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Chapter IV

A Variational Formalism for the Radiative Transfer Equation: Prelude to MODEL III

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1. Introduction

The MODEL III variational data assimilation model is the third of four general assimilation models designed to blend weather data measured from space-based platforms into the meteorological data mainstream in a way that maximizes the information content of the satellite data. Because there are many different observation locations and there are many instruments with different measurement error characteristics, it is also necessary to require that the blending be done to maximize the information content of the data and simultaneously to retain a dynamically consistent and reasonably accurate description of the state of the atmosphere. This is ideally a variational problem for which the data receive relative weights that are inversely proportional to measurement error and are adjusted to satisfy a set of dynamical equations that govern atmospheric processes. Because of the complexity of this type of variational problem, we have divided the problem into four variational models of increasing complexity. The first, MODEL I, includes as dynamical constraints the two horizontal momentum equations, the hydrostatic equation, and an integrated continuity equation. The second, MODEL II includes as dynamical constraints, the equations of MODEL I plus the thermodynamic equation for a dry atmosphere. MODEL III includes the equations of MODEL II plus the radiative transfer equation.

The advantage of MODEL III over the previous two models is that radiance, the atmospheric variable measured by satellite, becomes a dependent variable. In the previous versions, mean layer temperatures that had been retrieved from the radiances by some method, were included in the assimilation by substituting them in place of the rawinsonde temperatures. Now both rawinsonde temperatures and satellite radiances are included independently in the assimilation.

Our approach to the development of MODEL III has been to divide the problem into three steps of increasing complexity. Chapter IV deals with the first step, a variational version of the classical temperature retrieval problem that includes just the radiative transfer equation as a constraint. The radiances for each of the four TOVS MSU microwave channels are dependent variables. These plus temperature constitute a set a five adjustable variables. Each radiance is related to the temperature through its radiative transfer equation. There are therefore four dynamic constraints in this first variational problem.

Chapter V summarizes the second step which combines the four radiative transfer equations of the first step with the equations for a geostrophic and hydrostatic atmosphere. This step is intended to bring radiance into a three-dimensional balance with wind, height, and temperature. The use of the geostrophic approximation in place of the full set of primitive equations allows for an easier evaluation of how the inclusion of the radiative transfer equation increases the complexity of the variational equations.

The third and final step includes the four radiative transfer equations with the fully nonlinear set of primitive equations, ie., MODEL III.

2. A Variational Retrieval Algorithm

The radiative transfer equation is the only variational constraint. It takes the form

$$B-B_o w_o-\int_0^{\infty} w' T dz=0 \qquad (1)$$

where B is the brightness temperature as computed from radiance measured at the satellite and T is the mean layer temperature of an incremental depth of the atmosphere, dz. The weight, w_0, is the transmittance of the total atmosphere from the surface (where the surface brightness temperature, B_0, is measured) to the space-based observation platform. The weights, w', are proportional to the transmittance from some level within the atmosphere to the satellite. In order to make the variational derivations from (1) compatible with the larger set of variational equations in MODEL I and MODEL II, we will make the following modifications in (1). First, the brightness temperature is replaced by the skin temperature, T_0, and the weight, w_0, will become a skin level surface weight. Second, (1) is converted from the z to the sigma vertical coordinate. In this conversion,

$$\int_0^{\infty} w' T dz = \int_0^{\infty} w' T [f(T)]\, d\sigma = \int_0^{\infty} wT d\sigma \tag{2}$$

Now f(T), a small conversion term that results from the changeover to sigma coordinates, will be combined with the weights and not subjected to variation. This approach avoids complicated nonlinear equations that will otherwise arise through the variational formations. The f(T) and the weak temperature dependence in the weights will not be held constant however. At each step of a converging iterative process, the small temperature dependencies will be updated with adjusted temperatures. With these modifications, (1) becomes,

$$B - \int_0^{\infty} wT d\sigma = 0 \tag{3}$$

The next step is to bring (3) into dimensional compatibility with the more general variational models. Let,

$$T = \theta T' = \frac{gH}{R} T' \tag{4}$$

and

$$T' = T_R + \frac{F}{R_o} T'' \tag{5}$$

so that,

$$T = \frac{gH}{R}\left(T_R + \frac{F}{R_o}T''\right) \tag{6}$$

Here g is gravity, H=10 km is a reference height, R is the universal gas constant, F is the Froude number, and R_o is the Rossby number. The subscript R refers to a reference atmosphere and the notation " refers to departures from the reference atmosphere. Substitution of (6) in place of T in (3) gives,

$$B - \frac{gH}{R}\left[\int_0^\infty wT_R d\sigma + \frac{F}{R_o}\int_0^\infty wT'' d\sigma\right] = 0 \tag{7}$$

