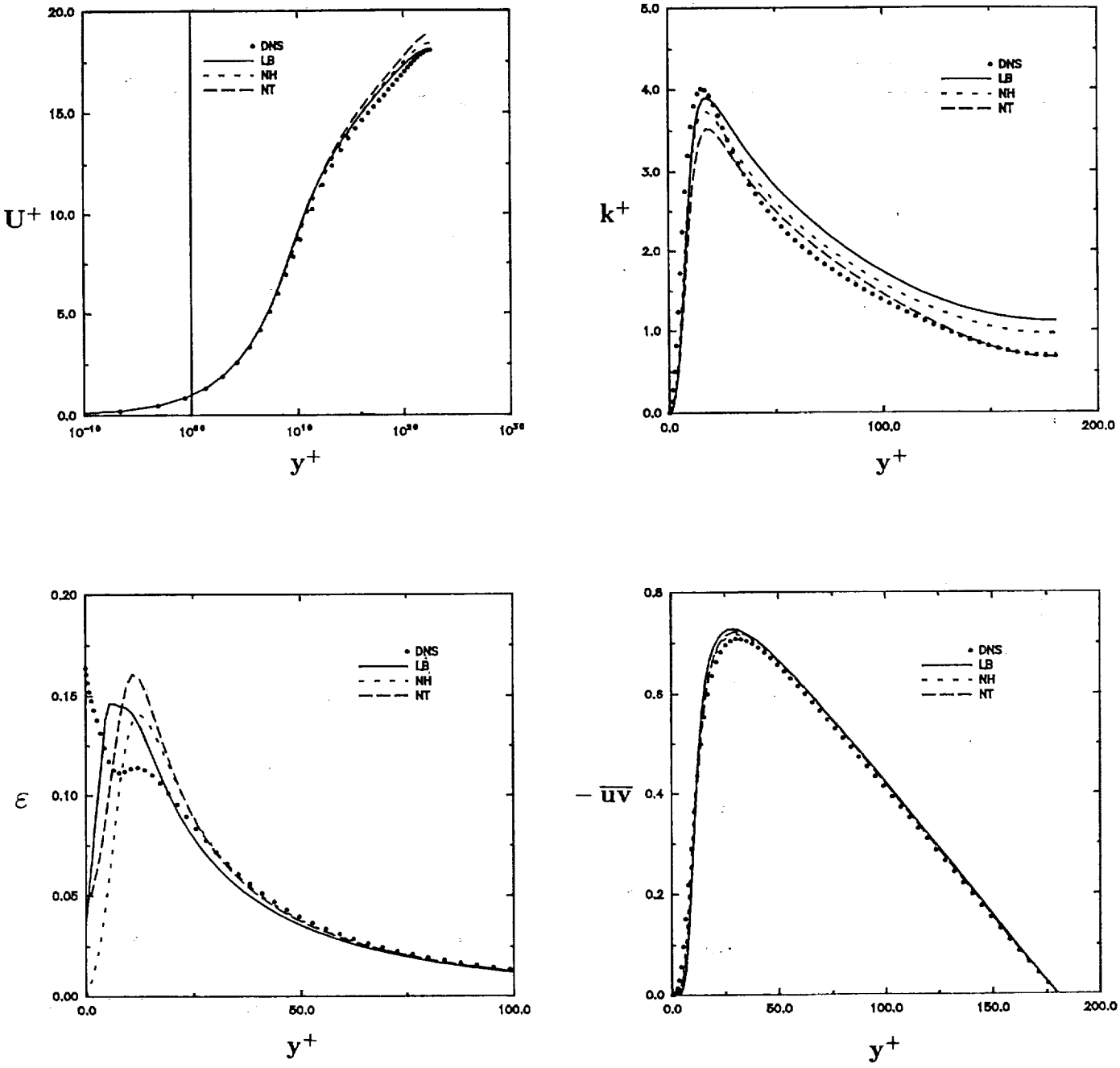


FIGURE 2
2-D Channel Flow $Re_\tau = 180$

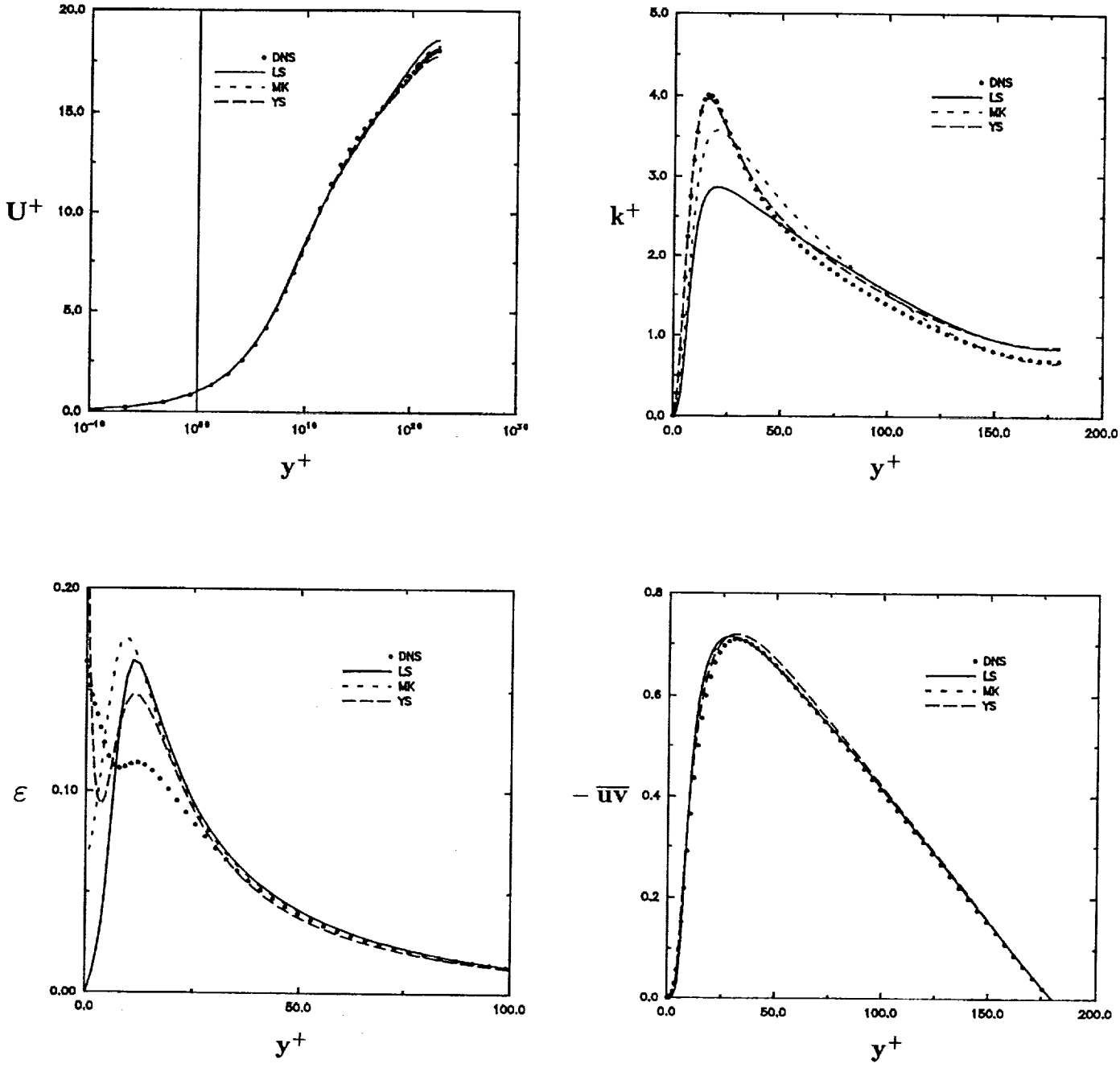


FIGURE 3
2-D Channel Flow $Re_\tau = 180$

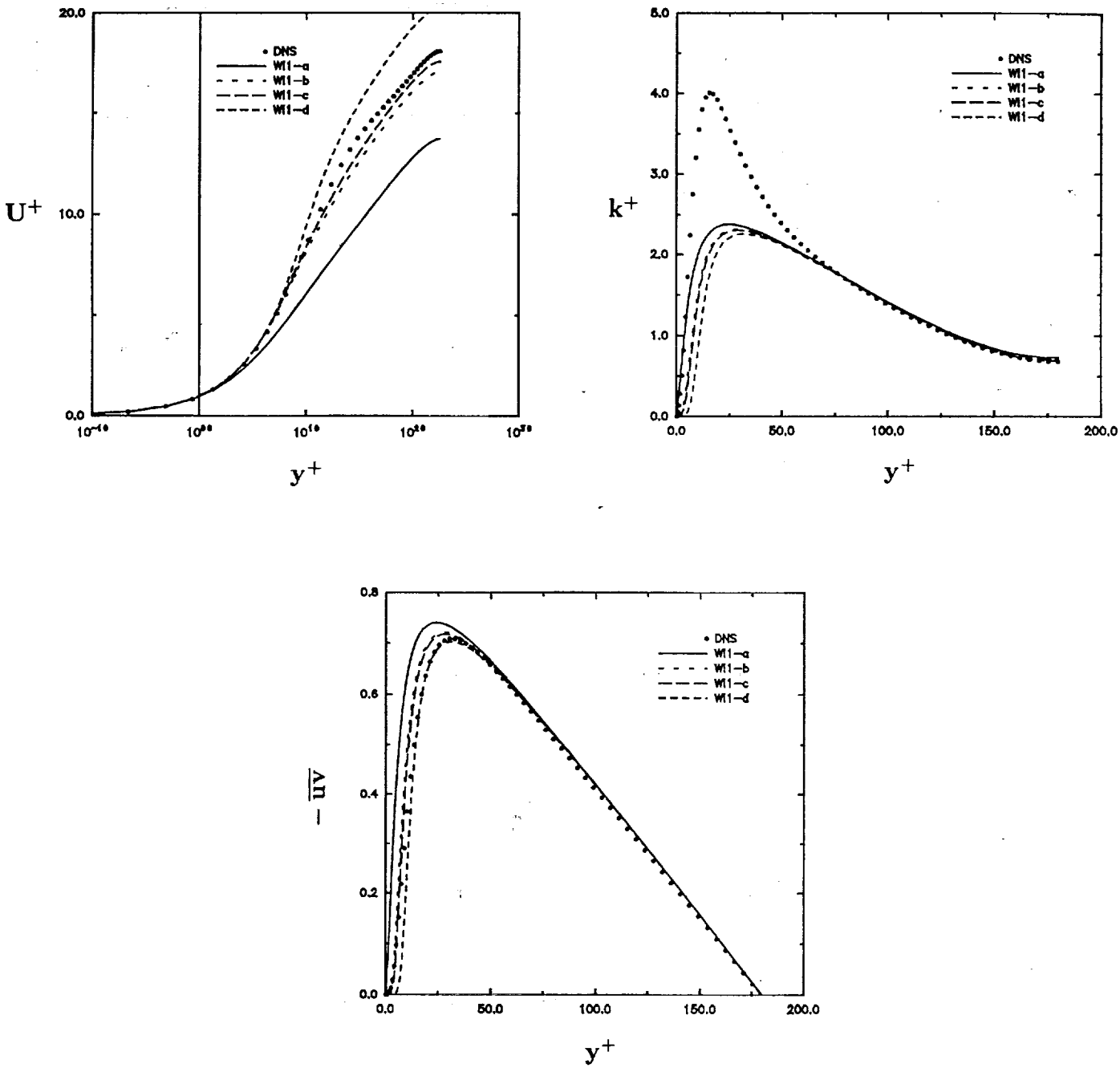


FIGURE 4
2-D Channel Flow $Re_\tau = 180$
a) $\omega_{wall} = 10^3$, b) $\omega_{wall} = 10^5$, c) $\omega_{wall} = 10^7$, d) $\omega_{wall} = 10^9$

