NUMERICAL SIMULATION OF
SWEPT-WING FLOWS

A Progress Report for

Graduate Program in Aeronautics

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ABSTRACT

This progress report describes our efforts over the past six months to computationally model the transition process characteristic of flows over swept wings. Specifically, the crossflow instability and crossflow/T-S wave interactions are analyzed through the numerical solution of the full three-dimensional Navier-Stokes equations including unsteadiness, curvature, and sweep. This approach is chosen because of the complexity of the problem and because it appears that linear stability theory is insufficient to explain the discrepancies between different experiments and between theory and experiments (e.g., steady vs. unsteady, interactions, ...). The leading-edge region of a swept wing is considered in a three-dimensional spatial simulation with random disturbances as the initial conditions.

An ultimate goal of the work is the computational modeling of the receptivity problem for the swept wing through the use of the same numerical techniques. Toward this end, as a parallel effort to the swept-wing computations mentioned in the first paragraph, we continue the computational modeling of the receptivity of the laminar boundary layer on a semi-infinite flat plate with an elliptic leading edge by a spatial simulation. The incompressible flow is simulated by solving the governing full Navier-Stokes equations in general curvilinear coordinates by a finite-difference method. First, the steady basic-state solution is obtained in a transient approach using spatially varying time steps. Then, small-amplitude time-harmonic oscillations of the freestream streamwise velocity or vorticity are applied as unsteady boundary conditions, and the governing equations are solved time-accurately to evaluate the spatial and temporal developments of the perturbation leading to instability waves (Tollmien-Schlichting waves) in the boundary layer. The effect of leading-edge radius on receptivity is determined.

The work has been and continues to be closely coordinated with the experimental program of Professor William Saric, also at Arizona State University, examining the same problems. Comparisons with the experiments at Arizona State University are necessary and an important integral part of this work.

Whenever appropriate, we will match our results from the spatial simulation with triple-deck theory. This is an important aspect of the work.
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1. INTRODUCTION

In this progress report, Section 2 contains a list of related experience and accomplishments. Section 3 contains a summary of the work using a spatial, full Navier-Stokes simulation to examine the crossflow instability on swept wings. Section 4 contains a summary of the work that uses a spatial, full Navier-Stokes simulation to determine the mechanisms of receptivity and the role of leading-edge radius. The personnel involved in this project are described in Section 5.

2. RELATED EXPERIENCE AND TECHNICAL ACCOMPLISHMENTS

In the past, 11 students were supervised, 22 publications were written, and 33 talks and lectures were given. The two students who are considered for the fellowship are two of the PhD students supervised.

Publications


Presentations


**Post Doctoral Associates**


**Ph.D. Students**


**MS Students**


**Undergraduate Students**


The technical accomplishments thus far are documented in the publications listed above.

A brief description follows.


"The Effects of Streamwise Vortices on Transition in the Plane Channel," B.A. Singer, H.L. Reed, and J.H. Ferziger, Physics of Fluids A, 1, 12, 1960-71, 1989. This paper shows why the experiments of Nishioka were unable to produce H-type breakdown predicted by theory. The value of the computations in examining the role of "freestream" disturbances in the flow is demonstrated.

"A Shear-Adaptive Solution of the Spatial Stability of Boundary Layers with Outflow Conditions," H. Haj-Hariri and H.L. Reed, in preparation. This work outlines the numerics and boundary conditions used in our spatial simulations of transition.


"Receptivity of the Boundary Layer on a Semi-Infinite Flat Plate with an Elliptic Leading Edge," N. Lin, H.L. Reed, and W.S. Saric, Arizona State University Report CEAS 90006, Sept. 1989. This report demonstrates the feasibility of numerically studying the receptivity problem and establishes the platform upon which our receptivity studies are based. This work represents the first successful numerical treatment of the receptivity problem!


3. CROSSFLOW INSTABILITY

Direct numerical simulations are playing an increasingly important role in the investigation of transition. In such simulations, the full Navier-Stokes equations are solved directly by employing numerical methods, such as finite-difference or spectral methods. This approach is widely applicable since it avoids many of the restrictions that usually have to be imposed in theoretical models. From recent developments, it is apparent that linear stability theory suffers from this; the discrepancies between theory and experiment (i.e. steady versus unsteady; the role of interactions; the role of roughness, curvature, and freestream disturbances) are currently unresolved for crossflow. It appears that stability theory is not well-posed; predicted N-factors can range from small to large for a given configuration depending on the version of theory used. The questions posed above must be addressed by computational simulations.

In this approach, in contrast to linear stability theory, no restrictions with respect to the form or amplitude of the disturbances have to be imposed, because no linearizations or special assumptions concerning the disturbances have to be made. Furthermore, this approach allows the realistic treatment of the space-amplified disturbances and no assumptions have to be made concerning the basic flow (such as that the flow be parallel). The basic idea of this method is to disturb an established basic flow by forced, time-dependent perturbations. Then the reaction of this flow, that is, the temporal and spatial development of the perturbations, is determined by the numerical solution of the complete Navier-Stokes equations.

The principal goal of the current research is therefore the spatial simulation of the process of laminar-turbulent transition in the leading-edge region of an infinitely long, swept wing. The existence of such a method will provide a tool which will enable computation to complement experimental contributions to further the understanding of the physics of these flows and, ultimately, will provide a tool for the prediction and modeling of these flows. This is an ambitious goal.

The object of this study is to investigate the crossflow instability on an infinite-span, 45° swept wing. The model wing consists of a 6:1 semi-elliptical nose followed by a flat plate. The 3-D unsteady incompressible Navier-Stokes equations are discretized by a Fourier-spectral/finite-difference scheme. Near the leading edge, low-amplitude, steady blowing and suction is introduced on the wall surface to provide the initial disturbance for stationary crossflow. We have succeeded in observing the stationary, co-rotating structure and have reported this in the last progress report. At present we are modifying linear stability theory to
include the effects of streamline curvature, so that better wavelength and neutral-point estimates may be made to provide input for the Navier-Stokes calculations. This work should be finished over the next two weeks and a major report written.

