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PC Programs for the Prediction of the Linear Stability Behavior of Liquid Propellant Propulsion Systems And Application to Current MSFC Rocket Engine Test Programs

VOLUME I

G. B. Doane III
W. Armstrong

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G. B. Doane, III
W.C. Armstrong

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SUMMARY

MSFC is participating in the current U.S. resurgence of interest in new liquid propellant rocket propulsion systems. In the course of this work modification of existing MSFC test stands is in process. As a part of the activation of the new liquid propulsion test facilities it is necessary to analyze total propulsion system stability. To accomplish this, several codes have been built to run on desktop 386 machines. These codes enable one to analyze the stability questions associated with the propellant feed systems. In addition, the work of Dr. Mitchell at Colorado State has been adapted to this computing environment and furnished along with the other codes. This latter inclusion furnishes those interested in high frequency oscillatory combustion behavior (that does not couple to the feed system) a set of code for investigations of proposed liquid rocket engines.

INTRODUCTION

As the decade of the 1980's ended there was a resurgence of interest in the United States in advanced liquid propulsion systems. This resurgence manifested itself in the inauguration of a number of programs by both the USAF and the NASA. These programs take the form of both analytical work to take advantage of the vast increases in computer power which occurred during the decade of the 80's and some experimental work with new or modified engines. At the Marshall Space Flight Center (MSFC/NASA) test stands which have lain dormant since the sixties are being activated and modified. Because of the extensive modification of the test stands e.g., new tanks and lines, and the new engine design(s) to come, the MSFC needs the capability to predict stability of the new stand and engine combination(s). In addition, as research continued it became apparent that several codes for intrinsic engine stability (i.e., the high frequency) investigations were available and could be adapted for execution on the same PC 386 class of machine as the other stability codes developed under this effort. Thus the thrust of this work has been to gather the specifics of the technology applying to all of the major anticipated stability problems attributed directly to a propulsion system that might occur on the new or revamped MSFC test stands, install them on 386 class desktop computers and apply them (as first users) to the new stands and engines taking form at MSFC.

References in this field are legion. Text books such as references one and two are available. A well known comprehensive coverage of the work available through the early 1970's edited by Messrs. D. T. Harrje and F. H. Reardon, is found in NASA SP-194 (reference three which contain 778 references). Discussions with engine manufacturers and with Dr. Reardon of the University of California in Sacramento reveal a consensus that stability prediction theory has not changed a great deal since SP-194 was published. Thus the low and intermediate frequency stability analyses are based upon that technology. We have drawn upon the very recent work of Professor Mitchell at Colorado State (USAF funded; see references four, five and six) for the high frequency stability modeling and prediction codes.

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In his two volume report, Professor Mitchell and his group have extended greatly their earlier work (see reference four). This report contains 54 references, a fair number of which are dated after those of SP-194 i.e., after circa 1968. In this latest work, combustion chambers of rectangular as well as circular cross section, are analyzed. In addition, quarter wave, Helmholtz and arbitrary geometry (L,T etc.) dampers are covered as are damping chamber liners. In this latest stability code virtually any combination of these devices may be present simultaneously in the combustion chamber in rather arbitrary locations e.g., there may be a partial liner in the combustion chamber while there are simultaneously resonators around the periphery of the injector end of the combustion chamber. Because Mitchell's work covers only high frequency stability questions dealing with combustion chamber design and not test stand design or modification it is of most interest to engine designers or those monitoring the development of new engine designs (or the modification of existing engine designs).

Professor Mitchell and the USAF (Mr. J. Levine AFSC Edwards Air Force Base) have been most helpful in providing access to his work especially by providing copies of his source code on computer media.

Our primary goal then was to create a family of programs running on tabletop computers that are of immediate use to engineers designing test stands destined for use with new liquid propellant propulsion systems. These programs have been formulated and installed in such a way that engineers without exhaustive backgrounds in fluid dynamics, stability theory and advanced mathematics will find them useful to the point of turning to and using them during the design and the checkout phases of their new propulsion hardware initiatives. In addition, we have concatenated to these programs those of Professor Mitchell, modified for the desktop environment, for the use of those people interested in high frequency combustion stability questions.

THEORY

Directly associated with an engine and propellant feed system there are generally taken to be three separate stability regimes. They are classified by the frequencies at which they tend to oscillate. Reasonably enough they are called the low, intermediate and high frequency regimes.

While in concept the analysis of all three regimes could be lumped into one extremely large set of code, in practice separate modules or programs are used for each regime. A further distinction between the regimes occurs in that, as will be developed, the low and intermediate regimes involve an intrinsic interaction between the combustion chamber processes and the propellant feedlines while the high frequency regime phenomena are usually confined to the immediate environs of the combustion chamber i.e. the combuster.

For these reasons the test stand designers may well want to pay particular attention to the two lower frequency stability regimes.

In what follows linear analysis is used. This is permissible if the quantities in question (e.g. mass flow rate and pressure) are assumed not to deviate very far from their set point (mean or average) values. Thus stability is evaluated, for example by investigating whether or not a pressure or mass flow rate, once perturbed, will return to its average or mean value i.e., whether or not the perturbation or small deviation vanishes with time. Because of this linear approach all the analytical tools of linear stability analyses are available. Also, because of this approach a number of analyses of any given hardware system will have to be performed; each corresponding to a different operating condition of the propulsion system. Different conditions would include such things as different amounts of liquid in the propellant tanks, operation with different mixture ratios and different valve settings.

THEORY OF PROPELLANT FEED LINES

To characterize dynamically a propellant feed system it is necessary to treat it in the same vein as other circuit problems. The circuit elements encountered may be categorized as follows. Distributed lines (incorporating the effects of distributed hydraulic capacitance and inertance), discrete hydraulic capacitors (used to model the engine manifolds and the propellant tanks), hydraulic resistors (which are used to characterize such things as the injectors through which propellant is introduced into the combustion chamber) and propellant pumps.

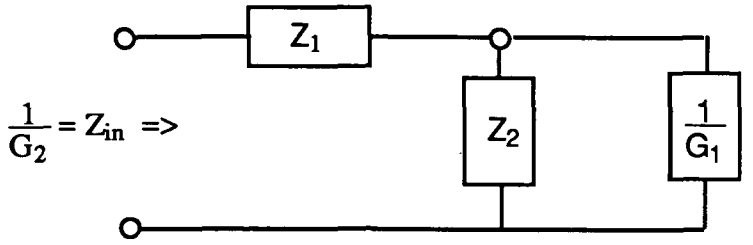
The methodology suggested in section 5.4.1 of SP-194 is used as follows. The admittance, G, at any point in the feed system is defined to be, in general, a complex quantity given as the ratio of the mass flow rate perturbation to the local pressure perturbation. Also, in general, any "downstream" admittance is a function of an "upstream" admittance. To illustrate the last point, it will be shown that the admittance of the feedline attached to a tank is a function of the admittance "looking into" the tank. With the foregoing in mind, the admittance as seen "looking" from the combustion chamber system toward the tank is formed by the following product of admittance ratios.

$$G_i = G_0 \left(\frac{G_1}{G_0}\right)\left(\frac{G_2}{G_1}\right) \cdots \left(\frac{G_m}{G_{m-1}}\right)\left(\frac{G_i}{G_m}\right)$$

where each ratio represents the admittance ratio associated with a given feedline circuit element. Typically one starts at the tank, considered to be a large, here constant pressure, liquid source and work one's way down the piping to and through the engine injectors. These admittance expressions are a key element in evaluating both the low and intermediate stability modes of the engine - propellant feed system.

In the work which follows immediately below, it is convenient to have at hand a general expression for the input admittance ratio of a certain network. This expression may be used to characterize all the combinations of discrete hydraulic circuit elements of interest. This general network is shown below

where the Z's represent linear impedance $\left(\frac{p'}{\dot{m}'}\right)$ and G's the linear admittance $\left(\frac{\dot{m}'}{p'}\right)$.



All the circuit elements will be taken to have sign convention corresponding to passive circuit elements. As will be pointed out below this needs very special interpretation in the case of a pump. Combining Z_2 and $\frac{1}{G_1}$ in parallel and then summing with Z_1 (after a little algebra) produces

$$\frac{G_2}{G_1} = \frac{\left(1+\frac{1}{Z_2G_1}\right)}{\left(1+\frac{Z_1}{Z_2}\right)+Z_1G_1}$$

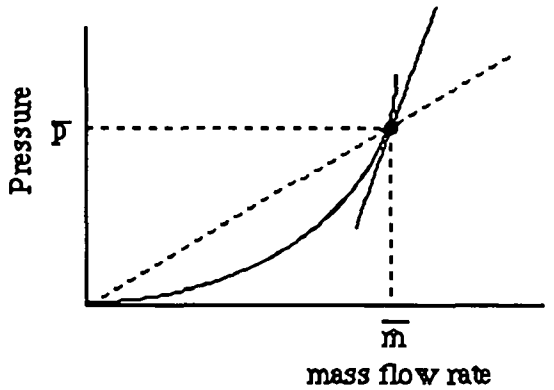
Note that if it is desired to open up a circuit element (i.e., no flow through it) then the appropriate Z is set equal to ∞ (or G to 0). If a circuit element is to present no obstacle to flow then its Z is set equal to 0 (or G to∞).

HYDRAULIC RESISTANCE

The first derivation will concern itself with an energy dissipating hydraulic resistance used to characterize an orifice. First consider the development of the linearized relationship between pressure drop and liquid flow through an orifice. This relationship is usually characterized by a square law relationship, i.e.

$$p = R\dot{m}^2$$

such that



$$R=\frac{\bar{p}}{(\bar{\dot{m}})^2}$$

The incremental resistance to flow at the operating point (defined by $\bar{p}$ and $\bar{\dot{m}}$)is taken to be the slope of the p versus $\dot{m}$ curve evaluated at $p=\bar{p}$ and $\dot{m}=\bar{\dot{m}}$.

Thus

$$r'_*=\frac{\partial p}{\partial \dot{m}}\Big|_{\dot{m}=\bar{\dot{m}}}^{p=\bar{p}}$$

and from before

$$R=\frac{\bar{p}}{(\bar{\dot{m}})^2}.$$

Substituting yields

$$r'_*=\frac{\partial}{\partial \dot{m}}(R\dot{m}^2)=2R\dot{m}\Big|_{\dot{m}=\bar{\dot{m}}}^{R(\bar{p},\bar{\dot{m}})}$$

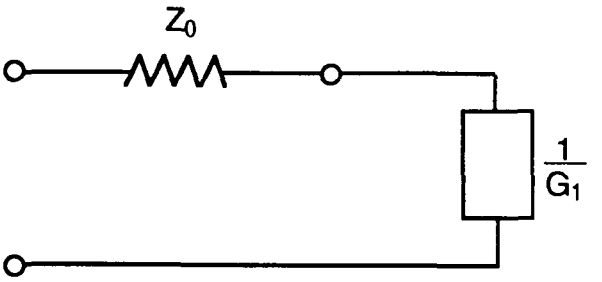
so

$$r'=2\left(\frac{\bar{p}}{\bar{\dot{m}}^2}\right)\bar{\dot{m}}=2\frac{\bar{p}}{\bar{\dot{m}}}$$

In all this work the pressure is normalized by the average combustion pressure $(\bar{p}_c)$ and the mass flow by the <u>total</u> propellant average mass flow $(\bar{\dot{m}})$, Thus the normalized resistance is

$$r' = \left(2\frac{\bar{p}}{\bar{\dot{m}}_L}\right)\left(\frac{\bar{\dot{m}}}{\bar{p}c}\right)$$

where $\bar{\dot{m}}_L$ is the average mass flow rate through the resistor. This derivation holds for a short orifice i.e., one in which the transit time of the fluid through the orifice is short. With this expression for resistance in hand, the admittance ratio is developed. A diagram is helpful at this point.



Orifice Circuit

From the figure it is seen that $Z_1=Z_0$ and $Z_2=\infty$ in the general formulation. Thus the input admittance ratio is

$$\frac{G_2}{G_1} = \frac{1}{1+Z_0G_1}$$

HYDRAULIC CAPACITANCE

Hydraulic capacitance is needed to account for the compressibility of the propellant liquids. For a lumped capacitance an analogous development to that of the resistor holds with the exception that time rates of change of pressure must be introduced i.e., the dynamics of fluid compression govern the dynamic behavior of the element. By definition the dynamics of a lumped capacitor are governed by

$$\frac{dp'}{dt} = \frac{\dot{m}'}{c}$$

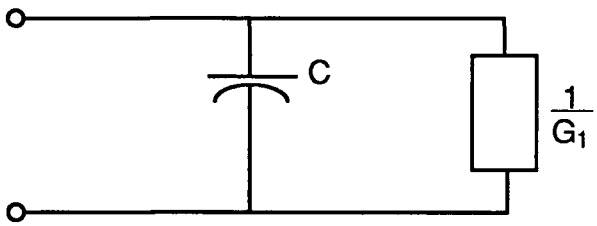
Under the assumption that forced solutions are of paramount interest here, the initial conditions applying to $\dot{p}'$ and $\dot{m}'$ are assumed zero yielding the LaPlace transformed equation as

$$\dot{p}(s) = \frac{\dot{m}'(s)}{c\,s}$$

where

$$s \triangleq \text{LaPlace Operator}$$
$$p'(s)=L[p'(t)] \text{ assumed to exist}$$
$$\dot{m}'(s)=L[\dot{m}'(t)] \text{ assumed to exist}$$

Once again a picture is helpful as



Hydraulic Capacitor Circuit

As before, from inspection, $Z_1=0$ and $Z_2=\frac{1}{cs}$. substitution yields

$$\frac{G_2}{G_1}=1+\frac{cs}{G_1}.$$

Two observations are in order. First consider the defining differential equation

$$\frac{dp'}{dt}=\frac{\dot{m}'}{c}.$$

From this it is evident that for c very large, as might occur with a propellant tank characterization, it will be difficult for p' to change. A second observation is evident from the transform of the differential equation i.e.,

$$p'(s)=\frac{\dot{m}'(s)}{cs}.$$

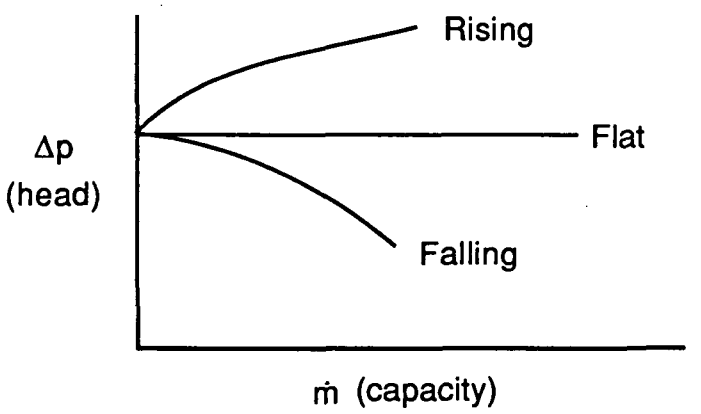
Letting s=jw to consider frequency response effects shows

$$p'(j\omega)=\frac{\dot{m}'(j\omega)}{j\omega c}.$$

Thus as $\omega \to 0$ $\dot{m}'(j\omega) \to 0$. When characterizing a tank then care must be taken if the frequency of interest is very low. In that case additional characterization should be considered e.g. perhaps an orifice or other purely real admittance should be added to the description.

PROPELLANT FEED PUMP

Following the methodology of SP-194 a lumped parameter characterization of a pump may be formulated. Some contemplation of a pump is first in order. The same pump may, depending upon the operating point, have either a rising, flat, or falling pressure change versus fluid flow characteristic. Graphically this may be shown as below.

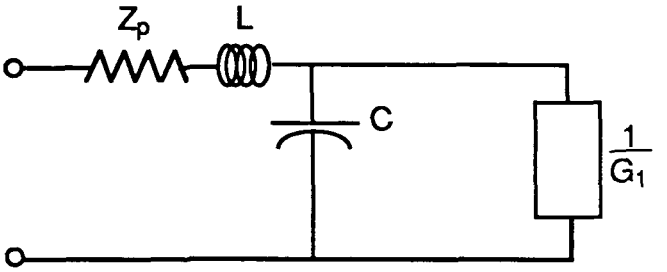


From the figure it is clear that the slope of

$$\frac{dp'}{d\dot{m}'}$$

may be greater, equal to or less than zero (this sign may very well vary with a given pump's operating point). Thus the pump impedance corresponding to the head-capacity relationship may be taken to be a resistor but one whose sign may not be "taken for granted". Clearly there is fluid in the pump. Therefore, it is necessary to associate with a dynamic pump model some inertance and capacitance. Numerical values of these quantities probably have to be gotten from dynamic testing. Such things as cavitation effects could cause values all

out of relationship to the nominal amount of fluid in the pump. The equivalent pump circuit is as shown.



Hydraulic Pump Circuit

By inspection

$$Z_1 = Ls + Z_p;\ Z_2 = \frac{1}{cs}$$

thus

$$\frac{G_2}{G_1} = \frac{1+\frac{cs}{G_1}}{1+(Z_p+Ls)cs+(Z_p+Ls)G_1} = \frac{1+\frac{cs}{G_1}}{1+(Ls+Z_p)(cs+G_1)}$$

or

$$\frac{G_2}{G_1} = \frac{1+\frac{cs}{G_1}}{1+Z_pG_1\left(1+\frac{cs}{G_1}\right)(1+tps)};\ t_p = \frac{L}{Z_p}$$

Note: in this formulation (to be consistent with the general formulation)

$$Z_p = \frac{\partial p}{\partial \dot{m}}\Big|_{\substack{p=\bar{p}\\ \dot{m}=\bar{\dot{m}}}} > 0$$

It may <u>very well</u> be that

$$Z_p = \frac{\partial p}{\partial \dot{m}}\Big|_{\substack{p=\bar{p}\\ \dot{m}=\bar{\dot{m}}}} < 0$$

In which case a negative number should be inserted for Z_P in the admittance ratio.

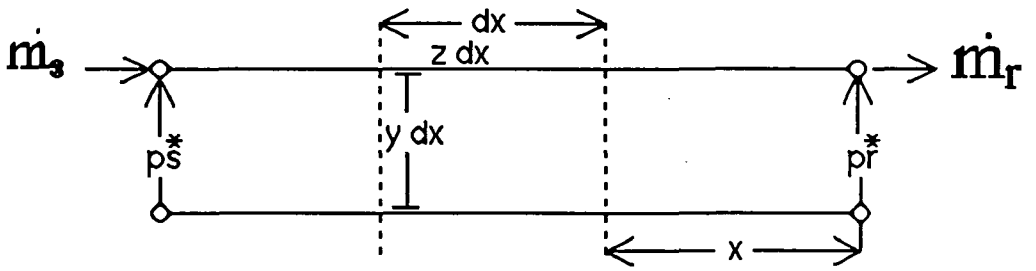
PROPELLANT DUCT

Another major non-active propellant delivery circuit element for which the admittance ratio is needed is the pipe or duct which carries the liquid from element to element. The following development is for a pipe of uniform cross section. Pipes of nonuniform cross section may well require the application of an ad hoc numerical solution for each one (there are some exceptions e.g., incompressible flow in a rigid wall pipe). The following development will be started including in the development the effect of a resistive pressure drop. However initial calculations on one proposed MSFC test stand design showed very low values of such drop and hence at the appropriate time they were dropped as being negligible. However the development can be readily expanded to include the resistive effect.

Partial differential equations may be used (for an alternate treatment see Appendix A) to treat the distributed nature of the feedline. One could, of course, approximate the feedlines by an interconnected set of discrete resistors, capacitors and inertances. But aside from the approximation (which could be made as negligible as desired by adding more elements) it is arguably more computationally efficient to use the results of the continuous representation.

The dynamic characterization of a constant cross section fluid duct with rigid walls will be accomplished with three objectives in mind. The first will be to characterize a lossless duct to obtain its admittance ratio. Then a short exposition will be included to indicate the straightforward way in which the analysis may be expanded to include dissipating (distributed resistance) effects. Lastly will be developed expressions for the standing pressure and flow waves to be found along a duct (pipe).

The dynamics of the pressure flow relations in a duct are governed by the one dimensional wave equation. The development is begun by deriving the wave equation. Refer to the figure which depicts an infinitesimally small segment of a uniform duct



where

$Z \overset{\Delta}{=}$ per unit length series impedance (resistance, inertance)

$y \overset{\Delta}{=}$ per unit length shunt admittance (typically conductance, capacitors)

note that x is measured from the receiving end of the duct.

By inspection the pressure drop across the element, along the duct is

$$dP = \dot{m}z\,dx \quad \text{or} \quad \frac{dP}{dx} = \dot{m}z$$

and as the liquid flows through the element there is a decrease

$$d\dot{m} = Py\,dx \quad \text{or} \quad \frac{d\dot{m}}{dx} = yP$$

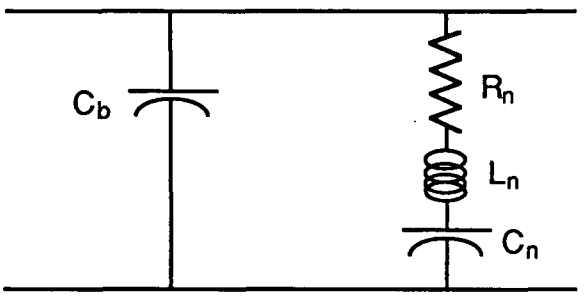
Note in these equations that in general $P, \dot{m}, y,$ and z are either functions of x or the LaPlace variable S or both. The most usually used expression for Z in transmission line network theory is probably

$$z = Ls + R$$

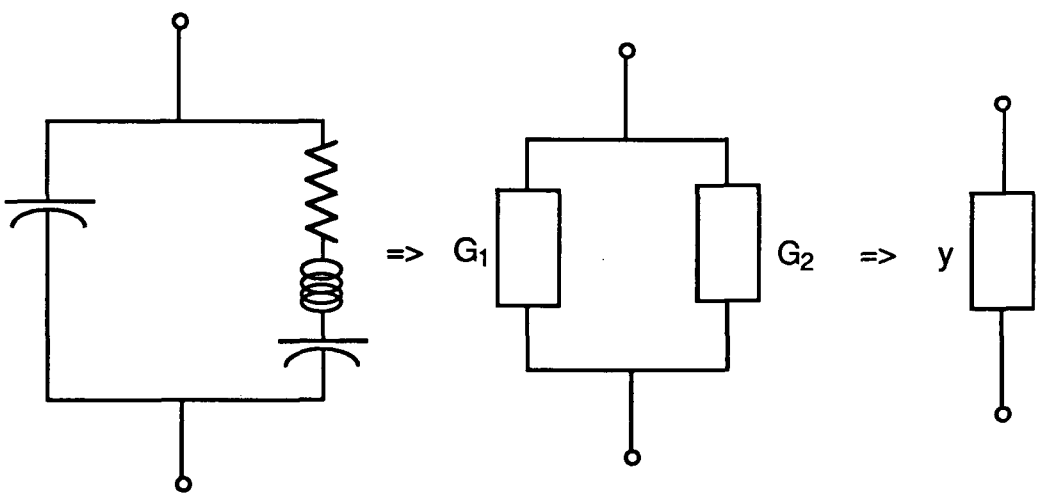
and for the admittance

$$y = G + cs$$

Some discussion of y is in order. The use of shunt capacity and conductance is in accord with usual network theory models. However, unless the duct leaked uniformly along its length, G, the conductance, would be set equal to zero for the purposes of this work. But it should be borne in mind that the use of other admittance functions has been suggested. For example, if a duct has internal damping such as might be provided by a liner which could be modeled as a set of distributed Helmholtz resonators then the following model has been proposal (in SP-194 on p. 274, Section 6.2.5.4) for the shunt admittance.



In this case y may be computed as follows.



$$y = C_b s + \frac{1}{R_n + L_n s + \frac{1}{C_n s}}$$

$$y = \frac{L_n C_n s^2 + R_n C_n s + 1}{C_n s} + C_b s$$

$$y=\frac{(L_nC_n+C_bC_n)s^2+R_nC_ns+1}{C_ns}$$

Returning to the overall derivation the derivative of each equation is taken with respect to x and the results substituted to obtain the one dimensional wave equations for pressure and flow as

$$\frac{d^2P}{dx^2}=zyP$$

$$\frac{d^2\dot{m}}{dx^2}=zy\dot{m}$$

Assume a solution for the pressure of the form

$$P=Ae^{\alpha x}$$

thus

$$\frac{dP}{dx} = A\alpha e^{\alpha x} \text{ and } \frac{d^2P}{dx^2} = A\alpha^2 e^{\alpha x} .$$

Substituting into the wave equation yields

$$\frac{d^2P}{dx^2} = A\alpha^2 e^{\alpha x} = zyAe^{\alpha x}$$

from which it is seen that

$$\alpha^2 = zy \text{ or } \alpha = \pm\sqrt{zy} .$$

Thus the traveling wave expression for pressure becomes

$$P = Ae^{\sqrt{zy}\,x} + Be^{-\sqrt{zy}\,x} .$$

Recalling the original expression

$$\dot{m} = \frac{1}{Z}\frac{dP}{dx}$$

and substituting for P the expression just obtained yields

$$\dot{m} = A\sqrt{\frac{y}{z}}\, e^{\sqrt{zy}\,x} - B\sqrt{\frac{y}{z}}\, e^{-\sqrt{zy}\,x}$$

The constants A and B are evaluated from the end conditions. Let P_r and $\dot{m}_r$ be the pressure and flow rate at the receiving end of the duct. Then recalling that here x is measured from the receiving and let x = 0 to obtain

$$P_r = A + B$$

$$\dot{m}_r = A\sqrt{\frac{y}{z}} - B\sqrt{\frac{y}{z}}$$

solve simultaneously for A and B

$$A = \frac{P_r}{2} + \sqrt{\frac{z}{y}}\,\frac{\dot{m}_r}{2}$$

$$B = \frac{P_r}{2} - \sqrt{\frac{z}{y}}\,\frac{\dot{m}_r}{2}\,.$$

Substituting into the expression for pressure gives

$$P = P_r\left[\frac{e^{x\sqrt{zy}} + e^{-x\sqrt{zy}}}{2}\right] + \dot{m}_r\sqrt{\frac{z}{y}}\left[\frac{e^{x\sqrt{zy}} - e^{-x\sqrt{zy}}}{2}\right]$$

$$P = P_r \cosh x\sqrt{zy} + \dot{m}_r\sqrt{\frac{z}{y}}\,\sinh x\sqrt{zy}$$

Similarly for the mass flow rate

$$\dot{m} = \dot{m}_r \cosh x\sqrt{zy} + P_r\sqrt{\frac{y}{z}}\,\sinh x\sqrt{zy}$$

These equations have been written when the distance is measured from the receiving end. These equations can be written in terms of sending end pressure and mass flow rate and with distance measured from the sending end provided -x is substituted for x in the equations above.

Because in the above equations positive x is measured back along the duct from the receiving end (with x = 0 at the receiving end), distances along the line (with x = 0 at the sending end), are to be regarded as -x. Recalling that cosh is an even function and sinh is an odd function then substituting -x for x yields

$$P = P_s \cosh x\sqrt{zy} - \dot{m}_s\sqrt{\frac{z}{y}} \sinh x\sqrt{zy}$$

$$\dot{m} = \dot{m}s \cosh x\sqrt{zy} - P_s\sqrt{\frac{y}{z}} \sinh x\sqrt{zy}$$

for a sending point |x| distance from the sending end. It is convenient to define the surge or characteristic line impedance as

$$Z_0 = \sqrt{\frac{z}{y}} .$$

Letting the length of the duct be l and the ratio of Pr to $\dot{m}_r$ be Zr then the input impedance of the duct as seen from the sending end is the quotient of P_s and $\dot{m}_s$.

$$P_s = P_r \cosh l\sqrt{zy} + \dot{m}_r Z_0 \sinh l\sqrt{zy}$$

$$\dot{m}_s = \dot{m}_r \cosh l\sqrt{zy} + \frac{P_r}{Z_0} \sinh l\sqrt{zy}$$

$$Z_s = \frac{P_s}{\dot{m}s} = Z_0\left[\frac{Z_r \cosh l\sqrt{zy} + Z_0 \sinh l\sqrt{zy}}{Z_0 \cosh l\sqrt{zy} + Z_r \sinh l\sqrt{zy}}\right]$$

To obtain the result in SP-194 (5.4.1-2) divide by $\cosh l\sqrt{zy}$

$$Z_s = Z_0\left[\frac{Z_r + Z_0 \tanh l\sqrt{zy}}{Z_0 + Z_r \tanh l\sqrt{zy}}\right]$$

and factor out Z_0 and divide by Z_r

$$\frac{Z_s}{Z_r} = \left[\frac{1 + \frac{Z_0}{Z_r} \tanh l\sqrt{zy}}{1 + \frac{Z_r}{Z_0} \tanh l\sqrt{zy}}\right].$$

Let

$$G_1 = \frac{1}{Z_r} \text{ and } G_2 = \frac{1}{Z_2}$$

then

$$\frac{Z_S}{Z_r} = \frac{G_1}{G_2}$$

or

$$\frac{G_2}{G_1} = \frac{1 + \frac{Z_r}{Z_0} \tanh \ell\sqrt{zy}}{1 + \frac{Z_0}{Z_r} \tanh \ell\sqrt{zy}} = \frac{1 + \frac{1}{G_1 Z_0} \tanh \ell\sqrt{zy}}{1 + G_1 Z_0 \tanh \ell\sqrt{zy}}$$

This is the desired admittance ratio which allows the duct to be treated in the same manner as a discrete element in the propellant feed line.

In modeling one proposed MSFC Test Stand piping layout a number of elbows were encountered. It has been shown by Jackson (as reported in SP-194, Section 3.2.2) that the inertance of an elbow can be regarded as the sum of two contributions. One of these is the inertance associated with the fluid treated as if it were flowing in a straight duct. The other is an increase in inertance caused by the duct's curvature. The analytical and experimental results of Jackson are shown as a curve of the latter contribution as a function of duct diameter and parameterized by the angle through which the duct is bent. The curve is reproduced here for convenience.

The approach taken by this author is to derive an equivalent lumped series circuit element. This element models the curved duct as a straight pipe with a decreased area and an increased length compared to a straight pipe having the length of the curved duct's center line and having a decreased cross section area compared to the bent duct. In this development a constant volume of the two ducts' fluid is maintained. The deviation is a follows. Note that the expressions for hydraulic inertance and capacitance for a lumped line are given by

$$L = l/A$$
$$C = \left(\frac{\rho}{k}\right) lA$$

Thus for two straight ducts of the same volume one may write expressions for inertance and capacitance for each as

$$C_1 = \left(\frac{\rho}{k}\right) l_1 A_1 \qquad C_2 = \left(\frac{\rho}{k}\right) l_2 A_2$$

$$L_1 = \frac{l_1}{A_1} \qquad L_2 = \frac{l_2}{A_2}$$

For a constant (lumped) volume of fluid the compressibility will be the same regardless of its form. Therefore,

$$C_1 = C_2$$

and thus

$$l_1 A_1 = l_2 A_2$$

or

$$l_2 = \left(\frac{A_1}{A_2}\right) l_1 \ .$$

Now let

$$L_2 = \lambda L_1$$

Where λ is defined by this equation. Then substituting $\dot{m}$ in the inertance relationship yields

$$\frac{l_2}{A_2} = \lambda \frac{l_1}{A_1}$$

from which

$$A_1 = \lambda \left(\frac{l_1}{l_2} \right) A_2$$

from before

$$l_2 = l_1 \left(\frac{A_1}{A_2} \right)$$

substituting

$$A_1 = \lambda \left(\frac{A_2}{A_1} \right) A_2 \ : \ A_1^2 = \lambda A_2^2$$

thus the "new area" of the equivalent line becomes

$$A_2 = \left(\frac{A_1}{\sqrt{\lambda}} \right)$$

and the "new" line length of the equivalent line is

$$l_2 = \left(\frac{l_1 A_1 \sqrt{\lambda}}{\sqrt{A_1}} \right) = l_1 \sqrt{\lambda}$$

All that remains is to evaluate λ . This is done by recalling

$$\lambda = \frac{L_2}{L_1}$$

where

$$L_1 = \frac{l_1}{A_1} \ ; \ L_1 + L'' = L_2$$

and L" is found from the graph. Our current experience indicates that because

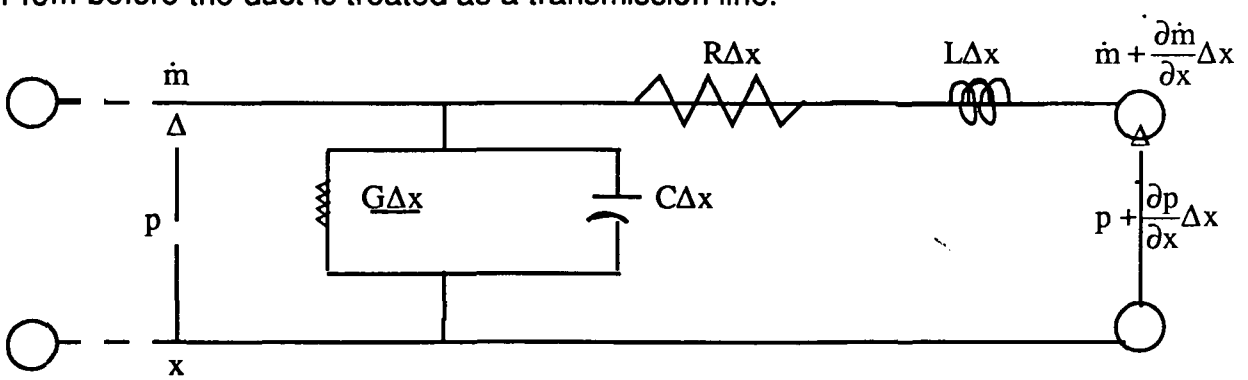
the effect of the added inertance is small especially at low frequencies it speeds computation to ignore this effect upon first analysis and then add its effect at the high frequencies at a later date to evaluate its effect upon stability.

FEEDLINE PRESSURE AND FLOW RELATIONSHIPS

During the course of this work the question of where to place pressure taps in the feedlines arose. This question came about because previous experience showed the importance of location when measuring pressure in a feedline. Fortunately all the methodology developed to this point is applicable. Recall that expressions for the admittance (and hence impedance) at any circuit element location (hydraulic capacitor, inertance, valve of duct end) "looking back" toward the tank have been developed. Similar methodology could be used to develop the admittance of impedance expressions "looking downstream" toward the engine's combuster. With these recollections in mind it is seen that it is possible to fashion a model of a duct being driven by an excitation source in series with a source or generation impedance and terminated in a load or receiving end impedance.

In the interest of additional pedagogy and to develop additional notation the pressure flow relations applicable to a duct will be rederived. However, the derivation will be done in such a way as to expose the functional relationships between pressure and flow and such things as excitation description, source and load impedances and the line parameters.

From before the duct is treated as a transmission line.



Recall the previous discussion about the various impedances e.g. in cases of smooth ducts it is expected that G ->o, R is small and the dynamics are dominated by L and C). If other conditions obtain then the methodology followed here may be used to solve the other case. Assuming the first term of the Taylor's Series expansion is adequate to describe the change in pressure and mass flow rate the differential equations for this infinitesimal length of duct are formulated by equating the source pressure (at x) to the pressure drop across the duct element summed with the pressure at the receiving end of the duct.

$$p(x,t)= R\Delta x\dot{m}(x,t) + L\Delta x\frac{\partial\dot{m}(x,t)}{\partial x} + p(x,t) + \frac{\partial p(x,t)}{\partial x}\Delta x$$

Likewise the equation for the flow rate relationship is formulated by equating the input flow rate to the element to that flowing through the shunt admittance summed with the flow leaving the element.

$$\dot{m}(x,t)=p(x,t)G\Delta x + c\Delta x\frac{\partial p(x,t)}{\partial t} + \dot{m}(x,t)+ \Delta x\frac{\partial\dot{m}(x,t)}{\partial x}$$

From these two equations it follows that

$$\frac{-\partial p(x,t)}{\partial x} = R\dot{m}(x,t) + L\frac{\partial\dot{m}(x,t)}{\partial t}$$

$$\frac{-\partial\dot{m}(x,t)}{\partial x} = Gp(x,t) + c\frac{\partial p(x,t)}{\partial t}$$

are the defining equations. To transform from the time domain to the frequency domain (and hence transform from differential equations in time to algebraic equations in the complex frequency variable s) the La Place transform is utilized. Initial conditions will be set equal to zero on heuristic grounds. Thus the effects of driven or particular solutions may be studied as the problem is formulated. If a given set of initial conditions were known the equations may be rederived to include them. The LaPlace transformed equations are

$$\frac{-\partial P(x,s)}{\partial x} = R\dot{m}(x,s) + Ls\dot{m}(x,s) = (Ls + R)\dot{m}(x,s)$$

$$\frac{-\partial\dot{m}(x,s)}{\partial x} = GP(x,s) + csP(x,s) = (cs + G)P(x,s)$$

Taking the partial derivative with respect to x of each of these equations and substituting the expression above for the resulting first partials yields.

$$\frac{\partial^2 P(x,s)}{\partial x^2} - (R + Ls)(G+cs)P(x,s)=0$$

$$\frac{\partial^2 \dot{m}(x,s)}{\partial x^2} - (R + Ls)(G+cs)\dot{m}(x,s)=0$$

now define

$n^2 = (Ls + R)(G + cs)$: the propagation constant and note that for a lossless line (a usual first approximation)

$$n^2 = Lcs^2$$

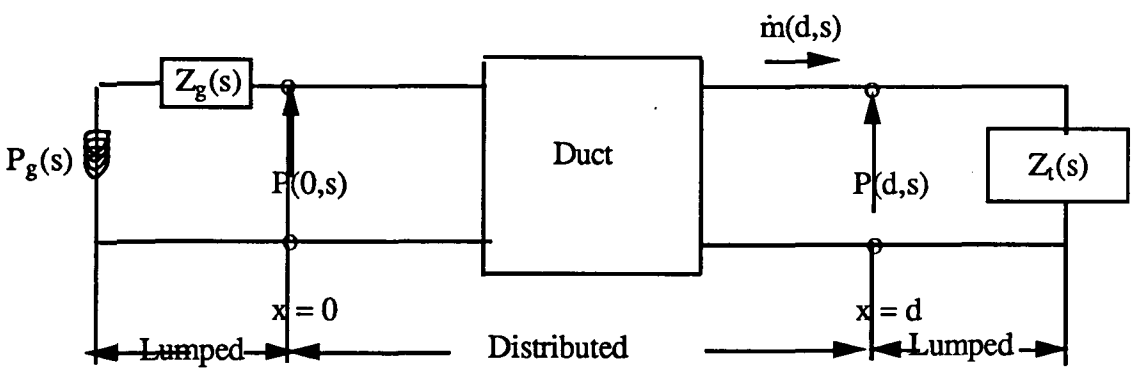
To achieve a solution, one is assumed, analogously with previous work.

$$\dot{m}(x,s) = A_1(s)e^{-nx} + B_1(s)e^{nx}$$

$$P(x,s) = A_2(s)e^{-nx} + B_2(s)e^{nx}$$

Note the functional arguments of the "constants." The "constants" (A's and B's) are evaluated from boundary conditions. Therefore, to evaluate them some specific configuration must be specified.

The configuration is chosen as shown below.



By inspection

$$P(0,s) = P_g(s) - Z_g\dot{m}(0,s) = A_2\ (s) + B_2(s)$$

$$P(d,s) = Z_t(s)\ \dot{m}(d,s)$$

Differentiate the assumed pressure solution with respect to x

$$\frac{\partial P(x,s)}{\partial x} = -nA_2(s)e^{-nx} + nB_2(s)e^{nx}$$

and substitute it into the original transformed PDE relating the rate of change of pressure to the mass flow rate to yield

$$\dot{m}(x,s) = \frac{1}{(Ls + R)}[nA_2(s)e^{-nx} - nB_2(s)e^{nx}]$$

Comparing this expression with the assumed solution for the mass flow rate produces the relationships among the "constants"

$$A_1(s) = \frac{nA_2(s)}{(Ls + R)}$$

$$B_1(s) = \frac{-nB_2(s)}{(Ls + R)}$$

These expressions for $A_1(s)$ and $B_1(s)$ may be substituted into the originally assumed solutions to yield

$$\dot{m}(x,s) = \frac{nA_2(s)}{(Ls + R)}e^{-nx} - \frac{nB_2(s)}{(Ls + R)}e^{nx}$$

$$P(x,s) = A_2(s)e^{-nx} + B_2(s)e^{nx}$$

Note that

$$\frac{n}{(Ls + R)} = \sqrt{\frac{(Ls + R)(Cs + G)}{(Ls + R)^2}} = \sqrt{\frac{(Cs + G)}{(Ls + R)}} = \frac{1}{z_o}$$

where z_o is defined to be the surge or characteristic impedance and

$$z_o = \sqrt{\frac{L}{C}} \quad \text{for a lossless line.}$$

using the definitions above produces

$$\dot{m}(x,s) = \frac{A_2(s)}{z_o}e^{-nx} - \frac{B_2(s)}{z_o}e^{nx}$$

$$P(x,s) = A_2(s)e^{-nx} + B_2(s)e^{nx}$$

The sending end boundary conditions are applied to yield

$$P(0,s) = P_g(s) - Z_g(s)\dot{m}(0,s) = P_g(s) - \frac{Z_g}{Z_o}[A_2(s) - B_2(s)] = A_2(s) + B_2(s)$$

and the receiving end boundary conditions yield

$$P(s,d) = Z_t\dot{m}(s,d) = \frac{Z_t}{Z_o}[A_2(s)e^{-nd} - B_2(s)e^{nd}]$$

These two simultaneous linear equations in the unknowns $A_2(s)$ and $B_2(s)$ may be solved to yield

$$A_2(s)= \frac{\frac{Z_o}{Z_o + Z_g}P_g(s)}{1 + MNe^{-2nd}}$$

$$B_2(s)=- \frac{(\frac{Z_o}{Z_o + Z_g})Ne^{-2nd}P_g(s)}{1 - MNe^{-2nd}}$$

where the reflection operators are defined by

$$N = \frac{Z_o - Z_t}{Z_o + Z_t}$$

$$M = \frac{Z_o - Z_g}{Z_o + Z_g}$$

Using these solutions for A_2 and B_2 and substituting them into the assumed solutions for pressure and flow rate produces the transfer functions between the pressure generator $P_g(s)$ and the pressure and flow perturbations at a given distance down the duct as

$$\frac{P(x,s)}{P_g(s)} = \frac{\frac{Z_o}{Z_o + Z_g}[e^{-nx} - (\frac{Z_o - Z_t}{Z_o + Z_t})e^{-n(2d-x)}]}{[1 - \{\frac{(Z_o - Z_t)(Z_o - Z_g)}{(Z_o + Z_t)(Z_o + Z_g)}\}e^{-2nd}]}$$

and

$$\frac{\dot{M}(x,s)}{P_g(s)} = \frac{\frac{1}{Z_o + Z_g}[e^{-nx} + (\frac{Z_o - Z_t}{Z_o + Z_t})e^{-n(2d-x)}]}{[1 - \{\frac{(Z_o - Z_t)(Z_o - Z_g)}{(Z_o + Z_t)(Z_o + Z_g)}\}e^{-2nd}]}$$

While there are special cases of interest (some considered below) it is seen, especially recalling that all the z's are functions of s ($s = \alpha + j\omega$), that the solutions following from this transfer function, given a generating function are

generally of a complicated nature. This fact leads to numerical investigations to explore given cases. But before investigating numerical approaches consider some special cases.

