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A NUMERICAL METHOD FOR UNSTEADY AERODYNAMICS VIA ACOUSTICS

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Abstract. Formal solutions to the wave equation may be conveniently described within the framework of generalized function theory. Here generalized function theory gives a formulation and formal solution of a wave equation describing oscillation of a flat plate from which a numerical method may be derived.

Summary. Wave equations describe vibrations and spatial perturbations ("waves") of physical terms away from certain ambient terms. In acoustics vibration terms are generally described as "sound" though they are of a wider range than audible sound. In general, think of wave equations as describing any moderately small variation in space or time. Acoustics and aeronautics parallel in that the governing equations of linearized compressible aerodynamics and acoustics are the same. The linear acoustic wave equation for the velocity potential $\phi(x,t)$ is

$$\frac{\partial^2 \phi}{\partial t^2} - \frac{1}{c^2}\frac{\partial^2 \phi}{\partial x^2} = 0 \tag{1}$$

where c is the "speed of sound." The nature and scope of the infinite possible solutions to (1) vary enormously. Usually a particular physical situation or other specifies "initial conditions" or "boundary conditions" which narrow the admissible solutions down to one. However, there is an enormous gulf between knowing that a unique solution is there and writing it down with a particular equation or function from which to calculate. There are many ingenious techniques for bridging this gulf. In particular, sound generated from physical situations which involve moving objects may be described by wave equations with "generalized derivatives" and have formal solutions which involve integrals of generalized functions. Generalized derivatives are a useful formal method roughly equivalent to a "weak" formulation [1], but with the benefit of derivative formalism in which the work of integration by parts is automatically incorporated. The generalized functions and derivatives are useful in describing discontinuous phenomena such as the intrusion of a wing in free space or a "shock" in flow quantities on the surface of the wing [2].

In thin airfoil theory the boundary conditions are specified on the mean chord surface [3]. In terms of generalized derivatives and linearized momentum equation $p = -\rho_0 \frac{\partial \phi}{\partial t}$, the wave equation becomes

$$\frac{\bar{\partial}^2 p}{\partial t^2} - \frac{1}{c^2}\frac{\bar{\partial}^2 p}{\partial x^2} = \frac{\bar{\partial}}{\partial t}[\rho_0 v_n |\nabla f_{mc}| \Delta(f_{mc})] + \bar{\nabla}\cdot[\Delta p \nabla K \delta(K)] \tag{2}$$

where f_{mc} is the mean chord surface, K is the mean chord surface and wake, v_n is the normal velocity due to thickness and the bars over the derivatives denote generalized derivatives [4]. This equation is similar to the Ffowcs Williams-Hawkings (FW-K)

$$\frac{\bar{\partial}^2 p}{\partial t^2} - \frac{1}{c^2}\frac{\bar{\partial}^2 p}{\partial x^2} = \frac{\bar{\partial}}{\partial t}[\rho_0 v_n |\nabla f| \Delta(f)] + \bar{\nabla}\cdot[p \nabla f \delta(f)] \tag{3}$$

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which is a formal restatement of the equations of continuity and momentum [5]. Since pressure is continuous across the wake, this may be rewritten

$$(4)\qquad \frac{\partial^2 p}{\partial t^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial x^2} = \frac{\partial}{\partial t}\left[\rho_0 v_n |\nabla f_{mc}| \Delta(f_{mc})\right] + \nabla \cdot \left[\Delta p \nabla f_{mc} \delta(f_{mc})\right].$$

Equation (3) has formal solution

$$(5)\quad 4\pi p(x,t) = \frac{1}{c}\frac{\partial}{\partial t}\int_{f_{mc}=0} \left(\frac{\rho_0 c v_n + \Delta p cos(\theta)}{r|1-M_r|}\right)_{\tau^*} dS + \int_{f_{mc}=0}\left(\frac{\Delta p cos(\theta)}{r^2|1-M_r|}\right)_{\tau^*} dS$$

Here τ^* is the emission time of a signal received by the observer at time t. Integration of (5) with respect to t and the identity $p = -\rho_0 \frac{\partial}{\partial t}\phi$ leaves

$$4\pi\rho_0\phi(x,t) = -\frac{1}{c}\int_{f_{mc}=0}\left(\frac{\rho_0 c v_n + \Delta p cos(\theta)}{r|1-M_r|}\right)_{\tau^*} dS + \int_{-\infty}^{t}\int_{f_{mc}=0}\left(\frac{\Delta p cos(\theta)}{r^2|1-M_r|}\right)_{\tau^{*\prime}} dS dt'.$$
(6)

Here $\tau^{*\prime}$ is the emission time of the signal received at time t'.

Generally (6) is separated into two terms. We are interested in the so called "lifting" terms (corresponding to aerodynamic lift), namely:

$$(7)\quad 4\pi\rho_0\phi(x,t) = -\frac{1}{c}\int_{f_{mc}=0}\left(\frac{\Delta p cos(\theta)}{r|1-M_r|}\right)_{\tau^*} dS + \int_{-\infty}^{t}\int_{f_{mc}=0}\left(\frac{\Delta p cos(\theta)}{r^2|1-M_r|}\right)_{\tau^{*\prime}} dS dt'.$$

