WEAK POSITIVE CLOUD-TO-GROUND FLASHES IN NORTHEASTERN COLORADO

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ABSTRACT

The frequency distributions of the peak magnetic field associated with the first detected return stroke of positive and negative cloud-to-ground (CG) flashes was studied using lightning data from northeastern Colorado. These data were obtained during 1985 with a medium- to high-gain network of three direction finders (DFs). The median signal strength of positive flashes was almost two times that of the negatives for flashes within 300 km of the DFs, which have an inherent detection-threshold bias that tends to discriminate against weak signals. This bias increases with range, and affects the detection of positive and negative flashes in different ways, because of the differing character of their distributions. Positive flashes appear to have a large percentage of signals clustered around very weak values that are lost to the medium-to-high gain Colorado detection system very quickly with increasing range. The resulting median for positive signals thus could appear to be much larger than the median for negative signals, which are more clustered around intermediate values. When only flashes very close to the DFs are considered, however, the two distributions have almost identical medians. The large percentage of weak positive signals detected close to the DFs has not been explored previously. They have been suggested to come from intracloud discharges and thus are improperly classified as CG flashes. Evidence in hand, however, points to their being real positive, albeit weak CG flashes. Whether or not they are real positive ground flashes, it is important to be aware of their presence in data from magnetic DF networks.

1. INTRODUCTION

Positive cloud-to-ground (CG) lightning flashes generally constitute a small fraction of the total number of flashes striking a given region during a year. They are, however, of considerable interest because of the large currents and charge transfers that have been measured in association with some of them [1, 2]. Positive CG flashes are also interesting because of the situations in which they occur. Thus, positive flashes have been principally studied in connection with lightning discharges to structures [1, 2] and with wintertime thunderstorms in Japan [3, 4] and the Scandinavian peninsula [5, 6]. However, they also have been found in spring thunderstorms in Oklahoma [7], and summer thunderstorms in Florida [8] and Montana [9].

With the establishment of magnetic direction-finder (DF) networks, positive flashes have been identified in large numbers in the northeastern U.S. [10-12], Oklahoma [7, 13-15], and Sweden [16, 17]. In the past, sample sizes from a particular region have been rather
limited, especially those pertaining to lightning discharges that did not occur to tall buildings and towers. The new networks provide the opportunity to obtain large samples of positive flashes to study their characteristics.

A network of three DFs of medium to high gain in northeastern Colorado was used in 1981 to study the effect of local topography on the location and timing of negative CG flashes in the region. Using the same network, data on positive CG flashes were also collected during 1985. The data will be used in this paper to explore the distribution of the peak amplitudes of the magnetic field associated with positive CG return strokes in northeastern Colorado. The main emphasis is to compare signal strength distributions of positive and negative flashes.

2. FLASH DATA

The lightning detection network used in this study consisted of three DFs of medium to high gain located in the vicinity of Denver in a relatively flat area adjoining the Front Range of the Rocky Mountains; the mean distance between DFs was 81 km. The network used the commercial lightning mapping system based on magnetic direction-finder technology manufactured by Lightning Location and Protection, Inc. (LLP) of Tucson, Arizona.

The peak amplitude of the magnetic field of each flash's first detected return stroke was recorded by the system as detected by each antenna. This amplitude for each DF was then multiplied by the distance between the DF and the flash, then an average was taken for all DFs that detected the flash. This average was then normalized to 100 km by dividing by 100. The normalized values are expressed in uncalibrated units of peak magnetic radiation. This parameter is related to the peak current in the first return stroke. No attempts have been made in this study either to calibrate the signal strength values or to relate them to peak current, as the main thrust of this paper is to examine the relative magnitudes of the signals from negative and positive CG flashes.

3. PREVIOUS MEASUREMENTS OF POSITIVE FLASH INTENSITY

It appears that positive strokes can have very large peak currents and peak magnetic radiation fields, which are larger than the maximum ones for negative strokes. Flashes with very weak signals, however, can also constitute a large percentage of all positive flashes. The average values seem to be larger for positives than for negatives. It should be realized, however, that direct measurements of peak stroke currents have been made only for positive flashes to tall structures. Most of those flashes are due to upward-propagating flashes triggered by the structures. To our knowledge, no direct measurements of peak stroke currents have been made for downward-propagating positive flashes to flat ground. The natural lightning observations by Brook et al. refer to charge transfers and the continuing currents of positive flashes. The peak signal observations by networks of DFs provide indications of the relative peak currents in positive and negative strokes, and allow for the study of large samples. However, because experience with these data is limited, they should be critically examined before conclusions are drawn from the frequency distributions of signal strength. The present paper reports a study with that purpose.