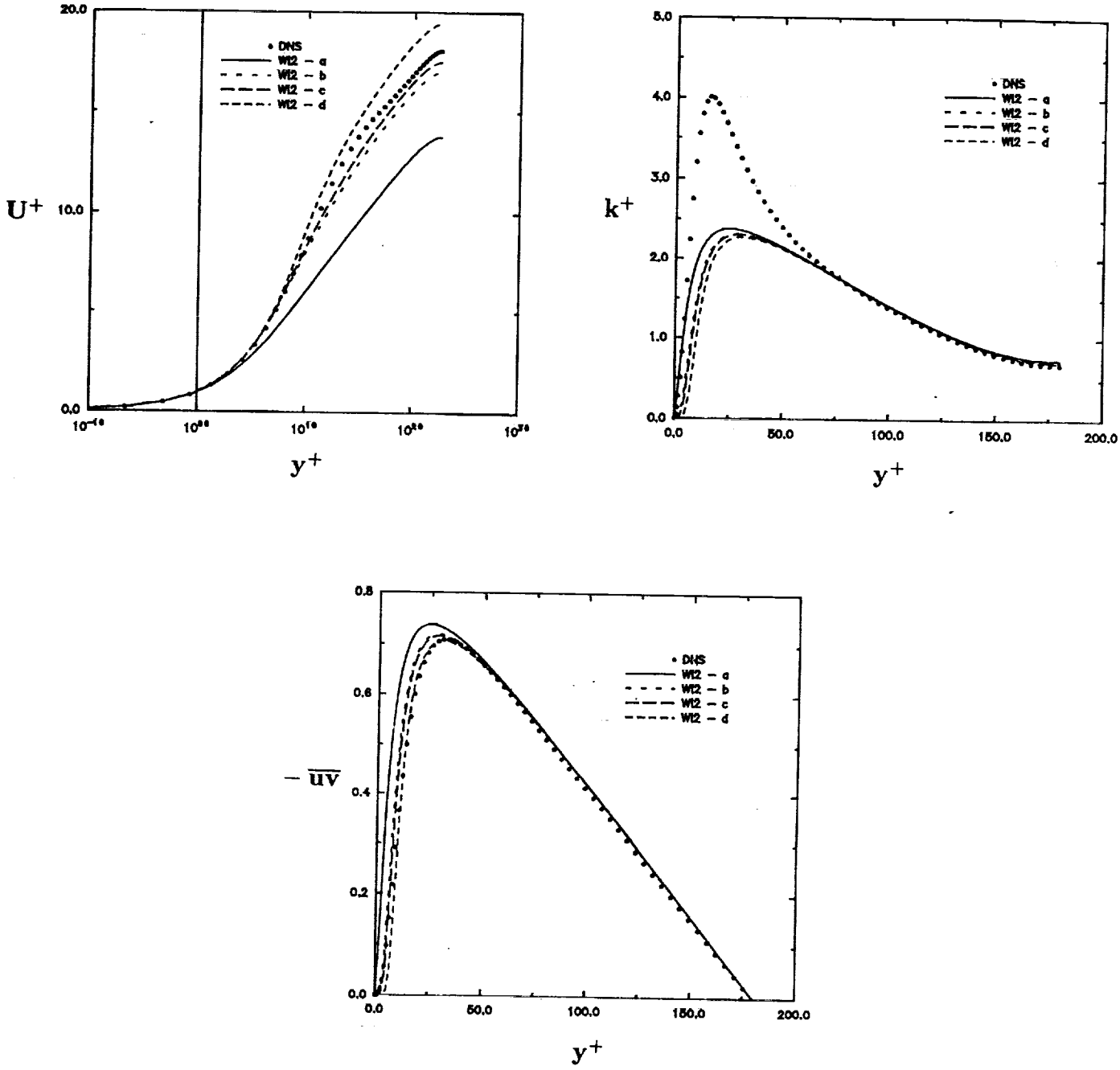


FIGURE 5
2-D Channel Flow $Re_\tau = 180$
a) $\omega_{wall} = 10^3$, b) $\omega_{wall} = 10^5$, c) $\omega_{wall} = 10^7$, d) $\omega_{wall} = 10^9$

