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# Suppressed Carrier Full-Spectrum Combining

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*A technique to accomplish full-spectrum arraying where all the telemetry power is put into the subcarrier sidebands (suppressed carrier) is described. The matched filter needed in each antenna prior to cross-correlation for deriving the coherence delay and phase offsets is an open-loop version of the telemetry phase-lock loop provided in the Advanced Digital Receiver. In analogy with a Costas-loop telemetry receiver, a "squaring loss" is derived, and a signal-to-noise ratio for the cross-correlation loop phase is presented.*

## I. Introduction

Normally, as a spacecraft travels farther from Earth and the telemetry signal-to-noise ratio (SNR) gets poorer, two system parameter trade-offs come into play. First, the telemetry modulation index is usually increased so more transmitter power is moved from the carrier to the telemetry signal, thereby improving telemetry SNR. This, of course, may result in a carrier signal that is significantly harder to acquire and track. The limit for this trade-off is full modulation where *no* carrier power is present. In this case, the carrier signal frequency must be acquired and tracked using a less-than-optimal Costas phase-lock-loop technique. The capability to Costas-loop track is not presently available in the DSN, but is planned as part of the new Block V receivers.

The second trade-off that comes into play is the rate at which telemetry data are transmitted back to Earth; this rate can be reduced, resulting in an improved SNR per telemetry bit. This has the unfortunate consequence of also reducing the total amount of data that can be returned during the critical encounter phase of a mission.

A technique often applied within the DSN to overcome these constraints (short of building larger antennas, even

lower noise receivers, or employing more advanced data encoding/decoding methods) is antenna arraying to enhance the SNR of the received telemetry [1]. Perhaps the most recent major applications of antenna arraying occurred during the Voyager 2 encounter with Uranus and the Voyager 2 encounter with Neptune. In each case, arraying provided adequate telemetry SNR employing data rates higher than would have been possible otherwise.

At least four different techniques, which depend on the details of the observing circumstance, have been used for arraying or combining the signals from several antennas. These include symbol-stream combining, baseband arraying, carrier arraying (or more correctly, carrier aiding), and full spectrum combining. In the various discussions of these techniques, the terms arraying and combining are usually used interchangeably.

The first technique, symbol-stream combining, works well when each ground-based antenna/receiver is capable of locking on the spacecraft telemetry stream and demodulating it down to the soft symbol stream (the raw telemetry data before the decision of whether a given bit is 1 or 0 is made). Each stream must then be delayed to line up

symbols, followed by weighting and combining. One major advantage of this technique is that the data stream from each antenna arrives at only the telemetry rate (typically on the order of 100 kbaud), facilitating its transmission to the combining location, either in real time or through a recording medium. The International Cometary Explorer (ICE) mission successfully used symbol-stream combining in a primary mode to gain approximately 2 dB in SNR [2]. A short time later, this combining technique was used on Voyager 2 to gain slightly less than 3 dB improvement [3]. More recently, symbol-stream combining was used as a backup for baseband arraying during the Voyager 2 encounter with Neptune [4].

The second technique, baseband arraying, works when the telemetry SNR is high enough to permit locking on and tracking of the carrier signal at each antenna, but too low to reliably maintain subcarrier lock and symbol sync. In this case, the baseband signal obtained after carrier demodulation is delayed, weighted, and added to a corresponding baseband signal from the other antenna. Then subcarrier demodulation and symbol sync are accomplished on the combined stream. The baseband bandwidth is on the order of 3 MHz and therefore somewhat more difficult to transmit or record for combining. During the Voyager 2 encounter with Uranus, this technique was used effectively with the Parkes Radio Astronomy antenna in Australia [5].

The third technique, carrier aiding, is useful when one antenna in the array can acquire and maintain carrier lock, but the other antennas cannot, either because they are smaller or have higher noise receivers. If the antennas to be arrayed are in proximity, the lock signal from the more sensitive antenna can be sent to the other antennas in real time and used to aid their locking on the telemetry carrier. The resulting baseband signals are then treated as they are in baseband arraying. This technique has been used quite recently to successfully array a 70-m antenna and a 34-m antenna at Goldstone while looking at the Pioneer 11 spacecraft.<sup>1</sup>

Finally, full-spectrum combining of an open-loop spacecraft telemetry signal can be used when the carrier is too weak to track, or when it is not convenient to track, at any single antenna. One form of this technique was used to coherently add the signal from all the antennas of the Very Large Array (VLA) during the Voyager 2 Neptune

encounter [6]. Following is a description of a significantly improved version of full-spectrum combining, with comments on how it differs from the original.

