TO: Distribution
FROM: Eric L. Christiansen/SN3
SUBJECT: Shield Sizing and Response Equations

REFERENCES:

This memorandum provides a consolidated list of meteoroid/debris shield equations which have been given in the referenced memorandums. In some cases, equations have been updated; thus, this memorandum supersedes Reference 1 (i.e., SN3-90-131). The equations in this memorandum are presented in two parts: (1) shield sizing equations which are used to produce preliminary estimates of shielding weights, and (2) response equations to describe the impact conditions (projectile size as a function of velocity, density, and impact angle) causing failure of a given shield that are to be used for probability analyses (such as in the modified BUMPER program). Specific equations are given that are applicable for the following types of shields: aluminum Whipple shields, Nextel multi-shock (MS) shields, and mesh double-bumper (MDB) shields.

These equations will be updated in the future as warranted by the results of additional HVI tests, analyses, and shield modelling.
Nomenclature

C
Speed of sound in target (km/sec)
d
projectile diameter (cm)
dc
critical projectile diameter (cm) causing failure
\
\delta
density (g/cm³)
H
Brinell hardness of target (BHN)
m
areal density (g/cm²)
M
projectile mass (g)
P
penetration depth (cm)
S
overall spacing between outer bumper and rear wall (cm)
\sigma
rear wall yield stress (ksi)
t
thickness (cm)
\Theta
impact angle (deg) measured from surface normal
V
projectile velocity (km/sec)
V_n
normal component of proj. velocity (km/sec) = V \cos \Theta

Subscripts:

b
bumper(s) [all bumpers in Multi-Shock (MS) shield, first & second bumper in Mesh Double-Bumper (MDB) shield]
I
intermediate layer
p
projectile
w
rear wall
1,2,3,4
individual bumpers

EQUATIONS FOR PRELIMINARY SHIELDING DESIGN

For the WP-2 preliminary design review (PDR), McDonnell Douglas Space Systems Company (MDSSC) selected an aluminum Whipple two-sheet shield for meteoroid and debris protection of WP-2 critical equipment. A simplified method was used by MDSSC to size the thicknesses of the bumper and rear wall of the shields and estimate shielding weights. A "design" particle size was calculated for each surface of a critical element from probability of no-failure requirements, environment models, surface area, and orientation considerations. The bumper and rear wall thicknesses for each surface were calculated based on the "design" particle size, assuming average orbital debris and meteoroid impact velocity (10 km/sec and 20 km/sec, respectively), debris and meteoroid densities of 2.8 g/cc and 0.5 g/cc, and a normal impact angle (i.e., \Theta = 0 deg.).

This approach is adequate for deriving estimates of shielding weights and for performing quick trade studies, but it is not suitable for verifying design adequacy or for assessing design options to a greater level of detail. However, because MDSSC and JSC organizations are using this simplified method for estimating shielding weights for Whipple and advanced shields, the following equations are provided based on recent hypervelocity impact (HVI)
test results. They will be updated in the future as warranted by the results of additional HVI tests, analyses, and shield modelling.

**Aluminum Whipple Shield**

**Equations:**

\[ t_b = 0.25 \frac{m_p}{\delta_b} = 0.25 \frac{d}{\delta_b} \]  
\[ t_w = c d^{0.5} (\delta_p \delta_b)^{1/6} M^{1/3} V_n/S^{0.5} (70/\sigma)^{0.5} \]

Where, in equation 2, coefficient \( c = 0.16 \) cm²/sec/g²³-km.

Bumper thickness, in Equation 1, has been adjusted by the ratio of projectile to bumper density. This change will result in reductions in the MDSSC weight estimate for bumpers on surfaces of critical equipment that are only exposed to the meteoroid flux (because meteoroids are low density). The coefficient in Equation 1 has been increased to 0.25 from the MDSSC PDR approach of using 0.20. This is required to reduce the possibility of underestimating the required rear wall thickness with small standoff distances (i.e., when \( S/d < 15 \)). If standoff distance is large (i.e., \( S/d > 30 \)), then the original 0.20 coefficient can be substituted without reduction in accuracy of Equation 2.