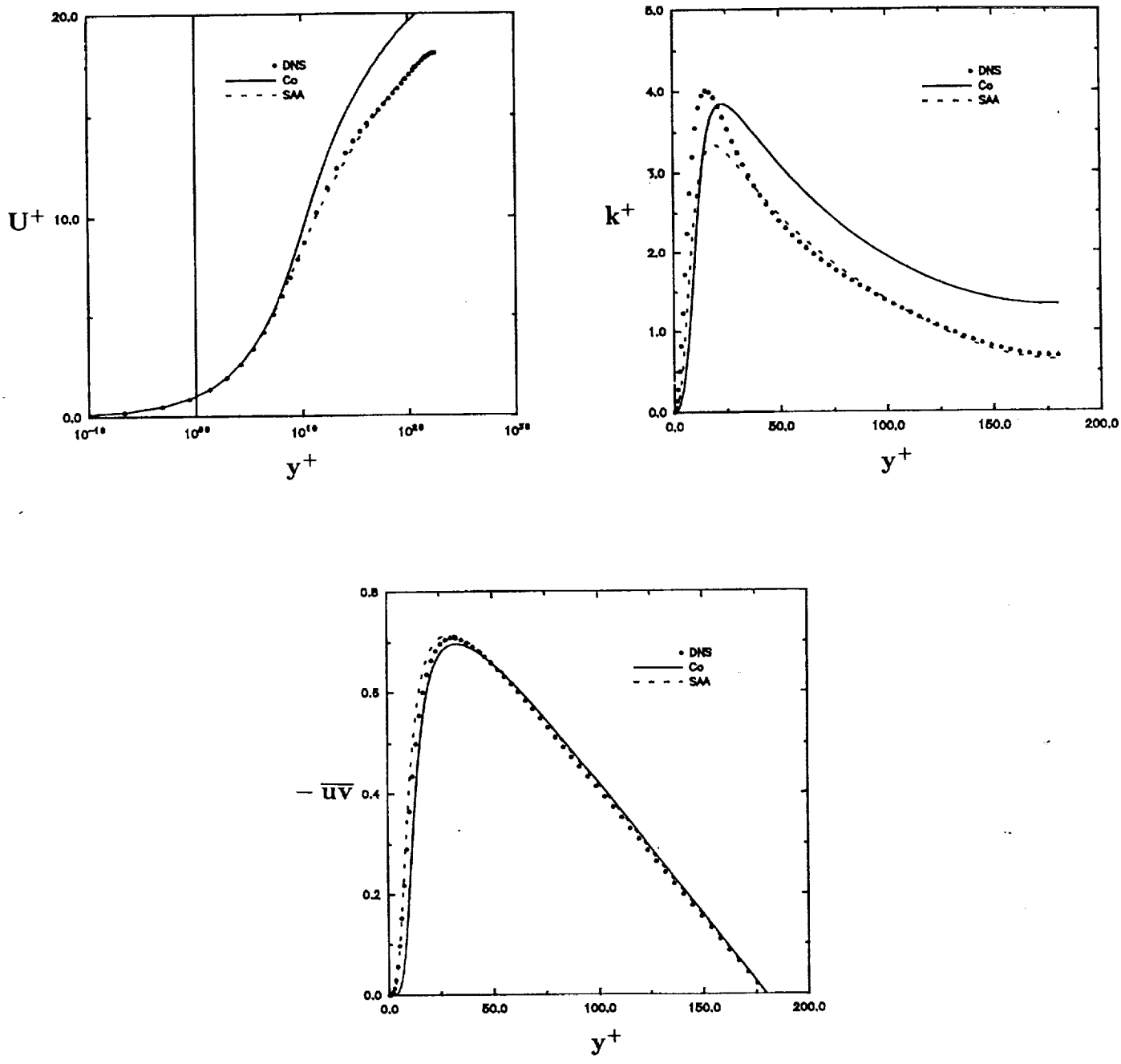


FIGURE 6
2-D Channel Flow $Re_\tau = 180$

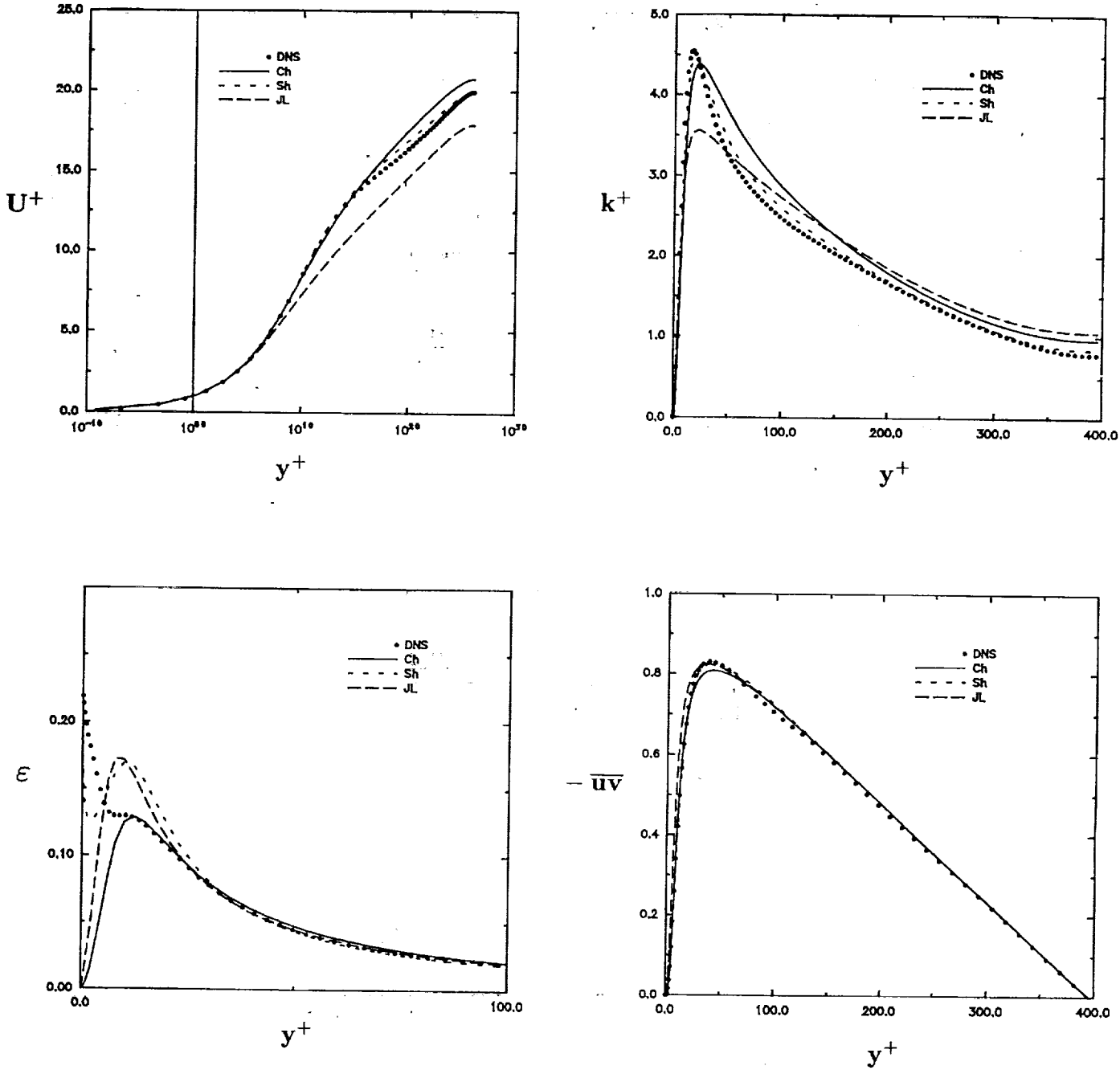


FIGURE 7
2-D Channel Flow $Re_\tau = 395$

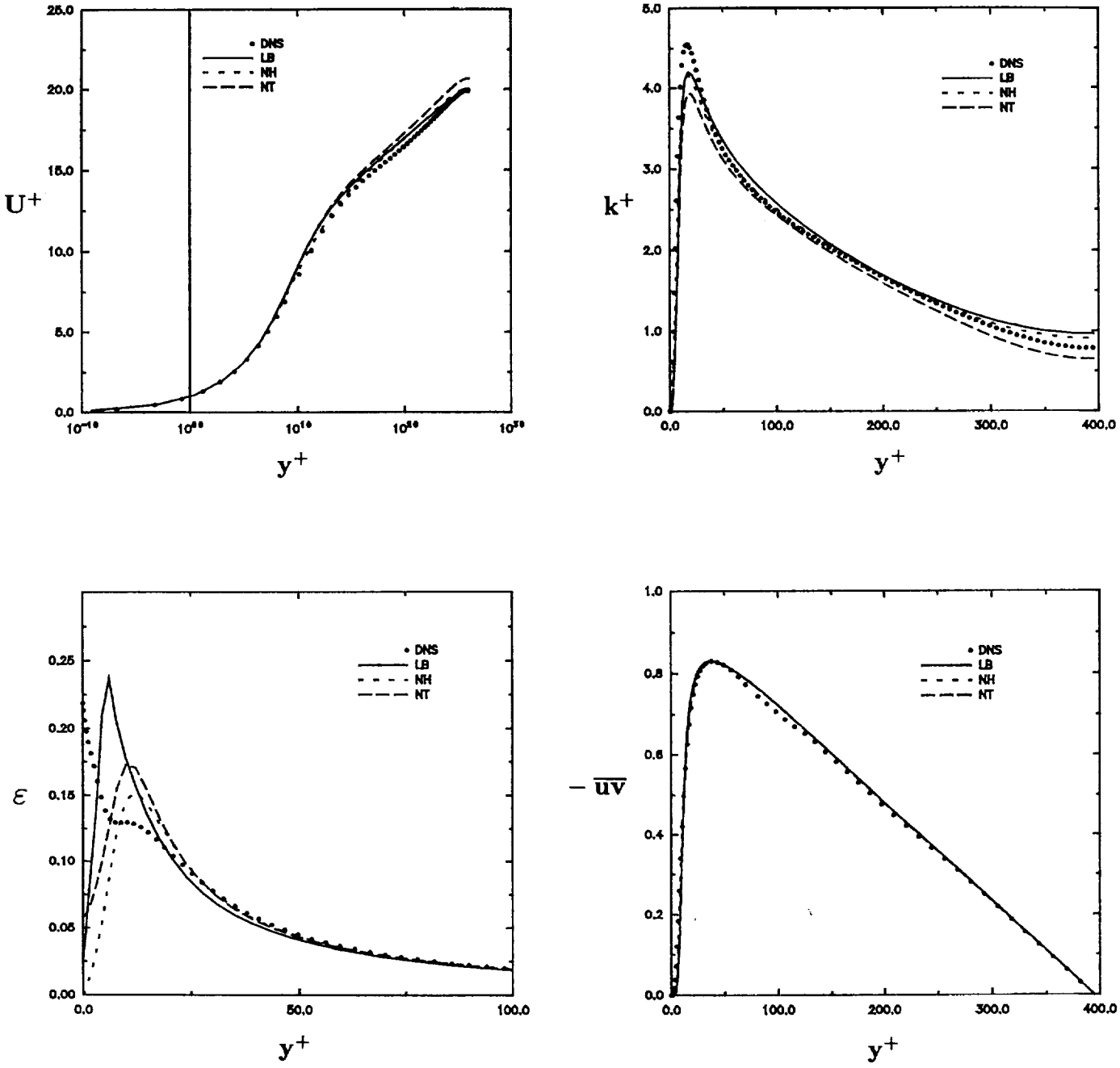


FIGURE 8
2-D Channel Flow $Re_\tau = 395$

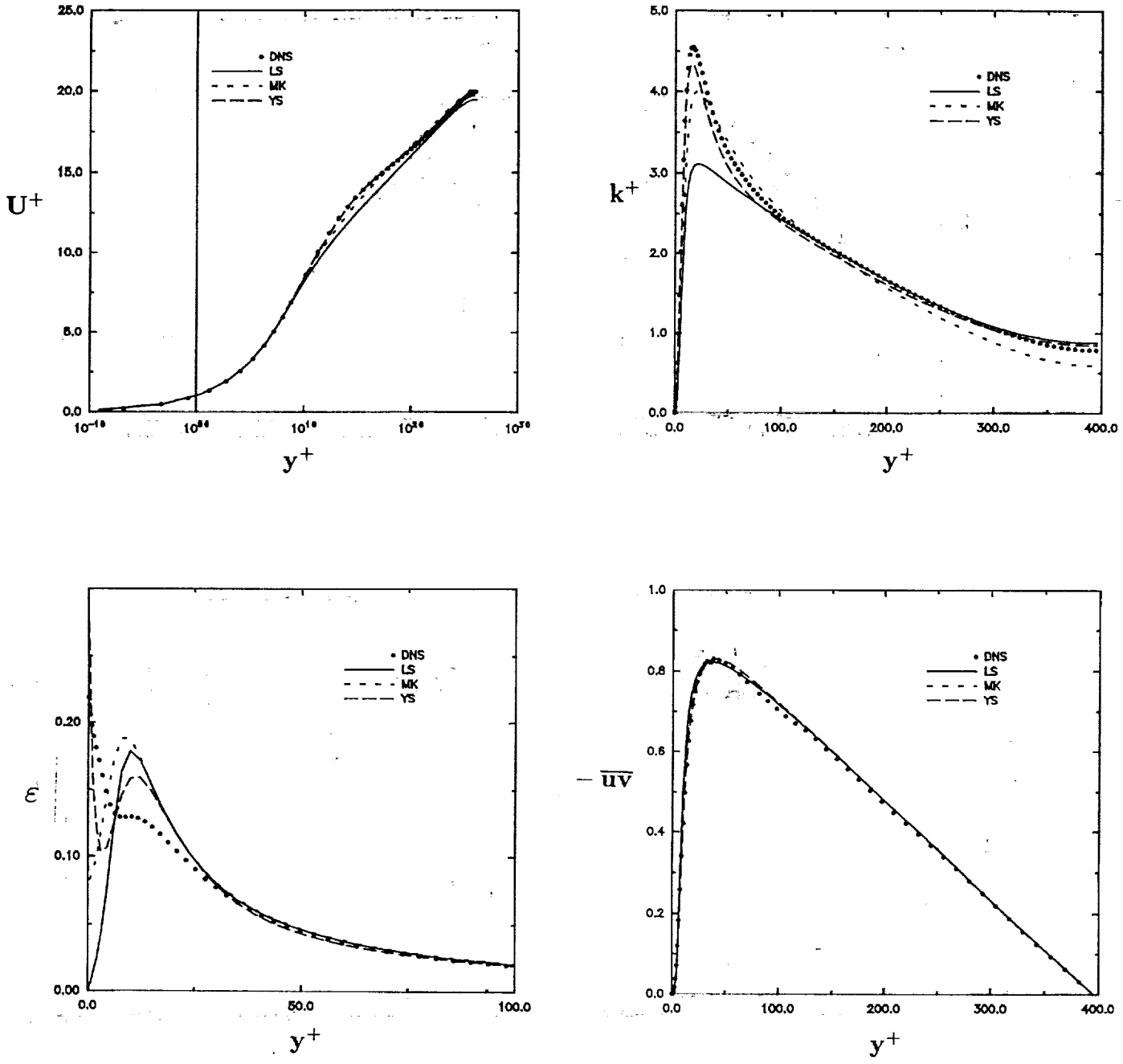


FIGURE 9
2-D Channel Flow $Re_\tau = 180$

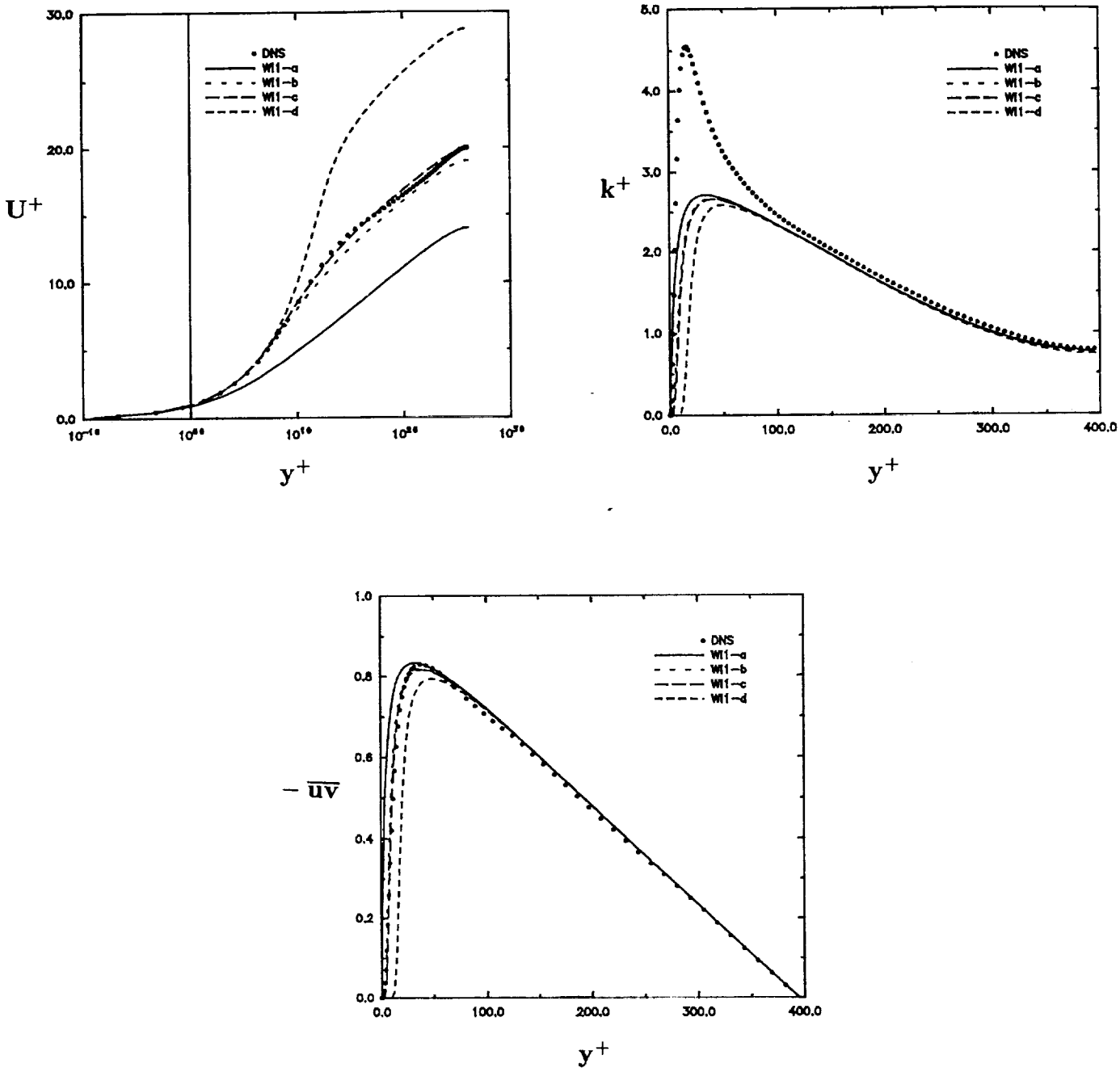


FIGURE 10
2-D Channel Flow $Re_\tau = 395$
a) $\omega_{wall} = 10^3$, b) $\omega_{wall} = 10^5$, c) $\omega_{wall} = 10^7$, d) $\omega_{wall} = 10^9$