4. SIGNAL STRENGTH DISTRIBUTIONS FROM COLORADO

REGIONAL SIGNAL STRENGTH DISTRIBUTIONS

Figure 1 portrays the cumulative frequency distribution of the signal strength of all positive and negative flashes lying within 300 km of at least one DF for 1985 in the Colorado network. Again, the signal strength is the magnitude of the peak magnetic radiation field associated with the first return stroke of a flash, in uncalibrated units and normalized to 100 km. Because of the large skewness and wide spread of the data, the distribution is plotted on a logarithmic scale in the ordinate. For the abscissa, a probability scale has been used.
According to Figure 1, the positive flashes have a larger median signal strength than the negatives (167 versus 94). Notice, however, that the negative frequency distribution curve is much more regular than the positive one and approximates much better a lognormal distribution (a straight line in this coordinate system). There is a larger percentage of flashes with very low peak signals in the positive than in the negative frequency distribution; notice how the positive curve lies to the right of the negative one on the left side of the graph. For example, the percentage of positive flashes with signal strengths less than 20 is 17 times that of the negatives. From 20 on, however, the percentage of weak flashes in the positive distribution decreases rapidly with increasing signal strength, so that eventually there is the same percentage of flashes with signal strengths less than 44, the crossover point, in both the negative and the positive distributions. The positive curve turns rapidly upward between signal strengths of 20 and 80, in relation to the negative curve. In that signal strength interval of the positive flash curve, there is a large percentage deficit of flashes compared with the distribution of the negatives. There are 24% fewer flashes with signal strength less than 80 in the positive distribution than in the negative. For signals strengths greater than 140 the two distributions have approximately the same slope, but because of the earlier deficit of intermediate values, the overall median of the positives is 79% more than that of the negatives. Actually, there are only three positive flashes in the sample with signal strengths greater than the largest negative signal. Compared with the negative frequency distribution, then, the positive distribution has the following characteristics:

- More flashes with weak signals (<20),
- A smaller percentage of intermediates (20-80),
- About an equal percentage of large ones (>80).

The present median values are of the same order of magnitude as those reported for the U.S. East Coast network [11], but are 29% (positive) and 27% (negative) lower. This might be due to the different DF gains of the two networks. The Colorado system is set half way between medium and high gain; the eastern system is set at high gain. Also, the present results agree with the observations [17] in Sweden, where many positive strokes had rather large or rather weak signals, but not many had intermediate strengths.

However, the frequency distributions of Figure 1 for a large region around the DFs should be considered with caution. Although these curves probably preserve the overall characteristics of the distribution of signal strength, they also contain sampling and measuring biases. For the correct comparison of the signal strengths of positive and negative flashes, these different biases and their effects on the measured distributions of signal strength should be explored. Figures 2 and 3 illustrate some of these biases for the Colorado network. The lower series of points in each figure portrays the variation with distance of the minimum of all the normalized signals detected at the same range by all DFs. The upper series of points represents the maximum, and the middle one the average. To produce these figures, each flash in the study area was assigned to a particular range increment relative to each DF that detected it. Thus, the same flash could have been assigned to two or three different range increments corresponding to the different DFs. For convenience, the
flashes falling in the same range increment with respect to all three DFs were considered together in determining the parameters of Figures 2 and 3. The range increments were selected in such a way that the areas of the annuli they define are always the same; thus, radially, they are of decreasing magnitude. In this way, range samples have roughly the same number of flashes (this is not exactly true, of course, because the density of flashes is not uniform in the area of study). This precaution is desirable when comparing the extreme values (maxima and minima) of samples from populations that have slowly rising and decaying frequency distributions. The box-and-whisker diagrams are discussed below.

