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EXTENSION OF AND IMPROVEMENTS TO THE ERDC LOW FREQUENCY MAGNETIC DIRECTION FINDING SYSTEM

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ABSTRACT

In 1985 a three station direction finding lightning location system was established in the UK. In this paper a description is given of its evolution from an experimental system to a five station commercial system providing both on-line and historical location information. A method of error reduction which enables simultaneous optimisation of strike location and calculation of aerial twist error is described, together with results of the analysis of four station data from 1989. It is demonstrated that one station with a twist error can produce an apparent bearing dependent error in a station without errors. Application of the bearing corrections has significantly improved the accuracy of the system. Average location errors of better than 1km can be acheived at a distance of 200km from the stations.

INTRODUCTION

Damage by lightning to the distribution systems of the UK regional electricity companies, together with the associated costs of interruption of supply to customers is a significant cost to the businesses. It has been estimated that, in the mid 1980's, this cost was of the order of £4 million per annum [1].

By 1985 an experimental three station direction finding (df) lightning location system was established in the UK. The positions of the stations and the coverage of the system for strikes of 7 kA and greater is shown in figure 1. Earlier work [2,3] had showed that the errors in wide band df systems could be attributed to the downcoming horizontally polarized wave produced by reflections at the ionosphere. The experimental system therefore operated at a VLF frequency of 2 kHz, which is the cut-off frequency for horizontally polarized waves propagating in a plane waveguide with a separation between horizontal conductors of 75 km. This distance is the effective daytime height of the ionosphere for VLF waves. The df stations used 3 vertical loops differing in azimuth by 120 degrees. This avoided the need for polarity sensing to determine the bearing quadrant, and also avoided the need for accurate measurement of very small voltages, since the largest two signals will not vary by more than 1:0.5 in relative magnitude because the bearing to a strike must always be within 60 degrees of the planes of at least two of the three loops.

The system was operated intermittently during 1985 and 1986. Raw bearing and signal strength data from the stations was collected by a DEC PDP11/73 computer, with analysis and location being performed off-line on a Prime mainframe. Simple error analysis, consisting of the calculation of the "strike error radius" (defined as the root mean square of the perpendicular distance of the located strike position to the bearings from each of the stations), together with a few observer reports indicated that the system was only accurate to between 15 and 20 km at night. The operating frequency was therefore reduced to 1070 Hz with a bandwidth of 350 Hz. A similar analysis of the 1987 data indicated a mean accuracy of location of 6 km.

SYSTEM EXPANSION AND IMPROVEMENTS

Following the improvement in performance, a decision was made to increase the coverage and accuracy of the system by adding more stations. Also, the value of the system for producing early warning of impending lightning was appreciated, and a decision was made to develop the system to locate and broadcast lightning strike data to customers in real time. During 1989, it became clear that the chosen sensitivity was inadequate, as a small number of strikes which had caused damage were sensed by only one or no stations. The threshold level at which the stations reported was therefore reduced. These improvements introduced a small number of problems to be overcome and these together with improved methods of location of strikes and error analysis are described below.

CHOICE OF NEW SITES

The original sites were chosen so that the overall level was within 1 degree over 1 km radius with mean level constant over 10 km. The geography of the UK, together with a requirement to position stations with a separation of 300 km, and approximately 1 km from 11 kV power lines has prevented the installation of stations on such favourable sites. The principle need for more stations was to increase coverage of Scotland. At least one station was required in northern or central Scotland, with a second in northern England or southern Scotland. The hilly nature of the north of England and Scotland in general has led to most urban development being restricted to the low-lying coastal areas. The consequent need for electricity in these areas has resulted in a high density of transmission lines and hence reduced the number of sites which would otherwise be acceptable. However, four potentially acceptable localities were found.

Upon more detailed surveys of the areas, two sites were chosen as more acceptable than the others. These sites were both in Forestry Commission land, minimizing the risk from future development. The first, Harwood forest is some miles north west of Newcastle upon Tyne, whilst the second, Rhynie forest, is some miles north west of Aberdeen. The mean level of the Harwood site slopes by 0.03 degrees SE to NW, with the overall level within 1.2 degrees. The mean level of the Rhynie site slopes by 0.12 degrees E to W with the overall level within 1.2 degrees. The effects of these departures from the "ideal" sites are discussed in the section on errors below. At present all 5 stations are operating. The position of the new stations and the present coverage for strikes of 5 kA and above is also shown in figure 1.