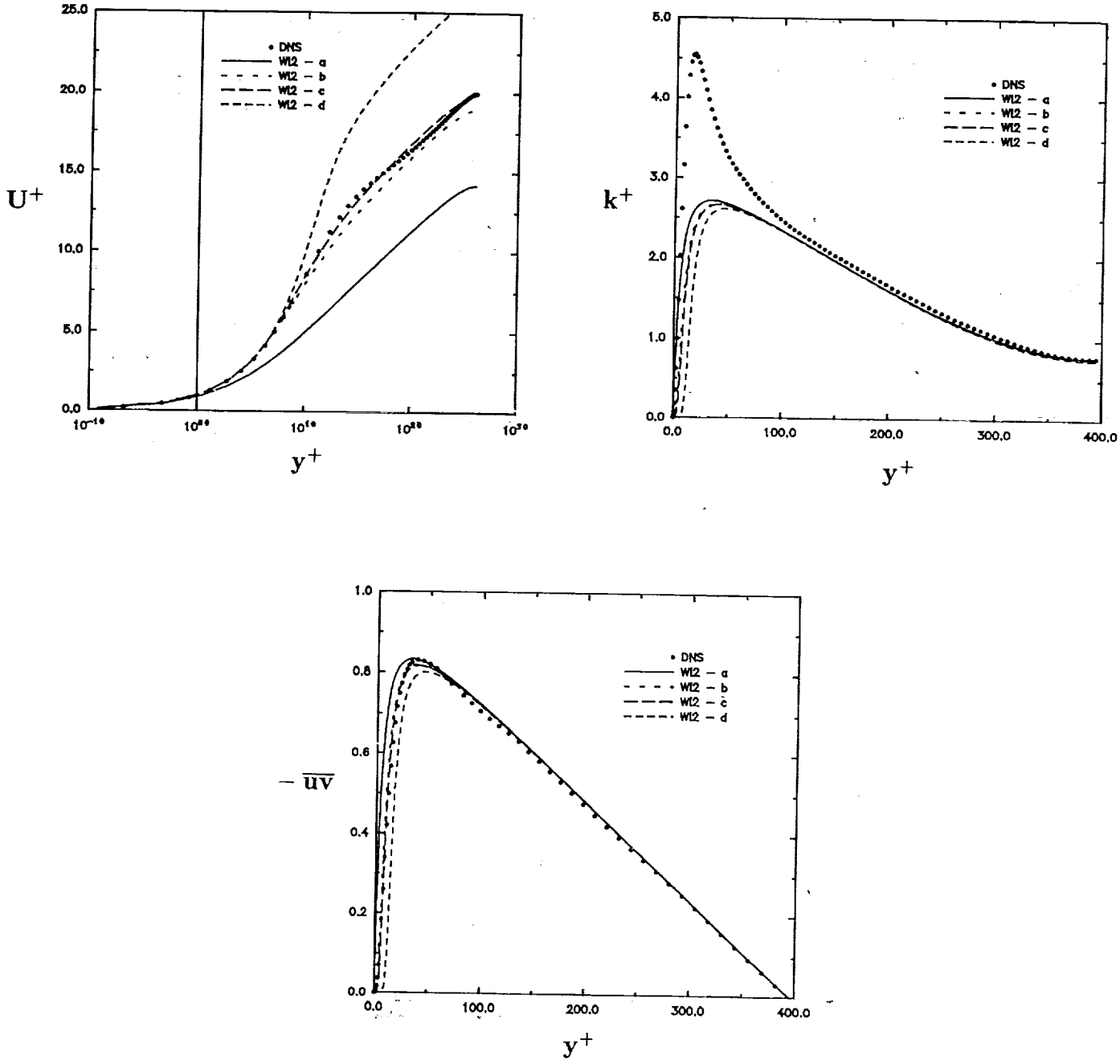


FIGURE 11
2-D Channel Flow $Re_\tau = 395$
a) $\omega_{wall} = 10^3$, b) $\omega_{wall} = 10^5$, c) $\omega_{wall} = 10^7$, d) $\omega_{wall} = 10^9$

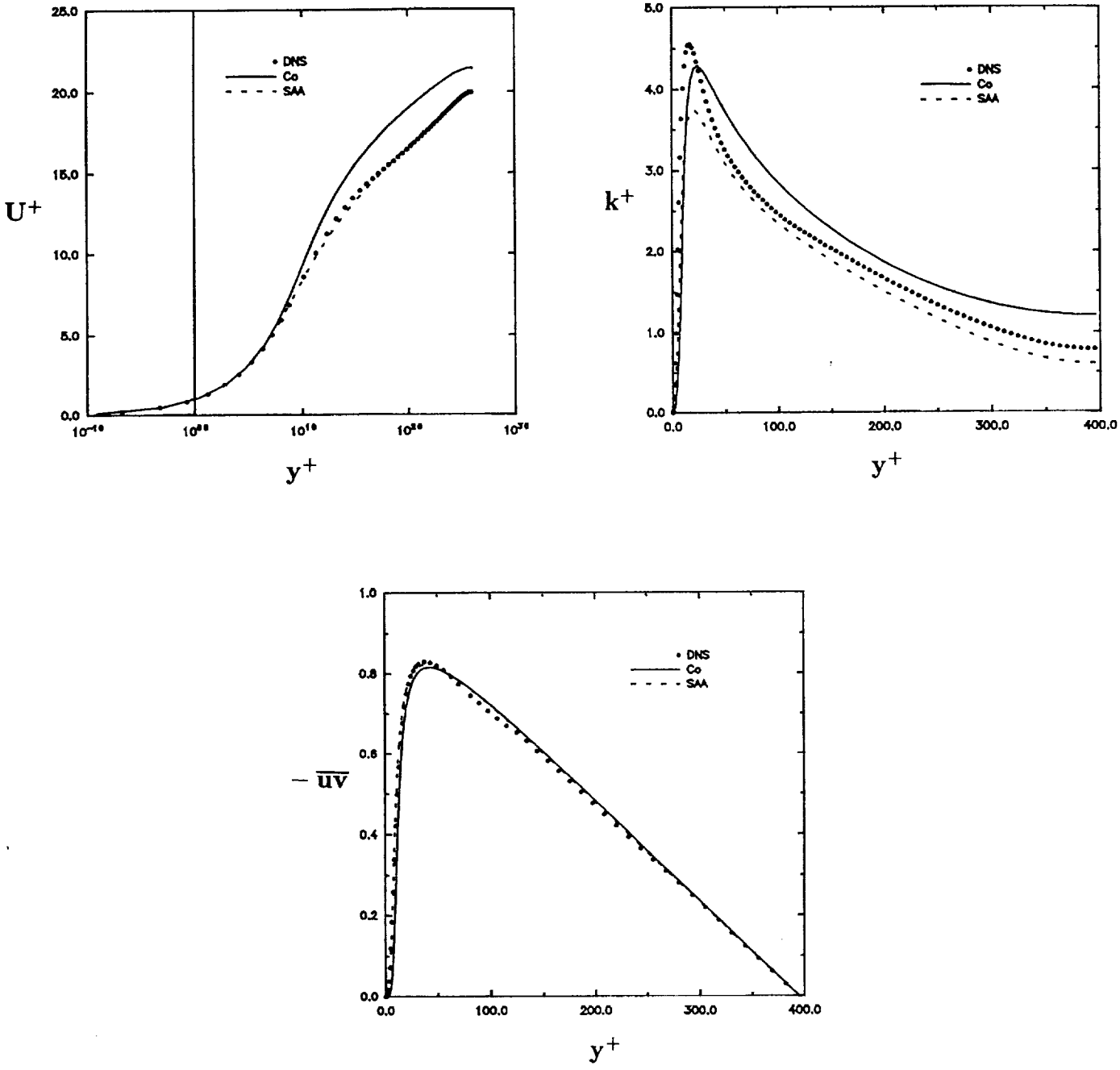


FIGURE 12
2-D Channel Flow $Re_\tau = 395$

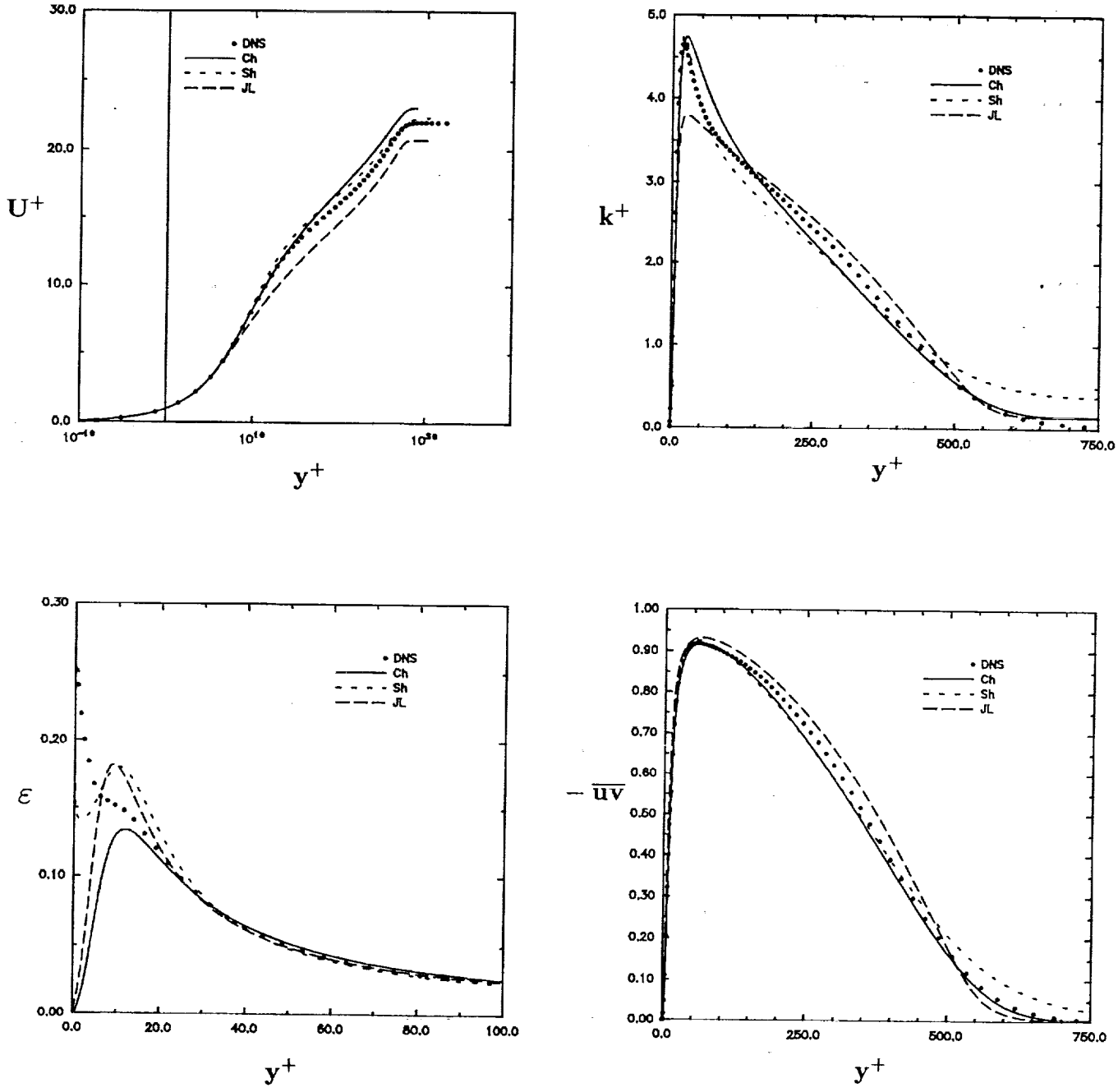


FIGURE 13
2-D Boundary Layer Flow $Re_\theta = 1410$

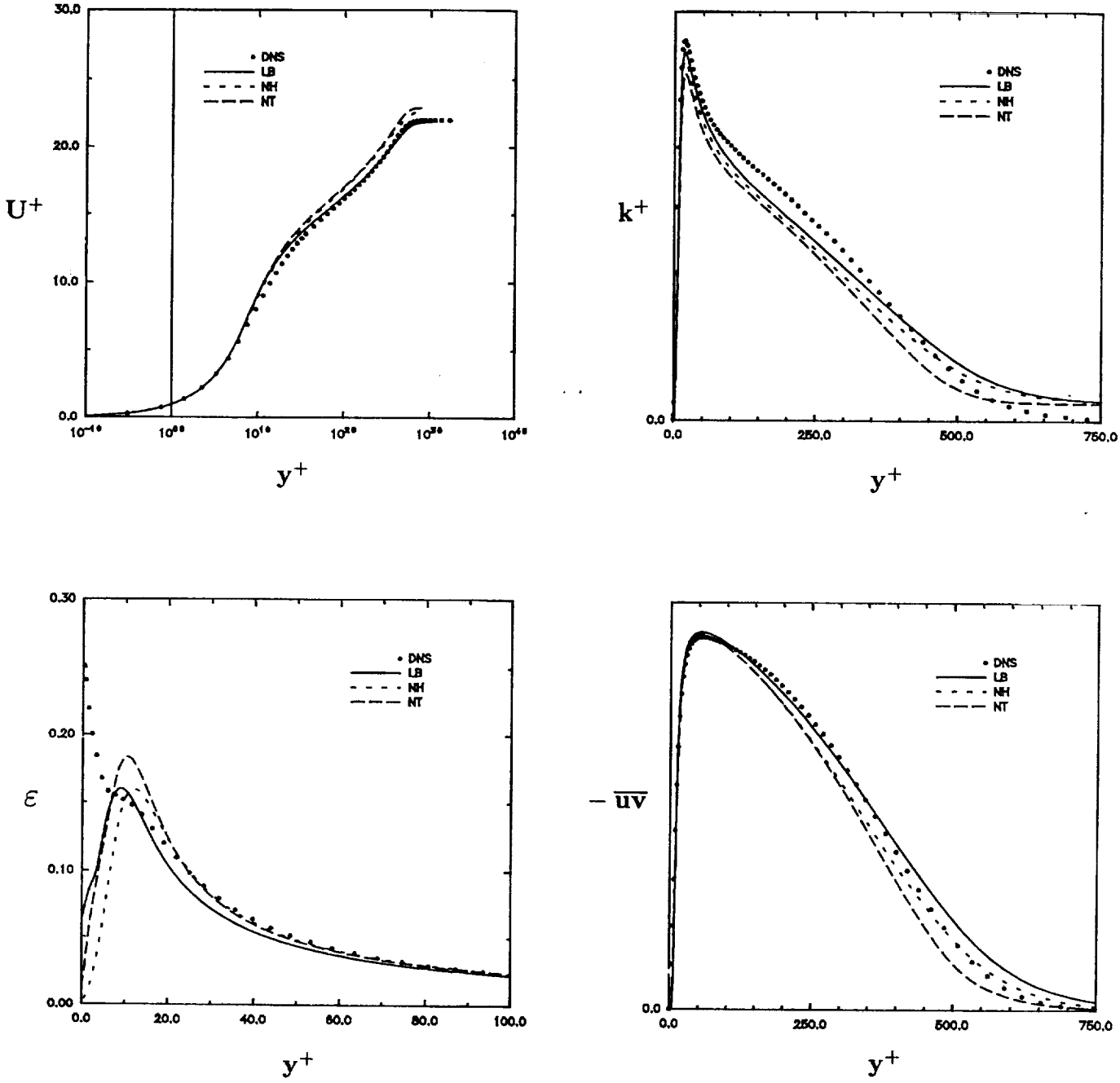


FIGURE 14
2-D Boundary Layer Flow $Re_\theta = 1410$

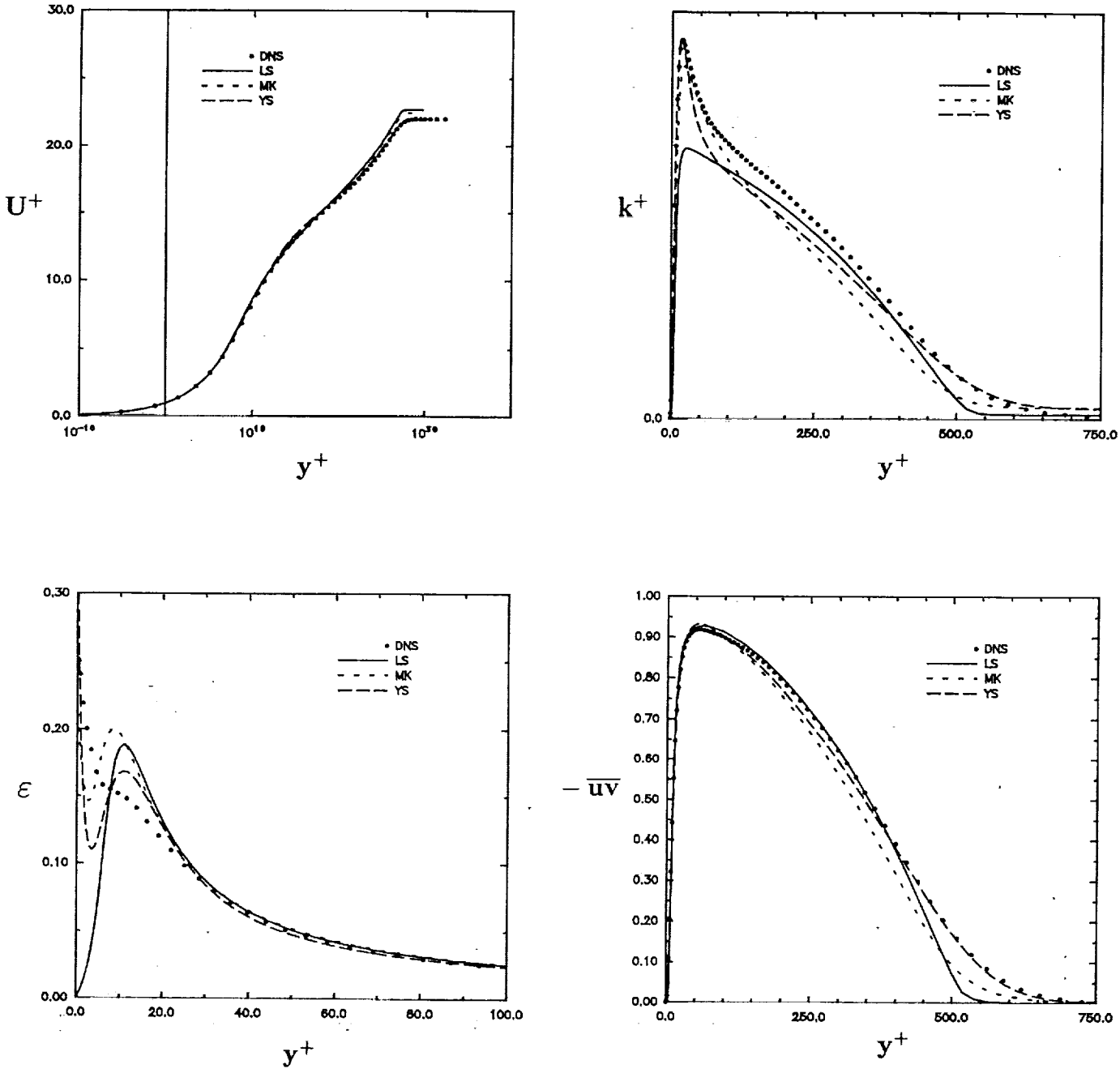