The most perfect (and impossible) case occurs when the duct is considered lossless and the reflection operators are zero (the matched load, lossless case i.e., $Z_g = Z_t = Z_o$). Under these assumptions the transfer functions yields

$$P(x,s) = P_g'(s)e^{-nx}$$

This indicates that an input pressure signal will travel down the duct undistorted in wave shape and only delayed in time (transport lag phenomenon) or, from the frequency domain viewpoint, there is introduced a phase lag proportional to the distance down the duct.

Another special case of some interest occurs when G = 0 and R << Ls. The problem to be attacked is that the quantity n contains s in a square root form.

$$n = \sqrt{(Ls + R)(G + Cs)}$$

Under this set of assumptions

$$n = \sqrt{(Ls + R)\,Cs} = s\sqrt{1 + \frac{R}{Ls}}\sqrt{LC} \cong \sqrt{LC}s\left(1 + \frac{R}{2Ls}\right)$$

or

$$n \cong \sqrt{LC}s + \frac{1}{2}R\left(\sqrt{\frac{C}{L}}\right)$$

Once again, assuming a matched load i.e. duct terminated in the surge or characteristic impedance Z_o and $Z_g = Z_o$ the pressure as a function of distance and the generation pressure is given by

$$P(x,s) = P_g'(s)e^{-\sqrt{LC}s}e^{-(\frac{1}{2})R\sqrt{\frac{C}{L}}x}$$

The inverse transform to get to the time domain is obtained by inspection as

$$P(t,x) = e^{-(\frac{1}{2})R\sqrt{\frac{C}{L}}x}\, P_g(t - \sqrt{LC}x)u(t - \sqrt{LC}x)$$

where $u(t-\sqrt{LC}x)$ is the familiar unit step function notation used as a sectioning function so that no time response occurs before $t=\sqrt{LC}x$ i.e. this is a causal system (therefore physically realizable). Thus this wave may be interpreted as an attenuated wave traveling at a speed $= \frac{1}{\sqrt{LC}}$, having zero value until $t > \sqrt{LC}x$. Notice that the form of this solution is the same as that encountered previously for the lossless line with the exception that here there is attenuation i.e. the wave maintains its shape as it moves along but it shrinks in amplitude.

There are other combinations of the duct parameters which yield a (possibly) attenuated but undistorted traveling wave. These other cases are not of particular interest here because they do not assume G to be zero. In any case a duct having parameters so designed is called a "distortionless" or "heaviside" duct.

Before moving away from duct characterization the subject of transient events will be addressed. There are a number of catastrophic events that could be postulated as transient events. However, the attention here will concern itself only with a step change to a different load. This might be used to model fairly rapid changes in valve settings (openings or closings). The general approach to handling any transient event of this generic type will always involve solving the given forced-solution for the first configurations' solution which in turn become initial conditions for the solution of the second configuration etc. To this point the solutions of the wave equation where these have involved transforms have assumed the initial condition to be zero (to get transfer functions, for instance). To relax this assumption return to the two first order PDE'S which define the pressure and mass flow rate dynamics when they are LaPlace transformed but this time include the initial conditions.

$$\frac{-\partial P(x,s)}{\partial x} = R\dot{m}(x,s) + Ls\dot{m}(x,s) - L\dot{m}(x,0)$$
$$\frac{-\partial \dot{m}(x,s)}{\partial x} = GP(x,s) + CsP(x,s) - CP(x,0)$$

It is of course recognized that the last term in each equation is caused by the indicated initial condition. These equations are now manipulated as before to yield the wave equations in mass flow rate and pressure.

$$\frac{\partial^2 P(x,s)}{\partial x^2} - n^2 P(x,s) = L\frac{\partial \dot{m}(x,0)}{\partial x} - C(Ls + R)P(x,0)$$

$$\frac{\partial^2 \dot{m}(x,s)}{\partial x^2} - n^2 \dot{m}(x,s) = C\frac{\partial P(x,0)}{\partial x} - L(Cs + G)\dot{m}(x,0)$$

Written in this form all the terms on the right hand sides (rhs) are due to the initial conditions. One now has a choice of how to proceed. Which way is chosen depends upon the particular analyst and what computational or other insightful factors are involved. The conceptually most straightforward way is to terminate one solution at the switch time, determine the pressure and mass flow rates in the duct , use these as the initial conditions for the new configuration (having written its equations) and solve another subsequent problem etc. There are other approaches. One of these is by the use of cancellation sources which model the pressure and/or flow rates of the switches after the configuration change. This is done by introducing <u>ideal</u> pressure (hence no generation source impedance) and ideal mass flow rate sources (hence infinite source impedance). This technique is especially useful for solving switching modeled configurations changes. It converts from a problem with initial conditions to two or more problems without initial conditions but whose total response is obtained by summing the response of all the problems.

Returning to the original question of where to place pressure taps in the line in order to measure standing waves in the duct some inspection of the p(x,s)/Pg(s) transfer function is in order. Standing waves are associated with steady state sinusoidal duct responses. This type of response may be examined by restricting s to the imaginary axis in the s-plane i.e. let s=jw. One notes in passing that transient information is not lost by so doing. Making an appeal to the Weierstrass Approximation Theorem allows one to think of the transfer functions as approximated by a ratio of polynomials in s, for a given x. Provided the resulting transfer function is analytic in the right hand plane and on the imaginary axis an exact inverse Fourier Transform may be written as

$$h(t) = \frac{2}{\pi}\int_0^{\infty} \{\text{Real } H(j\omega)\cos t\omega\}d\omega \quad t > 0$$

where h (t) is the impulse response of the system and

$$L[h(t)] = H(s)$$

Having let s=jw it is seen that the numerator of the duct transfer function consists of two terms of phasors which may be visualized as a magnitude and phase angle in the complex plane. The one term is

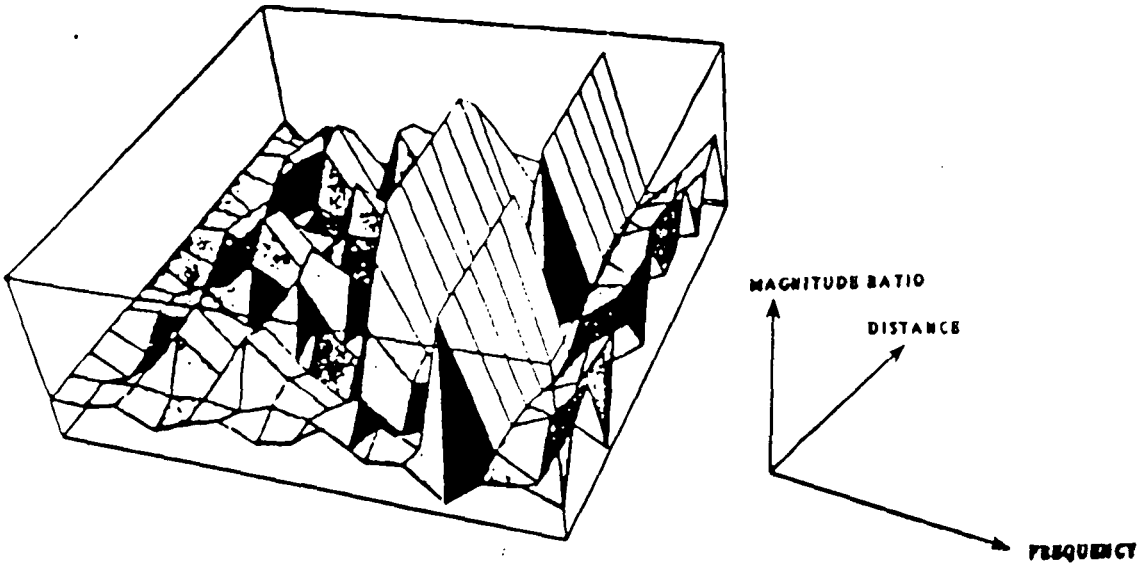
$$e^{-nx}$$

which for the example of a lossless line would be represented by a unit phasor at an angle determined by the frequency of interest, the duct parameters and the location chosen on the duct.

The second term is more involved but it also may be interpreted as a phasor in the complex plane for a given x. It is not unreasonable to expect that the difference of these two phasor quantities would experience maxima and minima. As a matter of fact for a lossless duct it may be shown that for a variety of conditions the pressure and mass flow are displaced along the duct by one quarter wavelength and both are zero at points separated by one half a wavelength.

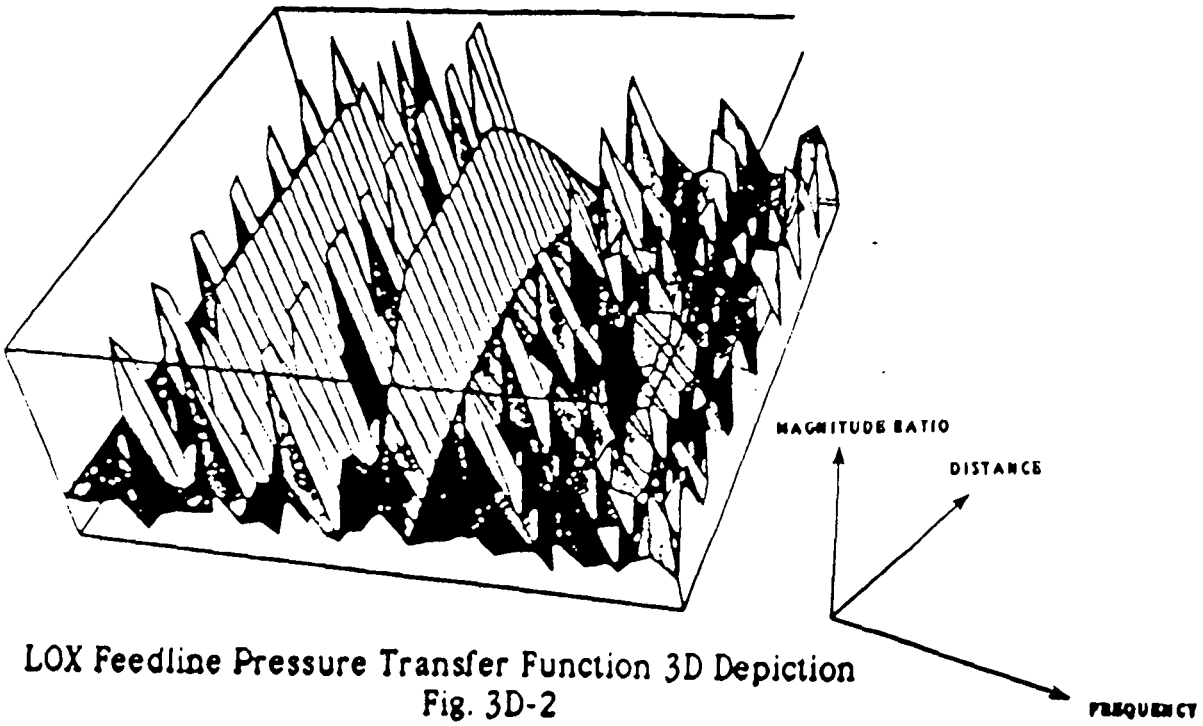
As a practical matter a specific configuration, one supplied by MSFC (a proposed LOX feedline), was used as an example to explore the transfer functions from a numerical standpoint. Two problems arise as far as visualizing the solution is concerned. One is that of presenting the results as a function of both position down the duct and as a function of frequency. The solution suggests itself as a three dimensional plot with transfer function magnitude, frequency and duct position as the three coordinate axes. Fortunately MATHEMATICA has a built in 3D plotting capability which has been enhanced in the latest release. In addition its contour plot capability proved to be extremely useful in determining the granularity of the solution. The second problem is that of the granularity. The duct chosen is 62 feet in length and somewhat arbitrarily a frequency range of zero to six thousand radians per second was chosen initially . At this point the age old question of granularity across the 2D grid of displacement versus frequency arose.

An initial range was chosen as shown with the results shown in the figure below.



LOX Feedline Pressure Transfer Function 3D Depiction
Fig. 3D-1

The number of points were then increased and Fig 3D-2 produced.



LOX Feedline Pressure Transfer Function 3D Depiction
Fig. 3D-2

Clearly a great deal more detail has arisen. The cost is clearly in sharply increased computing time and, eventually, saturated memory. At this point a contour plot of the data was run producing figure 3D-3.

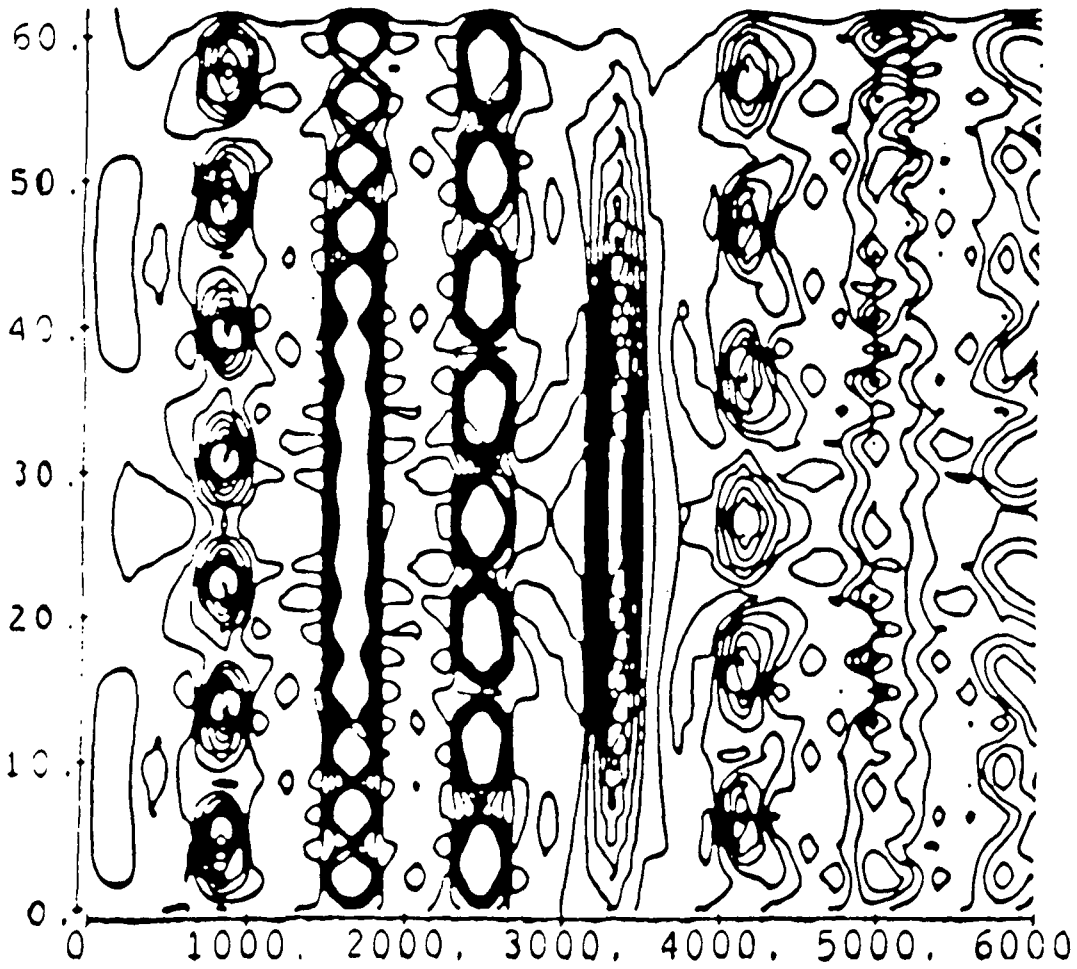
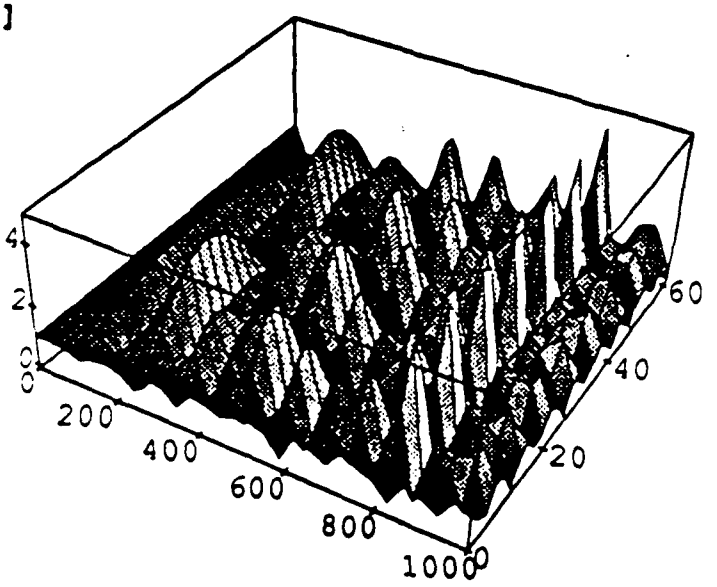


Fig. 3D-3

Note that many of the "ridge lines" of response peaks are smeared together e.g. the ones at 3250 and 1575 radians per second while it was unclear what was happening below about 750 radians per second. Consider that usually there is more energy in lower frequency modes than higher frequency ones and also that one usually expects a half wave to develop between the two ends of the duct. At this point the frequency range was reduced to zero to one thousand and the same number of points plotted as before. This yielded fig. 3D-4. This figure was created using an updated version of MATHEMATICA which allowed labeling the axes. Also the plotting code is included to illustrate the use of this program for this purpose.

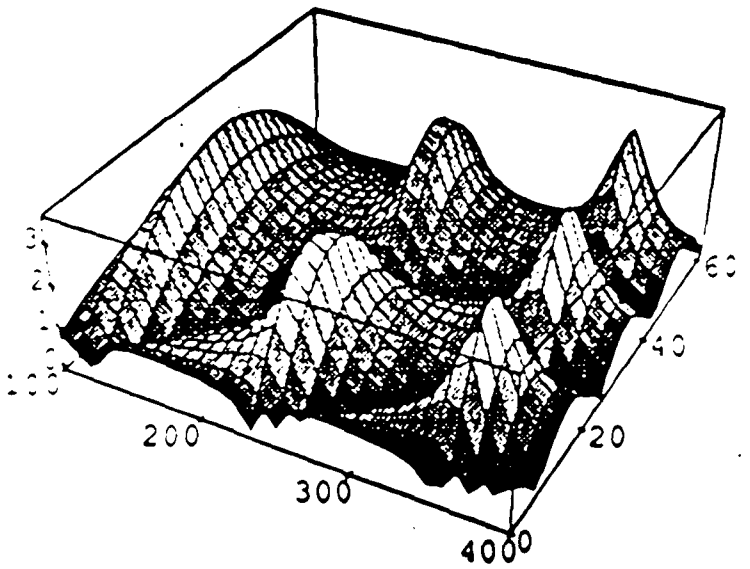
Fig. 3D-4

```
In[1]:=
Plot3D[Abs[((3.28+I*u*32.37*10^-4)/(4.33+I*u*32.37*10^-4))
*
(Exp[-1*I*4.3*10^-4*u*x]-((-1+I*2.76*u)/(1+I*u*2.76))*Exp[
-1*
I*u*4.3*10^-4*(124-x)])/(1-(((-1+I*u*2.76)/(1+I*u*2.76))*(
(2.23+I*u*.0034)/
(4.33+I*u*.0034))*Exp[-5.33*10^-2*I*u]))],{u,0,1000},{x,0,
62},PlotPoints
->30,PlotLabel->"LOX Feedline Pressure Transfer Function
3D Depiction"]
```



LOX Feedline Pressure Transfer Function 3D Depiction
Fig. 3D-4

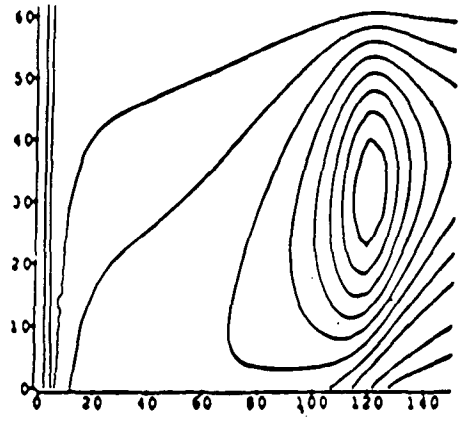
In this figure the half wave of pressure becomes discernable below 150 radians/sec and much better resolution of the other lower modes occurred. Therefore the upper frequency value was once again reduced, this time to 400 and fig. 3D-5 resulted.



LOX Feedline Pressure Transfer Function 3D Depiction
Fig. 3D-5

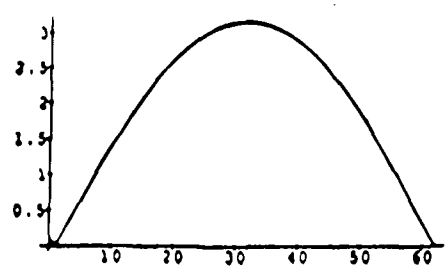
This figure clearly depicts the three lowest frequency modes in reasonable detail. These types of figures could be developed for any other band of frequencies of interest. To develop additional information including numerical data contour plots were made for the frequency ranges 0-150; 200-300; and 300-400/ In addition a series of 2D plots of amplitude versus length for the fixed frequencies 120, 240 and 358.3 radian per second were made. These latter plots were made to depict the standing pressure waves corresponding to the first three lowest modal or resonant frequencies. They have been grouped together below for ease of association and cross interpretation. Once again, for each type of plot one representative set of code is included.

```
In[5]:=
ContourPlot[Abs[((3.28+I*u*32.37*10^-4)/(4.33+I*u*32.37*10
^-4))*
(Exp[-1*I*4.3*10^-4*u*x]-((-1+I*2.76*u)/(1+I*u*2.76))*Exp[
-1*
I*u*4.3*10^-4*(124-x)])/(1-(((-1+I*u*2.76)/(1+I*u*2.76))*(
(2.23+I*u*.0034)/
(4.33+I*u*.0034))*Exp[-5.33*10^-2*I*u]))],{u,0,150},{x,0,6
2},PlotPoints
->30,PlotLabel->"LOX Feedline Pressure Transfer Function
3D Depiction"]
```

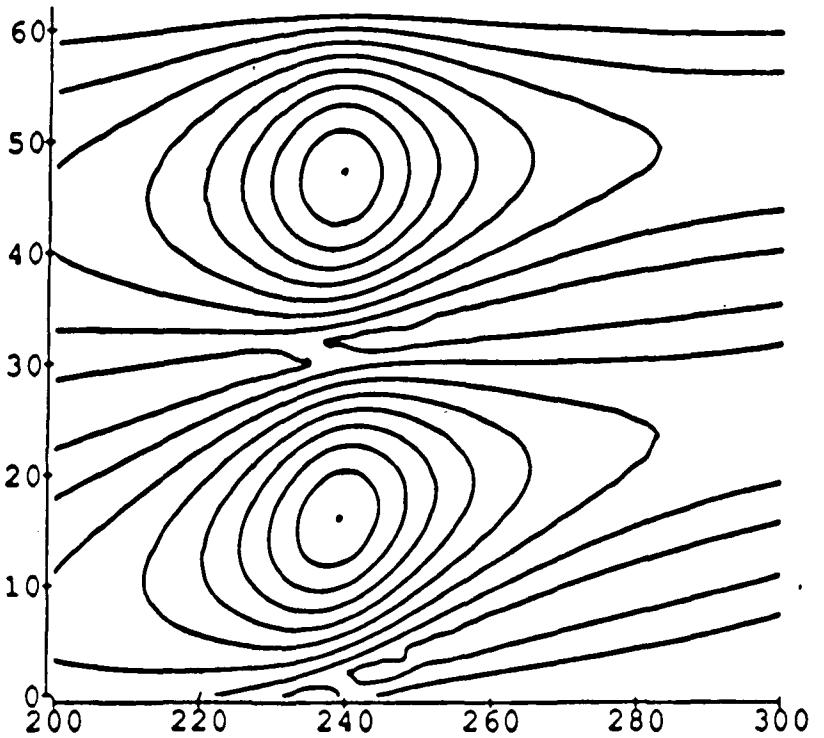


LOX Feedline Pressure Transfer Function Topological Depiction
Fundamental Frequency
Fig. 3D-6

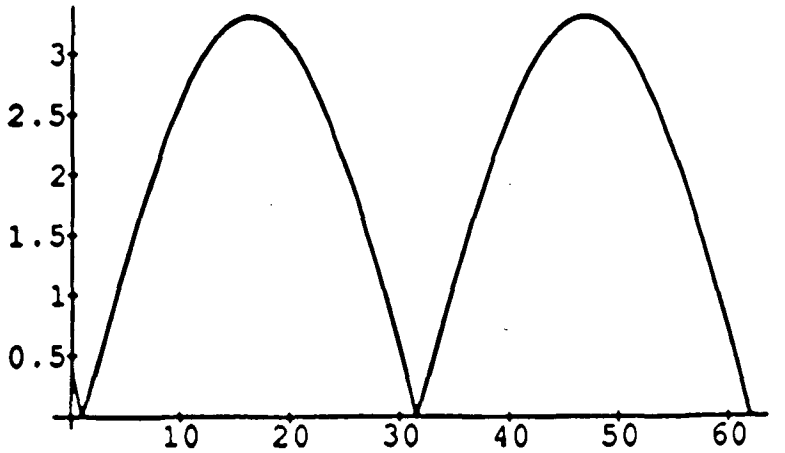
```
u=120
Plot[Abs[((3.28+I*u*32.37*10^-4)/(4.33+I*u*32.37*10^-4))*
(Exp[-1*I*4.3*10^-4*u*x]-((-1+I*2.76*u)/(1+I*u*2.76))*Exp[
-1*
I*u*4.3*10^-4*(124-x)])/(1-(((-1+I*u*2.76)/(1+I*u*2.76))*(
(2.23+I*u*.0034)/
(4.33+I*u*.0034))*Exp[-5.33*10^-2*I*u]))],{x,0,62},PlotPoi
nts
->30,PlotLabel->"LOX Feedline Pressure Transfer Function
2D Depiction"]
```



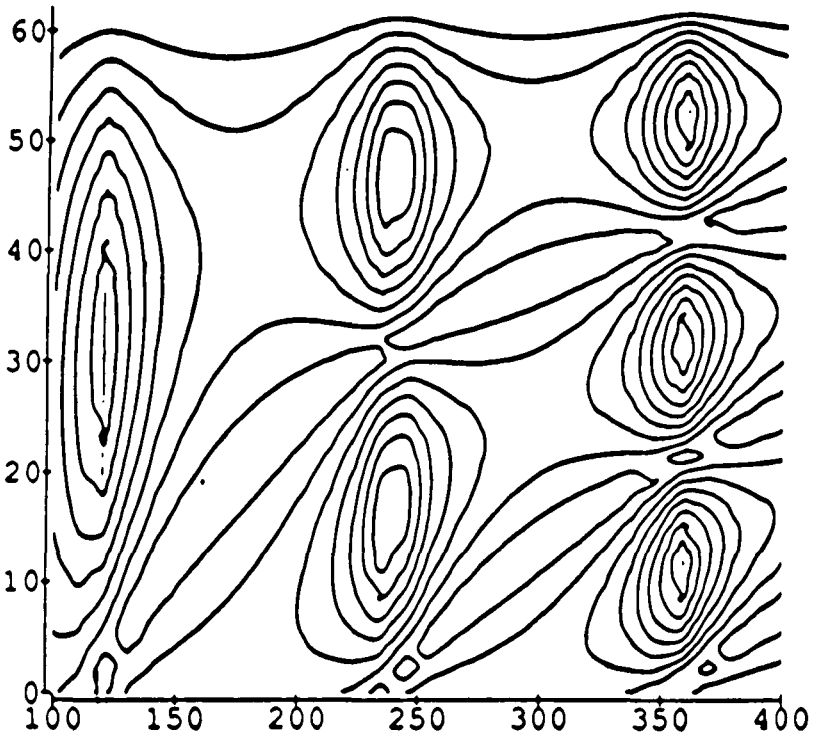
LOX Feedline Pressure Transfer Function 2D Depiction
Fundamental Frequency
Fig. 3D-7



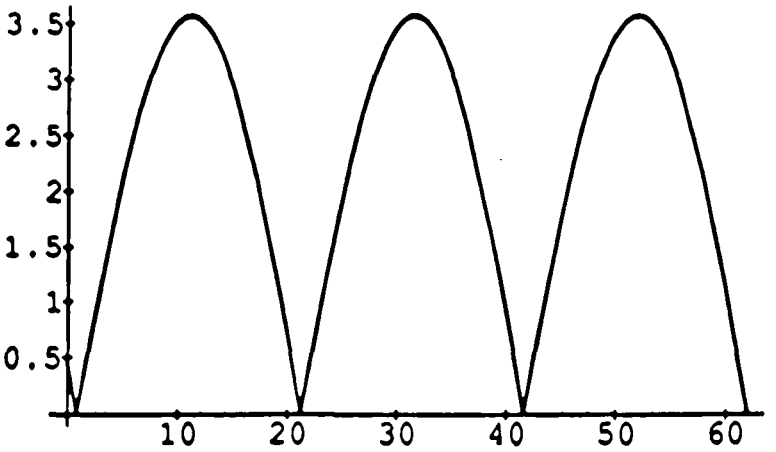
LOX Feedline Pressure Transfer Function Topological Depiction
First Harmonic
Fig. 3D-8



LOX Feedline Pressure Transfer Function 2D Depiction
First Harmonic
Fig. 3D-9



LOX Feedline Pressure Transfer Function Topological Depiction
Second Harmonic
Fig. 3D-10



LOX Feedline Pressure Transfer Function 2D Depiction
Second Harmonic
Fig. 3D-11

From these plots it is readily apparent that several instrumentation issues arise. Amongst them is the fact that instrumentation placed to measure a given mode may well miss another mode. For example, if it were desired to measure the lowest frequency modal pressure and the instrument tap is put at mid duct the pressure due to the next mode which nulls at mid duct would not be measured. Thus if it were desired to measure the next modes' pressure taps at 15 or 45 feet (approximately one quarter of the way down the duct) would be appropriate. And so forth for the other modes. Using the contour and 2D plots numerical data can be developed to aid in placing the taps. It also is helpful in specifying the bandwidth of the instrumentation system. It would seem to be good engineering practice to record the output of the instrumentation and perform a spectral analysis on it to confirm the modal frequency and to make sure that the desired modal pressures were measured and evaluated.

INTRODUCTION TO COMBUSTION STABILITY

Combustion driven or coupled stability questions associated with liquid fueled rocket engine systems have been amply shown to be accurately categorized as having a high frequency regime and a lower frequency regime. In addition the lower frequency regime is subdivided in two regimes, sometimes called the low and intermediate modes of oscillation.

The high frequency mode of oscillation is exclusively associated with a specific combuster design. It depends greatly on such things as injector design, combuster shape, fuel characteristics etc. A key element in analyzing this regime's effects are the acoustical resonant modes of the combuster. Indeed the first approximation of the high frequency combuster modes includes a classical acoustical analysis which is used as a good first approximation to an analysis which modifies the classical acoustical analysis to account for the effects of combustion. Thus high frequency combustion stability is basically the purview of the combuster designer rather than the test stand designer. However, to allow a monitoring capability a late version of Dr. Mitchell's (of Colorado State sponsored by the USAF under contract F046 11-86-k-0020) analysis software has been modified for and installed on the MSFC computers in EP-64. Sadly the current state of the art is such that "from scratch" predictive

stability analysis is impossible. This is because the detailed aerothermochemistry processes involved are not at all well understood. Even the equations that are available are complex and difficult to solve. Perhaps with time and the renewed interest in liquid propulsion in the USA (after an hiatus of twenty years) progress will be made. Various supercomputers e.g. the Cray YMP and the TMC Connection Machine (CM) are being employed to investigate various single aspects of the aerothermochemical processes. However, much is yet to be done even approach simulating an entire combuster with all its interconnected processes.

The state of the art for predicting the lower frequency oscillatory modes is farther advanced than that for the high frequency regime. While still depending on a number of factors e.g. n and τ that define the combustion effects in a from-outside-the-black-box or phenomenalistic viewpoint the ability to analyze and predict stability questions is reasonably well in hand. And is covered below. Unlike the high frequency oscillatory behavior which is almost exclusively the province of the combuster designer the lower frequency oscillating modes involve intimate coupling between the combuster and the propellant feed systems. Thus the lower frequency stability questions are very much the concern of the test stand designer (who must of course confer with the combuster designers or engine suppliers for certain engine or combuster parameters).

Thus it is seen that except for a much wider use of much more powerful computing capacity the state of the art in stability analysis stands not much advanced from where it was in 1970. Such manner as those of Von Karman, Crocco, Reardon and Tsien are associated with that era (along with many others of course, see SP-194). The authors of this report have talked to Dr. Reardon during the winter of 1989-90. He had recently set about seeing if a sequel to SP-194 could or should be written. He contacted various groups once again active in the field and concluded that not enough new technology had been developed at this point in time to warrant a new volume on the subject.

Diagrammatically the frequency separation of the modes and stability associated phenomena are shown in a general way in the figure below.

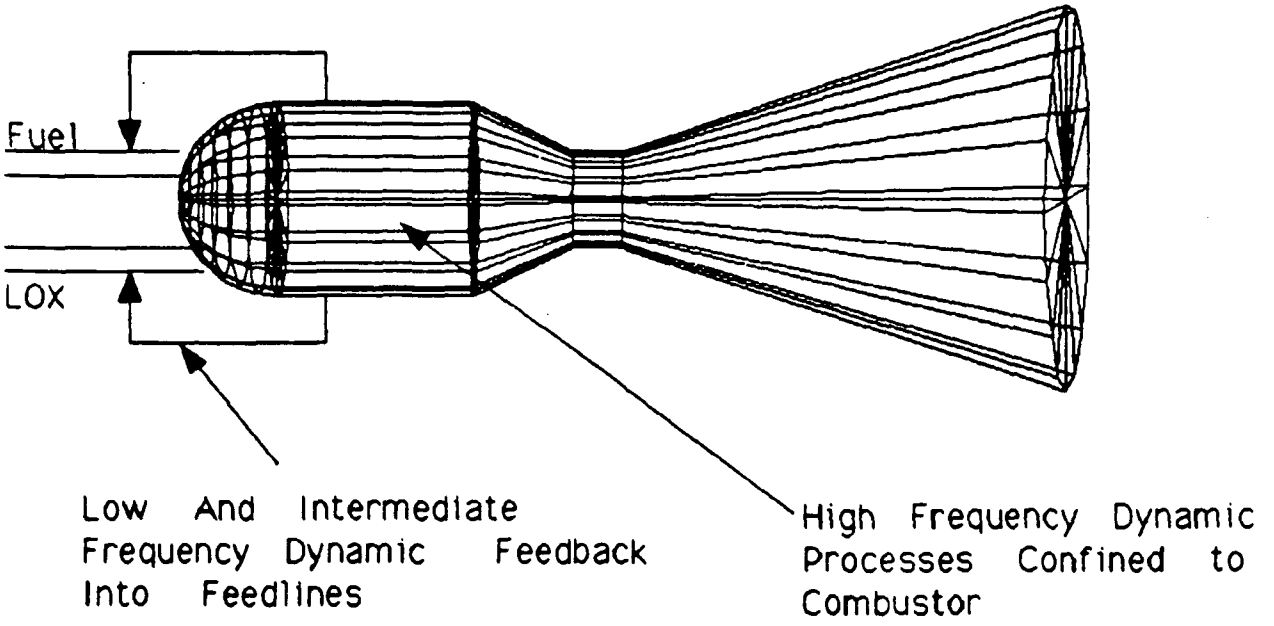


Diagram Depicting Where Dynamic Processes Occur and Their Interaction

LOW FREQUENCY STABILITY

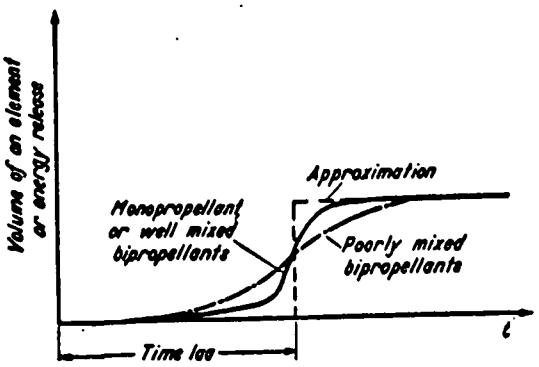
Liquid propellant rocket systems couple the propellant supply system (feed ducts, tanks etc) to the combuster through the injection process. Typically a number of nozzles are fed from a manifold with fluid pressurized to an average pressure substantially above that in the combuster so as to feed the propellant into the ignition process at a desired average rate. The specific requirements on the injectors and the conditioning of propellant for combustion vary from propellant to propellant but for bipropellant non-hypergolic types they may be summarized as follows.

(1) To obtain good atomization of the fluid it is necessary to assure a large surface area of contact between the liquid and the hot recirculating gases.
(2) A proper proportion between the mass of the propellant and the mass of the hot gases surrounding it must be obtained.
(3) A good renewal of the hot gases to activate surface exchanges must be assured.
(4) A fast, uniform as possible, mixing between the two propellants must be ensured.

Some contemplation of the injection and subsequent combustion process shows that as the propellants travel downstream in the combustion zone of the engine they initially are little effected by the hot gases surrounding them. However as they continue down the chamber they begin to mix, be heated by the environment and finally begin to burn. Thus in this Crocco style scenario it is seen that there is a time delay between the time of propellant injection and the time at which it burns or turns from a liquid phase into a gaseous one.

Crocco et al proposed a model, in wide use, in which the process described above is modeled approximately by a time delay or transport lag phenomena modeling the time it takes for the liquid propellant to enter the combuster travel downstream and (instantaneously) turn into gas. This phenomenon is shown in the figure (note the step function approximation).

Schematic diagram of energy release or volume of an element versus time



If the time delay from propellant injection to ignition is dominated by one of the propellants in a bipropellant system as is the case with, for example, LOX-RP1 (where the rapid evaporation rate of the LOX causes the RP1 evaporation rate to control the combustion process rates) then a single delay model is justified. If the delays of both constituents are comparable then a two delay model is to be used. Fortunately while it introduces some more computational complexity the inclusion of another delay is an easy, straight forward analytical thing to do, as will be seen below.

It is now necessary to consider how variations in combustion parameters couples in to the combustion process. This is necessary to establish a feedback mechanism or gain between perturbations in the process and the energy release due to combustion that can feed an instability if the proper conditions occur. Crocco does this in the following way. He assumes that the rate of processes at a given location and time are a function of physical factors such as pressure, temperature and other factors. He then assumes that all these factors influence the pressure and develops a functional relationship as follows. Using a Taylor's Series Expansion where over bars denote average values and primes denote small perturbations he writes

$$f(p,T,z,\ldots) = f(\bar{p},\bar{T},\bar{z}) + p'\frac{\partial \bar{f}}{\partial p} + T'\frac{\partial \bar{f}}{\partial T} + z'\frac{\partial \bar{f}}{\partial z} + \ldots$$

where the partials are evaluated at the respective average value of $\bar{p},\bar{T},\ldots$. Assuming that T and z are correlated to p then

$$T = T(p) \quad \text{and} \quad z = (p)$$

thus factoring the expression above yields

$$f(p,T,z,\ldots) = f(\overline{p},\overline{T},\overline{z},\ldots)\left[1 + p'\frac{1}{f}\left(\frac{\partial f}{\partial p} + \frac{\partial T\,\partial f}{\partial p\partial T} + \frac{\partial z\,\partial f}{\partial p\partial z} + \ldots\right)\right]$$

where the barrel quantity is to be evaluated at the average values for the quantities involved. At this point the quantity n is defined as the interaction index given by

$$n = \left[\frac{\overline{p}}{\overline{f}}\left(\frac{\partial f}{\partial p} + \frac{\partial T\,\partial f}{\partial p\partial T} + \frac{\partial z\,\partial f}{\partial p\partial z} + \ldots\right)\right]$$

Thus the instantaneous processes rate is written

$$f(p,T,Z\ldots)=f(\overline{p},\overline{T},\overline{Z}\ldots)[1+n\frac{\dot{p}}{p}]$$

If one assumes that the process rate is only proportional to a power of pressure then

$$f \propto p^n$$

from which as before

$$f(p) \propto f(\overline{p}) + p'\frac{\partial \overline{f}}{\partial p}$$

$$f(p) \propto f(\overline{p}) + p'np^{n-1}$$

$$f(p) \propto \overline{p}^n + p'n\overline{p}^{n-1}$$

$$f(p) \propto \overline{p}^n\left[1 + \frac{np'}{\overline{p}}\right]$$

Thus the same result as before results under the assumption of only pressure dependence. Therefore, under the assumption that the physical factors are correlated one can disregard the explicit effects of all the factors except that of the pressure. Crocco and Cheng propose a few examples of physical processes which suggest values for n. However historical design values seen to be the best source of quantitative data for n's value.