**FIGURE 2.** Variation with distance of minimum, average, and maximum of all normalized signals of negative flashes detected at the same range by all DFs. Note scale break in ordinate. Superimposed are box-and-whisker depictions of frequency distributions at related range intervals. Adjacent rectangles represent second and third quartiles; open circles represent maximum and minimum values. Range increments and boxes decrease in width with range because they represent annuli of equal areas.

**FIGURE 3.** Minimum, average and maximum signal strengths, as in Figure 2, but for positive flashes.

**MINIMUM DETECTABLE SIGNAL BIAS**

One of the most important sampling problems is the inevitable bias with range toward flashes with very strong signals. Because of the range dependency of the magnetic field of a flash, weak signals coming from distant ranges are not detected, as their magnetic fields reach the antenna with a strength that is below the threshold level of the DF. This level is set above the ambient noise level. As the distance increases, progressively stronger signals are lost in this way. This effect is illustrated by the lower curves in Figures 2 and 3, where the minimum detected signal strength increases linearly with radius for both positive and negative flashes. Notice the progressive erosion with range of detected weak signals indicated by the minimum curve. The minimum values for the positive flashes appear to be slightly larger than those for the negatives. This is so because the acceptance threshold, the minimum voltage that needs to be generated in the antennas before a signal is accepted for processing, was set by LLP at around 350 mV for positive flashes, and 120 mV.
for negatives. A higher threshold for the positives was set by LLP to eliminate weak positive signals that were coming from very distant flashes (>1000 km) that were assumed to be ionospheric (inverted) reflections of negative ones.

**DF SATURATION BIAS**

By contrast, the maximum values detected by the DFs (upper curves) do not seem to be affected as much with range as the minimum values (note a factor of ten difference between the upper and lower scales of the figures). These signals are way above the detection threshold, so that even when coming from 300 km they are still picked up by the DFs. Only three values in the positive curve are above the maximum value in the negative one. However, there is a slight drop in the values of the detected maximum signals from about 100 km inwards towards the DFs. This reduction could be due in part to another bias, the problem of DF saturation. Flashes striking close to the antennas can saturate the electronics of the DFs even if their normalized signals are not too high; the closer the flash position to a DF, the smaller the signal required to achieve saturation. Saturated detections were not considered in deriving the maximum signal curves. In Figures 2 and 3, the closest ring goes out to 39 km, and the maximum normalized signal observed within that ring for the negative flashes was 523. According to the linear decay of signal strength with range, that would give a value for the maximum absolute signal that a DF can detect before saturating of 1350 LLP units, which is a reasonable number for these systems. By 55 km (the extent of the second ring), that saturation threshold could tolerate a normalized signal of 739 before saturating the DF. The observed maximum for both positive and negative flashes in the second ring can be seen from Figures 2 and 3 to be considerably under that value. So, although the effect of saturation might account for some of the apparent reduction in maximum signal measured from 100 km inwards, the sampling variability and regional distribution of flashes are probably the major factors. A very sharp decrease in maximum signal values, however, could have been detected if the closest rings had been much smaller.

There is a striking difference between the curves depicting the average signal strengths (middle curves), in contrast to the basically similar character of the maximum and minimum curves of positive and negative flashes. The curve for negative values increases monotonically and smoothly with range, almost paralleling the curve for minimum values. The curve for the positives, however, although it starts out with an average that is similar to that of the negatives, jumps very quickly by 140 km to more than twice the value at close ranges (equivalent to that attained by the negatives at 300 km). From that point on, the increase is gradual and follows the increase in the minimum values. Thus, if the frequency distributions for negatives and positives are compared at close range (say, less than 70 km) they appear to have the same mean and extreme values. Throughout most of the region, however, the positives appear to have a very much larger mean, although the extremes are similar.