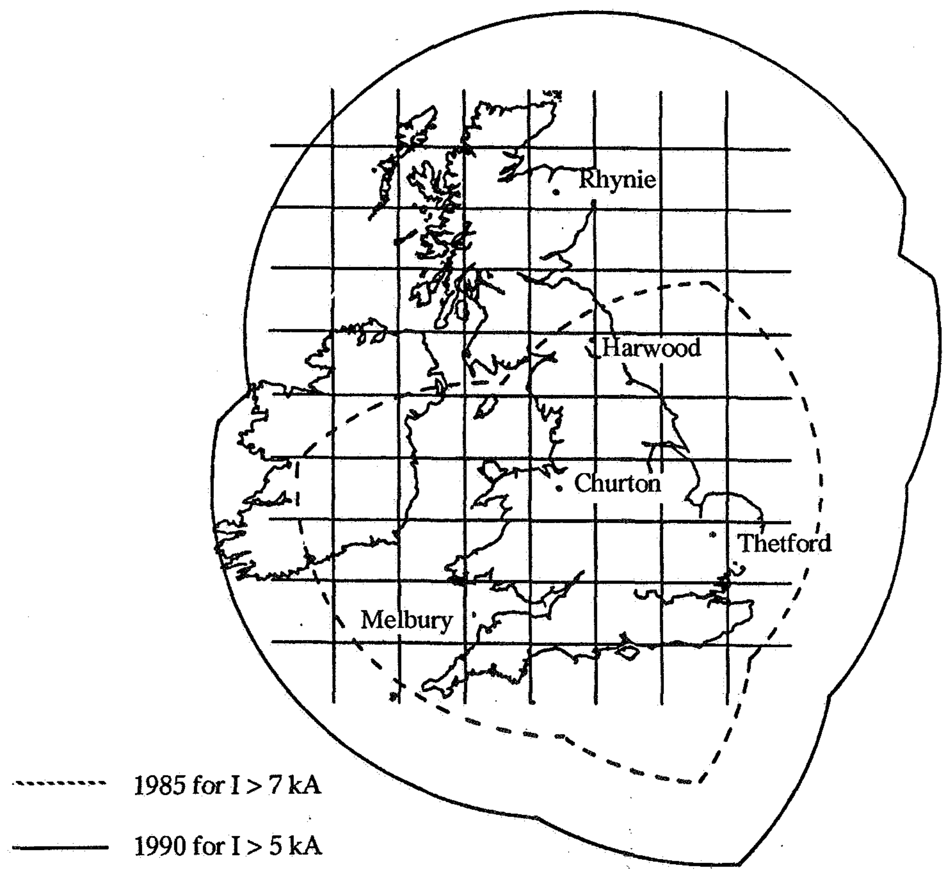

Figure 1. Position of DF Stations and coverage of system in 1985 and 1990

DEVELOPMENT OF THE REAL-TIME ON-LINE SYSTEM

Collection of data and calculation of location. The df stations can sense a strike, calculate bearings and signal strength, and transmit the encoded data at a rate of 40 per second. However, it soon became clear that the PDP11 computer could not collect, time tag and store the data, and in addition calculate the strike location at rates significantly above one per second. This was acceptable for an off-line system, providing the input buffers were not filled. An on-line system, operating at this mean rate, could result in location information being sent to customers up to 15 minutes after the strike occurred. This is unacceptable for a "real-time" system.

The duties of the central computer were therefore distributed. The PDP11 continued to collect, time tag and store the data, whilst calculation of the location and transmission to remote displays was performed by a Sun 3/160 computer under Unix. This operating system was chosen as it allowed development of the various components of the system as separate programs to be run as separate processes.

There are three main process running live on the Sun. Communication between the various processes is by a low level Unix method known as "sockets". The first process collects the packet of data as made up by the DEC. It stores the raw data on disc and passes it on to the analysing process. This calculates the location, produces an estimate of accuracy of location and current of the strike, then tests the data against a series of rules in order to assess whether the strike has been incorrectly located. These test were of great importance in the early days when only two stations were operating, and became of importance again when the threshold of the system was reduced.

The analyser passes the location data, together with date, time, accuracy, number of re-strikes, current and flags indicating the results of the rules' test to the third main process. This encodes the data for transmission to remote displays and transmits it. Simultaneous connection of the analyser to more than one "post analysis" process is possible. It is therefore possible to filter data to a customer's needs, for example restricting the area of coverage, without making any change to the analysis program.

This system design makes the system more robust, since failure of one process does not affect the others. It also is of great assistance when bringing a station on-line, whilst it is being calibrated. Two complete systems can be run on the same machine, one without the new station, one with, and the results compared in a separate process without interfering with the live system.

In 1989 the Sun was upgraded to a 4/260 10 MIPS machine. In addition all the processes were streamlined to make them more time efficient. The result of this was to increase the processing speed by a factor of eight. The continuous rate is now well in excess of that which the data circuits to customers are able to support. A second DEC and a second Sun computer have been added to the system. These are inter-switchable with the original computers to provide hardware redundancy. As a consequence of this, although damage to stations has occurred, collection of data from at least two stations has not been interrupted for two years and within this period, excluding problems with data circuits, customers have only been off-line for a total of 44 hours.

