AN INITIAL INVESTIGATION INTO METHODS OF COMPUTING TRANSONIC AERODYNAMIC SENSITIVITY COEFFICIENTS

aerospace engineering department

TEXAS A&M UNIVERSITY

Semiannual Progress Report

July 1991 - December 1991

TAMRF Report No. 5802-92-01
February 1992

NASA Grant No. NAG-1-793

Leland A. Carlson
Professor of Aerospace Engineering
Texas A&M University
College Station, TX 77843-3141

TEXAS ENGINEERING EXPERIMENT STATION
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I. Introduction

This report covers the period from July 1991 thru December 1992. During this reporting period, a method based upon the quasi-analytical approach has been developed for computing the aerodynamic sensitivity coefficients for three-dimensional wings in transonic and subsonic flow. In addition, the method computes for comparison purposes the aerodynamic sensitivity coefficients using the finite-difference approach. The accuracy and validity of the methods are currently under investigation.

II. Personnel

The individuals associated with the project during this reporting period have been Dr. Leland A. Carlson, Principal Investigator, and Hesham Elbanna, Graduate Research Assistant. Mr. Elbanna has been supported by the project during this period and will use the results of this research effort for his Ph.D. dissertation.

III. Research Progress

During the past six months, significant progress has been achieved in developing the quasi-analytical approach to obtaining aerodynamic sensitivity coefficients about wings in transonic flow. As mentioned above, a method and computer program has been developed which can compute such coefficients for wings in subsonic and transonic flow. In addition, the method also automatically computes for comparison purposes the aerodynamic sensitivity coefficients using the finite-difference approach.

The method consists of several fundamental components. The first is the transonic flow solver which is a three-dimensional full potential method using the zebra algorithm as developed by Carlson, Weed, and Anderson. The flow solver uses are Cartesian like grid and places the surface boundary conditions in the x-y plane. It is a very robust and efficient flow solver and should be adequate for the present studies.

The second portion determines the "analytical" sensitivity derivatives using the quasi-analytical approach. The code for this task was developed using Macsyma to determine appropriate derivatives, i.e. \( \partial R / \partial \phi \)'s and \( \partial R / \partial X_D \)'s, where \( R \) is the residual, \( \phi \) is the potential, and \( X_D \)'s are the design variables. The resultant set of algebraic equations for the medium grid being considered is 17500 x 17500. These algebraic equations are solved using an IBM3090 conjugate gradient solver. For the sensitivity coefficient solver, twelve basic design variables and six derived design variables are considered. The basic design variables are -- freestream Mach number, angle of attack,
maximum airfoil thickness, maximum airfoil camber, location of maximum camber, twist at four locations, wing tip leading edge streamwise coordinate, wing tip trailing edge streamwise coordinate, and wing tip spanwise coordinate. From these, the derived design variables considered are semi-span, wing area, aspect ratio, taper ratio, leading edge sweep angle, and trailing edge sweep angle.

In addition, to the quasi-analytical approach, the current computer code and method also computes sensitivity derivatives using the finite-difference approach. In this section, the input design variables are changed automatically and the transonic flow solver is re-run to obtain a new solution. The derivatives are then determined using $\Delta \phi / \Delta x_D$ for each grid point in the flow field. From these values, the aerodynamic sensitivity coefficients can then be determined.

The method also contains a section which creates and plots extensive graphical output. For example the $\delta C_p(x)/\delta x_D$ distribution and the $\delta C_l/\delta x_D$ is computed at twenty spanwise stations and plotted at user selected stations. Further, spanwise variations of section sensitivity derivatives are determined and plotted along with predicted $C_p(x)$ distributions determined using the computed sensitivity derivatives.

It should be noted that currently there are two versions of the quasi-analytical sensitivity derivative code. The first computes $\delta R / \delta \phi$'s ignoring any dependence of the potential flow upwind switch function on the $\phi$'s. This version is called the Nu = C version. The second scheme computes the $\partial R / \partial \phi$'s including the dependence of the upwind switching function on the $\phi$ values; and it is termed the Nu=f(\phi) version.

At this point it is recognized that neither approach is perfect or validated. Further, to date all calculations have been executed using only single precision arithmetic. Based upon previous two-dimensional studies, it is suspected that the finite difference approach may require double precision execution in order to yield correct values. Thus, in the comparisons which follow, the fact that the quasi-analytical approach and the finite difference method yield different values for the sensitivity derivatives does not necessarily imply that the quasi-analytical method is in error.

