EXPERIENCES AT LANGLEY RESEARCH CENTER IN THE APPLICATION OF OPTIMIZATION TECHNIQUES TO HELICOPTER AIRFRAMES FOR VIBRATION REDUCTION

T. Sreekanta Murthy and Raymond G. Kvaternik

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Langley Research Center
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Experiences at Langley Research Center in the Application of Optimization Techniques to Helicopter Airframes for Vibration Reduction

T. Sreekanta Murthy
Lockheed Engineering and Sciences Company
Hampton, Virginia

and

Raymond G. Kvaternik
NASA Langley Research Center
Hampton, Virginia

ABSTRACT

A NASA/industry rotorcraft structural dynamics program known as DAMVIBS was initiated at Langley Research Center in 1984 with the objective of establishing the technology base needed by the industry for developing an advanced finite-element-based vibrations design analysis capability for airframe structures. As a part of the in-house activities contributing to that program, a study was undertaken to investigate the use of formal, nonlinear programming-based, numerical optimization techniques for airframe vibrations design work. Considerable progress has been made in connection with that study since its inception in 1985. This paper presents a unified summary of the experiences and results of that study. The formulation and solution of airframe optimization problems are discussed. Particular attention is given to describing the implementation of a new computational procedure based on MSC/NASTRAN and CONMIN in a computer program system called DYNOPT for the optimization of airframes subject to strength, frequency, dynamic response, and fatigue constraints. The results from the application of the DYNOPT program to the Bell AH-1G helicopter are presented and discussed.

INTRODUCTION

All helicopters are prone to vibrations which can seriously degrade ride quality, reduce service life, and limit maximum speed in forward flight. Considerable progress has been made over the past forty years in reducing vibrations in helicopters. However, the level of vibration reduction achieved in newer helicopter designs has been, for the most part, either insufficient or only marginally acceptable in meeting the increasingly stringent vibration requirements which are being imposed. Even though excessive vibrations have plagued virtually all new helicopter developments, until recently, there has been little reliance on the use of vibration analyses during design to limit vibration. With only a few exceptions, helicopters have been designed to performance requirements while relying on past experience to "tinker out" excessive vibrations during the ground and flight testing phases of development. Most often, excessive vibrations have been reduced through the use of add-on vibration control devices at the expense of significant weight penalties associated with using them. Recently, however, there has emerged a consensus within the helicopter industry on the need to account for vibrations more rigorously during the design process. This need has resulted in the subject of helicopter vibrations receiving considerably increased attention in recent years (ref. 1). The goal (unofficially) set down by the industry is to achieve the vibration levels associated with jet aircraft, the so-called "jet smooth" ride (ref. 2). To achieve this goal will require the development of advanced design analysis methodologies and attendant computational procedures which properly and adequately take into account vibration requirements during all phases of the design process.

Recent research activities in the United States concerning the development of advanced design analysis methodologies to limit vibrations are focused in three major areas: (1) rotor system design, (2) control system design, and (3) airframe structural design. Various types of vibration analyses methodologies are evolving to support rotor and airframe design work. Rotor aeroelastic analysis codes are being implemented to evaluate design analysis of rotors. Methods for employing active control systems to suppress vibrations in both the rotor and the airframe are being studied. Design optimization analysis methods are being formulated to select rotor and airframe design parameters which yield low inherent vibrations.

At Langley Research Center, a NASA/Industry rotorcraft structural dynamics program known as DAMVIBS - Design Analysis Methods for VIBrationS (ref. 3) - was initiated in 1984 with the objective of establishing the
technology base needed by the industry for developing an advanced finite-element-based vibrations design analysis capability for airframe structures. As a part of the in-house activities contributing to that program, a study was undertaken to investigate the use of formal, nonlinear programming-based, numerical optimization techniques for airframe vibrations design work. Considerable progress has been made in connection with that study since its inception in 1985. The purpose of this paper is to present a unified summary of the experiences and results of that study.

The paper begins with a background section which discusses the vibratory loads of interest, current vibration reduction approaches, and relevant research activities. Key tasks involved in the optimization of airframe structures are described. The formulation and solution of airframe optimization vibration problems are discussed. The implementation of a new computational procedure based on MSC/NASTRAN and CONMIN in a computer program system called DYNOPT for the optimization of airframes subject to strength, frequency, dynamic response, and fatigue constraints is described. Major steps of the airframe design process are outlined. An optimization methodology which appears suited to airframe design work is described. Considerations needed in formulating optimization problems for both new and existing airframes are discussed. Finally, numerical results from the application of the DYNOPT program to the Bell AH-1G helicopter are presented and discussed.

BACKGROUND

Predominant Vibratory Loads

The predominant source of vibration in a helicopter arises from the oscillatory airloads acting on the blades of the main rotor. These loads are transmitted from the rotor through the hub and into the airframe where they produce objectionable vibrations. In steady-state level flight, the loads acting on the individual blades sum in such a way that the resultant forces and moments transmitted to the airframe occur at integer multiples of the product of the rotor rotational speed \( \Omega \) and number of blades \( N \). Thus, the airframe is subjected to steady-state rotor-induced forces which occur at the discrete frequencies \( N\Omega, 2N\Omega, 3N\Omega, \ldots, nN\Omega \). The dynamic characteristics of the rotor and the airframe and the way in which these two systems are coupled at the rotor hub determine the manner in which the helicopter airframe responds to the dynamic loads. Because the magnitude of the harmonic airloads generally decreases with increasing harmonic number, the lower harmonics of the loads occurring at \( N\Omega \) (and sometimes \( 2N\Omega \)) are usually more important with respect to vibrations than the higher harmonics.

