FAULT DETECTION AND ISOLATION FOR
MULTISENSOR NAVIGATION SYSTEMS

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SUMMARY

Increasing attention is being given to the problem of erroneous measurement data for multisensor navigation systems. A recursive estimator can be used in conjunction with a "snapshot" batch estimator to provide fault detection and isolation (FDI) for these systems. A recursive estimator uses past system states to form a new state estimate and compares it to the calculated state based on a new set of measurements. A "snapshot" batch estimator uses a set of measurements collected simultaneously and compares solutions based on subsets of measurements. The "snapshot" approach requires redundant measurements in order to detect and isolate faults. FDI is also referred to as Receiver Autonomous Integrity Monitoring (RAIM).

BACKGROUND

The objective is to detect and isolate sensor malfunctions which cause unacceptably large position errors using only inconsistency in the measurement data. Previously, FDI has been successfully applied to redundant inertial navigation systems (refs. 1-3). However, FDI can be used for any multisensor navigation system, including systems based on the Global Positioning System (GPS), Long Range Navigation (Loran-C) and the Global Navigation Satellite System (GLONASS).

A recursive estimator, such as the Kalman filter, uses the history of the user navigation state to form a predicted state estimate. If the difference between the predicted state estimate and the calculated state (based on a new set of measurements) is too large, a fault is declared. This approach is excellent for detecting step errors or rapidly growing ramp errors, and requires no redundant measurements. However, if a measurement ramp error with a small slope enters the system (caused by for instance an uncorrected clock drift in a GPS satellite), the Kalman filter may "smooth" the data rather than declare a fault. To solve this problem, a batch estimator is used since it does not depend on the history of the user state.

The "snapshot" batch estimator is based on a least squares solution which requires at least one redundant measurement for detection, and at least two redundant
measurements for isolation. Solutions calculated based on subsets of measurements can be compared to determine if inconsistency exists. Once a fault is declared, isolation is accomplished by applying detection techniques to subsets formed by leaving out one measurement at the time. This procedure is successful if no fault is found when the faulty measurement is omitted and a fault is declared for each subset containing the erroneous measurement. The focus of the remainder of this paper is on the characterization of "snapshot" batch estimators to perform FDI.

LEAST SQUARES FAULT DETECTION

One of the main input parameters to the fault detection algorithm is the alarm threshold, defined as the allowable horizontal radial error in the calculated user position. The ideal case would be to raise a flag only when this limit is exceeded and never raise a flag otherwise. However, since fault detection is performed in the presence of measurement noise and in a domain other than the solution space, it is only possible to detect a fault with a certain probability. Therefore, two undesirable events are possible - a false alarm and a missed detection. Two major parameters used in characterizing the performance of the fault detection algorithm are the probability of a false alarm ($P_{FA}$) and the probability of a missed detection ($P_{MD}$). As one might assume, it is desirable for these probabilities to be very small.

A fault is declared when a detection statistic exceeds a certain detection threshold ($T_D$). Two cases of a ramp error are shown in figure 1. In case I, $T_D$ is breached before the alarm threshold is crossed, causing a false alarm. As the position error grows, the false alarm becomes a correct alarm. In case II, the alarm threshold is exceeded before the detection threshold is exceeded, resulting in a missed detection. Eventually $T_D$ is crossed, causing a flag to be raised for a correct alarm. The normal operating state includes all circumstances where neither threshold is exceeded. For multisensor systems, this state should have a large probability of occurrence.

FAULT DETECTION ALGORITHM

A least squares approach can be used for fault detection. The linear relationship between the measurements and the user state is given by:

$$\mathbf{y} = \mathbf{H} \mathbf{\beta}$$

where:
- $\mathbf{y}$ = measurement vector
- $\mathbf{\beta}$ = user state vector
- $\mathbf{H}$ = data matrix

The dimension of $\mathbf{H}$ is n-by-m, where $n$ is the number of measurements and $m$ is the dimension of the user state vector. The user state vector $\mathbf{\beta}$ consists of the user
position coordinates and other navigation state elements such as clock offset with respect to, for instance, GPS time, as required by the navigation solution.