By way of introduction, note that the telemetry signal transmitted from a spacecraft can be represented by the equation [7]

$$S[t] = \sqrt{2P} \{ \cos \Delta \sin [\Omega_c t] + d[t] \sin \Delta \sum s_k[t] \}$$

where each term in the summation (odd  $k$  from 1 to  $\infty$ ) is

$$s_k[t] = (2/\pi)(1/k) \{ g_{uk} \sin [(\Omega_c + k\Omega_{sc})t] - g_{lk} \sin [(\Omega_c - k\Omega_{sc})t] \}$$

corresponding to the subcarrier sidebands. The amplitude corresponds to a signal total power of  $P$ . Figure 1 is a graph of this signal.

In these expressions,

$t$  = time

$\Omega_c$  = the transmitter carrier frequency

$\Omega_{sc}$  = the telemetry subcarrier frequency

$\Delta$  = the modulation index of the subcarrier

$k$  = the subcarrier harmonic number

$d[t]$  = the telemetry modulation

$g_{uk}$  = the upper sideband gain at frequency  $k$

$g_{lk}$  = the lower sideband gain at frequency  $-k$

$P$  = the signal total average power

For simplicity, assume the modulation index is 90 deg, so that only the suppressed carrier case is treated (this will likely be true when full-spectrum combining is used to put all possible power into the telemetry). Also, assume the gain factors are all unity so the summation over  $k$  can be treated as a square wave (the effects of this on the end result are minor). The signal then becomes

$$S[t] = \sqrt{2P} d[t] \cos [\Omega_c t] \mathbf{S}[\Omega_{sc} t] \quad (1)$$

where the outlined  $\mathbf{S}$  represents a square wave with zero crossings coincident with a sine wave of the same argument.

<sup>1</sup> T. Peng, "Carrier Array Demonstration at Goldstone with Pioneer 11," JPL Interoffice Memorandum 3393-90-62 (internal document), Jet Propulsion Laboratory, Pasadena, California, May 11, 1990.

The spacecraft signal in Eq. (1) is received by several antennas, located at various delays,  $\tau$ , from the spacecraft. These delays include geometric as well as nondispersive media and instrument components (as well as a large component corresponding to the distance from the spacecraft to the center of the Earth and common to all antennas, which will be ignored because it differences out in the final results). In addition, each antenna's receiver down-converts the signal to a carrier intermediate frequency,  $\Omega_{if} = (\Omega_c - \Omega_{lo})$ , using a local oscillator frequency,  $\Omega_{lo}$ . The combined effect on the signal received at the  $m$ th antenna is

$$S_m[t] = \sqrt{2Pd}[t - \tau_m] \cos[\Omega_{if}t - \Omega_c\tau_m + \theta_{om}] \times \mathbf{S}[\Omega_{sc}(t - \tau_m)] + N_m \quad (2)$$

where  $\theta_{om}$  is an unknown, slowly varying (its change in a time equal to the difference in the delays between the various antennas is negligible) residual phase due to various dispersive media and instrument effects.  $N_m$  is the noise of variance  $\sigma_{om}$  in the bandpass containing the signal.

The goal of the combining technique is to coherently add this signal from several antennas to obtain an improved SNR for telemetry extraction. To achieve this goal, the received signals must be delayed and phase shifted to correct for the relative effects between the various antennas that are represented in Eq. (2) before they can be added. In the following analysis, assume the signal (plus noise) modeled in Eq. (2) is digitally sampled at the Nyquist rate and processed with accuracy sufficient to ignore quantization and round-off effects.

## II. Delay and Phase Shift

Following the steps outlined in Fig. 2, begin by delaying the signal received at the  $m$ th antenna by  $\hat{\tau}'_m$ , based on a best a priori model,  $\tau'_m$ , of the antenna/spacecraft system (the caret on this delay signifies a quantized version of the model delay; the prime indicates a model delay). From this,

$$S_m[t] = \sqrt{2Pd}[t - \Delta_m] \cos[\Omega_{if}(t + \hat{\tau}'_m) - \Omega_c\tau_m + \theta_{om}] \times \mathbf{S}[\Omega_{sc}(t - \Delta_m)] \quad (3)$$

where the quantity  $\Delta_m = \tau_m - \hat{\tau}'_m$  is the residual between the actual delay and the value used in data processing.