Approaches to Vibration Reduction

Many approaches to the solution of the vibration problem have been proposed and studied. The most common approaches employed to reduce helicopter airframe vibrations include: (1) the modification of the main rotor system by altering blade stiffness and mass properties to bring about reduction in the magnitude of the resultant hub loads (refs. 4-6); (2) the use of active and passive vibration control devices to absorb and/or isolate the forces transmitted from the rotor hub to the airframe (refs. 7-8); and (3) the modification of the airframe structure to ensure that the natural frequencies of the airframe are well separated from the predominant rotor excitation frequencies to avoid resonance and to reduce the dynamic responses of the airframe under rotor-induced loads (refs. 9-10). Among these approaches, the approach involving main rotor modification requires extremely complex multi-disciplinary design trade-off studies for which optimization approaches are being developed, and the approach involving the use of active and passive vibration control devices has weight penalties. The approach involving airframe structural modification is gaining renewed attention in the design community, and efficient and practical methods to perform airframe structural modification are being sought.

Airframe Design for Vibration Reduction

The requirement for low vibratory response of the airframe necessitates: (1) Insuring that none of the major airframe natural frequencies is close to the predominant rotor exciting frequencies to avoid resonance; and (2) Reducing the magnitude of the dynamic response of the airframe under the combined action of the exciting forces and moments at the frequencies of interest. Low vibration design of an airframe requires knowledge of the airframe dynamic characteristics in terms of both its frequency response characteristics and its frequencies and mode shapes. In practice, airframe design involves repetitive structural design modifications to obtain the desired dynamic characteristics. The identification of the necessary structural modifications typically requires extensive analyses using large finite element models of the airframe structure and multi-dimensional searches in design variable space to determine the optimum sizes of the structural members. Airframe design is primarily based on engineering judgement and involves a tedious trial-and-error modification process. Selection of the best airframe that meets all design requirements, in particular the vibration requirements, is a difficult task. Therefore, there is a need for a systematic procedure which considers vibration requirements during the airframe design process by properly accounting for the various multi-disciplinary interactions that influence the design modifications. It would appear that structural optimization tools, if properly brought to bear by the design engineer, would go a long way toward achieving the goal of an analysis capability for designing a low vibration helicopter.

Recent Research In Airframe Optimization

The airframe structural optimization approach for helicopter vibration reduction has not been addressed much
in the past. The reported work is contained primarily in references 11-19. References 11-15 address the vibration reduction problem by modifying the airframe structure to tune the natural frequencies of the airframe and/or to reduce the responses under dynamic loads. Although the word "optimization" is used in these references, the work described there addresses the use of ad hoc methods as the basis for making structural modifications without the use of any formal optimization techniques. The methods described include those based on considerations such as the Vincent circle trace and the strain energy in a member. The use of nonlinear mathematical programming methods to tune airframe frequencies has only recently gained attention, and references 16-17 describe what are apparently the first applications of that method to finite-element models of airframes. Computer codes for using the nonlinear programming optimization methods for airframe vibration reduction are beginning to be developed by the rotorcraft industry (see, for example, refs. 18-19). Clearly, there is a need for further research to explore more fully the potential of optimization approaches for vibration reduction in helicopter airframes.

Focus of Airframe Optimization Research at Langley Research Center

As mentioned earlier, the objective of the airframe optimization research at Langley Research Center is to investigate the use of formal, nonlinear programming-based, numerical optimization techniques for airframe vibrations design work. The primary focus of this research study is directed toward: (1) identification and examination of key tasks involved in the application of optimization techniques to helicopter airframes; (2) development of practical computational procedures for optimization; (3) development of suitable optimization methodology which would be compatible with the airframe design process; and (4) application of the formulated computational procedures for optimization to real airframe structures. Some results of the airframe optimization research activities at Langley are reported in References 20-24.

KEY TASKS IN AIRFRAME OPTIMIZATION

The basic idea in airframe structural optimization for vibration reduction is to design the airframe in a way that the vibratory responses in the areas of interest are minimized. The application of the nonlinear mathematical programming approach typically involves formulation of the vibration problem to find a minimum value of an objective function, under a specified set of structural response constraints, and with prescribed bounds on the structural design variables. A solution to the optimization problem is sought via an iterative approach consisting of a sequence of computational tasks involving finite element analysis, sensitivity analysis, approximate analysis, and design change computations. The specific considerations required to accomplish these computational tasks, particularly with reference to the airframe optimization computations, are identified and discussed below.

Formulation of the Problem

A key task in the successful application of optimization techniques to helicopter airframes is the formulation of the optimization problem, which includes the establishment of a relevant set of design variables, an objective function, and constraints. An optimization problem is generally expressed in the form:

\[
\begin{align*}
\text{Minimize the objective function} & \quad F(b) \\
\text{subject to the constraints} & \quad g(b) \leq 0 \\
\text{and bounds on design variables} & \quad b_l \leq b \leq b_u
\end{align*}
\]

where \(b_l\) and \(b_u\) are the lower and upper bounds on the design variables \(b\).