**DIFFERENT RESPONSES OF POLARITY DISTRIBUTIONS TO BIASES**

Why is there such a difference between the two distributions at different ranges? Figure 3 displays box and whisker representations of the frequency distribution of the positive signal strength for different range intervals. Notice how the first two quartiles of the portrayed distribution closest to the DFs lie very low on the signal strength scale. The quartile levels rise rapidly in the second distribution, but in the next two the rise is gradual and parallels the rise in minimum value. Thus, it appears that the distribution of positive signal strengths observed closest to the DFs contains a large proportion of very weak signals. If the distribution of positive signals over the entire region was the same, such weak signals (most of the first quartile) would quickly become undetected with range, modifying greatly the character of the observed distribution. The negative flashes, however, have a more regular distribution that is not so skewed toward the weak signal strengths. Therefore, the effect of losing the weak signals with range would be reflected in a more gradual change in the distribution with
range. Recalling that the acceptance threshold for the positive flashes was about three times higher than for negatives, the true distribution of positive signals corresponding to the first box-and-whisker representation of the graph probably had an even higher percentage of weaker values. Also, because of the different acceptance thresholds, the erosion of weak values with range is much faster in the case of the positives. Even if the thresholds were the same, however, the effect of the detection bias would have been different for each polarity because of the even more marked disparity in the proportion of weak flashes between the two. Although, conceivably, there are geographical inhomogeneities in the signal strength distributions, these two different responses (both because of the different thresholds and the different basic distributions) to the detection bias of weak signals are obviously an important factor in explaining the difference between the curves depicting the average signal with range for positive and negative flashes.

**SUMMARY OF SAMPLING BIASES**

Thus, although the frequency distributions portrayed in Figure 1 for a large region around the DFs probably preserve the overall characteristics of the distribution of signal strength, they are biased toward flashes with strong signals and do not give enough weight to the weak flashes. Furthermore, since it appears that the positive and negative flashes have different signal strength distributions, especially for weak flashes, the range bias has quite different effects on the signal strength distributions of flashes of different polarities. In addition, there is a bias against large signals caused by the saturation of the DFs by flashes that strike close by. This effect is most important for flashes that fall within a few tens of kilometers from the DFs. Although those flashes are detected, their position and normalized signal strength cannot be accurately determined unless three DFs detect them (one saturated and two normal detections).

It should be emphasized, however, that the particular way in which the sampling and measuring biases affect the signal distributions with range depends on the gain of the network. The Colorado network used in this paper has a medium-to-high gain. In networks with lower gain, the range detection bias would produce an even more dramatic modification of the signal strength distributions with range. Networks with high gain would see weaker flashes much farther away and the modifications to the signal distributions would be noticed only at larger distances, although the saturation effect will be worse at high gain. On the other hand, lower gain networks are usually designed to cover small areas, while higher gain networks are intended to monitor regions of a large extent.

It is virtually impossible to obtain a totally unbiased sample of flashes detected by a DF network to study their signal strength distribution. However, one could obtain a sample from an area that is close to the DFs and thus minimize the effect of the range attenuation, but not so close to the DFs that the saturation problem is severe. Figure 4 shows a plot of the frequency distribution of such a sample for positive and negative signals in log-probability coordinates for flashes that lie no farther than 60 km from at least one DF and not closer than 20 km from any DF. Flashes that saturated one DF but were detected correctly by two others were included in the two samples, as in this case both flash location and normalized signal strength can be estimated from the two non-saturated DFs. Saturating flashes detected by only two DFs were not considered. The two resulting distributions are probably close to the unbiased ones, although some deterioration has

![Figure 4](image-url)
undoubtedly already occurred in the chosen interval. These frequency distributions, however, should correspond better to the original, unbiased populations of positive- and negative-flash signal strengths in the area of study. Thus, a comparison of positive and negative flashes based on these distributions should be more physically meaningful than one based on the distribution of flashes over the entire area covered by the network and contaminated by different sampling and measuring biases. Having obtained samples that are more representative of the true populations, we proceed to compare the signal strengths of negative and positive flashes.

5. COMPARISON OF POSITIVE AND NEGATIVE SIGNALS

As anticipated in Figures 2 and 3, the distributions of Figure 4 have basically the same median. The positive distribution, however, has a much larger percentage frequency of small signals, especially for signal strengths less than 20. Above that value, there is a rapid decrease in relative frequency that extends to a signal strength level of 100. After that point, the relative frequency of the larger values increases compared with the previous curve segment; observe how the curve turns toward the right compared with the segment between the signal strength values of 20 and 100. By comparison, the negative distribution seems much smoother and more regular.