In the following section, the viewgraphs used in a presentation to NASA Langley are reproduced. These chart effectively summarize the current state of the research and are indicative of the type of results which can be obtained from the present approach. While not definitely established, it is believed that the present Nu=f(\phi) version has the correct behavior and is the better of the two quasi-analytical versions. However, it is believed that it still contains some "errors", and this possibility is currently under investigation.
IV. Future Efforts

During the next reporting period, work will continue on developing the quasi-analytical approach and verifying its usefulness from a proof-of-concept viewpoint. In addition, it is planned to prepare and present a paper at the 1992 AIAA Applied Aerodynamics Meeting and at the 3rd Pan American Congress of Applied Mechanics.

V. Technical Monitor

The technical monitor for this project is Dr. Woodrow Whitlow, Jr., Unsteady Aerodynamics Branch, MS 173, NASA Langley Research Center. Dr. Whitlow replaces Dr. E. Carson Yates, Jr., who retired recently from the Interdisciplinary Research Office at NASA Langley.
CHARTS USED IN THE PROGRESS REPORT

PRESENTATION TO

NASA LANGLEY

ON

FEBRUARY 24, 1992
An Initial Investigation Into Methods of Computing Transonic Aerodynamic Sensitivity Coefficients

Progress Report
NASA Grant No. NAG 1-793

Leland A. Carlson
Professor of Aerospace Engineering
Texas A&M University
February 24, 1992
Important Points

This is a report on research in progress.

The computer codes are still under development.

The answers are not perfect. They may even be wrong.

There is still a lot of work to be accomplished.

Primary objective is investigate the quasi-analytical approach to aerodynamic sensitivity derivatives, determine methods for finding the derivatives using the QA approach, and establish its "validity" and range of applicability. I.E. PROOF OF CONCEPT.
Accomplishments

A method has been developed for three-dimensional flow to compute aerodynamic sensitivity coefficients using the quasi-analytical approach.

The method also computes for comparison aerodynamic sensitivity coefficients using the finite-difference approach.
Components of Method

Flow Solver

3-D full potential method using the Zebra algorithm as developed by Carlson, Weed, and Anderson. Zebra uses are Cartesian like grid and places surface boundary conditions in the x-y plane.

Good Idea?? Yes and No.

"Analytical" Sensitivity Derivatives

Based on quasi-analytical approach. Code developed using Macsyma to determine appropriate derivatives, $\partial R/\partial \phi$'s and $\partial R/\partial X_D$'s.

Resultant set of algebraic equations is about 17500 x 17500. Solved for $\partial \phi/\partial X_D$ values using IBM3090 conjugate gradient solver.
Components of Method  
(continued)

Finite-Difference Sensitivity Derivatives

Input design variables changed automatically and flow solver run to obtain new solution. Derivatives determined using $\Delta \phi/\Delta X_D$ for each grid point.

Graphical Output

$\partial C_p(x)/\partial X_D$ distribution and $\partial C_l/\partial X_D$ computed at 20 spanwise stations and plotted at selected stations.

Spanwise variations of section sensitivity derivatives determined and plotted.

Predicted $C_p(x)$ distributions determined using sensitivity derivatives and plotted.

Etc. Etc. Etc.
Twelve Basic Design Variables --

Freestream Mach Number
Angle of Attack
Maximum Airfoil Thickness
Maximum Camber
Location of Maximum Camber
Twist at Four Locations
Wing Tip Leading Edge Coordinate
Wing Tip Trailing Edge Coordinate
Wing Tip Coordinate

Six Derived Design Variables--

Semi-span
Wing area
Aspect Ratio
Taper Ratio
Leading Edge Sweep
Trailing Edge Sweep
Comments

Currently there are two versions of the quasi-analytical sensitivity derivative code. The first computes $\partial R/\partial \phi$'s ignoring any dependence of the potential flow upwind switch function on $\phi$'s. This is called the Nu = C version. The second computes $\partial R/\partial \phi$'s including the dependence of the upwind switching function on $\phi$'s. It is termed the Nu=f($\phi$) version.

Neither method is "perfect" or "validated".

Not all known "changes" have been included.

