EXPERIMENTAL VALIDATION OF STRUCTURAL OPTIMIZATION METHODS

Howard M. Adelman

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ABSTRACT

This paper addresses the topic of validating structural optimization methods by use of experimental results. The paper describes the need for validating the methods as a way of effecting a greater and an accelerated acceptance of formal optimization methods by practicing engineering designers. The range of validation strategies is defined which includes comparison of optimization results with more traditional design approaches, establishing the accuracy of analyses used, and finally experimental validation of the optimization results. The remainder of the paper describes examples of the use of experimental results to validate optimization techniques. The examples include experimental validation of the following: optimum design of a trussed beam; combined control-structure design of a cable-supported beam simulating an actively controlled space structure; minimum weight design of a beam with frequency constraints; minimization of the vibration response of helicopter rotor blade; minimum weight design of a turbine blade disk; aeroelastic optimization of an aircraft vertical fin; airfoil shape optimization for drag minimization; optimization of the shape of a hole in a plate for stress minimization; optimization to minimize beam dynamic response; and structural optimization of a low vibration helicopter rotor.

INTRODUCTION

The author has been engaged in a comprehensive program for applying mathematical optimization techniques to helicopter rotor blade design (Ref. 1). One of the important aspects of this work is to formulate strategies for validating the procedures and the results. One such strategy involves validating design methods based on the use of experimental results obtained by fabrication and testing specimens representing a baseline (nominal) design, an optimized design, and possibly some intermediate designs. The questions to be addressed by these tests are as follows:
(1) Did the optimization process produce a design with improved performance compared to that of the baseline design?

(2) Was the predicted performance of the designs verified by the tests?

(3) Are there any designs in the neighborhood of the predicted optimum which are better than the predicted optimum?

In developing this validation strategy, the author became involved in two activities which led to two seemingly unconnected but arguably related observations. The first activity was a review of the literature to ascertain to what extent previous investigators have attempted the type of validation outlined in items (1)–(3). The second activity was to obtain a critique from rotorcraft industry colleagues on the overall program plan for optimization of helicopter rotor blade design and the outlook for use of optimization methods. The literature review revealed a very limited catalog of documented instances where items (1)–(3) were addressed. Additionally, as part of the industry critique, the author found a pronounced "wait and see" attitude on the part of the industry as far as the outlook for near-term use of optimization techniques. It occurred to this writer that the two observations are very possibly related—that is, one reason for the less-than-enthusiastic response from industry is the paucity of demonstrated validation of optimization methods—especially by experimental means. The relationship between the two observations led to the idea for this paper.

The purpose of this paper is twofold: to review those few instances where optimization methods have been in some way validated by experimental means; and by citing these efforts, to inspire the optimization community to pursue more of this type of activity. It is this author's belief that such experimental validation will eventually be useful in effecting a more widespread use of optimization methods in industry. This increased use ought to be one of the principal goals of practitioners of optimization methodology.

As will be demonstrated in the paper, optimization validation experiments often serve a crucially important function of revealing unaccounted for failure mechanisms. For example, in one of the experiments reviewed in the paper, the optimization experiment showed that
structural failure occurred in buckling modes which were not included in the formulation of the constraint set. The test results were instrumental in the proper reformulation of the optimization problem.

The paper begins with a perspective on the overall problem of how to validate an optimization procedure. It is important in the overall picture to distinguish between validating optimization methods and validating the underlying disciplinary analyses. This distinction is discussed in some detail in the paper. Following this discussion, the paper reviews several cases of experimental validation of optimization procedures. These cases are as follows: optimum design of a trussed beam under static loads (Ref. 2); combined structure-control optimization of a simulated space structure (Ref. 3); minimum weight design of a beam with frequency constraints (Ref. 4); minimization of vibratory response of helicopter rotor blades (Refs. 5, 6); optimum shape of a turbine disk for minimum weight (Ref. 7); aeroelastic optimization of an aircraft vertical fin (Ref. 8); airfoil shape optimization for a helicopter rotor blade to minimize drag (Ref. 9); optimization of the shape of a hole in a plate for stress minimization (Ref. 10); optimization to minimize beam dynamic response (Ref. 11); and structural optimization of a low vibration helicopter rotor (Ref. 12).