FIGURE 15
2-D Boundary Layer Flow $Re_\theta = 1410$

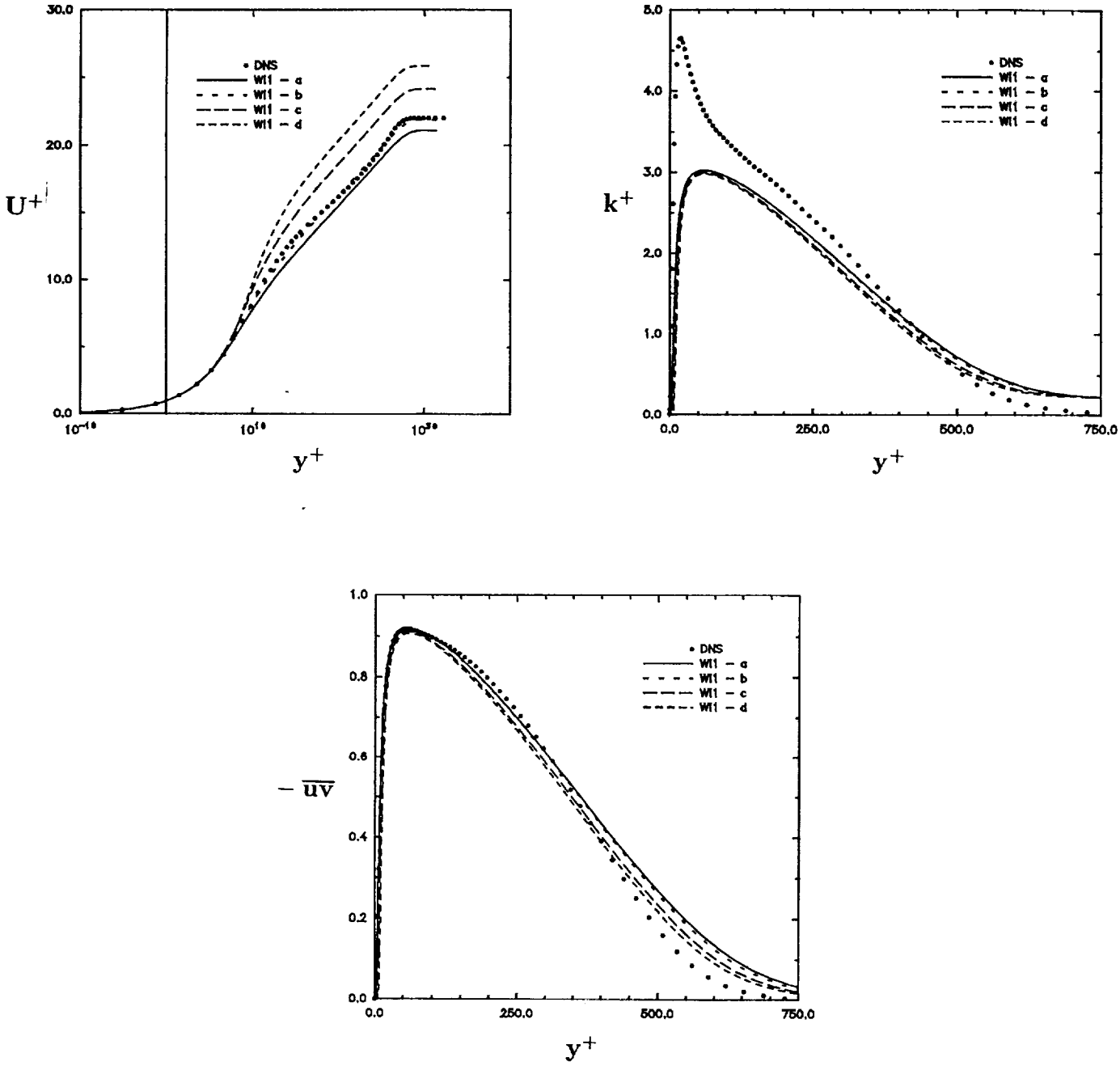


FIGURE 16
2-D Boundary Layer Flow $Re_\theta = 1410$

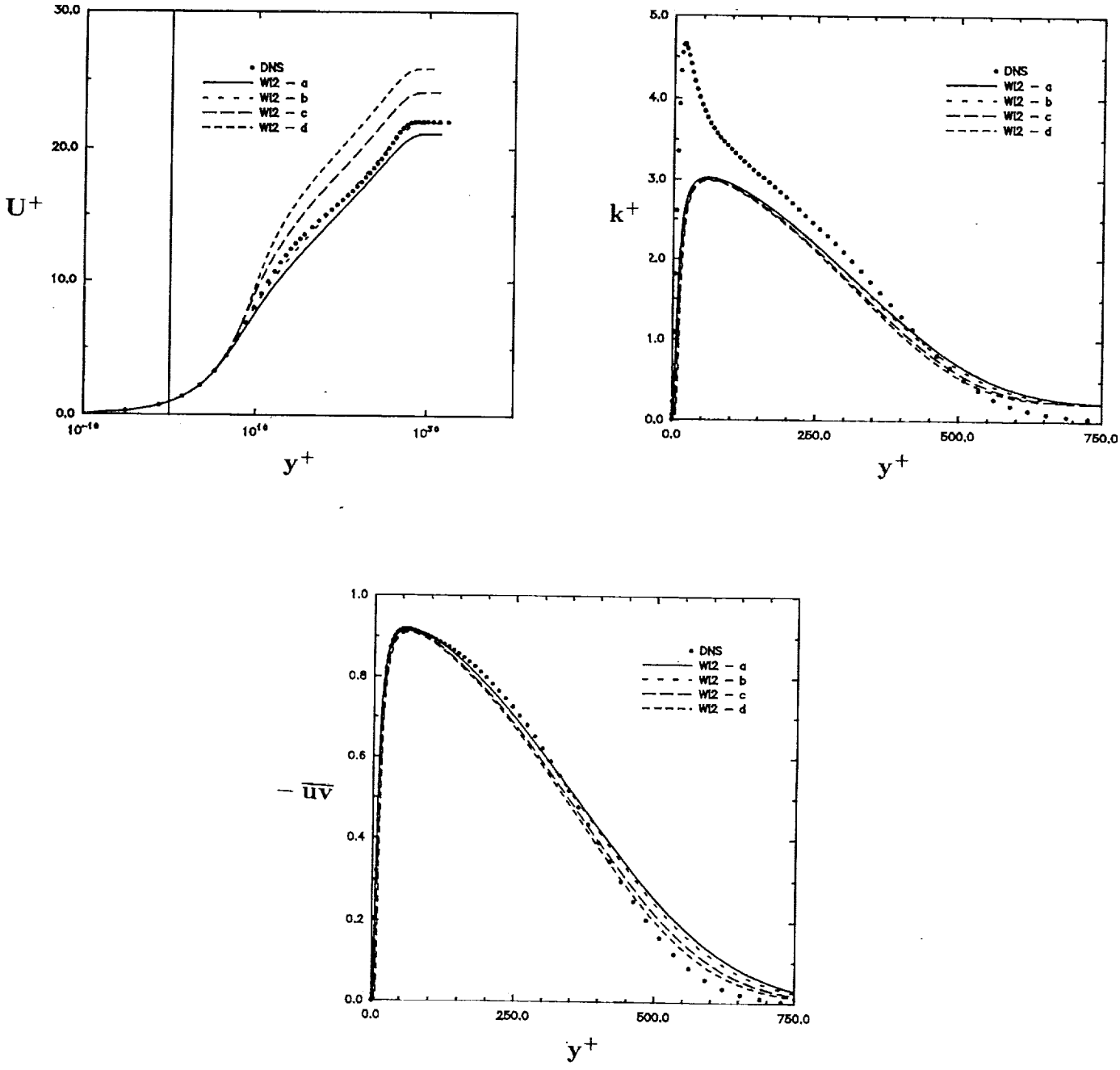


FIGURE 17
2-D Boundary Layer Flow $Re_\theta = 1410$

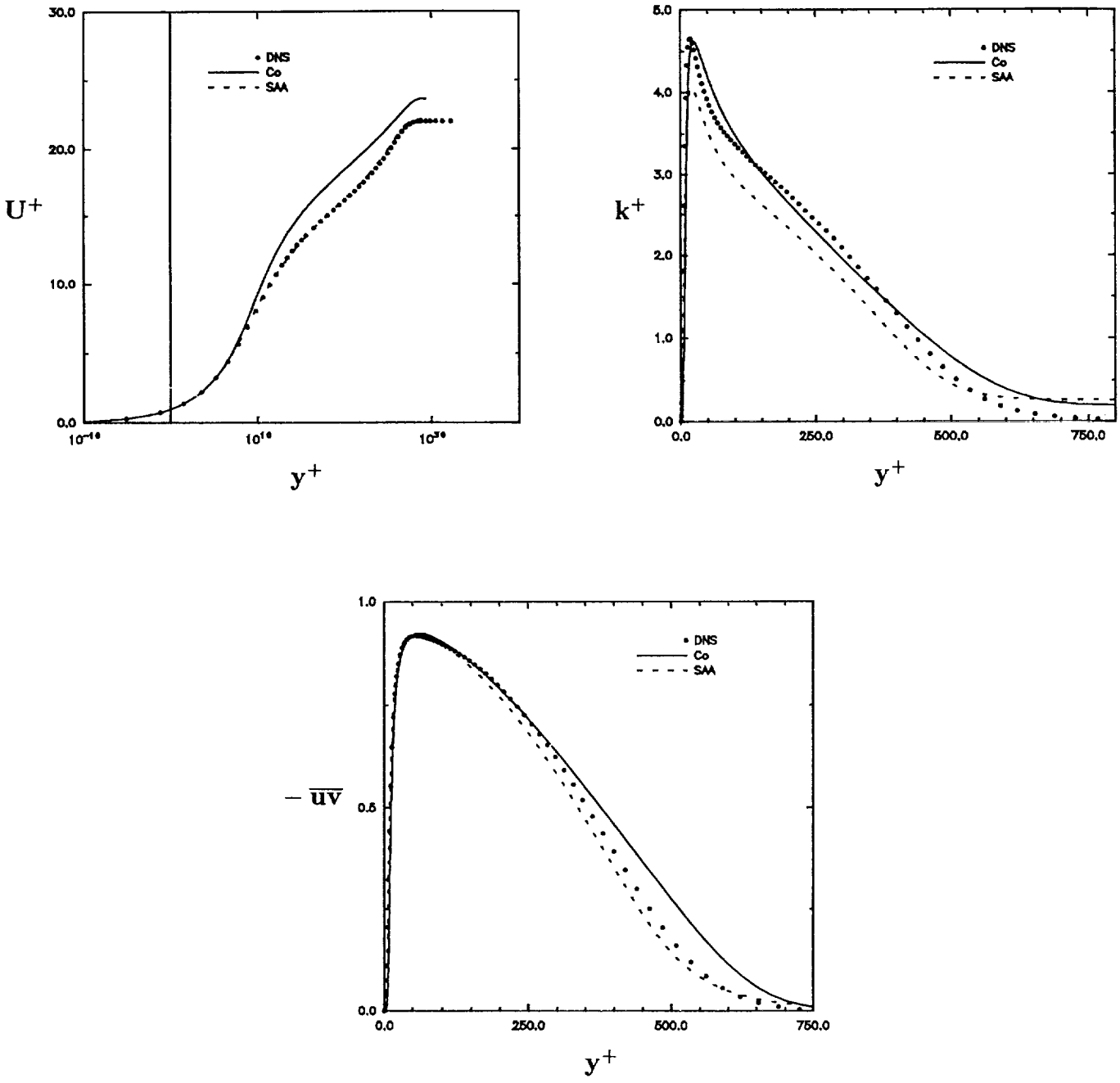
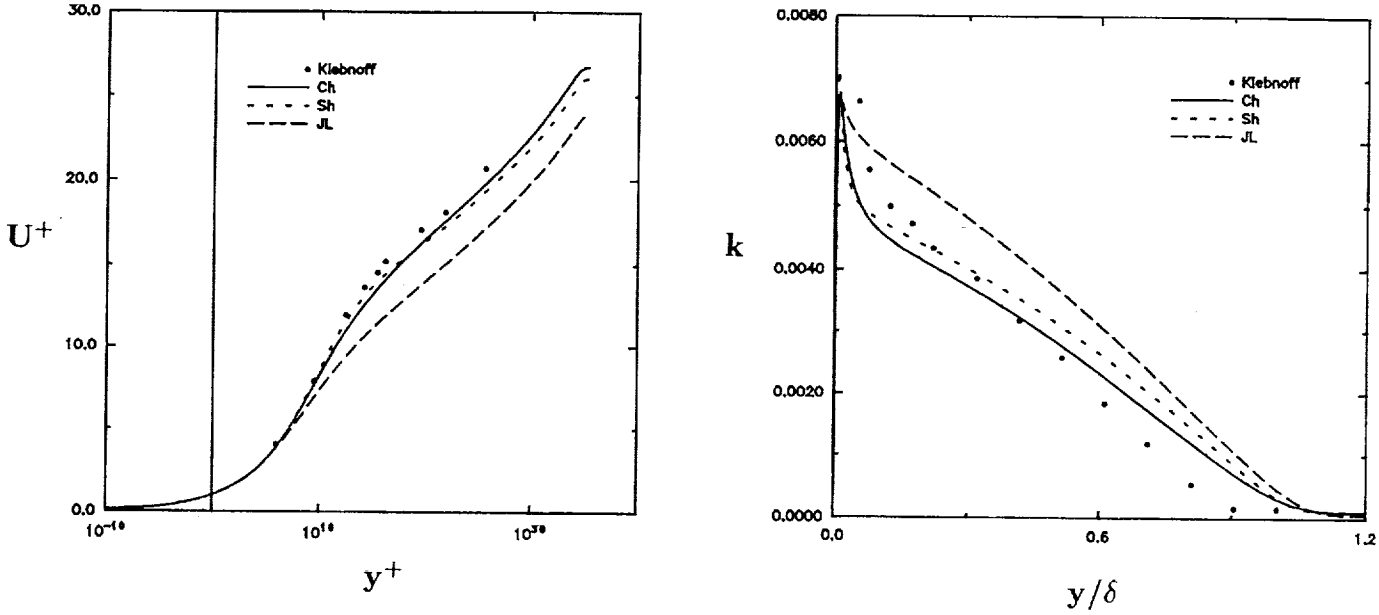