Equation 2 is a slightly modified version of the Cour-Palais Whipple equation ("non-optimum") which was used in the Apollo program to extrapolate test data to meteoroid impact conditions (B.G. Cour-Palais: "Meteoroid Protection by Multiwall Structures," AIAA Paper No. 69-372, 1969). The wall thickness calculated by Equation 2 is for a ballistic limit defined as no perforation or detached spall of the rear wall (corresponding to damage categories D1-D2, E1-E2, and F1-F3 as given in Figure 1). The Equation 2 coefficient was derived from HVI testing with aluminum, glass, and nylon projectiles that varied in diameter from 0.04 cm to 1.9 cm. Equation 2 is valid for normal component velocity (\( V_n \)) of greater than 7 km/sec, \( S/d \) ratios of greater than 15, and \( t_b/d \) ratios of greater than 0.15. Outside of these ranges, the equation potentially will underpredict rear wall thickness. Reference 2 contains more information on the derivation and applicability of these equations.

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**Nextel Multi-Shock (MS) Shield**

The multi-shock (MS) shield is an advanced, low-weight shielding alternative to the Whipple shield. Sizing equations for two types of MS shield are given in this section: (1) Nextel ceramic fabric MS bumpers with an aluminum rear wall and (2) An all-flexible shield consisting of Nextel MS bumpers with a Nextel
rear wall. The equations are based on four equal areal density Nextel bumpers, all equally spaced. In these equations, the combined areal density of all four Nextel bumpers is given by \( m_b \), and the overall spacing (from outermost bumper to the rear wall) is given by \( S \).

**Equations for Nextel bumpers and aluminum wall:**

\[
\begin{align*}
m_b &= 0.19 \ m_p = 0.19 \ d \ \delta_p \\
m_w &= 43.1 \ M \ V_n/S^2 \ (40/\sigma)^{0.5}
\end{align*}
\]

(3) (4)

**Equations for Nextel bumpers and Nextel wall:**

\[
\begin{align*}
m_b &= 0.19 \ m_p = 0.19 \ d \ \delta_p \\
m_w &= 43.6 \ M \ V_n/S^2
\end{align*}
\]

(5) (6)

These equations are slightly a modified version of the MS equations presented in B.G. Cour-Palais and J.L. Crews: "A Multi-Shock Concept for Spacecraft Shielding," International Journal of Impact Engineering, Vol.10, pp.135-146, 1990. The wall areal density calculated by Equations 4 and 6 is based on the ballistic limit criterion of preventing both perforation and detached spall (Damage Category: F1 and F3 in Figure 1). HVI testing with aluminum projectiles up to 1 cm have been performed on the Nextel bumper and aluminum wall MS configuration (Figure 2). The all Nextel MS shield has been demonstrated with aluminum projectiles up to 0.32 cm. These equations can be applied for normal component velocity \( V_n \) of greater than 6 km/sec and \( S/d \) ratios of greater than 15.

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**Mesh Double-Bumper (MDB) Shield**

The Mesh Double-Bumper (MDB) is another advanced shield that provides similar protection benefits as the MS shield. A schematic of the MDB shield is given in Figure 3. It was developed to show how additions of a mesh and high strength fabric to a Whipple shield could provide a large improvement in shielding protection capability. Impact testing at the JSC Hypervelocity Impact Research Laboratory has shown that a double bumper system with a mesh outer bumper exhibits superior performance than the same weight double bumper consisting of two continuous aluminum sheets. The following equations have been modified from those presented in E.L. Christiansen: "Advanced Meteoroid and Debris Shielding Concepts," AIAA Paper No. 90-1336, April 1990.
MDB Equations

For the mesh first bumper:

\[ m_1 = c_m d \delta_p \]  \hspace{1cm} (7)