In the formulation of the optimization problem, the objective function is typically the total weight of the airframe structural members expressed as a function of the design variables. The expression for the constraint functions are formulated based on the allowable limits on the vibration levels or equivalent structural forced response displacements at selected locations in the structure. The expression for the constraint functions can also be formulated to specify the allowable ranges in which the natural frequencies of the structure are to be placed to avoid resonances with the frequency of the excitation forces. The design variables are normally chosen as the cross-sectional sizes of the structural members which are to be varied within certain prescribed bounds to seek optimum values for the variables. A major issue in the formulation of the optimization problem arises from the fact that there is no unique way of formulating a given vibration reduction problem as an optimization problem because the design variables can be defined in several different ways, and because the objective functions and constraints can be written in many different forms. The formulation is also dependent on which phase of the design process is being addressed and on whether the airframe is a new design or an existing one which is to be modified. Because different formulations will yield different solutions to a vibration reduction problem, the various considerations needed in the formulation of alternative optimization problems need to be examined.

Finite Element Analysis

After formulating an optimization problem, an important task in the optimization solution is the finite element analysis of the airframe structure to compute structural responses which are subsequently used in the numerical evaluation of the objective and constraint functions. In the present study, the MSC/NASTRAN program (ref. 25) was chosen for the finite element analysis task because it is the code of choice for structural analysis in the rotorcraft industry. The finite element analysis computations require
considerable effort in managing the repetitive execution of large-sized finite element models and careful organization of the large amount of data which is associated with such models. Companion design models (ref. 26) need to be developed for the optimization computations. A design model may be defined as an organized collection of design variables, constraints, objective function, bounds on design variables, and linking of design variables. A design model is to optimization as a finite element model is to structural analysis. It should be noted that the development of a well-organized design model may require as much effort as the development of the underlying finite element model.

**Sensitivity Analysis**

Design sensitivity analysis consists of the computation of the sensitivity coefficients of the objective and constraint functions with respect to changes in the design variables. In the airframe optimization computation, the sensitivities of weight, natural frequencies, forced response displacements, and dynamic stresses are needed. The MSC/NASTRAN (Version 65) program (ref. 25) has the capability to provide sensitivities of the natural frequencies and static stress constraint functions. However, the sensitivity analysis for the constraints on the forced response displacements and weight are not available in the program and therefore had to be implemented via an user-developed DMAP (Direct Matrix Abstraction Program).

**Approximate Analysis**

Use of large finite element models of airframes in the repetitive structural analyses required during optimization iterations is computationally inefficient. Therefore, a crucial task in the successful application of optimization techniques to airframe structures is the development and the use of an efficient and accurate approximate analysis technique for evaluating the objective and constraint functions. Three of the most common types of approximate analysis techniques (ref. 27) - two based on a Taylor's series expansion and the other based on a hybrid constraint approximations - were chosen for use in the airframe optimization solution. It should be noted that while these approximate analysis techniques have been used in the solution of static optimization problems with some success, the accuracy and reliability of these techniques in static applications are still under investigation. These techniques have yet to be evaluated in dynamics applications, in particular to airframe dynamics problems.

**FORMULATIONS OF THE VIBRATION OPTIMIZATION PROBLEM**

Expressions for natural frequency, steady-state forced response displacement, and dynamic stress constraint functions, and the sensitivity derivatives of these functions, are presented and discussed here. In general, the objective and constraint functions can be used interchangably in formulating an optimization problem. Because the expressions for the objective and constraint functions have the same form for a given type of response, in the discussion below only the constraint expressions are given.

**Natural Frequency Constraints**

As discussed earlier, constraints on airframe natural frequencies are required to ensure that the frequencies are well-separated from the main rotor excitation frequencies to avoid resonance. The rotor-induced forcing frequencies are discrete frequencies $nN\Omega$ (where $n$ is an integer; $N$ is the number of blades; $\Omega$ is the rotor speed). The natural frequency constraints can be written as:

$$\omega_i \leq \omega \leq \omega_{iu}$$

(4)

where $\omega_i$ and $\omega_{iu}$ are the lower and upper bounds on the $i$th natural frequency $\omega_i$ of the airframe. The airframe natural frequencies are determined by solving the eigenvalue equation

$$[K - \lambda M] \Phi = 0$$

(5)

where $K$ and $M$ are the stiffness and mass matrices of the structure, $\lambda$ is the eigenvalue and is equal to the square of the frequency $\omega$, and $\Phi$ is the mode shape vector. The sensitivity derivatives of the frequency constraints are obtained by differentiating equation (5) with respect to the design variables vector $b$. After simplifying the differentiated terms by making use of the symmetry and orthogonality of matrices $K$ and $M$, and assuming unit generalized mass, the resultant expression for the sensitivity derivative is given by:

$$\frac{\partial \lambda}{\partial b} = \Phi^T [ \frac{\partial K}{\partial b} - \lambda \frac{\partial M}{\partial b} ] \Phi$$

(6)

**Steady-State Forced Response Constraints**

In addition to natural frequency constraints, constraints are required on the forced response amplitudes of the airframe. The forced response amplitudes are obtained by solving the equation:

$$M(b) \ddot{X}(b) + C(b) \dot{X}(b) + K(b) X(b) = F(t)$$

(7)

where $M$, $C$ and $K$ are the mass, damping and stiffness matrices, $F$ is a vector of steady-state harmonic forces acting on the top of the main rotor shaft, and $X$ is a vector of harmonic response displacements. The forced response constraints can be written as:

$$X - X_a \leq 0$$

(8)

where $X$ is the amplitude of displacement at a specified location in the airframe and $X_a$ is the allowable value. The sensitivities of the forced responses are obtained by differen-
The steady-state forced response and dynamic stress analysis. Assembly; (2) static finite program are illustrated in Figure 1. The operations in (ref. dynamic 65) operations in the computational steps used in the DYNOPT program are organized to perform the necessary computations upon reading the appropriate input data, and to operate on a database to store and retrieve data generated and stored at intermediate steps of the computation.