A Perspective On Verification Of Optimization Procedures

For the purpose of the current paper an optimization procedure is defined as an automated computerized method which combines a systematic search algorithm with one or more disciplinary analyses. The optimization procedure requires defining an objective function to be extremized, a set of behavior and/or side constraints to be satisfied, and a set of design variables which constitute a design space to be searched by the algorithm for the optimum design. The optimum design (final design) is the one which has the extreme value of the objective function while all constraints are satisfied.
The question arises of how to establish with a high degree of confidence that the above procedure will produce a design which will perform as predicted. In other words how do we "validate" the procedure? This issue is crucial to the eventual acceptance of the procedure in the industrial workplace. Validation is one of those terms which means different things to different people. To many observers and developers of optimization methods (probably a majority), validation is exclusively concerned with the accuracy of the analyses. This point of view is that validation of an optimization procedure need not go beyond a successful test of the accuracy of the analyses used in the optimization. This view ignores the importance of assuring the proper formulation of the design problem including the most appropriate objective function and constraints, and of verifying the inclusion of all appropriate analyses to assure that critical modes of failure are represented. Additionally, in a multidisciplinary optimization procedure, it is necessary to assure that the proper interactions among disciplines have been accounted for. Furthermore, to validate an automated optimization procedure to the satisfaction of potential users, it is essential to demonstrate that the procedure can produce a design which is at least as good and preferably superior to one from a conventional (non-automated) approach for the same specified requirements and conditions. Finally, validation of a design procedure requires that a design produced by such a procedure be demonstrated to produce at least an improved measure of performance than the initial trial or existing baseline design.

In view of the aforementioned observations, it is suggested that the task of establishing confidence in an optimization procedure actually has two parts (Fig. 1):

1. establish the accuracy of the underlying analyses—a task which is herein denoted *analysis correlation*

2. compare details and performance of designs from the optimization procedure with those from conventional designs or with results from testing of the designs—a task herein denoted *design validation*
Together the correlation and validation tasks constitute a *verification* of the optimization procedure—a goal which when achieved should provide confidence in the procedure.

To be more precise about the concepts being discussed, the following definitions and amplifications are offered:

**Analysis correlation** consists of analyses and associated experimental investigations to establish accuracy of analysis codes used in an optimization procedure. This task could include comparison of results with other codes as well as comparison of results with experimental measurements. It should be mentioned that if the analysis code in question is an existing established analysis package, it is hoped and anticipated that the correlation task would have already been accomplished by the code developer.

**Design validation** consists of analyses and associated experimental investigations to establish credibility of an optimization procedure. This task may be accomplished by comparison with conventional design methods and by comparison of results with test measurements.

It seems clear that both tasks are important to the verification process and that papers could be written on various aspects of the tasks. This paper however, is primarily focused on the experimental validation of optimization procedures. Consequently the remainder of this paper describes examples wherein developers of optimization procedures have employed experimental methods to validate their procedures.

**Design of A Trussed Beam**

In Reference 2 Cullimore and Larnach reported on a research activity based on designing two types of trussed beams against yielding. The layout of a grid of the trussed beams is shown in Figure 2. In Figure 2, detail A indicates the geometry of a beam and detail B shows a typical beam element. As shown schematically in Figure 3, the two beam types are: type 1 which has two diaphragms (vertical members) and type 2 which has three diaphragms. The structure shown in Figure 2 contains type 2 beams. The design approach in Reference 2 is to size the thickness of the head section and the bar diameter of the beam element shown
in Figure 4. Their procedure, although not a formal optimization approach, does contain enough aspects of analytical design to fall within the subject matter of the present article. The procedure begins with a trial design which has minimum gage dimensions and increases both dimensions until the stresses in the bars and the head section are within acceptable limits. The beams having these dimensions were then fabricated and tested.

The results are summarized in Table 1. For the type 1 beam, the test article yielded at a load of 28 kN compared to a predicted yield load of 26 kN. For the type 2 beam the test article was predicted to yield at load of 35.5 kN but failed at a load of 32 kN due to local buckling of the central diaphragm (fig. 3b). This mode of failure was not explicitly accounted for in the design formulation. This example demonstrates quite clearly the value of experimentation in uncovering failure modes which might have been omitted in a design formulation.