Refer figure 1 for the following. We assume that the mean chord surface is a flat plate moving with uniform velocity in direction shown. Assume that the pressure is piecewise of the form $\Delta p e^{i\Gamma t}$ over individual surface panels of the plate. This corresponds to an unsteady aerodynamic loading. On individual panels P_{ij} in the coordinate system shown in figure 1 the individual integrals restated in terms of normal velocity reduce to

$$4\pi v_n(x,t) = 1/c\frac{\partial}{\partial x_3}\int_{P_{ij}}\left(\frac{\Delta p cos(\theta)}{r|1-M_r|}\right)_{\tau^*} dS + \frac{\partial}{\partial x_3}\int_{-\infty}^{t}\int_{P_{ij}}\left(\frac{\Delta p cos(\theta)}{r^2|1-M_r|}\right)_{\tau^{*\prime}} dS dt'$$
(8)

where v_n is the velocity in the normal direction and M_r is the Mach number in the r-direction. Since we are interested in aerodynamic lifting, the observer point will be on the lifting surface and consequently (8) will be singular on panel on which the observer lies. Although divergent, a physically correct solution may be extracted when the integral is considered in the Cauchy principle value sense with corresponding "Hadamard finite part" [6]. After a good choice of coordinate systems, reducing $\frac{\partial}{\partial x_3}$ to a limit definition, using a principle value, and alot of summertime work, the integrals remarkably reduce to $\lim_{h\to 0^+}\frac{\partial}{\partial h}(I_1 + I_2)$ where

$$(9)\qquad I_1 = h/ce^{i\Gamma t}\int_{r_1}^{r_2}\frac{e^{-i\Gamma r/c}}{r}\left\{sin^{-1}\left[\frac{y_2(r)}{\sqrt{r^2-h^2}}\right] - sin^{-1}\left[\frac{y_1(r)}{\sqrt{r^2-h^2}}\right]\right\} dr$$

and

$$(10)\quad I_2 = h\int_{-\infty}^{t} e^{i\Gamma t'}\int_{r_1}^{r_2}\frac{e^{-i\Gamma r/c}}{r^2}\left\{sin^{-1}\left[\frac{y_2(r)}{\sqrt{r^2-h^2}}\right] - sin^{-1}\left[\frac{y_1(r)}{\sqrt{r^2-h^2}}\right]\right\} drdt'$$

(the endpoints r_i are the range of r over panel P_{ij} and quantities in the 2nd integral y_i and r depend on t'). We are now ready to consider the global effect of simultaneous observers in each panel. This leaves a system of algebraic equations which may be solved for unknowns Δp. At present a numerical method for this system is being formed with the aid of Mathematica.

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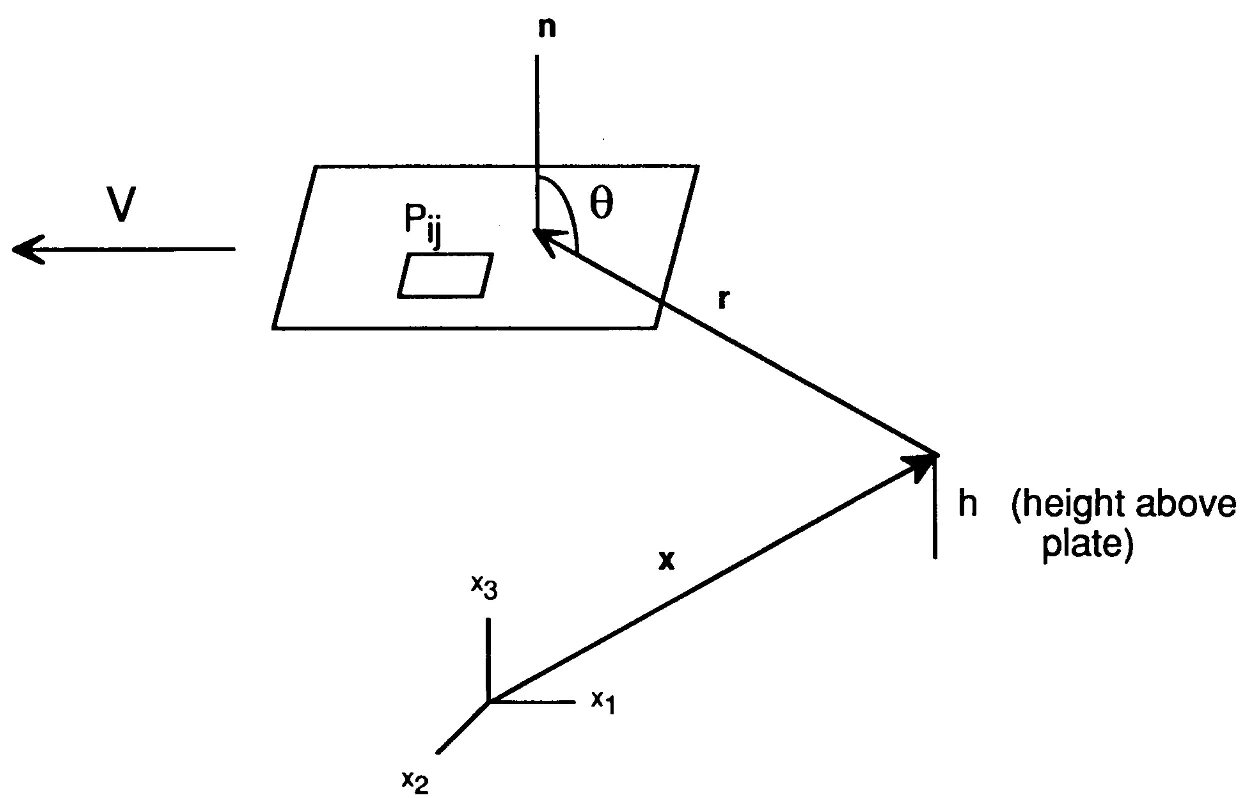

Figure 1