Target Correlation Effects

Neutron-Nucleus Total, Absorption, and Abrasion Cross Sections

Francis A. Cucinotta, Lawrence W. Townsend, and John W. Wilson

DECEMBER 1991
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Francis A. Cucinotta, Lawrence W. Townsend, and John W. Wilson
Langley Research Center
Hampton, Virginia

NASA Technical Memorandum 4314
Abstract

Second-order optical model solutions to the elastic scattering amplitude were used to evaluate total, absorption, and abrasion cross sections for neutron-nucleus scattering. Improved agreement with experimental data for total and absorption cross sections is found when compared with first-order (coherent approximation) solutions, especially below several hundred MeV. At higher energies, the first- and second-order solutions are similar. There are also large differences in abrasion cross-section calculations; these differences indicate a crucial role for cluster knock-out in the abrasion step.

I. Introduction

The assessment of radiation risk to humans, in both their natural and space environments, from cosmic radiation is currently an area of active investigation. Predictions of biological damage will ultimately require a knowledge of the particle fluence spectra at the endpoint of interest. In turn, these particle fluence spectra are determined from charged-particle transport codes, which must contain a description of all important physical processes that occur as the incident ions and subsequent-generation fragment nuclei pass through natural and protective radiation shielding.

Recent studies (refs. 1 and 2) have shown the importance of using accurate energy-dependent, nuclear-interaction cross sections for the determination of fluence spectra for high-energy nuclei. A theoretical model for the prediction of cross sections is extremely useful, as it cannot be expected that enough experiments will be performed for all the collision pairs and energies of interest in cosmic-ray studies. In previous studies (refs. 3 to 6), a nuclear-interaction theory based on an effective coupled-channel solution to the nuclear scattering problem was considered for the prediction of interaction cross sections. A first-order approximation to the elastic amplitude was applied within the eikonal approximation for the prediction of total, absorption, and abrasion cross sections and showed good agreement with available experimental data.

More recently, a second-order solution to the eikonal coupled-channels (ECC) model (refs. 7 to 9) was developed and was found to give improved accuracy over the first-order solutions in limited studies for several collision pairs and energies. In this report, neutron-nucleus total, absorption, and abrasion cross sections are considered by using the second-order ECC model in an effort to improve the prediction of nuclear-interaction cross sections at cosmic-ray energies.

In the first-order (coherent approximation) ECC model, all nuclear excitations are neglected in the evaluation of the elastic amplitude. The second-order solution is found in terms of the bordered interaction matrix, which includes all scatterings from the ground to excited states and subsequent decay to the ground state. However, cascades between excited states are neglected. The second-order solution can be expressed in terms of the two-particle form factors of the nuclear ground-state, which must include the effects of short-range correlations to accurately represent the large momentum-transfer region of the scattering. In this work, we use a simple model for the two-particle form factors based on the nuclear matter approximation. Following the procedure in reference 10, the combined effects of Pauli correlations and short-range dynamic correlations are taken into account through the use of an effective correlation length. To simplify the calculations, the zero-range approximation is used to evaluate the correlation terms. The effects of correlations between the projectile and target, to first order, are included following the method developed in reference 6.

The abrasion cross-section model is based on a geometric probability interpretation of the absorption cross section. The effects of target correlations on the abrasion cross section are also considered herein by redefining the geometric probability for nucleon removal in a manner consistent with the second-order ECC solution. Large differences are found in this manner between the first- and second-order solutions at the lowest energies considered in this report.

Section III contains details of the formalism of the second-order solution for the elastic amplitude and introduces the second-order model for the abrasion cross section. The physical inputs used in our calculations are also described. Comparisons are made in section IV between the first- and second-order solutions, and comparisons are made with experimental
data for calculations of absorption and abrasion cross sections.