**Dynamic Stress Constraints**

Because vibration and fatigue are closely related problems, the airframe structural dynamic design considerations are formulated to also insure that the dynamic stresses are within acceptable limits by proper sizing of the structural members. The dynamic stress constraint is formulated as:

\[
\sigma_i \leq \sigma \leq \sigma_{iu}
\]

where \( \sigma \) is the computed mean dynamic or fatigue stress and \( \sigma_i \) and \( \sigma_{iu} \) are the lower and upper bound stress for the \( i \)th structural member. The dynamic stresses are computed from the relation:

\[
\sigma = S X
\]

where \( S \) is the stress-displacement transformation matrix computed in a finite element analysis. The sensitivity of the stress constraint can be written as:

\[
\partial \sigma / \partial b = S \partial X / \partial b + \partial S / \partial b X
\]

where \( \partial X / \partial b \) is the derivative of the forced response displacement, and \( \partial S / \partial b \) is the derivative of the stress-displacement transformation matrix with respect to the design variables.

**DYNOPT COMPUTER PROGRAM**

A computer program called DYNOPT (DYnamics OPTimization) was developed to carry out the optimization computations based on the nonlinear mathematical programming approach. The DYNOPT code features a unique operational combination of the MSC/NASTRAN (Version 65) finite element structural analysis code (ref. 25) extended to include the calculation of steady-state forced response and dynamic stress sensitivities, and the CONMIN optimizer (ref. 28). The computational steps used in the DYNOPT program are illustrated in Figure 1. The operations in the finite element analysis are: (1) stiffness and mass matrix assembly; (2) static analysis; (3) frequency analysis; and (4) steady-state forced response and dynamic stress analysis. The operations in the sensitivity analysis are: (1) static sensitivity; (2) frequency sensitivity; (3) forced response displacement and dynamic stress sensitivity; and (4) weight sensitivity. The static and frequency sensitivity analyses are performed using solution sequences 51 and 53 in the MSC/NASTRAN program. The forced response displacement, dynamic stress, and weight sensitivity modules are newly developed using the Direct Matrix Abstraction Program (DMAP) language of NASTRAN. For repetitive objective and constraint function evaluations, three different approximation techniques have been incorporated into the DYNOPT program. These approximation techniques are based on the use of direct, reciprocal, and hybrid forms of the Taylor's series expansion in the design variables. Depending on the optimization problem and degree of nonlinearity of the objective and constraint functions, any of these approximation techniques are chosen. The method of feasible directions available in the CONMIN optimizer program is used for design change computations. The various computational steps are organized into several independent modules in the DYNOPT program. Each module of the DYNOPT program is organized to perform the necessary computations upon reading the appropriate input data, and to operate on a database to store and retrieve data generated and stored at intermediate steps of the computation.

![Figure 1. Computational Steps Used in the DYNOPT Program](image)
transferring data between programs and restarting the CONMIN optimizer after the NASTRAN reanalysis, and
(3) updating the NASTRAN input data for changes in element cross-sectional sizes and material properties
concerning to the design variable changes computed in the optimizer. Using interactive commands of
the computer operating system, the appropriate modules in the DYNOPT program can be selected for execution depending
on the types of objective and constraint functions specified for the optimization problem.

AIRFRAME DESIGN PROCESS

Before proceeding to the application of the DYNOPT program to a helicopter airframe, it is helpful to
recognize the different phases of a typical airframe design process. The primary phases of design are: 'Conceptual de-
sign', 'Preliminary design', 'Detailed design', and 'Ground and Flight test'. In the conceptual design phase, candidate
configurations of an airframe are evaluated through trade-off studies on weight, aerodynamics, mission, performance,
and stability. In the preliminary design phase, the configurations emerging from the conceptual design phase are
worked out in greater detail, including layout of major structural members and selection of materials. In the de-
tailed design phase, airframe members are sized based on strength, vibration, weight, and crash-worthiness require-
ments and the structural integrity is checked for various load cases within the flight envelope. In the ground and
flight test phase, modifications are made to the airframe, if necessary, to enable the helicopter to satisfy the design re-
quirements.

The practical application of optimization techniques to airframe vibration design work will require
attention to the specific nature of the work performed in each of the phases of the airframe design process. This is
necessary to ensure that any optimization-based design procedures which are developed are compatible with current
engineering design practice. An optimization methodology which appears suited to the aforementioned design process
is described in the appendix. A discussion of the major considerations needed in the application of optimization
techniques in the various phases of an airframe design process is also given in the appendix.

APPLICATION OF DYNOPT TO A HELICOPTER AIRFRAME

This section summarizes the application of the DYNOPT program to the Bell AH-1G helicopter airframe structure. The numerical results obtained from the application of the program are discussed to illustrate some of the essential computational tasks involved in applying the optimization methodology (as described in the appendix) in both the preliminary and detailed design phases of an airframe design.

AH-1G Helicopter Airframe Structure

The structure of the Bell AH-1G helicopter airframe (ref. 29) with its skin panels removed is shown in
Figure 2a. The airframe structure is composed of several major components - fuselage, tail boom, vertical fin, landing gear, main rotor pylon, main rotor shaft, wings and

(a) Airframe Structure with Skin Panels Removed

(b) Elastic-line Model

(c) Built-up Fuselage / Elastic-line tail boom Model

(d) Complete Built-up Model

Figure 2. Bell AH-1G Airframe Structure and Finite Element Models
wing-carry through structure. The gross weight of the AH-1G helicopter is 8399 lbs. This weight is composed of structural weight, non-structural weight, and useful weight items. The weight of the primary structure is about 1000 lbs.