Figures 5 and 6 show the distributions of Figure 4 in histogram form for the close-in range of 20 to 60 km. The positive distribution is highly skewed towards very small signal values, having the maximum frequency between 20 and 30. The negative distribution, on the other hand, has a maximum frequency at signal strengths between 60 and 70, and the skewness is not as large. It should also be kept in mind that because of the higher acceptance threshold for positive flashes mentioned above, the true proportion of weak positive signals could be even higher.

The positive distribution, then, appears to be markedly different in shape from the negative one, showing a large percentage of very weak flashes and a small percentage of intermediate ones. Although this result is hinted at in the observations of [17] in Sweden and [3] in Japan, to our knowledge, the importance of this high percentage of weak positive flashes in the distribution has not been emphasized before.

Before the physical nature of these weak positive flashes is discussed, however, it is important to consider how representative are the resulting frequency distributions of signal strength of the thunderstorms in northeast Colorado during the summer. As can be seen from Figures 5 and 6, the most marked difference between positive and negative flashes lies in the frequency of signals at or below 40 LLP units. A total of 216 positive flashes were

![Figure 5](image1.png)

**Figure 5.** Normalized peak signal strength for negative flashes within 20-60 km of a DF.

![Figure 6](image2.png)

**Figure 6.** Normalized peak signal strength, as in Figure 5, but for positive flashes.
observed in that range on 46 different summer
days. Thirteen of those days had three or more
of the weak positive flashes, and eight of
those 13 days had seven. The weak flashes
have their peak frequency between 1500 and
and 1600 MST, coinciding with the peak diurnal
convective activity in the region [18]. These
flashes are also fairly well distributed around
the DFs, although their frequency drops
dramatically with range after about 60 km
from the antennas as explained above in
connection with Figure 3. In addition, when the
frequency distributions of both positive and
negative flashes of all signal strengths are
computed for a ring of 20 to 60 km around each of
the three DFs separately, the three
distributions for each polarity are very similar
to each other and the two sets are very
similar to Figures 5 and 6. Thus, the weak
positive flashes are not the result of just a few
atypical storms in a few particular localities,
but come from thunderstorms that are part of
the regular diurnal cycle of convection
throughout the summer. Also, within the
region where they can be detected, the weak
positive flashes are geographically well
distributed in numbers and belong to fairly
representative signal strength distributions.

It has been suggested, however, that
the very weak positive signals close to the DFs
are not from cloud-to-ground flashes [17]. In the
absence of independent evidence showing that
these signals indeed come from cloud-to-ground
positive flashes, there are basically three
possible explanations for these results:

a. The weak signals do come from positive
CG flashes and because of their weak signal
strength are detected only when a flash occurs
very close to a DF.

b. The weak signals come from intracloud
discharges with waveshapes similar to in-
verted return strokes, are detected by the
positive stroke circuits in the DFs, and are
improperly classified as positive cloud-to-
ground lightning. Since these are intracloud
discharges, their signal amplitudes are small
relative to cloud-to-ground flashes and are
preferentially detected only near the DFs.
Because the waveshapes are similar to those of
real cloud-to-ground strokes, it would be very
difficult to discriminate against them except by
further increasing the acceptance threshold for
positive flashes. It is strange, however, that

similar intracloud discharges with negative
return stroke waveshapes are not being detected
(see frequency distributions in Figures 5
and 6). Also, it must be remembered that these
signals refer to flashes that were detected by
two or three DFs with a mean separation of
about 80 km. Very weak intracloud discharges
close to an antenna would not have been
detected by another at those distances.
Furthermore, a study [22] of the correctness of
the polarity assignments of direction-finding
equipment similar to the one used in the present
study but for a high gain system, concluded that
the acceptance criteria for positive flashes are
adequate, provided that the distances of the
flashes to the DFs are less than 600 km.
Another study [23] recently considered positive
flashes detected by a similar DF network and
examined the waveforms simultaneously
detected by an extremely low frequency (ELF)
system. The conclusion was that no more than
15% of the positive flashes detected by the DFs
with signal strengths of less than 50 are false
detections. The percentage is probably even
lower, the study concluded, because the very
weak positive flashes produce ELF signals that
are close to the ELF system’s noise level.

c. These are signals with waveshapes
that do not correspond to return strokes (in
contradistinction to “b” above) and should be
rejected by the positive stroke detection circuit
as not fitting the waveshape criteria for cloud-
to-ground strokes, but because of their small
signal strengths they are not recognized as such.
These signals could well come from very weak
but legitimate intracloud flashes which are
being improperly classified as CG flashes or
they could come from non-stroke discharges
such as k changes. In this case, a modification
to the waveshape criteria logic could filter out
these weak signals.