Where, in Equation 7, the coefficient, \( c_m \), can range from 0.035 to 0.057 without affecting the accuracy of the following equations. The mesh is composed of wires in a square pattern with a wire diameter to projectile diameter ratio of from 0.07 to 0.10. Generally, from 4 to 6 wires are "cut" by the diameter of the projectile. The first to second bumper spacing is: \( S_1 = 4 d \). The second bumper is a continuous aluminum sheet that is sized by the following equation:

\[ m_2 = 0.093 d \delta_p \]  \hspace{1cm} (8)

A high strength fabric intermediate layer (Spectra, Kevlar, etc.) is mounted at a short distance in front of the rear wall \( (S_3 = 4 d) \). For Spectra or Kevlar, the sizing equation is:

\[ m_i = 0.064 d \delta_p \]  \hspace{1cm} (9a)

Nextel has also been tested successfully as an intermediate layer. If Nextel is used, the sizing equation is:

\[ m_i = 0.095 d \delta_p \]  \hspace{1cm} (9b)

The rear wall sizing equation is:

\[ m_w = 34.8 \frac{M V_n}{S^2 (40/\sigma)^{0.5}} \]  \hspace{1cm} (10)

HVI testing of the mesh double-bumper (MDB) shield has been performed for 0.32 cm, 0.635 cm and 1 cm projectiles (Figure 2). These equations can be applied for normal component velocity \( (V_n) \) of greater than 6 km/sec and \( S/d \) ratios of more than 15. The wall areal density calculated by Equation 10 is based on the ballistic limit criterion of preventing perforation and detached spall (Damage Category: F1 and F3 in Figure 1).

Based on limited HVI testing, if no intermediate cloth layer is used with the MDB shield, the rear wall sizing equation is:

\[ m_w = 2.1 d^{1.5} \delta_p^{0.5} \frac{V/S (40/\sigma)^{0.5}} \]  \hspace{1cm} (11)

**Single Aluminum Sheet**

The following Cour-Palais cratering equations are recommended for predicting single wall penetration. For projectile density \( (\delta_p/\delta_t < 1.5) \), the penetration depth into a semi-infinite target
is:

\[ P_o = 5.24 \, d^{19/18} \, H^{0.25} \, (\delta_p/\delta_t)^{0.5} \, (V_n/C)^{2/3} \]  

(12a)

For projectile density \((\delta_p/\delta_t \geq 1.5)\):

\[ P_o = 5.24 \, d^{19/18} \, H^{0.25} \, (\delta_p/\delta_t)^{2/3} \, (V_n/C)^{2/3} \]  

(12b)

If there is attached spall, the penetration depth is greater than into a semi-infinite target:

\[ P = 1.05 \, P_o \]  

(13)

If there is detached spall, penetration depth can vary between 1.08 and 1.5 times the semi-infinite target penetration, i.e.:

\[ P = 1.08 \, P_o \text{ to } 1.5 \, P_o \]  

(14)

The plate thickness to prevent perforation, but not detached spall (Damage category B3):

\[ t = 1.8 \, P_o \]  

(15)

Plate thickness to prevent perforation and detached spall, but would allow attached spall (Damage category B2):

\[ t = 2.2 \, P_o \]  

(16)

Plate thickness to prevent perforation and incipient spall (Damage category B1):

\[ t = 3 \, P_o \]  

(17)

**EQUATIONS FOR PROBABILITY ANALYSES**

The velocity and directional distribution of the meteoroid and debris threat must be assessed against shield capabilities before the design process is complete (i.e., to verify that the shielding design meets the specified no-failure requirements). The MDSSC PDR shielding designs must be refined to account for the directional nature of debris and meteoroids, the complex response of the shielding to oblique and low speed impact, and to account for shadowing from nearby equipment.

This part of the memorandum provides ballistic limit equations for the Whipple, Multi-Shock (MS), and Mesh Double-Bumper (MDB) shields that can be used in probability analyses. The equations are in a form that relates critical particle diameter to fail a given structure with impact velocity and impact angle. The equations are consistent with the equations given previously, but
additional equations are given to cover the full range of on-orbit impact velocities and impact angles.

Hypervelocity impact testing is currently in progress to better define these ballistic limit equations. An update to these equations will be made after testing results have been analyzed.