On-Line Display terminals have been developed to provide a real-time graphical display of strikes. The display software runs on IBM PC and PS2 compatible machines within the Microsoft 'Windows' environment. Flashes are displayed within seconds of their occurrence in a window displaying an outline map of the British Isles and the north west coast of Continental Europe. The program is menu driven and includes facilities for pan and zoom of the display and sizing of the window. Various overlays are available displaying the 400 kV, 275 kV and 132 kV networks, power stations and sub-stations. User defined overlays are easy to produce and can be included in the display.

The strike position indicators are colour coded, representing the time of the strike. Data on any specific strike can be extracted by positioning cross hairs over the strike using the mouse. Clicking the mouse button opens a window which shows the details of the strike including the latitude, longitude and Ordinance Survey National Grid Eastings and Northings of the strike position; the uncertainty of the fix, the current of the strike, the date and time to the nearest hundredth of a second. In addition for restrikes a multiplicity counter is included. Hard copy of both the screen display and details of strikes within a given time period can be produced by a Microsoft 'Windows' supported printer.

Historical data can be replayed in a similar windowed display on the same machine without stopping the collection and display of live data.

Effects of the reduction in Threshold The analogue circuitry of the df stations have a dynamic range limited by external noise of 80 dB (power) from a reference of 22.5 nT magnetic flux density. The original working range was chosen to be 36 dB. This was chosen to limit the number of strikes detected at distances of above 1000 km, and also because of limitations in the resolution of the A/D converters. From work by Erskin [4], using the far field approximation for the ELF magnetic field due to lightning current given by Watt [5] and Uman [6] the minimum current against distance for a strike to be detected is shown in curve A figure 2. Using the flash current population from 1988 data [4] this gave a detection efficiency at 400 km of 99.5% (93% at 1000 km). The detection efficiency at 400 km using flash current population data measured by Berger et al. [7] was 99.5% for negative first strokes, 82% for negative following strokes and 95% for positive strokes. Using data of Popolansky [8] gave a detection efficiency of 94%.

However, there still remained incidents on the distribution network which were caused by strikes which were detected by one station only. The threshold was therefore reduced to 190 pT giving a dynamic range of 42 dB. This gives a minimum current for detection as shown in curve B figure 2. The detection efficiency is now 99.9% at 400 km and 99.5% at 1000 km (from [4]); greater than 99.9% for negative first strokes, 97% for negative following strokes, and greater than 95% for positive strokes all at 400km (from [7]); and approximately 98% for all strokes at 400km (from [8]). The coverage of the system for currents of 5kA and above is shown in figure 1.

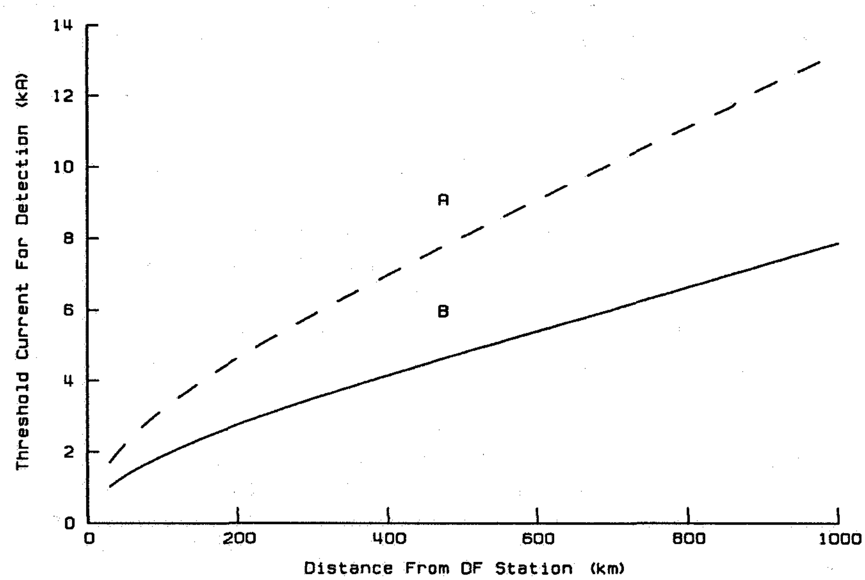

Figure 2. Effective range of DF stations as a function of strike current

Because the collection efficiency of the ERDC system is a function of distance from a station, the area over which strikes are detected is greater for strong strikes than for weak strikes. This effect is not allowed for in [4], and therefore the distribution may be weighted to larger currents. It is intended to analyse 1989 and 1990 data to determine the degree of weight resulting from this effect.

The greater sensitivity led to a much greater number of strikes in southern and central Europe being detected. Unfortunately the digitising errors from the weak signals caused large errors in the location of these strikes. The increase in number of stations meant it was possible for a strike to lie close to the baselines between three stations, a situation which was not possible with the three station system. There was also an indication that the approximation that the signal travels along a great circle was not sufficiently good for accurate location of strikes at distances in excess of 2000 km. The rules were therefore adjusted to prevent strikes occurring for example in northern Spain being incorrectly located in Brittany. The correct trapping of these strikes has been confirmed by correlation with data from the Meteorological Office.