Design of an Actively-Controlled Space Structure

In Reference 3, Haftka et. al. investigated the simultaneous structure/control system design of a beam test article with a single direct rate feedback actuator (fig. 5). The beam vibrates in the X-Y plane with rotational motion about the lower end. The actuator is located in the support bracket. The issue investigated in Reference 3 was whether a small modification in the structural mass or stiffness of the beam could lead to an appreciable decrease in required control gain. An optimization problem was formulated as follows: Minimize the control gain by manipulating design variables consisting of added lumped masses, while enforcing a lower limit constraint of 3 percent of critical damping on the first five modes of the structure.

Optimization results indicated that by adding non-structural masses totaling 10 percent of the structural mass, a decrease of 43 percent in control-system gain could still achieve the required damping. To experimentally verify this observation, two test articles were fabricated and tested: the baseline structure with no added masses; and a structure in which lumped masses totaling ten percent of the structural mass were added and the control system gain
was reduced by 43 percent. The tests consisted of measuring the modal damping ratios and frequencies for the first five modes of the test articles. As indicated in Table 2, the agreement between predicted and measured results are reasonably good for both configurations. The test also confirmed the prediction that an addition of 10 percent in the mass could achieve the same amount of damping (2.5 percent of critical) as the baseline structure with a 43 percent decrease in control system gain.

**Minimum Weight Dynamic Design of A Beam**

In Reference 4, Chen performed a minimum weight design of a free-free thin-walled beam. The constraints were a lower-bound on the fundamental bending frequency and a requirement that a node point for the fundamental bending mode be at a specified location. The beam was a thin-walled circular tube modeled by ten finite elements. The inner diameter of the tube was constant and the outer diameters of the finite elements making up the model were the design variables. The optimization approach was an optimality criterion method in which the Kuhn-Tucker conditions were satisfied by use of an iterative procedure.

An optimized beam design was obtained by use of the above method. Based on the final result, a test article was manufactured and tested as a free-free model. The mode shape, node locations and the fundamental frequency were measured. Comparisons of predictions and measurements were made and are shown in Figure 6. As indicated, the mode shape was in excellent agreement with the measured shape. The node location was required to be at 0.38 meters and was measured at 0.40 meters. Further, the frequency constraint required the frequency to be at least 318 Hz. The test indicated the frequency to be 306 Hz.

**Minimization of Vibration Response of Helicopter Rotor Blades**

In Reference 5 Weller and Davis described the experimental validation of optimization procedures for minimizing the vibratory shear response of helicopter rotor blades. Three procedures were examined—each based on a different criterion for reducing the response. The
three criteria were: reducing modal shears, reducing "modal vibration indices", and placing natural frequencies away from driving frequencies. Details of these criteria are described in Reference 5. In all three procedures the design variables were masses placed along the span of the blade.

The experimental validation was carried out as follows: a blade model was fabricated with provision for an insertable spar which contained 70 equally-spaced holes into which masses could be inserted. Designs in terms of optimum mass distributions were generated from each of the three optimization criteria. Masses were added to the insertable spar to model the optimum mass distributions and vibration tests were performed on each configuration. In the tests, 4/rev vertical hub shear was measured as a function of advance ratio for each of the designs and for the baseline blade which had no added mass.

A typical set of results from Reference 5 are shown in Figure 7, comparing designs from the modal shear approach, the modal vibration indices approach and the baseline design. The results were useful in two regards: first they provided a means for comparing the relative efficacy of the optimization criteria; second, they provided a validation of the optimization methods. Specifically, as seen from Figure 7, both the "minimized modal shears" designs and the "minimized vibration indices" designs have generally lower 4/rev shears than the baseline design thus validating the ability of both procedures to produce a reduced vibration design. Further, the minimized vibration indices design has lower shear values than the minimized modal shears designs. This suggests that the latter criteria may be more effective than the former. The work in Reference 5 and also a follow-on activity reported in Reference 6 are demonstrations of the use of experimental data to validate trends from optimization procedures. It should be pointed out that the results in Reference 5 do not include validation by direct comparison of predicted and measured design performance. Rather, the results validate the design procedures by showing that the optimized designs had lower vibration levels than the baseline design.
Optimum Shape Design of Rotating Disks

In Reference 7 Yu, Ma, and Chen reported an optimization investigation in which the shape of a turbine disk was optimized for minimum weight while satisfying constraints on stresses. The design variables consisted of coordinates of four “control nodes” on the surface of the disk. The shape of the disk was then defined by two circular arcs and a straight line passing through these nodes. The stress analysis was performed using a finite element analysis. Stress constraints were enforced on the mean radial stress, mean tangential stress, and the maximum tangential stress at the inner and outer radii of the disk. The optimization technique used is a standard nonlinear mathematical programming approach with a variable metric search algorithm.

