Bandwidth Efficient Coding for Satellite Communications*

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RESEARCH PURPOSE

The purpose of this research is to devise an error control coding scheme to achieve large coding gain and high reliability by using coded modulation with reduced decoding complexity.

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CODED MODULATION ALONE

- To achieve a 3 to 5 dB coding gain and moderate reliability, the decoding complexity is quite modest.

- In fact, to achieve a 3 dB coding gain, the decoding complexity is quite simple, no matter whether trellis coded modulation (TCM) or block coded modulation (BCM) is used.

- However, to achieve coding gains exceeding 5 dB, the decoding complexity increases drastically, and the implementation of the decoder becomes very expensive and unpractical.
A BASIC QUESTION

- How can we achieve large coding gains and high reliability by using coded modulation with reduced decoding complexity?
• Use coded modulation in conjunction with concatenated (or cascaded) coding.

• A good short bandwidth efficient modulation code (trellis or block) is used as the inner code and relatively powerful Reed-Solomon (RS) code is used as the outer code.

• With properly chosen inner and outer codes, a concatenated coded modulation scheme not only can achieve large coding gains and high reliability with good bandwidth efficiency but also can be practically implemented.

• This combination of coded modulation and concatenated coding really offers a way of achieving the best of three worlds, reliability and coding gain, bandwidth efficiency and decoding complexity.
PROPOSED SCHEME

- A concatenated (or cascaded) coded modulation scheme.

- For NASA high-speed satellite communications for large data file transfer where very high reliability is required.

- The outer code $C_2$ is an $(n_2, k_2)$ RS code with symbols from GF $(2^b)$.

- The outer code is interleaved to a depth of $m$.

- The inner code is a bandwidth efficient block M-ary PSK code of length $n_1$ and dimension $k_1 = mb$.

- Under the same research project, we have investigated concatenated coding with TCM inner codes.
THE OVERALL CONCATENATED CODED MODULATION SCHEME

- The outer code $C_2$ is an $(n_2, k_2)$ RS code over $\text{GF}(2^8)$, which is designed to correct $t_2$ or fewer symbol errors with $0 \leq t_2 \leq \lfloor (n_2 - k_2)/2 \rfloor$.

- The inner code $C_i$ is a $2^i$-PSK code of length $n_i$ and dimension

$$k_i = \sum_{i=1}^{i} k_{i},$$

where $k_i = mb$.

- The outer code $C_2$ is interleaved to a depth of $m$.

- The encoding consists of two stages, the outer and inner encodings.

- The decoding consists of two stages, the inner and outer decodings.

- When the receiver fails to decode a received block, the block is erased and the receiver raises a flag.

- In the event of an erasure, we could either request a re-transmission or accept the erroneous block with alarm.
Figure 1: A Segment-Array
ERROR PERFORMANCE
OF THE OVERALL SCHEME

- Let $P_c$, $P_e$ and $P_\text{inc}$ be the probabilities of a correct decoding, an erasure and an incorrect decoding for an entire received code block respectively.

- Lower bound on $P_c$ and upper bounds on $P_e$ and $P_\text{inc}$ have been derived for an AWGN channel.

- Let $\hat{P}_c$ denote a lower bound on $P_c$.

- Then $1 - \hat{P}_c$ is an upper bound on the total probability of a decoding failure and a decoding error.

- Let $\hat{P}_e$ denote an upper bound on $P_e$.

- The performance of the proposed concatenated coded modulation scheme is measured by the pair, $\hat{P}_e$ and $1 - \hat{P}_c$.

- We can compute the coding gains of the proposed scheme over the uncoded QPSK modulation system either in terms of decoded block-error rates or in terms of decoded bit-error rates.
• For data file transfer, the block-error rates should be used as the measure of the error performance of the scheme.

• There are two types of bit-error rates, denoted $P_{s1}$ and $P_{s2}$.

• $P_{s1}$ is computed based on the block error probability $P_e$ using the approximation,

\[ P_{s1} = \left(\frac{d}{2n_2}\right) \cdot P_e \]

• $P_{s1}$ is a measure of bit-error performance of the proposed scheme when retransmission is allowed.

• $P_{s2}$ is computed based on the total probability $1 - P_c$ of a decoding failure and a decoding error of a code block using the approximation,

\[ P_{s2} = \left(\frac{d}{2n_2}\right) \cdot (1 - P_c) \]

• $P_{s2}$ is used as the measure of bit-error performance of the scheme when retransmission is not available or allowed.
TWO SPECIFIC
CONCATENATED CODED MODULATION SCHEMES

SCHEME – I

• The outer code $C_2$ is the NASA standard (255,223) RS code over GF($2^8$) which has minimum distance 33. It is used to correct up to 16 symbol errors.

• The inner code $C_1$ is an 8-PSK code with $n_1 = 8$, $k_1 = 16$, $D[C_1] = 4$, $R[C_1] = 1$ and $\gamma[C_1] = 3$ dB (over uncoded QPSK).

• The outer code is interleaved to a depth of $m = 2$.

• The overall effective rate of the scheme is

$$R_{eff} = \frac{k_2}{n_2} \cdot R[C_1] = 0.875.$$ 

• The inner code has a 4-state trellis structure and can be decoded with a soft-decision Viterbi decoder.
ERROR PERFORMANCE

• With SNR = 9 dB/symbol (6.57 dB/infor. bit),

\[ P_e \leq 6.28 \times 10^{-26} \]
\[ 1 - P_e \leq 4.95 \times 10^{-16} \]

• With SNR = 10 dB/symbol (5.57 dB/infor. bit),

\[ P_e \leq 6.80 \times 10^{-4} \]

and \( 1 - P_e \) is very small.
CODING GAIN OVER QPSK

• At the block-error rate $= 10^{-7}$,

\[ G_b = 8 \text{ dB/symbol}. \]

• At the block-error rate $= 10^{-10}$,

\[ G_b = 9 \text{ dB/symbol}. \]

• At the bit-error rate $P_b = 10^{-12}$,

\[ G_b = 9.80 \text{ dB/symbol} \text{ (9.20 dB/infor. bit)}. \]

The required SNR to achieve $P_b = 10^{-12}$ is 7.10 dB/symbol (4.60 dB/infor. bit).
• At the bit-error rate $P_{e_2} = 10^{-8}$,

$$G_{e_2} = 5.52 \text{ dB/symbol (4.94 dB/infor. bit).}$$

The required SNR to achieve $P_{e_2} = 10^{-8}$ is 8.04 dB/symbol (5.61 dB/infor. bit).

• At the bit-error rate $P_{e_2} = 10^{-10}$,

$$G_{e_2} = 7.60 \text{ dB/symbol (7.02 dB/infor. bit).}$$

The required SNR to achieve $P_{e_2} = 10^{-10}$ is 8.50 dB/symbol (6.07 dB/infor. bit).
Figure 2 Error performance of the 4-state 8-PSK block code (the 4-th code in Table 1)
Figure 3 The total probability of a decoding failure and a decoding error for the concatenated coded modulation scheme with the (255,223) RS outer code and the 4-state 8-PSK block inner code (the 4-th code in Table 1)
Figure 4 The probability of a decoding error for the concatenated coded modulation scheme with the (255,223) RS outer code and the 4-state 8-PSK block inner code (the 4-th code in Table 1)
SCHEME – II

- The outer code is the NASA standard $[255, 223]$ RS code over $GF(2^s)$.

- The inner code $C_i$ is an 8–PSK code of length 16 and dimension $k_i = 36$ with $D[C_i] = 4$, $R[C_i] = 9/8$ and $\gamma[C_i] = 3.52$ dB (over uncoded QPSK).

- The outer code is interleaved to a depth of $m = 9$.

- The overall effective rate of the scheme is

$$R_{eff} = \frac{223}{255} \cdot \frac{9}{8} = 0.9838.$$
• The inner code has a 16-state trellis diagram which consists of two identical parallel 8-state trellis sub-diagrams with no cross connection between them.

• The probability of an incorrect decoding for this code is

\[
P_{ic}^{(1)} \leq 248 \text{erfc}(\sqrt{\rho}) + 1920 \text{erfc}(\sqrt{2(2-\sqrt{2})\rho}) + 30720 \text{erfc}(\frac{\sqrt{2(9-4\sqrt{2})\rho}}{2}) \\
+ 15360 \text{erfc}(\frac{\sqrt{2(8-3\sqrt{2})\rho}}{2}) + 16384 \text{erfc}(2\sqrt{2-\sqrt{2})\rho}) \\
+ 245760 \text{erfc}(\frac{\sqrt{3(8-3\sqrt{2})\rho}}{2}) + 262144 \text{erfc}(\frac{\sqrt{2(16-7\sqrt{2})\rho}}{2}).
\]

• At the $10^{-6}$ decoded block error rate, this inner code provides a 2.20 dB real coding gain over the uncoded QPSK.
ERROR PERFORMANCE

• With SNR = 10 dB/symbol (or 7.06 dB/infor. bit),

\[ P_e \leq 6.91 \times 10^{-41} \]
\[ 1 - P_e \leq 2.08 \times 10^{-12} \]
CODING GAIN

- At the block-error rate $= 10^{-7}$,

$$G_b = 7 \text{ dB/symbol}.$$  

- At the block-error rate $= 10^{-10}$,

$$G_b = 8 \text{ dB/symbol}.$$  

- At the bit-error rate $P_{b_1} = 10^{-31}$,

$$G_{b_1} = 15 \text{ dB/symbol}.$$  

- At the bit-error rate $P_{b_2} = 10^{-10}$,

$$G_{b_2} = 6.26 \text{ dB/symbol (6.19 dB/infor. bit).}$$
Figure 5  Error performance of the 16-state 8-PSK block inner code (the 5-th code in Table 1)
Figure 6  The total probability of a decoding failure and a decoding error for the concatenated coded modulation scheme with the (255,223) RS outer code and the 16-state 8-PSK block inner code (the 5-th code in Table 1)
Figure 7  The probability of a decoding error for the concatenated coded modulation scheme with the (255,223) RS outer code and the 16-state 8-PSK block inner code (the 5-th code in Table 1)
REM ARK

• The inner code decoder can be implemented to perform both decoding and erasure operations.

• In this case, the outer code decoder is devised to correct both symbol errors and erasures.
SESSION VI

TECHNOLOGY NEEDS/OPPORTUNITIES FOR FUTURE MISSIONS
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FUTURE COMMUNICATIONS SATELLITE APPLICATIONS
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