

Long-Term Evolution of a Planetesimal Swarm in the Vicinity of a Protoplanet

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Many models of planet formation involve scenarios in which one or a few large protoplanets interact with a swarm of much smaller planetesimals. In such scenarios, three-body perturbations by the protoplanet as well as mutual collisions and gravitational interactions between the swarm bodies are important in determining the velocity distribution of the swarm. We are developing a model to examine the effects of these processes on the evolution of a planetesimal swarm. The model consists of a combination of numerical integrations of the gravitational influence of one (or a few) massive protoplanets on swarm bodies together with a statistical treatment of the interactions between the planetesimals. Integrating the planetesimal orbits allows us to take into account effects that are difficult to model analytically or statistically, such as three-body collision cross-sections and resonant perturbations by the protoplanet, while using a statistical treatment for the particle-particle interactions allows us to use a large enough sample to obtain meaningful results.

Our model follows the interactions between planetesimals and protoplanets in a deterministic manner. Planetesimal orbits are numerically integrated using a predictor-corrector integrator on the three (or in some cases more) body problem. At intervals throughout the calculation, the orbital elements of the individual bodies will be used to develop a swarm density profile binned in both radius and height above the midplane. The associated velocity and size distributions will be recorded for each bin. Such a distribution can, in turn, be used to statistically compute the size and velocity evolution of individual planetesimals as they are affected by inelastic collisions and gravitational scattering with other swarm bodies. This will be accomplished using analytic and semi-analytic formulae analogous to those developed by Stewart and Wetherill (1988, *Icarus* 74, 542), but generalized to include the situation where eccentricities can be much larger than inclinations. In such a manner, we will be able to follow the growth of the protoplanet(s) and the evolution of the planetesimal swarm.

Thus far we have done simulations in which the planetesimals are only acted upon by the gravity of the star and the protoplanet. We have looked at both the low initial random velocity limit ($i_H = 0, e_H = 0$), and an intermediate initial velocity ($i_H = 1.0, e_H = 0.5$), where $i_H = \frac{ia}{R_H}$, $e_H = \frac{ea}{R_H}$, R_H is the protoplanet's Hill Sphere radius, and $e, i,$ and a are the particle's eccentricity, inclination, and semi-major axis, respectively. Our results are as follows:

- 1) The long-term collision probability varies slowly with protoplanet radius. Two different assumed protoplanet radii, $0.1R_H$ and $0.005R_H$ were tested. This factor of 20 change in the two-dimensional cross-section gave a change of less than a factor of 2 in the number of impacts in the longest low velocity run done so far (40 to 60 synodic periods). The intermediate velocity case gives a factor of 8 difference in the number

of impacts over the same period, but since this is a three-dimensional simulation the difference in cross-sections is 400.

2) Particles with semi-major axes within a few R_H of the protoplanet quickly impact the protoplanet or are perturbed into more distant orbits. If we define

$$b_H = |a_{\text{protoplanet}} - a_{\text{particle}}| / R_H,$$

then we find that most particles with $1.3 < b_H < 3.2$, including all of the material with $1.9 < b_H < 2.4$, collides with (passes $< 0.1R_H$ from) the protoplanet, while most of the bodies with larger b_H 's are perturbed outward. Particles with $b_H < 1.3$ are trapped in horseshoe orbits and don't approach the protoplanet.

3) In the far field ($b_H > 5$), the most striking features of the planetesimal distribution are the forced eccentricities near mean motion commensurabilities, particularly the $j/(j+1)$ resonances. These give localized bands of eccentricity up to an order of magnitude larger than the background value resulting from random perturbations by the protoplanet.

4) As indicated by the results of Greenzweig and Lissauer (1990, *Icarus* 87, 40), the inclination evolution does not keep pace with the eccentricity evolution. The intermediate velocity runs begin with $i = \frac{1}{2}e$; however, near field particles that are ejected to the far field have a much lower $i : e$ ratio.

This work is supported in part by NASA Planetary Geology and Geophysics grant NAGW-1107.