II. Symbols

- $A_f$: fragment mass number
- $A_P$: mass number of projectile nucleus
- $A_T$: mass number of target nucleus
- $B$: slope parameter
- $b$: impact parameter
- $C(q)$: correlation function
- $F^{(i)}$: projectile $i$-particle form factor
- $F$: nucleus-nucleus scattering amplitude
- $f_{NN}$: nucleon-nucleon scattering amplitude
- $G^{(i)}$: target $i$-particle form factor
- $k$: relative wave number
- $k_{NN}$: two-body relative wave number
- $l_c$: correlation length, 0.86 fm
- $m$: number of abraded nucleons
- $P(b)$: probability of not removing a nucleon
- $q$: momentum transfer vector
- $r$: internal nuclear coordinate
- $T$: kinetic energy of neutron, MeV
- $\alpha$: ratio of real part to imaginary part of forward two-body amplitude
- $\chi$: first-order eikonal phase
- $\Omega$: solid-angle phase
- $\rho_{CH}$: target charge density
- $\rho_0$: normalization of density
- $\sigma$: cross section
- $\chi$: first-order eikonal phase
- $\Omega$: solid-angle phase
- $\Gamma$: second-order eikonal phase
- $\rho_{CH}$: target charge density
- $\rho_0$: normalization of density
- $\chi$: first-order eikonal phase
- $\Omega$: solid-angle phase

Subscripts and superscripts:

- $abs$: absorption
- $c$: correlation
- $dir$: direct
- $ex$: exchange
- $el$: elastic
- $m$: abrasion
- $NN$: nucleon-nucleon
- $P$: projectile
- $T$: target
- tot: total

III. Elastic Channel

In the bordered interaction solution to the optical-model coupled-channel equations, the elastic amplitude is found (refs. 7 to 9) as

$$f(q) = \frac{-ik}{2\pi} \int d^2b \exp(-i q \cdot b) \{\exp[i \chi(b)] \cos \Gamma(b) - 1\}$$ (1)

where $k$ is the relative wave number in the overall center-of-mass frame, $q$ is the momentum transfer, and $b$ is the impact parameter. In equation (1), $\chi$ is the first-order eikonal phase, which represents the elastic matrix element, and $\Gamma$ is the correlation phase, which represents the summation over all double scatterings where states are excited from and then de-excited to the ground state. The first-order eikonal phase is written as

$$\chi(b) = \chi_{dir}(b) - \chi_{ex}(b)$$ (2)

where the exchange term takes into account correlation effects between projectile and target nucleons (ref. 6). These terms are written as

$$\chi_{dir}(b) = \frac{A_P A_T}{2\pi k_{NN}} \int d^2q \exp(iq \cdot b) F^{(i)}(-q)G^{(i)}(q) f_{NN}(q)$$ (3)

and

$$\chi_{ex}(b) = \frac{A_P A_T}{2\pi k_{NN}} \int d^2q \exp(iq \cdot b) F^{(i)}(-q)G^{(i)}(q)$$

$$\times \frac{1}{(2\pi)^2} \int d^2q' \exp(iq' \cdot b) f_{NN}(q + q') C(q')$$ (4)

where $F^{(1)}$ and $G^{(1)}$ are projectile and target ground-state one-body form factors, respectively, and $f_{NN}$ is the two-body amplitude parameterized as

$$f_{NN}(q) = \frac{\sigma(\alpha + i)}{4\pi} k_{NN} \exp\left(-\frac{1}{2}Bq^2\right)$$ (5)

where $k_{NN}$ is the relative wave number in the two-body system, $\sigma$ is the two-body cross section, $B$ is the slope parameter, and $\alpha$ is the ratio of the real part to the imaginary part of the forward, two-body amplitude. Values for the energy-dependent $\sigma$, $B$, $\alpha$, $B$, and $k_{NN}$ are tabulated in Table I.
and \( \alpha \) are found in references 4 and 5. The correlation factor is found as

\[
C(q) = \frac{1}{4} \left( \frac{\pi}{d} \right) \exp \left( -q^2/4d^2 \right)
\]

in reference 6 with \( d = 1.85 \text{ fm}^{-1} \).