The delay operation is followed by a phase shift that involves first the generation of a quadrature version of Eq. (3) using a hybrid and then a complex multiplication by an expression of the form

$$E_{pm}[t] = \exp[-j\theta_{pm}] \quad (4)$$

The final signal becomes

$$S_m[t] = \sqrt{2Pd}[t - \Delta_m] \exp[j\{\Omega_{if}(t + \hat{\tau}'_m) - \Omega_c\tau_m + \theta_m\}] \times \mathbf{S}[\Omega_{sc}(t - \Delta_m)] \quad (5)$$

where  $\theta_m = \theta_{om} - \theta_{pm}$  is the resulting phase offset of the signal. Note that the signal is now a complex quantity, having both in-phase and quadrature-phase components.

By adjusting  $\hat{\tau}'_m$  and  $\theta_{pm}$ , it is possible to bring the signals from each antenna into coherence for combining. An error signal will be generated by comparing the signal in Eq. (5) between pairs of antennas and then using this signal to control the delay and phase adjustment of the telemetry streams from these antennas relative to each other.

## III. Matched Filter

The first step toward obtaining an error signal is to perform a matched filtering operation on the signal. When implementing full spectrum arraying with the VLA during the Voyager Neptune encounter, this "matched" filtering consisted simply of narrowing the VLA receiver bandpass from its nominal 50 MHz down to about 8 MHz with base-band filters. While hardly optimal, such a procedure is acceptable if the SNR at each antenna is reasonable, as was the case for Voyager. However, in general one needs a more optimal approach. A true matched filtering (providing a better power SNR by as much as a factor of 8 MHz/43 kHz, or  $\sim 180$  in this case) is properly accomplished through the implementation of two procedures: (1) carrier demodulation followed by (2) subcarrier demodulation.

### A. Carrier Demodulation

Carrier demodulation is accomplished by coherently detecting the signal at the carrier intermediate frequency with a locally generated signal of the form

$$E_{cm}[t] = \exp[-j\{\Omega'_{if}(t + \hat{\tau}'_m) - \Omega'_c\tau'_m + \theta_{cm}\}] \quad (6)$$

where the primed quantities are the best guesses for their unprimed counterparts, and  $\theta_{cm}$  is an unknown arbitrary phase. The result of this process is a signal with real and imaginary parts representing the in-phase ( $I$ ) and quadrature-phase ( $Q$ ) components of the demodulated carrier. After low-pass filtering,

$$\langle I_{cm} \rangle = (1/2)\sqrt{2Pd}[t - \Delta_m] \cos[\Phi_{cm}]\mathbf{S}[\Omega_{sc}(t - \Delta_m)] \quad (7)$$

$$\langle Q_{cm} \rangle = (1/2)\sqrt{2Pd}[t - \Delta_m] \sin[\Phi_{cm}]\mathbf{S}[\Omega_{sc}(t - \Delta_m)] \quad (8)$$

where

$$\Phi_{cm} = (\Omega_c - \Omega'_c)t - \Omega_c\Delta_m + \Omega'_c\Delta'_m + \theta_m - \theta_{cm} \quad (9)$$

is the residual carrier demodulation phase and, with good modeling, is only a slowly varying function of time. In Eq. (9), the quantity  $\Delta'_m = \tau'_m - \hat{\tau}'_m$  is the model residual delay.

## B. Subcarrier Demodulation

Subcarrier demodulation is accomplished by coherently detecting the signals in Eqs. (7) and (8) at the subcarrier frequency using

$$E_{sm}[t] = j\mathbf{E}[-j\{\Omega'_{sc}(t - \Delta'_m) + \theta_{sm}\}] \quad (10)$$

where  $\Omega'_{sc}$  is the best guess of the subcarrier-oscillator frequency,  $\theta_{sm}$  is an unknown arbitrary phase, and the outlined  $\mathbf{E}$  represents a complex square wave with components  $\mathbf{C} + j\mathbf{S}$ . Again, the results of this process are signals with real and imaginary parts representing the in-phase and quadrature-phase components of this subcarrier demodulation. After filtering, there are four components:

$$\begin{aligned} \langle I_{cm}I_{sm} \rangle &= (1/2)\sqrt{2Pd}[t - \Delta_m] \\ &\times \cos[\Phi_{cm}]\mathbf{C}^+[\Phi_{sm}] + N_{II_m} \quad (11) \end{aligned}$$

$$\begin{aligned} \langle Q_{cm}I_{sm} \rangle &= (1/2)\sqrt{2Pd}[t - \Delta_m] \\ &\times \sin[\Phi_{cm}]\mathbf{C}^+[\Phi_{sm}] + N_{QI_m} \quad (12) \end{aligned}$$

$$\begin{aligned} \langle I_{cm}Q_{sm} \rangle &= (1/2)\sqrt{2Pd}[t - \Delta_m] \\ &\times \cos[\Phi_{cm}]\mathbf{S}^+[\Phi_{sm}] + N_{IQ_m} \quad (13) \end{aligned}$$

$$\begin{aligned} \langle Q_{cm}Q_{sm} \rangle &= (1/2)\sqrt{2Pd}[t - \Delta_m] \\ &\times \sin[\Phi_{cm}]\mathbf{S}^+[\Phi_{sm}] + N_{QQ_m} \quad (14) \end{aligned}$$

where

$$\Phi_{sm} = (\Omega_{sc} - \Omega'_{sc})t - \Omega_{sc}\Delta_m + \Omega'_{sc}\Delta'_m - \theta_{sm} \quad (15)$$

is the residual (slowly varying) subcarrier demodulation phase. Specifically, the real component of Eq. (10) when mixed with Eqs. (7) and (8) and then filtered gives Eqs. (11) and (12); the imaginary component of Eq. (10) when mixed with Eqs. (7) and (8) and then filtered gives Eqs. (13) and (14). The outlined  $\mathbf{C}^+$  and  $\mathbf{S}^+$  quantities represent the convolution of appropriate square waves (sine or cosine) as a function of their offset in phase; these waves have been lowpass filtered. They are, in fact, triangle waves as a function of this phase argument, with peak amplitude 1. The four independent noises,  $N_{II_m}$ ,  $N_{QI_m}$ ,  $N_{IQ_m}$ , and  $N_{QQ_m}$ , have been included explicitly at this point, each having a mean square variance of

$$\sigma_m^2 = (1/4)\sigma_m^2/n_a \quad (16)$$

In this equation, recall that  $\sigma_m$  is the noise deviation per data point (noise coherence interval) as represented by  $N_m$  in Eq. (2), while the 1/4 factor is due to averaging over the IF cycles and  $n_a$  is the number of data points over which the lowpass filters average. It is desirable that the value of  $n_a$  correspond to a length of time equivalent to a telemetry bit interval (the largest it can be when the telemetry, represented by the function  $d(t)$ , is unknown). The goal of the delay and phase modeling is to minimize the changes in residual delay and phase so this integration can be as long as possible before the cross-correlation is performed. Note that to achieve an  $n_a$  corresponding to a full telemetry bit interval, the symbol-sync on each of the data streams must be obtained independently. This is possible in the face of low SNR, since one can integrate over many telemetry bits to acquire sync.

## IV. Cross-Correlation

The final step in the detection process requires the "squaring" of the signal to eliminate the telemetry data,

$d[t]$ , which has an amplitude of  $\pm 1$  but an unknown code. This is accomplished by cross-multiplying, or correlating, the signals from pairs of antennas. Unfortunately, this is a noisy process because of the multiplying of noise by noise, and noise by signal that comes about when generating the desired signal-by-signal product. If the  $SNR_i$  of the incoming data is low ( $<1$ ), then the  $SNR_o$  of the outgoing signal will be a function of the square of  $SNR_i$ , which deteriorates rapidly with decreasing  $SNR_i$ . The loss of SNR due to this effect in a Costas loop is usually called "squaring loss." Here it more appropriately should be called "cross-correlation loss."