IMPROVED METHOD OF LOCATION

The original location method used plane geometry with corrections for the difference between true and grid north at the stations. The root mean square of the perpendicular distance of the location from the bearings was minimised. This is unsatisfactory when the variation in the distances from the strike to the stations is not small compared with the distances, and when the distance is greater than 300 km. From 1988 the locations were determined in spherical coordinates using the eigenvector method of Wangsness [9]. This is essentially the same as the method described by Orville [10]. The point which is least far from all the bearings in a least squares sense, is found by calculating the smallest eigenvalue λ of the equation

$$(A' \cdot A - \lambda I) \cdot x = 0$$

where I is the unit matrix, x is a unit vector from the origin in the direction of the strike, A' is the transpose of A, and A is the matrix of the unit normals to the bearing planes. All vectors are expressed in a spherical coordinate system with the origin at the centre of the earth. In 1990, following the increase in speed of the system, the method was improved by determining the distance to the strike from each station and repeating the calculation with the elements of A divided by the corresponding distance to give the optimum location of the strike. This is a first order approximation to minimizing the sum of the squares of the difference between the measured and optimum location azimuths.

The increase in number of stations produces redundancy in data for producing the fix and error assessment. Were all measurements to be made with equal accuracy then an increase in stations would naturally lead to an increase in accuracy. However, increased distance from a strike reduces signal strength and increases digitising errors, and also a given bearing error results in a linear error which increases with distance. These effects combine to make measurements at stations close to a strike more reliable then those further away. The latter effect is normalised by dividing the elements of A by the corresponding distance to the strike as described above.

ASSESSMENT AND REDUCTION OF SITE ERRORS

A brief review of the subject of site error estimation has been given by Passi & Lopez [9]. They divide the approaches to the problem into parametric and non-parametric methods. In the former method a two cycle sinusoid is fitted to the differences ε_i for each strike between the measured and optimum location azimuths. This is performed for each df station independently. In the latter method no functional form of the error is assumed and the mean value of ε_i is calculated for angular sectors independently for each d.f. station. Both of these approaches are amenable to feedback in the sense that the corrections found can be applied to the data set in order to produce better optimum locations, and the error assessment repeated. However, as pointed out in [9], errors determined by either of these methods are not the actual site errors of df stations, since they are determined from optimum positions which were biased by site errors. It is by no means certain that the solution will converge, indeed we have found that it is possible using this method to produce a small decrease in mean error radius, but at the cost of moving the locations to positions which are known to be in error. Passi and Lopez assumed that the errors in the bearings were induced by scattering objects near to a df station and that therefore the site errors could be represented by a harmonic series based on two cycle sinusoids. They showed that with this approximation it was possible to decouple the site error estimation from the localisation optimisation. They applied the method to a system with gross errors (up to 20 degrees) and showed convergence was possible using a function with two harmonics.

The dimensions of scattering objects must be a significant fraction of the wavelength of the radiation being scattered. The wavelength utilised by the ERDC system is 280 km. Hence the size of scattering objects is large and may be positioned at large distances from the d.f. station. There is no certainty that a back-scattered wave will have sufficient amplitude to interfere with the incoming wave at a station. It therefore cannot be assumed that the site error can be represented by harmonics of two cycle sinusoids unless very high harmonics are used.

SIMULTANEOUS OPTIMISATION OF FLASH LOCATION AND SITE ERROR ESTIMATION

The principles of the method used were derived by Waddington [10]. The true errors β_j of the jth station cause apparent errors ε_i to be measured at the ith station. Thus all stations can appear in error when in fact only one is, as is

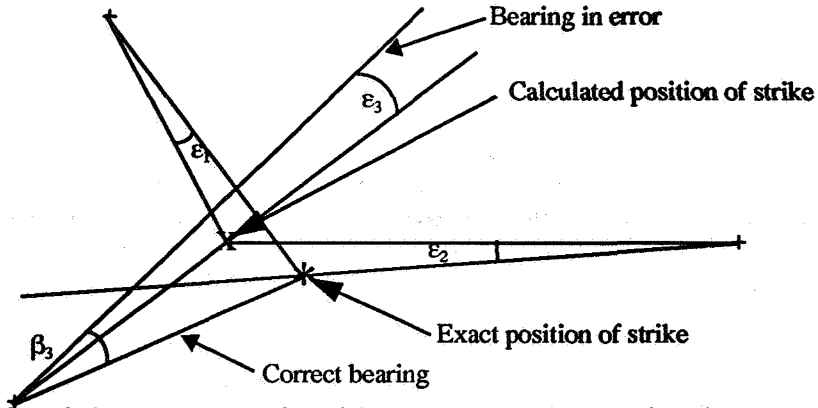

Figure 3. Definition of symbols and demonstration of the apparent error in all stations from a true error in one

shown in figure 3 for a three station system. It is this effect that can result in the calculation of incorrect corrections when not optimising the location simultaneously with calculation of site errors. It is clear from figure 3 that it is not possible to calculate the true errors from the apparent errors for one strike, although the reverse problem can be solved. However, it is possible to produce an estimate to the true errors if sufficient strikes are distributed over a large area. For an N station system the apparent errors ε_i of station i are related to the true errors β_j of station j by the equation:

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \ldots\varepsilon_N \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} & \ldots J_{1N} \\ J_{21} & J_{22} & \ldots J_{2N} \\ \ldots J_{N1} & \ldots J_{N2} & \ldots J_{NN} \end{bmatrix} \cdot \begin{bmatrix} \beta_1 \\ \beta_2 \\ \ldots\beta_N \end{bmatrix} \qquad \text{where} \qquad J_{ij} = w \cdot \frac{\partial \varepsilon_i}{\partial \beta_j}$$

This is the first term in a Taylor expansion. It is assumed that the apparent errors are a continuous function of the true errors and that both are small. The elements of the jacobian are calculated by deliberately introducing small errors to each of the measured bearings in turn and re-calculating the optimum location of the strike. The elements of the jacobian are multiplied by a weighting factor w which is related to the mean, over all reporting stations, of the probable instrumental error in the measurement of the bearings. This quantity is dependent on the strength of the wave at the stations. If a station has not reported then the weighting factor for that station is zero. Hence the method can be applied to strikes with reports from a variable number of stations, as long as there are three or more. In addition, strikes of varying strengths and hence accuracies can be used, without weak inaccurate strikes having a disproportionate influence on the calculated corrections.

There will be an equation of this type for every strike, giving n estimates to the jacobian for n strikes, and producing an overdetermined system of M = nN linear equations for a N station system. The M equations cannot be satisfied exactly, but the residual angular error over all strikes can be minimised in a least squares sense. That is, minimise:

$$R = \sqrt{\sum_{k=1,M} r_k^2} \qquad \text{where } r_k = \varepsilon_k - \sum_{j=1,N} J_{k,j} \cdot \beta_j$$

It can be shown [11] that the minimum value of R is given by the solutions β_j of the following equation:

$$\begin{bmatrix} J_{11} & J_{21} & \ldots J_{M1} \\ J_{12} & J_{22} & \ldots J_{M2} \\ \ldots J_{1N} & \ldots J_{2N} & \ldots J_{MN} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \ldots\varepsilon_M \end{bmatrix} = \begin{bmatrix} J_{11} & J_{21} & \ldots J_{M1} \\ J_{12} & J_{22} & \ldots J_{M2} \\ \ldots J_{1N} & \ldots J_{2N} & \ldots J_{MN} \end{bmatrix} \cdot \begin{bmatrix} J_{11} & J_{12} & \ldots J_{1N} \\ J_{21} & J_{22} & \ldots J_{2N} \\ \ldots J_{M1} & \ldots J_{M2} & \ldots J_{MN} \end{bmatrix} \cdot \begin{bmatrix} \beta_1 \\ \beta_2 \\ \ldots\beta_N \end{bmatrix}$$

It will be appreciated that no single optimised location has been used in determining the errors. Rather the effect of variation of a single station bearing on the error in the bearings from all other reporting stations, for each station, for each strike has been used in order to produce a global minimisation of errors over all strikes simultaneously. Once an estimate to the error has been found then the bearings can be corrected and the method repeated for a more refined estimate.

EVALUATION OF THE METHOD

Since it is difficult to obtain a statistically significant sample of ground truths with which to compare the validity of calculated corrections it is necessary to have great confidence both in the method used to find the corrections and in the data sample used. Hence the method was tested on the 1989 data set in the following manner. Firstly optimised locations were calculated from the raw data set. The bearings from the stations to these locations were then calculated to produce a new data set which gave negligible location errors. Errors were then added to these bearings. These took the form of a constant and a part which was random within a range determined by the instrumental error for the strength at a station for a given strike. The introduced errors were therefore very similar to those which would be expected in the original raw data, but in this case they were known. The constant part of the error was taken to be a fraction of a degree for all but one station, which was set at 4.5 degrees. This large figure was included to assess the ability of the method to find small errors on one station in the presence of gross errors from another. The results of the analysis are shown in the table below. It can be seen that the method was able to extract the errors within 0.03 degrees.