Test articles based on the initial and optimized designs were fabricated and tested. The results for the initial design are shown in Figure 8a. The disk shape is shown on the left and the distribution of radial and tangential stress components are shown on the right. There is excellent agreement between analysis and experiment for the stresses in this model. The results for the final design are shown in Figure 8b. Again the agreement between analysis and test is excellent. It is observed from comparisons of the initial and final designs that the optimized design has eliminated a considerable amount of excess material in the web of the structure, resulting in a 12 percent weight reduction from the initial design. Also, the peak stress (the tangential component at the outer radius) was reduced by about 29 percent.

Aeroelastic Tailoring of a Composite Fin

In Reference 8 Schneider et. al. reported on an analytical-experimental optimization study to validate the design of a composite fin for a wind-tunnel model. The mathematical model of the fin is shown in Figure 9(a). The problem was to minimize the weight of the fin with constraints on the flutter speed, static strength, and stiffness using design variables which consisted of the thicknesses of plies at preassigned angles. During the study three different optimization codes were used: two of the codes were based on optimality criteria concepts and were used to obtain preliminary designs. The third procedure was a nonlinear
mathematical programming method which was used to generate the final design. Analyses were carried out using finite element structural analyses and a linear panel method for the aerodynamics. The final design from the mathematical programming method was the basis for the fabrication and testing of a model (Fig. 9b) to validate the procedures.

Based on the optimization results, test articles were fabricated and tested. The following quantities were measured and compared with predictions: static deflections, side forces and yawing moments. These quantities all showed reasonably good agreement between test and predictions. For example, Figure 9c shows comparisons of side forces and yawing moments for three different values of dynamic pressure. The agreement between predictions and measurements are excellent for the side forces and good for the moments. For both quantities the trend with respect to side slip angle is very good. While the results in Reference 8 represent an excellent contribution and have demonstrated how to verify the predicted behavior of the final design, what was lacking in this work was a validation of the trends between a baseline design and a final design. It would have been valuable to have included tests for the baseline design in this study.

Shape Optimization of Helicopter Airfoils

In Reference 9, Reneaux and Allongue reported on an optimization procedure for helicopter rotor blade airfoils. The procedure was, specifically, to optimize the shape of an airfoil to minimize drag while satisfying upper limit constraints on the moment coefficient (to limit blade torsion). The design was formulated and solved as a nonlinear mathematical programming problem. The objective function was the drag coefficient Cx. The constraints were upper limits on the moment coefficients Cm. The airfoil shape was represented as a linear combination of shapes from a library of existing airfoils. The design variables were the participation coefficients of the shapes contributing to the airfoil shape. The optimization procedure employed was nonlinear mathematical programming using the method of feasible directions.
A set of blades whose airfoils were shaped by the design procedure were manufactured and tested in the S3MA wind tunnel of the ONERA. As indicated in Figure 10, the optimized airfoils exhibited improved performance relative to the initial airfoils. As indicated in Figure 10a, the drag coefficient and the moment coefficient of the advancing blade were both substantially reduced throughout the range of the flight speeds. As shown in Figure 10b, the lift coefficient for the retreating blade increased slightly over most of the speed range, but generally deviated little from the initial design. In Figure 11, the predicted and measured pressures over the airfoils for the advancing and retreating blades are shown. The figures show excellent agreement between predicted and measured values for both the advancing blade (M=0.776) and the retreating blade (M=0.398).

**Optimization of the Shape of a Hole in a Plate**

Reference 10 reports on a research activity in which the shape of a hole in a plate ("tall beam" as it is denoted in Reference 10) shown in Figure 12a, is optimized to obtain a desirable stress distribution. This stress distribution is one in which the tangential stress component is uniform around the periphery of the hole and the tensile component of stress is no larger than the corresponding stress in the plate without the hole. The design variables are the coordinates of points on the edge of the hole. The optimization method is a standard nonlinear mathematical programming technique. The stress calculations were performed using a finite element method. The model is shown in Figure 12b.