By taking some of the possible products (traceable from their names) of Eqs. (11) through (14) pair-wise for two antennas (the  $m$ th and the  $n$ th), for the signal-by-signal part,

$$\begin{aligned} \langle II_m QI_n \rangle &= (P/2)d^+ [|\Delta_m - \Delta_n|] \cos [\Phi_{cm}] \\ &\quad \times \sin [\Phi_{cn}] \mathbf{C}^+ [\Phi_{sm}] \mathbf{C}^+ [\Phi_{sn}] \end{aligned} \quad (17)$$

$$\begin{aligned} \langle QI_m II_n \rangle &= (P/2)d^+ [|\Delta_m - \Delta_n|] \sin [\Phi_{cm}] \\ &\quad \times \cos [\Phi_{cn}] \mathbf{C}^+ [\Phi_{sm}] \mathbf{C}^+ [\Phi_{sn}] \end{aligned} \quad (18)$$

$$\begin{aligned} \langle IQ_m QQ_n \rangle &= (P/2)d^+ [|\Delta_m - \Delta_n|] \cos [\Phi_{cm}] \\ &\quad \times \sin [\Phi_{cn}] \mathbf{S}^+ [\Phi_{sm}] \mathbf{S}^+ [\Phi_{sn}] \end{aligned} \quad (19)$$

$$\begin{aligned} \langle QQ_m IQ_n \rangle &= (P/2)d^+ [|\Delta_m - \Delta_n|] \sin [\Phi_{cm}] \\ &\quad \times \cos [\Phi_{cn}] \mathbf{S}^+ [\Phi_{sm}] \mathbf{S}^+ [\Phi_{sn}] \end{aligned} \quad (20)$$

where

$$d^+ [|\Delta_m - \Delta_n|] = \langle d[t - \Delta_m]d[t - \Delta_n] \rangle \quad (21)$$

is the time average of a convolution of the telemetry data as a function of the delay residuals. This function has a maximum value of 1 at the origin, and drops linearly to zero at an absolute delay difference of one telemetry bit length, assuming  $d[t]$  takes on values of  $\pm 1$ , and successive data bits are uncorrelated.

If Eq. (17) is subtracted from Eq. (18), and Eq. (19) is subtracted from Eq. (20),

$$\begin{aligned} Q_{cc} &= (P/2)d^+ [|\Delta_m - \Delta_n|] \sin [\Phi_{cm} - \Phi_{cn}] \\ &\quad \times \mathbf{C}^+ [\Phi_{sm}] \mathbf{C}^+ [\Phi_{sn}] \end{aligned}$$

$$\begin{aligned} Q_{ss} &= (P/2)d^+ [|\Delta_m - \Delta_n|] \sin [\Phi_{cm} - \Phi_{cn}] \\ &\quad \times \mathbf{S}^+ [\Phi_{sm}] \mathbf{S}^+ [\Phi_{sn}] \end{aligned}$$

When these two expressions are added,

$$\begin{aligned} Q_{\hat{c}} &= (P/2)d^+ [|\Delta_m - \Delta_n|] \sin [\Phi_{cm} - \Phi_{cn}] \\ &\quad \times \hat{\mathbf{C}}^+ [\Phi_{sm} - \Phi_{sn}] \end{aligned} \quad (22)$$

In this expression,  $\hat{\mathbf{C}}^+ [\Phi_{sm} - \Phi_{sn}]$  is the time average of the sum of  $\mathbf{C}^+ [\Phi_{sm}] \mathbf{C}^+ [\Phi_{sn}]$  and  $\mathbf{S}^+ [\Phi_{sm}] \mathbf{S}^+ [\Phi_{sn}]$  and is simply a function of the difference of  $\Phi_{sm}$  and  $\Phi_{sn}$ . In fact, this function is a slightly smoothed version of the triangle function represented by  $\mathbf{C}^+ [\Phi]$  (hence, the caret on  $\mathbf{C}^+$ ) with a maximum value of  $2/3$  at  $\Phi = 0$ .

By taking all other possible products of Eqs. (17) through (20) for two antennas and combining these in a fashion analogous to that used to derive Eq. (22),

$$\begin{aligned} I_{\hat{c}} &= (P/2)d^+ [|\Delta_m - \Delta_n|] \cos [\Phi_{cm} - \Phi_{cn}] \\ &\quad \times \hat{\mathbf{C}}^+ [\Phi_{sm} - \Phi_{sn}] \end{aligned} \quad (23)$$

$$\begin{aligned} Q_{\hat{s}} &= (P/2)d^+ [|\Delta_m - \Delta_n|] \sin [\Phi_{cm} - \Phi_{cn}] \\ &\quad \times \hat{\mathbf{S}}^+ [\Phi_{sm} - \Phi_{sn}] \end{aligned} \quad (24)$$

$$\begin{aligned} I_{\hat{s}} &= (P/2)d^+ [|\Delta_m - \Delta_n|] \cos [\Phi_{cm} - \Phi_{cn}] \\ &\quad \times \hat{\mathbf{S}}^+ [\Phi_{sm} - \Phi_{sn}] \end{aligned} \quad (25)$$