Station	Thetford	Churton	Melbury	Harwood
Constant part of fake error	0.44	0.1	-0.3	4.5
Extracted error	0.417	0.069	-0.323	4.427
Standard deviation	0.039	0.059	0.034	0.041

RESULTS OF ERROR ANALYSIS OF 1989 DATA

The angular differences, before corrections were applied, between the bearing from a station to the optimised location of a strike and the measured bearing at the station ε_i were collected in 2.5 degree bins for each of the four stations which were operating in 1989. The variations with bearing of the mean of the apparent errors are shown in figure 4. The error bars show one standard deviation from the mean. The standard deviation over all angles is quoted in each graph.

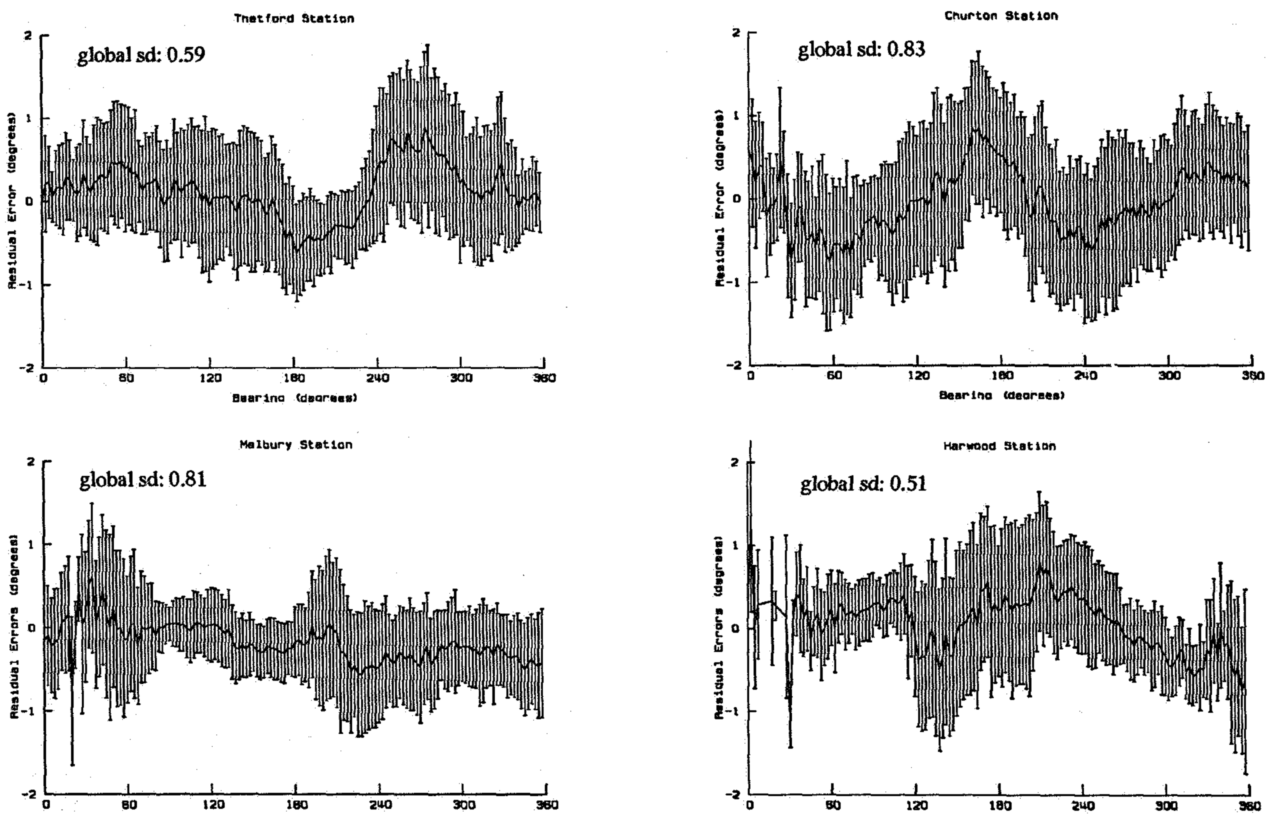

Figure 4. Variation of the apparent error in the bearing of a strike from each station as a function of the bearing.

It is apparent that a good fit to a two cycle sinusoid would be made by the data from Churton. Thetford, Melbury and Harwood also show features with a two cycle form. It will be noticed that the data for Harwood between 5 and 30 degrees is not as good as for the rest of the range, or for the other stations. This is most likely due to the geometry of the network of stations. The analysis requires a fix of a least three stations. One would expect a lower number of multi-station reports from this region, as it lies furthest from the Churton, Thetford and Melbury stations. However, there may be some meteorological reason for the lack of data in this region.

EVALUATION OF THE DEPENDENCE OF RESIDUALS ON BEARING

The bearing dependence of the mean residual in 2.5 degree bins after global correction for each station is shown in figure 5. The standard deviation over all angles is again quoted in each graph. The influence on location errors of correcting for the bearing dependent residual was investigated. This has not been subjected to the same critical test as described for the global error and therefore confidence in the results is not as great. Correcting for bearing dependent error and re-calculating the global error had only a small effect on the global correction of each station, as can be seen from the table below. Hence bearing dependent errors are a second order effect. The standard deviation over all angles was reduced by a similar amount upon correcting for the bearing dependent error, for all stations excepting Churton which was reduced to 0.54 degrees.