LOW FREQUENCY STABILITY MODELING

With the simplified models and concepts described above the tools are in hand to develop the first and simplest mode. suitable for stability investigations. The

overall approach taken is to work with small perturbations in propellant feed rate and combuster pressure. This local linearizations around a nominal engine operating point allows the use of all the techniques of linear stability theory e.g. Nyquist diagrams and Bode Plots. In what follows primes are used to denote perturbation variables and the pressure is normalized (divided by) the nominal or average combuster pressure and the mass flow rate by the total nominal or average propellant mass flow rate. Both normalizing factors are denoted by the average or superbar notation.

Assuming that both propellants undergo the same time delay (τ_f) to ignition and where b denotes burn, i injection, f fuel, ox oxidizer and t time one writes as follows for the propellant mass flow rates.

$$\dot{m}'_{oxb}(t) = \dot{m}'_{oxi}(t)(t-\tau_f)$$
$$\dot{m}'_{Fb}(t) = \dot{m}'_{Fi}(t)(t-\tau_f)$$

The admittance expressions of the feed systems which relate propellant delivery rate perturbation to combine for pressure perturbations (as seen from the combustion chambers) are developed in explicit from elsewhere in the report in terms of feedsystem parameters. Here from the combuster location they are expressed as

$$G_{ox} = \frac{-\bar{p}}{\bar{\dot{m}}}\frac{\dot{m}'_{oxi}}{p'}$$
$$G_F = \frac{-\bar{p}}{\bar{\dot{m}}}\frac{\dot{m}'_{fi}}{p'}$$

or rearranging

$$\frac{\dot{m}'_{oxi}}{G_{ox}} = \frac{-p'}{\bar{p}}$$
$$\frac{\dot{m}'_{Fi}}{G_F} = \frac{-p'}{\bar{p}}$$

Equating like quantities

$$\frac{\dot{m}'_{oxi}}{G_{ox}\bar{\dot{m}}} = \frac{-p'}{\bar{p}} = \frac{\dot{m}'_{Fi}}{G_F\bar{\dot{m}}}$$

These expressions link the propellant feedsystem to the combustion process via the chamber measure.

The use of perturbation analysis has been amply displayed in the foregoing by use of the prime notation, Taylor's Series etc. This approach will be enlarged upon below but before that the major assumptions governing low frequency dynamic behavior will be listed.

1. The gas pressure in the combuster is uniform and oscillates about a mean value (i.e. no wave motion is considered until the intermediate frequency oscillations are considered).

2. The temperature of the gas in the chamber is uniform and constant i.e. not effected by pressure oscillations.

3. The time lag from injection of propellant to the burned state is the same for all propellant elements.

4. Propellant feedsystem dynamics including wave motion must be taken into account.

5. Combustion is characterized by a delay.

As already pointed out the explicit expressions for the feedline admittances are in hand. Now the explicit relationships for the combustion process must be derived.

One relationship follows from the principle of mass conservation. It states that the rate of change of the mass of gas in the chamber is equal to the difference between that entering (i.e. burned) and that leaving the chamber (i.e. emitted)

$$V\frac{d\rho}{dt} = \dot{m}_b(t) - \dot{m}_e(t)$$

Perturbation quantities are now introduced.

$$\rho = \bar{\rho} + \rho' \;:\; \dot{m}_b(t) = \overline{\dot{m}_b} + \dot{m}_b' \;:\; \dot{m}_e = \overline{\dot{m}_e} + \dot{m}_e'$$

thus

$$\frac{\partial\rho}{\partial t} = \frac{\partial\rho'}{\partial t} \;:\; \dot{m}_b(t) = \overline{\dot{m}_b}\left(1 + \frac{\dot{m}_b'}{\overline{\dot{m}_b}}\right) \;:\; \dot{m}_e = \overline{\dot{m}_e}\left(1 + \frac{\dot{m}_e'}{\overline{\dot{m}_e}}\right)$$

now define

$$\theta \equiv \frac{\bar{\rho}V}{\overline{\dot{m}}}$$

Substitute into the conservation equations

$$V\frac{d\rho'}{dt} = \overline{\dot{m}_b} + \dot{m}_b' - \dot{m}_e' - \overline{\dot{m}_b}$$

but in the steady state as much mass on the average is leaving the engine as there is entering. Thus

$$\overline{\dot{m}_b} = \overline{\dot{m}_e}$$

and

$$V\frac{d\rho'}{dt} = \dot{m}_b' - \dot{m}_e'$$

Normalizing by the average mass flow rate and density values yields

$$V\frac{\overline{\rho}}{\overline{m}}\frac{\partial}{\partial t}\left(\frac{\rho'}{\overline{\rho}}\right) = \frac{\dot{m}_b'}{\overline{m}} - \frac{\dot{m}_e'}{\overline{m}}$$

The equation of state for a gas is (R is the gas constant)

$$p = \rho T R$$

The perturbation development is

$$p = \overline{p} + p' \ : \ \rho = \overline{\rho} + \rho' \ : \ T = \overline{T} + T'$$

substitute to obtain (second order infinitesimals are as usual set equal to zero)

$$(\overline{p} + p') = \left[\overline{\rho}\,\overline{T} + \overline{\rho}\,T' + \rho'\,\overline{T} + \rho'\,T'\right]R$$

from which

$$\overline{p} = \overline{\rho}\,\overline{T}R \quad : \quad p' = \left[\overline{\rho}\,T' + \overline{T}\rho'\right]R$$

and

$$\frac{\partial p'}{\partial t} = \left[\overline{\rho}\frac{\partial T'}{\partial t} + \overline{T}\frac{\partial \rho'}{\partial t}\right]R$$

dividing by $\overline{p}$ produces

$$\frac{\partial}{\partial t}\left(\frac{\rho'}{\overline{\rho}}\right) = \frac{\partial}{\partial t}\left(\frac{\rho'}{\overline{\rho}}\right) - \frac{\partial}{\partial t}\left(\frac{T'}{\overline{T}}\right)$$

where the last term on the right hand side of the equation is assumed zero. The literature suggests that good results may be obtained from this assumption and that appears to be, aside from mathematical tractability, its main justification.

From elementary theory describing the flow of a compressible gas through a nozzle (see reference 8, P. 8-37 equation (47) the mass flow rate of the emitted gas may be expressed as

$$\overline{\dot{m}_e} \propto \frac{p}{\sqrt{T}}$$

From this the perturbation analysis proceeds as

$$\overline{\dot{m}_e} + \dot{m}_e' = A \quad \frac{\overline{p} + p'}{\sqrt{\overline{T} + T'}}$$

but from perturbation theory

$$T' << \overline{T}$$

thus from the binomial theorem

$$\overline{\dot{m}_e} + \dot{m}_e' \doteq \frac{A}{\sqrt{\overline{T}}} \left\{ \overline{p} - \overline{p}\frac{T'}{2\overline{T}} + p' \right\}$$

from which

$$\frac{\dot{m}_e'}{\overline{\dot{m}_e}} = \frac{p'}{\overline{p}} - \frac{T'}{2\overline{T}}$$

An alternate ratio in terms of the characteristic rocket velocity (C*) to express the last term in the last equation is useful. From the definition of C*

$$C^* = \frac{p_c A_t}{\dot{m}}$$

from previous discussion

$$\dot{m} \propto \frac{p_c}{\sqrt{T}}$$

thus by substitution

$$C^* \propto \sqrt{T}$$

let

$$C^* = \overline{C^*} + C^{*'} = B\left[\left(\overline{T} + T'\right)\right]^{\frac{1}{2}} = B\left[\left(\overline{T}^{\frac{1}{2}} + \frac{1}{2}\frac{T'}{\overline{T}^{\frac{1}{2}}}\right)\right]$$

and then

$$\frac{C^{*'}}{\overline{C^*}} = \frac{1}{2}\frac{T'}{\overline{T}}$$

Using this relationship and the previous development to yield

$$\frac{\dot{m}_e'}{\overline{\dot{m}_e}} = \frac{p'}{\overline{p}} - \frac{C^{*'}}{\overline{C^*}}$$

Making all the substitutions developed to this point yields.

$$\theta_c \frac{\partial}{\partial t}\left(\frac{p'}{\overline{p}}\right) = \frac{\dot{m}_b'}{\overline{\dot{m}}} - \frac{p'}{\overline{p}} + \frac{C^{*'}}{\overline{C^*}}$$

It is now necessary to relate C* to the mixture ratio ($r = \dot{m}_o/\dot{m}_f$) and chamber pressure to continue the development. Evidence in the literature suggests that

$$C^* = f(p_c, r)$$

Using the expression for the total differential produces

$$C^{*'} = \left(\frac{\partial C^*}{\partial p_c}\right) p_c' + \left(\frac{\partial C^*}{\partial r}\right) r'$$

however other literature states that the first term on the right is negligible with respect to the second. Thus

$$C^{*'} \doteq r'\left(\frac{\partial C^*}{\partial r}\right)$$

Dividing by $\overline{c^*}$ yields the desired ratio

$$\frac{C^{*'}}{\overline{C^*}} = \frac{\bar{r}}{\overline{C^*}}\left(\frac{\partial C^*}{\partial r}\right)\frac{r'}{\bar{r}}$$

Now it is necessary to develop an expression for $\frac{r'}{\bar{r}}$. The rate at which propellant is being burned is given by the sum of the rates at which the oxidizer and the fuel burn.

$$\dot{m}_b \equiv \dot{m} = \dot{m}_{ob} + \dot{m}_{fb}$$

Performing the usual perturbation analysis and making use of the definition of the mixture ratio yields (after some tedium).

$$\frac{r'}{\bar{r}} = \frac{(1+\bar{r})}{\bar{r}}\frac{\dot{m}_{ob}'}{\bar{\dot{m}}} - \frac{(1+\bar{r})}{\bar{\dot{m}}}\dot{m}_{fb}'$$

This expression may be used in conjunction with the expression for the $c^{*'}/\overline{c^*}$ ratio to substitute into the differential equation developed earlier to produce the defining equation for low frequency stability investigations.

$$\theta_c\frac{\partial}{\partial t}\left(\frac{p'}{\bar{p}}\right) + \left(\frac{p'}{\bar{p}}\right) = \frac{1}{\bar{\dot{m}}}\left\{\left[1 + \frac{(1+\bar{r})}{\overline{C^*}}\left(\frac{\partial C^*}{\partial r}\right)\right]\dot{m}_{ob}' + \left[1 - \frac{\bar{r}(1+\bar{r})}{\overline{C^*}}\left(\frac{\partial C^*}{\partial r}\right)\right]\dot{m}_f'\right\}$$

A common way to investigate low frequency stability is by means of a Nyquist plot. To transform the equation above into a form suitable for such an investigation recall that

$$\dot{m}_{oxb}' = \dot{m}_{oi}'(t - \tau_f)$$
$$\dot{m}_{fb}' = \dot{m}_{fi}'(t - \tau_f)$$

Making use of the LaPlace transform to move to the complex s-plane and making all the substitutions indicated above yields the stability equation in a form suitable for the Nyquist plot.

$$\frac{e^{-\tau_f s}}{[\theta_c s + 1]}\left\{\left[1 + \frac{(1+\bar{r})}{\overline{C^*}}\left(\frac{\partial C^*}{\partial r}\right)\right]G_{ox} + \left[1 - \frac{\bar{r}(1+\bar{r})}{\overline{C^*}}\left(\frac{\partial C^*}{\partial r}\right)\right]G_f\right\} = -1$$

which is of the well known form (let s=j w)

$$K(jw) = -1$$

It is evident that not only does K(jw) encompass a transport lag but also includes possibly transcendental functions in the admittance functions (from the duct transfer

admittances). However it is well established that this presents no conceptual problem to applying the Nyquist procedure (see reference nine, p. 467, section 16). Realistically it does, however, require a computer to evaluate the contours of interest. One word of caution is in order. When using applications programs care should be taken to determine how the terms such as the transport lag or the complex hyperbolic functions are evaluated. For example, some programs use a Pade' approximation to the complex exponential. This approximation is only good over a limited frequency range. Other programs such as the new symbolic programs (e.g. MATHEMATICA) evaluate the various exponential functions without amplitude approximation although the treatment of phase angles greater than stipulated range(e.g. plus or minus pi) should be investigated.

Inspections of the stability equation suggests a simple extension to the case where both the fuel and the oxidizer time lags have to be accounted for separately. Merely multiply the transport lag exponential inside the brackets and note that each term is delayed. One term applies to the oxidizer, the other to the fuel. Now instead of one identical time lag being associated with each of the propellant constituents separate time lags may be associated with each constituent. This procedure yields the "double time lag model."

Also note in this development that n (the sensitivity index) does not appear. It would appear if the variability of the time delay term is taken into account. Indeed the literature suggests that in any but the low frequency investigations it must be taken into account. However, it may be neglected in the low frequency regime if the variability is small compared to the total time delay (see reference three, section 5.3).

Some examples of the Nyquist plots to be expected are exhibited in the Appendix titled FEEDLINE. In this appendix an increasing number of terms are included in an example problem to illustrate their cumulative effect. In this example the feedsystem admittance functions were calculated based on a proposed MSFC test stand design. Some of the engine parameters e.g.steady state mass flow rates were furnished by MSFC. However, other parameters such as the variation of C* with respect to mixture ratio were estimated from the general literature.

INTERMEDIATE FREQUENCY STABILITY MODELING

As previously indicated the next frequency range of interest is the intermediate one. In this regime the effects of combustion coupling with the feedsystem play a part as does the variability of the combustion time delay (hence introducing n directly). In addition the longitudinal oscillatory ("organ pipe") modes of the combustion chamber gases play a significant part in the stability analysis i.e. the chamber pressure etc is <u>not</u> considered to be constant throughout the chamber volume as was done in the low frequency stability analysis.

Because intermediate frequency stability in a given combuster propellant feed system combination depends upon the two combustion related parameters n and τ it is

customary to characterize graphically the stability condition of the system in terms of a parameter plane rather than a Nyquist plot. This plane (the n versus τ plane) contains contour separated regions denoting either stable or unstable system operation (in the small). Thus proximity to the stability boundary is to be avoided (keeping to the stable side, of course). An example of such a parameter plane is shown in the Code Development section.

Included immediately below is a synopsis of the intermediate frequency stability investigation and results. An extensive mathematical development leading to these results is contained in the Appendix titled "Intermediate Frequency Oscillations in a Liquid Propellant Rocket Nozzle."

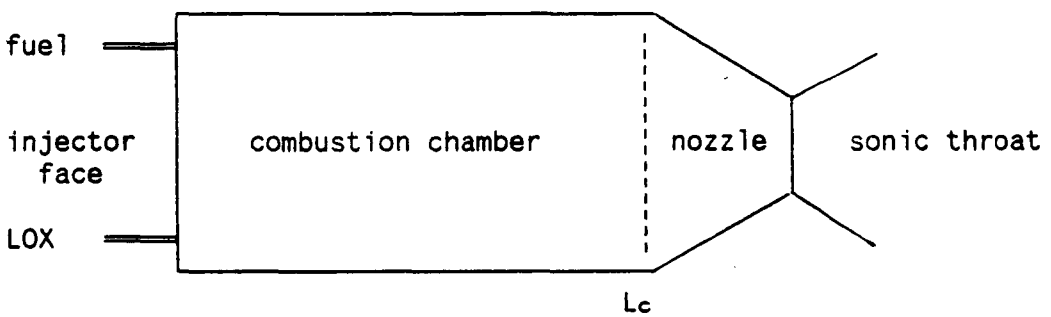
Intermediate Frequency Mode

Problem Description

The intermediate frequency mode couples the feed lines, combustion chamber, and the nozzle. Lower frequency modes couple only with the feed lines and the higher frequency modes couple only with the nozzle. Thus, for the intermediate mode, the whole system must be modeled.

The results of the low frequency mode study are used to determine the admittances for the fuel and LOX lines. The flow in combustion chamber is a complex phenomena and therefore some simplifying assumptions are made. The medium in the chamber consists of reactant gases, liquid oxidizer and fuel, and the gaseous products of combustion. This is represented as a two-phase mixture comprised of a mass-averaged gas comprising all species and a single mass-averaged liquid phase. The flow is inviscid (except for the existence of a droplet drag). The gases are thermally and calorically perfect. The liquid phase is well dispersed throughout the chamber. The variations of the energy (internal plus kinetic) of the liquid are neglected. And, for this analysis, there is no heat transfer to the walls.

The physical setup is illustrated by the schematic:



The engine consists of an injector face where the liquid propellants enter and the throat where the gaseous products exit. The rest of the surfaces are solid. Conservation requires that the mass of gaseous products leaving must equal the mass of the liquid propellants entering. The momentum of the two must also be equal. The energy of the products leaving must be equal to the energy of the propellants entering plus the energy due to chemical reaction and change of phase. The conservation equations are:

mass: $\partial\rho/\partial t + \nabla\cdot(\rho V) + \partial\rho_L{}^{\circ}/\partial t + \nabla\cdot(\rho_L{}^{\circ}V_L) = 0$

momentum: $\partial(\rho V)/\partial t + \nabla\cdot(\rho V\cdot V) + \nabla p = -\partial(\rho_L{}^{\circ}V_L)/\partial t - \nabla\cdot(\rho_L{}^{\circ}V_L\cdot V_L)$

energy: $\partial(\rho e_s)/\partial t + \nabla\cdot(\rho e_s V) + \partial(\rho_L{}^{\circ}e_{Ls})/\partial t + \nabla\cdot(\rho_L{}^{\circ}e_{Ls}V) + \nabla\cdot(pV) = E$

The boundary conditions may be expressed as

1. For the liquid phase, specify the injection velocity and the injection density
2. The admittance conditions are used as the boundary condition at the combustion chamber - nozzle interface
3. The condition at solid surfaces is the vanishing of the normal velocity component
4. There is no information passed upstream through the throat.

The mathematical development of the equations and boundary conditions are presented in the appendix titled "Intermediate Frequency Oscillations in a Liquid Propellant Rocket Nozzle" and will not be presented here. The non-dimensionalized equations based on small perturbation theory will be summarized.

Auxiliary equations:

$$X_1 = (\gamma - 1)\bar{u}u_0 + (1 + \bar{r})(\overline{dh_L/dr})(\bar{M}/s)e^{-s\tau_T}(G_{ox} - \bar{r}G_f)P_{00}$$

$$Y_1 = -\bar{u}p_0$$

$$Z_1 = (1/\gamma)\bar{u}p_0 + \bar{\rho}_L{}^0 u_{L0}$$

$$W_1 = 2\bar{u}u_0$$

$$M_1 = \bar{M}\{e^{-s\tau_T}[\mathcal{K}(s) + \mathcal{H}(s)]P_{00} - \mathcal{P}(s)p_0\}$$

where $\mathcal{K}(s) = (1 + s\tau_T)(G_{ox} + G_f)$ - effect of total mass flux oscillations

$\mathcal{H}(s) = (1 + \bar{r})[(\bar{r}/\bar{c}^*)(\partial c^*/\partial r) - n_r s\bar{\tau}](G_{ox} - \bar{r}G_f)/\bar{r}$ - effect of mixture ratio fluctuations

$$\mathcal{P}(s) = \bar{n}(1 - e^{s\tau})$$

$$p_0 = P_{00}\cosh(sx)$$

$$u_0 = -(1/\gamma)P_{00}\sinh(sx)$$

$$u_1 = Y_1 + \int_0^x \{[s(W_1 - X_1) + M_1]\cosh[s(x - x')] + s(Y_1 + Z_1)\sinh[s(x - x')]\}dx'$$

$$p_1/\gamma = -W_1 - \int_0^x \{[s(W_1 - X_1) + M_1]\sinh[s(x - x')] + s(Y_1 + Z_1)\cosh[s(x - x')]\}dx'$$

Applying the boundary conditions at L_c (as developed in section 3.6 of SP-194)

$$u'(L_c) + \mathcal{A}\, p'(L_c) + \mathcal{C}\, \sigma'(L_c) = 0$$

$$u' = u_0 + u_1$$

$$p' = p_0 + p_1$$

$$\sigma' = \sigma_0 + \sigma_1 = -(1/\bar{u})\int_0^x \{((\gamma - 1)/\gamma)p_0 + (1 + \bar{r})(\overline{dh_L/dr})(G_{ox}$$

$$- \bar{r}G_f)P_{00}e^{-s\tau_T}\}\bar{M}\, \exp[-s\int_{x'}^{x}(1/\bar{u})dx'']dx'$$

$\mathcal{C}$ is generally small compared with $\mathcal{A}$ and may be deleted for small perturbations. For rockets which have a cylindrical combustion chamber and conical nozzle, $\mathcal{A}$ may be approximated by $M(\gamma - 1)/(2\gamma)$, where M is the Mach Number at the intersection of the chamber and nozzle.

Parameter plane stability can be investigated by setting $\lambda = 0$ and regard ω as an independent variable. Then the boundary equation can be used as a relation between two of the engine design parameters or operating parameters (e.g. n vs τ). Values of ω in the intermediate mode may be chosen in the range

$$1/\bar{\tau}_T < \omega < 1/\bar{\tau}.$$

CODE DEVELOPMENT

Three FORTRAN codes are being delivered. These cover the three major modes of oscillation in a liquid propellant rocket: low frequency mode (feedline-combustion chamber interaction), intermediate frequency mode (feedline-combustion chamber-nozzle interaction), and high frequency mode (combustion chamber-nozzle interaction). Along with the codes, sample runs will also be included on the diskettes. Program listing of each of the codes will be furnished.

Low Frequency Program (FEEDLINE)

The code for the low frequency mode oscillations (FEEDLINE) is based on the MATHEMATICA program for the Macintosh as presented in appendix titled "A MATHMATICA File for Creating a Model of an Engine Feed System". The mathematical derivations are given in this report. The FORTRAN program was developed to generate input for the intermediate mode program and to allow the use of any PC compatible computer to run the code. Because the FORTRAN code is compiled and not interpreted, as with MATHEMATICIA, execution time is greatly reduced. The increased speed of the compiled program makes it a canidate for the primary code for the low frequency mode.

The code uses various files for the input and output of data. The input may be input by a data file, input interactively, or a combination of both. If the input is entered interactively, either completely or as changes to the data file, the input information is written to the input file for later runs or for use by IMODE, the intermediate frequency mode program. The file names used by the program are:

FUEL.DAT	- file with parameters for engine, tank, and fuel also with piping configuration,
LOX.DAT	- file with parameters for engine, tank, and lox also with piping configuration,
ADMIT.DAT	- output of admittances computed (print file),
ADMIT.PRN	- LOTUS file of admittances computed (plot file).

The code is interactive with the user, asking questions of the user and using the answers to determine what needs to be done. The first question asked is "IS THIS SETUP FOR FUEL OR OXIDIZER? ENTER F OR O.". This question is to determine whether to use FUEL.DAT for fuel or LOX.DAT for oxidizer. The other questions are asked of the user to read data from the file, input the data interactively, or to modify the data on the file. Once the system is completely described, then the admittance calculations are begun. This allows a range of frequencies to be run.

Certain results are output to the screen in order for the user to follow the progress of the calculations and possibly request additional frequency ranges to be run. Further output is directed to a print file (ADMIT.DAT) and a plot file (ADMIT.PRN). The print file may later be directed to a printer to obtain a hardcopy. The plot file may be used with LOTUS to obtain plots of admittance vs frequency. Either frequency or normalized frequency may be used for plotting as both are output to the plot file. A sample plot is included below.

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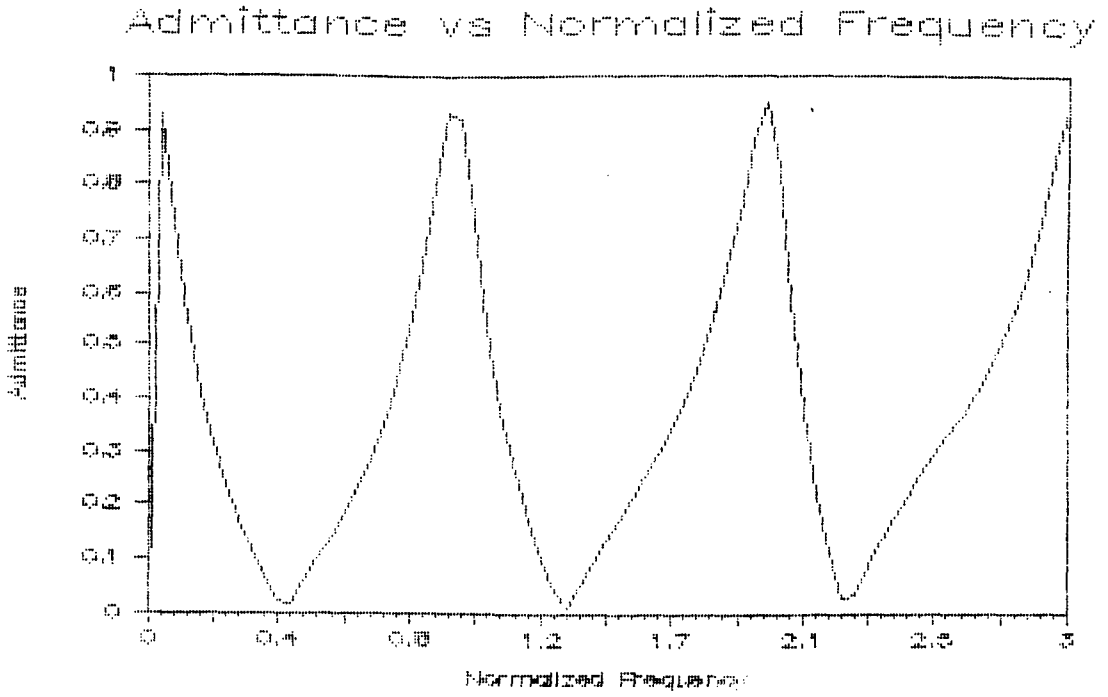

Intermediate Frequency Program (IMODE)

The code for the intermediate mode oscillations (IMODE) is an implementation of the equations presented in the appendix titled "Intermediate Frequency Oscillations in a Liquid Propellant Rocket Nozzle". The intermediate mode couples with the piping (using the results from FEEDLINE) and with the nozzle (using the combustion chamber - nozzle interface boundary condition). The program was designed to allow the parametric study of two variables at a given frequency of oscillation.

The program allows the user to specify an independent and a dependent variable. The dependent variable is iterated upon until the real part of the nozzle boundary condition is within 10^{-5} of zero. If it doesn't converge within 10 iterations, the current values are printed out and the user can decide whether to continue the iteration. By running through a series of values of the independent variable, data for a parameter plane stability plot is created. This may be plotted by using LOTUS. A sample n vs tau curve is included below.

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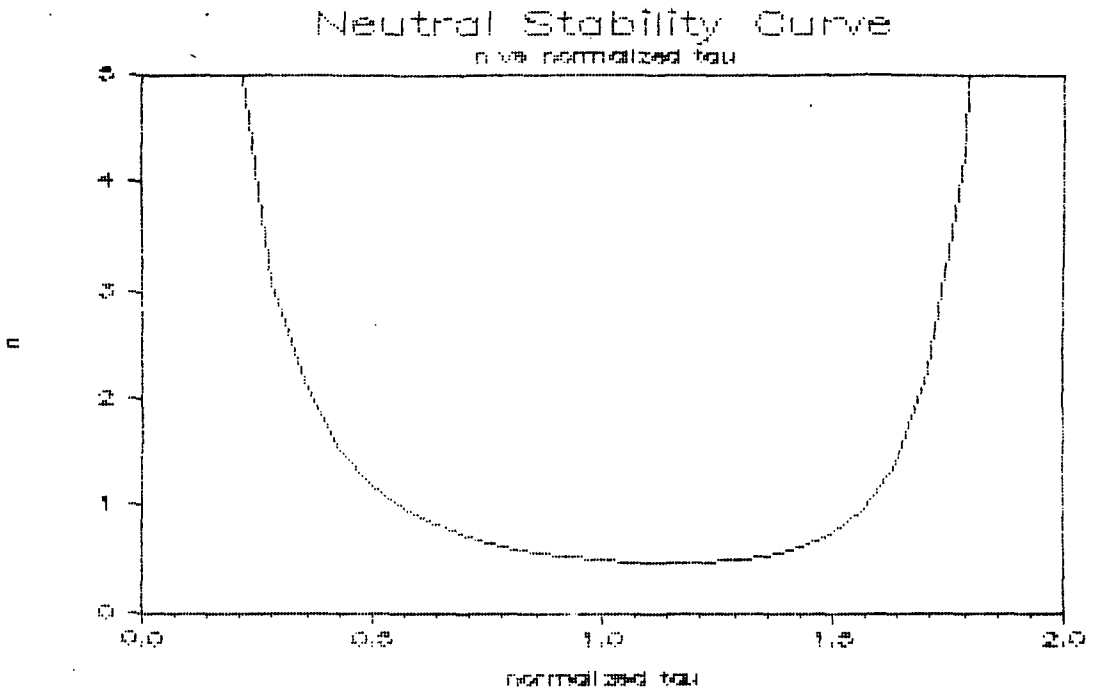

Five files are used by the program. Two are from FEEDLINE:

FUEL.DAT - file with parameters for engine, tank, and fuel also with piping configuration,

LOX.DAT - file with parameters for engine, tank, and lox also with piping configuration.

The other three files may have any names. The default names are:

IMODE.INP - file with parameters for the physical configuration,

IMODE.OUT - output of calculations (print file),

IMODE.PRN - LOTUS file with parametric values (plot file).

The user may use the default names or specify them at run time.

The program interactively allows the user to input values, modify values, change independent and dependent variables, change independent variable value, or exit program. Also, current values of input parameters may be listed or the names and dimensions (e.g. ft/sec) of them may be displayed.

High Frequency Program (FDORC)

The high frequency code was obtained from Dr. Mitchell at Colorado State University. The program was written for a VAX computer and has been modified to run on a PC. Three modes of oscillation are handled; radial, transverse, and longitudinal. No coupling with the piping is considered because of the high frequencies the program is designed to analyze. A complete description of the program is given in Refs. 5 and 6.

Several changes were made to the code to get it to run on a PC. In one subroutine, several variables were dimensioned by (0:100,0:100). However, in the instructions, a maximum of 10 was allowed for the upper dimension. Therefore, these were changed to (0:10,0:10) and the modified code would run on a PC. Without the changes, the compiled code required more than 640K of RAM. Also, a few other aesthetic changes were made.

Thirteen files are used by the program. The input files are:

PIN	- pressure input,
NOZIN	- input for nozzle admittance calculation,
ZONIN	- general combustor input,
CAV1DAT	- input for radially oriented absorbers,
CAV2DAT	- input for axially oriented absorbers,
RADIN	- radial cavity input,
AXIN	- axial cavity input.

The output files are:

ZDAT	- summary of results and echo of input,
ZNTAU	- summary of stability plot results,
NTPLOT	- contains n, tau data tabulated for plotting,
RADOUT	- results specific to radial absorbers,
AXOUT	- results specific to axial absorbers,
POUT	- contains desired pressure point calculations.

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4. Mitchell, C. E. and Eckert, K., A Simplified Computer Program for the Prediction of the Linear Stability Behavior of Liquid Propellant Combustors, BASA cr 3169, 1979.

5. Mitchell, C. E. , Stability Design Methodology Vol. 1, USAF Final Report AL-TR-89-041, 1989.

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7. Meisenholder, S. G. and Bickford, L. L., "Pogo" Analysis of the Saturn Propulsion System, Report AMDR 9635-037, Aerojet-General Corporation, 1967.

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1. Culick, F. E. C., "Combustion Instabilities in Liquid Fueled Propulsion Systems - An Overview", AGARD 72B PEP Meeting, October 6-7, 1988.

Nomenclature

G	- admittance $\left(\frac{\dot{m}'}{p'}\right)$
$\dot{m}'$	- perturbation in mass flow rate
p'	- perturbation in pressure
z	- impedance $\left(\frac{p'}{\dot{m}'}\right)$
p	- pressure
R	- ratio of pressure drop across a dissipative hydraulic circuit element e.g. an orifice to the square of the mass flow rate through the element
r'	- linearized value of pressure mass flow rate ratio evaluated at an operating point
$\overline{\dot{m}}$	- total propellant flow average value
$\overline{\dot{m}_L}$	- a propellant flow rate average value
L	- hudraulic circuit symbol for inertance
C	- hydraulic circuit symbol for capacitance
A	- cross sectional area of duct
l	- length of duct
ρ	- fluid density
k	- bulk modulus of fluid
PDE	- partial differential equation
S	- complex LaPlace operator
LOX	- liquid oxygen
RP1	- hydrocarbon fuel (kerosene like)
V	- volume of combustion chamber
T	- temperature on an absolute scale
t	- time
R	- gas constant in equation of state
C^*	- characteristic exhaust velocity
A_t	- nozzle throat area
r	- mixture ratio (oxidizer mass rate/ fuel mass rate)

APPENDIX I

Four Terminal Network Approximations to Duct Admittances

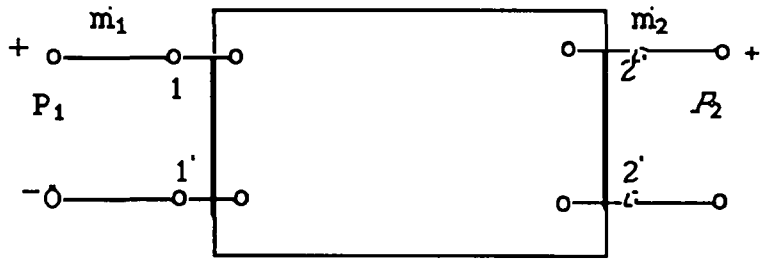
George B. Doane III

APPENDIX I

Four Terminal Network Approximations To Duct Admittances

In the main body of this report the transfer admittance of a duct is developed using a continuous distributed model. This is a good approach especially for uniform ducts. Another way to develop a model is to approximate the dynamics by assuming a finite element model. This is done by subdividing the duct into a number of pieces and then assuming that each piece is characterized by a lumped inertance, lumped capacitor and as applicable a lumped resistor. The number of elements to choose is an heuristic matter. Aerojet recommends choosing an element length eight times smaller than the wavelength of the highest harmonic of interest. A more conservative choice would be the oft-used order of magnitude. What with powerful computers as ubiquitous as they currently are an iterative approach in which successively smaller length elements are chosen until some figure of merit e.g. the nth harmonic frequency change is met would seem a tractable and rational way to assess the element length choice. On notes that in case the duct is not uniform but of arbitrary cross section versus position certainly the finite element model technique takes on considerable additional appeal.

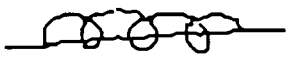
To treat the duct or indeed any cascade or tandem connection of elements use in made of four-terminal or two terminal pair network theory. This theory is well covered in the technical literature e.g. in the works of E.A. Guillemin but enough will be developed below for the purposes at hand. Consider the network specified in the figure below.



The indicated reference conditions (polarities or signs) for pressure and mass flow rate are an important part of the basic structure. The network in the box is arbitrary, except that it is composed of linear and bilateral elements and does not contain any pressure or mass flow rate sources or sinks. In the development that follows it is most convenient to use transform notation i.e. The LaPlace transform notation which converts time domain dynamics (differential equations) to s-domain algebraic ones. A summary of the zero initial condition hydraulic element relationship on interest here is given below.

Capacitor:

$$p(t) = \frac{1}{C}\int_{?}^{?} \dot{m}(t)dt \qquad P(s) = \frac{\dot{m}(s)}{Cs}$$



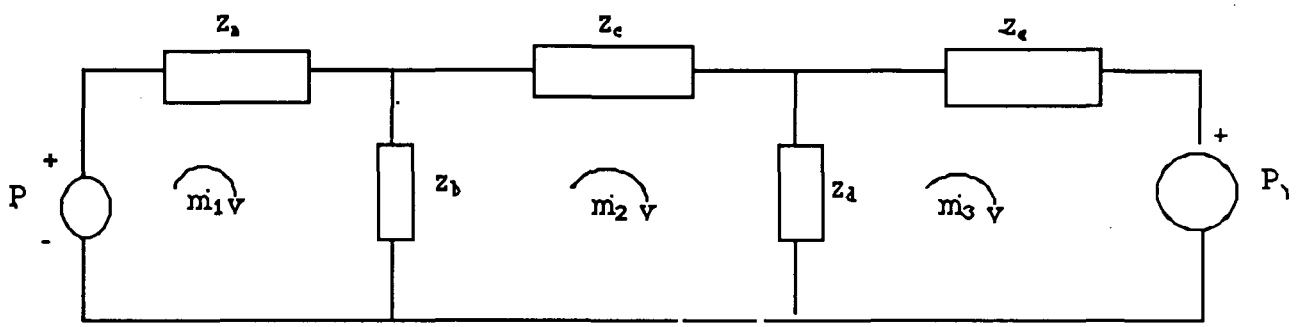
$$p(t) = \frac{L\dot{m}(t)}{dt} \qquad P(s) = Ls\dot{m}(s)$$

$$p(t) = R\dot{m}(t) \qquad P(s) = R\dot{m}(s)$$

Assuming a topologically flat network in the box it is possible to write the following set of equations.

$$\rho_{11}\dot{m}_1 + \rho_{12}\dot{m}_2 + \ldots + \rho_{1n}\dot{m}_n = P_1$$
$$\rho_{21}\dot{m}_1 + \rho_{22}\dot{m}_2 + \ldots + \rho_{2n}\dot{m}_n = -P_2$$
$$\rho_{31}\dot{m}_1 + \rho_{32}\dot{m}_2 + \ldots + \rho_{3n}\dot{m}_n = 0$$

The factors ρ_{pq} have the dimensions of hydraulic impedance. When p=q they are the sum of all impedances on a loop contour. When $p \neq q$,ρ_{pq} is + or - the impedances Using the "window pane" method of traversing all loops in the same direction e.g. clockwise the sign will be negative. As an example consider the 3 loop or mesh network below



$$\rho_{11} = z_a + z_b \qquad \rho_{12} = \rho_{21} = -z_b \qquad P_1 = P_a$$
$$\rho_{22} = z_b + z_c + z_d \qquad \rho_{13} = \rho_{31} = 0 \qquad P_2 = 0$$
$$\rho_{33} = z_d + z_e \qquad \rho_{23} = \rho_{32} = -z_d \qquad P_3 = -P_\gamma$$

Solving the set of equations consists of determining the k*th* mass flow rate according to

$$\dot{m}_k = \frac{\Delta 1kP_1}{\Delta} + \frac{\Delta 2kP_2}{\Delta} + \ldots + \frac{\Delta nkP_n}{\Delta}$$

where k = 1, 2,... N successively and Δ is the system determinant

$$\Delta = \begin{vmatrix} \rho_{11} & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{21} & \rho_{22} & \cdots & \rho_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \rho_{n1} & \rho_{n2} & \cdots & \rho_{nn} \end{vmatrix}$$

and the ρ_{pq} terms are the cofactors of Δ. The actual performance of these operations becomes quite tedious if N is greater than about 3. If it should ever

be required to determine them for larger order systems a symbolic computer language such as MATHEMATICA is suggested.

Choosing k = 1 and k = 2 yields a solution to the four terminal network problem as

$$\dot{m}_1 = \frac{\Delta_{11}}{\Delta}P_1 - \frac{\Delta_{21}}{\Delta}P_2$$
$$\dot{m}_2 = \frac{\Delta_{12}}{\Delta}P_1 - \frac{\Delta_{22}}{\Delta}P_2$$

It is conventional as well as convenient to define the following quantities.

$$y_{11} = \frac{\Delta_{11}}{\Delta}$$
$$y_{12} = y_{21} = \frac{\Delta_{12}}{\Delta}$$
$$y_{22} = \frac{\Delta_{22}}{\Delta}$$

So that the equations may be written as below.

$$\begin{aligned} \dot{m}_1 &= y_{11}P_1 - y_{12}P_2 \\ \dot{m}_2 &= y_{21}P_1 - y_{22}P_2 \end{aligned} \quad or \quad \begin{bmatrix} \dot{m}_1 \\ \dot{m}_2 \end{bmatrix} = \begin{bmatrix} y_{11} & -y_{12} \\ y_{21} & -y_{22} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$$

These expressions are called the y-system equations. If the y matrix formed from the y coefficients is inverted and left multiplied times the left and right sides of the equation above one obtains

$$\begin{aligned} P_1 &= z_{11}\dot{m}_1 - z_{12}\dot{m}_2 \\ P_2 &= z_{21}\dot{m}_1 - z_{22}\dot{m}_2 \end{aligned} \quad or \quad \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} z_{11} & -z_{12} \\ z_{21} & -z_{22} \end{bmatrix} \begin{bmatrix} \dot{m}_1 \\ \dot{m}_2 \end{bmatrix}$$

where

$$z_{11} = \frac{y_{22}}{|y|}$$

$$z_{12} = z_{21} = \frac{y_{12}}{|y|}$$

$$z_{22} = \frac{y_{11}}{|y|}$$

$$|y| = y_{11}y_{22} - y_{12}^2$$

This latter formulation is sometimes referred to as the z-system equations for a four terminal network.

Both the y and z equations are relationships among the four variable $P_1, P_2, \dot{m}_1, \dot{m}_2$. A third set of equations is possible. They may be derived from the y and z set as follows. From the second equation in the y set solve for P_1 as

$$P_1 = \frac{y_{22}}{y_{21}}P_2 + \frac{1}{y_{21}}\dot{m}_2$$

Now substitute this expression into the first y equation to yield

$$\dot{m}_1 = y_{11}\left(\frac{y_{22}}{y_{21}}P_2 + \frac{1}{y_{21}}\dot{m}_2\right) - y_{12}P_2$$

or

$$\dot{m}_1 = \frac{|y|}{y_{21}}P_2 + \frac{y_{11}}{y_{21}}\dot{m}_2 \quad (recall\ y_{12} = y_{21})$$

A similar procedure carried out on the z set yields

$$P_1 = \frac{z_{11}}{z_{21}}P_2 + \frac{|z|}{z_{21}}\dot{m}_2$$

$$\dot{m}_1 = \frac{1}{z_{21}}P_2 + \frac{z_{22}}{z_{21}}\dot{m}_2$$

Because there are now two sets of equations relating the vectors $[P_1\ \dot{m}_1]^T$ *and* $[P_2\ \dot{m}_2]^T$ both can be expressed as the same equation by defining

$$A = \frac{z_{11}}{z_{21}} = \frac{y_{22}}{y_{21}}$$
$$B = \frac{|z|}{z_{21}} = \frac{1}{y_{21}}$$
$$C = \frac{1}{z_{21}} = \frac{|y|}{y_{21}}$$
$$D = \frac{z_{22}}{z_{21}} = \frac{y_{11}}{y_{21}}$$

Thus a simple set of equations may be written in terms of the ABCD parameters as

$$P_1 = AP_2 + B\dot{m}_2$$
$$\dot{m}_1 = CP_2 + D\dot{m}_2$$

It is to be noted that the equations above yield descriptions of the four terminal network in terms of three y or three z coefficients whereas the general equation has four coefficients. This is because the ABCD are not mutually independent. This dependency can be expressed as

$$AD - BC = 1$$

as can be verified by direct substitution of the y's or z's.