4. RECEPTIVITY
EFFECT OF LEADING-EDGE GEOMETRY ON BOUNDARY-LAYER
RECEPTIVITY TO FREESTREAM SOUND

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ABSTRACT

The receptivity to freestream sound of the laminar boundary layer over a semi-infinite flat plate with an elliptic leading edge is simulated numerically. The incompressible flow past the flat plate is computed by solving the full Navier-Stokes equations in general curvilinear coordinates. A finite-difference method which is second-order accurate in space and time is used. Spatial and temporal developments of the Tollmien-Schlichting wave in the boundary layer, due to small-amplitude time-harmonic oscillations of the freestream velocity that closely simulate a sound wave travelling parallel to the plate, are observed. The effect of leading-edge curvature is studied by varying the aspect ratio of the ellipse. Boundary layer over the flat plate with a sharper leading edge is found to be less receptive. The relative contribution of the discontinuity in curvature at the ellipse-flat-plate juncture to receptivity is investigated by smoothing the juncture with a polynomial. Continuous curvature leads to less receptivity. A new geometry of the leading edge, a modified super-ellipse, which provides continuous curvature at the juncture with the flat plate, is used to study the effect of continuous curvature and inherent pressure gradient on receptivity.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>U</td>
<td>freestream velocity</td>
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<tr>
<td>L</td>
<td>half-thickness of the flat plate, minor radius of the ellipse</td>
</tr>
<tr>
<td>rₙ</td>
<td>leading-edge nose radius</td>
</tr>
<tr>
<td>v</td>
<td>kinematic viscosity</td>
</tr>
<tr>
<td>Re</td>
<td>Reynolds number, UL/v</td>
</tr>
<tr>
<td>Ω</td>
<td>frequency of oscillations</td>
</tr>
<tr>
<td>F</td>
<td>non-dimensional frequency parameter, Ωv/UL²</td>
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¹This work is supported by the Air Force Office of Scientific Research under Contract No. AFOSR-90-0234 and NASA/Langley Research Center under a Graduate Fellowship in Aeronautics.
1. Introduction

Receptivity, the process by which external disturbances lead to instabilities in shear flows, plays a vital role in the transition from laminar to turbulent flow. The importance of receptivity in prediction, modelling and control of transition has been recognized [Morkovin (1978), Reshotko (1984)] and can not be overemphasized. Substantial progress has been made in investigating receptivity of boundary layers. Discussions of recent developments in boundary-layer receptivity theory may be found in Goldstein and Hultgren (1989) and Kerschen (1990). A detailed review of some experiments on receptivity are presented in Nishioka and Morkovin (1986).

In the prediction of boundary-layer receptivity to freestream long-wavelength disturbances, theoretical investigations based on high-Reynolds-number asymptotic methods have identified that the conversion of freestream disturbances to TS instability waves takes place in the boundary layer where the mean flow exhibits rapid local variations in the streamwise direction [Kerschen (1990,1991), Goldstein (1983,1985)]. The discussions here concentrate on receptivity to sound in the leading-edge region.

A few experiments, using flat plates with elliptic leading edges, have been done on boundary-layer receptivity to freestream sound. In Leehey and Shapiro (1980) and Leehey et al. (1984) a leading edge with AR=6 was used. Acoustic receptivity for their flat plate was reduced from order one to essentially nothing after tipping the plate to obtain a zero mean-pressure gradient [Leehey et al. (1984)]. Results of Wlezien and Parekh (1990) and Parekh et al. (1991), using AR=6 and AR=24 (on a solid plate) indicate that the flat plate with a sharper leading edge is less receptive to a plane sound wave. In their...
experiments mean-pressure gradients are effectively zero and junctures between the leading edge and the flat plate are smoothed by a filler.

In terms of the computational modelling of the boundary-layer receptivity to long-wavelength acoustic waves, Kachanov et al. (1978) solved the incompressible flow over an infinitely thin flat plate, using the Navier-Stokes equations linearized for small disturbances. A freestream vortical disturbance and a transverse acoustic wave across the leading edge were considered. Murdock (1980) studied the receptivity to a plane parallel sound wave of an incompressible boundary layer over a flat plate with no thickness, and boundary layer over parabolic bodies (Murdock 1981). Receptivity was found to occur near the leading edge. A sharper leading edge (smaller nose radius, \( r_n \)) was reported to be more receptive. Hammerton & Kerschen (1991) found that the effect of nose bluntness was to decrease leading-edge receptivity for a plane acoustic wave propagating parallel to a parabolic body. It should be noted that these bodies have favorable pressure gradients everywhere. Gatski and Grosch (1987) solved the full incompressible Navier-Stokes equations for flow over an infinitely thin, semi-infinite flat plate. No clear development of the TS wave due to freestream oscillations was reported.

These results from experiments, computations and theories indicate that differences in not only parameters such as F and Re, but also details of leading-edge curvature, local and freestream steady/unsteady pressure gradients can affect receptivity greatly. In an attempt to investigate boundary-layer receptivity mechanisms to different freestream disturbances, a numerical code has been developed to compute the unsteady incompressible flow over a semi-infinite flat plate with an elliptic leading edge solving the full unsteady Navier-Stokes equations (Lin 1989). The use of a body-fitted curvilinear grid in this work enables us to perform direct simulations with a variety of leading-edge geometries.