The total cross-correlation noise on each of the components given in Eqs. (22) through (25) is due to the sum of the products of the various signals and noises explicitly shown in Eqs. (11) through (14). It is straightforward, but tedious, to show that these cross-correlation noise components are all independent of each other, have zero mean

value, and all have the same maximum mean square variance of

$$\sigma_x^2 = 4\sigma^2(P/6 + \sigma^2)/n_t \quad (26)$$

In Eq. (26),  $P$  is the power in the original telemetry signal,  $\sigma^2$  is the mean square noise variance per telemetry bit for the two uncorrelated signals making up this cross product from Eq. (16),

$$\sigma^2 = (\sigma_m^2 + \sigma_n^2)/2 \quad (27)$$

and  $n_t$  is the number of telemetry bits over which the cross-correlation sum is averaged.

## V. Residual Delay and Phase Estimation

Following the practice used in very long baseline interferometry (VLBI) cross-correlation, each of these signals can be determined for several delay lags (i.e., for several values of  $|\Delta_m - \Delta_n|$  around the best a priori estimate). Then, based on the signal amplitudes in each lag, the correct delay difference can be determined and used to adjust the delay of one of the data streams relative to the other to maintain data stream *delay* coherency. In actual fact, when the telemetry bits are many microseconds in length, the delay difference can be estimated a priori to a small fraction of this bit length, eliminating the need for multilag correlation.

The phase differences in Eqs. (22) through (25) can be written explicitly as

$$\begin{aligned} \Phi_{cm} - \Phi_{cn} &= \Omega'_c(\Delta'_m - \Delta'_n) - \Omega_c(\Delta_m - \Delta_n) \\ &+ \theta_{om} - \theta_{on} - \theta_{pm} \\ &+ \theta_{pn} - \theta_{cm} + \theta_{cn} \end{aligned} \quad (28)$$

and

$$\begin{aligned} \Phi_{sm} - \Phi_{sn} &= \Omega'_{sc}(\Delta'_m - \Delta'_n) - \Omega_{sc}(\Delta_m - \Delta_n) \\ &- \theta_{sm} + \theta_{sn} \end{aligned} \quad (29)$$

where  $\Delta_k = \tau_k - \hat{\tau}'_k$  and  $\Delta'_k = \tau'_k - \hat{\tau}'_k$  are the residual delay and the model residual delay for the  $k$ th antenna.

Assuming (1) the frequencies can be specified to within a few hertz, (2) the rate-of-change of the delay differentials is no greater than a few picoseconds per hour (quite easily obtained, based on VLBI experience), and (3) the instrument phases are indeed slowly varying (a few tens of millihertz), it is possible to estimate these phase differences and feed them back to the appropriate phase mixers in one of the antenna data streams to maintain data stream *phase* coherency. Note that for the carrier phase in Eq. (28), this correction can be accomplished in any one of three phase mixers, while for the subcarrier phase in Eq. (29), it must be performed in the subcarrier demodulation mixer.

In actual practice, the subcarrier phase residual in Eq. (29) can be kept small. This is because the delay residuals and the antenna clock offsets can usually be maintained at less than 25 nsec, which, for a subcarrier frequency on the order of 1 MHz, will result in no more than 10 deg of phase error. Therefore, the only residual that need be estimated is that for the carrier phase.

## VI. Signal-to-Noise Ratio

Estimating the noise in measuring the cross-correlation phase, as represented in Eq. (28), is completely analogous to determining the same quantity for a Costas phase-lock loop when tracking telemetry. As noted above, the Costas-loop algorithm involves the multiplying of quadrature signal components from a single antenna, while cross-correlation involves multiplying analogous signal components, but crosswise from a pair of antennas.

