Abstract
We have calculated the amplitude squeezing in the output of several conventionally pumped multi-level lasers. We present results which show that standard laser models can produce significantly squeezed outputs in certain parameter ranges.

Introduction
Production of non-classical light by lasers is an active field both theoretically and experimentally. Sub-Poissonian output has been predicted and observed from lasers in which a regular pumping mechanism reduces the population fluctuations in the lasing levels (Ref.1-4).

Recently we have found that rigorous solutions of conventionally pumped 3 and 4-level lasers predict amplitude squeezing in their output (Ref.5). This is contrary to standard laser theory which predicts Poissonian output far above threshold when the pumping is conventional (Ref.6,7). Our results are in agreement with those of Khazanov et al (Ref.8).

The basic requirement for sub-Poissonian output, without regular pumping, is that the sequence of levels involved in moving an electron from the lower lasing level to the upper lasing level contains at least two steps with approximately equal rates. One of these steps may be the pump itself. Any other rates must be faster. In previous multi-level treatments solutions have been obtained by assuming the pump rate is much slower than all other rates (Ref.6,7). Squeezing will not be seen under these conditions.

We present here results of squeezing spectra calculations for incoherently pumped 3-level and 4-level lasers and a coherently pumped 4-level laser (fig 1). The results highlight the basic effect and how it varies between the models. We also discuss a simple statistical model which illustrates the physical mechanism behind the squeezing.

Fig.1 Laser atomic level schemes. Incoherently pumped 3-level on the left and incoherently pumped (E=0) or coherently (Γ=0) 4-level on the right. The γij are spontaneous decay rates. g is the dipole coupling strength.
Squeezing Results
Using standard techniques (Ref.6,7) a master equation for the reduced density operator $\rho$ of the atoms and cavity is derived. We solve for the full quantum mechanics of the master equation by transforming it into an equivalent partial differential equation for the generalized $P$-function of Drummond and Gardiner (Ref.9). We make the usual approximation that the quantum fluctuations are small perturbations on the semiclassical steady state (Ref.2,9,10). The amplitude squeezing spectrum, $V$, of the laser output field is calculated in the usual way (Ref.10).

In Figure 2 we plot the spectral variance at the zero frequency local minimum of the spectrum as a function of pump rate for the three cases. The full spectra are approximately Lorentzians (in the region shown) with linewidths corresponding to that of the laser cavity. Laser phase diffusion has been ignored. Parameters have been chosen to show maximum squeezing. 0 is perfect squeezing and 1 is the coherent state spectral variance.

Squeezing is improved both by increasing the number of levels with similar rates and by using a coherent pump. The 3-level laser has maximum squeezing of 50% when the spontaneous decay rate $\gamma_{12}$ is double the pump rate ($\Gamma$). The incoherently pumped 4-level laser has maximum squeezing of 66% when $\Gamma = \gamma_{34} = 0.5\gamma_{12}$. The improvement due to the coherent pump is more significant. If $\sqrt{8}\bar{E} = \gamma_{34} = 0.5\gamma_{12}$, where $E$ is proportional to the coherent field strength, 80% squeezing is predicted.

Discussion
The origin of the squeezing can be understood in terms of the temporal behaviour of the electrons in individual atoms$^1$. The variance in the time the pump cycle takes to place an electron in the upper lasing level of an individual atom, $\Delta t^2$, and the spectral variance at zero frequency, $V_{\text{min}}$, are related by$^2$

\[ V_{\text{min}} = \frac{\Delta \bar{n}^2}{\bar{n}} = \frac{\Delta i^2}{(i)^2} \]  

(1)

where \( i \) is the mean time it takes for the electron to arrive in the upper lasing level, \( \Delta \bar{n} \) is the photon number variance of the output and \( \bar{n} \) is the mean number of photons. To obtain this result we have assumed the laser is well above threshold and has a strong enough dipole coupling such that the lasing transition time can be considered to have zero variance. Also we assume spontaneous emission out of the upper lasing level is negligible.

The right-hand side of (1) can be evaluated exactly. For an \((r+3)\)-level incoherently pumped laser the independence of the noise introduced in each step leads to the following expression

\[ V_{\text{min}} \approx \frac{(2/\gamma_L)^2 + (1/\gamma_1)^2 + \ldots + (1/\gamma_r)^2}{(2/\gamma_L + 1/\gamma_1 + \ldots + 1/\gamma_r + 1/\Gamma)^2} \]

where \( \gamma_L \) is the decay rate out of the lower lasing level, \( \Gamma \) is the pump rate and \( \gamma_1 \ldots \gamma_r \) are the rates of the intermediate steps. The rates are matched for optimum noise reduction when \( \Gamma = \gamma_1 = \ldots = \gamma_r = 0.5 \gamma_L \). The minimum value of \( V_{\text{min}} \) is then \( 1/(r+2) \). If the pump rate is much slower than all the other rates then \( V_{\text{min}} \rightarrow 1 \), i.e., Poissonian. This is the limit in which previous calculations were carried out.

For a certain range of pump rates a coherent step introduces less noise into the pump cycle than an incoherent step. This leads to superior squeezing in the coherently pumped laser.

**Summary**

We have presented a brief report of results we have obtained from rigorous solutions of conventionally pumped standard laser models. Contrary to established theory we find amplitude squeezing of the output beam is possible in certain parameter ranges. Physically we find the noise suppression is due to the independence of the noise introduced in the various steps involved in inverting the atoms.

We see no fundamental reason why lasers could not be built which operate in regimes meeting the requirement for squeezing.

**References**