FIGURE 18
2-D Boundary Layer Flow $Re_\theta = 1410$



2-D Boundary Layer Flow $Re_\theta = 7700$

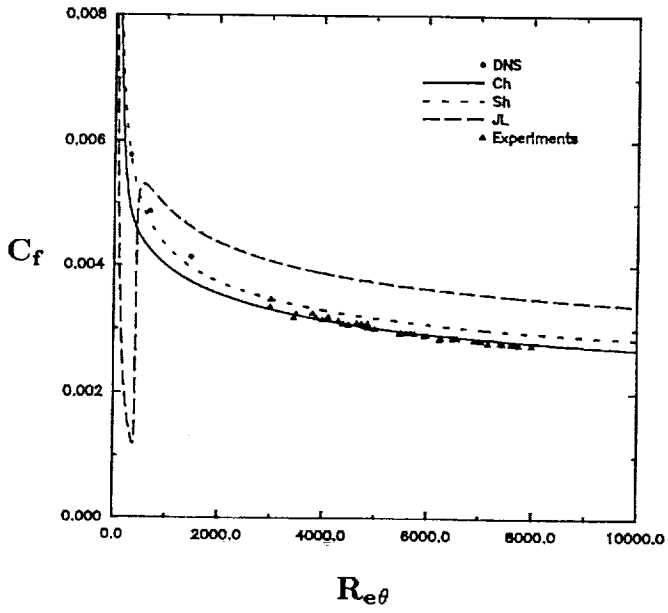
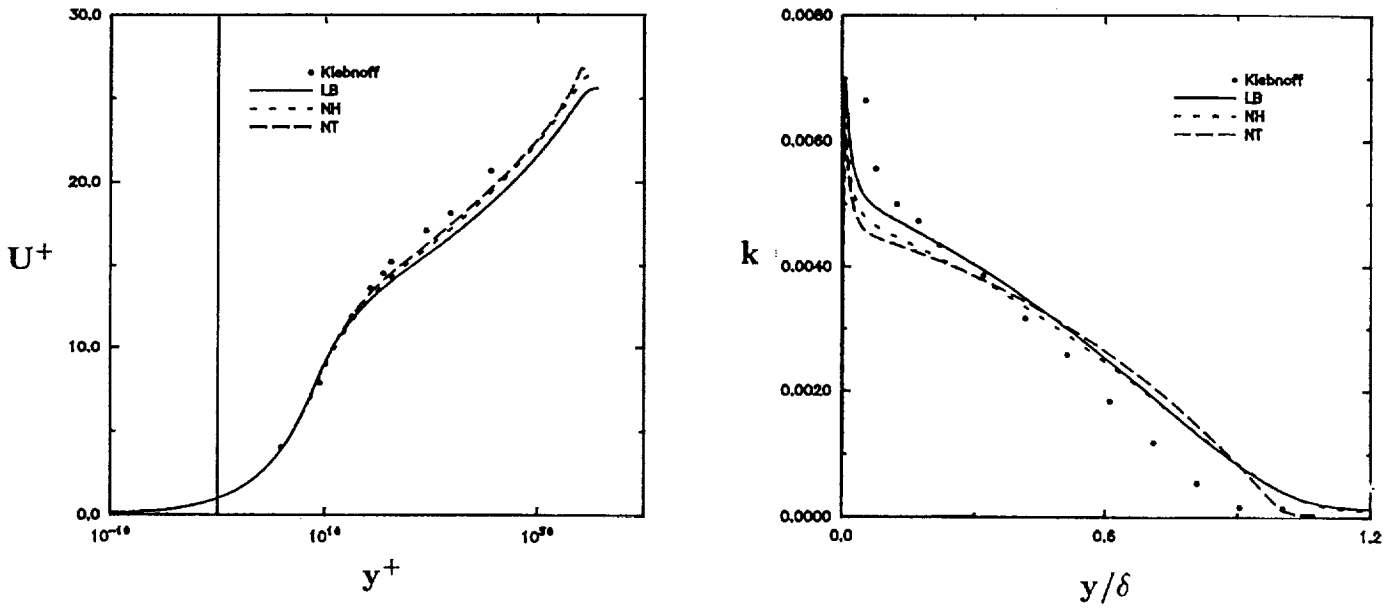


FIGURE 19
2-D Boundary Layer Flow



2-D Boundary Layer Flow $Re_\theta = 7700$

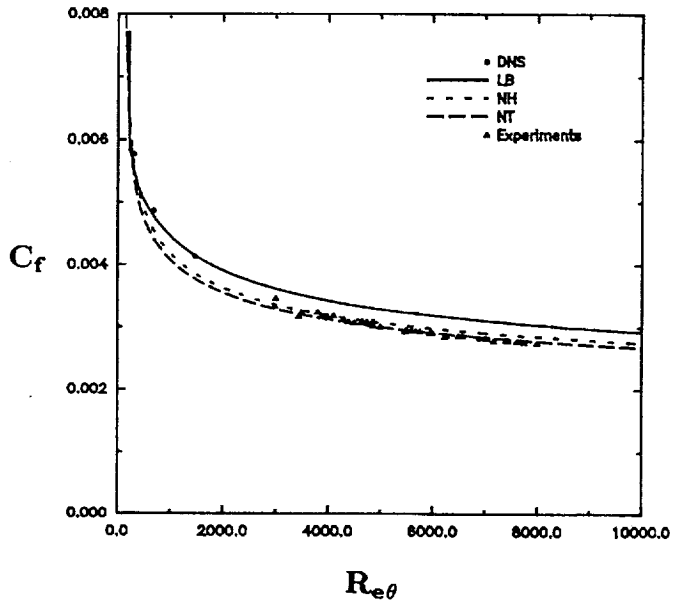
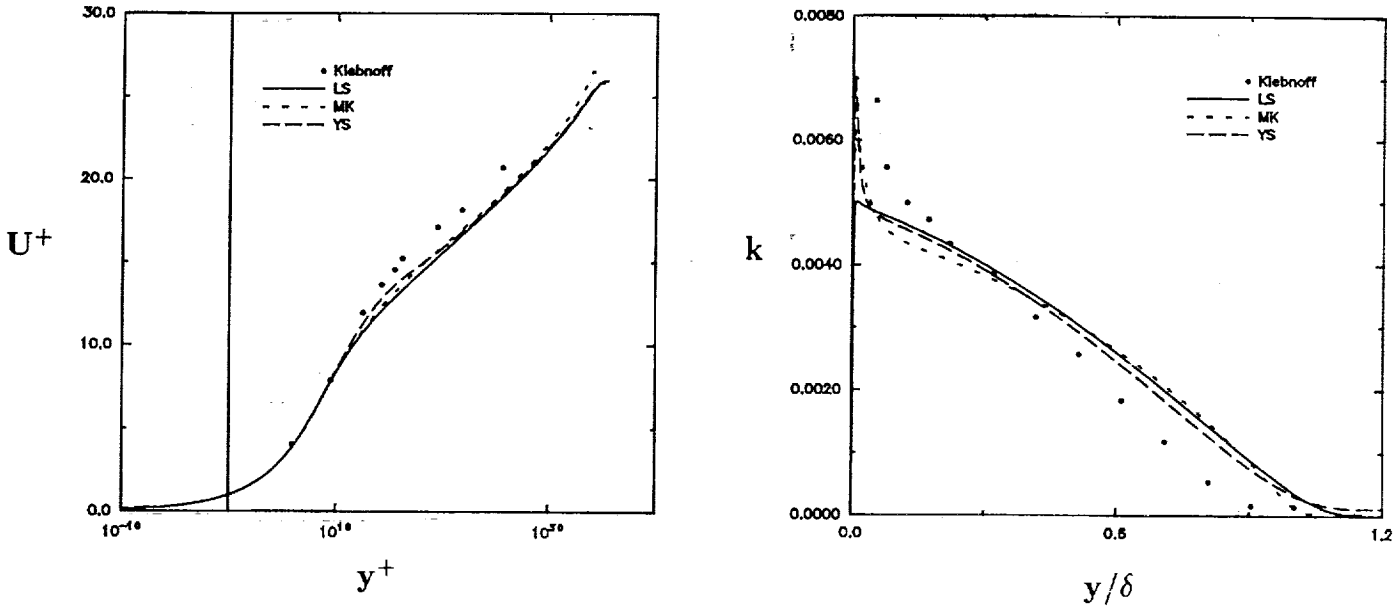


FIGURE 20
2-D Boundary Layer Flow



2-D Boundary Layer Flow $Re_\theta = 7700$

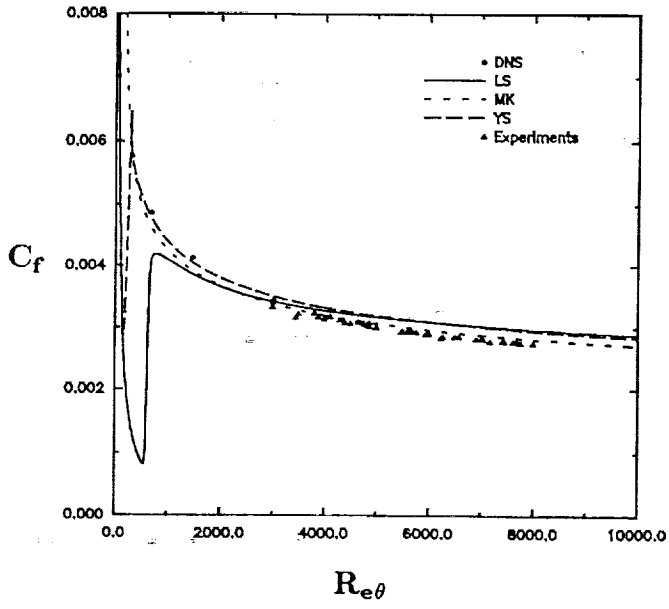
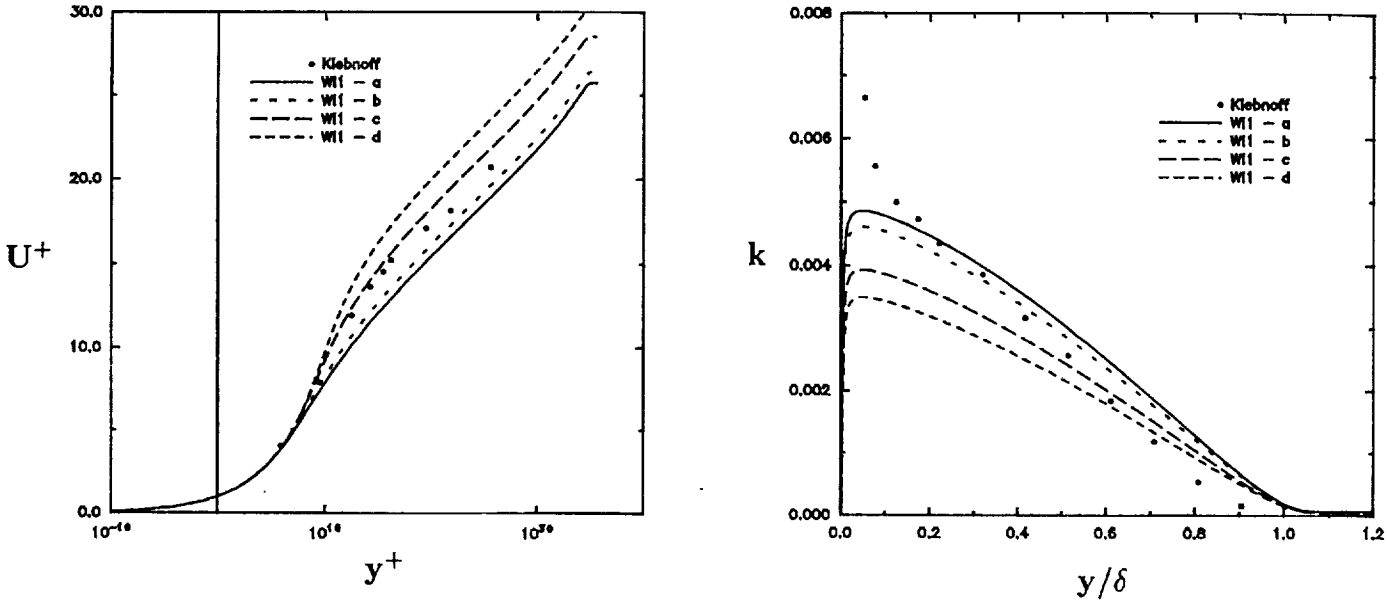