2. Numerical Formulation

The governing equations are the two-dimensional unsteady incompressible Navier-Stokes equations with vorticity and stream function as dependent variables. A C-type orthogonal grid is generated around the leading edge and the flat plate (Fig. 1). The boundary conditions are the usual no-slip and no-penetration conditions at the wall, inviscid freestream velocities at the farfield boundary, and numerical boundary conditions downstream (Fasel 1976). The equations and boundary conditions are written in general curvilinear coordinates and discretized in space and time, using second-order accurate finite
differences. The resulting system of equations is solved using a modified strongly implicit procedure of Schneider and Zedan (1981). Further details of the numerical formulation may be found in Lin (1989).

First, a basic-state solution is computed by solving the governing equations for steady, incompressible flow with a uniform freestream, using a transient approach and spatially varying time steps. Then the steady flow is disturbed by applying forced perturbations about the steady basic flow at the freestream as unsteady boundary conditions. The resulting unsteady flow and the temporal and spatial development of the perturbations are calculated by solving the unsteady governing equations time accurately. Here, time-harmonic small-amplitude oscillations of the freestream streamwise velocity about the steady flow, which closely simulate a sound wave travelling parallel to the flat plate in the incompressible limit, are used as freestream perturbations.

3. Results and Discussions

3.1. Steady-state solutions

Steady-state solutions are obtained for three different values of the aspect ratio (AR) of the semi-ellipse, i.e. AR = 3, 6 and 9. In all cases, the Reynolds number, based on half-thickness of the flat plate or the minor radius of the semi-ellipse L and the freestream velocity U, is chosen to be 2400. In modelling the semi-infinite flat plate the downstream computational boundary is located such that the branch 1 of the neutral stability curve for Blasius flow (according to linear parallel theory) is well within the computational domain. The farfield computational boundary is located at about 25 to 30 Blasius-boundary-layer thicknesses at the downstream region. The grid-point distribution in the streamwise direction is determined not only to ensure high resolution of the leading-edge region where the curvature is changing rapidly, and at the ellipse-flat-plate juncture, but also to have at least 10 to 20 grid points in one expected TS wavelength for unsteady calculations. The stretching in the normal direction is done to pack more grid points near the wall to resolve the boundary layer and the Stokes viscous layer of the unsteady solution. Numerical studies have been made to insure the adequacy of grid resolution and computational boundary locations.

Typical steady-state solutions are presented in Figures 2 and 3. The wall vorticity (Fig. 2) differs appreciably from that of the Blasius solution near the leading edge. The sharper leading edges exhibit a peak vorticity closer to the singular behavior of the Blasius vorticity,
as expected. Velocity vector profiles accordingly differ from Blasius profiles, having overshoots above the freestream velocity at the leading-edge region. The surface pressure coefficient distributions (Fig. 3) of the sharper leading edges have smaller pressure minima after acceleration around the leading edge, hence smaller magnitudes of adverse pressure gradient in the pressure-recovery region. After this recovery, the pressure coefficient is very close to the inviscid solution, with a very small and almost constant adverse pressure gradient downstream. The square of the displacement thickness varies linearly with x, after some distance from the leading edge (Lin et al. 1991). The virtual leading edges are located upstream of the actual ones, due to rapid thickening of the boundary layer in the pressure-recovery region.

3.2. Unsteady solutions

Unsteady calculations are carried out at different values of the nondimensional frequency parameter, i.e. F=333x10^{-6} and 230x10^{-6} for a leading edge with AR=3 (Lin 1989) and F=230x10^{-6}, and 110x10^{-6} for AR=6 and 9 (Lin et al. 1991). The amplitude of the freestream oscillations (or the amplitude of the sound wave, a) used in these calculations is either 10^{-4}U or 2x10^{-4}U. Detailed discussions of the temporal and spatial development of perturbations in the boundary layer are given in the above references. After the theoretical Stokes-wave solution is subtracted from the total perturbation obtained from the Navier-Stokes solution, the amplitude profiles of the remaining disturbance can be calculated. These profiles are found to gradually develop into typical TS-wave amplitude profiles (Fig. 4) after some distance from the ellipse-flat-plate juncture and are zero after the TS wavefront.

The streamwise variation of Fourier amplitude and phase (obtained by performing temporal Fast Fourier Transforms) for the u' disturbance velocities (along the j=9 grid line) are shown in Figures 5 and 6. These amplitude and phase variations are typical of a TS wave linearly superposed on a long-wavelength disturbance (Stokes wave). In the flat-plate region, the v' component of the disturbance response is mostly contributed by the TS wave. Therefore wavenumber information of the TS wave present in unsteady solutions is extracted by differentiating the Fourier phase of the v' in the streamwise direction. These wavenumbers are shown together with wavenumbers obtained by linear stability analysis of the Navier-Stokes basic-state solution in Figure 7. The locations of the branch I neutral stability curve predicted by the unsteady Navier-Stokes solution and by the linear stability analysis of the Navier-Stokes basic-state solution are close and occur
well upstream of the branch I based on the Blasius boundary layer. In Figure 8, a typical result of grid-refinement studies is shown. It is clear that 200x80 grid points are adequate (for \( F = 230 \times 10^{6} \)) to get accurate solutions in the leading-edge region and further downstream except the TS-wavefront region. A finer grid than 280x90 would be required to resolve smaller scales present in the leading wave-packet, which is beyond the scope of this work.

3.3. Effect of leading-edge geometry

The effect of leading-edge curvature on receptivity is investigated by varying the AR of the ellipse while keeping the half-thickness \( L \) (hence Re) the same. The amplitude of the TS wave is smaller for sharper leading edges. This can be seen clearly in surface plots of instantaneous streamwise disturbance velocity, \( u' \), after 4 cycles of forcing and after subtracting the Stokes wave (Fig. 9 and 10). The magnitude of receptivity, as defined by the maximum (in y) amplitude of \( u' \) (of the TS wave only) in the boundary layer to the amplitude of the freestream sound, \( a \), is found to be about 0.8, i.e. order one, for the AR=3 leading edge at \( F = 230 \times 10^{6} \), while the magnitude of receptivity is found to be of order \( 10^{-1} \) for the AR=9 leading edge. For the AR=6 plate, the magnitude is of the same order but approximately twice as large as the AR=9 plate. A sharper leading edge has a larger curvature at the nose, has a smaller (in magnitude) local adverse pressure gradient in the leading-edge region, and also has a smaller magnitude of discontinuity in curvature at the ellipse-flat-plate juncture. It is a combination of these factors that may contribute to reduced receptivity.