In as much as the motivation for this Appendix was to treat a duct as a succession of finite elements it is now necessary to demonstrate how to cascade networks. Actually use of the ABCD methodology renders it straightforward. Consider the figure

For each network the input-output relations are given as

$$\begin{bmatrix} P_1 \\ \dot{m}_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} P_2 \\ \dot{m}_2 \end{bmatrix}$$

$$\begin{bmatrix} P_1' \\ \dot{m}_1' \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} P_2' \\ \dot{m}_2' \end{bmatrix}$$

however

$$\begin{bmatrix} P_2 \\ \dot{m}_2 \end{bmatrix} = \begin{bmatrix} P_1' \\ \dot{m}_1' \end{bmatrix}$$

therefore

$$\begin{bmatrix} P_1 \\ \dot{m}_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} P_2' \\ \dot{m}_2' \end{bmatrix}$$

Multiplying out the matrices immediately to the right of the equals mark yields a two by two matrix equation relating the input and output pairs of the network. In the main body of work input impedance or its reciprocal, admittance, is used. Thus an expression for it is required. Given the overall expressions

$$P_1 = EP_2 + F\dot{m}_2$$
$$\dot{m}_1 = GP_2 + H\dot{m}_2$$

The impedance looking into the network at the I - I' terminals is given by

$$z_1 = \frac{P_1}{\dot{m}_1} = \frac{EP_2 + F\dot{m}_2}{GP_2 + H\dot{m}_2}$$

now let

$$z_r = \frac{P_2}{\dot{m}_2}$$

so that

$$z_1 = = \frac{Ez_r + F}{Gz_r + H}$$

is the desired result. EFG and H are obtained from the elements of a matrix resulting from multiplying together individual finite element transmission matrices. Thus depending upon the desired usage they could either be done symbolically or numerically (for a given frequency for example) by computer.

Perusal of an Aerojet report circa 1967 dealing with some propellant feedsystem analyses shows use of yet another alternate formulation. This formulation makes use of a propagation function derivable from the work above. Closer study shows it to produce a result directly parallel to the development in the main body of text in this report. The defining function for the propagation function is taken to be

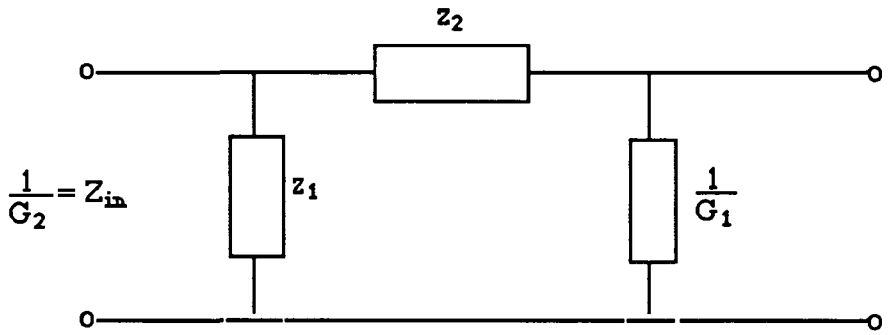
$$e^{\gamma} = \sqrt{AD} + \sqrt{BC}$$

or

$$\gamma = ln(\sqrt{AD} + \sqrt{BC})$$

Perhaps it is now apparent through the foregoing somewhat formal development that the transfer admittance technique used in the main text is highly related to this ABCD technique. As a matter of interest both formalisms will be used in the example given below.

As an example the lumped element model of a duct section element will be taken to have the same configuration as that used on an infinitesimal scale for the continuous distributed model.



$z_1 = \frac{1}{Cs}$

$z_2 = R + Ls$

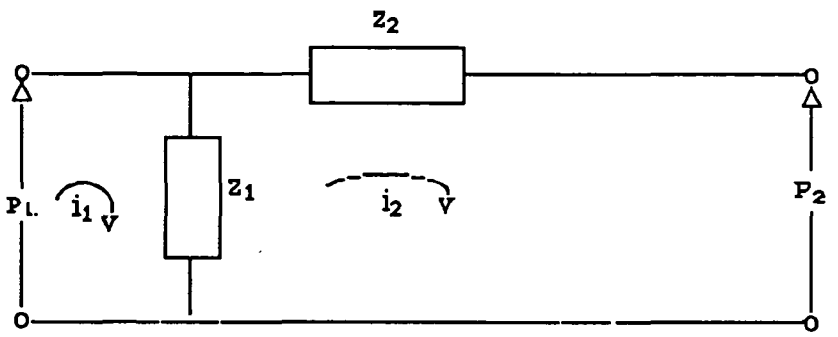
Using network reduction techniques as before one writes

$$z_{in} = \frac{1}{G_2} = z_1 \| z_2 \,\&\, \frac{1}{G_2}$$

$$z_{in} = \frac{(z_1)\left(z_2 + \frac{1}{G_1}\right)}{\left(z_1 + z_2 + \frac{1}{G_1}\right)} = \frac{(z_1)(1 + G_1 z_2)}{G_1\left(z_1 + z_2 + \frac{1}{G_1}\right)}$$

$$\frac{G_2}{G_1} = \frac{\left(z_1 + z_2 + \frac{1}{G_1}\right)}{z_1(1 + G_1 z_2)}$$

as the desired transfer admittance ratio for this element. To formulate this problem using the ABCD method designate loops and write equations as shown below.



$$P_1 = z_1 \dot{m}_1 - z_2 \dot{m}_2$$
$$-P_2 = -z_1 \dot{m}_1 + (z_1 + z_2)\dot{m}_2$$

or
$$\begin{bmatrix} z_1 & -z_1 \\ -z_1 & (z_1 + z_2) \end{bmatrix} \begin{bmatrix} \dot{m}_1 \\ \dot{m}_2 \end{bmatrix} = \begin{bmatrix} P_1 \\ -P_2 \end{bmatrix}$$

These equations are of the form used in the derivation. Therefore by matching coefficients

$$z_{11} = z_1 \qquad z_{12} = z_{21} = z_1 \qquad z_{22} = z_1 + z_2$$
$$[z] = z_1 z_2$$

so

$$A = \frac{z_{11}}{z_{21}} = 1 \qquad B = \frac{[z]}{z_{21}} = \frac{z_1 z_2}{z_{21}} = z_2$$

$$C = \frac{1}{z_{21}} = \frac{1}{z_1} \qquad D = \frac{z_{22}}{z_{21}} = \frac{(z_1 + z_2)}{z_1}$$

and from before

$$z_{\dot{m}} = \frac{Az_r + B}{Cz_r + D} = \frac{\dfrac{1}{G_1} + z_2}{\dfrac{1}{z_1 G_1} + \dfrac{(z_1 + z_2)}{z_1}} = \frac{z_1\left(z_2 + \dfrac{1}{G_1}\right)}{\left(z_1 + z_2 + \dfrac{1}{G_1}\right)}$$

$$G_2 = \frac{\left(z_1 + z_2 + \dfrac{1}{G_1}\right)}{z_1\left(z_2 + \dfrac{1}{G_1}\right)}$$

$$\frac{G_2}{G_1} = \frac{\left(z_1 + z_2 + \frac{1}{G_1}\right)}{z_1(1 + z_2 G_1)}$$

This result is of course precisely the same as previously developed from network reduction theory.

To summarize then if it is desired to cascade finite element models such as finite dimensional duct segments modeled as having finite lumped inertance, capacitance and resistance several techniques are available. Two are developed above. These are the transfer admittance approach as used previously and the ABCD approach. The use of either would be greatly facilitated in actual practice by use of computer based techniques to perform the manipulations (symbolic mathematics) and the numerical evaluations.

APPENDIX II

This Appendix was created in its present form by Ms. Robin Weir.

FEEDLINE

A MATHMATICA FILE FOR CREATING AN IMPEDANCE MODEL OF A PRESSURE FED LIQUID PROPELLANT ROCKET ENGINE FEED SYSTEM

DECEMBER 1989

About FEEDLINE

The FEEDLINE Mathmatica file can be used to create a model for an engine feed system. Using the approach found in document SP-194 by Harrje and Reardon, FEEDLINE calculates admittance ratios for each component of the feed system. These components may include the fuel tank, straight piping, bends in piping, valves, bellows, manifold and injector orifices. The file then calculates an overall admittance of the system (as seen from the engine), by multiplying these separate admittance ratios together. From this equation, frequency response plots such as Bode and Nyquist can be created to study low frequency instability (chugging), or it can be used as input to the intermediate combustion stability model developed separately.

Shown below is the analytical approach to FEEDLINE, using a simple system as in figure 1. It is important to note where each component "occurs" in the feed system, because each admittance ratio depends on the admittance ratios of the sections above it.

The equation for the overall admittance is:

$$G_i = G_0\left(\frac{G_1}{G_0}\right)\left(\frac{G_2}{G_1}\right)\left(\frac{G_3}{G_2}\right)\left(\frac{G_4}{G_3}\right)\left(\frac{G_5}{G_4}\right)\left(\frac{G_6}{G_5}\right)\left(\frac{G_7}{G_6}\right)\left(\frac{G_8}{G_7}\right)\left(\frac{G_9}{G_8}\right)\left(\frac{G_{10}}{G_9}\right)\left(\frac{G_{11}}{G_{10}}\right)\left(\frac{G_{12}}{G_{11}}\right)$$

where,

G_0 =admittance at the top of the fuel tank
$\frac{G_1}{G_0}$ =admittance ratio for the tank
$\frac{G_2}{G_1}$ =admittance ratio for line 1
$\frac{G_3}{G_2}$ =admittance ratio for the valve
$\frac{G_4}{G_3}$ =admittance ratio for line 2
$\frac{G_5}{G_4}$ =admittance ratio for bend 1
$\frac{G_6}{G_5}$ =admittance ratio for line 3
$\frac{G_7}{G_6}$ =admittance ratio for bend 2
$\frac{G_8}{G_7}$ =admittance ratio for line 4
$\frac{G_9}{G_8}$ =admittance ratio for the bellows
$\frac{G_{10}}{G_9}$ =admittance ratio for line 5

$\frac{G_{11}}{G_{10}}$=admittance ratio for the manifold

$\frac{G_{12}}{G_{11}}$=admittance ratio for the orifices

Since there is no fuel flowing into the top of the tank (for a pressure fed system),

$$G_0=0$$

therefore, by cancelation of terms,

$$G_i=G_1\left(\frac{G_2}{G_1}\right)\left(\frac{G_3}{G_2}\right)\left(\frac{G_4}{G_3}\right)\left(\frac{G_5}{G_4}\right)\left(\frac{G_6}{G_5}\right)\left(\frac{G_7}{G_6}\right)\left(\frac{G_8}{G_7}\right)\left(\frac{G_9}{G_8}\right)\left(\frac{G_{10}}{G_9}\right)\left(\frac{G_{11}}{G_{10}}\right)\left(\frac{G_{12}}{G_{11}}\right)$$

Each component of the feed system has a general equation for its admittance ratio. Therefore, it is easy to accommodate changes in the system by simply calculating the additional section's admittance ratio and inserting it in the proper place in the equation above. Adding a ratio, however, will cause the subscripts in the following ratios to change. Therefore, check all subscripts when an addition is made. Another important note to make is that several sections of the feed system may have similar parameter values. For example the diameter of all the straight lines may be the same, or the bends may have the same angles. Therefore, you may want to define a general parameter value that may be used in calculating several different admittance ratios. Finally, it is very important to use the correct units, or all calculations will be inaccurate. The general admittance ratio equations for various components, normalized with respect to the combustion chamber pressure, are shown below:

*Admittance ratio for the fuel tank:

$$\frac{G_1}{G_0}=1+sctank$$

for $G_0=0$,

$$G_1=sctank$$

ctank=capacitance associated with the fuel tank

$$ctank=\frac{den*vol*prchamb}{ktank*tflow}$$

den=density of fluid $\left(\frac{lb}{ft^3}\right)$

vol=volume of fuel tank (ft^3)

ktank=bulk modulus of fluid in tank $\left(\frac{lb}{ft^2}\right)$

prchamb=pressure inside combustion chamber $(\frac{lb}{ft^2})$

tflow=total flow rate of fluid inside engine $(\frac{lb}{s})$

s=jw

*Assume x&y are subscripts, y=x+1

*Admittance ratio for constant area feed line: (straight pipe)

$$\frac{G_y}{G_x}=\frac{1+(\frac{1}{G_x Zline})\tanh(stl)}{1+G_x Zline\tanh(stl)}$$

Zline=impedance associated with the feed line

$$Zline=\frac{a*tflow}{areal*prchamb*g}$$

a=velocity of sound in the fluid $(\frac{ft}{s})$

areal=area of feed line (ft^2)

g=acceleration of gravity $(32.2\ \frac{ft}{s^2})$

tl=time constant of line

tl=lenl/a (s)

lenl=length of line (ft)

*Admittance ratio for orifices:

$$\frac{G_y}{G_x}=\frac{1}{1+ZorG_x}$$

Zor=impedance associated with the orifices

$$Zor=\frac{2*dpror*tflow}{lflow*prchamb}$$

dpror=pressure drop across orifices $(\frac{lb}{ft^2})$

lflow=flow rate of fluid through the line $(\frac{lb}{s})$

*Note that this method (for orifices) may also be used to find the admittance ratio for a <u>valve</u>.

*Admittance ratio for bellows:

model as straight pipe for now

*Admittance ratio for manifold: (similar to model of fuel tank)

$$\frac{G_y}{G_x}=1+\frac{sCm}{G_x}$$

Cm=capacitance associated with manifold
(refer to equation used for ctank)

*Admittance ratio for bends in the line: From Dr. George Doane

In representing a bend in the feed line, you want to show an increase in inertance with constant fluid volume. Therefore, a bend is modeled as a straight pipe with a decreased area and an increased length. Assume that the bend is two feet long (one foot on each side). Therefore,

l1 = 2 (ft)
A1 = area of straight pipe (ft^2)

Recall, for hydraulic lines:

L=l/A	inertance
C=(p/k)lA	capacitance

therefore, we can say,

$$C_1=(\frac{p}{k})l_1A_1 \qquad C_2=(\frac{p}{k})l_2A_2$$
$$L_1=\frac{l_1}{A_1} \qquad L_2=\frac{l_2}{A_2}$$

for a constant fluid volume,

$$C_1=C_2$$

then,

$$l_1A_1=l_2A_2$$
$$l_2=(\frac{A_1}{A_2})l_1$$

now set $L_2=\lambda L_1$

where λ is a constant $=\frac{L_2}{L_1}$

then,

$$\frac{l_2}{A_2}=\lambda\frac{l_1}{A_1}$$

$$A_1 l_2=\lambda A_2 l_1$$

dividing by l_2:

$$A_1=\lambda A_2\left(\frac{l_1}{l_2}\right)$$

Using the information found previously,

$$l_2=l_1\frac{A_1}{A_2}$$

then,

$$A_1=\lambda\left(\frac{A_2}{A_1}\right)A_2$$

$$A_1^2=\lambda A_2^2$$

$$A_2=\frac{A_1}{\sqrt{\lambda}}$$ "new area of line"

$$l_2=\frac{l_1 A_1\sqrt{\lambda}}{A_1}=\sqrt{\lambda l_1}$$ "new length of line"

to find λ:

$$\lambda=\frac{L_2}{L_1}$$

where,

$$L_1=\frac{l_1}{A_1}$$

$$L_2=L_1+L''$$

L'' found from graph in SP194, P. 109 (see APPENDIX A)

Using FEEDLINE

The first step in using FEEDLINE is to break the system up into its various components so that each section will have a separate admittance ratio. By clearly indicating these divisions, you will avoid confusion in the latter calculations.

One of the most tedious tasks in using this model is obtaining the parameter values needed for calculations. One piece of information that is important in finding accurate values for viscosity, bulk modulus, etc.. is the pressure drop down the feed line. Disk 1 is set up to help with these calculations. First, however, the analytical approach is discussed below:

The first step is to calculate the Reynold's number in the line, which determines whether the flow is laminar or turbulent.

Form the equation on p. 81 of Fluid Mechanics:

$$N_R = \frac{den*vel*dia}{vis}$$

den=density of fluid $(\frac{lb}{ft^3})$

vel=velocity of fluid $(\frac{ft}{s})$

$$vel = \frac{lflow}{den*areal}$$

where,

lflow=flow rate of fluid in line $(\frac{lb}{s})$

areal=area of line (ft^2)

dia=diameter of line (ft)

vis=viscosity of fluid $(\frac{lb}{ft \cdot s})$

For a Reynold's number greater than 10,000, we can assume that the flow is turbulent.

The next step is to find an approximate friction factor associated with the Reynold's number. This can be done using the chart in Appendix B, which comes from p. 195 of Fluid Mechanics. Assume that we are using smooth pipes.

We then want to find the head loss in each section using the equation found on p. 93 of Fluid Mechanics:

$$head=\frac{fr*len*vel^2}{dia*2*g}$$

where,

fr=friction factor

len=length of section (ft)

With this information, we can calculate the pressure drop in the pipe, and the pressure drop due to friction.

For the pressure drop in the pipe we use the equation from p. 98 of Fluid Mechanics:

$$dprl=\frac{den*lenl}{144}$$

And, for the pressure drop due to friction, the equation on p. 98 of Fluid Mechanics:

$$dprfr=\frac{head*den}{144}$$

With these values for each component of the feed system, we can find the total pressure drop, which in turn, allows us to find more accurate values of the bulk modulus of the fluid, viscosity of the fluid, etc...

On the computer

Insert DISK1 to calculate the pressure drop in the line. Click on to the FEEDLINE1 disk icon and then four Mathmatica icons will appear to choose from. Each file is set up for various systems. There are two LOX systems, one with bends and one without bends, and two RP-1 systems, one with bends and one without bends. Choose the appropriate file by clicking onto the icon, and Mathmatica will then be started. You are now ready to make changes or add new parameters to the file.

*It is important to remember, when entering these parameters, that you want to evaluate each entry as a separate piece of information. To do this, press shift/return after you are finished working on the entry, and a bracket should appear on the right side of the screen as shown below:

den = 53.3

Note that in the example above, the bracket has an extra stem on the upper side. This indicates that the cell is INACTIVE. If the entry on the screen appears with a bracket containing only one stem rather than two stems as shown below,

den = 53.3

then this cell is ACTIVE.

In order for Mathmatica to recognize this cell and use the information in it, you want the cell to be ACTIVE. To activate an INACTIVE cell, click onto the cell and go up to CELLS. Under CELLS, the word INACTIVE should appear with a check mark beside it. Go down to the word INACTIVE and highlight it. This will make the check mark disappear, and the cell will be ACTIVE and ready to be evaluated by Mathmatica.

*In all of these files, general parameter abbreviations have been designated with meanings and units shown in the list in APPENDIX C.

*Once again, it is important that the proper units are used in defining these parameters, or the calculations will be inaccurate.

For the pressure calculations on DISK1, you must first find a value for the friction factor. Notice that the file already contains values for the friction factors. However, those factors are associated with the parameter values initially used, therefore, if any parameter value is changed, you must evaluate the notebook again with the new values entered. The easiest way to do this is to first make the parameter changes, checking that all those cells are ACTIVE. Then, you want to go to the top of the screen under SPECIAL, and choose EVALUATE NOTEBOOK. Mathmatica will then evaluate every ACTIVE cell in the notebook. You then want to look to see what the new Reynold's numbers are. Using these values and assuming that the pipes are smooth, use the chart in APPENDIX B to find the associated friction factors. Once you have those values, go back to your file and enter them in the proper place. Now, go under SPECIAL and pick EVALUATE NOTEBOOK again. Mathmatica will evaluate every ACTIVE cell, and will give you the new pressure drop calculations.

*Remember that you must have pressure drop equations for every section of the feed system. Make sure that each equation contains the parameters with the proper subscripts associated with that section.

With the information from this file, you can go to various references (see the list in APPENDIX D) to find values for the bulk modulus, viscosity, etc...

Once you have found all the necessary parameters, you are ready to use DISK2 to calculate the admittance ratios.

Insert DISK2 and click on to the FEEDLINE2 disk icon. As before, you will see four icons, representing the four different systems. Choose the appropriate icon and the Mathmatica program will begin.

You will then see a list of parameters similar to those in the files on DISK1. In the same manner as before, you may make changes in these values, checking that each cell is still ACTIVE. After making these changes, you should once again, check all the equations to make sure that the proper subscripts are inserted, and that each section is included. You may then evaluate the entire notebook by going up to SPECIAL, and choosing EVALUATE NOTEBOOK. Another option is to evaluate each entry step by step, by pressing shift/return for each entry.

*The Mathmatica file that you are working on may contain plot commands from previous runs. Since these plots require a lot of memory and time to run, it is best to make these cells INACTIVE when they are not needed, so that Mathmatica will ignore them when you choose EVALUATE NOTEBOOK.

Frequency Plots

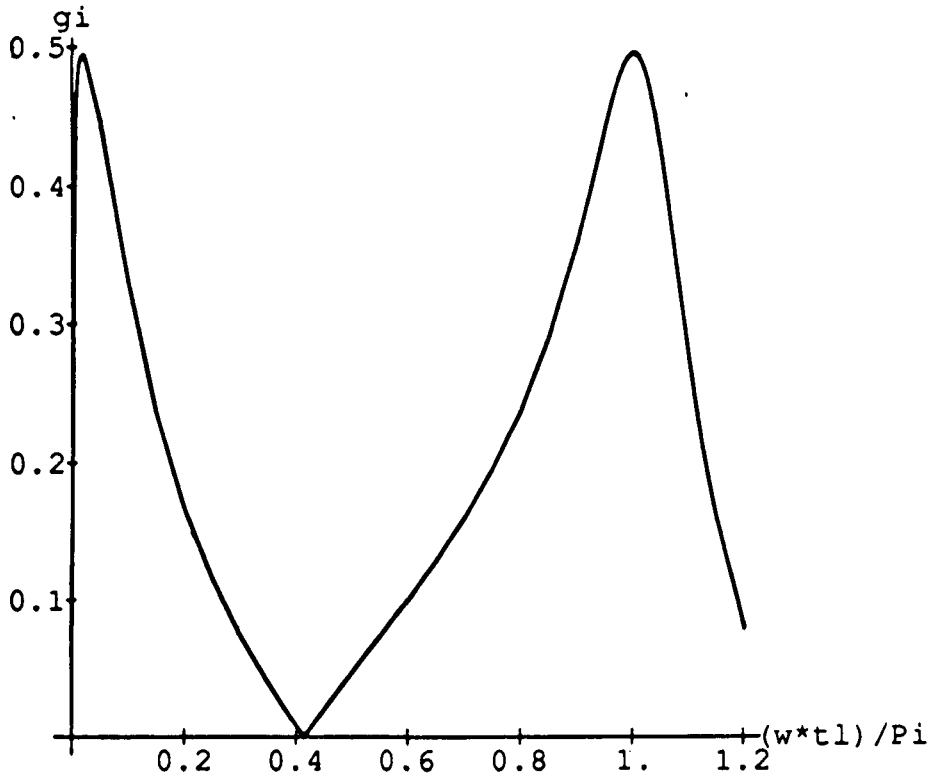
We have found in running these files, the feed systems including the bends require a tremendous amount of calculations to produce a plot. This is because of the larger number of admittance ratios in these systems. When trying to produce frequency plots of these systems, the kernel of Mathmatica is usually destroyed due to lack of memory. We realize that the cause of this problem stems from the fact that in the initial setup of these files, Mathmatica will not remember the previous calculations it has made. Therefore, it recalculates every equation over each time a new admittance ratio is figured. One can see from this that the time and memory required for calculation as admittance ratios are added grows exponentially. Efforts are still being made to write commands that will cause Mathmatica to remember the values it has already found, thus using less memory and speeding up the time needed. An approach that has been used to get frequency plots is discussed next.

Once you have entered all necessary parameters (using the systems without bends), you may plot your results in different forms of frequency response plots. These plots, as said before, require a lot of time and memory to run. Therefore, it is best that the commands are only evaluated one at a time (using shift/return), rather than using EVALUATE NOTEBOOK.

Now you are ready to make some plots. Some examples are shown.

This plot gives you the magnitude of your admittance versus frequency:

```
pgi5 = ParametricPlot[{(w*tl)/Pi,Abs[gi[w]]},{w,.0001,1000},
        AxesLabel->{"(w*tl)/Pi","gi"}]
```



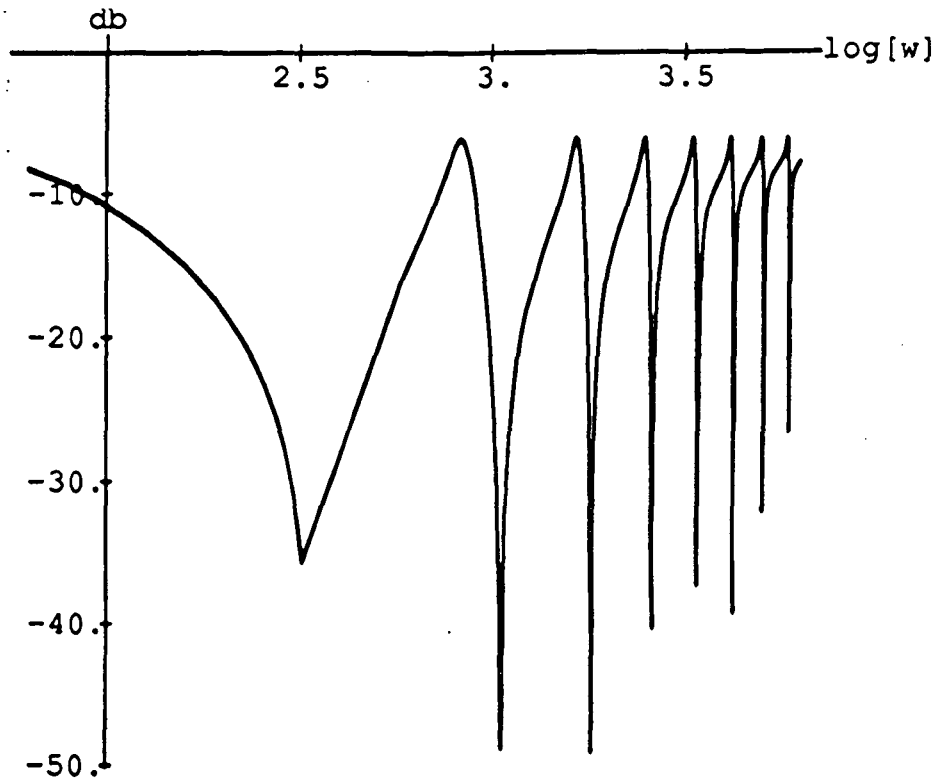

```
-Graphics-
```

The basic form for a plot of this type is:

Parametric Plot [{x, y}, {x, x min, x max}]

This plot gives you the admittance ratio in decibels versus w on a log scale:

```
pgi2 =ParametricPlot[{Log[10,w],20*Log[10,Abs[gi[w]]]},{w,62.8,6280},
        AxesLabel->{"log[w]","db"}]
```

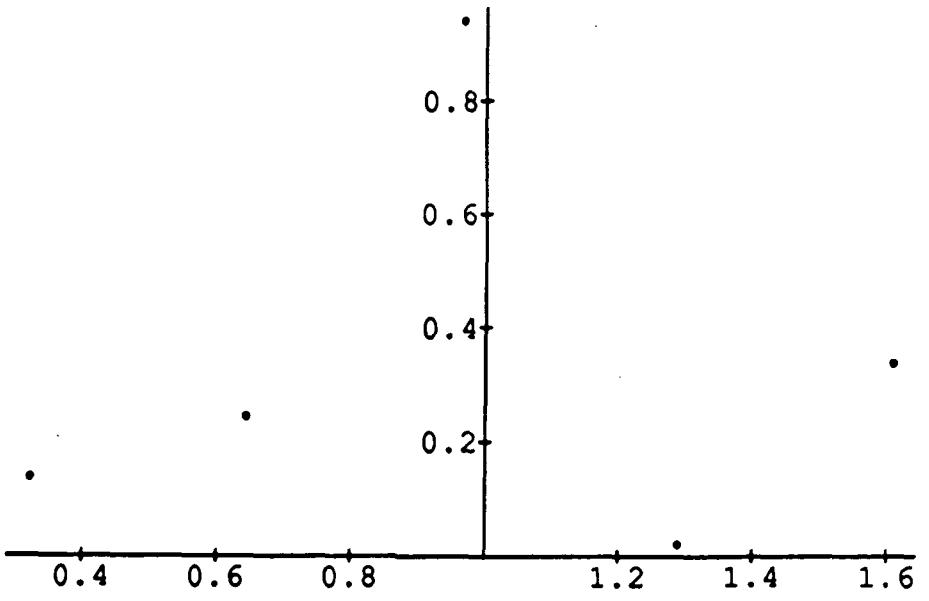



```
-Graphics-
```

*Note that the previous plots were made using the feed systems <u>without</u> bends. Since the time required for plotting the feed systems with bends is so great, you may just want to create a table of a few points which can then be plotted. An example follows.

As a note, this table containing only five points required about 30 or 45 minutes to be produced:

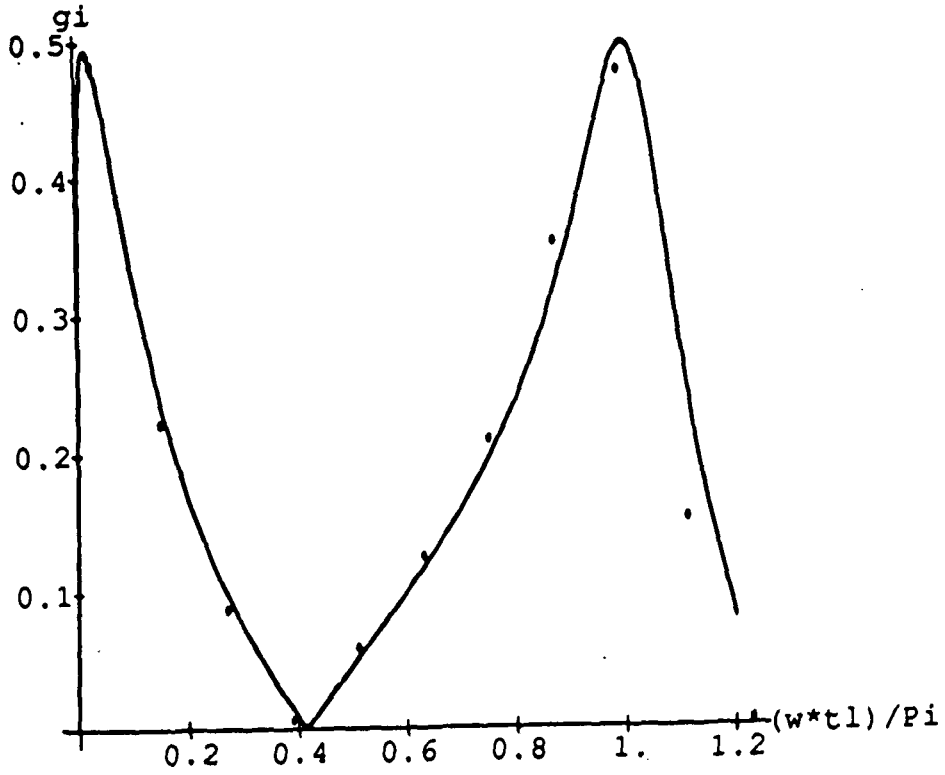
```
tgiwb = Table[{w*.0202284/3.14,Abs[gi[w]]},{w,50,250,50}]

{{0.322108, 0.137589}, {0.644217, 0.244106}, {0.966325, 0.941706},

  {1.28843, 0.0210737}, {1.61054, 0.338609}}

ptgiwb = ListPlot[tgiwb]
```




```
-Graphics-
```

Even though the table may not produce a meaningful plot, when compared with the plot of the system without bends, you can get a good idea of its form. This example uses the show command to put plots on top of each other.

```
Show[pgi5,pgiwb]
```




```
-Graphics-
```

Nyquist Plots

Using the overall admittance equations for both the LOX and RP-1 feed systems, you may create Nyquist plots to aid in stability checking, etc.....

To do this, you first need to get both overall admittance equations into one file, in their INPUT format. Insert DISK 2 and get into either the LOX or RP-1 file. To get an equation, enter,

gi[w] (shift/return)

This will give an equation for the overall admittance. Since the equation will initially be in the output format, you may first want to unformat it to save space. To do this, click onto the cell and go up to CELLS. Under CELLS, highlight FORMATTED and this should make the check mark disappear, and the equation will be condensed. Now COPY the equation over. Once it is copied, you want the cell to be in the INPUT form by going up to STYLE,going down to CELL STYLE and highlighting INPUT. Now, set the equation equal to gi in the form:

gi [w_] : = equation

COPY the equation again in its INPUT form onto the clipboard. Close the file on DISK 2, eject DISK 2 and then insert DISK 3. Open up one of the Nyquist files and PASTE the equation into the file. You will need to rename the equation to something unique, such as

gilox[w_] or girpl [w_]

since both admittance equations are called gi[w_] on DISK 2. Save the file, close it, and eject DISK 3. Insert DISK 2, once again, get into the other feed system file, and as before, COPY the overall admittance equation (in its INPUT form) onto the clipboard. Eject DISK 2, insert DISK 3, get into the same file as before and PASTE the other equation into the file. Once you have renamed the second equation, you are ready to form some Nyquist Plots.

*NOTE: There are other methods of transferring the admittance equations to the Nyquist files. One may be to COPY one equation into the same file as the other. Then you would copy both equations simultaneously onto the clipboard for transfer to DISK 3. The choice of method is up to the user.

The Nyquist plots are basically just a plot of the real versus imaginary plots of an equation. Some examples that have been calculated follows:

This plot is for the equation:

$$x1[\omega_] := \frac{2\, e^{-j\omega \tau_T}}{1 + \Theta_c * j\omega}$$

where,

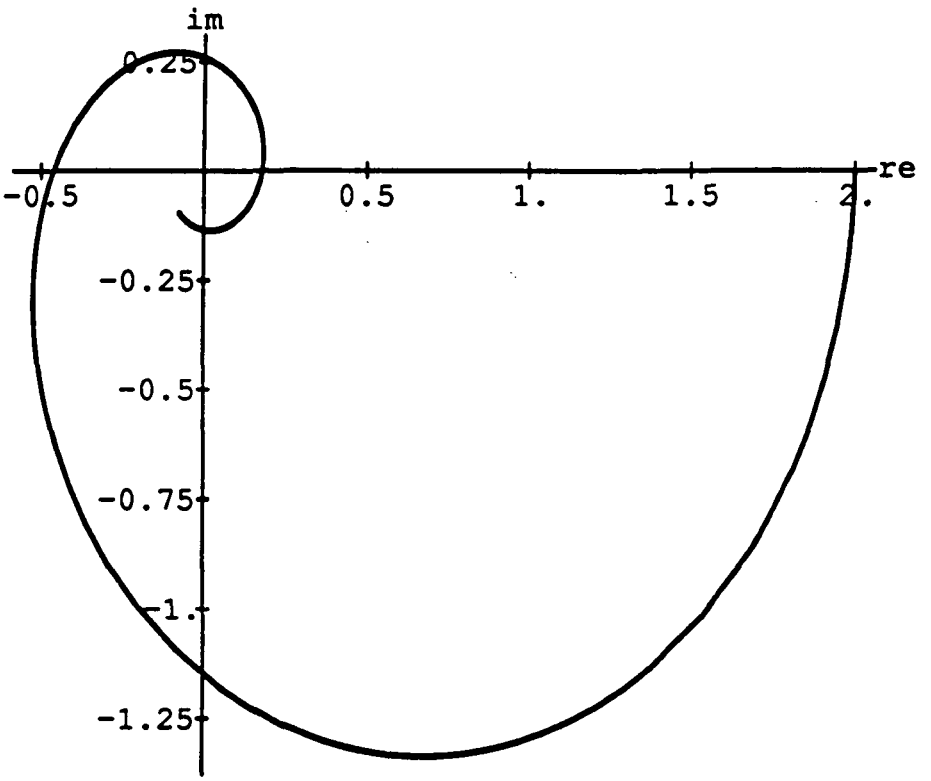
Θ_c, τ_T are constants

In[11]:=

```
gi[w_] := 2*Exp[-I*w*taut]/(1+(thetac*I*w))
```

In[12]:=

```
pgi = ParametricPlot[{Re[gi[w]],Im[gi[w]]},{w,0.01,7000},
      AxesLabel -> {"re","im"}]
```



Out[12]=

-Graphics-

This plot is for the equation:

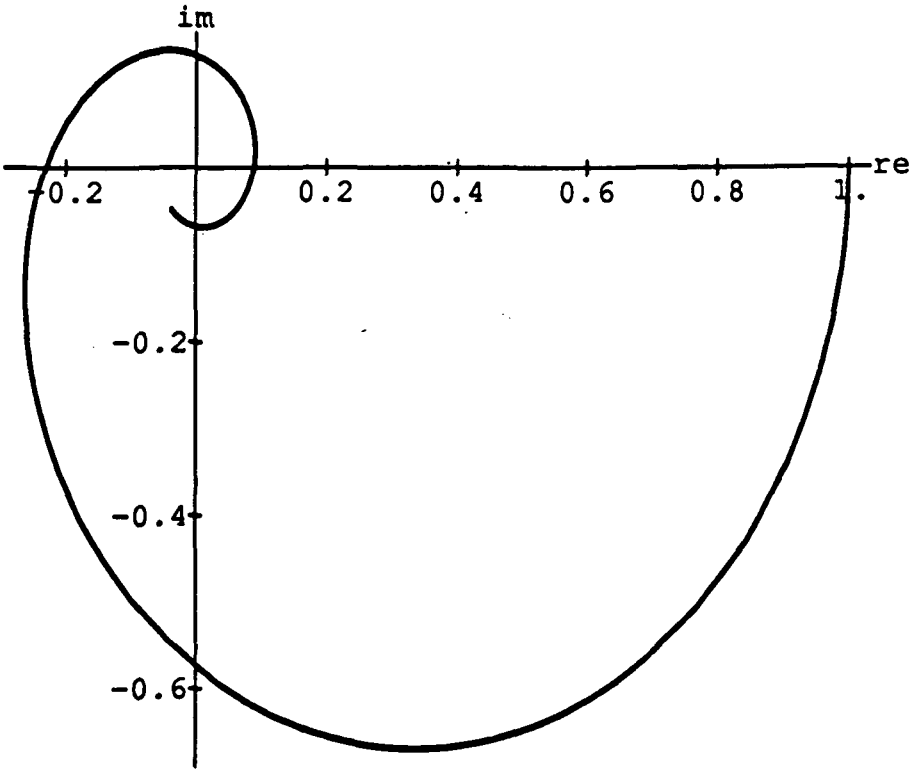
$$x2[\omega_]:= \frac{e^{-j\omega\tau_T}}{1+\Theta_c * j\omega}\left\{\left[1+\frac{(1+\bar{r})}{\bar{c}^*}\left(\frac{dc^*}{dr}\right)\right]gilox[\omega]+1\right\}$$

where,

$\tau_T,\ \Theta_c,\ \bar{r},\ \bar{c}^*,\ \frac{dc^*}{dr}$ are constants

In[20]:=

```
pgi2 = ParametricPlot[{Re[gi[w]],Im[gi[w]]},{w,0.01,7000},
       AxesLabel -> {"re","im"}]
```

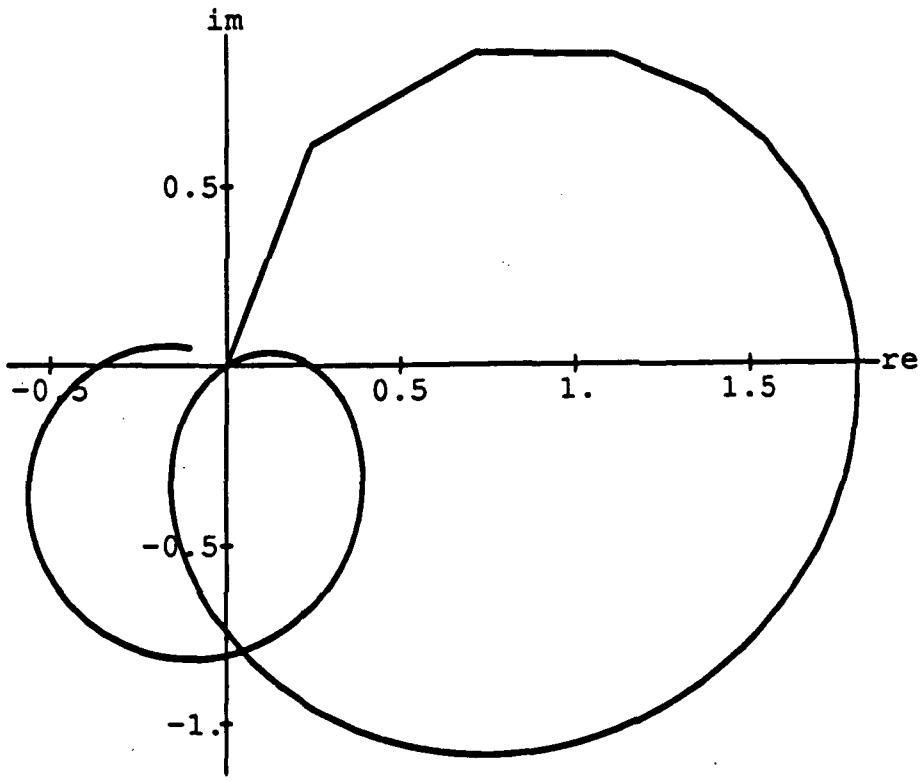


A final example shows the plot of the equation:

$$x_3[\omega_] := \frac{e^{-j\omega\tau_T}}{1+\theta_c \bar{r} j\omega}\left[\left(1+\frac{(1+\bar{r})}{\bar{c}^*}\left(\frac{dc^*}{dr}\right)\right) gilox + \left(1-\frac{\bar{r}(\bar{r}+1)}{\bar{c}^*}\left(\frac{dc^*}{dr}\right)\right) girpl\right]$$

In[23]:=

```
pgi3a = ParametricPlot[{Re[gi[w]],Im[gi[w]]},{w,0.01,1000},
        AxesLabel -> {"re","im"}]
```



The process described in the previous pages was used in modeling the feed systems shown in APPENDIX E. Since a detailed explanation has already been given, a short example will show the calculation of some of the parameters for the LOX feed system:

for LOX:

line 1:

lenl1 = 17.97 (ft)
dial1 = 1.04 (ft) = diameter for all lines
areal1 = .852 (ft^2) = area for all lines

line 2:

lenl2 = 8.138 (ft)

line 3:

lenl3 = 32.51 (ft)

line 4

lenl4 = 8.652 (ft)

line 5:

lenl5 = 4.12 (ft)

75° bend: (assume 1 foot on each side)

R_2 = .52 (ft) (inner radius of bend)
R_1 = 1.56 (ft) (outer radius of bend)

R_2/R_1 = .333

from chart in APPENDIX A:

$$\frac{L''A}{R_1-R_2} = .17$$

$$L'' = \frac{(.17)(1.56-.52)}{.852} = .208 \text{ (ft)}$$

$$L_1 = L' = \frac{2}{.852} = 2.35 \text{ (ft)}$$

$$L_2 = L' + L'' = 2.56 \text{ (ft)}$$

$$\lambda = \frac{L_2}{L_1} = \frac{2.56}{2.35} = 1.09$$

l1 = 2 (ft) (assumed length of bend)

$$lenb75 = \lambda l_1 = (1.09)(2) = 2.18\ (ft)$$

$$Areab75 = area1/\sqrt{\lambda} = .852/\sqrt{1.09} = .816\ (ft^2)$$

From AEROJET DATA:

$$flowrate_{LOX} = 1\ flow_{LOX} = 2264\ lb\ m/s$$

$$flowrate_{RP1} = 1\ flow_{RP1} = 848\ lb\ m/s$$

$$tflow = 1\ flow_{LOX} + 1\ flow_{RP1} = 3112\ lb\ m/s$$

*density and viscosity of fluid found in references in APPENDIX D.

$$den = 72.13\ (lb/ft^3)$$

$$vis = 14.32*10^{-5} (lb/ft\ s)$$

With this information, you can run the file for pressure drop calculations and then find remaining parameters.