In the results, you will see solutions from various versions of each approach.
Current Situation

We are suffering from a bad case of

IO

"Information Overload"
MEDIUM GRID  45  30  16

NACA 4-DIGIT SECTION

MACH NUMBER  
ANGLE OF ATTACK  
AIRFOIL MAX THICKNESS  0.06  
AIRFOIL MAX CAMBER  0.01  
LOCATION OF MAX CAMBER  0.40

Figure 1 -- Conditions for Subcritical Test Case
WING PLANFORM:

ONERA M6

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<th>Value</th>
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Figure 2 -- Wing Planform Used for All Test Cases
Figure 3 -- $C_p$ Distribution for Subcritical Test Case, $C_p^* = -0.435$
Fig. 4A -- Quasi-Analytical and Finite Diff. Basic Sensitivity Derivatives

$M_{\infty} = 0.8$, AOA = 1°, Nu = f(\phi) in QA, $\Delta X_D = 10^{-3}$
Fig. 4B -- Quasi-Analytical and Finite Diff. Basic Sensitivity Derivatives

\( M_{\infty} = 0.8 \), \( \text{AOA} = 1^\circ \), \( \text{Nu} = f(\phi) \) in QA, \( \Delta X_D = 10^{-3} \)
Fig. 5 -- Derived Sensitivity Derivatives

$M_0 = 0.8$, AOA = 1°, Nu = $f(\phi)$ in QA, $\Delta X_D = 10^{-3}$
MEDIUM GRID  45  30  16

NACA  4-DIGIT SECTION

MACH NUMBER  0.80
ANGLE OF ATTACK  3.00
AIRFOIL MAX THICKNESS  0.06
AIRFOIL MAX CAMBER  0.01
LOCATION OF MAX CAMBER  0.40

Fig. 6 -- Conditions for First Supercritical Test Case
Fig. 7 -- $C_p$ Distribution for Supercritical Test Case, $C_p^* = -0.435$
Fig. 8A -- Quasi-Analytical and Finite Diff. Basic Sensitivity Derivatives

$M_{00} = 0.8$, $\text{AOA} = 3^\circ$, $\text{Nu} = f(\Phi)$ in QA, $\Delta X_D = 10^{-3}$
Fig. 8B -- Quasi-Analytical and Finite Diff. Basic Sensitivity Derivatives

\( M_{oo} = 0.8, \ AOA = 3^\circ, \ Nu = f(\phi) \) in QA, \( \Delta X_D = 10^{-3} \)
Fig. 9 -- Derived Sensitivity Derivatives

\( \eta(12) = 0.564 \)

\( M_{oo} = 0.8, \ AOA = 3^\circ, \ Nu = f(\phi) \) in QA, \( \Delta X_D = 10^{-3} \)
MEDIUM GRID 45 30 16

NACA 4-DIGIT SECTION

MACH NUMBER 0.82
ANGLE OF ATTACK 3.00
AIRFOIL MAX THICKNESS 0.06
AIRFOIL MAX CAMBER 0.01
LOCATION OF MAX CAMBER 0.40

Fig. 10 -- Conditions for Second Supercritical Test Case
Fig. 11 -- $C_p$ Distribution for Supercritical Test Case, $C_p^* = -0.379$
Fig. 12A -- Quasi-Analytical and Finite Diff. Basic Sensitivity Derivatives

\( M_{oo} = 0.82, \text{ AOA} = 3^\circ, Nu = f(\phi) \text{ in QA}, \Delta X_D = 10^{-3} \)
Fig. 12B -- Quasi-Analytical and Finite Diff. Basic Sensitivity Derivatives

\( M_{\infty} = 0.82, \ AOA = 3^\circ, \ Nu = f(\phi) \) in QA, \( \Delta X_D = 10^{-3} \)
Fig. 13 -- Derived Sensitivity Derivatives

\( M_{\infty} = 0.82, \quad AOA = 3^\circ, \quad Nu = f(\phi) \) in QA, \( \Delta X_D = 10^{-3} \)
MEDIUM GRID  45 30 16

NACA 4-DIGIT SECTION

MACH NUMBER            0.84
ANGLE OF ATTACK        3.00
AIRFOIL MAX THICKNESS  0.06
AIRFOIL MAX CAMBER     0.01
LOCATION OF MAX CAMBER 0.40