As for the effect of frequency parameter \( F \) (for the same AR), a lower value of \( F \) is found to give a larger amplitude of \( u' \) near the juncture. This trend is in qualitative agreement with the theory of Goldstein (1985, 1989). It should be mentioned that for a fixed nose radius \( r_n \), lower \( F \) means lower Strouhal number, \( S_n \), hence the contribution from the nose bluntness should be higher according to results of Hammerton and Kerschen (1991) based on parabolic bodies. Farther downstream, the TS wave amplitude is smaller for a lower \( F \) since damping rates before the first neutral-stability point are stronger.

The wavelength conversion process necessary for receptivity to freestream long-wavelength disturbances is illustrated in plots of instantaneous disturbance streamlines (Fig.11 and 12). A disturbance structure which has a larger length scale than the TS wavelength and extends well beyond the boundary layer in the normal direction can be
observed at the leading edge. This structure has about twice the TS wavelength in the case of F=110x10^-6 (see also Fig. 6). At the edge of the steady boundary layer, it can be considered as unsteady pressure gradient A(x) input to the boundary layer, in light of discussions given by Nishioka and Morkovin (1986). One may relate it to eigen-solutions of the unsteady boundary layer as described in Goldstein (1983).

The asymptotic theories of Goldstein and Hultgren (1989), in comparison with the experiments of Leehey and Shapiro (1980) predicted that discontinuity in curvature at the juncture contributes more to order-one receptivity, while the disturbances incited due to leading-edge adjustment of the boundary layer (Goldstein 1983) have a much smaller contribution due to their rapid decay. To determine the effect of the discontinuity in surface curvature, a portion of the surface at the juncture region of the AR=6 leading edge is replaced by a polynomial, making the curvature continuous everywhere. Figure 13 illustrates the surface-curvature variation before and after this smoothing. The inherent change in the pressure gradient due to smoothing along the surface is small and an enlarged view of the pressure gradient near the juncture is given in Figure 14. Then steady and unsteady results are obtained at F=230x10^-6, using the same grid resolution and time-step size. The two solutions obtained with continuous and discontinuous curvature are compared in Figure 15. It is clear that discontinuous curvature enhances receptivity.

In order to investigate the effect of continuous curvature and inherent pressure gradient of the leading-edge, a new leading-edge geometry based on a super-ellipse is considered. The shape of this modified super-ellipse (MSE) is given by the following formula.

\[
[1 - x/(AR L)]^{m(x)} + [y/L]^n = 1, \quad 0 < x/L < AR
\]

\[
m(x) = 2 + [x/(AR L)]^2 \quad \text{and} \quad n = 2
\]

For a usual super-ellipse, both m and n are constants. Several leading-edge geometries can be designed by using different constant values for m and n. These super-ellipses will have the advantage of a continuous curvature (zero) at the juncture with the flat plate as long as m is greater than or equal to 3 at x/L=AR. The MSE, with m(x) given above, has the further advantage of having a nose radius and geometry (hence a pressure distribution) close to that of an ordinary ellipse with m=2 and n=2 (Fig. 16).

Variation of steady wall pressure gradient for the AR=6 MSE is given in Figure 14. See Figure 12.b for contours of instantaneous disturbance streamlines. Plots of instantaneous wall disturbance vorticity at F=230x10^-6 (Fig. 17) show that the MSE geometry is as
receptive as an ellipse (with a discontinuous curvature) of the same
AR. The MSE has a steeper adverse pressure gradient near the juncture,
although the curvature is continuous. This indicates that a rapid
(continuous) change in this adverse pressure gradient region is as
important as the discontinuity in curvature.

Further calculations at different Re, F, AR and different
leading-edge geometries, comparisons with results of different
numerical schemes and grid generation methods, local comparisons
with available theories and direct comparisons with experimental
results, should be carried out in the near future.

4. Conclusions

The receptivity of the laminar boundary layer over a semi-
infinite flat plate at the leading-edge region is investigated by direct
numerical solution of the full Navier-Stokes equations. The leading-
edge curvature and finite thickness of the flat plate are included by
using body-fitted coordinates.

We are able to observe both temporal and spatial initial
developments of the TS wave in the boundary layer due to time-
harmonic oscillations of the freestream streamwise velocity. The
receptivity occurs in the leading-edge region where rapid adjustments
of the basic flow exist. In this region the variation of curvature, the
adjustment of the growing boundary layer, the discontinuity in surface
curvature and the inherent local pressure gradients introduce length
scales to the thin layer of oscillating vorticity imposed by the long-
wave-length freestream disturbances. This leads to the development of
a TS wave in the boundary layer and its propagation downstream.

The magnitude of total receptivity (measured after the
leading-edge-flat-plate juncture) depends on the leading-edge radius
of curvature, with sharper leading edges being less receptive to plane
parallel sound waves. This is not in conflict with the results of
Hammerton and Kerschen (1991) (based on a parabolic body) since the
nose radius r_n considered here is relatively large so that contribution
from nose bluntness alone will be very small. Hence effects of change of
curvature and adverse pressure gradient near the ellipse-flat-plate
juncture will dominate the total disturbance response. The contribution
from the discontinuity in curvature to receptivity is found to be
substantial, making up almost 50 percent of the total receptivity for
the AR=6 leading edge at F=230x10^{6}. A new leading-edge geometry for
flat plates, based on a modified super ellipse, is proposed to eliminate
discontinuous curvature as a source of receptivity so that we can
concentrate on the effects of continuous leading-edge curvature and the associated adverse pressure gradient.