A companion experimental optimization is carried out based on a method perfected and reported in numerous articles by Durelli. The technique consists of using a photoelastic technique to guide the machining of the hole. By observing the photoelastic fringes, an operator can remove material from low-stressed regions of the plate to achieve the stress pattern corresponding to the required conditions (i.e. uniform tangential stress and normal stress below a prescribed value).

Comparison of designs from the optimization procedure and the photoelastic technique are given in Figure 13. The shapes of the hole from the two procedures are remarkably close.
Furthermore, comparison of stress results from the two designs indicate a difference of less than ten percent in the maximum values and almost identical stress distributions.

**Minimization Of Beam Vibratory Response**

In Reference 11, Watts and Starkey reported on an investigation in which an optimization procedure was developed to minimize the response of a beam while avoiding excessive cost of the structural modifications. The design variables were the thicknesses of the beam at ten lengthwise locations shown in Figure 14. The objective function was the sum of the dynamic amplification factors (referred to as the response function in Ref. 11) and a measure of the cost (a linear combination of the design variables). The only constraints in the problem were side constraints (upper and lower bounds on the design variables). Consequently, a univariate search algorithm was able to be employed to minimize the objective function. The optimization procedure was applied to a test problem in which the excitation was close to the fifth natural frequency of the beam, thus assuring that the response was nearly a pure unimodal response of the fifth mode. A comparison of the response for the initial design and final design is shown in Figure 15. It was observed in Reference 11, that the response was predicted to be reduced by 37 percent (from 0.86 to 0.54) as a result of the optimization.

A test article based on the design was fabricated and tested. The first set of tests consisted of a modal survey which verified that the frequencies were accurately predicted for both an initial (uniform) beam and the optimized design. For example, the frequencies for the optimized design are shown in Figure 16a. Next, the test article was excited by the force used in the optimization example and the responses compared. The results of this comparison are shown in Figure 16b, from which it is observed that the predicted and measured responses of the optimized beam are in excellent agreement. Although there are no direct measurements comparing the responses of the original and optimized beam designs, it is clear that taken together, Figures 15 and 16 validate the optimization procedure and its ability to significantly reduce the vibration amplitude of the beam.
Structural Optimization of a Low-Vibration Helicopter Rotor

In Reference 12, Young and Tarzanin reported on a study in which a Mach-scaled helicopter rotor was designed for minimum vibratory loading. In the optimization procedure, the objective function was a combination of vibratory shear forces and moments at the blade hub. The design variables were the stiffness and mass distributions of the blade. The constraints included limits on rotor mass and blade droop. The optimization procedure consisted of a rotor dynamic analysis code combined with a nonlinear programming method. The rotor was designed and the resulting configuration was fabricated and tested. Also fabricated and tested was a reference rotor which served as the baseline design for the optimization.

Figure 17a depicts a typical comparison of test results for the baseline and optimized rotors. The graph shows the variation of vertical hub load at a frequency of four times the rotor rotational speed (denoted 4P) as a function of nondimensional forward flight speed (denoted by advance ratio). It is observed that the optimized rotor has a significant reduction in vibratory load for the speed range of interest. This tends to validate the ability of the optimization procedure to produce a reduced vibration rotor. Figure 17b shows the corresponding calculated values of vibratory loads for both the baseline and optimized rotors. Comparing the results in Figures 17a and 17b gives an indication of the fidelity of the predicted dynamic behavior. The analysis predicts the same basic trend of load vs. speed, as well as the improvement of the optimized rotor relative to the baseline rotor. However, considerable differences exist between analysis and test results at corresponding speeds. This discrepancy is not unexpected (it is recognized throughout the rotorcraft community that improved loads prediction methodology is needed) and should in no way detract from the importance of the contribution of Reference 12 as a good example of validation of an optimization procedure.
CONCLUDING REMARKS

This paper has addressed the topic of validating optimization methods by use of experimental results. The paper described the need for validating the methods as a way of effecting a greater and an accelerated acceptance of formal optimization methods by practicing engineering designers. Thus, a principal purpose of the paper was to stimulate additional work in the area of experimental validation by demonstrating by examples from the optimization literature that such validation is feasible and that valuable information is obtained from such studies. It was suggested that experimental studies can be valuable in answering the following questions relative to optimization procedures:

(1) Did the optimization procedure produce a design with improved performance compared to that of the baseline design?