Fig. 14 -- Conditions for Third Supercritical Test Case
Fig. 15 -- $C_p$ Distribution for Supercritical Test Case, $C_p^* = -0.327$
Fig. 16A -- Quasi-Analytical and Finite Diff. Basic Sensitivity Derivatives

$M_{oo} = 0.84$, $AOA = 3^\circ$, $Nu = f(\phi)$ in QA, $\Delta X_D = 10^{-3}$
Fig. 16B -- Quasi-Analytical and Finite Diff. Basic Sensitivity Derivatives

\[ \eta(12) = 0.564 \]

METHOD

- Finite-Difference
- Quasi-Analytical

\[ \Delta X_D = 10^{-3} \]

\[ M_\infty = 0.84, \ \text{AOA} = 3^\circ, \ \text{Nu} = f(\phi) \] in QA,
Fig. 17 -- Spanwise Variation of Sensitivity Derivatives, $M_{oo} = 0.84$,

$\Delta \phi = 3^\circ$, $Nu = f(\phi)$ in QA, $\Delta X_D = 10^{-3}$

<table>
<thead>
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<th>$\partial C_1 / \partial M_{\partial C_1 / \partial M}$</th>
<th>$\partial C_1 / \partial a_{\partial C_1 / \partial a}$</th>
<th>$\partial C_1 / \partial T_{\partial C_1 / \partial T}$</th>
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<td><img src="image17" alt="Graph" /></td>
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</tbody>
</table>

$\partial C_1 / \partial M = 0.765$, $0.383$

$\partial C_1 / \partial a = 4.642$, $4.384$

$\partial C_1 / \partial T = 1.067$, $0.861$
Fig. 18 -- Derived Sensitivity Derivatives

$M_{oo} = 0.84$, $\text{AOA} = 3^\circ$, $\text{Nu} = f(\phi)$ in QA, $\Delta X_D = 10^{-3}$
Fig. 19 -- Spanwise Variation of Derived Sensitivity Derivatives,

\( M_{\infty} = 0.84, \ AOA = 3^\circ, \ Nu = f(\phi) \) in QA, \( \Delta X_D = 10^{-3} \)
Fig. 20A - Quasi-Analytical and Finite Diff. Basic Sensitivity Derivatives

$M_0 = 0.84$, AOA = 30°, Nu = C in QA, $\Delta X_D = 10^{-3}$
Fig. 20B -- Quasi-Analytical and Finite Diff. Basic Sensitivity Derivatives

$M_{oo} = 0.84$, $\text{AOA} = 3^\circ$, $\text{Nu} = C$ in QA, $\Delta X_D = 10^{-3}$
Fig. 22 -- Derived Sensitivity Derivatives

\[ \eta(12) = 0.564 \]

METHOD

\[ \begin{align*}
0.047 & \quad \text{Finite-Difference} \\
0.051 & \quad \text{Quasi-Analytical}
\end{align*} \]

\[ \begin{align*}
0.137 & \quad \text{Finite-Difference} \\
0.149 & \quad \text{Quasi-Analytical}
\end{align*} \]

\[ \begin{align*}
0.000 & \quad \text{Finite-Difference} \\
0.000 & \quad \text{Quasi-Analytical}
\end{align*} \]

\( M_0 = 0.84, \quad \text{AOA} = 30^\circ, \quad \text{Nu} = \text{C in QA}, \quad \Delta X_D = 10^{-3} \)
Fig. 23 -- Spanwise Variation of Derived Sensitivity Derivatives,
$M_{00} = 0.84$, AOA = $3^\circ$, Nu = C in QA, $\Delta X_D = 10^{-3}$
NOTE

Comparison of Execution Times

Derivatives by Finite Differences = 9.8

Derivatives by QA Nu = C version = 3.9

Derivatives by QA Nu = f(\phi) version = 5.8
Question

Which version is "correct"?

\[ \text{Nu} = f(\phi) \]

or

\[ \text{Nu} = C \]

To determine, consider some of our previous 2-D results.
Figs. 25 -- Two-Dimensional Results at $M_{\infty} = 0.82$
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Figs. 25 -- Two-Dimensional Results at $M_{\infty} = 0.82$
Fig. 12  
Figs. 25 -- Two-Dimensional Results at $M_{oo} = 0.82$
Figs. 26 -- Two-Dimensional Results at $M_\infty = 0.84$
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Fig. 26 -- Two-Dimensional Results at $M_{\infty} = 0.84$
Figs. 26 -- Two-Dimensional Results at $M_{oo} = 0.84$
METHOD

Finite-Difference

Quasi-Analytical

Fig. 12  
Figs. 26 -- Two-Dimensional Results at $M_{oo} = 0.84$
Conclusions

The Nu =f(ϕ) version of the QA method has the same behavior and trends as our previous 2-D results.