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Reshotko, E. 1984 'Environment and receptivity', AGARD Report No. 709 (Special course on stability and transition of laminar flows) VKI, Brussels.


Figure 1. Generated C-type grid; AR=6, Re=2400.
Figure 2. Steady-state vorticity distribution along the wall; Re=2400.

Figure 3. Surface pressure coefficient Cp; Re=2400.
Figure 4. Amplitude profiles of $u'/U$ taken during the fourth cycle at consecutive downstream locations, after the Stokes wave is subtracted; $AR=3$, $F=230\times10^{-6}$, $a/U=10^{-4}$. 
Figure 5. Amplitude of streamwise disturbance velocity, $u'/U$ vs. $x$; AR=6, Re=2400.

Figure 6. Phase of streamwise disturbance velocity, $u'/U$ vs. $x$; AR=6, Re=2400.
Figure 7. Variation of wave number $\alpha L$ (of $\psi'$) vs. $x$, taken after 6 cycles; $AR=6$, $F=230 \times 10^{-6}$.

Figure 8. Instantaneous disturbance wall vorticity $\omega' L/U$ obtained with two different grid resolutions after 4 cycles of forcing; $AR=6$, $F=230 \times 10^{-6}$, $a/U=10^{-4}$.
Figure 9. Surface plots of $u'/a$ after 4 cycles of forcing, after the Stokes wave is subtracted; $f=230\times10^{-6}$, $AR=3$, 6 and 9.
Figure 10. Surface plots of $u'/a$ after 4 cycles of forcing, after the Stokes wave is subtracted; $F=110 \times 10^{-6}$, $AR = 6$ and $9$.

Figure 11. Contours of instantaneous disturbance streamlines after 4 cycles of forcing; $AR=9$, $F=230 \times 10^{-6}$ and $110 \times 10^{-6}$.
Figure 12.a. Contours of instantaneous disturbance streamlines after 4 cycles of forcing; AR=6, $F=230 \times 10^{-6}$ and $110 \times 10^{-6}$.

Figure 12.b. Contours of instantaneous disturbance streamlines after 5 cycles at $F=230 \times 10^{-6}$ and after 4 cycles at $F=110 \times 10^{-6}$; MSE, AR=6.
Figure 13. Variation of curvature of the AR=6 leading edge near the juncture.

Figure 14. Enlarged view of the steady pressure gradient along the surface, near the juncture; AR=6, Re=2400.
Figure 15. Amplitude and phase of disturbance wall vorticity $\omega \cdot L/ U$ taken after 6 cycles of forcing: $AR=6$, $F=230 \times 10^{-6}$, $a/ U= 10^{-4}$. 
Figure 16. Shapes of the super-ellipse with $m=3$, $n=2$ and the modified super-ellipse, $AR=6$.

Figure 17. Instantaneous disturbance vorticity $\omega' L/U$ along the wall, after 4 cycles of forcing; $AR=6$, $F=230 \times 10^{-6}$, $a/U=10^{-4}$. 
Leading-edge Receptivity to a Vortical Freestream Disturbance: A Numerical Analysis

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ABSTRACT

The receptivity to freestream vorticity of the boundary layer over a flat plate with an elliptic leading edge is investigated numerically. The flow is simulated by solving the incompressible Navier-Stokes system in general curvilinear coordinates with the vorticity and stream function as dependent variables. A finite-difference scheme which is second-order accurate in both space and time is used. As a first step, the steady basic-state solution is computed. Then a small amplitude vortical disturbance is introduced at the upstream boundary and the governing equations are solved time-accurately to evaluate the spatial and temporal growth of the perturbations leading to instability waves (Tollmien-Schlichting waves) inside the boundary layer. Preliminary results for a symmetric, 2-D disturbance reveal the presence of TS waves aft of the flat-plate/ellipse juncture for an aspect ratio 6 leading edge subject to a non-dimensionai forcing frequency of 230.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>U</td>
<td>freestream velocity</td>
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<tr>
<td>L</td>
<td>half-thickness of the plate</td>
</tr>
<tr>
<td>\nu</td>
<td>kinematic viscosity</td>
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<tr>
<td>Re</td>
<td>Reynolds number, UL/\nu</td>
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<tr>
<td>\omega</td>
<td>frequency of oscillations</td>
</tr>
<tr>
<td>F</td>
<td>dimensionless frequency parameter, \omega/\nuU^2x10^6</td>
</tr>
<tr>
<td>AR</td>
<td>aspect ratio of the semi-ellipse</td>
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<tr>
<td>TS</td>
<td>Tollmien-Schlichting</td>
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This work is supported by the Air Force Office of Scientific Research and NASA Langley Research Center.
1. Introduction

Boundary-layer stability and transition remains one of the major unsolved research areas in fluid mechanics at the time of this publication. It is of considerable importance with regard to a myriad of technological applications. Linear stability theory has been applied extensively in the design of laminar flow control (LFC) systems to delay or even eliminate boundary-layer transition. LFC can provide major increases in the performance of both aerodynamic and hydrodynamic systems (Bushnell and Malik, 1985). Transition modeling, which encompasses the accurate prediction of the location of transition and the mechanisms which induce it, is crucial to flow systems which have both laminar and turbulent regions. In a low-disturbance environment, transition in boundary-layer flows is usually initiated by the unbounded growth or instability of small disturbances in the boundary layer. Boundary-layer stability theory describes the development of these self-excited or free oscillations in the laminar boundary layer. However, these free or self-excited oscillations are usually initiated by some externally forced disturbances such as freestream turbulence or sound.