Further, we partition B = B_R + B_m and define

$$B_R = \frac{gH}{R}\int_0^\infty T_R d\sigma \tag{8}$$

It follows then, that

$$B'' = \frac{gH}{R}\frac{F}{R_o}B_m \tag{9}$$

Finally, upon suppression of the double primes, the radiative transfer equation becomes,

$$B-\sum_{k=1}^{K} w_k T_k = 0 \tag{10}$$

Now there are four TOVS microwave channels each with an independent measurement of the brightness temperature. Let B_j be the brightness temperature perturbation for the jth channel. The J constraining equations are,

$$m_j = B_j - \sum_{k=1}^{K} w_{kj} T_k = 0 \tag{11}$$

The functional to be minimized is

$$F = \int I d\sigma \tag{12}$$

where

$$I = \sum_{k=1}^{K} \pi_k (T_k - T_k^o)^2 + \sum_{j=1}^{J} \pi_j (B_j - B_j^o)^2 + 2\sum_{j=1}^{J} \lambda_j m_j \tag{13}$$

Performing the variations upon T and B as shown by Achtemeier, et al. (1986) yields the following Euler-Lagrange equations,

$$\delta T: \quad \pi_k (T_k - T_k^o) - \sum_{j=1}^{J} w_{kj} \lambda_j = 0 \tag{14}$$

for each k and,

$$\delta B: \quad \pi_j(B_j - B_j^o) + \lambda_j = 0 \tag{15}$$

for each j. Variation upon the J Lagrange multipliers restore the original constraints (11). These equations are linear and may be easily reduced to one diagnostic equation in temperature. First, eliminate reference to the Lagrangian multipliers by substituting (15) into (14). Then substituting for B_j gives the adjusted temperature as a function of weight functions and observed variables,

$$\pi_k T_k + \sum_{i=1}^{K} \sum_{j=1}^{J} \pi_j W_{kj} W_{ij} T_i - F_k = 0 \tag{16}$$

for each k. Here

$$F_k = \pi_k T_k^o + \sum_{j=1}^{J} \pi_j B_j^o \tag{17}$$

Equation (16) can be easily solved with a standard matrix inversion package to retrieve the variationally adjusted temperature profile. At most two cycles with the weight functions updated with adjusted temperatures are required for convergence to a final adjusted temperature.

4. Results

In order to properly interpret the results of the example of variational adjustment with the radiative transfer equation, one must be aware that three sets of weights appear in (16). The weights, w_{ij}, are the transmittance weights for the ith level and the jth microwave channel. They are not subject to the variational adjustment and remain unchanged with the exception of minor adjustments for temperature sensitivity. The variational weights, π_j and π_k, carry the relative importance of the jth microwave channel and the temperature at the kth level. It is the choice of the variational weights that are important in interpreting the results.

Consider a temperature profile that is to be retrieved from MSU brightness temperatures. It is to be made halfway between two rawinsonde sites. The rawinsonde soundings are given by A and B in Fig. 1. Sounding A is cold up to the tropopause (about 220 mb) and then it becomes isothermal up to 60 mb. Sounding B is warm from the surface to 170 mb and then becomes colder than A in the layer from 170 mb to 60 mb. Its tropopause is located at 100 mb.

The first guess or "observed" sounding that will enter into the temperature part of the variational analysis is the mean of A and B. It is given by M in Fig. 1. Now suppose that the true sounding is given by T. Note that M=T from the surface to 230 mb and from 50 mb to the top.

Next, the brightness temperatures, B_j, were calculated from (10) using the true temperature sounding. Thus the B_j° that enter (17) are true and the T_k° are approximate. However, only the temperatures between 100-230 mb need adjustment. The observational error for the temperature was 0.7 K and the weight accorded to the temperature was,

$$\pi_k = \frac{1}{2\sigma_T^2} = 1.0 \quad \textbf{(18)}$$

Fig. 2 shows the results of three retrievals between 500 mb and the top. The dashed line is the difference M-T between the true and first guess temperature soundings. The other curves are the differences between the adjusted and the true temperatures for π_j that ranged in values from 10 to 100 to 1000. Note that the weights for the four MSU channels and hence the brightness temperatures were always equal.