Using the quantities given in Eqs. (11) and (12) from the same (e.g.,  $m$ th) antenna, the Costas-loop equivalent of Eq. (22) can be derived:

$$Q_c = (P/4)d^+ [0] \sin [2\Phi_{cm}] \mathbf{C}^+ [\Phi_{sm}] \quad (30)$$

In an analogy with Eq. (26), the mean square variance for this Costas component is

$$\sigma_c^2 = \sigma_m^2(P/2 + \sigma_m^2)/n_t \quad (31)$$

From these two equations, the signal-to-noise ratio for measuring the carrier phase can be derived:

$$SNR_c = \rho / \{1 + 1/(2R_d)\} = \rho S_{Lc} \quad (32)$$

where  $\rho = 2P n_a n_t / \sigma_o^2$  is the equivalent SNR in the final loop bandwidth,  $R_d = P n_a / \sigma_o^2$  is the data (symbol) SNR,

and  $S_{Lc}$  is the squaring loss as a function of  $R_d$  due to multiplication of signal by noise, and noise by noise, in the Costas-loop process:

$$S_{Lc}[R_d] = 1/\{1 + 1/(2R_d)\} \quad (33)$$

Following an analogous procedure using Eqs. (22) and (26), the SNR for measuring the cross-correlation phase can be derived:

$$SNR_x = \rho (1/3)[1/\{1 + 1/(2R_d/3)\}] = \rho S_{Lx} \quad (34)$$

The squaring loss here is the same as that for the Costas loop, except for scaling:

$$S_{Lx}[R_d] = (1/3)S_{Lc}[(1/3)R_d] \quad (35)$$

The larger loss in the cross-correlation case is due to the fact that the function  $\hat{\mathbf{C}}^+$  in Eq. (22) has a peak value of 2/3, while the function  $\mathbf{C}^+$  in Eq. (30) has a peak value of 1. The only way to overcome this loss is by making  $n_i$ , the number of symbols over which integration takes place, larger. This is analogous to making the Costas loop bandwidth narrower.

Figure 3 presents plots of these two squaring losses as a function of  $R_d$ . It is evident that to obtain an SNR for the cross-correlation phase equal to that for the Costas-loop phase, integration must be at least eight times longer when  $R_d$  is less than 1.

Because of instability in the frequency of the spacecraft transmitter, increasing the integration time (or equivalently, narrowing the loop bandwidth) of the Costas-loop case beyond about 1 sec is not possible without serious SNR loss. But for the cross-correlation case, this instability largely "common-modes out" from one antenna to the other, and therefore integration periods up to 100 sec are easily obtained with only slight SNR degradation. The limit on this integration time is set by the differential tropospheric effects between the two antennas.

## VII. Conclusions

A technique has been outlined for performing full-spectrum combining of telemetry signals. It is possible to implement a device to accomplish this type of arraying by using the DSN Advanced Receiver (ARX). ARX components would serve as the matched filters while new hardware would have to be added to provide the incoming delay and phase rotation. The real practicality for the use of full-spectrum combining in the DSN ultimately rests on the difficulty and expense of implementing these matched filters. It is expected that continued advancement in VLSI technology will easily provide inexpensive solutions in the near future.

An alternative approach for a demonstration of the basic ideas would be to record telemetry data with the DSN wide channel bandwidth (WCB) VLBI system and use the Block II VLBI correlator to accomplish the combining. Only a slight modification in hardware together with some software upgrades would be needed to test the technique. The one-bit sampling of this system, however, would limit its value as a combiner in any real application.