Receptivity, the process by and through which these external-forced disturbances enter the boundary layer and lead to unstable waves, thus plays an important role in stability and transition studies. Much of what is understood regarding the nature of boundary-layer receptivity stems from the high-Reynolds-number asymptotic analysis conducted first by Goldstein and colleagues (1983,1985,1989) and extended by Kerschen and colleagues(1988,1990,1991). They and their colleagues have identified that the conversion of the typically long-wavelength freestream disturbances to TS instability waves occurs in regions of the boundary layer where there exist rapid local variations of geometry and/or local flowfield in the streamwise direction. Much
of their work has concentrated on the effect of acoustic disturbances in the leading-edge region and regions with short-scale variations in surface geometry. Kerschen (1991) has presented results of such an analysis for the case of a convected gust. Their analysis has been used to aid in the interpretation of experimental results obtained, in general, on flat-plate models having elliptic leading edges. Nishioka and Morkovin (1986) present a detailed review of some receptivity experiments. Recent work by Kendall (1991) in which he studies the receptivity problem for the case of freestream turbulence (FST) and ongoing work by Parekh et al. (1991) also concentrate on similar geometries of varying aspect ratio. Kendall has seen evidence of a TS structure but experimentalists have, in general, found it difficult to isolate a TS component in their measured signal. Much of the difficulty may stem from the complex nature of the background flow found in even the most sophisticated wind tunnels and therefore in quantifying the true disturbance environment (Saric, 1990).

With the advent of powerful supercomputers and numerical algorithms, the numerical solution of the full unsteady Navier-Stokes equations is fast becoming an integral tool for the study of the receptivity phenomena. One advantage of this technique is that one has complete control of the disturbance environment and is therefore able to isolate particular kinds of disturbances and study their individual effects (Lin, 1989). There have been several successful numerical works in receptivity to local disturbances, e.g. Fasel et al. (1987).

In the case of freestream disturbances, Kachanov et al. (1978) solved the incompressible flow over an infinitely thin, semi-infinite flat plate, using a finite-difference method to solve the Navier-Stokes equations linearized for small disturbances. A von-Karman vortex street with its core far from the boundary layer and transverse acoustic waves across the leading edge were considered. Murdock (1980) solved the full Navier-Stokes equations in parabolic coordinates for flow over an infinitely thin flat plate, using a spectral finite-difference method and in a 1981 paper presented results for the flow over parabolic bodies (with favorable pressure gradient) Gatski and Grosch (1987) solved the full incompressible Navier-Stokes equations using finite differences, for flow over an infinitely thin, semi-infinite flat plate. Plane longitudinal sound waves were considered, however no clear conclusions as to the development of the TS wave were presented. Lin (1989) numerically simulated the receptivity of the flow over a semi-infinite flat plate with an elliptic leading edge to a plane acoustic disturbance for a series of leading edge aspect ratios. His results indicated a development of TS waves in the boundary layer, with amplitudes which decreased with increasing aspect ratio. Lin et al. (1991) also investigated the role of the discontinuity in curvature present at the
ellipse flat-plate juncture and found that for a smoothed juncture-curve, the amplitude of the TS waves was roughly halved.

To our knowledge, unsteady flow over a flat plate including the leading-edge curvature has not been solved numerically for the case of freestream vorticity. This leading-edge curvature region, with inherent pressure gradients and large tangential velocity component, has been shown by Lin (1989, 1991) to play a vital role in receptivity to freestream disturbances. In this work, an attempt is made to numerically simulate the receptivity of a two-dimensional boundary layer to freestream vorticity and a brief description of the approach taken is presented in the next section.

2. Numerical Formulation

The present work is an attempt to investigate the initial stage, receptivity, of the transition process on a semi-infinite flat plate for an incompressible flow, by direct numerical solution of the governing Navier-Stokes equations, here cast with vorticity and stream function as dependent variables. Following the work of Lin (1989), a C-type orthogonal grid is generated around the elliptic leading edge and flat plate (Figure 1). This mesh is structured in such a way as to allow the introduction of a small, localized vortical disturbance at the far-field boundary near the basic-state stagnation line (Figures 2a&b), yet maintain sufficient resolution of the flat-plate and juncture regions where the development of the instability waves is expected. As the surface curvature is changing rapidly (and is in fact discontinuous at the juncture) the mesh is particularly fine in this region. A hyperbolic-tangent stretching function is used to ensure that no fewer than 12 points in the streamwise direction lie within the expected TS wavelength all along the flat plate. The equations and boundary conditions are written in general curvilinear coordinates and discretized in space and time, using second order finite differences. The Modified Strongly Implicit Procedure (MSIP) of Schneider and Zedan (1981) is used and is found to be robust and stable for reasonably small time steps.

Initially, the basic-state flow is calculated presuming the following flow conditions: the flow is steady, incompressible with a uniform freestream. The basic-state boundary conditions are as follows: the no-slip and no-penetration conditions are applied at the wall, a symmetric boundary condition along what amounts to the basic-state stagnation line (a zero incidence case is considered) is employed, and inviscid velocities are specified at the far-field boundaries. At the downstream boundary, a homogeneous second-derivative boundary condition following Fasel (1976) is used.
The basic-state, upon application of the afore-mentioned boundary conditions, is solved using a transient approach and spatially varying time steps. Once a basic-state solution is obtained, a small-amplitude \((u'_{\text{max}}/U=4 \times 10^{-3})\), periodic, localized disturbance is applied as an unsteady boundary condition at the far-field boundary. The resulting unsteady flow and the temporal and spatial development of the perturbations are determined by solving the governing equations subject to this modified boundary condition time accurately. Current results are presented in the following section.