**Analysis Models of AH-1G Airframe**

Three different finite element models of the airframe are available (Fig. 2). However, in the optimization studies discussed here only two of these models were used: the elastic-line (or 'stick') model (ref. 14) shown in Figure 2b and the detailed (or 'built-up') model (Fig. 2c) shown in Figure 2c.

In the airframe stick model (Fig. 2b), the fuselage, tail boom, wings and rotor shaft structure were modeled with beam elements. The MSC/NASTRAN finite element model consists of 42 beam elements, 13 scalar spring elements and 12 rigid bar elements. There are 56 grid points in the model for a total of 336 degrees of freedom. A consistent mass representation is employed to model the weight of the primary structure in the airframe. A finite element analysis was carried out to determine the natural frequencies and mode shapes of the stick model of the airframe.

The fuselage and wing structures in the airframe built-up model (Fig. 2c) were modeled primarily with rods, shear panels and membrane elements. The tail boom, vertical fin and tail rotor shaft were modeled with beam elements in the same manner as they were in the stick model. The MSC/NASTRAN finite element model of the airframe consists of a total of 2954 finite elements which includes: 2001 rods, 197 beams, 340 shear panels, 243 triangular membranes, 160 quadrilateral membranes and 13 scalar spring elements. There are 504 grid points for a total of 3024 degrees of freedom. The natural frequencies computed using the stick model are within 10% of the frequencies of the built-up model for the modes of interest here.

**Design Models**

Two different design models of the airframe (see Figs. 3 and 4) were developed for the optimization studies using the DYNOPT program. The model shown in Figure 3 might be appropriate for use in preliminary design while the model shown in Figure 4 might be appropriate for use in detail design. In the discussions that follow, these models are referred to as the preliminary design model and the detailed design model, respectively.

The preliminary design model (Fig. 3) takes as design variables the overall depth (d) of the cross-section of the primary structure at several stations along the airframe. The design model chosen reflects the several different types of cross-sections which comprise the primary structure of the airframe. The locations of structural members acting as stiffeners are indicated by solid dots while the location of flanges, webs, and skins are indicated by solid lines. In the design model, the depth of these sections in the fuselage and the tail boom was allowed to vary while holding fixed the sizes of the stiffeners, flanges, webs, and skins. The design model has a total of 46 design variables. An empirical relationship between the design variables of the design model (Fig. 3) and the element section properties of the finite element model (Fig. 2b) was established to update the NASTRAN bulk data deck during optimization iterations. For the numerical studies, the design variables were bounded to within ±50% of their initial values.

![Figure 3. Preliminary Design Model Used for Optimization Studies](image)

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Optimization Using the Preliminary Design Model

Three different optimization problems were formulated using the preliminary design model of the airframe. These optimization problems are used to demonstrate the application of the DYNOPT code for tuning the airframe natural frequencies, and for reducing dynamic stresses in the structural members.

First Problem:

In the first optimization problem, the objective function was the weight of the primary structure of the airframe. Constraints were imposed on the natural frequencies corresponding to the pylon pitching ($\omega_1$), pylon rolling ($\omega_2$), first vertical bending ($\omega_3$), second vertical bending ($\omega_4$), and torsional ($\omega_5$) modes. The first few natural frequencies and the corresponding mode shapes are shown in Figure 5. The objective function and the constraints are given by equations 13 through 15:

$$ F = \sum \rho_i A_i L_i \quad i=1,46 \quad (13) $$

$$ g_{iL} = \omega_i - \omega_{iL} \leq 0, \quad i=1,5 \quad (14) $$

$$ g_{iu} = \omega_i - \omega_{iu} \leq 0, \quad i=1,5 \quad (15) $$

where $\rho_i$ is the material density, $A_i$ is the area of the cross-section and $L_i$ is the length of the beam element. Subscripts $L$ and $u$ indicate lower and upper bounds on the natural frequencies. In equations 14 and 15, the lower bounds on the frequencies are 2.5, 3.5, 5.0, 15.0, and 20 Hz and the upper bounds are 3.5, 4.5, 11.0, 20.0 and 25.0 Hz.

The sensitivity coefficients for the frequency constraints were computed using the DYNOPT program. Figure 6 shows the distribution of sensitivity coefficients for the constraints on the first and second vertical bending mode frequencies. The sensitivity coefficients indicate that the design variables in the rear fuselage and most of the tail boom would be effective in changing the frequency of the first vertical bending mode. The figure also indicates

Figure 6. Sensitivity of Weight and Natural Frequency
that the design variables in both the fuselage and tail boom would be effective in changing the frequency of the second vertical bending mode. The weight gradients of the airframe were also computed and are shown in the figure. It is seen that the design variables in the central and rear fuselage structures would have a significant effect in changing the weight of the airframe.