Three cases exist:

1) \( n < m \): Underdetermined system
2) \( n = m \): Exactly determined system
3) \( n > m \): Overdetermined system

Algorithms for managing the redundant measurements in case 3, an overdetermined system, form the basis of fault detection. In the presence of redundant signals, a parity equation can be derived from equation (1). First, a QR factorization is performed on the data matrix \( H \) (ref. 4):

\[
H = QR
\]  

(2)

This factorizes \( H \) into an orthogonal matrix \( Q \) \((Q^TQ = I)\) and an upper triangular matrix \( R \). \( R \) contains \((n-m)\) rows of zeros, reflecting that \( H \) includes data from redundant measurements. Substituting equation (2) for \( H \) in equation (1) yields:

\[
\chi = QR\beta
\]

\[
Q^T\chi = Q^TQR\beta
\]

(3)

\[
Q^T\chi = R\beta
\]

Let \( R \) be partitioned into an \( m \)-by-\( m \) upper triangular matrix \( U \) and \((n-m)\) rows of zeros, denoted by \( 0 \). Partition \( Q^T \) conformably into \( Q_1 \) and \( Q_2 \).

\[
\begin{pmatrix}
Q_1 & y_1 \\
\vdots & \vdots \\
Q_2 & y_n \\
\end{pmatrix}
= 
\begin{pmatrix}
U & \beta_1 \\
\vdots & \vdots \\
0 & \beta_{m}
\end{pmatrix}
\]

(4)

The least squares solution is given by:

\[
\beta = U^{-1}Q_1\chi
\]

(5)

Note that \( U \) is nonsingular due to the way \( R \) is partitioned. The parity equation is:

\[
Q_2\chi = 0
\]

(6)

Since \( \chi \) contains measurement errors such as noise \((e)\) and measurement biases \((b)\), a parity vector \((\mathbf{p})\) can be defined as (replace \( \chi \) by \( \chi + e + b \)):
Although the measurement noise and bias errors are not known, their components in parity space are given by equation (7). The parity vector can be used as a detection function for declaring faults.

\[ p = Q_2 y + Q_2 \varepsilon + Q_2 b \]
\[ p = Q_2 \varepsilon + Q_2 b \]  \hspace{1cm} (7)

PARITY SPACE AND ESTIMATION SPACE

As an example, consider a scenario where one redundant measurement is available. In this case, the parity vector becomes a scalar. The detection statistic is given by \(|p|\), which is assumed to be normally distributed. Figure 2 shows the distribution of \(p\) when no bias exists in any of the measurements. A fault is declared when the detection threshold \((T_D)\) is exceeded. Note that integrating the area under the tails outside \(T_D\) yields the probability of a false alarm in parity space.

Figure 3 illustrates the existence of a bias in one of the measurements. The distribution of the detection statistic \(p\) is shifted over, and the area under the curve within the limits of \(T_D\) is the probability of a missed detection in parity space.

If an alarm is raised in parity space, it can either be a correct alarm or a false alarm in estimation space. If no alarm is raised in parity space, it can either be normal operation or a missed detection in estimation space. Thus, the definitions of a false alarm and a missed detection are slightly changed. References 5 and 6 contain a detailed explanation of the relation between parity space and estimation space.

FAULT ISOLATION CONCEPTS

Let \(n\) be the number of available measurements. Then \(k\) detection functions are formed \((k = 1, \ldots, n; \text{one for each measurement})\), and an alarm is raised if any \(|d_k| > T_D\). Once alarm status has been reached, the next step is to attempt isolation. At least two redundant measurements are required for this. The process uses the fault detection algorithm applied to all subsets created by leaving out one measurement at the time. By omitting the failed measurement, the detection functions for that subset should all lie within \(T_D\). By omitting a healthy measurement, at least one detection function should exceed \(T_D\).

The number of detection functions used for isolation is \(n(n-1)\). Let \(d_{m,k}\) be the detection function for the \(k^{th}\) satellite with satellite \(m\) omitted (note that \(k=m\) does not exist). Successful isolation occurs when:
1) If m = failed satellite, all \( |d_{m,k}| < T_D \)

and 2) If m ≠ failed satellite, at least one \( |d_{m,k}| > T_D \)

Figure 4 shows the fault detection and isolation state diagram. From the diagram, it can be seen that the probability of a false alarm should be very small, because it results in either removing a healthy measurement or in system unavailability. The latter is very undesirable for a sole means of navigation system. Furthermore, the probability of a wrong isolation should also be minimized. The goal of the current research is to define the transitional probabilities of the state diagram given the required state probabilities for navigation systems.

**CONCLUSIONS**

A fault detection and isolation algorithm is presented for use in a multisensor navigation system. A state diagram has been developed which incorporates all important system states for the FDI process. Efforts continue on the quantification of state transition probabilities such that navigation system requirements for fault detection and isolation can be satisfied.

**REFERENCES**


Figure 1. Example of the impact of slowly growing measurement errors on fault detection.

Figure 2. Probability Density Function for $p$ - no measurement bias.
Figure 3. Probability Density Function for $p$ - with measurement bias.

Figure 4. Fault Detection and Isolation State Diagram.

MD = Missed Detection
FA = False Alarm
CA = Correct Alarm
FI = False Isolation
WI = Wrong Isolation
CI = Correct Isolation