3. Results and Discussion

The steady-state solution is obtained for an aspect-ratio \((AR)=6\) semi-ellipse. The Reynolds number \((Re)\), based upon the half-thickness of the leading edge and the freestream velocity \(U\) is 2400. The downstream boundary of the computational domain is chosen such that the branch 1 of the neutral stability curve (according to linear parallel theory) lies well within the computational domain. The farfield computational boundary is placed such that it is 20 boundary-layer thicknesses from the plate in the downstream region. The mesh size used in the present analysis is 260 points in the tangential direction and 100 points in the normal direction, although a calculation for a mesh of 360x100 yielded an identical result for the basic-state calculation.

Wall vorticity is presented in Figure 3. Nearly Blasius flow is recovered in the flat-plate region as evidenced by the streamwise velocity profiles depicted in Figure 4.

The unsteady calculation was carried out for a dimensionless frequency parameter \(F=230\). The forcing amplitude applied has a maximum streamwise velocity perturbation of \((u'_{\text{max}}/U)=4 \times 10^{-3}\). The disturbance was introduced over a small region just above the stagnation line along the farfield boundary in the form of a periodic boundary condition cast in the following form:

\[
w' = -A \left[ \frac{4(\eta - \eta_0) - 2}{\sigma} \right] e^{-(\eta - \eta_0)^2/\sigma} \sin(\omega t) \tag{1}\]

Note that this represents the vorticity associated with the introduction of a perturbation of velocity normal to the farfield boundary. The velocity perturbation is specified such that it is symmetrical about a specified location along the farfield boundary, \(\eta_0\), so that no net mass is introduced into the domain. The disturbance was found to propagate downstream, as expected, at the convective...
speed and as such the wavefront of the disturbance reached the
downstream boundary during the 6th cycle of forcing. Figure 5 is a
presentation of instantaneous disturbance streamlines observed at the
end of a series of cycles of forcing which reveal the quasi-steady
structure of the solution over much of the plate following 6 cycles. A
temporal Fast Fourier Transform (FFT) analysis conducted over several
cycles also confirmed the quasi-steady nature of the flow after 6 cycles.
In figure 6 typical u'-velocity profiles are displayed and in figure 7 the
change of u' with X is presented. From these figures it is evident that
the signal is a complex combination of more than one wave. In an
attempt to extract some information regarding the specific components
of this signal, some educated assumptions were applied. Kerschen
points out that the freestream vorticity will decay exponentially as
one approaches the wall. This is evident in the streamwise u'
plots(figure 7). Note the change in signal wavelength with distance
from the wall. Near the wall, the signal wavelength is less than half
of that found in the freestream and, qualitatively, the wavelength of
the signal appears to approach that of the TS wave. Thus, to better
assess the nature of the disturbance, spatial correlations with signals of
freestream and predicted TS wavelength were conducted. In figure 8, a
series of correlations between the total signal and a signal with a
wavelength based on forcing frequency and freestream velocity at
different Y-locations is presented. Clearly, this correlation supports
our supposition that the direct contribution of the freestream
perturbation to the total signal near the wall is minimal. If one then,
as a first guess, presumes the signal wavelength one point from the wall
to be that of the TS wave(a length as it turns out very close to that
which is predicted by linear theory), then one can use the correlation
with this wavelength to determine the phase of this portion of the
signal. Phase information for this portion, hereafter referred to as the
TS portion of the total signal, is presented in figure 9 for a
representative X-location aft of the juncture. Note the 180° phase shift
which is associated with the TS wave. A comparison with the total
signal phase(obtained from a temporal FFT over one cycle of forcing)
with that obtained for the freestream and TS components is displayed
in figure 10. Note that the total signal and the 'TS' are in-phase near
the wall, as expected, but as one moves away from the wall, the
freestream portion of the signal begins to dominate. The correlation can
also give a rough estimate of the signal amplitude, which again
displays the characteristic TS mode shape (Figure 11). If one uses the
phase information obtained from the spatial correlations in concert
with the temporal FFT of the total signal, an estimate of the
respective component amplitudes which accounts for signal modulation
with X may be obtained. Amplitude plots are presented in Figure 12 for
a series of X-stations using this technique. As a measure of the receptivity of the disturbance, one may compare the ratio of the maximum TS amplitude to that of the disturbance in the freestream. From this analysis we find that for an AR=6 leading edge at a forcing frequency $F=230$, the receptivity is of order $10^{-1}$.

4. Conclusions

Preliminary results from a numerical simulation of the receptivity of a two-dimensional laminar boundary layer to a symmetric vorticity wave in the freestream suggest the presence of TS waves aft of the ellipse-flat-plate juncture. While the signal decomposition technique is imprecise, the receptivity, based on maximum TS amplitude to freestream amplitude, is estimated to be of order $10^{-1}$. In addition to a more detailed analysis of the current result to include a comprehensive grid study and an analysis of the leading edge flowfield, studies of the effect of disturbance position, amplitude, frequency, leading-edge aspect ratio, and the effect of juncture curvature are planned. Similar cases will be investigated for the particularly interesting case of an asymmetric disturbance. Also in progress is a repeat of this case on a computational domain which extends past branch II.

Acknowledgements

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References


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Figure 1. C-grid used in computations.

Figure 2a. Blow-up of disturbance input region.
Figure 2b. Schematic of disturbance velocity profile at input location. $(V$ has both a $u'$ and $v'$ component).

Figure 3. Basic state wall vorticity.
Figure 4. Streamwise basic-state velocity profiles. The body surface is located at Y=1.
Figure 5. Disturbance Streamfunction contours after 5th, 6th and 7th cycles of forcing.
Figure 6. Streamwise-perturbation velocity profiles after 9 cycles of forcing.
Figure 7. Streamwise-perturbation velocity profiles after 9 cycles of forcing.
Figure 8. Spatial correlation of the total signal with a signal of freestream wavelength.
Figure 9. Phase of TS component of signal based on correlation of total signal with a signal of TS wavelength.