The history of the objective function and constraints is plotted in Figure 7. During the optimization iterations, the frequency constraints were satisfied and the optimizer computed the design changes by reducing the value of the objective function. In the first iteration, the airframe structural weight reduced from 1000 lbs. to 987 lbs. In the second iteration, the frequency constraints were still within bounds and the airframe weight reduced to 986 lbs. In this iteration, the weight reduction was small compared with the previous iteration because of the decreased step size used in the optimizer. In the third iteration, the airframe weight was reduced to 937 lbs. In this iteration, the constraint on the frequency of the first vertical bending mode reached its lower bound and the constraints on the other frequencies were still within their bounds. In the fourth, fifth, and sixth iterations the constraints on pylon pitching, rolling, first and second vertical bending and the torsion modes became active. Many of the design variables in the rear fuselage region reached their lower bounds. The weight of the airframe reduced to 900 lbs. in the sixth iteration. Subsequent iterations indicated insignificant changes in the design variables and the computations were terminated. A comparison of the initial and final values of the design variables is shown in Figure 8. It can be seen that the design variables were reduced in the rear fuselage and forward tail-boom structure whereas they increased in the forward fuselage and rear tail-boom structure.

The iteration history of the objective function and constraints for this problem are shown in Figure 9. In Figure 9, the element numbers are designated by the grid point number of their end points. For example, the element connecting grid points 26 and 27 is designated element 2627. In the first two iterations, the structural weight and the dynamic stresses reduced slightly because of the small step size in these iterations. In the 3rd and 4th iterations, however, there are considerable changes in the structural weight and stresses due to the larger step size in the optimizer. In the final iteration, the structural weight is 1034 lbs., which is 34 lbs. higher than the initial weight. The design variables increased by 8 to 11% of the initial design in the rear fuselage and 3 to 4% in the center fuselage and tail-boom region. This increase in the design variables (and hence the structural weight) is a result of the optimizer seeking a feasible design space to satisfy the dynamic stress constraints. It should be noted that optimization control parameters such as the step size, push-off factor (that pushes the violated constraints into feasible region) and the participation coefficient (indicating the degree to which the design is to be pushed to the feasible region) had a significant influence in the optimization solution.

**Figure 8. Initial and Final Design of the Primary Structure**

**Second Problem:**

In the second optimization problem, the objective function was taken to be the weight of the primary structure and the constraints were imposed on the dynamic stresses in the structural members (Eq. 10). The dynamic stresses in the structural members were computed for the case of a vertical excitation force at the rotor hub having a magnitude of 1,000 lbs. acting at a frequency of 10.8 Hz. (2/rev). The upper bound stress constraints in the tail-boom elements (see fig. 2b) connected by grid points (16-17), (18-19), (20-21), (22-23), (24-25), and (26-27) are 10.0 psi., 10.0 psi., 5.0 psi., 4.0 psi., and 5.0 psi., respectively. The bending stresses corresponding to those elements at the initial design are 6.91 psi., 7.44 psi., 6.28 psi., 5.64 psi., 6.50 psi., and 2.22 psi., respectively.

The iteration history of the objective function and constraints for this problem are shown in Figure 9. In Figure 9, the element numbers are designated by the grid point number of their end points. For example, the element connecting grid points 26 and 27 is designated element 2627. In the first two iterations, the structural weight and the dynamic stresses reduced slightly because of the small step size in these iterations. In the 3rd and 4th iterations, however, there are considerable changes in the structural weight and stresses due to the larger step size in the optimizer. In the final iteration, the structural weight is 1034 lbs., which is 34 lbs. higher than the initial weight. The design variables increased by 8 to 11% of the initial design in the rear fuselage and 3 to 4% in the center fuselage and tail-boom region. This increase in the design variables (and hence the structural weight) is a result of the optimizer seeking a feasible design space to satisfy the dynamic stress constraints. It should be noted that optimization control parameters such as the step size, push-off factor (that pushes the violated constraints into feasible region) and the participation coefficient (indicating the degree to which the design is to be pushed to the feasible region) had a significant influence in the optimization solution.

**Figure 9. Optimization History for the Preliminary Design Model (Second Problem)**
Third Problem:

In the third optimization problem, the objective function was again the structural weight, but the constraints on the natural frequencies and some of the constraints on the dynamic stresses from the first two problems were simultaneously imposed on the airframe. Therefore, this problem is similar to the first problem with 4 additional constraints on the dynamic stresses. It should be remarked that the natural frequency and dynamic stress constraints were imposed independently of one another in the previous two optimization problems. The upper bound stress constraints in tail-boom elements (see Fig. 2b) connected by grid points 21 and 22 is designated element 2122. The bending stresses corresponding to those elements at the initial design are 6.91psi., 7.44psi., 6.97psi., and 5.74psi., respectively. Thus, there are 10 natural frequency constraints and 4 dynamic stress constraints in the optimization problem.

Figure 10 shows the optimization iteration history for the objective and the constraint functions. In Figure 10, the element numbers are designated by the grid point number of their end points. For example, the element connecting grid points 21 and 22 is designated element 2122. The structural weight remains almost the same in the various iterations. Recall that the weight decreased in the presence of frequency constraints in the first problem and the weight increased in the presence of dynamic stress constraints in the second problem. In the first few iterations, the dynamic stresses in the elements exceeded their constraint limits, however, the stresses were reduced below their limiting values in the subsequent iterations. The natural frequency constraints (not shown) remained within the feasible region during the iterations.

Optimization Using the Detailed Design Model

A different problem was formulated to illustrate the optimization of the AH-1G helicopter airframe using the detailed design model (Fig. 4). In this problem, the forced response displacement at selected locations in the airframe was used to formulate both the objective function as well as the constraint functions. The finite element model shown in Figure 2c was used for the analyses. The forced response displacements at various locations in the airframe were computed for a force of 1000 lb. at a frequency of 10.8 Hz (2/rev) acting vertically at the top of the main rotor shaft. The objective function was the forced response displacement at the pilot seat location $X_p$ (which location in the built-up model of Figure 2c approximately corresponds to grid point 8 in the stick model of Figure 2b). Constraints were imposed on the forced response displacements at the nose (grid point 2), gunner (grid point 4), engine (grid point 60), tail boom (grid point 24) and fin (grid point 30) locations. These constraints are all of the form $g_i = X_i/X_a - 1$. The constraint limits $X_a$ at the nose, gunner, and the engine locations were 0.0025in. and those at the tail boom and fin locations were 0.005in.