Fig. 2 shows that increasing the brightness temperature weights progressively reduced the differences between the adjusted and true temperature soundings but by only 2.5 K. However, the retrievals also spread the adjustments throughout the depth of the sounding. Therefore, improvements where the M-T residuals were nonzero were offset by degraded temperatures throughout the reminder of the sounding - the errors being almost 2 K at 250 mb with lesser error elsewhere.

A more extensive analysis of the behavior of (16) found that the retrievals were sensitive to the vertical distribution of the weights for the temperature hence the errors of observation for the temperature. If there existed some independent observations that could be used to estimate the accuracy of the first guess temperature as a function of height, then the retrievals could be focused into those locations where the M-T residuals were greatest. Consider possible accuracy functions given in Fig. 3. The effective temperature error at 150 mb is doubled by f(1) and is tripled by f(2). Therefore, the weights accorded to the temperature there are decreased by a factor of four for f(1) and a factor of nine for f(2).

Fig. 4 shows the residuals between the adjusted and true temperature profiles for the three retrievals when the accuracy function f(1) was applied to the temperature weights. The initial residual has been reduced by approximately 6 K. Fig. 5 shows the results for f(2). Additional reductions in the residuals over f(1) results were found between 150 and 100 mb. Fig. 6 summarizes the resulting temperature soundings for f(0), f(1), and f(2) if the weights for the brightness temperatures were $\pi_j = 1000$. The improvement of f(2) over f(1) is apparent between 150 and 100 mb but elsewhere the differences between the two retrievals are only a few tenths of a degree. This suggests that it is the shape of the accuracy function, not the magnitude, that determines where the variational adjustment will be focused.

Fig. 7 shows part of the temperature soundings T and M between 250 mb and 50 mb. The curve identified by V1 is the sounding that was obtained with the conditions that the weights for the first guess temperature were constant with height. The sounding V2 results from the application of f(2) to the temperature weights.

The first step in the variational analysis of the radiative transfer equation succeeded in producing a variational algorithm that could be used to retrieve temperature from the four MSU channel brightness temperatures given a first guess temperature sounding. The results showed that the variational retrievals were subject to the same limitations as are retrievals by other methods, inability to accurately resolve temperatures near the tropopause spreads error though the whole retrieved sounding, unless some temperature accuracy function is employed to focus the retrieval. The identification of a data set that could be used for a temperature accuracy function and the derivation of the same is beyond the scope of this study.

REFERENCE

Achtemeier, G. L., H. T. Ochs, III, S. Q. Kidder, R. W. Scott, J. Chen, D. Isard, and B. Chance, 1986: A variational assimilation method for satellite and conventional data: Development of basic model for diagnosis of cyclone systems. NASA Con. Rept. 3981, 223 pp.

FIGURE CAPTIONS

Figure 1. Two typical temperature soundings A and B; the mean of A and B, sounding M; and true temperature sounding T used for sensitivity studies of variational temperature retrievals.

Figure 2. Dashed line: differences between the mean or first guess temperature sounding and true sounding. Solid lines: differences between variational temperature retrievals and true temperature sounding for the following choices of brightness temperature weights; sounding 1 (10), sounding 2 (100), sounding 3 (1000).