## References

- [1] J. W. Layland, A. M. Ruskin, D. A. Bathker, R. C. Rydgig, D. W. Brown, B. D. Madsen, R. C. Clauss, G. S. Levy, S. J. Kerridge, M. J. Klein, C. E. Kohlhase, J. I. Molinder, R. D. Shaffer, and M. R. Traxler, "Interagency Array Study Report," *TDA Progress Report 42-74*, vol. April-June 1983, Jet Propulsion Laboratory, Pasadena, California, pp. 117-148, August 15, 1983.
- [2] W. J. Hurd, F. Pollara, M. D. Russell, and B. Siev, "Intercontinental Antenna Arraying by Symbol Stream Combining at ICE Giacobini-Zinner Encounter," *TDA Progress Report 42-84*, vol. October-December 1985, Jet Propulsion Laboratory, Pasadena, California, pp. 220-228, February 15, 1986.
- [3] W. J. Hurd, J. Rabkin, M. D. Russell, B. Siev, H. W. Cooper, T. O. Anderson, and P. U. Winter, "Antenna Arraying of Voyager Telemetry Signals by Symbol Stream Combining," *TDA Progress Report 42-86*, vol. April-June 1986, Jet Propulsion Laboratory, Pasadena, California, pp. 131-142, August 15, 1986.
- [4] D. W. Brown, W. D. Brundage, J. S. Ulvestad, S. S. Kent, and K. P. Bartos, "Interagency Telemetry Arraying for Voyager Neptune Encounter," *TDA Progress Report 42-102*, vol. April-June 1990, Jet Propulsion Laboratory, Pasadena, California, pp. 91-118, August 15, 1990.
- [5] D. W. Brown, H. W. Cooper, J. W. Armstrong, and S. S. Kent, "Parkes-CDSCC Telemetry Array: Equipment Design," *TDA Progress Report 42-85*, vol. January-March 1986, Jet Propulsion Laboratory, Pasadena, California, pp. 85-110, May 15, 1986.
- [6] J. S. Ulvestad, "Phasing the Antennas of the Very Large Array for Reception of Telemetry From Voyager 2 at Neptune Encounter," *TDA Progress Report 42-94*, vol. April-June 1988, Jet Propulsion Laboratory, Pasadena, California, pp. 257-273, August 15, 1988.
- [7] J. Yuen, ed., *Deep Space Telecommunications System Engineering*, JPL Publication 82-76, Jet Propulsion Laboratory, Pasadena, California, p. 191, July 1982.

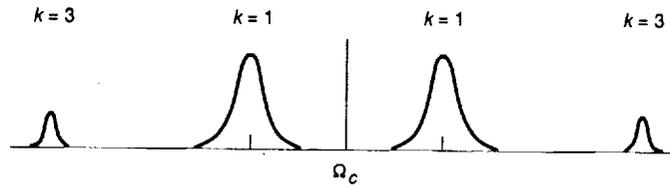


Fig. 1. Spacecraft telemetry signal.

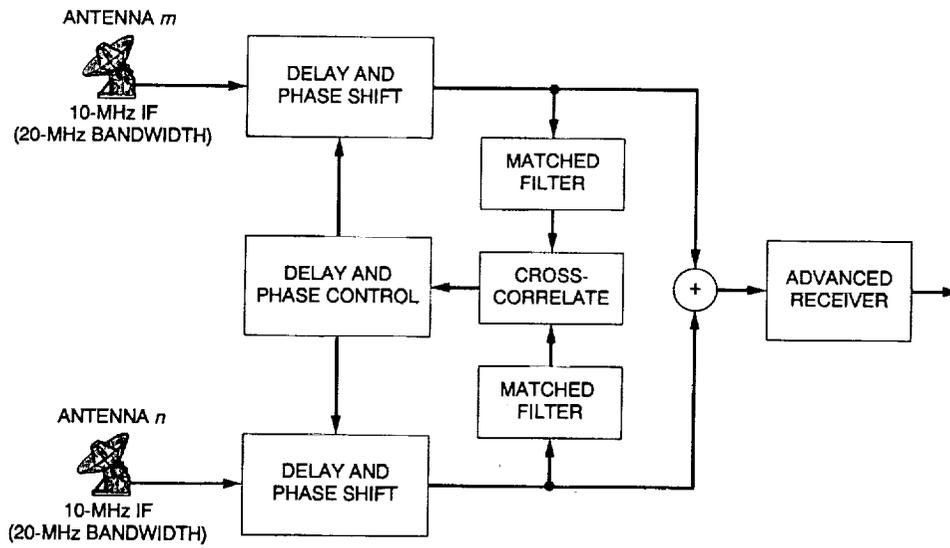


Fig. 2. Delay and phase shift.

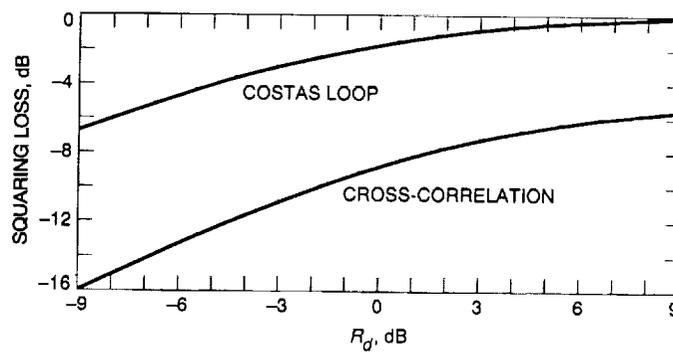


Fig. 3. Squaring loss.