Figure 10 shows the distribution of the sensitivity coefficients for the forced response displacement at the pilot seat with respect to the fuselage and tail-boom design variables. A comparison of the magnitudes of the sensitivity coefficients in the figure indicates that the response is an order of magnitude more sensitive to changes in the design variables in the tail boom portion of the airframe than with respect to changes in the design variables in the fuselage portion of the airframe. This comparison indicates that the design variables in the tail boom would be more effective in reducing the response at the pilot seat location.

![Figure 10. Optimization History for the Preliminary Design Model (Third Problem)](image)

![Figure 11. Sensitivity of Forced Response Displacement Constraint](image)
The optimization history of the objective and constraint functions in the various iterations are shown in Figure 12. The objective function was reduced, indicating a trend of decreasing vibration response at the pilot seat. As shown in the figure, all constraints are satisfied throughout the optimization process except for iteration one at the gunner location. During the iterations, the constraints at the gunner and nose location were reduced, indicating a reduction of the vibration response at those locations; however, the constraint values at the fin, tail and engine locations increased gradually.

SUMMARY AND CONCLUDING REMARKS

The paper has summarized the experiences and results from a study undertaken at the Langley Research Center to investigate the use of formal, nonlinear programming-based numerical optimization technique for helicopter airframe vibration reduction. The objective and constraint functions and the sensitivity expressions used in the formulation of airframe vibration optimization problems were presented and discussed. Implementation of a new computational procedure based on MSC/NASTRAN and CONMIN in a computer program system called DYNOPT for optimization of airframes subject to strength, frequency, dynamic response and dynamic stress constraints was described. An optimization methodology was proposed which is thought to provide a new way of applying formal optimization techniques during the various phases of the airframe design process. Numerical results obtained from the application of the DYNOPT optimization code to the Bell AH-1G helicopter airframe were discussed.

Specifically, this study has:

1. Provided significant insight into how the practical problem of airframe vibration reduction can be posed as an optimization problem by examining the various considerations needed in the formulation of optimization problems for both new and existing airframes.

2. Demonstrated the use of the DYNOPT optimization code for tuning airframe natural frequencies and for reducing weight, dynamic stresses, and vibration amplitudes in the airframe structure by performing numerical computations on both preliminary and detailed design models of the Bell AH-1G helicopter airframe.

3. Defined an optimization methodology which simplifies the complex task of airframe optimization by dividing the optimization computations into several independent and sequential tasks organized according to the various phases of the airframe design process.

The subject optimization studies have provided considerable practical experience in the systematic application of formal optimization techniques to the problem of vibration reduction in helicopter airframe structures. The results of these applications are quite encouraging. However, more application experience on other airframe models is needed both to assess the proposed optimization methodology and to establish clearly the exact role which optimization can play in the airframe structural design process.

REFERENCES


APPENDIX

CONSIDERATIONS IN FORMULATING OPTIMIZATION PROBLEMS

A Proposed Methodology for Optimization

The work involved in the various phases of an airframe design encompasses many disciplinary areas. In such a multi-disciplinary and multi-phase design environment, the formulation of a single optimization problem applicable to all phases of design would be an extremely complex and difficult task. Therefore, a simpler approach to both the formulation and solution of the vibration optimization problem is needed. In an attempt to meet this need, an optimization methodology was identified which is thought to provide a new way of applying formal optimization techniques during the various phases of the airframe design process.

Basically, the methodology involves the formulation and solution of separate optimization problems, one corresponding to each of the phases of the airframe design process: conceptual design, preliminary design, detail design, and ground and flight test, as depicted in Figure 13. In the methodology, the necessary optimization analyses required for the different phases are sequentially organized as depicted in the figure. The optimization tasks, such as the formulation of the problem, structural analysis, sensitivity analysis, and design change computations, are independently performed in each phase as indicated by the flow diagram in each block of the figure. The optimization formulation in each block includes a set of design variables, an objective function, and constraints that are appropriate to that particular phase. The formulation differs from one design phase to another. In a given design phase, necessary analyses are performed to evaluate the objective function, the constraints and the sensitivity derivatives required for the solution of the optimization problem at that particular stage of design. These results are used in a nonlinear programming algorithm for determining the design changes necessary to solve the optimization problem. The optimum design solution determined in one phase is used as an "initial" design in the subsequent phase. Additional and/or redefined design variables and constraints are included as required in each subsequent phase. In this process, the airframe design obtained from the last phase is the "best" or optimum one. The assumption implied here (which is probably reasonable in a practical design situation) is that the design obtained from each phase is successively improved in subsequent phases to obtain an optimum design in the final phase. The sequential and independent organization of optimization computations in this approach avoids the need for complex numerical procedures to link the computations from the different phases. It should be noted that the optimization methodology which is being proposed here falls under what might be termed a multi-phase approach to optimization rather than the multi-level decomposition approach which has also been proposed (see, for example, ref. 30).

Considerations for New Airframes:

Consideration required in formulating optimization problems in the conceptual, preliminary, detailed, and ground and flight test phases of a new airframe design are identified and discussed below.