Differences from QA results and FD results are larger in present 3-D cases than in previous 2-D cases. While, current results are all single precision, these differences indicate that "errors" probably still exist in the method.

Question

What is the influence of the size of \( \Delta X_D \) on the finite difference results?
Fig. 27A -- Effect of $\Delta M_{00}$ Size on Fin. Diff. Derivatives
Fig. 27B -- Effect of $\Delta M_{oo}$ Size on Fin. Diff. Derivatives
$M_d = 0.84$
$\alpha = 3^\circ$
$\eta = 0.564$

$$\frac{\partial C_l}{\partial M_\infty}$$

**Fig. 28** Effect of $\Delta M_\infty$ Size on $\partial C_l/\partial M_\infty$ by Fin. Diff.
Fig. 29A -- Effect of ΔAOA Size on Fin. Diff. Derivatives
Fig. 29B -- Effect of ΔAOA Size on Fin. Diff. Derivatives
Fig. 30 -- Effect of ΔAOA Size on $\partial C_l/\partial \text{AOA}$ by Fin. Diff.
Fig. 31A -- Effect of $\Delta T$ Size on Fin. Diff. Derivatives
Fig. 31B -- Effect of ΔT Size on Fin. Diff. Derivatives
Fig. 32 -- Effect of $\Delta T$ Size on $\frac{\partial C_I}{\partial T}$ by Fin. Diff.
Fig. 33A -- Effect of ΔC Size on Fin. Diff. Derivatives
Fig. 33B — Effect of ΔC Size on Fin. Diff. Derivatives
Fig. 34 – Effect of ΔC Size on ∂C_1/∂C by Fin. Diff.
Fig. 35A -- Effect of ΔL Size on Fin. Diff. Derivatives
Fig. 35B -- Effect of ΔL Size on Fin. Diff. Derivatives
Fig. 36 -- Effect of ΔL Size on $\partial C_1/\partial L$ by Fin. Diff.
Question

What is the ability of the present aerodynamic sensitivity coefficients to predict $C_p(x)$ distributions away from the nominal point? i.e.

$$C_p(x,y,X_D+\Delta X_D) = C_p(x,y,X_D) + \left(\frac{\partial C_p}{\partial X_D}\right)_{x,y} \cdot \Delta X_D$$

To be fair, the sensitivity derivative in the finite difference case must be evaluated at a $(\Delta X_D)_I$ different from $\Delta X_D$. Also, the $\Delta X_D$'s must be reasonable. For the following, in the FD cases, the derivatives were computed using 0.001 increments.
Fig. 37 -- Comparison of $C_p$ Predictions using QA Derivatives at Nominal Point with Actual Results, $Nu = f(\phi)$ in QA.
Fig. 38 -- Comparison of $C_p$ Predictions using QA Derivatives at Nominal Point with Actual Results, Nu = C in QA
Fig. 39A -- Comparison of Cp Predictions Using QA with Nu variable, QA with Nu constant, and Fin. Diff. Derivatives at Nominal Point with Actual Results.
Fig. 39B -- Comparison of Cp Predictions Using QA with Nu variable, QA with Nu constant, and Fin. Diff. Derivatives at Nominal Point with Actual Results.
Conclusions

Believe \( \text{Nu} = f(\Phi) \) version is better of the two. However, it still needs improvement. Plan to have a reasonable version by summer.

Currently having Macsyma computer problems.

If know \( \Delta X_D \) needed, a finite difference approach will obviously yield correct results. If don't know \( \Delta X_D \), QA approach will give reasonable estimates of aerodynamic sensitivity derivatives.

Present code needs some further generalization. Depends upon funding situation, new student learning method, etc.

Might be desirable to "repeat" process with extended small disturbance equations. Matrix elements would be easier to determine. Result might be more general.