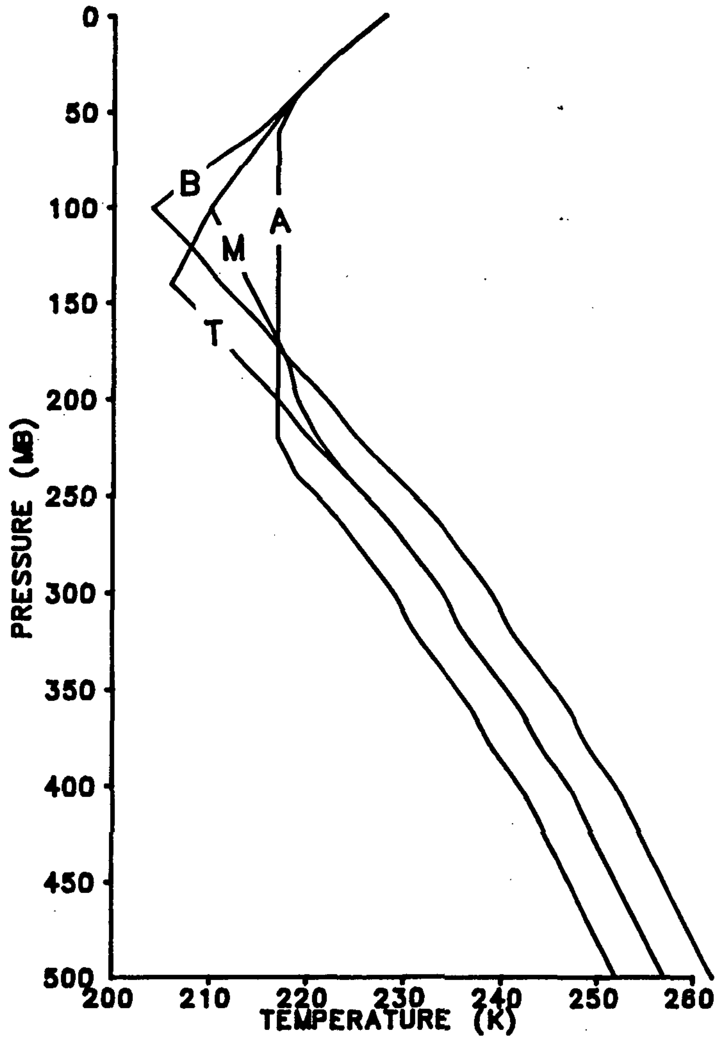
Figure 3. Curves for hypothesized temperature accuracy functions.

Figure 4. Same as Fig. 2 but for f(1).

Figure 5. Same as Fig. 4 but for f(2).

Figure 6. Differences between first guess and true temperature (dashed line) and variational temperature retrievals and true temperature for brightness temperature weights equal to 1000 for f(0), f(1), and f(2).

Figure 7. Parts of temperature soundings T and M between 250 mb and 50 mb. Sounding V1 is temperature retrieval with f(0) and sounding V2 is temperature retrieval with f(2).



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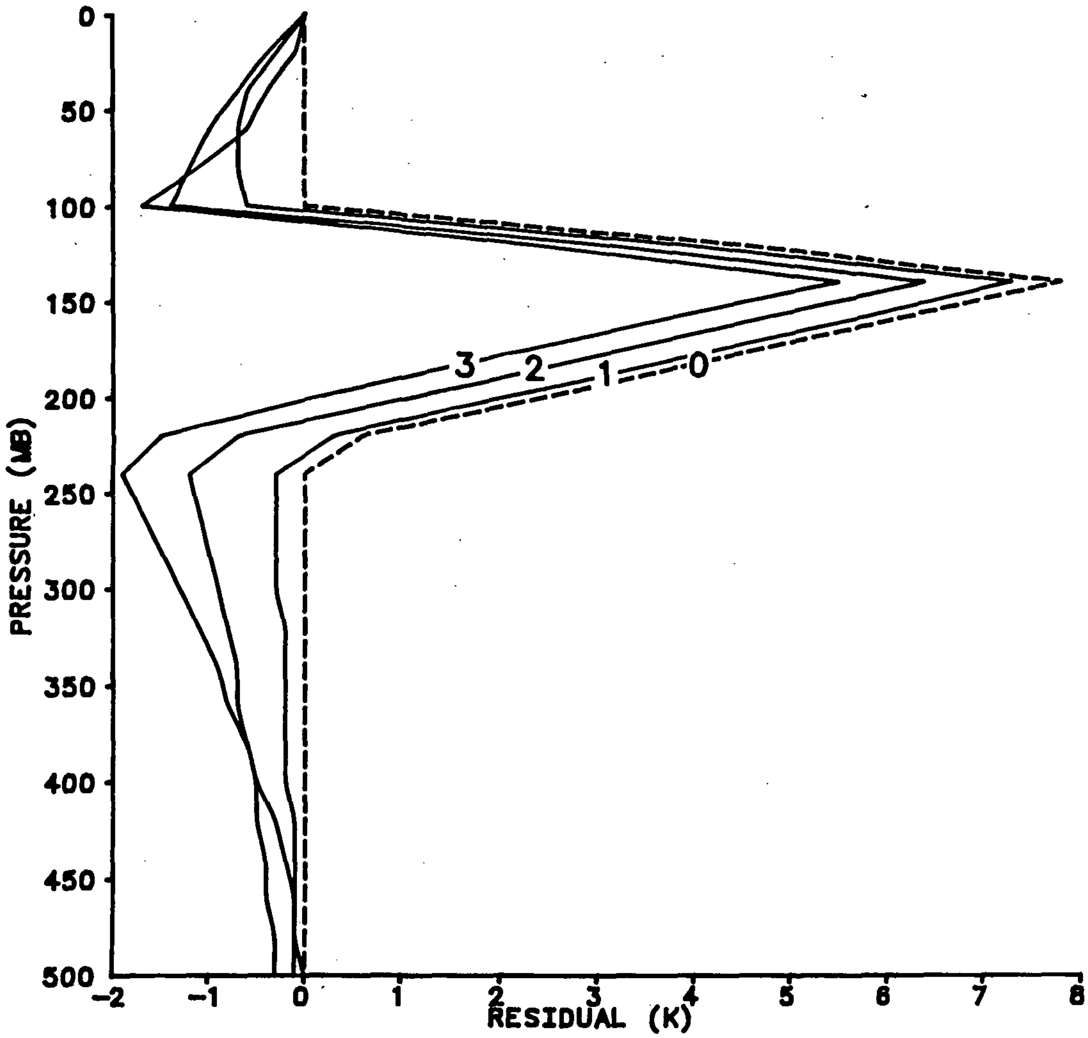