Aluminum Whipple Shield

This shield consists of an aluminum bumper and aluminum rear wall. A set of three ballistic limit equations that covers the three primary penetration regimes is given below. The three penetration regimes are based on normal component velocities with penetration of the rear wall occurring by molten material, vapor, and possibly solid particulates at normal component velocities above 7 km/sec; a fragmenting projectile regime between 3 km/sec and 7 km/sec; and a non-fragmenting projectile ballistic regime below 3 km/sec.

For $V_n \geq 7$ km/sec:

$$d_c = 3.918 t_w^{2/3} \delta_p^{-1/3} \delta_b^{-1/9} (V \cos \theta)^{-2/3} S^{1/3} (\sigma/70)^{1/3}$$

(18)

For $3$ km/sec < $V_n < 7$ km/sec:

$$d_c = \left\{ \left[ (t_w (\sigma/40)^{0.5} + t_b)/(1.248 \delta_p^{0.5} \cos \theta) \right]^{18/19} \right. \times \\
(1.75 - (V \cos \theta)/4) + \left\{ [1.071 t_w^{2/3} \delta_p^{-1/3} \delta_b^{-1/9} S^{1/3} (\sigma/70)^{1/3}] \times \right. \\
((V \cos \theta)/4 - 0.75) \right\}$$

(19)

For $V_n \leq 3$ km/sec:

$$d_c = \left[ (t_w (\sigma/40)^{0.5} + t_b)/(0.6 (\cos \theta)^{5/3} \delta_p^{0.5} V^{2/3}) \right]^{18/19}$$

(20)

Multi-Shock (MS) Shield

The following MS shield ballistic limit equations are valid for a shield consisting of 4 Nextel bumpers and an aluminum rear wall, with equal spacing between sheets. In these equations, the overall spacing from the first, outer-most, bumper to the rear wall is given by "S".
For $V_n \geq 6 \text{ km/sec}$:

$$d_c = 0.354 \ t_w^{1/3} \ \delta_p^{-1/3} \ \delta_w^{1/3} \ (V \cos \theta)^{-1/3} \ S^{2/3} \ (\sigma/40)^{1/6} \quad (21)$$

For $3 \text{ km/sec} < V_n < 6 \text{ km/sec}$:

$$d_c = \{[(t_w \ (\sigma/40)^{0.5} + 0.37 \ m_b)/(0.624 \ \delta_p^{0.5} \ \cos \theta)]^{18/19} \ * \\
(2 - (V \cos \theta)/3)\} + \{[0.1948 \ t_w^{1/3} \ \delta_p^{-1/3} \ \delta_w^{1/3} \ S^{2/3} \ (\sigma/40)^{1/6}] \ * \\
((V \cos \theta)/3 - 1)\} \quad (22)$$

For $V_n \leq 3 \text{ km/sec}$:

$$d_c = [(t_w \ (\sigma/40)^{0.5} + 0.37 \ m_b)/(0.3 \ (\cos \theta)^{5/3} \ \delta_p^{0.5} \ V^{2/3})]^{18/19} \quad (23)$$

Figure 4 shows the results of applying the above equations for a MS shield consisting of four Nextel AF26 bumpers (each with an areal density of 0.043 g/cm$^2$) and a 0.020" (0.0508 cm) Al 2024-T3 rear wall, with 1" (2.54 cm) between each sheet, 4" (10.16 cm) overall spacing. This plot shows that a 3.18 mm (1/8") aluminum projectile impacting at 6.5 km/sec and normal impact angle will be on the ballistic limit of the shield, while the shield will stop a 1.25 mm projectile in a normal impact at 3 km/sec.

Mesh Double-Bumper

The following MDB equations are based on a mesh double-bumper shield using either Kevlar or Spectra cloth as an intermediate layer.

For $V_n \geq 6 \text{ km/sec}$:

$$d_c = 0.38 \ t_w^{1/3} \ \delta_p^{-1/3} \ \delta_w^{1/3} \ (V \cos \theta)^{-1/3} \ S^{2/3} \ (\sigma/40)^{1/6} \quad (24)$$

For $3 \text{ km/sec} < V_n < 6 \text{ km/sec}$:

$$d_c = \{[(t_w \ (\sigma/40)^{0.5} + 0.37 \ \Sigma m_{p+1})/(0.83 \ \delta_p^{0.5} \ \cos \theta)]^{18/19} \ * \\
(2 - (V \cos \theta)/3)\} + \{[0.209 \ t_w^{1/3} \ \delta_p^{-1/3} \ \delta_w^{1/3} \ S^{2/3} \ (\sigma/40)^{1/6}] \ * \\
((V \cos \theta)/3 - 1)\} \quad (25)$$
For $V_n \leq 3$ km/sec:

$$d_c = \left[ \left( t_w \left( \sigma/40 \right)^{0.5} + 0.37 \Sigma_{b+1} \right) / \left( 0.4 (\cos \theta)^{5/3} \delta_p^{0.5} V^{2/3} \right) \right]^{(18/19)} \quad (26)$$
**Class**

**CATEGORY A: SINGLE CRATER PATTERN - LOW VELOCITY**

PROJECTILE REMAINS INTACT

A1  • NO PERFORATION OR REAR SURFACE DEFORMATION
     • CRATER DIAMETER APPROXIMATE SIZE OF PROJECTILE

A2  • NO PENETRATION
     • CRACKS OR SPLITTING MAY BE PRESENT
     • REAR SURFACE DEFORMATION

A3  • PENETRATION
     • HOLE DIAMETER APPROXIMATE SIZE OF PROJECTILE

**CATEGORY B: SINGLE CRATER PATTERN - HYPERVELOCITY**

PROJECTILE REMAINS INTACT

B1  • NO PERFORATION OR REAR SPALL
     • SINGLE ROUNDED CRATER
     • FRONT SURFACE LIP OR SPALLATION

B2  • NO PERFORATION, BUT WITH ATTACHED REAR SPALL
     • SINGLE ROUNDED CRATER
     • FRONT SURFACE LIP OR SPALLATION

B3  • NO PERFORATION, BUT WITH DETACHED REAR SPALL
     • SINGLE ROUNDED CRATER
     • FRONT SURFACE LIP OR SPALLATION
     • LIGHT TIGHT

B4  • PERFORATION DUE TO CRATER AND REAR SPALL MEETING
     (HOLE DIAMETER < 2 mm)
     • FRONT SURFACE LIP OR SPALLATION
     • NOT LIGHT TIGHT

B5  • PENETRATION
     • HOLE FORMED BY CRATER AND DETACHED SPALL
     (HOLE DIAMETER ≥ 2 mm)
     • FRONT AND REAR SURFACE LIPS OR SPALLATION

**FIGURE 1. Damage Classification for Shielded Metallic Targets**

REF. Dahl and Cour-Palais: "Standardization of Impact Damage Classification and Measurements for Metallic Targets", 1990
**Category C: Multiple Crater Pattern**

- C1: No perforation, rear surface deformation or spall
  - Random surface craters, pitting or erosion
- C2: No perforation, but with attached spall(s) or rear surface deformation
  - Random surface craters, pitting, or erosion
- C3: No perforation, but with detached spall(s)
  - Random surface craters, pitting or erosion
  - Light tight
- C4: Perforation
  - Cracks or small hole(s) (all hole diameters < 2 mm)
  - Not light tight
- C5: Penetration
  - Large hole(s) (applicable if any hole diameter ≥ 2 mm)

**Category D: Central Crater Pattern**

- D1: No perforation or rear spall
  - Central surface crater, pitting or erosion
- D2: No perforation, but with attached spall
  - Central surface crater, pitting, or erosion
- D3: No perforation, but with detached spall
  - Central surface crater, pitting or erosion
  - Light tight
- D4: Perforation
  - Cracks or small hole(s) due to crater and spall meeting (all hole diameters < 2 mm)
  - Not light tight
- D5: Penetration
  - Large hole(s) formed by crater and detached spall
  (applicable if any hole diameter ≥ 2 mm)

**Figure 1. Damage Classification for Shielded Metallic Targets (Cont.)**

REF. Dahl and Cour-Palais: "Standardization of Impact Damage Classification and Measurements for Metallic Targets", 1990
**Category E: Ring Crater Pattern**

- E1: No perforation or rear spall
- Ring craters surround central surface crater, pitting, or erosion

- E2: No perforation
- Ring craters with spall pimples attached and/or central spall attached
- Central surface crater, pitting, or erosion

- E3: No perforation
- Ring craters with spall pimples detached and/or central spall detached
- Central surface crater, pitting, or erosion
- Light tight

- E4: Perforation
- Hole(s) due to crater(s) and spall(s) meeting
- Not light tight

- E5: Penetration
- Large hole punched out due to ring perforations and impulsive load

**Category F: Non-Particulate Impulsive Loading**

- F1: No perforation or rear spall
  - Surface pitting or molten splash

- F2: No perforation
  - Spall present, attached or detached
  - Surface pitting or molten splash

- F3: No perforation
  - Dented, but intact
  - Surface pitting or molten splash
  - Light tight

- F4: Perforation
  - Dented and split
  - Surface pitting or molten splash
  - Not light tight

- F5: Penetration by impulsive load failure
  - Petalled hole
  - Surface pitting or molten splash

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**Figure 1. Damage Classification for Shielded Metallic Targets (Cont.)**

REF. Dahl and Cour-Palais: "Standardization of Impact Damage Classification and Measurements for Metallic Targets", 1990
Shield Mass Per Unit Area
(No perforation of rear wall)

- 3.2 mm Aluminum Projectile (45 mg), Normal Impact, 6.5 km/sec
  — Monolithic Aluminum Plate: 3.53 g/cm².
- 3.2 mm Aluminum Projectile (45 mg), Normal Impact, 10 cm Spacing
  — Whipple Shield: 0.60 g/cm².
  — Nextel MS Shield: 0.29 g/cm²
  — Mesh Double-Bumper: 0.26 g/cm².
- 3.2 mm Aluminum Projectile (45 mg), 45° Impact, 10 cm Spacing
  — Whipple Shield: 1.22 g/cm².
  — Nextel MS Shield: 0.31 g/cm²
  — Mesh Double-Bumper: 0.36 g/cm².
- 3.2 mm Aluminum Projectile (45 mg), Normal Impact, 5 cm Spacing
  — Whipple Shield: 0.80 g/cm².
  — Nextel MS Shield: 0.52 g/cm²
  — Mesh Double-Bumper: 0.42 g/cm².
- 9.5 mm Aluminum Projectile (1.3 g), Normal Impact, 30 cm Spacing
  — Whipple Shield: 1.35 g/cm².
  — Nextel MS Shield: 0.97 g/cm²
  — Mesh Double-Bumper: 1.08 g/cm²
- 6.4 mm Aluminum Projectile (0.37 g), Normal Impact, 20 cm Spacing
  — Whipple Shield: 0.96 g/cm²
  — Mesh Double-Bumper: 0.64 g/cm²

(all impacts at ~6.5 km/sec)
Mesh Double-Bumper Shield

- Aluminum Mesh:
  - Mass efficient method to disrupt projectile.
  - Greater spread of debris cloud results from impacts on mesh — reduces performance degradation at smaller spacings.
  - Fine mesh used. Small projectiles passing unhindered through mesh easily defeated by remaining shield elements.
  - Improvement over equal-weight aluminum double-bumpers.

- Second bumper used to deliver second shock to remaining fragments.

- Intermediate layer of high-strength fabric (Spectra, Kevlar, Nextel, etc.) used to slow debris cloud and decrease impulsive loading on back sheet.

- Development status: spacing/areal densities optimized, preliminary sizing relationships formulated, scale-up tests performed, alternative materials and oblique impacts studied.

- Future development: ballistic limit investigations and additional material optimization (Al fabric, flexible second bumper, alternative intermediate and backwall materials).

- Augmentation for protection from high-density particles: consider steel mesh or fabric.

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