The one-body form factor is written in terms of the charge form factor as

\[
F^{(1)}(q) = F_{CH}(q)/F_p(q)
\]

where \( F_p \) is the proton form factor. For a harmonic well distribution,

\[
F_{CH} = (1 - sq^2) \exp \left( -aq^2 \right)
\]

where values for parameters \( s \) and \( a \) are from reference 5. For nuclei where a Woods-Saxon density is appropriate \((A_T \geq 20)\),

\[
\rho^{(r)}_{CH} = \frac{\rho_0}{1 + \exp \left[ \frac{r - R}{c} \right]}
\]

An exact Fourier transform to obtain the charge form factor may be found in a series solution (ref. (11))

\[
F_{CH}(q) = \frac{4\pi}{q} \rho_0 \phi(q)
\]

where

\[
\phi(q) = \pi R c \left\{ \frac{-\cos(Rq)}{\sinh(\pi cq)} + \frac{\pi c \sin(Rq) \coth(\pi cq)}{R} \right\}
\]

\[
- \frac{2c}{\pi R} \sum_{m=1}^{\infty} (-1)^m \frac{mcq \exp(-mR/c)}{[(cq)^2 + m^2]^2}
\]

The series in equation (11) converges rapidly, and the first three or four terms are accurate for most applications. Values for the parameters \( c \) and \( R \) are taken from reference 5.

The second-order phase \( \Gamma \) was defined in references 7 to 9 and, for nucleon-nucleus scattering, reduces to

\[
\Gamma^2(b) = A_T \left( \frac{1}{2\pi k_{NN}} \right)^2 \int d^2qd^2q' \exp(iq \cdot b) \exp(iq' \cdot b')
\times f_{NN}(q)f_{NN}(q') \left[ -A_T G^{(1)}(q)G^{(1)}(q') \right.
\]

\[
+ \left( A_T - 1 \right) G^{(2)}(q, q') \right]
\]

where \( G^{(2)} \) is the ground-state two-body form factor of the target. To simplify the evaluation of equation (12), the zero-range approximation (refs. 12 and 13) is used, where

\[
r^2(b) = -2 \left( \frac{2\pi A_T f_{NN}(0)}{k_{NN}} \right)^2 l_c \int_{-\infty}^{\infty} ds \int_{0}^{2} \gamma_c \left[ \frac{1}{A_T} \gamma \right] \exp \left( -\gamma_c b \right)
\]

and where the correlation length \( l_c \) is 0.86 fm, and \( z \) is the \( z \)-component of \( r \).

The total cross section is found from the elastic amplitude by using the optical theorem as follows:

\[
\sigma_{tot} = \frac{4\pi}{k} \left( \int f(q = 0) \right)
\]

Equations (1) and (14) show that

\[
\sigma_{tot} = 4\pi \int_0^\infty db \left\{ 1 - \frac{1}{2} \exp \left[ -2\text{Im}(\chi + \Gamma) \right] \cos \left[ \text{Re}(\chi + \Gamma) \right] \right\}
\]

The total absorption cross section is found as follows:

\[
\sigma_{abs} = \sigma_{el} + \sigma_{cl}
\]

where \( \sigma_{el} \) is the total elastic cross section. Integrating equation (1) by using \( d\Omega \approx d^2q/k^2 \) and by using equations (15) and (16) yields

\[
\sigma_{abs} = 2\pi \int_0^\infty db \left\{ 1 - \frac{1}{2} \exp(-2\text{Im}\chi) \right\}
\times \left[ \cosh(2\text{Im}\Gamma) + \cos(2\text{Re}\Gamma) \right]
\]

All the results of this section (i.e., eqs. (1), (15), and (17)) reduce to the coherent approximation in the limit \( \Gamma \to 0 \).

Another quantity of interest is the abrasion cross section, which gives the probability of knockout of any number of nucleons. As in references 4, 14, and 15, the absorption cross section is written as

\[
\sigma_{abs} = 2\pi \int_0^\infty db \left\{ 1 - [P(b)]^{A_T} \right\}
\]

where \( P(b) \) is the probability, as a function of impact parameter, of not removing a target nucleon. For the second-order solution, it follows from equations (17) and (18) that

\[
[P(b)]^{A_T} = \frac{1}{2} \exp(-2\text{Im}\chi) \left[ \cosh(2\text{Im}\Gamma) + \cos(2\text{Re}\Gamma) \right]
\]
and the cross section for removal of \( m \) target nucleons is written as

\[
\sigma_m = \left( \frac{A_T}{m} \right) 2\pi \int_0^\infty db \int_0^1 \left[ 1 - P(b) \right]^m \left[ P(b) \right]^{A_T - m} \, dB
\]

(20)

Because of the dependence of \( \Gamma^2 \) on \( f_{NN}(0) \), equations (17) and (20) depend on values of \( \alpha \). The analogous coherent model solutions are independent of \( \alpha \).