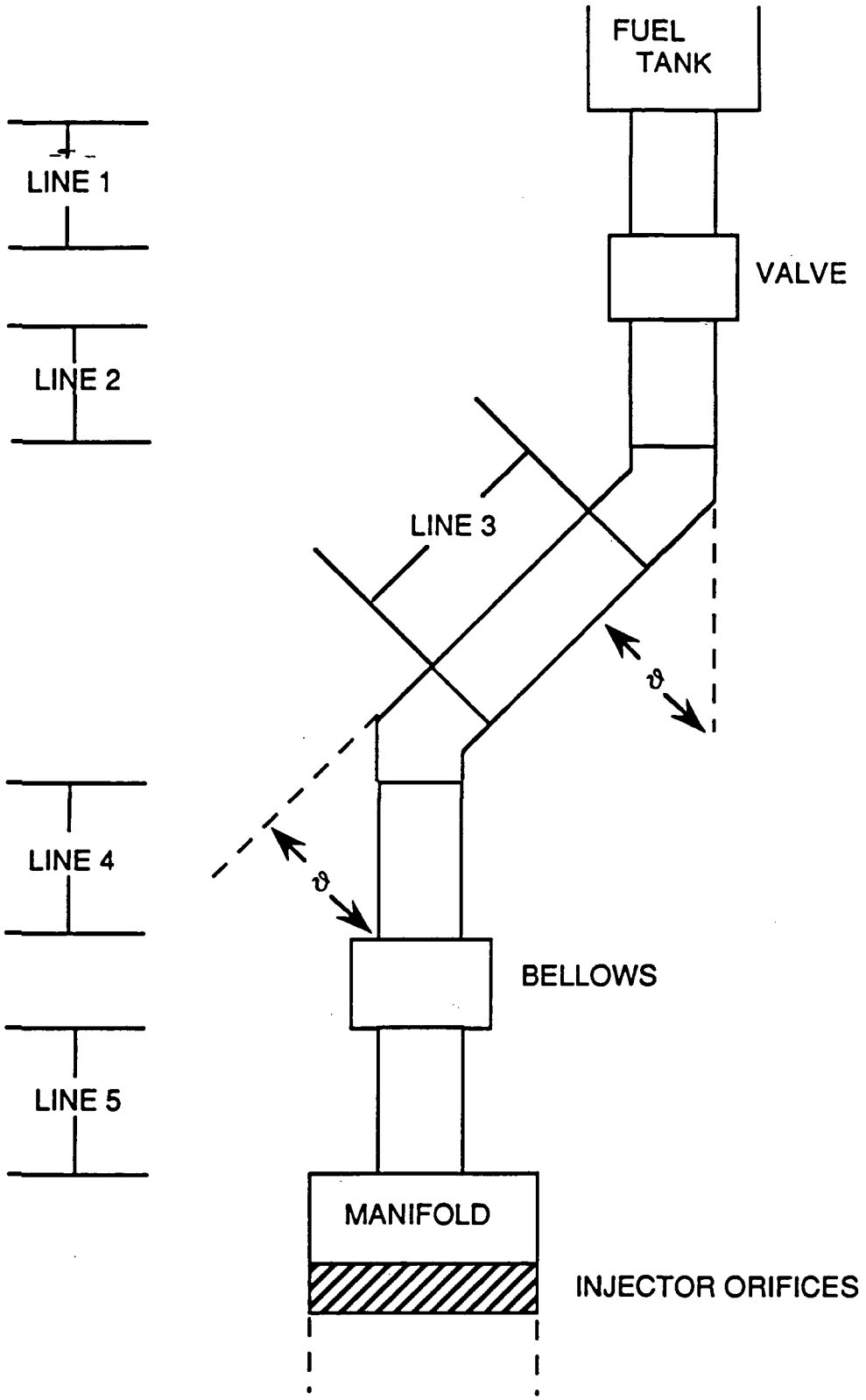
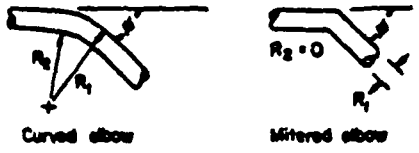

FUEL TANK
VALVE
LINE 1
LINE 2
LINE 3
LINE 4
LINE 5
BELLOWS
MANIFOLD
INJECTOR ORIFICES

FIGURE 1



$$\frac{\mathcal{L}''A}{R_1 - R_2}$$

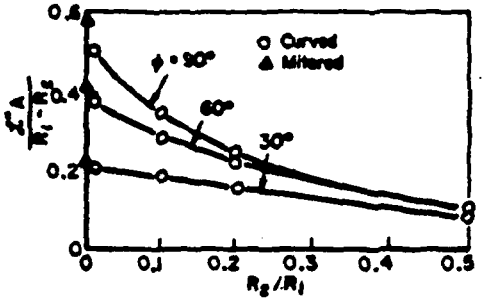


FIGURE 3.2.2c.—Inertance $\mathcal{L}''$ due to duct curvature.

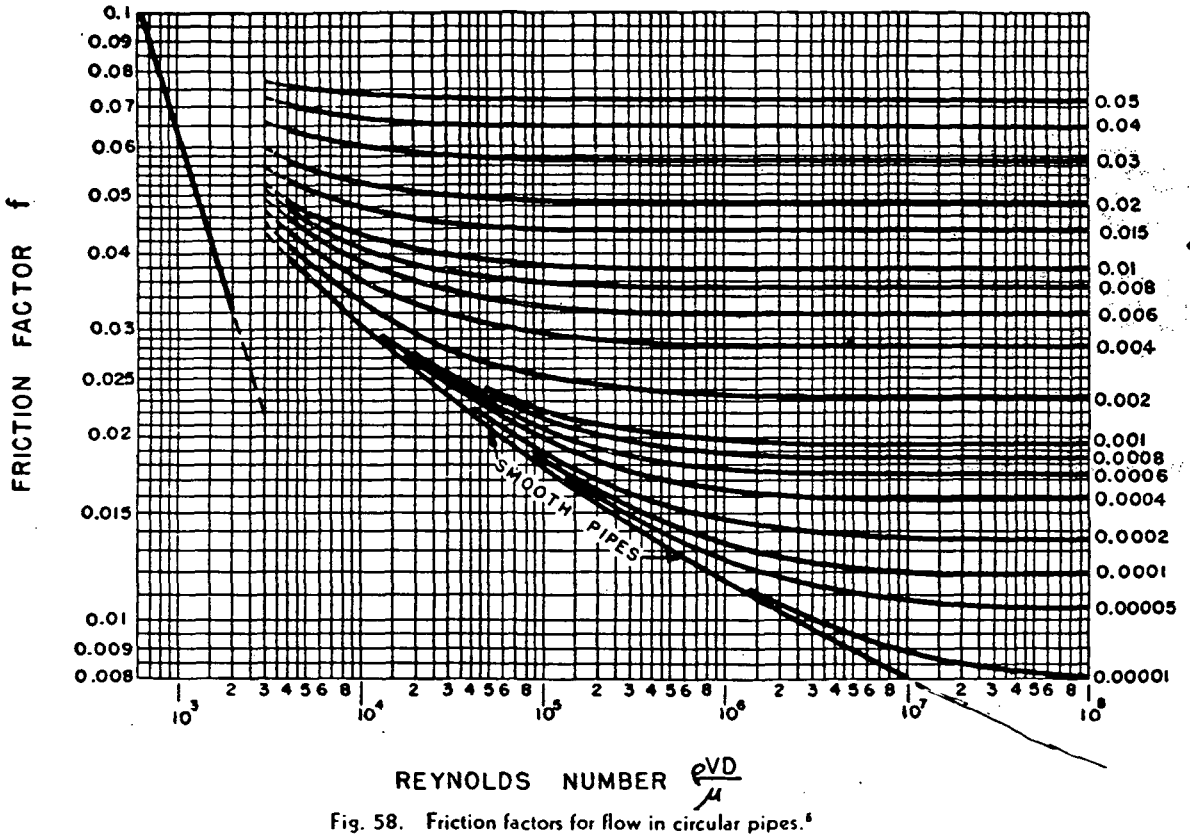


Fig. 58. Friction factors for flow in circular pipes.[5]

[5] Adapted from "Friction Factors for Pipe Flow," by L. F. Moody. *A.S.M.E. Transactions*, vol. 66, no. 8, November, 1944, page 671.

den	-	density of fluid (lb/ft^3)
vis	-	absolute viscosity of fluid (lb/ft•s)
areal	-	area of line (ft^2)
areab	-	area of bend (ft^2)
dial	-	diameter of line (ft)
diab	-	diameter of bend (ft)
lenl	-	length of line (ft)
lenb	-	length of bend (ft)
g	-	acceleration of gravity (32.2 ft/s^2)
a	-	velocity of sound in fluid (ft/s)
lflow	-	flowrate of fluid through the line (lb/s)
tflow	-	total flowrate of fluid to chamber (lb/s)
voltank	-	volume of tank (ft^3)
volman	-	volume of manifold (ft^3)
prchamb	-	combustion chamber pressure (lb/ft^2)
dpror	-	pressure drop across orifices (lb/ft^2)
vell	-	velocity of fluid in line (ft/s)
velb	-	velocity of fluid in bend (ft/s)
reynolds	-	Reynold's number
frl	-	friction factor in line
frb	-	friction factor in bend
headl	-	head loss through line (ft)
headb	-	head loss through bend (ft)
dprfr	-	pressure drop due to friction (lb/ft^2)
dprlin	-	pressure drop in line without friction (lb/ft^2)
kman	-	bulk modulus of fluid at manifold pressure (lb/ft^2)
ktank	-	bulk modulus of fluid at tank pressure (lb/ft^2)
cman	-	capacitance associated with manifold (sec)
ctank	-	capacitance associated with tank (sec)
g	-	admittance of section
r	-	admittance ratio of each section
zline	-	impedance associated with line
zbend	-	impedance associated with bend
zor	-	impedance associated with orifice
tl or tb	-	time constant (sec)
gi	-	overall admittance seen from engine

References for FEEDLINE Parameters

1. AEROJET, "Characterisctics of RP-1 Rocket Fuel," Sacremento, California, Libraray Aerojet-General Corporation Liquid Rocket Plant, February 14, 1957

2. Binder, R. C., Fluid Mechaics, New York, Prentice-Hall Inc., 1950, 361 p

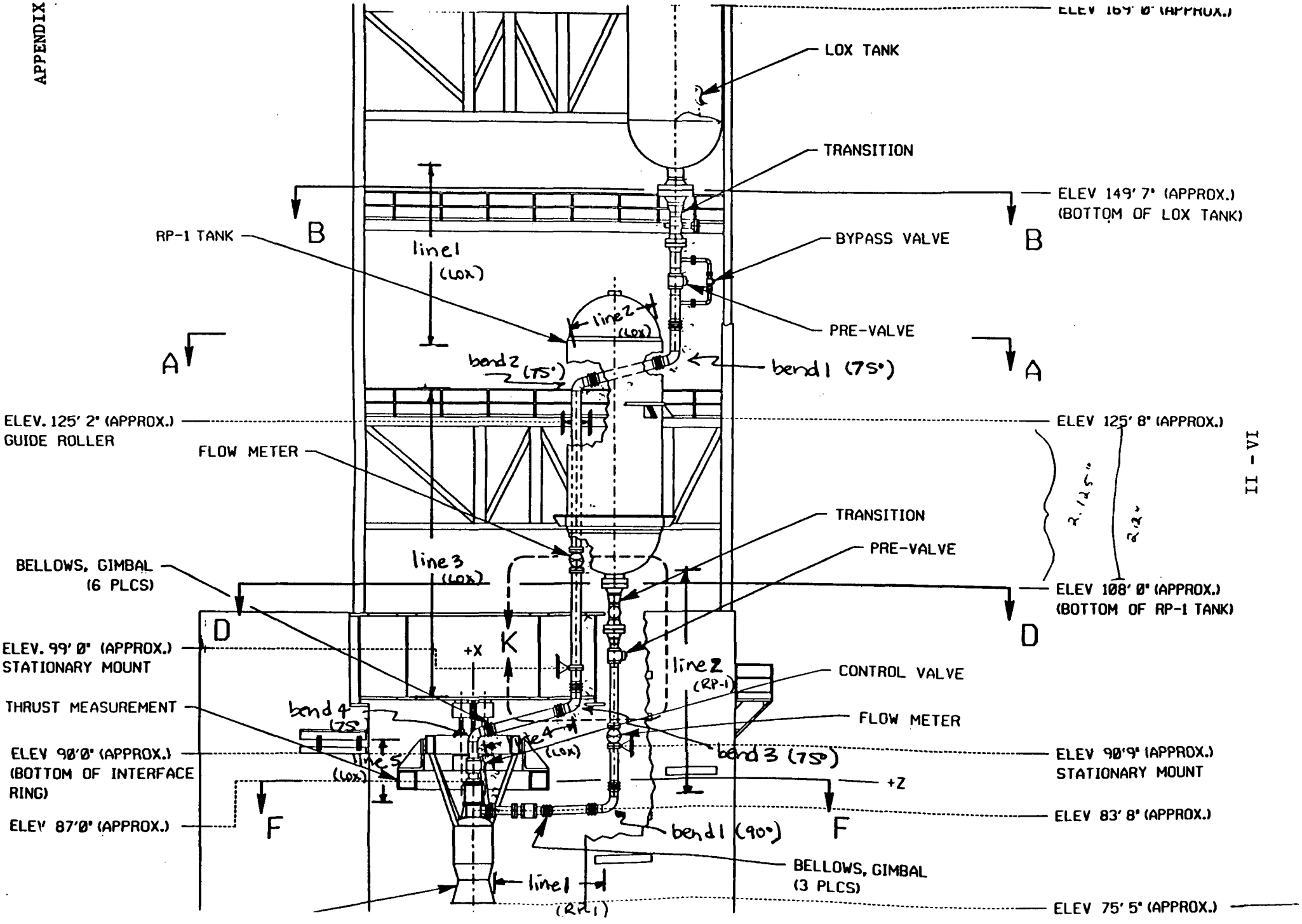
3. Chemical Propulsion Information Agency (CPIA), "Summary of Properties," Laurel, Maryland, The Johns Hopkins University Applied Physics Laboratory, November, 1987

4. Harrje, D. T. and Reardon, F. H., "Liquid Propellant Combustion Instability," NASA SP-194 (1972)

5. McCarty, Robert D. and Weber, Lloyd A., "Thermophysical Properties of Oxygen from the Freezing Liquid Line to 600 K for Pressures to 5000 PSIA," NBS Technical Note 384 (1971)

6 Von Doehren, Paul J., "Propellant Handbook," Edwards, California, Air Force Rocket Propulsion Laboratory, Research and Technology Division, January, 1966

*Other parameters were obtained from NASA data


ELEV 169' 0" (APPROX.)
LOX TANK
TRANSITION
ELEV 149' 7" (APPROX.)
(BOTTOM OF LOX TANK)
B
B
RP-1 TANK
line1
(LOX)
BYPASS VALVE
line2
(LOX)
PRE-VALVE
A
A
bend2 (75°)
bend1 (75°)
ELEV. 125' 2" (APPROX.)
GUIDE ROLLER
ELEV 125' 8" (APPROX.)
FLOW METER
2.125"
2.12"
TRANSITION
PRE-VALVE
BELLOWS, GIMBAL
(6 PLCS)
line3
(LOX)
ELEV 108' 0" (APPROX.)
(BOTTOM OF RP-1 TANK)
D
D
ELEV. 99' 0" (APPROX.)
STATIONARY MOUNT
+X
K
line2
(RP-1)
CONTROL VALVE
THRUST MEASUREMENT
bend4
(75°)
FLOW METER
line4
(LOX)
ELEV 90'0" (APPROX.)
(BOTTOM OF INTERFACE
RING)
line5
(LOX)
bend3 (75°)
ELEV 90'9" (APPROX.)
STATIONARY MOUNT
+Z
F
F
ELEV 87'0" (APPROX.)
ELEV 83' 8" (APPROX.)
bend1 (90°)
BELLOWS, GIMBAL
(3 PLCS)
line1
(RP-1)
ELEV 75' 5" (APPROX.)

APPENDIX III

Intermediate Frequency Oscillations in a Liquid Propellant Rocket Nozzle

Wilbur C. Armstrong

Intermediate Frequency Oscillations in a Liquid Propellant Rocket Nozzle

Wilbur C. Armstrong

December 1, 1989

Intermediate Frequency Oscillations

in a Liquid Propellant Rocket Nozzle

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Intermediate Frequency Oscillations in a Liquid Propellant Rocket Nozzle

Wilbur C. Armstrong

Introduction

Oscillations in a liquid propulsion rocket nozzle have been of interest since World War II. Possibly they were of interest before then when liquid rockets first began to be studied (e.g. Dr. Goddard). During the 50's and 60's much progress was made in understanding some of the problems of low frequency and intermediate frequency oscillations. In 1941 von Karman suggested introducing a time lag as way to explain combustion instabilities. Crocco (ref. 1) and others developed and applied the idea of a combustion time lag. This use of a time lag is used in the analysis of this report.

A volume edited by Harrje and Reardon (ref. 2) was published in 1972 which presented much of the work done on oscillations in a liquid rocket (SP-194). Chapters 3, 4, and 5 form the basis for this paper. An effort will be made to develop the fluid dynamic equations and fill in some of the missing mathematics. Also, SP-194 has several typographical errors which are difficult to find without a detailed following of the mathematics. The emphasis will be on the intermediate mode of oscillation; however, the initial development is applicable to all modes.

In the 1970's, much work was done by Mitchell and associates at Colorado State University with high frequency instability. A report by Mitchell and Eckert (ref. 3) describes a simplified computer program for predicting linear instabilities. In the early eighties their work slowed down. However, it has resumed and Mitchell has a new computer program which should be released soon.

At an AGARD meeting in October, 1988, Culick (ref. 4) presented an overview paper on combustion instabilities in liquid propulsion systems. Included is an excellent bibliography for rockets, thrust augmentors, ramjet engines, and passive and active control of instabilities.

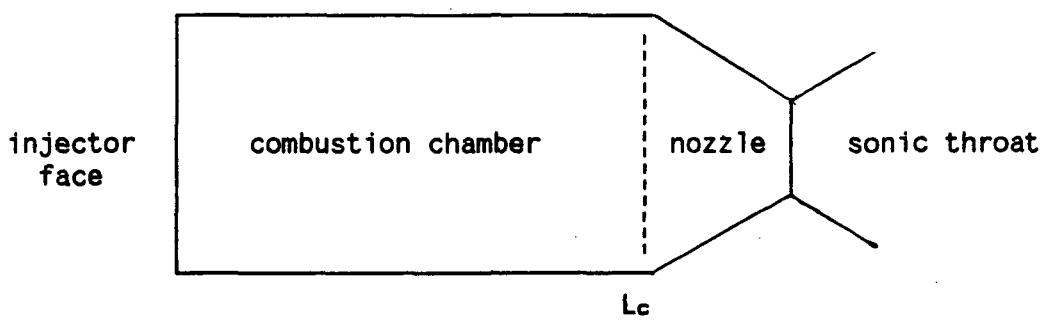
Problem Description

The flow in a rocket nozzle is a complex phenomena and therefore some simplifying assumptions will be made. The medium in the chamber consists of reactant gases, liquid oxidizer and fuel, and the gaseous products of combustion. This will be represented as a two-phase mixture comprised of a mass-averaged gas comprising all species, identified by no subscript, and a single mass-averaged liquid phase identified by the subscript $(\ )_L$. The flow is inviscid (except for the existence of a droplet drag). The gases are thermally and calorically perfect. The liquid phase is well dispersed throughout the chamber. The variations of the energy (internal plus kinetic) of the liquid are neglected. And, for this analysis, there is no heat transfer to the walls.
The assumptions made allows the use of the following:

equation of state: $p = \rho RT$

speed of sound $a = \sqrt{\gamma p/\rho} = \sqrt{\gamma RT}$

The rocket is considered to be comprised of two parts: the combustion chamber where the Mach number is low and the nozzle where the Mach number increases to 1.0 at the throat. The effect of the fuel lines will be initially ignored, but will be added later in the discussion. The physical setup is as follows:



The rocket motor consists of an injector face where the liquid propellants enter and the throat where the gaseous products exit. The rest of the surfaces are solid. Conservation requires that the mass of gaseous products leaving must equal the mass of the liquid propellants entering. The momentum of the two must also be equal. The energy of the products leaving must be equal to the energy of the propellants entering plus the energy due to chemical reaction and change of phase. The conservation equations are:

mass: $\partial\rho/\partial t + \nabla\cdot(\rho V) + \partial\rho_L{}^{\circ}/\partial t + \nabla\cdot(\rho_L{}^{\circ}V_L) = 0$ (1)

momentum: $\partial(\rho V)/\partial t + \nabla\cdot(\rho V\cdot V) + \nabla p = -\partial(\rho_L{}^{\circ}V_L)/\partial t - \nabla\cdot(\rho_L{}^{\circ}V_L\cdot V_L)$ (2)

energy: $\partial(\rho e_s)/\partial t + \nabla\cdot(\rho e_s V) + \partial(\rho_L{}^{\circ}e_{Ls})/\partial t + \nabla\cdot(\rho_L{}^{\circ}e_{Ls}V) + \nabla\cdot(pV) = E$ (3)

The boundary conditions may be expressed as

1. For the liquid phase, assign the injection velocity and the injection density
2. The admittance conditions are used as the boundary condition at the combustion chamber - nozzle interface
3. The condition at solid surfaces is the vanishing of the normal velocity component
4. There is no information passed upstream through the throat.

The last boundary condition is more conveniently used by computing the nozzle admittance conditions at the chamber-nozzle interface and applying the conditions there.

Fluid Flow Equations for Liquid Rocket

The conservation equations (1-3) are used to study the oscillations in a liquid propellant rocket. First the equations are written in a different form. Then they are non-dimensionalized. Next, small perturbation theory is applied and finally the one-dimensional longitudinal equations are solved.

The conservation of mass equation may be rewritten and M defined.

$$\partial\rho/\partial t + \nabla\cdot(\rho V) = M = -\partial\rho_L{}^{\circ}/\partial t - \nabla\cdot(\rho_L{}^{\circ}V_L) \tag{4}$$

Expand the momentum equation

$$V(\partial\rho/\partial t + \nabla\cdot(\rho V)) + \rho\partial V/\partial t + \rho V\cdot\nabla V + \nabla p =$$

$$-V_L(\partial\rho_L{}^{\circ}/\partial t - \nabla\cdot(\rho_L{}^{\circ}V_L)) - \rho_L{}^{\circ}\partial V_L/\partial t - \rho_L{}^{\circ}V_L\cdot\nabla V_L$$

and substitute M where appropriate.

$$\rho\partial V/\partial t + \rho V\cdot\nabla V + \nabla p = M(V_L - V) - \rho_L{}^{\circ}\partial V_L/\partial t - \rho_L{}^{\circ}V_L\cdot\nabla V_L \tag{5}$$

The next step in the development of these equations is to non-dimensionalize by use of reference values. Pressure, density, temperature, etc. are divided by their values at the injector face. Velocities are divided by the speed of sound at the injector face, distances by the length of the combustion chamber, and time by (length of combustion chamber) / (speed of sound at injector face). Thus

$$p = p_{dim}/p_r,\ u = u_{dim}/a_r,\ x = x_{dim}/L_c,\ t = t_{dim}/(L_c/a_r),$$
$$M = M_{dim}/(\rho_r a_r/L_c),\ e = e_{dim}/h_r,\ \text{etc.}$$

Substituting into equation (4) we obtain

$$\partial(\rho\rho_r)/\partial(tL_c/a_r) + \nabla\cdot(\rho\rho_r V a_r) = M(\rho_r a_r/L_c) = -\partial(\rho_L{}^\circ\rho_r)/\partial(tL_c/a_r) - \nabla\cdot(\rho_L{}^\circ\rho_r V_L a_r)$$

$$(\rho_r a_r/L_c)[\partial\rho/\partial t + \nabla\cdot(\rho V)] = M(\rho_r a_r/L_c) = -(\rho_r a_r/L_c)[\partial\rho_L{}^\circ/\partial t + \nabla\cdot(\rho_L{}^\circ V_L)]$$

$$\partial\rho/\partial t + \nabla\cdot(\rho V) = M = -\partial\rho_L{}^\circ/\partial t - \nabla\cdot(\rho_L{}^\circ V_L). \qquad (6)$$

Substituting into equation (5) we obtain

$$\rho\rho_r\partial(Va_r)/\partial(tL_c/a_r) + \rho\rho_r Va_r\cdot\nabla(Va_r) + \nabla\cdot(pp_r)$$

$$= M(\rho_r a_r/L_c)(V_L a_r - Va_r) - \rho_L{}^\circ\rho_r(\partial(V_L a_r)/\partial(tL_c/a_r) - \rho_L{}^\circ\rho_r V_L a_r\cdot\nabla(V_L a_r)$$

$$\rho_r a_r{}^2/L_c\{\rho\partial V/\partial t + \rho V\cdot\nabla V + [p_r/(\rho_r a_r{}^2)]\nabla p\}$$

$$= \rho_r a_r{}^2/L_c\{M(V_L - V) - \rho_L{}^\circ\partial V_L/\partial t - \rho_L{}^\circ V_L\cdot\nabla V_L\}$$

but, $a_r{}^2 = \gamma p_r/\rho_r$ giving

$$\rho\partial V/\partial t + \rho V\cdot\nabla V + (1/\gamma)\nabla p = M(V_L - V) - \rho_L{}^\circ\partial V_L/\partial t - \rho_L{}^\circ V_L\cdot\nabla V_L \qquad (7)$$

Substituting into equation (3) we obtain

$$\partial(\rho\rho_r e_s h_r)/\partial(tL_c/a_r) + \nabla\cdot(\rho\rho_r e_s h_r Va_r)/L_c + \partial(\rho_L{}^\circ\rho_r e_{Ls} h_r)/\partial(tL_c/a_r)$$

$$+ \nabla\cdot(\rho_L{}^\circ\rho_r e_{Ls} h_r Va_r)/L_c + \nabla\cdot(pp_r Va_r)/L_c = EE_r$$

$$\rho_r h_r a_r/L_c[\partial(\rho e_s)/\partial t + \nabla\cdot(\rho e_s V) + \partial(\rho_L{}^\circ e_{Ls})/\partial t + \nabla\cdot(\rho_L{}^\circ e_{Ls} V_L)]$$

$$+ (p_r a_r/L_c)\nabla\cdot(pV) = EE_r$$

$$\partial(\rho e_s)/\partial t + \nabla\cdot(\rho e_s V) + \partial(\rho_L{}^\circ e_{Ls})/\partial t + \nabla\cdot(\rho_L{}^\circ e_{Ls} V_L) + \nabla\cdot(pV) = E$$

The assumption was made that the internal energy of the liquid did not vary with time or space. Also that $e_s = h_s - p/\rho$. Thus,

$$\partial(\rho e_s)/\partial t + \nabla\cdot[\rho(h_s - p/\rho)V] + e_{Ls}[\partial(\rho_L{}^\circ)/\partial t + \nabla\cdot(\rho_L{}^\circ V_L)] + \nabla\cdot(pV) = E$$

$$\partial(\rho e_s)/\partial t + \nabla\cdot(\rho h_s V) - \nabla\cdot(pV) - Me_{Ls} + \nabla\cdot(pV) = E$$

If e_s is rewritten as internal energy and chemical energy, then

$$\partial(\rho e_s) + \nabla\cdot(\rho h_s V) = Me_{Ls}$$

The normalization process makes $e = T/\gamma$, $h_s = T_s$, $e_{Ls} = 1$, $T_s = T + \frac{1}{2}(\gamma - 1)V^2$.

$$\rho e_s = \rho e + \tfrac{1}{2}(\gamma - 1)\rho V^2 = \rho e + \rho(T_s - T) = \rho T_s + \rho(e - T) = \rho T_s + \rho(T/\gamma - T)$$

$$= \rho T_s + \rho T(1/\gamma - 1) = \rho T_s - p(\gamma - 1)/\gamma$$

Substitute this into above equation to obtain

$$\partial[\rho T_s - ((\gamma - 1)/\gamma)p]/\partial t + \nabla\cdot(\rho T_s V) = M \tag{8}$$

Substituting into the equation of state we obtain

$$pp_r = \rho\rho_r RTT_r$$

$$pp_r = (\rho_r RT_r)\rho T$$

$$p = \rho T \tag{9}$$

<u>Small Perturbation Theory</u>

If the perturbations of such things as pressure and velocity are small with respect to the mean (steady-state) values, the parameters may be represented in the form

$$p = \bar{p} + \mathrm{Real}(p'e^{st}) \tag{10}$$

where $\bar{p}$ is the mean value, p' is the perturbation value, and s is in the form $s = \lambda + i\omega$. The perturbation value is a function of location only, the time variation is taken care of by e^{st}. Substituting into equation (6) to obtain

$$\partial(\bar{\rho} + \rho'e^{st})/\partial t + \nabla\cdot[(\bar{\rho} + \rho'e^{st})(\bar{V} + V'e^{st})] = \bar{M} + M'e^{st}$$

$$\partial\bar{\rho}/\partial t + s\rho'e^{st} + \nabla\cdot[\overline{\rho V} + \bar{\rho}V'e^{st} + \rho'\bar{V}e^{st} + \rho'V'e^{2st}) = \bar{M} + Me^{st}$$

$$\partial\bar{\rho}/\partial t + \nabla\cdot(\overline{\rho V}) + e^{st}[s\rho' + \nabla\cdot(\bar{\rho}V') + \nabla\cdot(\rho'\bar{V}) + e^{st}\nabla\cdot(\rho'V')] = \bar{M} + M'e^{st}$$

Subtract M from both sides, divide by e^{st}, and set higher order terms to zero.

$$s\rho' + \nabla\cdot(\bar{\rho}V') + \nabla\cdot(\rho'\bar{V}) = M' \tag{11a}$$

similarly,

$$-s\rho_L{}^{\circ\prime} - \nabla\cdot(\bar{\rho}_L{}^{\circ}V_L') - \nabla\cdot(\rho_L{}^{\circ\prime}\bar{V}_L) = M' \tag{11b}$$

Rewrite equation (7) without M and substitute the perturbation equations to obtain

$$\partial(\rho V)/\partial t + \nabla\cdot(\rho V^2) + (1/\gamma)\nabla\cdot p = -\partial(\rho_L{}^{\circ}V_L)/\partial t - \nabla\cdot(\rho_L{}^{\circ}V_L{}^2)$$

$$\partial[(\bar{\rho} + \rho'e^{st})(\bar{V} + V'e^{st})]/\partial t + \nabla\cdot[(\bar{\rho} + \rho'e^{st})(\bar{V} + V'e^{st})(\bar{V} + V'e^{st})]$$

$$+ (1/\gamma)\nabla\cdot(\bar{p} + p'e^{st}) = -\partial[(\bar{\rho}_L{}^{\circ} + \rho_L{}^{\circ\prime}e^{st})(\bar{V}_L + V_L'e^{st})]/\partial t$$

$$- \nabla\cdot[(\bar{\rho}_L{}^{\circ} + \rho_L{}^{\circ\prime}e^{st})(\bar{V}_L + V_L'e^{st})(\bar{V}_L + V_L'e^{st})]$$

$$\partial(\overline{\rho V} + \bar{\rho}V'e^{st} + \rho'\bar{V}e^{st} + \rho'V'e^{2st})/\partial t + \nabla\cdot(\overline{\rho V^2} + 2\overline{\rho VV'}e^{st} + \bar{\rho}V'^2e^{2st}$$

$$+ \rho'\bar{V}^2e^{st} + 2\rho'\bar{V}V'e^{2st} + \rho'V'^2e^{3st}) + (1/\gamma)\nabla\cdot\bar{p} + (1/\gamma)e^{st}\nabla\cdot p'$$

$$= -\partial(\bar{\rho}_L{}^{\circ}\bar{V}_L + \bar{\rho}_L{}^{\circ}V_L'e^{st} + \rho_L{}^{\circ\prime}\bar{V}_Le^{st} + \rho_L{}^{\circ\prime}V_L'e^{2st})/\partial t - \nabla\cdot(\bar{\rho}_L{}^{\circ}\bar{V}_L{}^2$$

$$+ 2\bar{\rho}_L{}^{\circ}\bar{V}_LV_L'e^{st} + \bar{\rho}_L{}^{\circ}V'_L{}^2e^{2st} + \rho_L{}^{\circ\prime}\bar{V}_L{}^2e^{st} + 2\rho_L{}^{\circ\prime}\bar{V}_LV_L'e^{2st}$$

$$+ \rho_L{}^{\circ\prime}V_L'^2e^{3st})$$

set higher order terms to zero

$$\{\partial(\overline{\rho V}) + \nabla\cdot(\overline{\rho V^2}) + (1/\gamma)\nabla\cdot\bar{p}\} + e^{st}[s\bar{\rho}V' + s\rho'\bar{V} + \nabla\cdot(2\overline{\rho VV'} + \rho'\bar{V}^2) + (1/\gamma)\nabla\cdot p']$$

$$= -\{\partial(\bar{\rho}_L{}^{\circ}\bar{V}_L) + \nabla\cdot(\bar{\rho}_L{}^{\circ}\bar{V}_L{}^2)\} - e^{st}[s\bar{\rho}_L{}^{\circ}V_L' + s\rho_L{}^{\circ\prime}\bar{V}_L + \nabla\cdot(2\bar{\rho}_L{}^{\circ}\bar{V}_LV_L'$$

$$+ \rho_L{}^{\circ\prime}\bar{V}_L{}^2)]$$

the terms in {} are equal by equation (3) and may be deleted

$$s\bar{\rho}V' + s\rho'\bar{V} + \nabla\cdot(2\overline{\rho VV'} + \rho'\bar{V}^2) + (1/\gamma)\nabla\cdot p' = -s\bar{\rho}_L{}^{\circ}V_L' - s\rho_L{}^{\circ\prime}\bar{V}_L$$

$$- \nabla\cdot(2\bar{\rho}_L{}^{\circ}\bar{V}_LV_L' - \rho_L{}^{\circ\prime}\bar{V}_L{}^2)$$

$$s(\bar{\rho}V' + \rho'\bar{V}) + \nabla\cdot(2\overline{\rho VV'} + \rho'\bar{V}^2) + (1/\gamma)\nabla\cdot p' = -s(\bar{\rho}_L{}^{\circ}V_L' + \rho_L{}^{\circ\prime}\bar{V}_L) \qquad (12)$$

$$- \nabla\cdot(2\bar{\rho}_L{}^{\circ}\bar{V}_LV_L' - \rho_L{}^{\circ\prime}\bar{V}_L{}^2)$$

Substitute perturbation equations into equation (8) to obtain

$$\partial[(\bar{\rho} + \rho'e^{st})(\bar{T}_s + T_s'e^{st}) - ((\gamma - 1)/\gamma)(\bar{p} + p'e^{st})]/\partial t$$

$$+ \nabla\cdot[(\bar{\rho} + \rho'e^{st})(\bar{T}_s + T_s'e^{st})(\bar{V} + V'e^{st})] = \bar{M} + M'e^{st}$$

$$\partial[\overline{\rho T_s} + \bar{\rho}T_s'e^{st} + \rho'\bar{T}_se^{st} + \rho'T_s'e^{2st}) - ((\gamma - 1)/\gamma)(\bar{p} + p'e^{st})]/\partial t$$

$$+ \nabla\cdot[(\overline{\rho T_s}\bar{V} + \overline{\rho T_s}V'e^{st} + \bar{\rho}T_s'\bar{V}e^{st} + \bar{\rho}T_s'V'e^{2st} + \rho'\bar{T}_s\bar{V}e^{st} + \rho'\bar{T}_sV'e^{2st}$$

$$+ \rho'T_s'\bar{V}e^{2st} + \rho'T_s'V'e^{3st}] = \bar{M} + M'e^{st}$$

$$\{\partial[\overline{\rho T_s} - ((\gamma - 1)/\gamma)\bar{p})]/\partial t + \nabla\cdot(\overline{\rho T_s}\bar{V})\} + e^{st}[s(\bar{\rho}T_s' + \rho'\bar{T}_s - ((\gamma - 1)/\gamma)p')$$

$$+ \nabla\cdot(\overline{\rho T_s}V' + \bar{\rho}T_s'\bar{V} + \rho'\bar{T}_s\bar{V})] = \bar{M} + M'e^{st}$$

items in {} are $\bar{M}$ by equation (8)

$$s[\bar{\rho}T_s' + \rho'\bar{T}_s - ((\gamma - 1)/\gamma)p'] + \nabla\cdot(\overline{\rho T_s}V' + \bar{\rho}T_s'\bar{V} + \rho'\bar{T}_s\bar{V})] = M'$$

but, $\bar{T}_s = 1$ and $s\rho' + \nabla\cdot(\bar{\rho}V') + \nabla\cdot(\rho'\bar{V}) = M'$ by equation (11a)

thus,

$$s[\bar{\rho}T_s' - ((\gamma - 1)/\gamma)p'] + \nabla\cdot(\overline{\rho V}T_s') = 0 \tag{13}$$

The energy equation can also be written in terms of entropy (σ). Entropy is defined by the equation

$$\rho = p^{(1/\gamma)}e^{-\sigma}$$

Substituting the perturbation equations gives

$$\bar{\rho} + \rho' e^{st} = (\bar{p} + p' e^{st})^{(1/\gamma)}\exp[-(\bar{\sigma} + \sigma' e^{st})]$$

$$\bar{\rho} + \rho' e^{st} = \bar{p}^{(1/\gamma)}(1 + p'/\bar{p}\ e^{st})^{(1/\gamma)}\exp(-\bar{\sigma})\exp(-\sigma' e^{st})$$

Divide by $\bar{\rho}$

$$1 + \rho'/\bar{\rho}\ e^{st} = (1 + p'/\bar{p}\ e^{st})^{(1/\gamma)}\exp(-\sigma' e^{st})$$

Replace the factors on the RHS with their series representation and retain only up to the first order terms

$$1 + \rho'/\bar{\rho}\ e^{st} = [1 + (1/\gamma)p'/\bar{p}\ e^{st}](1 - \sigma' e^{st})$$

$$1 + \rho'/\bar{\rho}\ e^{st} = 1 - \sigma' e^{st} + (1/\gamma)p'/\bar{p}\ e^{st} - (1/\gamma)p'/\bar{p}\sigma' e^{2st})$$

$$\rho'/\bar{\rho} = -\sigma' + (1/\gamma)p'/\bar{p}$$

$$\rho'\bar{p}/\bar{\rho} = -p\sigma' + p'/\gamma$$

$$\bar{p}\sigma' = p'/\gamma - \bar{T}\rho'$$

Substitute this into equation (13)

$$s[\bar{\rho}T' + (\gamma - 1)\overline{\rho V}\cdot V' - ((\gamma - 1)/\gamma)p'] + \nabla\cdot\{\overline{\rho V}[T' + (\gamma - 1)\bar{V}\cdot V']\} = 0$$

$$s[\bar{p}\sigma' + (\gamma - 1)\overline{\rho V}\cdot V'] + \nabla\cdot\{\bar{V}[\bar{p}\sigma' + ((\gamma - 1)/\gamma)p' + (\gamma - 1)\overline{\rho V}\cdot V']\} = 0$$

$$s[\bar{p}\sigma' + (\gamma - 1)\overline{\rho V}\cdot V'] + \nabla\cdot\{\bar{V}[\bar{p}\sigma' + (\gamma - 1)\overline{\rho V}\cdot V']\} = -((\gamma - 1)/\gamma)\nabla\cdot(p'\bar{V}) \tag{14}$$