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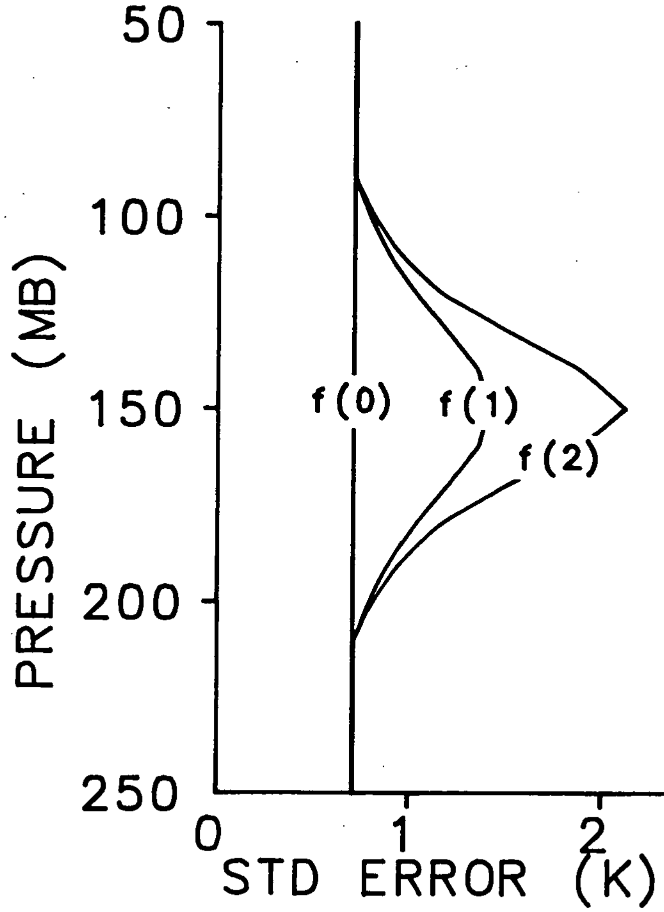