Station	Thetford	Churton	Melbury	Harwood
Global Correction before Bearing dependent correction	0.95	0.60	-0.31	0.67
Global Correction after Bearing dependent correction	0.92	0.62	-0.33	0.67

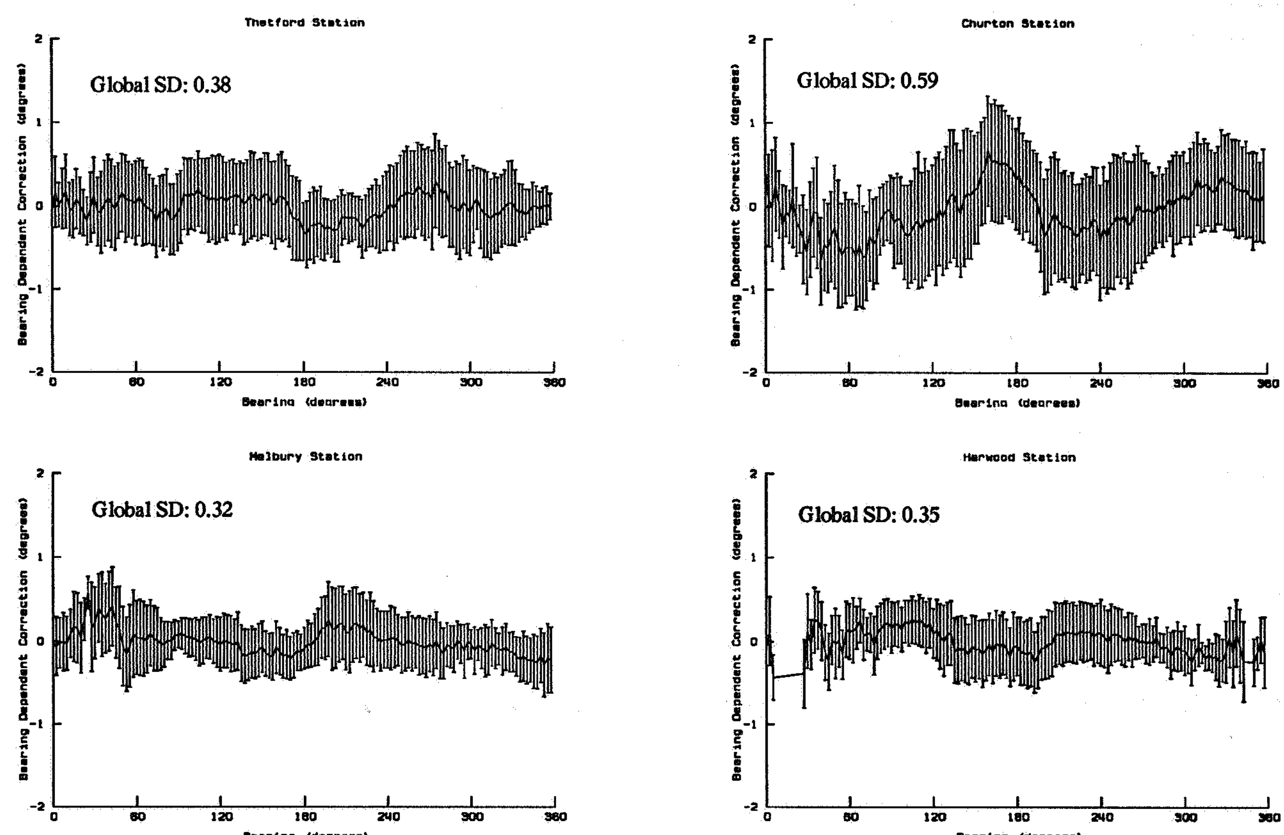

Figure 5. Variation with bearing of the apparent station error after global corrections applied

The effect on the accuracy of the location fixes resulting from the application of these corrections was marked. The mean rms linear error was calculated over all strikes in 100km square areas. In the centre of the network (at a distance of 200km from the stations) the mean rms linear errors decreased to less than 1km.