More work will have to be done to
explain the large percentage of weak positive
detections. In view of the growing importance
of lightning data and the proliferation of
lightning detection networks, the problem
should be given serious consideration when the
data are used for operational and research
applications. If the weak positive signals are
shown to come from real cloud-to-ground
positive flashes, it should be realized that
there is a larger proportion of weak, positive
flashes to ground than would appear from
previous studies and that, when all things are considered, the median signal strength values are about the same for positive and negative flashes. It also should be realized that, at least for medium- and low-gain systems, storms near the DFs would appear to have a larger proportion of positives flashes than would storms farther away. The details of the deformation with range of the frequency distributions of signal strength would depend on the particular configuration of the network and to some extent on the conductivity characteristics of the surrounding terrain.

If the weak signals are shown to be from intracloud or cloud-to-cloud discharges (with return-stroke-like waveshapes or not), then it should be realized that, as far as positive flashes are concerned, data from closer than about 100 km from a DF might give an erroneous picture of cloud-to-ground lightning activity.

6. CONCLUSION AND DISCUSSION

Using the lightning data of one entire year from northeastern Colorado, a comparison has been made of the magnitude of the peak signal strength of the first return strokes of positive and negative flashes. When the data from a large region around the DF (300 km) are considered together, the positives appear to have a median signal strength that is almost twice as large (1.8 times) as that of the negatives. The overall sample, however, tends to indicate that the two resulting distributions are quite different in regard to the very weak signals. These signal strength distributions based on positive and negative flashes from widely different distances, however, contain some sampling and measuring biases. One of the most important sampling problems is the inevitable bias with range against flashes with weak signals. This range bias has very different effects on the sampling of flashes of different polarity owing to the basic difference in the skewness and spread of the original distributions. In addition, there is a bias against large signals caused by the saturation of the DFs by flashes that strike close by.

The particular way in which the different sampling and measuring biases affect the signal distributions with range depends on the gain of the network. The Colorado network used in this paper has a medium-to-high gain. In networks with lower gain, the range detection bias would produce an even more dramatic modification of the signal strength distributions with range. Networks with high gain would see weaker flashes much farther away and the modifications to the signal distributions would be noticed only at larger distances.

Frequency distributions of the peak signals of flashes observed a small distance from the DFs best reveal the fundamental differences between negatives and positives. Both appear to have the same median signal strengths, but the positive sample has a larger proportion of very small signals. The positive sample lacks the large percentage of signals of intermediate values that are frequent in the negative group. The two distributions are more similar in their relative frequency of large values, but the positive sample, although it is about 15 times smaller, contains the largest values of peak signal.

That some positive strikes can transfer to earth very large charges and have very large currents is not denied by the present data set. It should also be kept in mind that positive strokes have longer time to peak current, and large continuing currents following the peak discharge. However, once the range bias is taken into account by considering only flashes close to the DFs, the present Colorado data tend to indicate that, relative to the negative, the positive distribution appears to have a larger percentage of flashes with very small values, a smaller percentage yielding intermediate ones, about the same proportion of large values, and a few flashes with values that are larger than any in the negative distribution. The medians of the two distributions, however, are about the same.

The larger percentage of weak signals detected close to the DFs has not been emphasized earlier, nor are there any independent data obtained by other methods in Colorado for studying their physical nature. The results of [23], however, lend weight to their being real cloud-to-ground positive flashes. In any case, the results reported here should be taken into account when positive
flash data from magnetic direction finders are used, especially for medium-and low-gain systems. It appears that frequency distributions for positive and negative flash signal strength are indeed different and that the detection bias with range, inherent in the system, produces different results as it operates on the different distributions.

ACKNOWLEDGEMENTS

The Program for Regional Observing and Forecasting Services (PROFS) of NOAA in Boulder, Colorado operated the network and collected the lightning data used in this study. The assistance of R. Ortiz of NSSL, Boulder, in plotting a portion of the data is appreciated.

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