Conceptual Design

Although vibrations considerations are not generally addressed in the conceptual design phase, the proposed optimization methodology includes it because there appears to be some potential for optimization to influence the airframe design for vibrations in this phase. Design variables that can be used in the formulation of the optimization problem in the conceptual design phase are depicted in Figure 14. An initial rough estimate of the
vibration characteristics of an airframe can be made in the conceptual design phase based on the knowledge of the past vibration history of a similar class of helicopters. Then configuration details such as the number of blades, rotor speed, flight loads/speed, gross weight, airframe shape and dimensions (Figs. 14a-b), layout of large non-structural masses such as the engine, transmission, fuel, and payload (Figs. 14c-d) can be used to further estimate vibration characteristics. It is expected that even with a basic attention to vibration characteristics, some potential vibration problems could be identified and reduced. Considerations in formulating an optimization problem in this phase should, therefore, be based on those configuration aspects that directly or indirectly influence airframe vibrations. The use of configuration design variables for vibration minimization necessitates consideration of the multi-disciplinary aspects of airframe design involving aerodynamics, layout of components, airframe shape and dimensions, weight, and stability. The modification of a configuration has a direct influence on airframe vibrations because the configuration is directly associated with the distribution of airframe structural stiffness and mass which affects the vibration characteristics. Vibration characteristics will also change significantly with any changes in the location of large mass components in the airframe.

Preliminary Design

In the preliminary design phase, the primary load paths in the airframe are determined, the arrangement of major load-carrying members are established, and the materials are selected. Candidate design variables in the preliminary design phase (Fig. 15) could include the following: the layout of major structural members such as bulkheads, beams, longerons and stringers (s, l); material properties of the primary structural members (E); cross-sectional area (A) and moment-of-inertia (I) of major structural members; and overall cross-sectional geometry of the primary structure defined by the distribution of the breadth (b) and depth (d) of the built-up-structural members which carry the major loads of the helicopter. Simple 'elastic-line' or 'stick' models of the airframe (such as shown in Fig. 2b) are usually developed for vibration analysis based on approximate distributions of stiffness and mass of the airframe. Airframe vibration characteristics obtained from such simplified models are much better than those estimated during the conceptual phase of design. Therefore, it is possible to include more detailed vibration considerations in the formulation of an optimization problem in this phase of airframe design. It would appear that the use of optimization in this phase of design has considerable potential to influence the airframe design for minimum vibrations.

Detailed Design

The formulation of an optimization problem in the detailed design phase allows for the consideration of constraints evaluated from more detailed discipline-oriented analyses of vibrations, strength, and weight of the airframe. Candidate design variables in this phase are the cross-sec-
tional dimensions of structural members such as the width and the thickness of stringer sections, the depth of beams, and the thickness of panels (Fig. 16). In this phase, the details of the thousands of structural members comprising the airframe and their layout are available. Complete built-up finite element models (such as shown in Fig. 2d) having better representation of the structural, material and geometric properties of an airframe can be developed and used to compute much improved estimates of the structural strength, vibration responses, and weight of the airframe. The use of optimization in this phase is thought to have good potential for influencing airframe design for minimum vibration.

Ground and Flight Test

In practice, any serious attempt to address airframe vibrations usually begins late in the helicopter design process - after actual vibration problems are identified during ground and flight test. Severe vibrations in specific areas of the helicopter such as the tail boom, landing gear, and engine supports could possibly be identified in the ground and flight test phase. Vibration alleviation in such areas can be addressed through local structural modifications of the airframe. Considerations to formulate the optimization problem for the ground and flight test phase should be based on the specific vibration problems identified in this phase. Because of the limited choice of design variables, and also because of the narrow bounds which would be placed on the allowable changes in the design variables, it may be difficult to find a feasible optimization solution to the vibration-minimization problem. Therefore, the use of optimization in this phase of design would probably have only a limited potential to influence the design for vibration reduction.

Considerations for Existing Airframes

Considerations needed in the formulation of an optimization problem for an existing airframe were found to be different from those needed in the design of a new airframe. In the case of an existing airframe structure, it is important to note that major modifications to the airframe structure are generally not permissible and that only small modifications to a few structural members can probably be made. In the formulation of the problem, this restriction in the allowable structural modifications needs to be considered by imposing narrow bounds on the allowable changes in the structural design variables. This restriction could severely limit the structural changes that could be made to reduce the vibration to the desired level.

Figure 16. Design Variables in Detailed Design Phase
Experiences at Langley Research Center in the Application of Optimization Techniques to Helicopter Airframes for Vibration Reduction

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<td>A NASA/industry rotorcraft structural dynamics program known as DAMVIBS was initiated at Langley Research Center in 1984 with the objective of establishing the technology base needed by the industry for developing an advanced finite-element-based vibrations design analysis capability for airframe structures. As a part of the in-house activities contributing to that program, a study was undertaken to investigate the use of formal, nonlinear programming-based, numerical optimization techniques for airframe vibrations design work. Considerable progress has been made in connection with that study since its inception in 1985. This paper presents a unified summary of the experiences and results of that study. The formulation and solution of airframe optimization problems are discussed. Particular attention is given to describing the implementation of a new computational procedure based on MSC/NASTRAN and CONMIN in a computer program system called DYNOPT for the optimization of airframes subject to strength, frequency, dynamic response, and fatigue constraints. The results from the application of the DYNOPT program to the Bell AH-1G helicopter are presented and discussed.</td>
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