Now apply the perturbation equations to equation (9)

$$\bar{p} + p' e^{st} = (\bar{\rho} + \rho' e^{st})(\bar{T} + T' e^{st})$$

$$\bar{p} + p' e^{st} = \overline{\rho T} + \bar{\rho}T' e^{st} + \rho'\bar{T}e^{st} + \rho' T' e^{2st}$$

$$p' = \rho'\bar{T} + \bar{\rho}T'$$

which may be rewritten as

$$\bar{\rho}T' - ((\gamma - 1)/\gamma)p' = p'/\gamma - \rho'\bar{T} \tag{15}$$

Also,

$$\bar{T}_s + T_s'e^{st} = \bar{T} + T'e^{st} + \tfrac{1}{2}(\gamma - 1)(\bar{V} + V'e^{st})(\bar{V} + V'e^{st})$$

$$\bar{T}_s + T_s'e^{st} = \bar{T} + T'e^{st} + \tfrac{1}{2}(\gamma - 1)(\bar{V}\bar{V} + 2\bar{V}V'e^{st} + V'V'e^{2st})$$

$$T_s' = T' + (\gamma - 1)\bar{V}V' \tag{16}$$

Substitute equation (16) into equation (13)

$$s[\bar{\rho}T' + (\gamma - 1)\overline{\rho V}V' - ((\gamma - 1)/\gamma)p'] + \nabla\cdot[\overline{\rho V}T' + (\gamma - 1)\overline{\rho V^2}V'] = 0$$

use equation (15) to obtain

$$s(p'/\gamma - \rho'\bar{T}) + s(\gamma - 1)\overline{\rho V}V' + \nabla\cdot[\bar{V}(\bar{\rho}T' + (\gamma - 1)\overline{\rho V}V')] = 0$$

$$s(p'/\gamma - \rho'\bar{T}) + s(\gamma - 1)\overline{\rho V}V' + \nabla\cdot[\bar{V}(p' - \rho'\bar{T} + (\gamma - 1)\overline{\rho V}V')] = 0$$

$$s(\rho' - p'/\gamma) = s(1 - \bar{T})\rho' + s(\gamma - 1)\overline{\rho V}V' + \nabla\cdot[\bar{V}(p' - \rho'\bar{T} + (\gamma - 1)\overline{\rho V}V')]$$

Let $X = (\gamma - 1)\overline{\rho V}\cdot V' + (1 - \bar{T})\rho'$

$$s(\rho' - p'/\gamma) = sX + \nabla\cdot[\bar{V}p' - \bar{V}\rho'\bar{T} + (\gamma - 1)\overline{\rho V}(\bar{V}\cdot V')]$$

$$s(\rho' - p'/\gamma) = sX + \nabla\cdot[\bar{V}p' + (1 - \bar{T})\rho'\bar{V} + (\gamma - 1)\overline{\rho V}(\bar{V}\cdot V') - \rho'\bar{V}]$$

Let $Y = -\bar{V}p' + (1 - \bar{\rho})V' - (\gamma - 1)\overline{\rho V}(\bar{V}\cdot V') - (1 - \bar{T})\bar{V}\rho'$

$$s(\rho' - p'/\gamma) = sX - \nabla\cdot Y + \nabla\cdot V' - \nabla\cdot(\bar{\rho}V') - \nabla\cdot(\rho'\bar{V})$$

or,

$$s\rho' = sp'/\gamma + sX - \nabla\cdot Y + \nabla\cdot V' - \nabla\cdot(\bar{\rho}V') - \nabla\cdot(\rho'\bar{V}) \tag{17}$$

Use equation (17) to eliminate ρ' from LHS of equations (11a) and (12)

$$sp'/\gamma + sX - \nabla\cdot Y + \nabla\cdot V' - \nabla\cdot(\bar{\rho}V') - \nabla\cdot(\rho'\bar{V}) + \nabla\cdot(\rho\bar{V}') + \nabla\cdot(\rho'\bar{V}) = M'$$

$$sp'/\gamma + \nabla\cdot V' = -sX + \nabla\cdot Y + M' \tag{18}$$

and now consider equation (12)

$$s(\bar{\rho}V' + \rho'\bar{V}) + \nabla\cdot(2\overline{\rho V}V' + \rho'\bar{V}^2) + (1/\gamma)\nabla\cdot p' = -s(\bar{\rho}_L{}^{\circ}V_L' + \rho_L{}^{\circ\prime}\bar{V}_L)$$

$$- \nabla\cdot(2\bar{\rho}_L{}^{\circ}\bar{V}_L V_L' - \rho_L{}^{\circ\prime}\bar{V}_L{}^2)$$

Add sV' to both sides and rearrange

$$sV' + (1/\gamma)\nabla\cdot p' = -s[\bar{V}\rho' + \bar{\rho}_L{}^{\circ}V_L' + \rho_L{}^{\circ\prime}\bar{V}_L - (1 - \bar{\rho})V'] - \nabla\cdot[2\overline{\rho V}V' + \rho'\bar{V}^2]$$

$$- \nabla\cdot(2\bar{\rho}_L{}^{\circ}\bar{V}_L V_L' - \rho_L{}^{\circ\prime}\bar{V}_L{}^2)$$

Let $Z = \bar{V}\rho' + \bar{\rho}_L{}^{\circ}V_L' + \bar{V}_L\rho_L{}^{\circ\prime} - (1 - \bar{\rho})V'$

and $W = 2\overline{\rho V}V' + 2\bar{\rho}_L{}^{\circ}\bar{V}_L V_L' + \bar{V}^2\rho' + \bar{V}_L{}^2\rho_L{}^{\circ\prime}$

Thus,

$$sV' + (1/\gamma)\nabla\cdot p' = -sZ - \nabla\cdot W \tag{19}$$

The development of the mass burning rate perturbation is presented in section 5.3.1 of SP-194 and will not be given here. The results are

$$M' = \bar{M}\{e^{-s\bar{\tau}_T}[\mathcal{K}(s) + \mathcal{H}(s)] - \mathcal{P}(s)\}p'/\bar{p}$$

where $\mathcal{K}(s) = (1 + s\tau_T)(G_{ox} + G_f)$ - effect of total mass flux oscillations

$\mathcal{H}(s) = (1 + \bar{r})[(\bar{r}/\bar{c}^*)(\partial c^*/\partial r) - n_r s\bar{\tau}](G_{ox} - \bar{r}G_f)/\bar{r}$ - effect of mixture ratio fluctuations

$\mathcal{P}(s) = n(1 - e^{s\bar{\tau}})$

<u>One Dimensional Equations</u>

When $\bar{u}$ and u' are small with respect to the sonic velocity, and thus higher order terms may be deleted from the equations. This makes the equations linear in terms of the perturbations. Thus, terms such as

$(1 - \bar{p})$, $(1 - \bar{T})$, etc. are small and may be deleted.

This also allows the superposition of solutions.

Thus,

$$p' = p_0 + p_1 + \ldots\ldots.$$
$$u' = u_0 + u_1 + \ldots\ldots.$$

where all terms of order higher than 1 will be dropped.

Consider now the intermediate mode of oscillation. This mode interacts with the feed line system. This mode is a longitudinal oscillation and we need to consider variations in the x - direction only. This simplifies the equations

zeroth order: $(s/\gamma)p_0 + du_0/dx = 0$ (20a)

$su_0 + (1/\gamma)dp_0/dx = 0$ (20b)

first order: $(s/\gamma)p_1 + du_1/dx = -sX_1 + dY_1/dx + M_1$ (21a)

$su_1 + (1/\gamma)dp_1/dx = -sZ_1 - dW_1/dx$ (21b)

where $X_1 \approx (\gamma - 1)\bar{u}u_0 + (1 + \bar{r})(\overline{dh_L/dr})(\bar{M}/s)e^{-s\tau_T}(G_{0x} - \bar{r}G_{\bar{r}})P_{00}$ (22)

$Y_1 \approx -\bar{u}p_0$ (23)

$Z_1 \approx (1/\gamma)\bar{u}p_0 + \bar{\rho}_L{}^0 u_{L0}$ (24)

$W_1 \approx 2\bar{u}u_0$ (25)

$M_1 \approx \bar{M}\{e^{-s\tau_T}[\mathcal{K}(s) + \mathcal{H}(s)]P_{00} - \mathcal{P}(s)p_0\}$ (26)

Note: A term has been added to X to account for fluctuating enthalpy (h_L) of the liquid propellants resulting from mixture ratio oscillations.

The zeroth order equations are homogeneous and may be solved in a straight forward manner. Use the second equation to remove u_0 from the first equation. This results in an equation in the form

$$(D^2 - s^2)p_0 = 0$$

$$p_0 = A\cosh(sx) + B\sinh(sx)$$

$$u_0 = -(1/\gamma)[B\cosh(sx) + A\sinh(sx)]$$

Apply the boundary conditions that at $x = 0$, $p' = P_{00}$ and $u' = 0$.

$$p_0 = P_{00}\cosh(sx) \quad (27)$$

$$u_0 = -(1/\gamma)P_{00}\sinh(sx) \quad (28)$$

The first order equations are non-homogeneous and are a bit more difficult to solve. First obtain an equation in terms of one variable. For example, u_1. This may be written as

$$(D^2 - s^2)u_1 = RHS$$

where RHS is the right hand side of the equation. In this case,

$$RHS = d(-sX_1 + dY_1/dx + M_1)/dx + s(sZ_1 + dW_1/dx)$$

which is non-homogeneous and may be solved by assuming a solution in the form

$$u_1 = v_1 \cosh(sx) + v_2 \sinh(sx)$$

then take first and second derivatives

$$Du_1 = sv_1 \sinh(sx) + sv_2 \cosh(sx)$$

$$+\cosh(sx)dv_1/dx + \sinh(sx)dv_2/dx$$

where the second line is set equal to zero.

$$D^2u_1 = s^2v_1 \sinh(sx) + s^2v_2 \cosh(sx)$$

$$+s \sinh(sx)dv_1/dx + s \cosh(sx)dv_2/dx$$

The second line is equal to RHS because when substituted into the equation

$$(D^2 - s^2)u_1 = RHS$$

everything disappears except the second line. Thus, we have two equations in two unknowns.

$$\cosh(sx)dv_1/dx + \sinh(sx)dv_2/dx = 0$$

$$s[\sinh(sx)dv_1/dx + \cosh(sx)dv_2/dx] = RHS$$

Solving these two equations for dv_1/dx and dv_2/dx

$$s[\cosh^2(sx) - \sinh^2(sx)]dv_2/dx = RHS \cosh(sx)$$

$$dv_2 = (1/s)RHS \cosh(sx)dx$$

$$dv_1 = -(1/s)RHS \sinh(sx)dx$$

thus,

$$u_1 = -\cosh(sx)(1/s)\int_0^x RHS \sinh(sx)dx + \sinh(sx)(1/s)\int_0^x RHS \cosh(sx)dx$$

Substitute for RHS

$$u_1 = -\cosh(sx)(1/s)\int_0^x [d(-sX_1 + \nabla\cdot Y_1 + M_1)/dx + s^2Z_1 + s\nabla\cdot W_1]\sinh(sx)dx$$

$$+ \sinh(sx)(1/s)\int_0^x [d(-sX_1 + \nabla\cdot Y_1 + M_1)/dx + s^2Z_1 + s\nabla\cdot W_1]\cosh(sx)dx$$

$$u_1 = -\cosh(sx)(1/s)\int_0^x [d(-sX_1 + \nabla\cdot Y_1 + M_1)/dx]\sinh(sx)dx$$

$$+ \sinh(sx)(1/s)\int_0^x [d(-sX_1 + \nabla\cdot Y_1 + M_1)/dx]\cosh(sx)dx$$

$$- \cosh(sx)\int_0^x [sZ_1 + \nabla\cdot W_1]\sinh(sx)dx + \sinh(sx)\int_0^x [sZ_1 + \nabla\cdot W_1]\cosh(sx)dx$$

let $u = \sinh(sx)$ and $dv = [d(-sX_1 + \nabla\cdot Y_1 + M_1)/dx]dx$

then $\int_0^x udv = (-sX_1 + \nabla\cdot Y_1 + M_1)\sinh(sx) - s\int_0^x (-sX_1 + \nabla\cdot Y_1 + M_1)\cosh(sx)dx$

substituting into equation for u_1

$$u_1 = -\cosh(sx)[(1/s)(-sX_1 + \nabla\cdot Y_1 + M_1)\sinh(sx) - \int_0^x (-sX_1 + \nabla\cdot Y_1 + M_1)\cosh(sx)dx]$$

$$+ \sinh(sx)[(1/s)(-sX_1 + \nabla\cdot Y_1 + M_1)\cosh(sx) - \int_0^x (-sX_1 + \nabla\cdot Y_1 + M_1)\sinh(sx)dx]$$

$$- \cosh(sx)\int_0^x [sZ_1 + \nabla\cdot W_1]\sinh(sx)dx + \sinh(sx)\int_0^x [sZ_1 + \nabla\cdot W_1]\cosh(sx)dx$$

$$u_1 = \cosh(sx)\int_0^x (-sX_1 + \nabla\cdot Y_1 + M_1)\cosh(sx)dx - \sinh(sx)\int_0^x (-sX_1 + \nabla\cdot Y_1 + M_1)\sinh(sx)dx$$

$$- \cosh(sx)\int_0^x [sZ_1 + \nabla\cdot W_1]\sinh(sx)dx + \sinh(sx)\int_0^x [sZ_1 + \nabla\cdot W_1]\cosh(sx)dx$$

$$u_1 = \cosh(sx)\int_0^x [s(W_1 - X_1) + M_1 + (\nabla\cdot Y_1 - sW_1)]\cosh(sx)dx$$

$$- \sinh(sx)\int_0^x [s(W_1 - X_1) + M_1 + (\nabla\cdot Y_1 - sW_1)]\sinh(sx)dx$$

$$- \cosh(sx)\int_0^x [s(Y_1 + Z_1) + (\nabla\cdot W_1 - sY_1)]\sinh(sx)dx$$

$$- \sinh(sx)\int_0^x [s(Y_1 + Z_1) + (\nabla\cdot W_1 - sY_1)]\cosh(sx)dx$$

$$u_1 = \cosh(sx)\int_0^x [s(W_1 - X_1) + M_1]\cosh(sx)dx - \sinh(sx)\int_0^x [s(W_1 - X_1) + M_1]\sinh(sx)dx$$

$$- \cosh(sx)\int_0^x s(Y_1 + Z_1)\sinh(sx)dx + \sinh(sx)\int_0^x s(Y_1 + Z_1)\cosh(sx)dx$$

$$+ \cosh(sx)\int_0^x (\nabla\cdot Y_1 - sW_1)\cosh(sx)dx - \sinh(sx)\int_0^x (\nabla\cdot Y_1 - sW_1)\sinh(sx)dx$$

$$- \cosh(sx)\int_0^x (\nabla\cdot W_1 - sY_1)\sinh(sx)dx + \sinh(sx)\int_0^x (\nabla\cdot W_1 - sY_1)\cosh(sx)dx$$

$$u_1 = \int_0^x [s(W_1 - X_1) + M_1][\cosh(sx)\cosh(sx') - \sinh(sx)\sinh(sx')]dx'$$

$$- \int_0^x s(Y_1 + Z_1)[\cosh(sx)\sinh(sx') - \sinh(sx)\cosh(sx')]dx'$$

$$+ \cosh(sx)\int_0^x (\nabla\cdot Y_1 - sW_1)\cosh(sx)dx - \sinh(sx)\int_0^x (\nabla\cdot Y_1 - sW_1)\sinh(sx)dx$$

$$- \cosh(sx)\int_0^x (\nabla\cdot W_1 - sY_1)\sinh(sx)dx + \sinh(sx)\int_0^x (\nabla\cdot W_1 - sY_1)\cosh(sx)dx$$

$$u_1 = \int_0^x \{[s(W_1 - X_1) + M_1]\cosh[s(x - x')] + s(Y_1 + Z_1)\sinh[s(x - x')]\}dx' + C$$

$$\text{where } C = \cosh(sx)\int_0^x (\nabla\cdot Y_1 - sW_1)\cosh(sx)dx - \sinh(sx)\int_0^x (\nabla\cdot Y_1 - sW_1)\sinh(sx)dx$$

$$- \cosh(sx)\int_0^x (\nabla\cdot W_1 - sY_1)\sinh(sx)dx + \sinh(sx)\int_0^x (\nabla\cdot W_1 - sY_1)\cosh(sx)dx$$

$$\nabla\cdot Y_1 = dY_1/dx = -d(\bar{u}p_o)/dx = -\bar{u}dp_o/dx - p_o d\bar{u}/dx = -\bar{u}sP_{oo}\sinh(sx)$$

$$- P_{oo}\cosh(sx)d\bar{u}/dx$$

$$\nabla\cdot W_1 = dW_1/dx = 2d(\bar{u}u_o)/dx = 2\bar{u}du_o/dx + u_o d\bar{u}/dx = -2\bar{u}(s/\gamma)P_{oo}\cosh(sx)$$

$$- (2/\gamma)P_{oo}\sinh(sx)d\bar{u}/dx$$

substituting into equation for C

$$C = \cosh(sx)\int_0^x [-((\gamma - 2)/\gamma)sP_{oo}\bar{u}\sinh(sx) - P_{oo}\cosh(sx)d\bar{u}/dx]\cosh(sx)dx$$

$$- \sinh(sx)\int_0^x [-((\gamma - 2)/\gamma)sP_{oo}\bar{u}\sinh(sx) - P_{oo}\cosh(sx)d\bar{u}/dx]\sinh(sx)dx$$

$$- \cosh(sx)\int_0^x [-((\gamma - 2)/\gamma)sP_{oo}\bar{u}\cosh(sx) - (2/\gamma)P_{oo}\sinh(sx)d\bar{u}/dx]\sinh(sx)dx$$

$$+ \sinh(sx)\int_0^x [-((\gamma - 2)/\gamma)sP_{oo}\bar{u}\cosh(sx) - (2/\gamma)P_{oo}\sinh(sx)d\bar{u}/dx]\cosh(sx)dx$$

$$C = -((\gamma - 2)/\gamma)sP_{oo}\{\cosh(sx)\int_0^x \bar{u}\ \sinh(sx)\cosh(sx)dx - \sinh(sx)\int_0^x \bar{u}\ \sinh^2(sx)dx$$

$$+ \{\cosh(sx)\int_0^x \bar{u}\ \cosh(sx)\sinh(sx)dx - \sinh(sx)\int_0^x \bar{u}\ \cosh^2(sx)dx\}$$

$$-P_{oo}\{\cosh(sx)\int_0^x (d\bar{u}/dx)\ \cosh^2(sx)dx - \sinh(sx)\int_0^x (d\bar{u}/dx)\cosh(sx)\sinh(sx)dx$$

$$- \cosh(sx)(2/\gamma)\int_0^x (d\bar{u}/dx)\ \sinh^2(sx)dx + \sinh(sx)(2/\gamma)\int_0^x (d\bar{u}/dx)\sinh(sx)\cosh(sx)dx\}$$

$$C = -((\gamma - 2)/\gamma)sP_{oo}\{\cosh(sx)\int_0^x \bar{u}\ \sinh(2sx)dx - \sinh(sx)\int_0^x \bar{u}\ \cosh(2sx)dx\}$$

$$-P_{oo}\{\cosh(sx)\int_0^x (d\bar{u}/dx)[1 + \sinh^2(sx)-(2/\gamma)\sinh^2(sx)]dx$$

$$- ((\gamma - 2)/\gamma)\ \sinh(sx)\int_0^x (d\bar{u}/dx)\ \sinh(sx)\ \cosh(sx)dx\}$$

$$C = -((\gamma - 2)/\gamma)sP_{oo}\{\cosh(sx)\int_0^x \bar{u}\ \sinh(2sx)dx - \sinh(sx)\int_0^x \bar{u}\ \cosh(2sx)dx\}$$

$$-P_{oo}\cosh(sx)\int_0^x (d\bar{u}/dx)dx - ((\gamma - 2)/\gamma)P_{oo}\{\cosh(sx)\int_0^x (d\bar{u}/dx)\sinh^2(sx)dx$$

$$- ((\gamma - 2)/\gamma)\ \sinh(sx)\int_0^x (d\bar{u}/dx)\ \sinh(sx)\ \cosh(sx)dx\}$$

Consider $\int_0^x (d\bar{u}/dx)\sinh^2(sx)dx$

let $u = \sinh^2(sx)$ and $dv = (d\bar{u}/dx)dx \qquad v = \bar{u}$

$$\bar{u}\ \sinh^2(sx) - 2s\int_0^x \bar{u}\ \sinh(sx)\cosh(sx)dx = \bar{u}\ \sinh^2(sx) - s\int_0^x \bar{u}\ \sinh(2sx)dx$$

Now, consider $\int_0^x (d\bar{u}/dx)\sinh(sx)\cosh(sx)dx$

let $u = \sinh(sx)\cosh(sx)$ and $dv = (d\bar{u}/dx)dx \qquad v = \bar{u}$

$$\bar{u}\sinh(sx)\cosh(sx) - s\int_0^x \bar{u}\sinh^2(sx)dx - s\int_0^x \bar{u}\cosh^2(sx)dx$$

$$\bar{u}\sinh(sx)\cosh(sx) - s\int_0^x \bar{u}\cosh(2sx)dx$$

thus,

$$C = -((\gamma - 2)/\gamma)sP_{oo}\{\cosh(sx)\int_0^x \bar{u}\sinh(2sx)dx - \sinh(sx)\int_0^x \bar{u}\cosh(2sx)dx\}$$

$$-P_{oo}\bar{u}\cosh(sx) - ((\gamma - 2)/\gamma)P_{oo}\{\cosh(sx)[\bar{u}\sinh^2(sx)$$

$$- s\int_0^x \bar{u}\sinh(2sx)dx] - \sinh(sx)[\bar{u}\sinh(sx)\cosh(sx) - s\int_0^x \bar{u}\cosh(2sx)dx]\}$$

$C = Y_1$

or,

$$u_1 = Y_1 + \int_0^x \{[s(W_1 - X_1) + M_1]\cosh[s(x - x')]$$

$$+ s(Y_1 + Z_1)\sinh[s(x - x')]\}dx' \tag{29}$$

Substituting du_1/dx into equation (21a)

$$(s/\gamma)p_1 + dY_1/dx + sW_1 - sX_1 + M_1 + s\{\sinh(sx)\int_0^x [s(W_1-X_1) + M_1]\cosh(sx)dx$$

$$- \cosh(sx)\int_0^x [s(W_1 - X_1) + M_1]\sinh(sx)dx + \cosh(sx)\int_0^x s(Y_1 + Z_1)\cosh(sx)dx$$

$$- \sinh(sx)\int_0^x s(Y_1 + Z_1)\sinh(sx)dx\} = -sX_1 + dY_1/dx + M_1$$

$$(s/\gamma)p_1 + sW_1 = s\{\int_0^x [s(W_1 - X_1) + M_1][\sinh(sx)\cosh(sx') - \cosh(sx)\sinh(sx')]dx'$$

$$+ \int_0^x s(Y_1 + Z_1)[\cosh(sx)\cosh(sx') - \sinh(sx)\sinh(sx')]dx'\}$$

$$p_1/\gamma = -W_1 - \int_0^x \{[s(W_1 - X_1) + M_1]\sinh[s(x - x')]$$

$$+ s(Y_1 + Z_1)\cosh[s(x - x')]\}dx' \qquad (30)$$

Applying the boundary conditions at L_c (as developed in section 3.6 of SP-194)

$$u'(L_c) + \mathcal{A}\, p'(L_c) + \mathcal{C}\, \sigma'(L_c) = 0 \qquad (31)$$

$$\text{where } \sigma' = \sigma_0 + \sigma_1 = -(1/\bar{u})\int_0^x \{((\gamma - 1)/\gamma)p_0 + (1 + \bar{r})(\overline{dh_L/dr})(G_{ox}$$

$$- \bar{r}G_f)P_{00}e^{-s\tau_T}\}\bar{M}\exp[-s\int_{x'}^x (1/\bar{u})dx'']dx' \qquad (32)$$

$\mathcal{C}$ is generally small compared with $\mathcal{A}$ and may be deleted for small perturbations. For rockets which have a cylindrical combustion chamber and conical nozzle, $\mathcal{A}$ may be approximated by $M(\gamma - 1)/(2\gamma)$, where M is the Mach Number at the intersection of the chamber and nozzle.

When the perturbation equations are substituted into the boundary equation (31), a complex equation for s is obtained. The solution for s is difficult. However, stability can be investigated by setting $\lambda = 0$ and regard ω as an independent variable. Then equation (31) can be used as a relation between two of the engine design parameters or operating parameters. Values of ω in the intermediate mode may be chosen between

$$1/\bar{\tau}_T < \omega < 1/\bar{\tau}.$$

Summary

The equations to analyze intermediate mode instabilities have been developed. In some cases with long and laborious mathematics. The pertinent equations are scattered throughout the report, therefore they have been grouped together for the convenience of the reader.

$$X_1 = (\gamma - 1)\bar{u}u_0 + (1 + \bar{r})(\overline{dh_L/dr})(\bar{M}/s)e^{-s\tau_T}(G_{ox} - \bar{r}G_f)P_{oo} \tag{22}$$

$$Y_1 = -\bar{u}p_0 \tag{23}$$

$$Z_1 = (1/\gamma)\bar{u}p_0 + \bar{\rho}_L{}^0 u_{L0} \tag{24}$$

$$W_1 = 2\bar{u}u_0 \tag{25}$$

$$M_1 = \bar{M}\{e^{-s\tau_T}[\mathcal{K}(s) + \mathcal{H}(s)]P_{oo} - \mathcal{P}(s)p_0\} \tag{26}$$

$$p_0 = P_{oo}\cosh(sx) \tag{27}$$

$$u_0 = -(1/\gamma)P_{oo}\sinh(sx) \tag{28}$$

$$u_1 = Y_1 + \int_0^x \{[s(W_1 - X_1) + M_1]\cosh[s(x - x')] + s(Y_1 + Z_1)\sinh[s(x - x')]\}dx' \tag{29}$$

$$p_1/\gamma = -W_1 - \int_0^x \{[s(W_1 - X_1) + M_1]\sinh[s(x - x')] + s(Y_1 + Z_1)\cosh[s(x - x')]\}dx' \tag{30}$$

$$u'(L_c) + \mathcal{A}\, p'(L_c) + \mathcal{C}\, \sigma'(L_c) = 0 \tag{31}$$

$$\sigma' = \sigma_0 + \sigma_1 = -(1/\bar{u})\int_0^x \{((\gamma - 1)/\gamma)p_0 + (1 + \bar{r})(\overline{dh_L/dr})(G_{ox} - \bar{r}G_f)P_{oo}e^{-s\tau_T}\}\bar{M}\exp[-s\int_{x'}^x (1/\bar{u})dx'']dx' \tag{32}$$

Recall that $u' = u_0 + u_1$ and $p' = p_0 + p_1$.

Recommendations and Future Plans

The equations have been developed for obtaining stability limit curves as a function of two variables. A computer program may be written for a PC to solve the equations for various combinations of variables (e.g. n vs τ). If any parameter, such as mixture ratio (r), is changed, new stability limits could be computed and compared to the old.

A FORTRAN program for the PC will be written to determine the stability limits as a function of n and τ. After the program is checked-out, plans will be made to extend the program to allow the use of other parameters to investigate stability.

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Nomenclature

a - speed of sound

a_r - reference speed of sound

$\mathcal{A}$ - nozzle pressure admittance coefficient

$\mathcal{C}$ - nozzle entropy admittance coefficient

D - differential operator

e - static internal energy

e_{Ls} - stagnation internal energy of liquid

e_s - stagnation internal energy $e_s = e + \frac{1}{2}V^2$ dimensional
$e_s = e + \frac{1}{2}(\gamma - 1)V^2$ non-dimensional

E - energy released per unit time per unit volume in the combustion chamber due to chemical reaction and change of phase

G_{ox} - Oxidizer injection admittance

G_f - Fuel injection admittance

h_L - enthalpy of liquid

h_s - stagnation enthalpy $h_s = e_s + p/\rho$

$\mathcal{H}(s)$ - combustion response function for mixture ratio oscillations

$\mathcal{K}(s)$ - combustion response function for mass flow oscillations

L_c - length of combustion chamber

M - combustion response

M_1 - first order term of combustion response perturbation

n - pressure interaction index

n_r - mixture ratio interaction index

p - pressure

p_{dim} - dimensional pressure

p_r - reference pressure

p_0 - zeroth order term of pressure perturbation

p_1 - first order term of pressure perturbation

P_{00} - maximum pressure at injection face

$\mathcal{P}(s)$ - combustion response function for pressure

R - gas constant

RHS - right hand side of differential equation

r - mixture ratio

s - oscillatory term in perturbation equation

T - temperature

t - time

u - axial component of gas velocity

u_0 - zeroth order term of velocity perturbation

u_1 - first order term of velocity perturbation

u_{L0} - axial component of liquid velocity

V - velocity vector of gas

V_L - velocity vector of liquid

W - function defined for equation (19)

W_1 - first order term of W

X - function defined for equation (18)

X_1 - first order term of X

Y - function defined for equation (18)

Y_1 - first order term of Y

Z - function defined for equation (19)

Z_1 - first order term of Z

γ - ratio of specific heats

∂ - partial derivative

λ - damping of oscillation

ρ - density of gas

ρ_L - mass of liquid per unit chamber volume

σ - entropy

τ - sensitive time lag

τ_T - total time lag

ω - frequency of oscillation

∇ - nabla operator

Mean values are indicated by $^{-}$ (e.g. $\bar{p}$ - mean pressure)

Perturbation values are indicated by ' (e.g. p' - perturbation pressure)