(2) Was the predicted performance of the design verified by the tests?

(3) Are there any designs in the neighborhood of the predicted optimum which are better than the predicted optimum?

The paper defined the range of validation strategies which includes comparison of optimization results with more traditional design approaches, establishing the accuracy of analyses used, and finally experimental validation of the optimization results. The remainder of the paper described examples of the use of experimental results to validate optimization techniques. The examples included experimental validation of the following optimization procedures: optimum design of a trussed beam; combined control-structure design of a cable-supported beam simulating an actively controlled space structure; minimum weight design of a beam with frequency constraints; minimization of the vibration response of helicopter rotor blade; minimum weight design of a turbine blade disk; aeroelastic optimization of an aircraft vertical fin; airfoil shape optimization for drag minimization; optimization of the shape of a hole in a plate for stress minimization; optimization to minimize beam dynamic response; and structural optimization of a low vibration helicopter rotor.
Reviewing the content of the above papers indicated that question (2) was addressed in all of the papers, question (1) was addressed in fewer than half of the papers, and question (3) was not addressed at all. This suggests that additional future emphasis needs to be concentrated on the latter question. Finally, it was demonstrated that optimization validation experiments can serve to reveal the existence of unaccounted for failure mechanisms. These failure modes thus revealed may then be added to the set of constraints leading to an improved optimization formulation.

REFERENCES


Table 1. - Predicted vs. Measured Failure Loads
For Optimized Trussed-Beam (Ref. 2)

<table>
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<th>Beam Type</th>
<th>Predicted Load At First Yield</th>
<th>Actual Load at First Yield</th>
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<td>Type 1</td>
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<td>Type 2</td>
<td>35.5 kN</td>
<td>32 kN</td>
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Table 2. - Experimental and Theoretical Damping and Frequency Values for Controlled-Structure Problem of Ref. 3

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Figure 1.- Verification of Optimization Procedures
Figure 2.- Trussed-Beam System Layout (Ref. 2)

Detail A Shown in Figure 3.
Detail B Shown in Figure 4.
Figure 3.- Types of Truss-Beams Used in Study of Reference 2. (Detail A in Figure 2)
Figure 4.- Detail of Beam Element Shape and Identification of Design Variables For Trussed Beam Optimization Study of Reference 2. (Detail B in Figure 2).

t = head section thickness

d = bar diameter
Figure 5.- Test Article Used in Optimization Study of Reference 3.
Required frequency  318 Hz
Measured frequency  306 Hz

\( \bigtriangleup \) Calculation point

\( \bullet \) Test point

Figure 6.- Optimized vs. Measured Mode Shape and Frequency For Beam Designed for Specified Frequency (From Reference 4).
Figure 7.- Measured Vibratory Shear Loads for Baseline and Optimized Rotor Blade Designs (From Reference 5).
Figure 8.- Calculated vs. Measured Stresses For Turbine Disk (From Reference 7).
(a) Mathematical Model

(b) Test Article

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(c) Analysis/Test Comparison

Figure 9.- Optimization of a Fin (Ref 8).
Figure 10.- Airfoil Designed for Minimum Drag (Ref. 9)
Figure 11.- Distribution of Calculated and Measured Pressure. Optimized Airfoil. Upper and Lower Surfaces.
Figure 12.- Problem Definition and Model for Optimization of the Shape of a Hole in a Plate (Reference 10).

Figure 12.- Problem Definition and Model for Optimization of the Shape of a Hole in a Plate (Reference 10).
(a) Photoelastic Method

(b) Optimization

Figure 13. - Optimum Shape of Hole in Plate (Ref. 10).
Figure 14.- Model of Beam Used in Optimization to Minimize Dynamic Response (Reference 11).
Figure 15.- Comparison of the Fifth Mode Shape, Before and After Beam Modifications.
(b) Mode 5 Eigenvector of Optimized Beam.

(a) Natural Frequencies for Optimized Beam.

Figure 16.- Beam Designed for Minimum Vibratory Response (Ref. 11).
Figure 17.- Calculated and Measured 4P Vertical Hub Loads (From Ref. 12)
Experimental Validation of Structural Optimization Methods

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<td>Vibration</td>
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Unclassified

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12a. DISTRIBUTION / AVAILABILITY STATEMENT
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12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)