Figure 10. Comparison of component phases to total signal phase.
Figure 11. Estimate of TS amplitude based on spatial correlation with signal of TS wavelength.
Figure 12. Estimate of TS amplitude profiles using temporal FFT signal and correlation phase angles.
5. RESOURCES AND PERSONNEL

One of the principal strengths of our team at Arizona State University is its broad skills in analysis, computations, and experiments. We facilitate day-to-day communication between the computational work and the experimental work through two IRIS Graphics Workstations (3030 and 3130) and two DEC 5000 Workstations. The system, with state-of-the-art, real-time, three-dimensional, color-graphics software (PLOT3D), is equipped with an extensive multi-user and multi-task environment with twelve serial lines. Users are able to share the same data base or experimental information. This provides the heart of the interaction of the analytical, computational, and experimental research.

In addition to the super computers at NASA facilities and Princeton/NSF Consortium, the network includes access to the IBM 4341/VM and Harris/VS computers, the IBM 3090 Class VI machine, and the Cray XMP on campus as well as the MASSCOMP. The College of Engineering at ASU is currently also equipped with several VAX/780 and VAX/785 minicomputers exclusively for research purposes (each office and laboratory has a hard-wired RS232 interface). These minicomputers are excellent systems for program development. The IRIS and DEC machines can access all the features available in those minicomputers through the existing local area networking (Ethernet) on the campus. Furthermore, the system can communicate directly with NASA research facilities to share information through telephone couplings. The full array of computer capabilities from super-mini to super-super is in place for the research.

Ray-Sing Lin, a PhD student, and Nay Lin, a PhD student, are the two candidates for the Fellowship. Ray, Nay, and Tom Buret (also a PhD student) each spent one month of the summer of 1991 at Langley Research Center working on these and similar problems, interacting with NASA and ICASE personnel.

The principal investigator for this work is Helen L. Reed, Associate Professor of Mechanical and Aerospace Engineering. Professor Reed has spent the last ten years conducting theoretical and computational research on problems of boundary-layer stability specifically applied to the ACEE/LFC programs. She also spent one month of the summer of 1991 at NASA/Langley Research Center. Her resume is attached as Appendix I.
HELEN LOUISE REED

1. EDUCATION


2. AREA OF TEACHING AND RESEARCH


3. POSITIONS HELD

Aug. 1985 - present, Associate Professor (Tenure awarded Apr. 1988), ASU.
Sept. 1982-Aug. 1985, Assistant Professor, Stanford Univ.
Jan. 1982-Aug. 1982, Assistant Professor (Non-tenure track), VPI&SU.
Summer 1976, Mathematics Aid, NASA/Langley Research Center.

4. HONORS / DISTINCTIONS

Phi Beta Kappa
Recipient of a NASA fellowship, 1976
Outstanding Summer Employee Award from NASA/Langley Research Center, 1976
Torrey Award for Excellence in Mathematics, Goucher College, 1977
Outstanding Achievement Award from NASA/Langley Research Center, 1978
Cunningham Fellowship Award from Virginia Polytechnic Institute & State Univ., 1981
Presidential Young Investigator Award, National Science Foundation, 1984
AIAA Excellence in Teaching Award, Arizona State University, Fall 1988
Professor of the Year, Pi Tau Sigma, Arizona State University, 1988-1989
Associate Fellow, American Institute of Aeronautics and Astronautics, December 1990
Faculty Awards for Women in Science and Engineering, National Science Foundation, 1991

5. PUBLICATIONS


"Development and Decay of a Pressure-Driven, Unsteady, Three-Dimensional Flow Separation," R.W. Henk, H.L. Reed, submit. JFM.

"On the Linear Stability of Supersonic Cone Boundary Layers," G.K. Stuckert, H.L. Reed, submit. AIAA J.

"Linear Disturbances in Hypersonic, Chemically Reacting Shock Layers," G.K. Stuckert, H.L. Reed, submit. AIAA J.


31 refereed national conference proceedings papers (5 invited)
6 books and 7 articles edited
7 technical reports

6. PROFESSIONAL SERVICE

Member of Presidential Young Investigator Workshop on U.S. Engineering, Mathematics, and Science Education for the Year 2010 and Beyond, Washington, D.C., November 4-6, 1990. This is an advisory group to President Bush’s Science Advisor, Allen Bromley, concerning the directions U.S. education must take in the preparation and training of the U.S.’s future scientists and lay people.

Member of National Academy of Sciences/National Research Council Aerodynamics Panel which is a part of the Committee on Aeronautical Technologies of the Aeronautics and Space Engineering Board, Commission on Engineering and Technical Systems, November 1990-March 1992. This is the advisory group to NASA and the U.S. Congress concerning the directions NASA must take in order to enable the U.S. to remain competitive in the world arena.

Member of U.S. National Transition Study Group under the direction of Eli Reshotko, 1984-Present.

Associate Editor, Annual Review of Fluid Mechanics, 1986-Present. With the work divided equally among John Lumley, Milton Van Dyke, and myself, we are responsible for complete selection, refereeing, and editorial correction of all articles in each yearly volume.


Member of Fluid Mechanics Technical Committee of the Applied Mechanics Division of the ASME, 1984-Present.

Member of Steering Committee for National Fluid Dynamics Congress, June 1988-Present.

Chairperson, 2nd Annual *Arizona Fluid Mechanics Conference*, Arizona State University, April 4-5, 1986.
Chair of 21 various symposia and sessions at international conferences.

**CONSULTING**

Westinghouse Electric and Naval Underwater Sea Center (1981)
Pratt and Whitney (1986-1991)
ICASE Consultant, NASA/Langley Research Center (Current)
Colorado Research Development Corporation (Current)