APPENDIX IV

FORTRAN Listing of IMODE

Wilbur C. Armstrong

Listing of Program IMODE

```
      PROGRAM MAIN
C
C            IMODE - INTERMEDIATE MODE OSCILLATIONS
C
C                VARIABLES IN COMMON /CMPVAL/      COMPLEX
C
C       X1 - FIRST ORDER TERM OF X
C       Y1 - FIRST ORDER TERM OF Y
C       Z1 - FIRST ORDER TERM OF Z
C       W1 - FIRST ORDER TERM OF W
C       M1 - FIRST ORDER TERM OF M
C       P0 - ZEROTH ORDER TERM OF PRESSURE
C       P1 - FIRST ORDER TERM OF PRESSURE
C       U0 - ZEROTH ORDER TERM OF VELOCITY
C       U1 - FIRST ORDER TERM OF VELOCITY
C      RFH - COMBUSTION RESPONSE FUNCTION FOR MIXTURE RATIO
C      RFK - COMPUSTION RESPONSE FUNCTION FOR MASS FLOW
C      RFP - COMBUSTION RESPONSE FUNCTION FOR PRESSURE
C        S - LAMDA + MU i - PERTURBATION OSCILLATION          13
C       GF - FUEL INJECTION ADMITTANCE                        14
C      GOX - OXIDIZER INJECTION ADMITTANCE                    15
C      RFA - NOZZLE PRESSURE ADMITTANCE COEFFICIENT           16
C      RFC - NOZZLE ENTROPY ADMITTANCE COEFFICIENT            17
C
C
C                VARIABLES IN COMMON /RELVAL/      REAL
C
C        N - PRESSURE INTERACTION INDEX                        1
C      TAU - SENSITIVE TIME LAG                                2
C     DTAU - DELTA TIME LAG (TOTAL TIME LAG = TAU + DTAU)      3
C       NR - ENTHALPY INTERACTION INDEX                       4
C     RBAR - MEAN MIXTURE RATIO                               5
C     MBAR - MEAN COMBUSTION RESPONSE FUNCTION                6
C    GAMMA - RATIO OF SPECIFIC HEATS                          7
C      P00 - MAXIMUM PRESSURE AT INJECTION FACE               8
C    DHLDR - CHANGE IN ENTHALPY WITH CHANGE IN MIXTURE RATIO  9
C    CSTAR - CHARACTERISTIC VELOCITY AT COMBUSTOR EXIT       10
C    DCSDR - CHANGE IN CSTAR WITH CHANGE IN MIXTURE RATIO    11
C    RHOLO - MASS OF LIQUID PER UNIT CHAMBER VOLUME          12
C      ULO - AXIAL COMPONENT OF LIQUID VELOCITY              13
C    LAMDA - DAMPING OF PERTURBATION
C       MU - FREQUENCY OF PERTURBATION
C     TAUT - TOTAL TIME LAG
C UBAR(50) - VELOCITY ALONG AXIS
C XBAR(50) - X LOCATIONS ALONG AXIS
C      XLC - X LOCATION OF CHAMBER-NOZZLE INTERFACE (1.0?)
C
C
C                VARIABLES IN COMMON /DIMVAL/      REAL
C
C        ND - PRESSURE INTERACTION INDEX                                1
C      TAUD - SENSITIVE TIME LAG                            SEC         2
C     DTAUD - DELTA TIME LAG (TOTAL TIME LAG = TAU + DTAU)  SEC         3
```

```
C       NRD - ENTHALPY INTERACTION INDEX                                  4
C    LAMDAD - DAMPING OF PERTURBATION                                     5
C       MUD - FREQUENCY OF PERTURBATION                                   6
C     CDIAM - CHAMBER DIAMETER                                 FT         7
C     TDIAM - THROAT DIAMETER                                  FT         8
C      XLCD - X LOCATION OF CHAMBER-NOZZLE INTERFACE           FT         9
C    GAMMAD - RATIO OF SPECIFIC HEATS                                    10
C      RGAS - GAS CONSTANT                              FT^2/SEC^2/°R    11
C      POOD - MAXIMUM PRESSURE AT INJECTION FACE             LBF/FT^2    12
C     MBARD - MEAN COMBUSTION RESPONSE FUNCTION              LBM/SEC     13
C     RBARD - MEAN MIXTURE RATIO                                         14
C    DCSDRD - CHANGE IN CSTAR/CHANGE IN MIXTURE RATIO        FT/SEC      15
C    DHLDRD - CHANGE IN ENTHALPY/CHANGE IN MIXTURE RATIO    FT^2/SEC^2   16
C    RHOLOD - MASS OF LIQUID PER UNIT CHAMBER VOLUME         LBM/FT^3    17
C      ULOD - AXIAL COMPONENT OF LIQUID VELOCITY             FT/SEC      18
C  PBAR(50) - PRESSURE ALONG AXIS                            LBF/FT^2
C  TBAR(50) - TEMPERATURE ALONG AXIS                         °R
C XBARD(50) - X LOCATIONS ALONG AXIS                         FT
C
C
C                VARIABLES IN COMMON /INTVAL/      INTEGER
C
C     NVAL - NUMBER OF INPUT POINTS ALONG AXIS
C
C
C                VARIABLES IN COMMON /RESULTS/     COMPLEX
C
C       PP - P' = P0 + P1
C       UP - U' = U0 + U1
C     SIGP - SIG' = SIG0 + SIG1
C     FUNB - BOUNDARY FUNCTION = U' + RFA * P' + RFC * SIG'
C
C
C
C       II - NUMBER OF INDEPENDENT VARIABLE
C       ID - NUMBER OF DEPENDENT VARIABLE
C
C           NUMBER            VARIABLE
C              1                 N
C              2                TAU
C              3               DTAU
C              4                NR
C              5              LAMDA
C              6                MU
C              7              CDIAM
C              8              TDIAM
C              9               XLC
C             10              GAMMA
C             11              RGAS
C             12               POO
C             13              MBAR
C             14              RBAR
C             15              DCSDR
C             16              DHLDR
C             17              RHOLO
C             18               ULO
```

```
C             19                PCHMB
C             20                TCHMB
C
C                   SUBROUTINES
C
C    ADMIT - COMPUTES ADMITTANCE FOR FUEL AND LOX
C    BOUND - EVALUATES THE BOUNDARY FUNCTION FUNB
C     EVAL - EVALUATES PARAMETERS AT A GIVEN X LOCATION
C     FUEL - OBTAINS ADMITTANCE FOR FUEL LINE
C     ITER - ITERATES FOR DEPENDENT VARIABLE (REAL BC - ORIGINAL)
C      LOX - OBTAINS ADMITTANCE FOR LOX LINE
C   NONDIM - NON-DIMENSIONALIZES VARIABLES
C   READIN - READS INPUT DATA
C   SETVAL - SET VALUES FROM ITERATED VARIABLES
C   SETVAR - SET ITERATED VARIABLES FROM VALUES
C    ZREAD - READS FREE FORMAT INPUT
C
C                   COMPLEX FUNCTIONS
C
C    CCOSH - COMPUTES COMPLEX HYPERBOLIC COSH
C    CSINH - COMPUTES COMPLEX HYPERBOLIC SINH
C    CTANH - COMPUTES COMPLEX HYPERBOLIC TANH
C      FP1 - EVALUATES P1
C    FSIGP - EVALUATES SIG'
C      FU1 - EVALUATES U1
C
C
      COMMON /CMPVAL/X1,Y1,Z1,W1,M1,P0,P1,U0,U1,RFH,RFK,RFP,
     *               S,GF,GOX,RFA,RFC
      COMMON /RELVAL/N,TAU,DTAU,NR,RBAR,MBAR,GAMMA,P00,DHLDR,CSTAR,
     *        DCSDR,RHOLO,ULO,LAMDA,MU,TAUT,UBAR(50),XBAR(50),XLC
      COMMON /RESULT/PP,UP,SIGP,FUNB
      COMMON /INTVAL/NVAL
      COMMON /DIMVAL/HOLDD(20),XBARD(50),PBAR(50),TBAR(50)
      COMMON /TITL/TITLE
      REAL MBAR,N,NR,LAMDA,MU,RVAR(13)
      COMPLEX S,X1,Y1,Z1,W1,M1,P0,P1,U0,U1,GF,GOX,RFH,RFK,RFP,RFA,RFC
      COMPLEX FP1,FU1,FSIGP,PP,UP,SIGP,FUNB,CSINH,CCOSH,CVAR(17)
      EQUIVALENCE (N,RVAR(1)),(X1,CVAR(1))
      CHARACTER*8 VAR(20),VARP(20)
      CHARACTER*1 ANS
      CHARACTER*70 TITLE
      CHARACTER*24 ROCIN,ROCOUT,ROCPLT
      DATA VAR /'     N =','   TAU =','  DTAU =','    NR =','  LAMDA =',
     *          '    MU =',' CDIAM =',' TDIAM =','   XLC =',' GAMMA =',
     *          ' RGAS =','   P00 =','  MBAR =','  RBAR =',' DCSDR =',
     *          ' DHLDR =',' RHOLO =','   ULO =',' PCHMB =',' TCHMB ='/
      DATA VARP/'   N    ','  TAU   ','  DTAU  ','   NR   ','  LAMDA  ',
     *          '   MU   ',' CDIAM  ',' TDIAM  ','  XLC   ',' GAMMA  ',
     *          ' RGAS  ','  P00   ','  MBAR  ','  RBAR  ',' DCSDR  ',
     *          ' DHLDR  ',' RHOLO  ','  ULO   ',' PCHMB  ',' TCHMB  '/
      DATA TOL/.0001/
    1 FORMAT(A8,1PE13.5,2X,A8,E13.5,'   FUNB=',2E13.5)
    2 FORMAT(A)
    3 FORMAT(/3X,A8,5X,A8,5X,' FUNB(R)',5X,' FUNB(I)'/)
    4 FORMAT(1P6E13.5)
```

```
 5 FORMAT(' ')
 6 FORMAT('"',A,'"')
 7 FORMAT(2X,'"',A8,'"',3X,'"',A8,'"')
   WRITE(*,*)' ARE THE FILES YOU ARE USING'
   WRITE(*,*)'    IMODE.INP - INPUT DATA'
   WRITE(*,*)'    IMODE.OUT - OUTPUT DATA'
   WRITE(*,*)'    IMODE.PRN - LOTUS PLOT DATA'
   WRITE(*,'(A\)')'        ENTER Y OR N '
   READ(*,2)ANS
   IF(ANS.EQ.'Y'.OR.ANS.EQ.'y')  THEN
    OPEN(5,FILE='IMODE.INP')
    OPEN(6,FILE='IMODE.OUT',STATUS='NEW')
    OPEN(7,FILE='IMODE.PRN',STATUS='NEW')
   ELSE
    WRITE(*,'(A\)')'  ENTER NAME OF FILE CONTAINING INPUT '
    READ(*,2)ROCIN
    OPEN(5,FILE=ROCIN)
    WRITE(*,'(A\)')'  ENTER NAME OF FILE FOR OUTPUT '
    READ(*,2)ROCOUT
    OPEN(6,FILE=ROCOUT,STATUS='NEW')
    WRITE(*,'(A\)')'  ENTER NAME OF FILE FOR LOTUS PLOT '
    READ(*,2)ROCPLT
    OPEN(7,FILE=ROCPLT,STATUS='NEW')
   ENDIF
   XLC=1.0
   WRITE(*,*)' '
   WRITE(*,*)' '
   WRITE(*,*)' '
   WRITE(*,*)' '
   WRITE(*,*)' '
   WRITE(*,*)' '
   WRITE(*,*)'                    Welcome to IMODE'
   WRITE(*,*)' '
   WRITE(*,*)'         Intermediate Mode Rocket Stability Aide'
   WRITE(*,*)' '
   WRITE(*,*)'  There are three types of input, rocket parameters,'
   WRITE(*,*)'  Oxidizer feed parameters, and fuel feed parameters,'
   WRITE(*,*)'   Each may be read from files or from the keyboard'
   WRITE(*,*)' '
   WRITE(*,*)'          File Name                  Input'
   WRITE(*,*)' '
   WRITE(*,*)'   IMODE.INP or NAME read in   Rocket Parameters  '
   WRITE(*,*)'          LOX.DAT              Oxidizer Parameters'
   WRITE(*,*)'          FUEL.DAT             Fuel Parameters    '
   WRITE(*,*)' '
   WRITE(*,*)' If keyboard entry, you will be prompted for values'
   GO TO 21
20 CONTINUE
   WRITE(*,*)' '
   WRITE(*,'(A\)')' Do you want to run another case? Enter Y or N '
   READ(*,2)ANS
   IF(ANS.EQ.'N'.OR.ANS.EQ.'n')  STOP
21 CONTINUE
   CALL READIN
   WRITE(*,*)' '
   WRITE(*,*)'   Codes for independent and dependent variables'
```

```
      WRITE(*,*)'  CODE    Variable Name    CODE    Variable Name'
      WRITE(*,*)'    1          N             2          TAU      '
      WRITE(*,*)'    3         DTAU           4           NR      '
      WRITE(*,*)'    5        LAMDA           6           MU      '
      WRITE(*,*)'    7        CDIAM           8         TDIAM     '
      WRITE(*,*)'    9         XLC           10         GAMMA     '
      WRITE(*,*)'   11         RGAS          12          POO      '
      WRITE(*,*)'   13         MBAR          14          RBAR     '
      WRITE(*,*)'   15        DCSDR          16         DHLDR     '
      WRITE(*,*)'   17        RHOLO          18          ULO      '
      WRITE(*,*)'   19        PCHMB          20         TCHMB     '
      WRITE(*,*)' '
      WRITE(*,'(A\)')' Enter codes for independent and dependent variabl
     *es '
   22 CONTINUE
      READ(*,*)II,ID
      IBAD=0
      IF(II.LT.1.OR.II.GT.20)  THEN
       IBAD=1
       WRITE(*,*)'  Code for Independent Variable OUT OF RANGE'
      ENDIF
      IF(ID.LT.1.OR.ID.GT.20)  THEN
       IBAD=1
       WRITE(*,*)'  Code for Dependent Variable OUT OF RANGE'
      ENDIF
      IF(IBAD.EQ.1)  THEN
       WRITE(*,'(A\)')'   Enter codes again '
       GO TO 22
      ENDIF
      WRITE(*,*)' '
      WRITE(6,5)
      WRITE(6,2)TITLE
      WRITE(6,3)VARP(II),VARP(ID)
      WRITE(7,6)TITLE
      WRITE(7,7)VARP(II),VARP(ID)
   23 CONTINUE
      CALL ITER(ID,TOL)
      WRITE(6,4)HOLDD(II),HOLDD(ID),FUNB
      WRITE(7,4)HOLDD(II),HOLDD(ID)
      WRITE(*,1)VAR(II),HOLDD(II),VAR(ID),HOLDD(ID),FUNB
      WRITE(*,'(A\)')
     *  ' Enter new value for independent variable (-999 to stop) '
      READ(*,*,END=99)VAR1
      IF(VAR1.EQ.-999.0)  GO TO 20
      CALL SETVAR(VAR1,II)
      GO TO 23
   99 CONTINUE
      STOP
      END
```

```
      SUBROUTINE ADMIT(S,GADM,A,AREA,CMAN,CTANK,DPROR,L,LFLOW,PCHMB,
     *                 SECTN,SEGMN,TFLOW)
C
C       THIS ROUTINE WILL COMPUTE THE ADMITTANCE RATIO FROM THE TANK TO
C       THE COMBUSTION CHAMBER
C
C           A = VELOCITY OF SOUND IN THE FLUID (FT/S)
C     AREA(I) = AREA OF FEED LINE (FT^2)
C        CMAN = CAPACITANCE ASSOCIATED WITH THE MANIFOLD
C       CTANK = CAPACITANCE ASSOCIATED WITH THE TANK
C       DPROR = PRESSURE DROP ACROSS ORIFICES (LB/FT^2)
C        G(I) = ADMITTANCE FROM TANK THROUGH PIPE I
C        GADM = OVERALL ADMITTANCE AS SEEN FROM ENGINE
C        GRAV = ACCELERATION OF GRAVITY (32.2 FT/S^2)
C        L(I) = LENGTH OF LINE (FT)
C       LFLOW = FLOW RATE OF FLUID THROUGH THE LINE(LB/S)
C       PCHMB = PRESSURE INSIDE COMBUSTION CHAMBER (LB/FT^2)
C           S = COMPLEX FREQUENCY OF OSCILLATION
C    SECTN(I) = TYPE OF LINE
C       SEGMN = NUMBER OF SECTIONS OF LINE
C       TFLOW = TOTAL FLOW RATE OF FLUID INSIDE ENGINE(LB/S)
C          TL = TIME CONSTANT
C       ZLINE = IMPEDANCE ASSOCIATED WITH THE FEED LINE
C         ZOR = IMPEDANCE ASSOCIATED WITH THE ORIFICES
C
      COMPLEX CTANH,G(76),GADM,S,W
      REAL AREA(75),L(75),LFLOW
      INTEGER SEGMN,SECTN(75)
      DATA GRAV/32.2/
      W=S*A*3.141593
      G(1)=CTANK*W
      GADM=G(1)+1.0
      ZTOP=A*TFLOW/(GRAV*PCHMB)
      ZOR=2.0*DPROR*TFLOW/(LFLOW*PCHMB)
      DO 22 I=2,SEGMN+1
       IF(SECTN(I-1).EQ.1)  THEN
        ZLINE=ZTOP/AREA(I-1)
        TL=L(I-1)/A
        G(I)=(1.0+CTANH(W*TL)/(G(I-1)*ZLINE))/(1.0+G(I-1)*ZLINE*
     *       CTANH(W*TL))
        GO TO 21
       ELSEIF(SECTN(I-1).EQ.2)  THEN
        G(I)=1.0+(CMAN*W/G(I-1))
        GO TO 21
       ELSEIF(SECTN(I-1).EQ.3)  THEN
        G(I)=1.0/(1.0+ZOR*G(I-1))
       ENDIF
   21 CONTINUE
       GADM=GADM*G(I)
       G(I)=G(I)*G(I-1)
   22 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE BOUND(PP,UP,SIGP,FUNB)
      COMMON /CMPVAL/X1,Y1,Z1,W1,M1,P0,P1,U0,U1,RFH,RFK,RFP,
     *              S,GF,GOX,RFA,RFC
      COMMON /RELVAL/N,TAU,DTAU,NR,RBAR,MBAR,GAMMA,P00,DHLDR,CSTAR,
     *        DCSDR,RHOLO,ULO,LAMDA,MU,TAUT,UBAR(50),XBAR(50),XLC
      COMMON /INTVAL/NVAL
      REAL MBAR,N,NR,LAMDA,MU
      COMPLEX S,X1,Y1,Z1,W1,M1,P0,P1,U0,GF,GOX,U1,RFH,RFK,RFP,RFA,RFC
      COMPLEX FP1,FU1,FSIGP,PP,UP,SIGP,FUNB,CSINH,CCOSH
C         EVALUATE PP,UP,SIGP, AND FUNB
      P1=FP1(XLC)
      U1=FU1(XLC)
      P0=P00*CCOSH(S*XLC)
      U0=-(1.0/GAMMA)*P00*CSINH(S*XLC)
      PP=P0+P1
      UP=U0+U1
      SIGP=FSIGP(XLC)
      FUNB=UP+RFA*PP+RFC*SIGP
      RETURN
      END
```

```
COMPLEX FUNCTION CCOSH(S)
COMPLEX S
REAL LAMDA, MU
LAMDA=REAL(S)
MU=AIMAG(S)
COSHR=COSH(LAMDA)*COS(MU)
COSHI=SINH(LAMDA)*SIN(MU)
CCOSH=CMPLX(COSHR,COSHI)
RETURN
END
```

```
COMPLEX FUNCTION CSINH(S)
COMPLEX S
REAL LAMDA, MU
LAMDA=REAL(S)
MU=AIMAG(S)
SINHR=SINH(LAMDA)*COS(MU)
SINHI=COSH(LAMDA)*SIN(MU)
CSINH=CMPLX(SINHR,SINHI)
RETURN
END
```

```
COMPLEX FUNCTION CTANH(S)
COMPLEX S,CTANN,CTAND,CSINH,CCOSH
CTANN=CSINH(S)
CTAND=CCOSH(S)
CTANH=(0.0,0.0)
IF(CTAND.NE.0.0)  CTANH=CTANN/CTAND
RETURN
END
```

```
      SUBROUTINE EVAL(X)
      COMMON /CMPVAL/X1,Y1,Z1,W1,M1,P0,P1,U0,U1,RFH,RFK,RFP,
     *              S,GF,GOX,RFA,RFC
      COMMON /RELVAL/N,TAU,DTAU,NR,RBAR,MBAR,GAMMA,P00,DHLDR,CSTAR,
     *        DCSDR,RHOLO,ULO,LAMDA,MU,TAUT,UBAR(50),XBAR(50),XLC
      COMMON /INTVAL/NVAL
      REAL MBAR,N,NR,LAMDA,MU
      COMPLEX S,X1,Y1,Z1,W1,M1,P0,P1,U0,U1,GF,GOX,RFH,RFK,RFP,RFA,RFC
      COMPLEX CSINH,CCOSH
C         EVALUATE EVERYTHING EXCEPT PP,UP,SIGP
      IF(NVAL.EQ.1)  THEN
       UB=UBAR(1)
       GO TO 23
      ENDIF
      DO 21 I=2,NVAL
       IF(X.LE.XBAR(I))  GO TO 22
   21 CONTINUE
      UB=UBAR(NVAL)
      GO TO 23
   22 CONTINUE
      FAC=(X-XBAR(I-1))/(XBAR(I)-XBAR(I-1))
      UB=UBAR(I-1)+FAC*(UBAR(I)-UBAR(I-1))
   23 CONTINUE
      RFH=(1.0+RBAR)*((RBAR/CSTAR)*DCSDR-NR*S*TAU)*(GOX
     *    -RBAR*GF)/RBAR
      RFK=(1.0+S*TAUT)*(GOX+GF)
      RFP=N*(1.0-CEXP(S*TAU))
      P0=P00*CCOSH(S*X)
      U0=-(1.0/GAMMA)*P00*CSINH(S*X)
      X1=(GAMMA-1.0)*UB*U0+(1.0+RBAR)*DHLDR*(MBAR/S)
     *   *CEXP(-S*TAUT)*(GOX-RBAR*GF)*P00
      Y1=-UB*P0
      Z1=(1.0/GAMMA)*UB*P0+RHOLO*ULO
      W1=2.0*UB*U0
      M1=MBAR*(CEXP(-S*TAUT)*(RFK+RFH)*P00-RFP*P0)
      RETURN
      END
```

```
      COMPLEX FUNCTION FP1(XL)
      COMMON /CMPVAL/X1,Y1,Z1,W1,M1,PO,P1,UO,U1,RFH,RFK,RFP,
     *              S,GF,GOX,RFA,RFC
      COMMON /RELVAL/N,TAU,DTAU,NR,RBAR,MBAR,GAMMA,POO,DHLDR,CSTAR,
     *        DCSDR,RHOLO,ULO,LAMDA,MU,TAUT,UBAR(50),XBAR(50),XLC
      COMMON /INTVAL/NVAL
      REAL MBAR,N,NR,LAMDA,MU
      COMPLEX S,X1,Y1,Z1,W1,M1,PO,P1,UO,U1,GF,GOX,RFH,RFK,RFP,RFA,RFC
      COMPLEX CSINH,CCOSH
      COMPLEX VINT
C         EVALUATE P1
      DX=XL/50.0
      FP1=CMPLX(0.0,0.0)
      DO 23 I=1,51
       X=(I-1)*DX
       CALL EVAL(X)
       VINT=(S*(W1-X1)+M1)*CSINH(S*(XL-X))
     *      +S*(Y1+Z1)*CCOSH(S*(XL-X))
       IF(I.EQ.1.OR.I.EQ.51)  THEN
        FP1=FP1+0.5*VINT*DX
       ELSE
        FP1=FP1+VINT*DX
       ENDIF
   23 CONTINUE
      FP1=-GAMMA*(W1+FP1)
      RETURN
      END
```

```
      COMPLEX FUNCTION FSIGP(XL)
      COMMON /CMPVAL/X1,Y1,Z1,W1,M1,P0,P1,U0,U1,RFH,RFK,RFP,
     *                S,GF,GOX,RFA,RFC
      COMMON /RELVAL/N,TAU,DTAU,NR,RBAR,MBAR,GAMMA,P00,DHLDR,CSTAR,
     *         DCSDR,RHOLO,ULO,LAMDA,MU,TAUT,UBAR(50),XBAR(50),XLC
      COMMON /INTVAL/NVAL
      REAL MBAR,N,NR,LAMDA,MU
      COMPLEX S,X1,Y1,Z1,W1,M1,P0,P1,U0,U1,GF,GOX,RFH,RFK,RFP,RFA,RFC
      COMPLEX CSINH,CCOSH
      REAL UB(51)
      COMPLEX VINT(51),VVINT(51),FSIG2,FCON
C         EVALUATE FSIGP  (INTEGRATION NOT CHANGED YET)
      DX=XL/50.0
      DO 23 I=1,51
       X=(I-1)*DX
       IF(NVAL.EQ.1)  THEN
        UB(I)=UBAR(1)
        GO TO 23
       ENDIF
       DO 21 II=2,NVAL
        IF(X.LE.XBAR(II))  GO TO 22
   21 CONTINUE
       II=NVAL
   22 CONTINUE
       FAC=(X-XBAR(II-1))/(XBAR(II)-XBAR(II-1))
       UB(I)=UBAR(II-1)+FAC*(UBAR(II)-UBAR(II-1))
   23 CONTINUE
      DO 24 I=1,51
       X=(I-1)*DX
       CALL EVAL(X)
       VINT(I)=((GAMMA-1.0)/GAMMA)*P0
       VVINT(I)=1.0/UB(I)
   24 CONTINUE
      FCON=(1.0+RBAR)*DHLDR*(GOX-RBAR*GF)*P00
     *     *CEXP(-S*TAUT)
      DO 26 I=1,51
       FSIG2=CMPLX(0.0,0.0)
       DO 25 J=I,51
        IF(J.EQ.I.OR.J.EQ.51)  THEN
         FSIG2=FSIG2+0.5*VVINT(J)*DX
        ELSE
         FSIG2=FSIG2+VVINT(J)*DX
        ENDIF
   25 CONTINUE
       FSIG2=CEXP(-S*FSIG2)
       VINT(I)=(VINT(I)+FCON)*MBAR*FSIG2
   26 CONTINUE
      FSIGP=CMPLX(0.0,0.0)
      DO 27 I=1,51
       IF(I.EQ.1.OR.I.EQ.51)  THEN
        FSIGP=FSIGP+0.5*VINT(I)*DX
       ELSE
        FSIGP=FSIGP+VINT(I)*DX
       ENDIF
   27 CONTINUE
      FSIGP=-FSIGP/UB(51)
```

RETURN
END

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```
      SUBROUTINE FUEL(S,GF)
C
C           A = VELOCITY OF SOUND IN THE FLUID (FT/S)
C     AREA(I) = AREA OF FEED LINE (FT^2)
C        CMAN = CAPACITANCE ASSOCIATED WITH THE MANIFOLD
C       CTANK = CAPACITANCE ASSOCIATED WITH THE FUEL TANK
C        DENS = DENSITY OF FLUID (LBM/FT^3)
C      DIA(I) = DIAMETER OF LINE (FT)
C       DPROR = PRESSURE DROP ACROSS ORIFICES (LB/FT^2)
C          GF = OVERALL ADMITTANCE OF FUEL AS SEEN FROM ENGINE
C        KMAN = BULK MODULUS OF FLUID AT MANIFOLD PRESSURE (LBF/FT^2)
C       KTANK = BULK MODULUS OF FLUID IN TANK (LBF/FT^2)
C        L(I) = LENGTH OF LINE (FT)
C       LFLOW = FLOW RATE OF FLUID THROUGH THE LINE(LB/S)
C       PCHMB = PRESSURE INSIDE COMBUSTION CHAMBER (LB/FT^2)
C           S = COMPLEX FREQUENCY OF OSCILLATION
C    SECTN(I) = TYPE OF LINE
C       SEGMN = NUMBER OF SECTIONS OF LINE
C       TFLOW = TOTAL FLOW RATE OF FLUID INSIDE ENGINE(LB/S)
C         VOL = VOLUME OF FUEL TANK (FT^3)
C       VOLMF = VOLUME OF MANIFOLD (FT^3)
C
      COMMON /PIPES/PFACE,TFACE
      COMPLEX GF,S
      REAL AREA(75),DIA(75),L(75),KMAN,KTANK,LFLOW
      INTEGER SEGMN,SECTN(75)
      DATA ISTRT/0/
    1 FORMAT(F15.0)
    2 FORMAT(I5,3F15.5)
      IF(ISTRT.EQ.0)  THEN
       ISTRT=1
       OPEN(UNIT=11,FILE='FUEL.DAT')
       READ(11,1)DENS
       READ(11,1)TFLOW
       READ(11,1)VOLMF
       READ(11,1)KMAN
       READ(11,1)PCHMB
       READ(11,1)DPROR
       READ(11,1)VOL
       READ(11,1)LFLOW
       READ(11,1)KTANK
       READ(11,1)A
       READ(11,1)CTANK
       READ(11,1)CMAN
       READ(11,2)SEGMN
       READ(11,2)(SECTN(I),L(I),AREA(I),DIA(I),I=1,SEGMN)
      ENDIF
      FLOWL=LFLOW*TFACE/TFLOW
      CTANK=(DENS*VOL*PFACE)/(KTANK*TFACE)
      CMAN=(DENS*VOLMF*PFACE)/(KMAN*TFACE)
      CALL ADMIT(S,GF,A,AREA,CMAN,CTANK,DPROR,L,FLOWL,PFACE,
     *           SECTN,SEGMN,TFACE)
      RETURN
      END
```

```
      COMPLEX FUNCTION FU1(XL)
      COMMON /CMPVAL/X1,Y1,Z1,W1,M1,PO,P1,UO,U1,RFH,RFK,RFP,
     *              S,GF,GOX,RFA,RFC
      COMMON /RELVAL/N,TAU,DTAU,NR,RBAR,MBAR,GAMMA,POO,DHLDR,CSTAR,
     *        DCSDR,RHOLO,ULO,LAMDA,MU,TAUT,UBAR(50),XBAR(50),XLC
      COMMON /INTVAL/NVAL
      REAL MBAR,N,NR,LAMDA,MU
      COMPLEX S,X1,Y1,Z1,W1,M1,PO,P1,UO,U1,GF,GOX,RFH,RFK,RFP,RFA,RFC
      COMPLEX CSINH,CCOSH
      COMPLEX VINT
C         EVALUATE U1
      DX=XL/50.0
      FU1=CMPLX(0.0,0.0)
      DO 23 I=1,51
       X=(I-1)*DX
       CALL EVAL(X)
       VINT=(S*(W1-X1)+M1)*CCOSH(S*(XL-X))
     *      +S*(Y1+Z1)*CSINH(S*(XL-X))
       IF(I.EQ.1.OR.I.EQ.51)  THEN
        FU1=FU1+0.5*VINT*DX
       ELSE
        FU1=FU1+VINT*DX
       ENDIF
   23 CONTINUE
      FU1=Y1+FU1
      RETURN
      END
```

```
      SUBROUTINE ITER(ID,TOL)
C
C        ITERATE FOR DEPENDENT VARIABLE
C
      COMMON /CMPVAL/X1,Y1,Z1,W1,M1,PO,P1,UO,U1,RFH,RFK,RFP,
     *               S,GF,GOX,RFA,RFC
      COMMON /RELVAL/N,TAU,DTAU,NR,RBAR,MBAR,GAMMA,POO,DHLDR,CSTAR,
     *        DCSDR,RHOLO,ULO,LAMDA,MU,TAUT,UBAR(50),XBAR(50),XLC
      COMMON /INTVAL/NVAL
      COMMON /RESULT/PP,UP,SIGP,FUNB
      REAL MBAR,N,NR,LAMDA,MU,RVAR(13)
      COMPLEX S,X1,Y1,Z1,W1,M1,PO,P1,UO,U1,GF,GOX,RFH,RFK,RFP,RFA,RFC
      COMPLEX FP1,FU1,FSIGP,PP,UP,SIGP,FUNB,CSINH,CCOSH,CVAR(17)
      EQUIVALENCE (N,RVAR(1)),(X1,CVAR(1))
      CALL SETVAL(VAL1,ID)
      CALL BOUND(PP,UP,SIGP,FUNB)
      FUN1=REAL(FUNB)
      IF(ABS(FUN1).LE.TOL)  GO TO 22
      VAL2=1.01*VAL1
      IF(VAL1.EQ.0)  VAL2=0.01
      CALL SETVAR(VAL2,ID)
      CALL BOUND(PP,UP,SIGP,FUNB)
      FUN2=REAL(FUNB)
      IF(FUN1.EQ.FUN2)  THEN
       VAL=VAL1+VAL2
      ELSE
       VAL=VAL1-FUN1*(VAL2-VAL1)/(FUN2-FUN1)
      ENDIF
      DO 21 I=1,10
       CALL SETVAR(VAL,ID)
       CALL BOUND(PP,UP,SIGP,FUNB)
       FUN=REAL(FUNB)
       IF(ABS(FUN).LE.TOL)  GO TO 22
       IF(ABS(FUN).LT.ABS(FUN1))  THEN
        FUN2=FUN1
        FUN1=FUN
        VAL2=VAL1
        VAL1=VAL
       ELSE
        FUN1=FUN2
        FUN2=FUN
        VAL1=VAL2
        VAL2=VAL
       ENDIF
       IF(FUN1.EQ.FUN2)  THEN
        VAL=VAL1+VAL2
       ELSE
        VAL=VAL1-FUN1*(VAL2-VAL1)/(FUN2-FUN1)
       ENDIF
   21 CONTINUE
   22 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE LOX(S,GOX)
C
C           A = VELOCITY OF SOUND IN THE FLUID (FT/S)
C     AREA(I) = AREA OF FEED LINE (FT^2)
C        CMAN = CAPACITANCE ASSOCIATED WITH THE MANIFOLD
C       CTANK = CAPACITANCE ASSOCIATED WITH THE LOX TANK
C        DENS = DENSITY OF FLUID (LBM/FT^3)
C      DIA(I) = DIAMETER OF LINE (FT)
C       DPROR = PRESSURE DROP ACROSS ORIFICES (LB/FT^2)
C         GOX = OVERALL ADMITTANCE OF LOX AS SEEN FROM ENGINE
C        KMAN = BULK MODULUS OF FLUID AT MANIFOLD PRESSURE (LBF/FT^2)
C       KTANK = BULK MODULUS OF FLUID IN TANK (LBF/FT^2)
C        L(I) = LENGTH OF LINE (FT)
C       LFLOW = FLOW RATE OF FLUID THROUGH THE LINE(LB/S)
C       PCHMB = PRESSURE INSIDE COMBUSTION CHAMBER (LB/FT^2)
C           S = COMPLEX FREQUENCY OF OSCILLATION
C    SECTN(I) = TYPE OF LINE
C       SEGMN = NUMBER OF SECTIONS OF LINE
C       TFLOW = TOTAL FLOW RATE OF FLUID INSIDE ENGINE(LB/S)
C         VOL = VOLUME OF FUEL TANK (FT^3)
C       VOLMF = VOLUME OF MANIFOLD (FT^3)
C
      COMMON /PIPES/PFACE,TFACE
      COMPLEX GOX,S
      REAL AREA(75),DIA(75),L(75),KMAN,KTANK,LFLOW
      INTEGER SEGMN,SECTN(75)
      DATA ISTRT/0/
    1 FORMAT(F15.0)
    2 FORMAT(I5,3F15.0)
      IF(ISTRT.EQ.0)  THEN
       ISTRT=1
       OPEN(UNIT=10,FILE='LOX.DAT')
       READ(10,1)DENS
       READ(10,1)TFLOW
       READ(10,1)VOLMF
       READ(10,1)KMAN
       READ(10,1)PCHMB
       READ(10,1)DPROR
       READ(10,1)VOL
       READ(10,1)LFLOW
       READ(10,1)KTANK
       READ(10,1)A
       READ(10,1)CTANK
       READ(10,1)CMAN
       READ(10,2)SEGMN
       READ(10,2)(SECTN(I),L(I),AREA(I),DIA(I),I=1,SEGMN)
      ENDIF
      FLOWL=LFLOW*TFACE/TFLOW
      CTANK=(DENS*VOL*PFACE)/(KTANK*TFACE)
      CMAN=(DENS*VOLMF*PFACE)/(KMAN*TFACE)
      CALL ADMIT(S,GOX,A,AREA,CMAN,CTANK,DPROR,L,FLOWL,PFACE,
     *           SECTN,SEGMN,TFACE)
      RETURN
      END
```

```
      SUBROUTINE NONDIM(HOLD)
C
C        ROUTINE TO NON-DIMENSIONALIZE INPUT PARAMETERS
C
      COMMON /CMPVAL/X1,Y1,Z1,W1,M1,PO,P1,UO,U1,RFH,RFK,RFP,
     *               S,GF,GOX,RFA,RFC
      COMMON /RELVAL/N,TAU,DTAU,NR,RBAR,MBAR,GAMMA,POO,DHLDR,CSTAR,
     *        DCSDR,RHOLO,ULO,LAMDA,MU,TAUT,UBAR(50),XBAR(50),XLC
      COMMON /INTVAL/NVAL
      COMMON /DIMVAL/HOLDD(20),XBARD(50),PBAR(50),TBAR(50)
      COMMON /PIPES/PFACE,TFACE
      COMMON /TITL/TITLE
      REAL MBAR,N,NR,LAMDA,MU,RVAR(15)
      REAL MBARD,ND,NRD,LAMDAD,MUD
      REAL HOLD(20),UBARD(50),RHOBAR(50)
      COMPLEX S,X1,Y1,Z1,W1,M1,PO,P1,UO,U1,GF,GOX,RFH,RFK,RFP,RFA,RFC
      COMPLEX CVAR(17)
      CHARACTER*8 VAR(13),VARD(20)
      CHARACTER*70 TITLE
      EQUIVALENCE (N,RVAR(1)),(X1,CVAR(1))
      EQUIVALENCE
     *    (ND,HOLDD(1)),(TAUD,HOLDD(2)),(DTAUD,HOLDD(3)),
     *    (NRD,HOLDD(4)),(LAMDAD,HOLDD(5)),(MUD,HOLDD(6)),
     *    (CDIAM,HOLDD(7)),(TDIAM,HOLDD(8)),(XLCD,HOLDD(9)),
     *    (GAMMAD,HOLDD(10)),(RGAS,HOLDD(11)),(POOD,HOLDD(12)),
     *    (MBARD,HOLDD(13)),(RBARD,HOLDD(14)),(DCSDRD,HOLDD(15)),
     *    (DHLDRD,HOLDD(16)),(RHOLOD,HOLDD(17)),(ULOD,HOLDD(18)),
     *    (PCHMB,HOLDD(19)),(TCHMB,HOLDD(20))
      DATA VAR/'      N=','    TAU=','   DTAU=','     NR=','   RBAR=',
     *         '   MBAR=','  GAMMA=','    POO=','  DHLDR=','  CSTAR=',
     *         '  DCSDR=','  RHOLO=','    ULO='/
      DATA VARD/'     N =','   TAU =','  DTAU =','    NR =',' LAMDA =',
     *          '    MU =',' CDIAM =',' TDIAM =','   XLC =',' GAMMA =',
     *          '  RGAS =','   POO =','  MBAR =','  RBAR =',' DCSDR =',
     *          ' DHLDR =',' RHOLO =','   ULO =',' PCHMB =',' TCHMB ='/
      DATA PI/3.141593/,GC/32.174/
    1 FORMAT(A)
    2 FORMAT(A8,1PE13.5,2X,A8,E13.5,2X,A8,E13.5)
    3 FORMAT(' ')
C
C          N     - HOLD(1)
C          TAU   - HOLD(2)
C          DTAU  - HOLD(3)
C          NR    - HOLD(4)
C          LAMDA - HOLD(5)
C          MU    - HOLD(6)
C          CDIAM - HOLD(7)
C          TDIAM - HOLD(8)
C          XLC   - HOLD(9)
C          GAMMA - HOLD(10)
C          RGAS  - HOLD(11)
C          POO   - HOLD(12)
C          MBAR  - HOLD(13)
C          RBAR  - HOLD(14)
C          DCSDR - HOLD(15)
C          DHLDR - HOLD(16)
```

```
C          RHOLO - HOLD(17)
C          ULO   - HOLD(18)
C          PCHMB - HOLD(19)
C          TCHMB - HOLD(20)
C          PBAR  - PBAR
C          TBAR  - TBAR
C          XBAR  - XBARD
C
C          PCHMB = PBAR(1)
C          TFLOW = LFLOW(LOX) + LFLOW(FUEL)
C          LFLOW = LINE FLOW OF LOX OR FUEL
C
      DO 21 I=1,20
       HOLDD(I)=HOLD(I)
   21 CONTINUE
      IF(PCHMB.NE.PBAR(1))  THEN
       FAC=PCHMB/PBAR(1)
       DO 22 I=1,NVAL
        PBAR(I)=FAC*PBAR(I)
   22 CONTINUE
      ENDIF
      IF(TCHMB.NE.TBAR(1))  THEN
       FAC=TCHMB/TBAR(1)
       DO 23 I=1,NVAL
        TBAR(I)=FAC*TBAR(I)
   23 CONTINUE
      ENDIF
      CAREA=0.25*PI*CDIAM**2
      WRITE(6,3)
      WRITE(6,*)'  CAREA=',CAREA
      TAREA=0.25*PI*TDIAM**2
      WRITE(6,*)'  TAREA=',TAREA
      PFACE=PBAR(1)
      PEXIT=PBAR(NVAL)
      TFACE=MBARD
      ASTAR=SQRT(GAMMAD*RGAS*TBAR(1))
      WRITE(6,*)'  ASTAR=',ASTAR
      CSTARD=PEXIT*TAREA*GC/MBARD
      WRITE(6,*)' CSTARD=',CSTARD
      DO 24 I=1,NVAL
       RHOBAR(I)=PBAR(I)*GC/(RGAS*TBAR(I))
      WRITE(6,*)' RHOBAR=',RHOBAR(I)
       UBARD(I)=MBARD/(RHOBAR(I)*CAREA)
      WRITE(6,*)'  UBARD=',UBARD(I)
   24 CONTINUE
      N=ND
      TAU=TAUD*ASTAR/XLCD
      DTAU=DTAUD*ASTAR/XLCD
      TAUT=TAU+DTAU
      NR=NRD
      RBAR=RBARD
      MBAR=MBARD/(RHOBAR(1)*ASTAR*CAREA/XLCD)
      GAMMA=GAMMAD
      POO=POOD/PBAR(1)
      DHLDR=DHLDRD
      CSTAR=CSTARD/ASTAR
```

```
      DCSDR=DCSDRD/ASTAR
      RHOLO=RHOLOD/RHOBAR(1)
      ULO=ULOD/ASTAR
      LAMDA=LAMDAD*XLCD/ASTAR
      MU=MUD*XLCD*PI/ASTAR
      XLC=1.0
      DO 25 I=1,NVAL
       XBAR(I)=XBARD(I)/XLCD
       UBAR(I)=UBARD(I)/ASTAR
   25 CONTINUE
      S=CMPLX(LAMDA,MU)
      CALL FUEL(S,GF)
      CALL LOX(S,GOX)
      RFAR=(GAMMA-1.0)*UBAR(1)/(2.0*GAMMA)
      RFA=CMPLX(RFAR,0.0)
      RFC=CMPLX(0.0,0.0)
      WRITE(*,*)' '
      WRITE(*,1)TITLE
      WRITE(*,*)'                          DIMENSIONAL VARIABLES'
      WRITE(*,'('' NVAL='',I5)')NVAL
      WRITE(*,'('' XBAR='',1P4E13.5/(8X,4E13.5))')(XBARD(I),I=1,NVAL)
      WRITE(*,'('' UBAR='',1P4E13.5/(8X,4E13.5))')(UBARD(I),I=1,NVAL)
      WRITE(*,2)(VARD(I),HOLDD(I),I=1,20)
      WRITE(6,3)
      WRITE(6,1)TITLE
      WRITE(6,3)
      WRITE(6,*)'                          DIMENSIONAL VARIABLES'
      WRITE(6,'('' NVAL='',I5)')NVAL
      WRITE(6,'('' XBAR='',1P4E13.5/(8X,4E13.5))')(XBARD(I),I=1,NVAL)
      WRITE(6,'('' UBAR='',1P4E13.5/(8X,4E13.5))')(UBARD(I),I=1,NVAL)
      WRITE(6,2)(VARD(I),HOLDD(I),I=1,20)
      WRITE(*,*)'                       NON-DIMENSIONAL VARIABLES'
      WRITE(*,'('' NVAL='',I5)')NVAL
      WRITE(*,'('' XBAR='',1P4E13.5/(8X,4E13.5))')(XBAR(I),I=1,NVAL)
      WRITE(*,'('' UBAR='',1P4E13.5/(8X,4E13.5))')(UBAR(I),I=1,NVAL)
      WRITE(*,'('''    S='',1P2E13.5)')LAMDA,MU
      WRITE(*,2)(VAR(I),RVAR(I),I=1,13)
      WRITE(*,'('''   GF='',1P2E13.5,5X,''    GOX='',2E13.5)')GF,GOX
      WRITE(*,'('''  RFA='',1P2E13.5,5X,''    RFC='',2E13.5)')RFA,RFC
      WRITE(6,3)
      WRITE(6,*)'                       NON-DIMENSIONAL VARIABLES'
      WRITE(6,'('' NVAL='',I5)')NVAL
      WRITE(6,'('' XBAR='',1P4E13.5/(8X,4E13.5))')(XBAR(I),I=1,NVAL)
      WRITE(6,'('' UBAR='',1P4E13.5/(8X,4E13.5))')(UBAR(I),I=1,NVAL)
      WRITE(6,'('''    S='',1P2E13.5)')LAMDA,MU
      WRITE(6,2)(VAR(I),RVAR(I),I=1,13)
      WRITE(6,'('''   GF='',1P2E13.5,5X,''    GOX='',2E13.5)')GF,GOX
      WRITE(6,'('''  RFA='',1P2E13.5,5X,''    RFC='',2E13.5)')RFA,RFC
      WRITE(*,'(A\)')' Hit ENTER to continue '
      READ(*,*)
      RETURN
      END
```

```
      SUBROUTINE READIN
C
C                 INPUT PARAMETERS
C
C
C                 VARIABLES IN COMMON /DIMVAL/      REAL
C
C        ND - PRESSURE INTERACTION INDEX                                 1
C      TAUD - SENSITIVE TIME LAG                              SEC        2
C     DTAUD - DELTA TIME LAG (TOTAL TIME LAG = TAU + DTAU)    SEC        3
C       NRD - ENTHALPY INTERACTION INDEX                                 4
C    LAMDAD - DAMPING OF PERTURBATION                                   5
C       MUD - FREQUENCY OF PERTURBATION                                 6
C     CDIAM - CHAMBER DIAMETER                                 FT        7
C     TDIAM - THROAT DIAMETER                                  FT        8
C      XLCD - X LOCATION OF CHAMBER-NOZZLE INTERFACE           FT        9
C    GAMMAD - RATIO OF SPECIFIC HEATS                                   10
C      RGAS - GAS CONSTANT                              FT^2/SEC^2/°R   11
C      POOD - MAXIMUM PRESSURE AT INJECTION FACE             LBF/FT^2   12
C     MBARD - MEAN COMBUSTION RESPONSE FUNCTION              LBM/SEC    13
C     RBARD - MEAN MIXTURE RATIO                                        14
C    DCSDRD - CHANGE IN CSTAR/CHANGE IN MIXTURE RATIO        FT/SEC     15
C    DHLDRD - CHANGE IN ENTHALPY/CHANGE IN MIXTURE RATIO    FT^2/SEC^2  16
C    RHOLOD - MASS OF LIQUID PER UNIT CHAMBER VOLUME         LBM/FT^3   17
C      ULOD - AXIAL COMPONENT OF LIQUID VELOCITY             FT/SEC     18
C  PBAR(50) - PRESSURE ALONG AXIS                            LBF/FT^2
C  TBAR(50) - TEMPERATURE ALONG AXIS                         °R
C XBARD(50) - X LOCATIONS ALONG AXIS                         FT
C
C
      COMMON /CMPVAL/X1,Y1,Z1,W1,M1,PO,P1,UO,U1,RFH,RFK,RFP,
     *               S,GF,GOX,RFA,RFC
      COMMON /RELVAL/N,TAU,DTAU,NR,RBAR,MBAR,GAMMA,POO,DHLDR,CSTAR,
     *        DCSDR,RHOLO,ULO,LAMDA,MU,TAUT,UBAR(50),XBAR(50),XLC
      COMMON /INTVAL/NVAL
      COMMON /DIMVAL/HOLDD(20),XBARD(50),PBAR(50),TBAR(50)
      COMMON /TITL/TITLE
      REAL MBAR,N,NR,LAMDA,MU,RVAR(15)
      REAL MBARD,ND,NRD,LAMDAD,MUD,HOLD(20)
      COMPLEX S,X1,Y1,Z1,W1,M1,PO,P1,UO,U1,GF,GOX,RFH,RFK,RFP,RFA,RFC
      COMPLEX CVAR(17)
      EQUIVALENCE (N,RVAR(1)),(X1,CVAR(1))
      EQUIVALENCE (ND,HOLD(1)),(TAUD,HOLD(2)),(DTAUD,HOLD(3)),
     *            (NRD,HOLD(4)),(LAMDAD,HOLD(5)),(MUD,HOLD(6)),
     *            (CDIAM,HOLD(7)),(TDIAM,HOLD(8)),(XLCD,HOLD(9)),
     *            (GAMMAD,HOLD(10)),(RGAS,HOLD(11)),(POOD,HOLD(12)),
     *            (MBARD,HOLD(13)),(RBARD,HOLD(14)),(DCSDRD,HOLD(15)),
     *            (DHLDRD,HOLD(16)),(RHOLOD,HOLD(17)),(ULOD,HOLD(18)),
     *            (PCHMB,HOLD(19)),(TCHMB,HOLD(20))
      CHARACTER*8 VAR(20),VARP(20),VARL(20),NAME
      CHARACTER*1 ANS
      CHARACTER*70 TITLE
      DATA IGO/0/
      DATA VAR /'    ND =','  TAUD =',' DTAUD =','   NRD =','LAMDAD =',
     *          '   MUD =',' CDIAM =',' TDIAM =','  XLCD =','GAMMAD =',
     *          '  RGAS =','  POOD =',' MBARD =',' RBARD =','DCSDRD =',
```

```
 *          'DHLDRD =','RHOLOD =','  ULOD =',' PCHMB =',' TCHMB ='/
  DATA VARP/'ND      ','TAUD    ','DTAUD   ','NRD     ','LAMDAD  ',
 *          'MUD     ','CDIAM   ','TDIAM   ','XLCD    ','GAMMAD  ',
 *          'RGAS    ','POOD    ','MBARD   ','RBARD   ','DCSDRD  ',
 *          'DHLDRD  ','RHOLOD  ','ULOD    ','PCHMB   ','TCHMB   '/
  DATA VARL/'nd      ','taud    ','dtaud   ','nrd     ','lamdad  ',
 *          'mud     ','cdiam   ','tdiam   ','xlcd    ','gammad  ',
 *          'rgas    ','p00d    ','mbard   ','rbard   ','dcsdrd  ',
 *          'dhldrd  ','rholod  ','ulod    ','pchmb   ','tchmb   '/
1 FORMAT(16I5)
2 FORMAT(4E15.6)
3 FORMAT(3E15.6)
4 FORMAT(A)
5 FORMAT(' Enter X (ft), P (lbf/ft^2), and T (°R) for point ',
 *         I3,'  ')
6 FORMAT(1P4E15.6)
7 FORMAT(2X,A8,2X,A8,2X,A8,2X,A8,2X,A8)
8 FORMAT(2X,A8,1PE13.5,2X,A8,E13.5,2X,A8,E13.5)
9 FORMAT(1P3E15.6)
  IF(IGO.EQ.1)  THEN
   WRITE(*,'(A\)')' Do you wish to use old data with or without chan
 *ges? Y or N '
   READ(*,4)ANS
   IF(ANS.EQ.'Y'.OR.ANS.EQ.'y')  GO TO 24
  ENDIF
  IGO=1
  WRITE(*,*)' '
  WRITE(*,'(A\)')' Is your rocket input on file? Y OR N '
  READ(*,4)ANS
  IF(ANS.EQ.'Y'.OR.ANS.EQ.'y')  THEN
   WRITE(*,'(A\)')' Does the file need to be rewound? Y OR N '
   READ(*,4)ANS
   IF(ANS.EQ.'Y'.OR.ANS.EQ.'y')  REWIND 5
   READ(5,4,END=99)TITLE
   READ(5,1,END=99)NVAL
   IF(NVAL.EQ.0)  GO TO 99
   READ(5,3)(XBARD(I),PBAR(I),TBAR(I),I=1,NVAL)
   PCHMB=PBAR(1)
   TCHMB=TBAR(1)
   READ(5,2)ND,TAUD,DTAUD,NRD
   READ(5,2)LAMDAD,MUD
   READ(5,2)CDIAM,TDIAM,XLCD
   READ(5,2)GAMMAD,RGAS,POOD
   READ(5,2)MBARD,RBARD
   READ(5,2)DCSDRD,DHLDRD,RHOLOD,ULOD
  ELSE
   WRITE(*,'(A\)')' How many points along centerline? '
   READ(*,*,END=99)NVAL
   IF(NVAL.EQ.0)  GO TO 99
   DO 21 I=1,NVAL
    WRITE(*,5)I
    READ(*,*)XBARD(I),PBAR(I),TBAR(I)
21 CONTINUE
   PCHMB=PBAR(1)
   TCHMB=TBAR(1)
   WRITE(*,*)' Enter Title'
```