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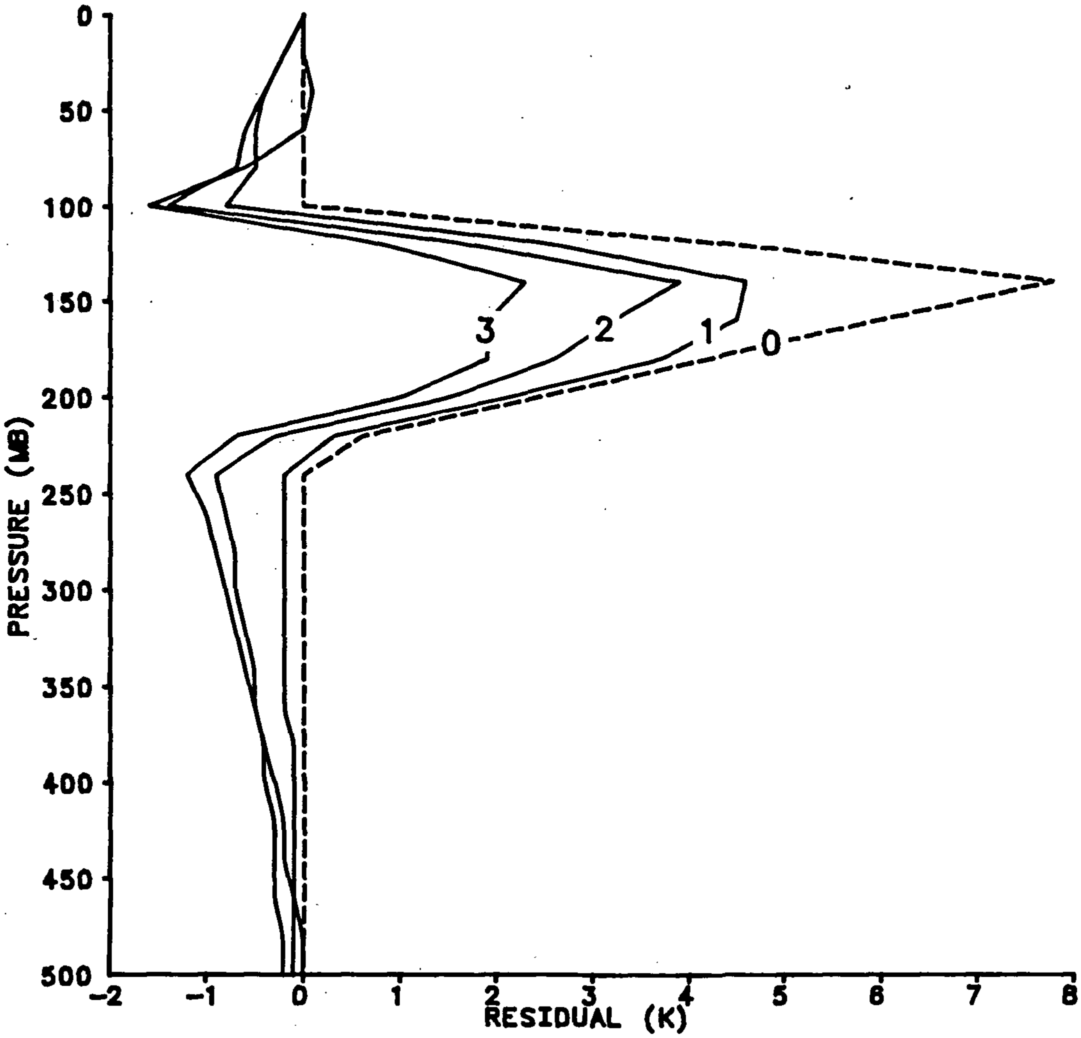


Figure 4. Same as Fig. 2 but for f(1).