FIGURE 21
2-D Boundary Layer Flow



2-D Boundary Layer Flow $Re_\theta = 7700$

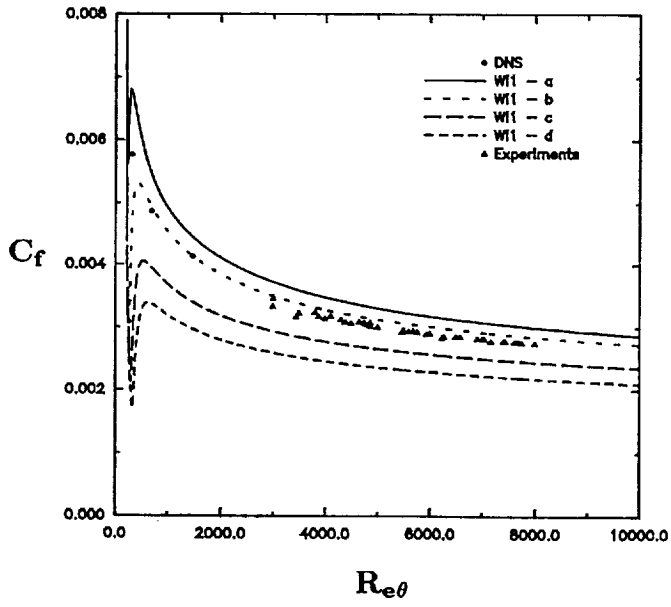
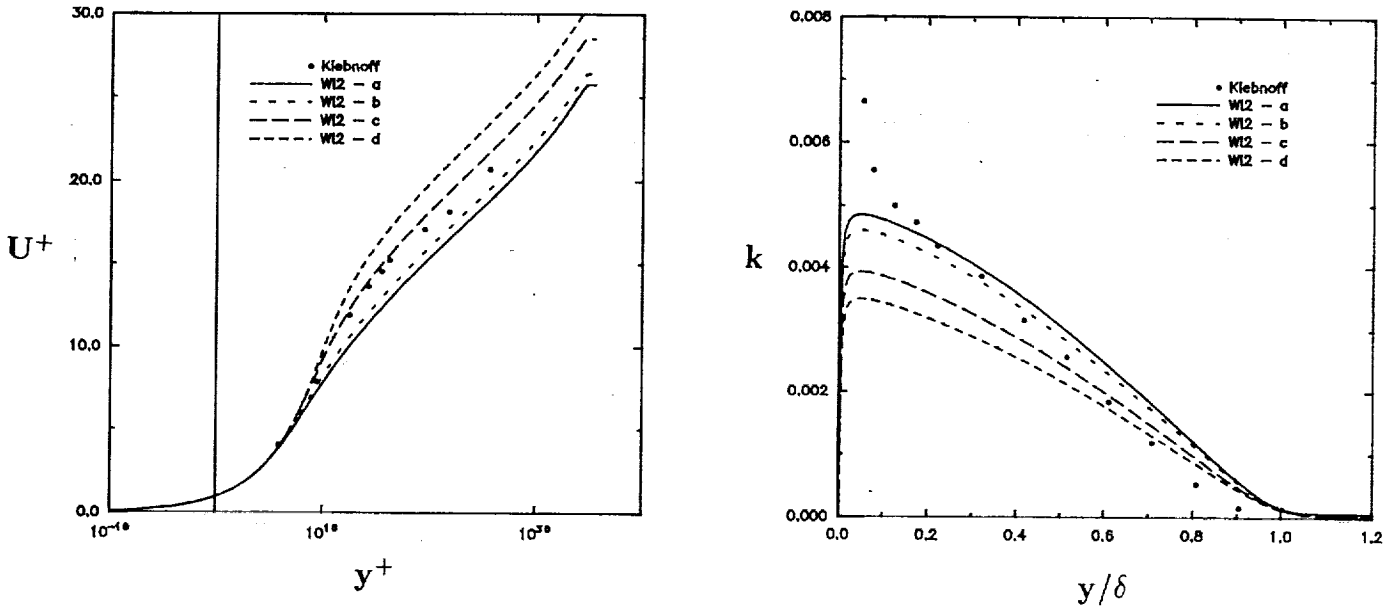


FIGURE 22
2-D Boundary Layer Flow



2-D Boundary Layer Flow $Re_\theta = 7700$

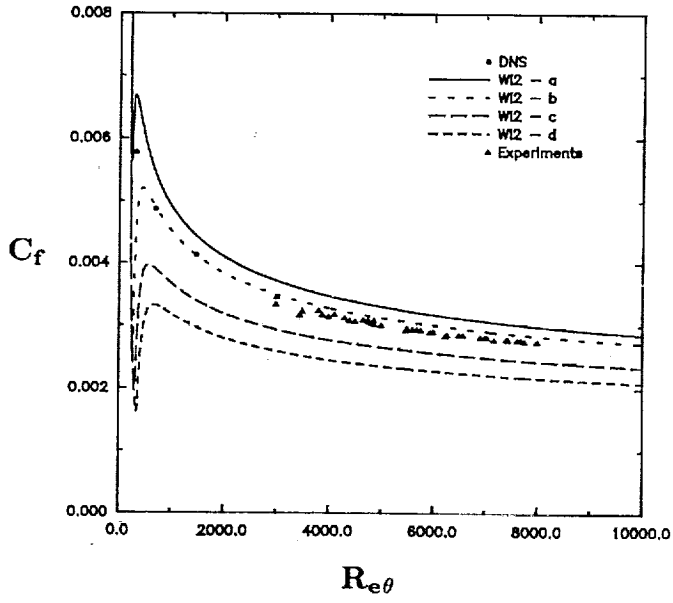
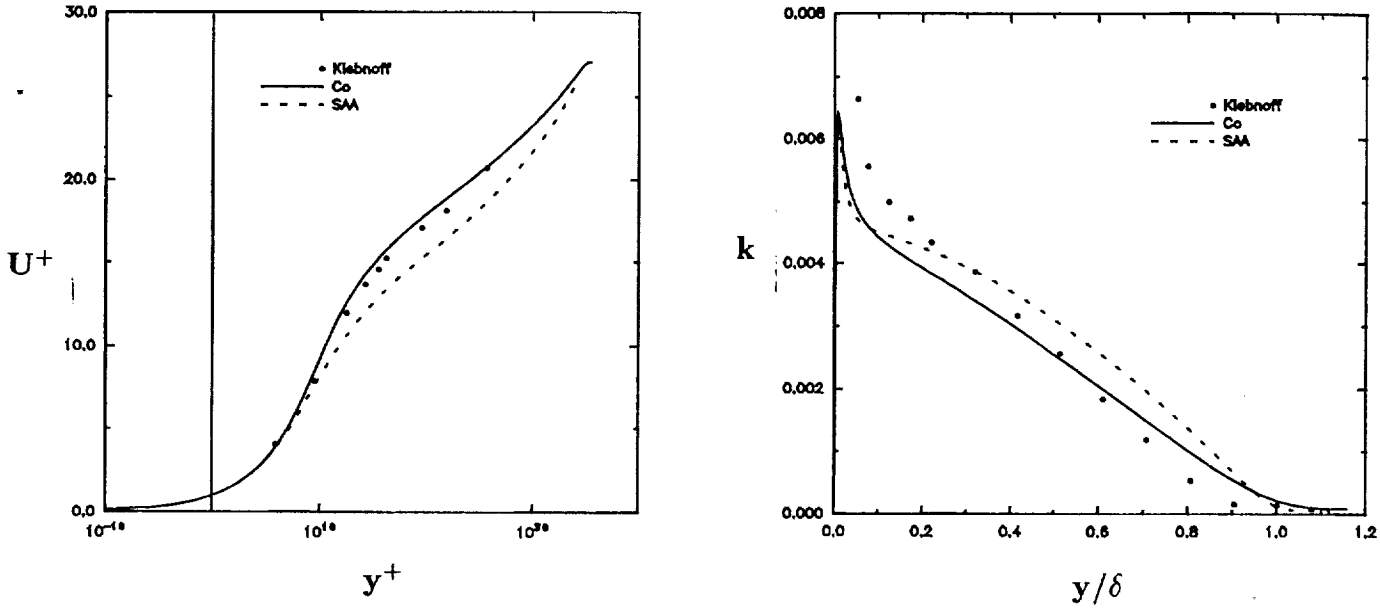


FIGURE 23
2-D Boundary Layer Flow



2-D Boundary Layer Flow $Re_\theta = 7700$

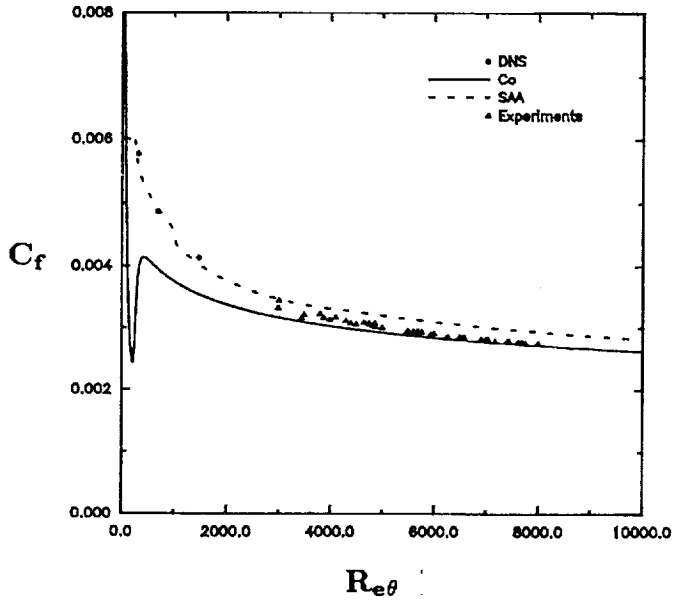


FIGURE 24
2-D Boundary Layer Flow

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13. ABSTRACT *(Maximum 200 words)*

Several two-equation models have been proposed and tested against benchmark flows by various researchers. For each study, different numerical methods or codes were used to obtain the results which were reported to be an improvement over other models. However, these comparisons may be overshadowed by the different numerical schemes used to obtain the results. With this in mind, several existing two-equation turbulence models, including $k-\epsilon$, $k-\tau$, $k-\omega$ and $q-\omega$ models, are implemented into a common flow solver code for near wall turbulent flows. Calculations were carried out for low Reynolds number, two-dimensional, fully developed channel and boundary layer flows. The quality of each model is based on several criterion including robustness and accuracy of predicting the turbulent quantities.

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