It is apparent that, following the global corrections, only the bearing dependent corrections for Churton station are double sinusoid in form. The most striking reduction in errors is shown by Harwood. The errors have changed from having a strong apparent bearing dependency, albeit with large scatter, to having small scatter and negligible bearing dependency. Melbury's errors show the same small scale dependence on bearing, with the large scale trend to greater negative errors between 120 degrees and 360 degrees removed by the global correction. An analysis of a three station system similar to that shown in figure 3 has been made. Churton and Harwood stations were assumed to be without errors. Bearings from each of the stations to 10000 strikes evenly spaced in latitude and longitude were calculated. The extracted twist error was then subtracted from the bearings from Thetford. The effective errors introduced on Harwood station by this twist error were calculated and are shown in figure 6. These should be compared with the graph for Harwood station in figure 4. The similarity in the functional dependence of the residual error on bearing is striking in view of the simplicity of the model. It is likely that the changes in the Melbury and Harwood errors are mainly due to the correction of the large twist error which was found in the Thetford station, and that Churton Station does display a real bearing dependent error. It is also evident that great care should be taken in applying bearing dependent corrections which have been extracted from the apparent errors of a station without first extracting the global twist errors in the manner described in this paper.

It is interesting that the shortcomings of the Harwood site have not had the expected effect on the errors of the station. It is probable that, were a good sample of data to have been collected between bearings of 5 and 30 degrees, Harwood would display lower errors than the other stations. Churton is sited on level ground in agricultural pasture that extends more than 10km in every direction. However it displays the greatest bearing dependent residual after correction for twist error on all the stations. This unlikely result is probably due to the long wavelength used by the system. Local potential scattering centres are of less importance than features with dimensions that are a significant fraction of a wavelength in size.

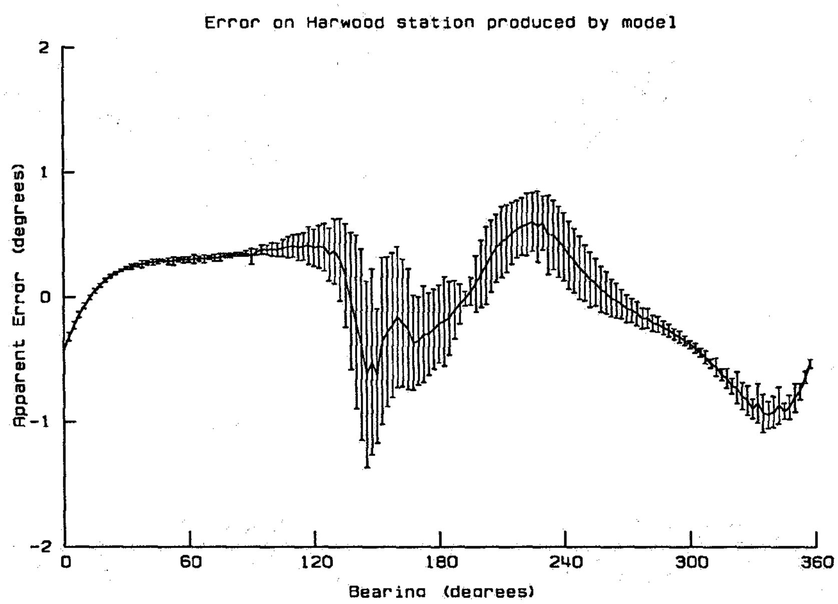

Figure 6. Apparent errors introduced on Harwood station by a twist error in Thetford station. Only Harwood, Thetford and Churton stations contributing to the analysis of 10000 simulated strikes.

IMPROVEMENTS IN PROGRESS

Although great improvements have been made in the accuracy of the system through analysis of historical data there remains a random residual which is of the order of the mean digitising error taken over all strengths of signal. Although the resolution is 0.05 degrees for the strongest signals, it decreases as the signal strength is reduced, as a consequence of the method of analogue to digital conversion of the signal. Examination of the rms residual as a function of signal strength indicates that 12 bit resolution over the entire range would reduce the rms random residual by a further factor of two. A replacement A/D converter circuit which is designed to give better than 12 bit resolution over a full dynamic range of 60 dB (power) is presently being fitted to the stations.

The present system uses a central clock for timing and is therefore dependent upon the propagation time for data along a private circuit to remain constant, hence repeat request error correction is not possible. The system is therefore very sensitive to the quality of the data circuit. Modification of the stations to include local clocks to remove this dependency is also underway. This will ultimately make the aged DEC computers redundant, and will allow more flexibility in choice of communication medium and position of future stations.

CONCLUSIONS

The development of the system from a three station system of limited coverage to a five station system with increased coverage, reliability and accuracy has been described. Particular attention has been paid to the analysis of station errors. A method of calculating the global twist errors of all stations simultaneously with optimising strike locations for all the strikes recorded by the stations, and hence extracting the true bearing dependent errors has been described.

The results of applying this method to four station data for 1989 have been presented. Application of the bearing corrections has significantly improved the accuracy of the system. Average location errors of better than 1km can be acheived at a distance of 200km from the stations. It is anticipated that further significant improvements in the accuracy of the system will result from the replacement of the existing A/D converters.

It is apparent that a degree of caution is needed when applying bearing dependent corrections which have been extracted from the apparent errors of a station without first extracting the global twist errors in the manner described in this paper.

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