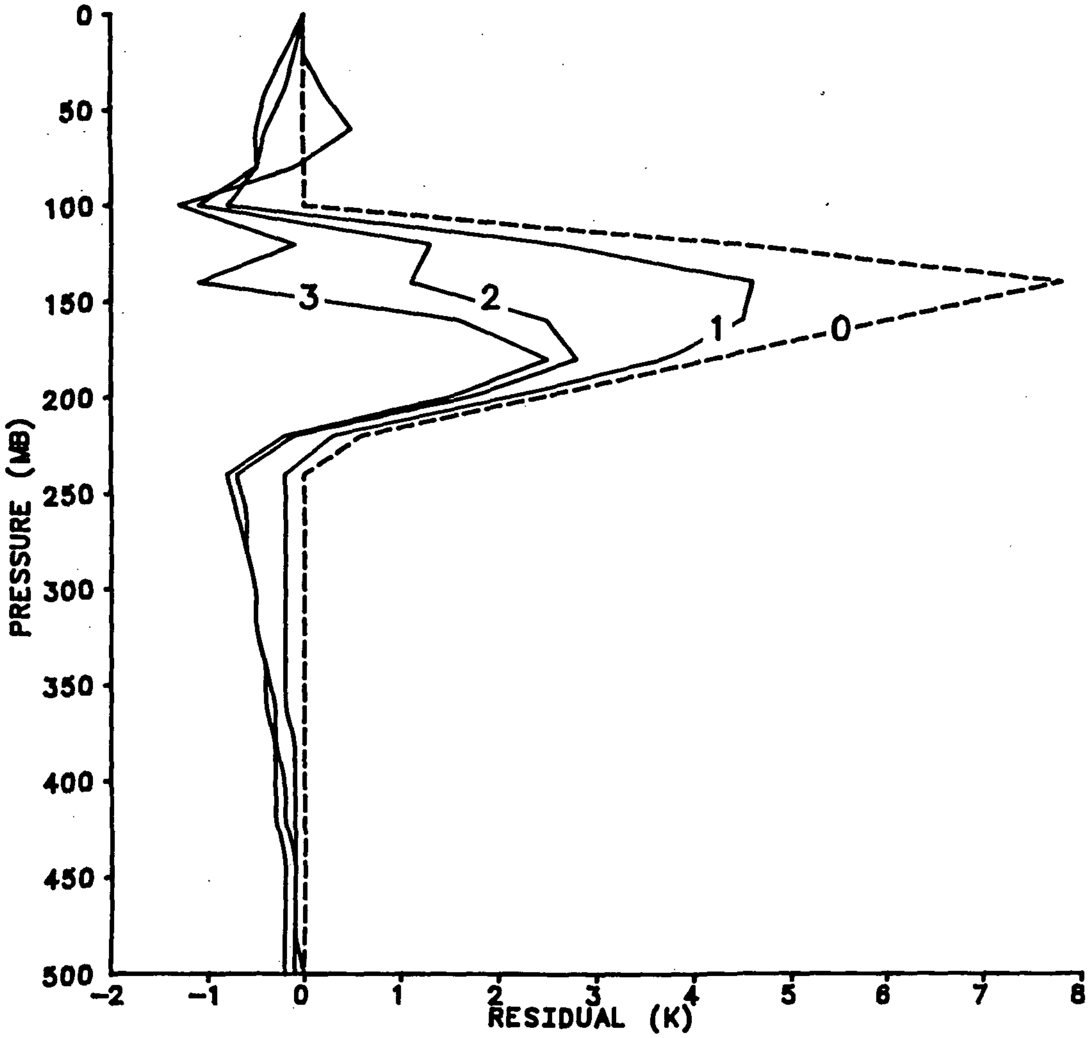
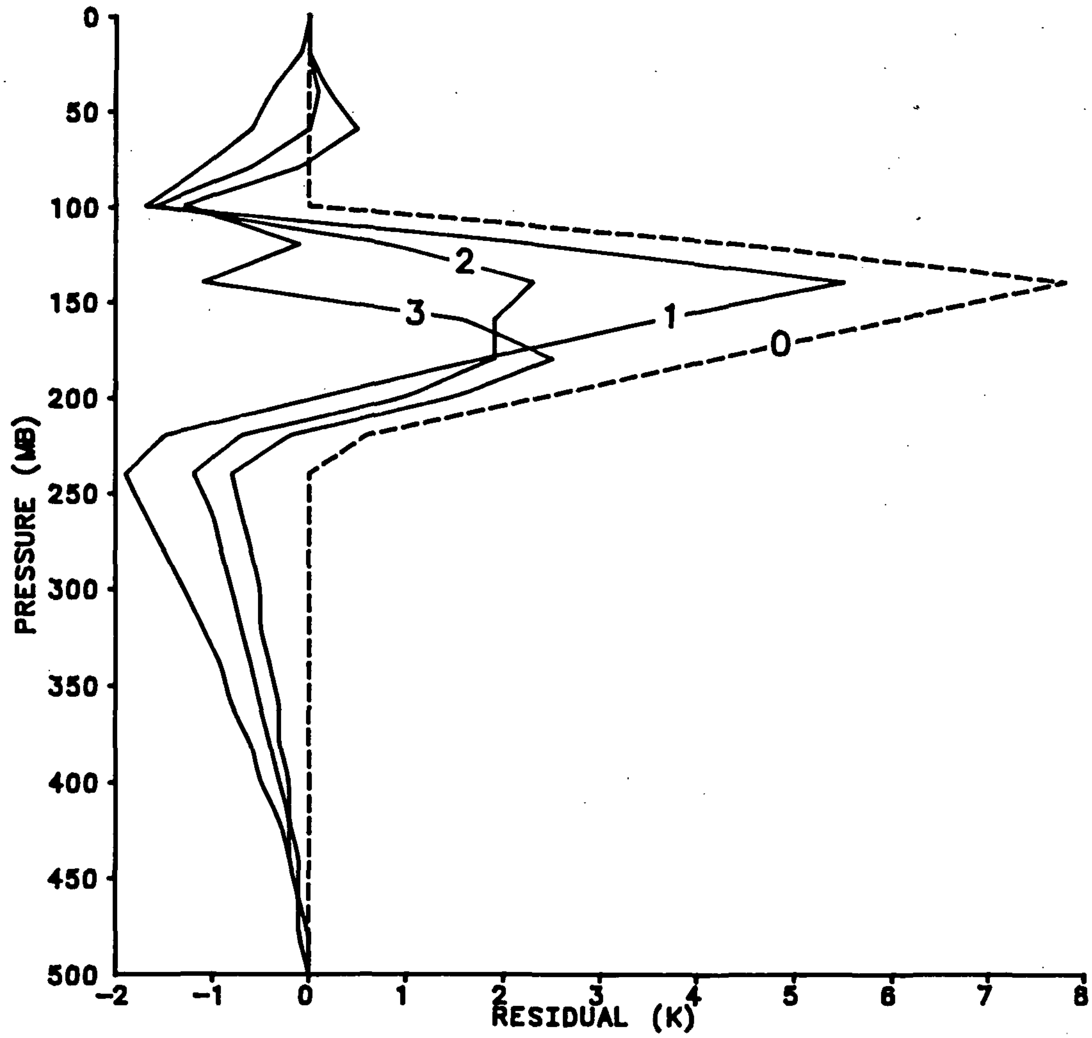