Of special note is the question of whether the abrasion model as represented by equation (20) is valid when correlations among target nucleons are present. The abrasion model is essentially a "classical geometric" picture of nucleon knockout. The original model is closely related to an independent particle assumption (ref. 16), where cluster knockout is not considered. Correlation effects lead to such cluster knockouts. (See ref. 17.) Equation (20) does not address cluster effects and assumes only that nucleons are removed. Numerical calculations are considered next. Deviations between first- and second-order abrasion cross sections suggest regions where the abrasion model as represented by equation (20) is questionable.

IV. Results

Comparisons between calculations and experimental data are shown in figures 1 to 6 for the total and absorption cross section as a function of neutron energy. The solid line is the second-order model, the dashed line is the first-order model, and the experimental data (error bars) are from reference 18. Above a few hundred MeV, the calculations give similar results; below 300 MeV, the second-order solutions show better agreement with the data. The representation of \( \Gamma \) as given in equation (13) may not be valid below 100 MeV because of the closure assumption that is invoked (ref. 8).

The differences observed at low energies between first- and second-order solutions are large for abrasion cross-section calculations. (See figs. 7 to 9.) Differences of a factor of 2 or greater vary with the number of nucleons that are abraded. Results for abrasion cross sections at 100, 200, and 300 MeV are shown in tables 1 to 3. Clearly, redefining \( P(b) \) to be consistent with the second-order optical model leads to a drastically different partitioning of abraded nucleons than with the previously studied coherent approximation.

V. Concluding Remarks

The second-order solution to eikonal coupled-channel equations leads to improved predictions of total and absorption cross sections for neutron-nucleus scattering. Using an abrasion probability function consistent with the second-order elastic amplitude leads to dramatically altered predictions of abrasion cross sections below 300 MeV neutron energy. The importance of future study of cluster knockouts in the abrasion step is clearly demonstrated. It would be interesting to extend the calculations presented herein to other projectiles. Calculations with ablation effects should be made and compared with experimental data for fragmentation cross sections.

VI. References


11. Maung, Khin Maung; Deutchman, P. A.; and Royalty, W. D.: Integrals Involving the Three-Parameter Fermi


Table 1. Abrasion Cross Sections for Neutron $^{12}$C

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Table 2. Abrasion Cross Sections for Neutron $^{27}$Al

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Table 3. Abrasion Cross Sections for Neutron $^{64}$Cu

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Figure 1. Total cross section for neutron $^{12}$C scattering versus neutron energy. Error bars are experimental values.

Figure 2. Total cross section for neutron $^{27}$Al scattering versus neutron energy. Error bars are experimental values.
Figure 3. Total cross section for neutron $^{64}$Cu scattering versus neutron energy. Error bars are experimental values.

Figure 4. Absorption cross section for neutron $^{12}$C scattering versus neutron energy. Error bars are experimental values.
Figure 5. Absorption cross section for neutron $^{27}$Al scattering versus neutron energy. Error bars are experimental values.

Figure 6. Absorption cross section for neutron $^{64}$Cu scattering versus neutron energy. Error bars are experimental values.
Figure 7. Abrasion cross section for neutron $^{12}$C scattering versus neutron energy.
Figure 7. Concluded.

(c) Four-nucleon removal.
Figure 8. Abrasion cross section for neutron $^{27}$Al scattering versus neutron energy.
(c) Four-nucleon removal.

Figure 8. Concluded.
Figure 9. Abrasion cross section for neutron $^{64}\text{Cu}$ scattering versus neutron energy.
Figure 9. Concluded.

(c) Three-nucleon removal.

(d) Four-nucleon removal.
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**Authors:** Francis A. Cucinotta, Lawrence W. Townsend, and John W. Wilson

**Performing Organization:** NASA Langley Research Center, Hampton, VA 23665-5225

**Sponsoring/Monitoring Agency:** National Aeronautics and Space Administration, Washington, DC 20546-0001

**Funding Numbers:** WU 199-04-16-11

**Performing Organization Report Number:** L-16955

**Sponsoring/Monitoring Agency Report Number:** NASA TM-4314

**Distribution/Availability Statement:** Unclassified Unlimited

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