```
      READ(*,4)TITLE
      WRITE(*,*)' Enter N (pressure interaction index) and NR',
     *          ' (enthalpy interaction index)'
      READ(*,*)ND,NR
      WRITE(*,*)' Enter TAU (sensitive time lag - sec) and DTAU',
     *          ' (invarient time lag - sec)'
      READ(*,*)TAUD,DTAUD
      WRITE(*,*)' Enter LAMDA and MU (real and imaginary parts',
     *          ' of frequency'
      READ(*,*)LAMDAD,MUD
      WRITE(*,*)' Enter XLCD (length of combustion chamber - ft)'
      READ(*,*)XLCD
      WRITE(*,*)' Enter CDIAM (chamber diameter - ft) and TDIAM',
     *          ' (throat diameter - ft)'
      READ(*,*)CDIAM,TDIAM
      WRITE(*,*)' Enter GAMMA (ratio of specific heats), RGAS',
     *          ' (gas constant - ft^2/sec^2/°R)'
      READ(*,*)GAMMAD,RGAS
      WRITE(*,*)' Enter POO (maximum overpressure - lbf/ft^2)'
      READ(*,*)POOD
      WRITE(*,*)' Enter MBAR (mean combustion response function -',
     *          ' lbm/sec)'
      WRITE(*,*)'   and RBAR (mean mixture ratio)'
      READ(*,*)MBARD,RBARD
      WRITE(*,*)' Enter DCSDR (dc*/dr - ft/sec) and DHLDR',
     *          ' (dh/dr - ft^2/sec^2)'
      READ(*,*)DCSDRD,DHLDRD
      WRITE(*,*)' Enter RHOLO (mass of liquid/unit chamber vol -',
     *          'lbm/ft^3)'
      WRITE(*,*)'   and ULO (axial component of liquid velocity',
     *          ' - ft/sec)'
      READ(*,*)RHOLOD,ULOD
      WRITE(5,4)TITLE
      WRITE(5,1)NVAL
      WRITE(5,9)(XBARD(I),PBAR(I),TBAR(I),I=1,NVAL)
      WRITE(5,6)ND,TAUD,DTAUD,NR
      WRITE(5,6)LAMDAD,MUD
      WRITE(5,6)CDIAM,TDIAM,XLCD
      WRITE(5,6)GAMMAD,RGAS,POOD
      WRITE(5,6)MBARD,RBARD
      WRITE(5,6)DCSDRD,DHLDRD,RHOLOD,ULOD
     ENDIF
     CALL NONDIM(HOLD)
     RETURN
24   CONTINUE
     WRITE(*,'(A\)')'  are there any changes? Y or N '
     READ(*,4)ANS
     IF(ANS.EQ.'N'.OR.ANS.EQ.'n')  THEN
      CALL NONDIM(HOLD)
      RETURN
     ENDIF
     WRITE(*,'(A\)')'  Do you wish to change title? Y or N '
     READ(*,4)ANS
     IF(ANS.EQ.'Y'.OR.ANS.EQ.'y')  THEN
      WRITE(*,*)' Enter Title'
      READ(*,4)TITLE
```

```
      ENDIF
      GO TO 29
   27 CONTINUE
      WRITE(*,*)'   VARIABLE NAMES AND DESCRIPTIONS'
      WRITE(*,*)' '
      WRITE(*,*)' ND     - pressure interaction index'
      WRITE(*,*)' TAUD   - sensitive time lag                    sec'
      WRITE(*,*)' DTAUD  - invarient time lag                    sec'
      WRITE(*,*)' NRD    - enthalpy interaction index'
      WRITE(*,*)' LAMDAD - damping of perturbation'
      WRITE(*,*)' MUD    - frequency of perturbation'
      WRITE(*,*)' CDIAM  - chamber diameter                      ft'
      WRITE(*,*)' TDIAM  - throat diameter                       ft'
      WRITE(*,*)' XLCD   - length of combustion chamber          ft'
      WRITE(*,*)' GAMMAD - ratio of specific heats'
      WRITE(*,*)' RGAS   - gas constant                          ',
     *          '(ft/sec)^2/°R'
      WRITE(*,*)' POOD   - maximum pressure                      ',
     *          'lbf/ft^2'
      WRITE(*,*)' MBARD  - mean combustion response funct.       ',
     *          'lbm/sec'
      WRITE(*,*)' RBARD  - mean mixture ratio'
      WRITE(*,*)' DCSDRD - d(c*)/d(mixture ratio)                ft/sec'
      WRITE(*,*)' DHLDRD - d(enthalpy)/d(mixture ratio)          ',
     *          'ft^2/sec^2'
      WRITE(*,*)' RHOLOD - mass of liquid/unit chamber volume    ',
     *          'lbm/ft^3'
      WRITE(*,*)' ULOD   - axial component of liquid velocity    ft/sec'
      WRITE(*,*)' PCHMB  - chamber pressure at injector          ',
     *          'lbf/ft^2'
      WRITE(*,*)' TCHMB  - chamber temperature                   °R'
      WRITE(*,*)' '
      GO TO 30
   28 CONTINUE
      WRITE(*,*)'   VARIABLE NAMES AND VALUES'
      WRITE(*,*)' '
      WRITE(*,8)(VAR(I),HOLD(I),I=1,20)
   29 CONTINUE
      WRITE(*,*)' '
      WRITE(*,*)' Enter ? to print variable names & descriptions'
      WRITE(*,*)'       # to print variable names & values'
      WRITE(*,*)'       END when all changes have been made'
      WRITE(*,*)' '
   30 CONTINUE
      WRITE(*,'(A\)')'   Enter variable name and new value, END, ?, or #
     * '
      CALL ZREAD(NAME,VALUE)
      IF(NAME.EQ.'?')  GO TO 27
      IF(NAME.EQ.'#')  GO TO 28
      IF(NAME.EQ.'END'.OR.NAME.EQ.'end')  THEN
       CALL NONDIM(HOLD)
       RETURN
      ENDIF
      DO 31 II=1,20
       I=II
       IF(NAME.EQ.VARP(I).OR.NAME.EQ.VARL(I))  GO TO 32
```

```
31 CONTINUE
   WRITE(*,*)'     Invalid name, try again'
   GO TO 27
32 CONTINUE
   HOLD(I)=VALUE
   GO TO 30
99 CONTINUE
   STOP
   END
```

```
SUBROUTINE SETVAL(VAL,ID)
COMMON /DIMVAL/HOLDD(20),XBARD(50),PBAR(50),TBAR(50)
VAL=HOLDD(ID)
RETURN
END
```

```
      SUBROUTINE SETVAR(VAL,ID)
      COMMON /CMPVAL/X1,Y1,Z1,W1,M1,P0,P1,U0,U1,RFH,RFK,RFP,
     *          S,GF,GOX,RFA,RFC
      COMMON /RELVAL/N,TAU,DTAU,NR,RBAR,MBAR,GAMMA,P00,DHLDR,CSTAR,
     *         DCSDR,RHOLO,ULO,LAMDA,MU,TAUT,UBAR(50),XBAR(50),XLC
      COMMON /RESULT/PP,UP,SIGP,FUNB
      COMMON /INTVAL/NVAL
      COMMON /DIMVAL/HOLDD(20),XBARD(50),PBAR(50),TBAR(50)
      REAL MBAR,N,NR,LAMDA,MU,RVAR(13)
      REAL MBARD,ND,NRD,LAMDAD,MUD
      COMPLEX S,X1,Y1,Z1,W1,M1,P0,P1,U0,U1,GF,GOX,RFH,RFK,RFP,RFA,RFC
      COMPLEX FP1,FU1,FSIGP,PP,UP,SIGP,FUNB,CSINH,CCOSH,CVAR(17)
      EQUIVALENCE (N,RVAR(1)),(X1,CVAR(1))
      EQUIVALENCE
     *    (ND,HOLDD(1)),(TAUD,HOLDD(2)),(DTAUD,HOLDD(3)),
     *    (NRD,HOLDD(4)),(LAMDAD,HOLDD(5)),(MUD,HOLDD(6)),
     *    (CDIAM,HOLDD(7)),(TDIAM,HOLDD(8)),(XLCD,HOLDD(9)),
     *    (GAMMAD,HOLDD(10)),(RGAS,HOLDD(11)),(P00D,HOLDD(12)),
     *    (MBARD,HOLDD(13)),(RBARD,HOLDD(14)),(DCSDRD,HOLDD(15)),
     *    (DHLDRD,HOLDD(16)),(RHOLOD,HOLDD(17)),(ULOD,HOLDD(18)),
     *    (PCHMB,HOLDD(19)),(TCHMB,HOLDD(20))
      DATA PI/3.141593/,GC/32.174/
      HOLDD(ID)=VAL
      IF(ID.EQ.1)  THEN
C                          ND
       N=ND
       RETURN
      ENDIF
      IF(ID.EQ.2)  THEN
C                          TAUD
       ASTAR=SQRT(GAMMAD*RGAS*TBAR(1))
       TAU=TAUD*ASTAR/XLCD
       TAUT=TAU+DTAU
       RETURN
      ENDIF
      IF(ID.EQ.3)  THEN
C                          DTAUD
       ASTAR=SQRT(GAMMAD*RGAS*TBAR(1))
       DTAU=DTAUD*ASTAR/XLCD
       TAUT=TAU+DTAU
       RETURN
      ENDIF
      IF(ID.EQ.4)  THEN
C                          NRD
       NR=NRD
       RETURN
      ENDIF
      IF(ID.EQ.5)  THEN
C                          LAMDAD
       ASTAR=SQRT(GAMMAD*RGAS*TBAR(1))
       LAMDA=LAMDAD*XLCD/ASTAR
       S=CMPLX(LAMDA,MU)
       RETURN
      ENDIF
      IF(ID.EQ.6)  THEN
C                          MUD
```

```
      ASTAR=SQRT(GAMMAD*RGAS*TBAR(1))
      MU=MUD*XLCD*PI/ASTAR
      S=CMPLX(LAMDA,MU)
      RETURN
     ENDIF
     IF(ID.EQ.7)  THEN
C                          CDIAM
      CAREA=0.25*PI*CDIAM**2
      ASTAR=SQRT(GAMMAD*RGAS*TBAR(1))
      DO 21 I=1,NVAL
       RHOBAR=PBAR(I)*GC/(RGAS*TBAR(I))
       UBARD=MBARD/(RHOBAR*CAREA)
       UBAR(I)=UBARD/ASTAR
   21 CONTINUE
      RETURN
     ENDIF
     IF(ID.EQ.8)  THEN
C                          TDIAM
      TAREA=0.25*PI*TDIAM**2
      ASTAR=SQRT(GAMMAD*RGAS*TBAR(1))
      CSTARD=PBAR(NVAL)*TAREA*GC/MBARD
      CSTAR=CSTARD/ASTAR
      RETURN
     ENDIF
     IF(ID.EQ.9)  THEN
C                          XLCD
      ASTAR=SQRT(GAMMAD*RGAS*TBAR(1))
      TAU=TAUD*ASTAR/XLCD
      DTAU=DTAUD*ASTAR/XLCD
      TAUT=TAU+DTAU
      LAMDA=LAMDAD*XLCD/ASTAR
      MU=MUD*XLCD*PI/ASTAR
      S=CMPLX(LAMDA,MU)
      DO 22 I=1,NVAL
       XBAR(I)=XBARD(I)/XLCD
   22 CONTINUE
      RETURN
     ENDIF
     IF(ID.EQ.10)  THEN
C                          GAMMAD
      GAMMA=GAMMAD
      CAREA=0.25*PI*CDIAM**2
      TAREA=0.25*PI*TDIAM**2
      ASTAR=SQRT(GAMMAD*RGAS*TBAR(1))
      TAU=TAUD*ASTAR/XLCD
      DTAU=DTAUD*ASTAR/XLCD
      TAUT=TAU+DTAU
      LAMDA=LAMDAD*XLCD/ASTAR
      MU=MUD*XLCD*PI/ASTAR
      S=CMPLX(LAMDA,MU)
      ULO=ULOD/ASTAR
      DCSDR=DCSDRD/ASTAR
      RHOB1=PBAR(1)*GC/(RGAS*TBAR(1))
      MBAR=MBARD/(RHOB1*ASTAR*CAREA/XLCD)
      CSTARD=PBAR(NVAL)*TAREA*GC/MBARD
      CSTAR=CSTARD/ASTAR
```

```
      DO 23 I=1,NVAL
       RHOBAR=PBAR(I)*GC/(RGAS*TBAR(I))
       UBARD=MBARD/(RHOBAR*CAREA)
       UBAR(I)=UBARD/ASTAR
   23 CONTINUE
      RETURN
     ENDIF
     IF(ID.EQ.11)  THEN
C                           RGAS
      CAREA=0.25*PI*CDIAM**2
      TAREA=0.25*PI*TDIAM**2
      ASTAR=SQRT(GAMMAD*RGAS*TBAR(1))
      TAU=TAUD*ASTAR/XLCD
      DTAU=DTAUD*ASTAR/XLCD
      TAUT=TAU+DTAU
      LAMDA=LAMDAD*XLCD/ASTAR
      MU=MUD*XLCD*PI/ASTAR
      S=CMPLX(LAMDA,MU)
      ULO=ULOD/ASTAR
      DCSDR=DCSDRD/ASTAR
      RHOB1=PBAR(1)*GC/(RGAS*TBAR(1))
      RHOLO=RHOLOD/RHOB1
      MBAR=MBARD/(RHOB1*ASTAR*CAREA/XLCD)
      CSTARD=PBAR(NVAL)*TAREA*GC/MBARD
      CSTAR=CSTARD/ASTAR
      DO 24 I=1,NVAL
       RHOBAR=PBAR(I)*GC/(RGAS*TBAR(I))
       UBARD=MBARD/(RHOBAR*CAREA)
       UBAR(I)=UBARD/ASTAR
   24 CONTINUE
      RETURN
     ENDIF
     IF(ID.EQ.12)  THEN
C                           POOD
      POO=POOD/PCHMB
      RETURN
     ENDIF
     IF(ID.EQ.13)  THEN
C                           MBARD
      CAREA=0.25*PI*CDIAM**2
      TAREA=0.25*PI*TDIAM**2
      ASTAR=SQRT(GAMMAD*RGAS*TBAR(1))
      RHOB1=PBAR(1)*GC/(RGAS*TBAR(1))
      MBAR=MBARD/(RHOB1*ASTAR*CAREA/XLCD)
      CSTARD=PBAR(NVAL)*TAREA*GC/MBARD
      CSTAR=CSTARD/ASTAR
      DO 25 I=1,NVAL
       RHOBAR=PBAR(I)*GC/(RGAS*TBAR(I))
       UBARD=MBARD/(RHOBAR*CAREA)
       UBAR(I)=UBARD/ASTAR
   25 CONTINUE
      RETURN
     ENDIF
     IF(ID.EQ.14)  THEN
C                           RBARD
      RBAR=RBARD
```

```
      RETURN
     ENDIF
     IF(ID.EQ.15)  THEN
C                          DCSDRD
      ASTAR=SQRT(GAMMAD*RGAS*TBAR(1))
      DCSDR=DCSDRD/ASTAR
      RETURN
     ENDIF
     IF(ID.EQ.16)  THEN
C                          DHLDRD
      DHLDR=DHLDRD
      RETURN
     ENDIF
     IF(ID.EQ.17)  THEN
C                          RHOLOD
      RHOB1=PBAR(1)*GC/(RGAS*TBAR(1))
      RHOLO=RHOLOD/RHOB1
      RETURN
     ENDIF
     IF(ID.EQ.18)  THEN
C                          ULOD
      ASTAR=SQRT(GAMMAD*RGAS*TBAR(1))
      ULO=ULOD/ASTAR
      RETURN
     ENDIF
     IF(ID.EQ.19)  THEN
C                          PCHMB
      CAREA=0.25*PI*CDIAM**2
      TAREA=0.25*PI*TDIAM**2
      ASTAR=SQRT(GAMMAD*RGAS*TBAR(1))
      FAC=PCHMB/PBAR(1)
      DO 26 I=1,NVAL
       PBAR(I)=FAC*PBAR(I)
       RHOBAR=PBAR(I)*GC/(RGAS*TBAR(I))
       UBARD=MBARD/(RHOBAR*CAREA)
       UBAR(I)=UBARD/ASTAR
   26 CONTINUE
      CSTARD=PBAR(NVAL)*TAREA*GC/MBARD
      CSTAR=CSTARD/ASTAR
      RHOB1=PBAR(1)*GC/(RGAS*TBAR(1))
      RHOLO=RHOLOD/RHOB1
      MBAR=MBARD/(RHOB1*ASTAR*CAREA/XLCD)
      POO=POOD/PCHMB
      RETURN
     ENDIF
     IF(ID.EQ.20)  THEN
C                          TCHMB
      DO 27 I=1,NVAL
       TBAR(I)=FAC*TBAR(I)
   27 CONTINUE
      CAREA=0.25*PI*CDIAM**2
      TAREA=0.25*PI*TDIAM**2
      ASTAR=SQRT(GAMMAD*RGAS*TBAR(1))
      FAC=TCHMB/TBAR(1)
      DO 28 I=1,NVAL
       RHOBAR=PBAR(I)*GC/(RGAS*TBAR(I))
```

```
      UBARD=MBARD/(RHOBAR*CAREA)
      UBAR(I)=UBARD/ASTAR
28 CONTINUE
    CSTARD=PBAR(NVAL)*TAREA*GC/MBARD
    CSTAR=CSTARD/ASTAR
    RHOB1=PBAR(1)*GC/(RGAS*TBAR(1))
    RHOLO=RHOLOD/RHOB1
    MBAR=MBARD/(RHOB1*ASTAR*CAREA/XLCD)
   ENDIF
   RETURN
   END
```

```
      SUBROUTINE ZREAD(NAME,VALUE)
C       SUBROUTINE TO READ NAMES AND VALUES
      CHARACTER*1 NAME(8)
      CHARACTER*1 CARD(80),PLUS,MINUS,PERIOD,LE,E,NUMBER(10)
      CHARACTER*1 LEND(3),CEND(3),POUND,QUEST,BLK,COMMA
      CHARACTER*80 DCARD
      EQUIVALENCE (CARD(1),DCARD)
      DATA PLUS/'+'/,MINUS/'-'/,PERIOD/'.'/,LE/'e'/,E/'E'/,BLK/' '/
      DATA NUMBER/'0','1','2','3','4','5','6','7','8','9'/,COMMA/','/
      DATA LEND/'e','n','d'/,CEND/'E','N','D'/,POUND/'#'/,QUEST/'?'/
    1 FORMAT(A)
      DO 21 I=1,8
       NAME(I)=BLK
   21 CONTINUE
      READ(*,1)DCARD
      IF(CARD(1).EQ.POUND)  THEN
       NAME(1)=POUND
       RETURN
      ENDIF
      IF(CARD(1).EQ.QUEST)  THEN
       NAME(1)=QUEST
       RETURN
      ENDIF
      DO 22 I=1,3
       IF(CARD(I).NE.LEND(I).AND.CARD(I).NE.CEND(I))  GO TO 23
       NAME(I)=CEND(I)
   22 CONTINUE
      RETURN
   23 CONTINUE
      DO 24 I=1,8
       II=I
       IF(CARD(I).EQ.BLK.OR.CARD(I).EQ.COMMA)  GO TO 25
       NAME(I)=CARD(I)
   24 CONTINUE
   25 CONTINUE
      DO 26 I=II,80
       ID=I
       IF(CARD(I).NE.BLK.AND.CARD(I).NE.COMMA)  GO TO 27
   26 CONTINUE
      VALUE=0.0
      WRITE(*,*)'   NO VALUE GIVEN, ZERO ASSUMED'
      RETURN
   27 CONTINUE
      SIGN=1.0
      IF(CARD(ID).EQ.MINUS)  THEN
       SIGN=-1.0
       ID=ID+1
      ELSEIF(CARD(ID).EQ.PLUS)  THEN
       ID=ID+1
      ENDIF
      WHOLE=0.0
      DO 30 I=ID,80
       II=I
       IF(CARD(I).EQ.PERIOD)  GO TO 31
       IF(CARD(I).EQ.PLUS)  GO TO 36
       IF(CARD(I).EQ.MINUS)  GO TO 36
```

```
      IF(CARD(I).EQ.E.OR.CARD(I).EQ.LE)  GO TO 35
      DO 28 J=1,10
       JJ=J-1
       IF(CARD(I).EQ.NUMBER(J))  GO TO 29
28    CONTINUE
      VALUE=SIGN*WHOLE
      IF(CARD(I).EQ.BLK)  RETURN
      WRITE(*,*)'  INPUT ERROR, VALUE SET TO ZERO'
      VALUE=0.0
      RETURN
29    CONTINUE
      WHOLE=WHOLE*10.0+JJ
30    CONTINUE
      VALUE=SIGN*WHOLE
      RETURN
31    CONTINUE
      ID=II+1
      FRACT=0.0
      ICOUNT=0
      DO 34 I=ID,80
       ICOUNT=ICOUNT+1
       II=I
       IF(CARD(I).EQ.PERIOD)  THEN
        WRITE(*,*)'  INPUT ERROR, VALUE SET TO ZERO'
        VALUE=0.0
        RETURN
       ENDIF
       IF(CARD(I).EQ.PLUS)  GO TO 36
       IF(CARD(I).EQ.MINUS)  GO TO 36
       IF(CARD(I).EQ.E.OR.CARD(I).EQ.LE)  GO TO 35
       DO 32 J=1,10
        JJ=J-1
        IF(CARD(I).EQ.NUMBER(J))  GO TO 33
32    CONTINUE
       VALUE=SIGN*(WHOLE+FRACT)
       IF(CARD(I).EQ.BLK)  RETURN
       WRITE(*,*)'  INPUT ERROR, VALUE SET TO ZERO'
       VALUE=0.0
       RETURN
33    CONTINUE
       FRACT=FRACT+JJ/10.0**ICOUNT
34    CONTINUE
      VALUE=SIGN*(WHOLE+FRACT)
      RETURN
35    CONTINUE
      II=II+1
36    CONTINUE
      VALUE=SIGN*(WHOLE+FRACT)
      SIGN=1.0
      IF(CARD(II).EQ.MINUS)  THEN
       SIGN=-1.0
       II=II+1
      ELSEIF(CARD(II).EQ.PLUS)  THEN
       II=II+1
      ENDIF
      WHOLE=0.0
```

```
      DO 39 I=II,80
       DO 37 J=1,10
        JJ=J-1
        IF(CARD(I).EQ.NUMBER(J))  GO TO 38
37    CONTINUE
       VALUE=VALUE*10.0**(SIGN*WHOLE)
       IF(CARD(I).EQ.BLK)  RETURN
       WRITE(*,*)'  INPUT ERROR, VALUE SET TO ZERO'
       VALUE=0.0
       RETURN
38    CONTINUE
       WHOLE=WHOLE*10.0+JJ
39    CONTINUE
      VALUE=VALUE*10.0**(SIGN*WHOLE)
      RETURN
      END
```

APPENDIX V

FORTRAN Listing of FEEDLINE

Wilbur C. Armstrong

Listing of Program FEEDLINE

```
C         PROGRAM TO CALCULATE ADMITTANCE COEFFICIENTS FROM TANK
C         THROUGH FEEDLINES TO ROCKET CHAMBER
C
C           A = VELOCITY OF SOUND IN THE FLUID (FT/S)
C     AREA(I) = AREA OF FEED LINE (FT^2)
C        CMAN = CAPACITANCE ASSOCIATED WITH THE MANIFOLD
C       CTANK = CAPACITANCE ASSOCIATED WITH THE FUEL TANK
C        DENS = DENSITY OF FLUID (LB/FT^3)
C      DIA(I) = DIAMETER OF LINE
C       DPROR = PRESSURE DROP ACROSS ORIFICES (LB/FT^2)
C        G(I) = OVERALL ADMITTANCE AS SEEN FROM ENGINE
C        GRAV = ACCELERATION OF GRAVITY (32.2 FT/S^2)
C       INERT = INERTANCE DUE TO DUCT CURVATURE
C        KMAN = BULK MODULUS OF FLUID AT MANIFOLD PRESSURE (LB/FT^2)
C       KTANK = BULK MODULUS OF FLUID IN TANK (LB/FT^2)
C        L(I) = LENGTH OF LINE (FT)
C       LFLOW = FLOW RATE OF FLUID THROUGH THE LINE(LB/S)
C       PCHMB = PRESSURE INSIDE COMBUSTION CHAMBER (LB/FT^2)
C       RATIO = RATIO OF INNER RADIUS TO OUTER RADIUS
C       TFLOW = TOTAL FLOW RATE OF FLUID INSIDE ENGINE(LB/S)
C          TL = TIME CONSTANT
C         VOL = VOLUME OF FUEL TANK (FT^3)
C       VOLMF = VOLUME OF MANIFOLD (FT^3)
C       ZLINE = IMPEDANCE ASSOCIATED WITH THE FEED LINE
C         ZOR = IMPEDANCE ASSOCIATED WITH THE ORIFICES
C
      COMPLEX G(76),CTANH,G1,S
      REAL AREA(75),DIA(75),L(75),INERT,INRAD,KMAN,KTANK,LBEND,LFLOW,
     *     LFREQ,LPRME,MAG,NEWLN,MAG1
      INTEGER N(75),SECTN(75),PTS,RSPON,SECT,SEGMN
      CHARACTER ANS*1
      CHARACTER*20 TITLE,TITLF,TITLO
      DATA GRAV/32.2/,PI/3.1415927/
      DATA TITLF/'     FUEL LINE      '/
      DATA TITLO/'    OXYGEN LINE     '/
    1 FORMAT(E15.6)
    2 FORMAT(I5,3E15.6)
    3 FORMAT(1P4E15.6)
    4 FORMAT(1PE13.5,'  (',E12.5,',',E12.5,')  (',E12.5,',',E12.5,')')
    5 FORMAT(/'       FREQ',8X,'FREQ-NORM',9X,'G(R)',11X,'G(I)'/)
    6 FORMAT(/2x,'"   FREQ"',7X,'"FREQ-NORM"',5X,'"   /G1/"',6X,
     *        '"   /G/"'/)
    7 FORMAT('"',A,'"')
    8 FORMAT(1PE15.6)
    9 FORMAT(I5,1P3E15.6)
      WRITE(*,'(A\)')'  IS THIS SETUP FOR FUEL OR OXIDIZER? ENTER F OR O
     *. '
      READ(*,'(A)')ANS
      IF(ANS.EQ.'F'.OR.ANS.EQ.'f')  THEN
       TITLE=TITLF
       OPEN(UNIT=11,FILE='FUEL.DAT',STATUS='OLD')
      ELSE
       TITLE=TITLO
```

```
 OPEN(UNIT=11,FILE='LOX.DAT',STATUS='OLD')
ENDIF
OPEN(UNIT=12,FILE='ADMIT.DAT',STATUS='NEW')
OPEN(UNIT=7,FILE='ADMIT.PRN',STATUS='NEW')
WRITE(*,'(A\)')'IF THERE IS DATA STORED ENTER YES '
READ(*,'(A)')ANS
IF(ANS.EQ.'N'.OR.ANS.EQ.'n')  THEN
 RSPON=3
 GO TO 21
ENDIF
READ(11,1)DENS
READ(11,1)TFLOW
READ(11,1)VOLMF
READ(11,1)KMAN
READ(11,1)PCHMB
READ(11,1)DPROR
READ(11,1)VOL
READ(11,1)LFLOW
READ(11,1)KTANK
READ(11,1)A
READ(11,1)CTANK
READ(11,1)CMAN
READ(11,2)SEGMN
READ(11,2)(SECTN(I),L(I),AREA(I),DIA(I),I=1,SEGMN)
C
C     The first stage in this program is to define the parameters then
C     we will begin the initial calculations. Because these parameters
C     are as likely to change as not, a provision is made to update the
C     parameters if necessary.
C
WRITE(12,*)' '
WRITE(12,*)TITLE
WRITE(12,*)' '
WRITE(12,*)'PRESENT CONDITIONS ARE AS FOLLOWS:'
WRITE(12,*)'DENS=',DENS
WRITE(12,*)'FUEL TANK VOLUME=',VOL
WRITE(12,*)'MANIFOLD VOLUME=',VOLMF
WRITE(12,*)'ENGINE CHAMBER PRESSURE=',PCHMB
WRITE(12,*)'PRESSURE DROP ACROSS ORIFICE=',DPROR
WRITE(12,*)'TOTAL FLOW RATE=',TFLOW
WRITE(12,*)'LINE FLOW RATE=',LFLOW
WRITE(12,*)'BULK MOD. OF FUEL TANK=',KTANK
WRITE(12,*)'BULK MOD. OF MANIFOLD=',KMAN
WRITE(12,*)'VELOCITY OF SOUND IN FLUID=',A
WRITE(12,*)'CAPACITANCE OF FUEL TANK=',CTANK
WRITE(12,*)'CAPACITANCE OF MANIFOLD=',CMAN
WRITE(12,*)' STATUS   LENGTH          AREA          DIAMETER'
WRITE(12,9)(SECTN(I),L(I),AREA(I),DIA(I),I=1,SEGMN)
WRITE(12,*)' '
WRITE(*,*)' '
WRITE(*,*)TITLE
WRITE(*,*)' '
WRITE(*,*)'PRESENT CONDITIONS ARE AS FOLLOWS:'
WRITE(*,*)'DENS=',DENS
WRITE(*,*)'FUEL TANK VOLUME=',VOL
WRITE(*,*)'MANIFOLD VOLUME=',VOLMF
```

```
      WRITE(*,*)'ENGINE CHAMBER PRESSURE=',PCHMB
      WRITE(*,*)'PRESSURE DROP ACROSS ORIFICE=',DPROR
      WRITE(*,*)'TOTAL FLOW RATE=',TFLOW
      WRITE(*,*)'LINE FLOW RATE=',LFLOW
      WRITE(*,*)'BULK MOD. OF FUEL TANK=',KTANK
      WRITE(*,*)'BULK MOD. OF MANIFOLD=',KMAN
      WRITE(*,*)'VELOCITY OF SOUND IN FLUID=',A
      WRITE(*,*)'CAPACITANCE OF FUEL TANK=',CTANK
      WRITE(*,*)'CAPACITANCE OF MANIFOLD=',CMAN
      WRITE(*,*)' STATUS   LENGTH          AREA          DIAMETER'
      WRITE(*,9)(SECTN(I),L(I),AREA(I),DIA(I),I=1,SEGMN)
      WRITE(*,*)' If revisions on the design have been made'
      WRITE(*,*)' (changes in fuel, pipelength,diameter,bends, etc.)'
      WRITE(*,'(A\)')' Please enter yes for revisions or no to continue.
     * '
      READ(*,'(A)')ANS
      IF(ANS.EQ.'N'.OR.ANS.EQ.'n')  GO TO 28
      WRITE(*,*)'CHANGES IN FUEL LINE SPECIFICATIONS ENTER 1'
      WRITE(*,*)'CHANGES IN ENGINE,TANK, AND FUEL SPECS. ENTER 2'
      WRITE(*,*)'CHANGES FOR BOTH FUEL LINE AND ENGINE SPECS. ENTER 3'
      READ(*,*)RSPON
      REWIND 11
      IF(RSPON.EQ.2)  GO TO 27
C
C    AT THIS POINT, YOU NEED TO FURNISH ME WITH SOME INFORMATION, LIKE
C    WHAT ARE THE LENGTHS AND AREAS OF THE SEGMENTS, HOW MANY BENDS ARE
C    THERE, WHERE ARE THEY LOCATED ON THE PIPE, WHAT IS THE ANGLE THAT
C    THEY MAINTAIN, WHAT IS THE RADIUS FOR THE BEND, AND ARE THERE
C    VALVES, WHERE ARE THEY LOCATED, ETC.
C
   21 CONTINUE
C
C ***    REMEMBER TO RESOLVE AMBIGUITY CONCERNING THE LENGTHS FOR
C     MANIFOLDS AND ORIFICES, SINCE THE PARAMETERS FOR THEIR
C     IMPEDENCE WILL BE INPUT.  2-22-90
C
      WRITE(*,'(A\)')'HOW MANY SEGMENTS IS THE PIPE BROKEN INTO? '
      READ(*,*)SEGMN
      L(I)=0.0
      AREA(I)=0.0
      DIA(I)=0.0
      SECTN(I)=0
      DO 26 I=1,SEGMN
       WRITE(*,*)'SPECIFY 0 for bend, 1 FOR STRAIGHT PIPE,'
       WRITE(*,*)' 2 FOR MANIFOLD, OR 3 FOR ORIFICE'
       WRITE(*,*)'IN ORDER,SPECIFY LENGTH,AREA, AND DIAMETER OF'
       WRITE(*,*)'EACH SEGMENT'
       WRITE(*,*)'EX. INPUT: 1,LENGTH,AREA,DIAMETER'
       WRITE(*,*)'   OUTPUT: 1 LENGTH AREA DIAMETER'
       READ(*,*) SECT,VALUE,AREAB,DIME
       IF(SECT.EQ.0)  GO TO 24
       IF(SECT.EQ.1)  GO TO 22
       IF(SECT.EQ.2)  THEN
        WRITE(*,'(A\)')'DENSITY OF FLUID '
        READ(*,*)DENS
        WRITE(*,'(A\)')'TOTAL FLOW RATE '
```

```
      READ(*,*)TFLOW
      WRITE(*,'(A\)')'VOLUME FLUID IN MANIFOLD '
      READ(*,*)VOLMF
      WRITE(*,'(A\)')'BULK MODULUS OF FLUID IN MANIFOLD '
      READ(*,*)KMAN
      WRITE(*,'(A\)')'ENGINE CHAMBER PRESSURE '
      READ(*,*)PCHMB
      CMAN=(DENS*VOLMF*PCHMB)/(KMAN*TFLOW)
      WRITE(*,*)'MANIFOLD CAPACITANCE=',CMAN
      VALUE=0.0
      AREAB=0.0
      DIME=0.0
      SECT=2
      GO TO 23
     ELSEIF(SECT.EQ.3)  THEN
      WRITE(*,'(A\)')'PRESSURE DROP ACROSS ORIFICE '
      READ(*,*)DPROR
      GO TO 22
     ENDIF
   22 CONTINUE
     L(I)=VALUE
     AREA(I)=AREAB
     SECTN(I)=SECT
     DIA(I)=DIME
   23 CONTINUE
     WRITE(*,*)SECTN(I),L(I),AREA(I),DIA(I)
     GO TO 26
   24 CONTINUE
C
C       THIS PART OF THE PROGRAM FINDS NEW LENGTHS AND AREAS FOR
C       SECTIONS OF THE PIPE WITH BENDS IN THEM BY MAKING USE OF
C       THE IDEA THAT A BEND CAN BE MODELED AS A STRAIGHT PIPE WITH
C       DECREASED AREA AND INCREASED LENGTH. THIS SECTION LOOKS AT
C       TWO CASES: MITERED AND ELBOW BENDS.
C
     WRITE(*,'(A\)')'PLEASE ENTER LENGTH OF BEND (E.G. 1/3 LENGTH) '
     READ(*,*)LBEND
     WRITE(*,*)'THE INNER AND OUTER RADIUS OF THE BEND FROM THE LOCUS'
     WRITE(*,*)' AN INNER RADIUS OF 0.0 DENOTES A MITERED BEND'
     READ(*,*)INRAD,OTRAD
     WRITE(*,'(A\)')'ENTER THE INITIAL AREA OF THE PIPE '
     READ(*,*)ARBND
     WRITE(*,*)LBEND,INRAD,'FT',OTRAD,'FT',ARBND,'FT^2'
     WRITE(*,'(A\)')'ENTER THE ANGLE OF THE BEND '
     READ(*,*)BEND
     RATIO=INRAD/OTRAD
     X=RATIO
     WRITE(*,*)BEND,'DEGREES'
     CALL GINERT(BEND,X,Y)
   25 CONTINUE
     INERT=(Y*(OTRAD-INRAD))/ARBND
     LPRME=LBEND/ARBND
     NEWLN=LPRME+INERT
     GAMMA=NEWLN/LPRME
     L(I)=GAMMA*LBEND
     AREA(I)=ARBND/SQRT(GAMMA)
```

```
   26 CONTINUE
   27 CONTINUE
      IF(RSPON.EQ.1)  GO TO 28
       IF(RSPON.EQ.2)  THEN
        WRITE(*,'(A\)')'ENTER FUEL DENSITY '
        READ(*,*)DENS
        WRITE(*,'(A\)')'ENTER TOTAL FLOW RATE INSIDE ENGINE '
        READ(*,*)TFLOW
        WRITE(*,'(A\)')'ENTER MANIFOLD VOLUME '
        READ(*,*)VOLMF
        WRITE(*,'(A\)')'ENTER BULK MODULUS OF FLUID INSIDE MANIFOLD '
        READ(*,*)KMAN
        WRITE(*,'(A\)')'ENTER CHAMBER PRESSURE IN ENGINE '
        READ(*,*)PCHMB
        WRITE(*,'(A\)')'ENTER PRESSURE DROP ACROSS ORIFICE '
        READ(*,*)DPROR
       ENDIF
       WRITE(*,'(A\)')'ENTER FUEL TANK VOLUME '
       READ(*,*)VOL
       WRITE(*,'(A\)')'ENTER FLOW RATE INSIDE LINE '
       READ(*,*)LFLOW
       WRITE(*,'(A\)')'ENTER BULK MODULUS OF FLUID INSIDE TANK '
       READ(*,*)KTANK
       WRITE(*,'(A\)')'ENTER VELOCITY OF SOUND IN FLUID '
       READ(*,*)A
       CTANK=(DENS*VOL*PCHMB)/(KTANK*TFLOW)
       CMAN=(DENS*VOLMF*PCHMB)/(KMAN*TFLOW)
       WRITE(11,8)DENS
       WRITE(11,8)TFLOW
       WRITE(11,8)VOLMF
       WRITE(11,8)KMAN
       WRITE(11,8)PCHMB
       WRITE(11,8)DPROR
       WRITE(11,8)VOL
       WRITE(11,8)LFLOW
       WRITE(11,8)KTANK
       WRITE(11,8)A
       WRITE(11,8)CTANK
       WRITE(11,8)CMAN
       WRITE(11,9)SEGMN
       WRITE(11,9)SECTN(I),L(I),AREA(I),DIA(I)
C
C       THIS SECTION COMPUTES THE NEW ADMITTANCE OVER VARYING FREQUENCIES.
C
   28 CONTINUE
      WRITE(*,*)'ENTER RANGE OF FREQUENCIES EX: 1,2,10'
      WRITE(*,*)'LOW FREQ=1 HIGH FREQ=2 #PTS=10'
      READ(*,*)LFREQ,HFREQ,PTS
      IF(PTS.LE.1)  GO TO 32
C
C       THIS SECTION WILL COMPUTE THE ADMITTANCE RATIO FOR THE FUEL TANK
C       AND THEN IT WILL COMPUTE THE ADMITTANCE RATIOS FOR EACH SEGMENT,
C       SINCE THERE ARE L(I) I=1,SEGMN LENGTHS, THEN THERE WILL BE AT LEAST
C       AS MANY ADMITTANCE RATIOS, THEREFORE I AM SETTING UP AN ARRAY FOR
C       EACH LENGTH L(I) HAVING AN ADMITTANCE RATIO G(I).
C
```

```
      SSIZE=(HFREQ-LFREQ)/(PTS-1)
      WRITE(*,*)'STEP SIZE',SSIZE
      WRITE(*,*) ' '
      WRITE(*,*) '  FREQUENCY                    G                          G1'
      WRITE(*,*) ' '
      WRITE(12,5)
      WRITE(7,7)TITLE
      WRITE(7,6)
      ZTOP=A*TFLOW/(GRAV*PCHMB)
      ZOR=2.0*DPROR*TFLOW/(LFLOW*PCHMB)
      TLT=0.0
      DO 29 I=1,SEGMN
      TLT=TLT+L(I)
   29 CONTINUE
      TLT=TLT/(PI*A)
      DO 31 K=1,PTS
       W=LFREQ+SSIZE*(K-1)
       S=CMPLX(0.0,W)
       G(1)=CTANK*S
       G1=G(1)+1.0
       DO 30 I=2,SEGMN+1
        IF(SECTN(I-1).EQ.1)  THEN
         ZLINE=ZTOP/AREA(I-1)
         TL=L(I-1)/A
         G(I)=(1.0+CTANH(S*TL)/(G(I-1)*ZLINE))/(1.0+G(I-1)*ZLINE*
     *        CTANH(S*TL))
        ELSEIF(SECTN(I-1).EQ.2)  THEN
         G(I)=1.0+CMAN*S/G(I-1)
        ELSEIF(SECTN(I-1).EQ.3)  THEN
         G(I)=1.0/(1.0+ZOR*G(I-1))
        ENDIF
        G1=G1*G(I)
        G(I)=G(I)*G(I-1)
   30 CONTINUE
       MAG=CABS(G(SEGMN+1))
       MAG1=CABS(G1)
       WN=W*TLT
       WRITE(12,3)W,WN,G(SEGMN+1)
       WRITE(7,3)W,WN,MAG1,MAG
       WRITE(*,4)W,G(SEGMN+1),G1
   31 CONTINUE
   32 CONTINUE
      WRITE(*,'(A\)')'TO CONTINUE ENTER YES '
      READ(*,'(A)')ANS
      IF(ANS.EQ.'Y'.OR.ANS.EQ.'y')  GO TO 28
      STOP
      END
```

```
      COMPLEX FUNCTION CTANH(S)
      COMPLEX CCOSH,CSINH,S
      CTANH=CSINH(S)/CCOSH(S)
      RETURN
      END


      COMPLEX FUNCTION CSINH(S)
      COMPLEX S
      REAL LAMDA, MU
      LAMDA=REAL(S)
      MU=AIMAG(S)
      SINHR=SINH(LAMDA)*COS(MU)
      SINHI=COSH(LAMDA)*SIN(MU)
      CSINH=CMPLX(SINHR,SINHI)
      RETURN
      END


      COMPLEX FUNCTION CCOSH(S)
      COMPLEX S
      REAL LAMDA, MU
      LAMDA=REAL(S)
      MU=AIMAG(S)
      COSHR=COSH(LAMDA)*COS(MU)
      COSHI=SINH(LAMDA)*SIN(MU)
      CCOSH=CMPLX(COSHR,COSHI)
      RETURN
      END


      SUBROUTINE GINERT(BEND,X,Y)
C
C    A0,A1,A2 = INERTANCE FIT FOR GIVEN BEND ANGLE
C        B(I) = COEFFICIENT ARRAY OF INERTANCE FIT
C        BEND = ANGLE OF BEND (DEGREES)
C           X = RATIO OF INNER RADIUS TO OUTER RADIUS (RI/RO)
C           Y = INERTANCE
C
      DIMENSION B(3)
      DATA B/0.0,0.7877014E-02,-0.2814679E-04/
      A=B(1)+(B(2)+B(3)*BEND)*BEND
      Y=A*(X-1.0)**2
      RETURN
      END
```

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