Figure 5. Same as Fig. 4 but for f(2).



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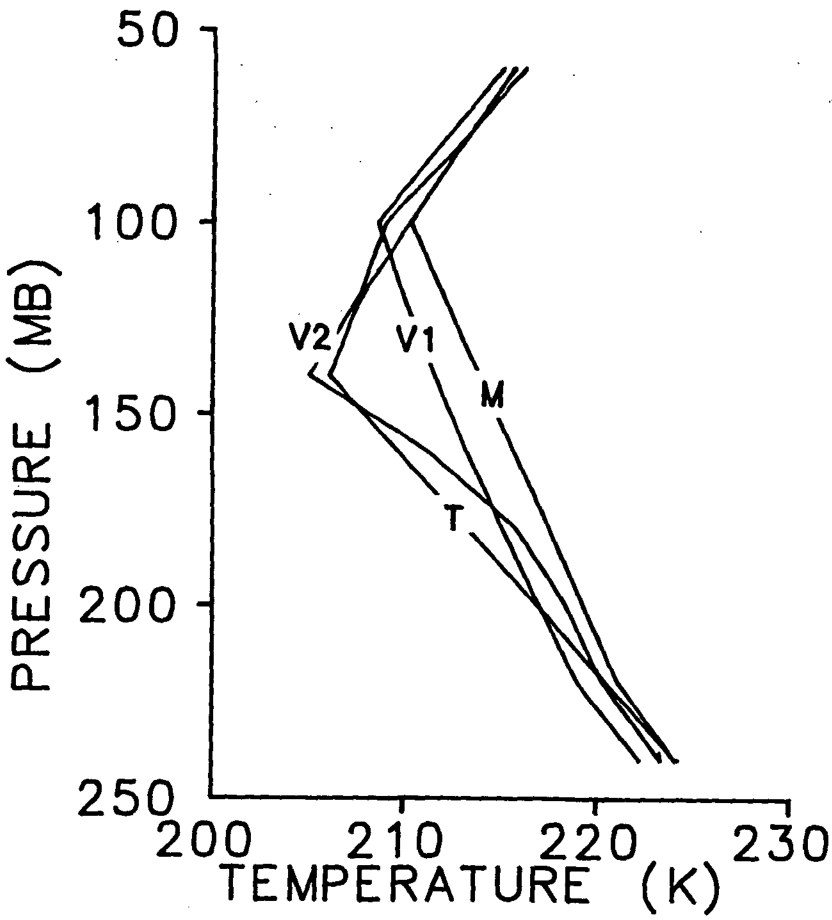

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