Tidal Disruption of Inviscid Protoplanets

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Roche showed that equilibrium is impossible for a small fluid body synchronously orbiting a primary within a critical radius now termed the Roche limit. Roche's static criterion has been extended to bodies with tensile strength [1], and has been used to argue that even bodies on hyperbolic orbits would be tidally disrupted within the Roche limit [10,11]. Tidal disruption of orbitally unbound bodies is a potentially important process for planetary formation through collisional accumulation, because the area of the Roche limit is considerably larger than the physical cross section of a protoplanet. Tidal disruption of unbound bodies is also the basis for the disintegrative capture model of lunar origin [10,11]. Because there is only a limited amount of time for tidal forces to act on an unbound body, a dynamic rather than a static analysis is required to determine the outcome.

Several previous studies have been made of dynamical tidal disruption. Protoplanets with strong dissipation (e.g., solid or partially molten bodies) do not undergo tidal disruption, even for grazing encounters [8,9]. The case for inviscid (e.g., molten) bodies, however, has been in dispute, with one model implying disruption within a modified Roche limit [7] and another disclaiming the possibility of disruption [5]. Another model followed the orbits of a hypothetically disrupted body [11], but did not include the body's self-gravity.

Considerable insight into the dynamics of tidal disruption can be gained from a simple model based on comparing the velocity dispersion expected to be produced in a body by tidal forces with the escape velocity for segments of the body to be removed to infinity. While crude, this analysis does include the self-gravity of the body, which is the only agent capable of resisting tidal disruption in an inviscid body. We have considered three different models of disruption: (1) test particles leaving a sphere; (2) hemispherical breakup [6]; and (3) distortion of a cylinder of radius \( R_c \) and half-length \( R \) with \( \epsilon = R_c / R \) being constant as \( R \rightarrow \infty \). For each of these models, the criterion for tidal disruption is the same except for a factor \( c \):

\[
\left(1 - \frac{R}{r_p}\right)^{-1/2} - 1 > \left(\frac{M}{cM'}\right)^{1/2} \left(\frac{r_p}{R}\right)^{1/2},
\]

where primes denote the primary, \( r_p \) is the perigee radius, \( R \) is the protoplanet's radius, \( M \) is the protoplanet mass, and \( c \) is 1 for (1), 8 for (2), and \( \epsilon \) for (3). In the limit of a small protoplanet \( (R << r_p) \), the criterion for case (1) becomes \( r_p < 0.63R'(\rho' / \rho)^{1/3} \), which shows that a small protoplanet cannot be tidally disrupted in this approximation for \( \rho' \sim \rho \); \( r_p < 0.63R' \) requires a collision. Roche's criterion has a similar form but with a factor of 2.5 instead of 0.63. The analytical criteria show that a massive protoplanet \( (M/M' \rightarrow 1) \) is tidally stable. For fixed protoplanet mass, as the perigee radius decreases, tidal disruption becomes possible if the protoplanet can disrupt before colliding with the primary. This can only occur for bodies less massive than \( \sim 0.01 - 0.1M_\oplus \).
Because of the limitations of these analytical models, we have used a smoothed particle hydrodynamics (SPH) code to model the tidal disruption process. The code is basically the same as the one used to model giant impacts [2]; here we simply choose impact parameters large enough to avoid collisions. The primary and secondary both have iron cores and silicate mantles, and are initially isothermal at a molten temperature. Previous lunar formation calculations with the SPH code have shown that inviscid, Mars-sized bodies do not suffer tidal disruption by the Earth, even during glancing collisions, so we have restricted our models to lower mass protoplanets, specifically $0.01M_\oplus$. Two parameters have been varied, the distance ($r_P$) between the two centers of mass at closest approach, and the velocity at infinity ($v_\infty$). Because of the boring nature of non-disruption models, our models have focused on parameters that do lead to tidal disruption.

Based on the analytical and numerical models, our conclusions may be summarized as follows. Protoplanets with masses greater than $\sim 0.1 M_\oplus$ do not suffer tidal disruption, even for grazing incidence, parabolic orbits. Smaller mass ($\sim 0.01M_\oplus$) inviscid protoplanets will be at least partially tidally disrupted if $r_P < 1.5R_\oplus$ for $v_\infty > 2 \text{ km s}^{-1}$. Up to half of the mass of a $0.01 M_\oplus$ protoplanet may impact the Earth if $v_\infty \leq 2 \text{ km s}^{-1}$ and $r_P < 2R_\oplus$. However, very little mass is captured in Earth orbit. Because typical protoplanets in the late phases of terrestrial planet accumulation are thought to have had $v_\infty \sim 10 \text{ km s}^{-1}$, tidal disruption was probably rare [5,8,9]. Tidal torques are efficient at producing rotational spin-up, rotational instability, and mass shedding [3]. Lunar formation through tidal disruption of a single protolunar body and capture of the debris into Earth orbit [10,11] appears impossible, and through multiple bodies very unlikely. Tidal disruption only occurs for relatively small bodies, and very little of their mass is injected into orbit. Orbital capture of a lunar mass would require many events, all favorably aligned, as well as having $v_\infty \sim 0$, both of which are unlikely [4]. Lunar formation following a giant impact appears to be preferable [2].

References: