Formal Design and Verification of a Reliable Computing Platform For Real-Time Control

Phase 2 Results

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1 Introduction

NASA is engaged in a major research effort towards the development of a practical validation and verification methodology for digital fly-by-wire control systems. Researchers at NASA Langley Research Center (LaRC) are exploring formal verification as a candidate technology for the elimination of design errors in such systems. In previous reports [1, 2, 3], we put forward a high level architecture for a reliable computing platform (RCP) based on fault-tolerant computing principles. Central to this work is the use of formal methods for the verification of a fault-tolerant operating system that schedules and executes the application tasks of a digital flight control system. Phase 1 of this effort established results about the high level design of RCP. This report presents our Phase 2 results, which carry the design, specification, and verification of RCP to lower levels of abstraction.

The major goal of this work is to produce a verified real-time computing platform, both hardware and operating system software, which is useful for a wide variety of control-system applications. Toward this goal, the operating system provides a user interface that "hides" the implementation details of the system such as the redundant processors, voting, clock synchronization, etc. We adopt a very abstract model of real-time computation, introduce three levels of decomposition of the model towards a physical realization, and rigorously prove that the decomposition correctly implements the model. Specifications and proofs have been mechanized using the EHDM verification system [4].

A major goal of the RCP design is to enable the system to recover from the effects of transient faults. More than their analog predecessors, digital flight control systems are vulnerable to external phenomena that can temporarily affect the system without permanently damaging the physical hardware. External phenomena such as electromagnetic interference (EMI) can flip the bits in a processor's memory or temporarily affect an ALU. EMI can come from many sources such as cosmic radiation, lightning or High Intensity Radiated Fields (HIRF). There is growing concern over the effects of HIRF on flight control systems. In the FAA Digital Systems Validation Handbook - volume II [5], we find:

A number of European military aircraft fatal accidents have been attributed to High Energy Radio Frequency (HERF). A digital fly-by-wire military Tornado aircraft and crew were lost during a tactical training strafing attack in Germany. The loss was attributed to HERF when the aircraft flew through a high intensity Radio Frequency (RF) field. The civil/military aviation industry has very limited experience or data directed to accidents caused by electromagnetic transients and/or radiation. The present criteria, specifications, and procedures are being reevaluated. The HERF fields apparently upset the digital flight control system of the Tornado which was qualified to a very low electromagnetic Environment (EME) standard.

While composite materials may offer significant advantages in strength, weight, and cost, they provide less electromagnetic shielding than aluminum. The use

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1In fly-by-wire aircraft the direct mechanical and hydraulic linkages between the pilot and actuators of the system are replaced with digital computers. These digital computers are being used to control life critical functions such as the engines, sensors, fuel systems and actuators.

2The term HERF has largely been replaced in current usage by the newer term HIRF.
of solid-state digital technology in flight-critical systems create major challenges to prevent transient susceptibility and upset in both civil and military aircraft. Therefore, the Civil Aviation Authority (CAA), United Kingdom (U.K.) and the Federal Aviation Administration (FAA), United States (U.S.) voiced concern relative to emerging technology aircraft and systems.

The RCP system is designed to automatically flush the effects of transients periodically, as long as the effect of a transient is not massive; that is, simultaneously affecting a majority of the redundant processors in the system. Of course, there is no hope of recovery if the system designed to overcome transient faults contains a design flaw. Consequently, a major emphasis in this work has been the development of techniques that mathematically show when the desired recovery properties have been achieved. The advantages of this approach are significant:

- Confidence in the system does not rely primarily on end-to-end testing, which can never establish the absence of some rare design flaw (yet more frequent than $10^{-9}$ [6]) that can crash the system [7].

- Minimizes the need for experimental analysis of the effects of EMI or HIRF on a digital processor. The probability of occurrence of a transient fault must be experimentally determined, but it is not necessary to obtain detailed information about how a transient fault propagates errors in a digital processor.

- The role of experimentation is determined by the assumptions of the mathematical verification. The testing of the system can be concentrated at the regions where the design proofs interface with the physical implementation.

### 1.1 Design of the Reliable Computing Platform

Traditionally, the operating system function in flight control systems has been implemented as an executive (or main program) that invokes subroutines implementing the application tasks. For ultra-reliable systems, the additional responsibility of providing fault tolerance and undergoing validation makes this approach questionable. We propose a well-defined operating system that provides the applications software developer a reliable mechanism for dispatching periodic tasks on a fault-tolerant computing base that appears to him as a single ultra-reliable processor.

Our system design objective is to minimize the amount of experimental testing required and maximize our ability to reason mathematically about correctness. The following design decisions have been made toward that end:

- the system is non-reconfigurable
- the system is frame-synchronous
- the scheduling is static, non-preemptive
- internal voting is used to recover the state of a processor affected by a transient fault

Future work will concentrate on the massive transient and techniques to detect and restart a massively upset system.
A four-level hierarchical decomposition of the reliable computing platform is shown in figure 1.

The top level of the hierarchy describes the operating system as a function that sequentially invokes application tasks. This view of the operating system will be referred to as the *uniprocessor model*, which is formalized as a state transition system in section 3.2 and forms the basis of the specification for the RCP.

Fault tolerance is achieved by voting results computed by the replicated processors operating on the same inputs. Interactive consistency checks on sensor inputs and voting of actuator outputs require synchronization of the replicated processors. The second level in the hierarchy describes the operating system as a synchronous system where each replicated processor executes the same application tasks. The existence of a global time base, an interactive consistency mechanism and a reliable voting mechanism are assumed at this level. The formal details of the model, specified as a state transition system, are described in section 3.3.

Although not anticipated during the Phase 1 effort, another layer of refinement was inserted before the introduction of asynchrony. Level 3 of the hierarchy breaks a frame into four sequential phases. This allows a more explicit modeling of interprocessor communication and the time phasing of computation, communication, and voting. The use of this intermediate model avoids introducing these issues along with those of real time, thus preventing an overload of details in the proof process.

At the fourth level, the assumptions of the synchronous model must be discharged. Rushby and von Henke [8] report on the formal verification of Lamport and Melliar-Smith's [9] interactive-convergence clock synchronization algorithm. This algorithm can serve as a foundation for the implementation of the replicated system as a collection of asynchronously operating processors. Dedicated hardware implementations of the clock synchronization function are a long-term goal.

Final realization of the reliable computing platform is the subject of the Phase 3 effort. The research activity will culminate in a detailed design and prototype implementation.
Figure 2: Generic hardware architecture.

Figure 2 depicts the generic hardware architecture assumed for implementing the replicated system. Single-source sensor inputs are distributed by special purpose hardware executing a Byzantine agreement algorithm. Replicated actuator outputs are all delivered in parallel to the actuators, where force-sum voting occurs. Interprocessor communication links allow replicated processors to exchange and vote on the results of task computations. As previously suggested, clock synchronization hardware may be added to the architecture as well.

1.2 Overview of Results

Before presenting the complete details, we provide an overview of the major formalizations and results for the reliable computing platform. In accordance with accepted terminology, we consider a fault to be a condition in which a piece of hardware is not operating within its specifications due to physical malfunction, and an error to be an incorrect computation result or system output. When a fault occurs, errors may or may not be produced. Although fault-tolerant architectures offer a high degree of immunity from hardware faults, there is a limit to how many simultaneous faults can be tolerated. Unless this limit is exceeded during system operation, the system will mask the occurrence of errors so that the system as a whole produces no computation errors. If the limit is exceeded, however, the system might produce erroneous results.
The primary mechanism for tolerating faults is voting of redundant computation results. Voting can take place at a number of locations in the system and associated with each choice are various tradeoffs. If voting occurs only at the actuators and the internal state of the system (contained in volatile memory) is never subjected to a vote, a single transient fault can permanently corrupt the state of a good processor. This is an unacceptable approach since field data indicates that transient faults are significantly more likely than permanent faults [10]. An alternative voting strategy is to vote the entire system state at frequent intervals. This approach quickly purges the effects of transient faults from the system; however, the computational overhead for this approach may be prohibitive. There is a trade-off between the rate of recovery from transient faults and the frequency of voting. The more frequent the voting, the faster the recovery from transients, but at the price of increased computational overhead. We observe that voting need only occur for a system state that is not recoverable from sensor inputs. A sparse voting approach can accomplish recovery from the effects of transient faults at greatly reduced overhead, but involves increased design complexity. The formal models presented here provide an abstract characterization of the voting requirements for a fault-tolerant system that purges the effects of transient faults.

The proofs we construct are implicitly conditional to account for the situation of limited fault tolerance. The main results we establish can be expressed by the following formula:

\[ W(r_1, \ldots, r_n) \supset s = V(r_1, \ldots, r_n) \]

where \( W \) is a predicate to define a minimal working hardware subset over time, \( s \) is the uniprocessor model's system results, \( r_1, \ldots, r_n \) are the results of the replicated processors, and \( V \) is a function that selects the properly voted values at each step. Moreover, asynchronous operation is assumed at the lowest specification layer. In this case, we further establish that if the minimal working hardware includes an adequate number of nonfaulty clocks, and clock synchronization is maintained, then the voted outputs continue to match those of higher level specifications. Thus, as long as the system hardware does not experience an unusually heavy burst of component faults, the proof establishes that no erroneous operation will occur at the system level. Individual replicates may produce errors, but they will be out-voted by replicates producing correct results.

If the condition \( W \) were true 100% of the time, the system would never fail. Unfortunately, real devices are imperfect and this cannot be achieved in practice. The design of the fault-tolerant architecture must ensure that condition \( W \) holds with high probability; typically, the goal is \( P(W) \geq 1 - 10^{-9} \) for a 10 hour mission. This condition provides a vital connection between the reliability model and the formal correctness proofs. The proofs conditionally establish that system output is not erroneous as long as \( W \) holds, and the reliability model predicts that \( W \) will hold with adequately high probability.

In the formal development to follow, we model the possible occurrence of component hardware faults and the unknown nature of computation results produced under such conditions. It is important to note that this modeling is for specification purposes only and reflects no self-cognition on the part of the running system. We assume a nonreconfigurable architecture that is capable of masking the effects of faults, but makes no attempt to detect or diagnose those faults. Each replicate is computing independently and continues to operate the best it can under faulty conditions; it has no knowledge of its own faultiness or that of
its peers. Wherever the formal specifications consider the two cases of whether a processor is faulty or not, it is important to remember that this case analysis is not performed by the running system. Also, it is important to realize that transient-fault recovery is a process that is continually in effect, even when there have been no fault occurrences. Each processor in the system continually votes and replaces its state with voted values. Thus, the transient fault recovery process does not require fault detection.

1.3 Previous Efforts

Many techniques for implementing fault-tolerance through redundancy have been developed over the past decade, e.g. SIFT [11], FTMP [12], FTP [13], MAFT [14], and MARS [15]. An often overlooked but significant factor in the development process is the approach to system verification. In SIFT and MAFT, serious consideration was given to the need to mathematically reason about the system. In FTMP and FTP, the verification concept was almost exclusively testing.

Among previous efforts, only the SIFT project attempted to use formal methods [16]. Although the SIFT operating system was never completely verified [17], the concept of Byzantine Generals algorithms was developed [18] as was the first fault-tolerant clock synchronization algorithm with a mathematical performance proof [9]. Other theoretical investigations have also addressed the problems of replicated systems [19].

Some recent work at SRI International has focused on problems related to the style of fault-tolerant computing adopted by RCP. Rushby has studied a fault masking and transient recovery model and created a formalization of it using EiDLM [20, 21]. In addition, Shankar has undertaken the formalization of a general scheme for modeling fault-tolerant clock synchronization algorithms [22, 23].

2 Specification Hierarchy and Verification Approach

This section outlines the general methods used in the RCP specifications and proofs. Detailed discussions of the actual specifications appear in later sections.

2.1 The State Machine Approach to Specification

The specification of the Reliable Computing Platform (RCP) is based upon a state-machine method. The behavior of the system is described by specifying an initial state and the allowable transitions from one state to another. The specification of the transition must determine (or constrain) the allowable destination states in terms of the current state and current inputs. One way of doing this is to specify the transition as a function:

\[ f_{\text{trans}} : \text{state} \times \text{input} \rightarrow \text{state} \]

This is an appealing method when it can be used. A second method is to specify the transition as a mathematical relation between the current state, the input and the new state. One way
to specify a mathematical relation is to define it using a function from the current state, the current input and the new state to a boolean:

\[ R : \text{state} \times \text{input} \times \text{state} \rightarrow \text{boolean} \]

The function \( R \) is true precisely when the relation holds and false, otherwise. The meaning is as follows: a transition from the current state to the new state can occur only when the relation is true. Although the concept is simple it is somewhat awkward to use at first. Consider the function \( g \) defined by \( g(x) = (x + 4)^2 \).

In relational form this function might be expressed by:

\[ R(x, y) = [y = (x + 4)^2] \]

The latter form is more awkward than the former when a purely functional relationship exists between \( x \) and \( y \). However, a relational approach has some advantages over a functional approach for the specification of complex system behavior. In particular, nondeterminism can be accommodated in a specification by only partially constraining system behavior. For example, if \( R \) is changed to the following:

\[ R(x, y) = [x > 0 \implies y = (x + 4)^2] \]

the value of \( y \) is specified only for positive values of \( x \). In other cases, any value of \( y \) would stand in the relation \( R \) to \( x \). Such partially constrained specifications are very natural for modeling fault-tolerant systems. It allows us to say nothing about the behavior of failed components, thereby enabling proved results to hold no matter what behavior is exhibited by failed components during system operation.

The relation \( R \) would be described as follows in the EHDM specification language:

\[
R: \text{function[number, number -> bool]} = \\
(\text{LAMBDA } x,y: (x > 0 \text{ IMPLIES } y = (x+4)*(x+4)))
\]

The first line declares that \( R \) is a function from number \( \times \) number to the set of booleans (bool). The second line uses lambda notation to define the body of the function.

It should also be noted that the modeling approach used in this paper is not based upon a finite state machine technique. Some of the components of the state takes values from infinite domains. Therefore, verification tools such as STATEMATE [24] or MCB [25] are not applicable to our specifications.

2.2 Specifying Behavior in the Presence Of Faults

The specification of the RCP system is given in relational form. This enables one to leave unspecified the behavior of a faulty component. Consider the example below.

\[
R_{\text{tran}} : \text{function[State, State \rightarrow bool]} = \\
(\lambda s, t : \text{nonfaulty}(s(i)) \supset t(i) = f(s(i)))
\]
In the relation $R_{\text{tran}}$, if component $i$ of state $s$ is nonfaulty, then component $i$ of the next state $t$ is constrained to be equal to $f(s(i))$. For other values of $i$, that is, when $s(i)$ is faulty, the next state value $t(i)$ is unspecified. Any behavior of the faulty component is acceptable in the specification defined by $R_{\text{tran}}$.

An alternative approach is to define the transition as a partially-specified function:

$$f_{\text{tran}} : \text{function}[\text{State} \rightarrow \text{State}]$$

$$\text{tran}_\text{ax} : \text{Axiom } \text{nonfaulty}(s(i)) \supset f_{\text{tran}}(s)(i) = g(s(i))$$

This approach does not fit within the definitional structure of EIIDM. Therefore, one must use an axiom to specify properties of a total, but partially defined function. This leads to a large number of axioms at the base of the proofs and significantly increases the possibility of inconsistency in the axiom set.

2.3 The Specification Hierarchy

The RCP specification consists of four separate models of the system: Uniprocessor System (US), Replicated Synchronous (RS), Distributed Synchronous (DS), Distributed Asynchronous (DA). Each of these specifications is in some sense complete; however, they are at different levels of abstraction and describe the behavior of the system with different degrees of detail. The US model is the most abstract and defines the behavior of the system using a single uninterpreted definition. The RS level supplies more detail. The computation is replicated on multiple processors but the data exchange and voting is captured in one transition. The next level, the DS level, introduces even more detail. Explicit buffers for data exchange are modeled and the transition of the RS level is decomposed into 4 sub-transitions. The DA level introduces time, and different clock times on each of the separate processors.4

1. **Uniprocessor System layer** (US). As in the Phase 1 report [1], this constitutes the top-level specification of the functional system behavior defined in terms of an idealized, fault-free computation mechanism. This specification is the correctness criterion to be met by all lower level designs. The top level of the hierarchy describes the operating system as a function that performs an arbitrary, application-specific computation.

2. **Replicated Synchronous layer** (RS). This layer corresponds to level 2 of the Phase 1 report. Processors are replicated and the state machine makes global transitions as if all processors were perfectly synchronized. Interprocessor communication is hidden and not explicitly modeled at this layer. Suitable mappings are provided to enable proofs that the RS layer satisfies the US layer specification. Fault tolerance is achieved using exact-match voting on the results computed by the replicated processors operating on the same inputs. Exact match voting depends on two additional system activities: (1) single source input data must be sent to the redundant sites in a consistent manner to ensure that each redundant processor uses exactly the same inputs during its

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4Due to the difficulties associated with reasoning about asynchronous systems, it was desirable to perform as much of the design and verification using a synchronous model as possible. Thus, only at level 4 is time explicitly introduced.
computations, and (2) the redundant processing sites must synchronize for the vote. Interactive consistency can be achieved on sensor inputs by use of Byzantine-resilient algorithms [18], which are probably best implemented in custom hardware. To ensure absence of single-point failures, electrically isolated processors cannot share a single clock. Thus, a fault-tolerant implementation of the uniprocessor model must ultimately be an asynchronous distributed system. However, the introduction of a fault-tolerant clock synchronization algorithm, at the DA layer of the hierarchy, enables the upper level designs to be performed as if the system were synchronous.

3. **Distributed Synchronous layer (DS).** Next, the interprocessor communication mechanism is modeled and transitions for the RS layer machine are broken into a series of subtransitions. Activity on the separate processors is still assumed to occur synchronously. Interprocessor communication is accomplished using a simple mailbox scheme. Each processor has a mailbox with bins to store incoming messages from each of the other processors of the system. It also has an outgoing box that is used to broadcast data to all of the other processors in the system. The DS machine must be shown to implement the RS machine.

4. **Distributed Asynchronous layer (DA).** Finally, the lowest layer relaxes the assumption of synchrony and allows each processor to run on its own independent clock. Clock time and real time are introduced into the modeling formalism. The DA machine must be shown to implement the DS machine provided an underlying clock synchronization mechanism is in place.

The basic design strategy is to use a fault-tolerant clock synchronization algorithm as the foundation of the operating system. The synchronization algorithm provides a global time base for the system. Although the synchronization is not perfect it is possible to develop a reliable communications scheme where the clocks of the system are skewed relative to each other, albeit within a strict known upper bound. For all working clocks \( p \) and \( q \), the synchronization algorithm provides the following key property:

\[
|c_p(T') - c_q(T)| < \delta
\]

assuming that the number of faulty clocks, say \( m \), does not exceed \((n_{rep}-1)/3\), where \( n_{rep} \) is the number of replicated processors. This property enables a simple communications protocol to be established whereby the receiver waits until \( \text{maxb} + \delta \) after a pre-determined broadcast time before reading a message, where \( \text{maxb} \) is the maximum communication delay.

Each processor in the system executes the same set of application tasks every cycle. A cycle consists of the minimum number of frames necessary to define a continuously repeating task schedule. Each frame is \( \text{frame.time} \) units of time long. A frame is further decomposed into 4 phases. These are the compute, broadcast, vote and sync phases. During the compute phase, all of the applications tasks scheduled for this frame are executed. The results of all tasks that are to be voted this frame are then loaded into the outgoing mailbox. During the next phase, the broadcast phase, the system merely waits a sufficient amount of time to allow all of the messages to be delivered. As mentioned above, this delay must be greater than \( \text{maxb} + \delta \). During the vote phase, each processor retrieves all of the replicated data
from each processor and performs a voting operation. Typically, this operation is a majority vote on each of the selected state elements. The processor then replaces its local memory with the voted values. It is crucial that the vote phase is triggered by an interrupt and all of the vote and state-update code be stored in ROM. This will enable the system to recover from a transient even when the program counter has been affected by a transient fault. Furthermore, the use of ROM is necessary to ensure that the code itself is not affected by a transient. During the final phase, the sync phase, the clock synchronization algorithm is executed. Although conceptually this can be performed in either software or hardware, we intend to use a hardware implementation.

2.4 Extended State Machine Model

Formalizing the behavior of the Distributed Asynchronous layer requires a means of incorporating time. We accomplish this by formulating an extended state machine model that includes a notion of local clock time for each processor. It also recognizes several types of transitions or operations that can be invoked by each processor. The type of operation dictates which special constraints are imposed on state transitions for certain components.

The time-extended state machine model we use allows for autonomous local clocks on each processor to be modeled using snapshots of clock time coinciding with state transitions. Clock values represent the time at which the last transition occurred (time current state was entered). If a state was entered by processor \( p \) at time \( T \) and is occupied for a duration \( D \), the next transition occurs for \( p \) at time \( T + D \) and this clock value is recorded for \( p \) in the next state. A function \( c_p(T) \) is assumed to map local clock values for processor \( p \) into real time. \( c_p(T) \) is a specification-only function; it is not implemented by the system.

Clocks may become skewed in real time. Consequently, the occurrence of corresponding events on different processors may be skewed in real time. A state transition for the DA state machine corresponds to an aggregate transition in which each processor experiences a particular event, such as completing one phase of a frame and beginning the next. Each processor may experience the event at different real times and even different clock times if duration values are not identical.

The DA model is based on a specialized kind of state machine tailored to the needs of an asynchronous system of replicated processors. The intended interpretation is that each component of the state models the local state of one processor and its associated hardware. Each processor is assumed to have a local clock running independently of all the others. Interprocessor communication is achieved by one class of transition that performs a simultaneous broadcast of a portion of the local state variables to all the other processors. Broadcast values are assumed to arrive in the destination mailboxes within a bounded amount of real time \( \text{maxb} \).

The four classes of transitions are defined as follows:

---

5In the design specifications, these implementation details are not explicitly specified. However, it is clear that in order to successfully implement the models and prove that the implementation performs as specified, such implementation constructs will be needed. These issues will be explored in detail in future work.

6We will use the now standard convention of representing clock time with capital letters and real time with lower case letters.
1. **L**: Purely local processing that involves no broadcast communication or reading of the mailboxes.

2. **B**: Broadcast communication where a send is initiated when the state is entered and must be completed before the next transition.

3. **R**: Local processing that involves no send operations, but does include reading of mailbox values.

4. **C**: Clock synchronization operations that may cause the local clock to be adjusted and appear to be discontinuous.

   We make the simplifying assumption that the duration spent in each state, except those of type C, is nominally a fixed amount of clock time. Allowances need to be made, however, for small variations in the actual clock time used by real processors. Thus if $\nu$ is the maximum rate of variation and $D_I, D_A$ are the intended and actual durations, then $|D_A - D_I| \leq \nu D_I$ must hold.

### 2.5 The Proof Method

The proof method is a variation of the classical algebraic technique of showing that a homomorphism exists. Such a proof can be visualized as showing that a diagram "commutes" (figure 3). The system is described at two levels of abstraction, which will be referred to as the top and bottom levels for convenience. The top level consists of a current state $s'$, a destination state, $t'$ and a transition that relates the two. The properties of the transition are given as a mathematical relation, $\mathcal{N}_{top}(s', t')$. Similarly, the bottom level consists of a state $s$, a destination state, $t$ and a transition that relates the two. The properties of the transition are given as a mathematical relation, $\mathcal{N}_{bottom}(s, t)$. The state values at the bottom level are related to the state values at the top level by way of a mapping function, $\text{map}$. To establish that the bottom level implements the top level one must show that the diagram commutes:

$$\mathcal{N}_{bottom}(s, t) \supseteq \mathcal{N}_{top}(\text{map}(s), \text{map}(t))$$
where $map(s) = s'$ and $map(t) = t'$ in the diagram. One must also show that initial states map up:

$$I_{bottom}(s) \supseteq I_{top}(map(s))$$

An additional consideration in constructing such proofs is that only states reachable from an initial state are relevant. Thus, it suffices to prove a conditional form of commutativity that assumes transitions always begin from reachable states. A weaker form of the theorem is then called for:

$$reachable(s) \land N_{bottom}(s, t) \supseteq N_{top}(map(s), map(t))$$

This form enables proofs that proceed by first establishing state invariants. Each invariant is shown to hold for all reachable states and then invoked as a lemma in the main proof.

Figure 4 shows the complete state machine hierarchy and the relationships of transitions within the aggregate model. By performing three layer-to-layer state machine implementation proofs, the states of DA, the lowest layer, are shown to correctly map to those of US, the highest layer. This means that any implementation satisfying the DA specification will likewise satisfy US under our chosen interpretation.
3 US/RS Specification

Up to now we have dealt only with general methods. Next we present the RCP specifications as developed using the EHDM language. An index at the end of this report indicates page numbers where each specification identifier and special symbol is defined in the text. The complete EHDM specifications can be found in Appendix A.

3.1 Preliminary Definitions

The US and RS specifications are expressed in terms of some primitive type definitions. First, we must establish a "domain" or type to represent the complete computation state of a processor. This domain is called Pstate. It is declared in EHDM as

```
Pstate: Type (* computation state of a single processor *)
```

Thus, all of the state information subject to computation has been collapsed into a single type Pstate. Similarly, inputs denotes the domain of external system inputs (sensors), and outputs the domain of output values that will be sent to the actuators of the system. These domains are named by the following EHDM declarations:

```
inputs: Type (* type of external sensor input *)
outputs: Type (* actuator output type *)
```

The number of processors in the system is declared as an arbitrary, positive constant, nrep:

```
nrep: nat (* number of replicated processors *)
```

The constraint on nrep's value is expressed by the following axiom

```
processors_exist_ax: Axiom nrep > 0
```

is a requirement that the system have at least one processor. Nearly all symbolic constants we introduce will have similar constraints imposed on them.

At the RS level and below, information is exchanged among processors via some interprocessor communication mechanism. Additional types are needed to describe the information units involved, being based on a mailbox model of communication. First, we introduce a domain of values for each bin in the mailboxes:

```
MB : Type (* mailbox exchange type *)
```

Then we construct a type for a complete mailbox on a processor:

```
MBvec: Type = array [processors] of MB
```

This scheme provides one slot in the mailbox array for each replicated processor.
3.2 US Specification

The US specification is very simple:

\[
\begin{align*}
  s, t &: \textbf{Var} \text{ Pstate} \\
  u &: \textbf{Var} \text{ inputs} \\
  \mathcal{N}_{us} &: \text{Definition function}[\text{Pstate, Pstate, inputs} \rightarrow \text{bool}] = \\
  &\quad (\lambda s, t, u : t = f_c(u, s))
\end{align*}
\]

The function \( \mathcal{N}_{us} \) defines a mathematical relation between a current state and a final state, i.e., it defines the transition relation. For this model, the transition condition is captured by a function: \( f_c(u, s) \), i.e., the computation performed by the uniprocessor system is deterministic and thus can be modeled by a function \( f_c : \text{inputs} \times \text{Pstate} \rightarrow \text{Pstate} \). To fit the relational, nondeterministic state machine model we let the state transition relation \( \mathcal{N}_{us}(s, t, u) \) hold iff \( t = f_c(u, s) \).

External system outputs are selected from the values computed by \( f_c \). The function \( f_s : \text{Pstate} \rightarrow \text{outputs} \) denotes the selection of state variable values to be sent to the actuators. The type \( \text{outputs} \) represents a composite of actuator output types.

Although there is no explicit mention of time in the US model, it is intended that a transition correspond to one frame of the execution cycle (i.e., the schedule).

The uninterpreted constant \( \text{initial\_proc\_state} \) represents the initial \( \text{Pstate} \) value from which computation begins.

\[
\text{initial\_us}: \text{function}[\text{Pstate} \rightarrow \text{bool}] = (\lambda s : s = \text{initial\_proc\_state})
\]

\( \text{initial\_us} \) is expressed in predicate form for consistency with the overall relational method of specification, although in this case the initial state value is unique.

3.3 RS Specification

At the RS layer of design, the state is replicated and a postprocessing step is added after computation. This step represents the voting of state variables and thus may be selectively applied. It suffices to encapsulate the entire voting process under a single function of the global state. Nonetheless, it is better to split voting into two parts to facilitate refinement to the DS layer. Another difference introduced at this layer is that the state transition relation needs to be conditioned on the nonfaulty status of each processor.

The global state at this level has type \( \text{RSstate} \). This is a vector of length \( n_{rep} \) where each component of the vector defines the state of a specific processor. Each processor in the system can be faulty or nonfaulty as a function of time measured in frames. The local processor "state" must not only reflect the computation state but indicate whether or not a processor is faulty. Such status information about faultiness is included for the purpose of modeling system behavior. An actual system component would be unable to maintain this status and it is understood that this part of the state exists only to model operational behavior and is not an implemented part of the system. Specification of the state type is as follows:
rs_proc_state: Type = Record healthy : nat,  
proc_state : Pstate  
end record

RSstate: Type = array [processors] of rs_proc_state

The state of a single processor is given by a record named rs_proc_state. The first field of the record is healthy, which is 0 when a processor is faulty. Otherwise, it indicates the (unbounded) number of state transitions since the last transient fault. Its value is one greater than the number of prior nonfaulty frames. A permanent fault is indicated by a perpetual value of 0. A processor that is recovering from a transient fault is indicated by a value of healthy less than the recovery period, denoted by the constant recovery_period. This constant is determined by details of the application task schedule and the voting pattern used for transient recovery. A processor is said to be working whenever healthy ≥ recovery_period.

The second field of the record is the computation state of the processor. It takes values from the same domain as used in the US specification. The complete state at this level, RSstate, is a vector (or array) of these records.

Two uninterpreted functions are assumed to express specifications that involve selective voting on portions of the computation state. Their role is described more fully in section 3.5.

fs: function[Pstate → MB] (* state selection for voting *)
fv: function[Pstate, MBvec → Pstate] (* voting and overwriting *)

These two functions split up the selective voting process to mirror what happens in the RCP architecture. First, fs is used to select a subset of the state components to be voted during the current frame. The choice of which components to vote is assumed to depend on the computation state. It maps into the type MB, which stands for a mailbox item. Second, the function fv takes the current state value and overwrites selected portions of it with voted values derived from a vector of mailbox items. Voting is performed on a component-by-component basis, that is, applied to each task state separately, rather than applied to entire mailbox contents. Note that selection via fs need not be a mere projection, but could involve more complex data transformations such as adding checksums to ensure integrity during transmission.

Given this background, the transition relation, Nr,s, can be defined:

Nr,s: Definition function[RSstate, RSstate, inputs → bool] =
( λ s, t, u : (∃ h : (∀ i : 
(s(i)).healthy > 0

⇒ good_values_sent(s, u, h(i)) ∧ voted_final_state(s, t, u, h, i)))
∧ allowable_faults(s, t))

This relation is defined in terms of three subfunctions: good_values_sent, voted_final_state, and allowable_faults. The first aspect of this definition to note is that the relation holds only when allowable_faults is true. This corresponds to the “Maximum Fault Assumption” discussed in [1], namely that a majority of processors have been working up to the current time. The next thing to notice is that the transition relation is defined in terms of a conjunction good_values_sent(s,u,h(i)) ∧ voted_final_state(s,t,u,h,i)). The meaning is intuitive: the
outputs produced by the good processors are contained in the vector $h$ (i.e., $h(i)$ is derived from the value produced on processor $i$), and the final state $t$ is obtained by voting the $h$ values. Let us look at the \texttt{voted\_final\_state} relation first.

\begin{align*}
\textbf{voted\_final\_state}: \text{function}[\text{RSstate, RSstate, inputs, MBmatrix, processors} \rightarrow \text{bool}] \\
= (\lambda s, t, u, h, i : t(i).proc\_state = f_v(f_c(u, s(i).proc\_state), h(i)))
\end{align*}

Processor $i$ is initially in state $s(i)$. If it is nonfaulty ($s(i).healthy > 0$), then its transition to the state $t(i)$ observes the following constraint:

\begin{align*}
t(i).proc\_state = f_v(f_c(u, s(i).proc\_state), h(i))
\end{align*}

Otherwise, the behavior of the processor is not defined (i.e., a known mathematical relation is not given). The change to the processor state is defined using two functions: $f_c, f_v$. The function $f_c$ is the same function used in the US specification. The function $f_v$ operates on the updated computation state and values obtained from the other processors to produce a new state. The idea is that the new state is obtained by replacing local values with voted values.

The values sent by the other processors must satisfy the following relation:

\begin{align*}
\textbf{good\_values\_sent}: \text{function}[\text{RSstate, inputs, MBvec} \rightarrow \text{bool}] = \\
(\lambda s, u, w : (\forall j : (s(j)).healthy > 0 \supset w(j) = f_s(f_c(u, s(j).proc\_state))))
\end{align*}

This relation constrains the $h(i)$ values used in the definition of the $N_s$ transition relation. Although this function is called with $h(i)$ as an argument, its formal parameter is named $w$. There is one $w$ value for each processor, which is used to model that processor’s mailboxes. If the sending processor $j$ is nonfaulty ($s(j).healthy > 0$), then the value in the receiving mailbox $w$ is given by

\begin{align*}
f_s(f_c(u, s(j).proc\_state)).
\end{align*}

The function $f_s$ selects which portion of the total state is to be voted. Note that since it is a function of the (complete) state, it can differ as a function of the frame, i.e., different data are voted during different frames.

The \texttt{allowable\_faults} function is defined as follows:

\begin{align*}
\textbf{allowable\_faults}: \text{function}[\text{RSstate, RSstate} \rightarrow \text{bool}] = \\
(\lambda s, t : \text{maj\_working}(t) \\
\quad \land (\forall i : t(i).healthy > 0 \supset t(i).healthy = 1 + s(i).healthy))
\end{align*}

This function enforces the restriction imposed by the Maximum Fault Assumption, namely that all reachable states must have a majority of working processors. The condition is expressed in terms of the function \texttt{maj\_working} and its subordinates:

\begin{align*}
\textbf{maj\_condition}: \text{function}[\text{set[processors]} \rightarrow \text{bool}] = \\
(\lambda A : 2 \ast \text{card(A)} > \text{card(fullset[processors])})
\end{align*}
\[
\text{working_proc: function[RSstate, processors → bool] =}
\quad (\lambda s, p : (s(p)).healthy \geq \text{recovery_period})
\]

\[
\text{working_set: function[RSstate → set[processors]] =}
\quad (\lambda s : (\lambda p : \text{working_proc}(s, p)))
\]

\[
\text{maj_working: function[RSstate → bool] =}
\quad (\lambda t : \text{maj_condition(working_set}(t)))
\]

The `working_set` function gives the set of working processors for the current replicated state. The cardinality of this set is then the number of working processors. (Note that sets are usually represented in EHDM by predicates on the element type. Thus, \((\lambda x : P(x))\) denotes the set \(\{x|P(x)\}\).) The relation `allowable_faults` is defined whenever the destination state contains a majority of working processors. It also states that if a processor is nonfaulty for the current frame then the next state's value of healthy equals the previous state's value plus one.

The initial state predicate `initial_rs` sets each element of the RS state array to the same value with the healthy field equal to `recovery_period` and the `proc_state` field equal to `initial_proc_state`.

\[
\text{initial_rs: function[RSstate → bool] =}
\quad (\lambda s : (\forall p : s(p).healthy = \text{recovery_period} \land s(p).proc_state = \text{initial_proc_state}))
\]

The constant `recovery_period` is the number of frames required to fully recover a processor's state after experiencing a transient fault. By initializing all healthy fields to this value, we are starting the system with all processors `working`.

### 3.4 Actuator Outputs

The nature of actuator outputs in the RCP application deserves special attention. In the uniprocessor case, an output is produced during each frame and sent to the actuators and no ambiguity exists. In a replicated system, however, multiple actuator values are produced and sent during each frame. Each nonfaulty processor `p` sends actuator values given by `f_a(rs(p),proc_state)`. There are `nrep` sets of actuator values delivered in parallel, some of which may be copies of previous values for processors that have failed in such a way as to stop generating new values.

It is understood that actuator outputs may be sent through one or more hardware `voting planes` before arriving at the actuators themselves. Other types of signal transformations may be applied to actuator lines between the output drivers and termination points. Additionally, some kind of `force-sum voting` typically is applied at the actuators to mask the presence of errors in one or more channels. All of this activity seeks to ensure that actuators perform as directed by a consensus of processors. These special-purpose requirements of the application leave us unable to completely reflect the proper constraints in the correctness criteria. However, we can use the majority function to map replicated output values into the single actuator output value that would be produced by an ideal uniprocessor. This captures the effect of voting planes and approximates the effect of force-sum voting at the actuators.
To show that replicated actuator outputs can be mapped into a single actuator output, we reason as follows. At the RS level, there are $n_{rep}$ actuator values given by $f_a(rs(p).proc.state)$ for $p = 1, \ldots, n_{rep}$. In section 4, a property of RS states is described that asserts that a majority exists among the $proc.state$ values. In other words, a majority of values in \{rs(p).proc.state\} equal $maj(rs)$. Therefore, a majority of $f_a(rs(p).proc.state)$ values exists and is equal to $f_a(maj(rs))$. Since $maj(rs)$, the mapped value of an RS state, is equal to the corresponding US state, this shows that a majority of RS actuator outputs match the value produced by the fault-free US machine.

Note that various additional requirements may be necessary, but are regarded as peculiar to the nature of an RCP application. Hence they must be imposed as correctness criteria beyond those necessary to show that one state machine properly implements another. The intended use of replicated actuator outputs is not contained in the state machine models and may necessitate the use of additional, application-specific correctness conditions.

### 3.5 Generic Fault-Tolerant Computing

To model a very general class of fault-tolerant, real-time computing schemes, we seek to parameterize the specifications as much as possible. This parameterization takes the form of a set of uninterpreted constants, types, and functions along with axioms to constrain their values. Some instances have already been introduced. The function $f_e$, for example, represents any computation that can be modeled as a function mapping from inputs and current state into a new state. As hardware redundancy and transient fault recovery are added to the specifications, additional types and functions are needed to express system behavior.

#### 3.5.1 State Model for Transient Fault Recovery

Thus far, we have not concerned ourselves with the internal structure of the computation state $Pstate$. However, to capture the concept of recovering this state information piecewise, it is necessary to make some minimal assumptions about the structure of a $Pstate$ value.

```plaintext
control.state: Type (* portion of state used to control or schedule computation activities, e.g., frame counter *)
cell: Type (* index for components of computation state *)
cell.state: Type (* information content of computation state components *)
```

We assume the state contains a control portion, used to schedule and manage computation, and a vector of $cells$, each individually accessible and holding application-specific state information. A sample instantiation of these types is that found in our previous report [1]: the control state is a frame counter and the cells represent the outputs of task instances in the task schedule. Unlike our previous model, however, the more general framework allows a system to maintain state information further back than just the previous execution of a schedule cell.

Also assumed is the existence of access functions to extract and manipulate these items from a $Pstate$ value.
succ: function[control_state → control_state]   (* next control state *)
f_k: function[Pstate → control_state]   (* extracts control state *)
f_t: function[Pstate, cell → cell_state]   (* extracts cell (e.g. task) state *)

As described in section 3.3, two additional functions are assumed to express specifi-
cations that involve selective voting on portions of the computation state. The functions
f_k : Pstate → MB and f_v : Pstate × MBvec → Pstate were introduced to model the selective
voting process applied by each processor. f_k selects which portions of the computation
results are subject to voting. f_v takes these selected values from the replicated processors and
replaces the required portions of the current state with voted values.

For every voting scheme used for transient fault recovery within RCP, we must be able to
determine when the state components have been recovered from voted values. This condition
is expressed in terms of the current control state and the number of nonfaulty frames since
the last transient fault. Two uninterpreted functions are provided for this purpose.

rec: function[cell, control_state, nat → bool]

The predicate rec(c, K, H) is true iff cell c’s state should have been recovered when in control
state K with healthy frame count H. Recall that we use a healthy count of one to indicate
that the current frame is nonfaulty, but the previous frame was faulty. This means that
H - 1 healthy frames have occurred prior to the current one.

dep: function[cell, cell, control_state → bool]

The predicate dep(c, d, K) indicates that cell c’s value in the next state depends on cell d’s
value in the current state, when in control state K. This notion of dependency is different
from the notion of computational dependency; it determines which cells need to be recovered
in the current frame on the recovering processor for cell c’s value to be considered recovered
at the end of the current frame. If cell c is voted during K, or its computation takes only
sensor inputs, there is no dependency. If c is not computed during K, c depends only on its
own previous value. Otherwise, c depends on one or more cells for its new value.

One derived function is used in the axioms. It asserts that two states X and Y agree on
all the corresponding cells on which cell c depends.

dep_agree: function[cell, control_state, Pstate, Pstate → bool] =
(λ c, K, X, Y : (∀ d : dep(c, d, K) ∪ f_t(X, d) = f_t(Y, d)))

3.5.2 Transient Recovery Axioms

Having postulated several functions that characterize a generic fault-tolerant computing
application, it is necessary to introduce axioms that sufficiently constrain these functions.
Once concrete definitions for the functions have been chosen, these axioms must be proved
to follow as theorems for the RCP results to hold for a given application. The eight axioms
are presented below.

succ_ax: Axiom f_k(f_t(u, ps)) = succ(f_k(ps))
The first axiom states the simple condition that \( f_c \) computes the successor of its control state component.

Three axioms give properties of the function \( \text{rec} \).

- **full.recovery**: Axiom \( II \geq \text{recovery.period} \supset \text{rec}(c, K, II) \)
- **initial.recovery**: Axiom \( \text{rec}(c, K, II) \supset II > 2 \)
- **dep.recovery**: Axiom \( \text{rec}(c, \text{succ}(K), II + 1) \land \text{dep}(c, d, K) \supset \text{rec}(d, K, II) \)

First, we require that after the recovery period has transpired, all cells should be considered recovered by \( \text{rec} \). Second, it takes a minimum of two frames to recover a cell. (This is necessary because one frame is used to recover the control state. In some applications, it may be possible to recover cells in one frame, but our proof approach does not accommodate those cases and the more conservative minimum of two is used.) Third, if cell \( c \) is to be recovered in the next state, all cells it depends on must be recovered in the current state.

- **components.equal**: Axiom \( f_k(X) = f_k(Y) \land (\forall c : f_k(X, c) = f_k(Y, c)) \supset X = Y \)

This axiom, which is a type of extensionality axiom, requires that the control state and cell state values form an exhaustive partition of a \( \text{Pstate} \) value.

Two axioms capture the key conditions for recovery of individual state components.

- **control.recovered**: Axiom \( \text{maj.condition}(A) \land (\forall p : p \in A \supset w(p) = f_s(ps)) \supset f_k(f_v(Y, w)) = f_k(ps) \)
- **cell.recovered**: Axiom \( \text{maj.condition}(A) \land (\forall p : p \in A \supset w(p) = f_s(f_c(u, ps))) \land f_k(X) = K \land f_k(ps) = K \land \text{dep.agree}(c, K, X, ps) \supset f_k(f_v(f_c(u, X), w), c) = f_k(f_c(u, ps), c) \)

The first axiom requires that the control state component be recovered after every frame. Thus, \( f_c \) must vote the control state unconditionally and update the \( \text{Pstate} \) value accordingly. The conditions in the antecedent state that for a majority of processors, their mailbox items must match the value selected by the function \( f_s \). The other axiom gives the required condition for recovering an individual cell state value. All cell values that \( c \) depends on must already agree with the majority value. After voting with \( f_v \), the function \( f_t \) must extract a cell state that matches that of the consensus.

- **vote.maj**: Axiom \( \text{maj.condition}(A) \land (\forall p : p \in A \supset w(p) = f_s(ps)) \supset f_v(ps, w) = ps \)

The final axiom expresses the additional requirement on \( f_v \) that if a majority of processors agree on selected mailbox values derived from state \( ps \), then \( f_v \) applied to \( ps \) preserves the value \( ps \). In other words, once a \( \text{Pstate} \) value has been fully recovered, it will stay that way in the face of subsequent voting.
3.5.3 Sample Interpretations of Theory

The proofs of section 4 make use of the foregoing axioms to establish that the RS specification correctly implements the US specification. A valid interpretation of the model provides definitions for the uninterpreted types and functions that are ultimately used to prove the axioms as theorems of the interpreted theory. To maintain the generality of our model and its applicability to a wide range of designs, we do not provide any standard interpretations. Nevertheless, it is desirable to carry out the exercise to establish that the axioms are consistent and can be satisfied for reasonable interpretations.

Two sample interpretations were constructed based on voting schemes introduced in the Phase 1 report [1]. Definitions for the basic concepts of a static, task-based scheduling system were formalized first. Included were the notions of cells as being derived from a frame, subframe pair, and state components to record both the frame counter as well as task outputs. Task execution according to a fixed, repeating schedule was assumed. Definitions were also provided for the continuous voting and cyclic voting schemes [1]. In both cases, the transient recovery axioms were proved using Ei IDM. A preliminary form of these specifications are given in Appendix B.

Carrying out the proofs required several changes to the module structure embodied in the specifications of Appendix A. For this reason, the specifications in Appendix B have not yet been integrated with the specifications of Appendix A. Additional work is required to integrate these provisional interpretations into the existing framework. The proofs conducted thus far were performed simply to demonstrate that the axioms could be satisfied and are thus consistent.

The continuous voting scheme requires that all state components are voted during each frame. Hence transient recovery is nearly immediate. Formalizations for this case are very simple and the proofs are trivial. The cyclic voting scheme represents the typical case where state components are voted in the frame they are produced. A cell’s value is not voted during frames where it is not recomputed. Formalization in this case is somewhat more involved and the proofs require a bit more effort. The proofs and supporting lemmas comprise about two pages of Ei IDM specifications. A few selected definitions for the cyclic voting functions are shown below.

\[
\begin{align*}
\text{f}_c &: \text{function[Pstate} \rightarrow \text{MB]} = \\
& \quad (\lambda ps : \text{ps with } \{(\text{control}) := \text{ps.control}, (\text{cells}) := \\
& \quad \quad \text{cell.apply}(\lambda c : \text{ps.cells}(c)), \\
& \quad \quad \text{ps.control}, \\
& \quad \quad \text{null.cell.array}, \\
& \quad \quad \text{num.cells})])
\end{align*}
\]

\[
\begin{align*}
\text{f}_v &: \text{function[Pstate, MBvec} \rightarrow \text{Pstate]} = \\
& \quad (\lambda ps, w : \text{ps with } \{(\text{control}) := \text{k.maj}(w), (\text{cells}) := \\
& \quad \quad \text{cell.apply}(\lambda c : \text{t.maj}(w, c)), \\
& \quad \quad \text{ps.control}, \\
& \quad \quad \text{ps.cells}, \\
& \quad \quad \text{num.cells})])
\end{align*}
\]
rec: function[cell, control_state, nat \rightarrow bool] =
( \lambda c, k, H : H
  > 1 + ( if k = cell_frame(c)
      then schedule_length
      else mod_minus(k, cell_frame(c))
    end if))

dep: function[cell, cell, control_state \rightarrow bool] =
( \lambda c, d, k : cell_frame(c) \neq k \land c = d)

A few supporting definitions are omitted; these functions are presented merely to show the general order of complexity involved.

4 RS to US Proof

Proving that the RS state machine correctly implements the US state machine involves introducing a mapping between states of the two machines. The function RSmap defines the required mapping, namely the majority of Pstate values over all the processors.

RSmap: function[RSstate \rightarrow Pstate] = ( \lambda rs : maj(rs))

maj: function[RSstate \rightarrow Pstate]

maj.ax: Axiom ( \exists A : maj_condition(A) \land ( \forall p : p \in A \supset (rs(p)).proc_state = us))

maj(rs) = us

The two theorems required to establish that RS implements US are the following.

frame_commutes: Theorem reachable(s) \land N_{rs}(s, t, u) \supset N_{us}(RSmap(s), RSmap(t), u)

initial_maps: Theorem initial_rs(s) \supset initial_us(RSmap(s))

The theorem frame_commutes, depicted in figure 5, shows that a successive pair of reachable RS states can be mapped by RSmap into a successive pair of US states. The theorem initial_maps shows that an initial RS state can be mapped into an initial US state.

The notion of state reachability is used to express the theorem frame_commutes. This concept is formalized as follows:7

rs_measure: function[RSstate, nat \rightarrow nat] == ( \lambda rs, k : k)
 reachable_in_n: function[RSstate, nat \rightarrow bool] =
( \lambda t, k : if k = 0
  then initial_rs(t)
  else ( \exists s, u : reachable_in_n(s, k - 1) \land N_{rs}(s, t, u))
 end if) by rs_measure
 reachable: function[RSstate \rightarrow bool] = ( \lambda t : ( \exists k : reachable_in_n(t, k)))
Proofs for the two main theorems are supported by a handful of lemmas. The most important is a state invariant that relates values of various state components to their corresponding consensus values.

\[
\text{state.invariant: function[\text{RSstate} \rightarrow \text{bool}] =}
\]
\[
(\lambda \text{rs.prop} : (\forall t : \text{reachable}(t) \supset \text{rs.prop}(t)))
\]

\[
\text{state.rec.inv: Lemma state.invariant(state_recovery)}
\]

\[
\text{control.recovery: function[\text{RSstate} \rightarrow \text{bool}] =}
\]
\[
(\lambda s : (\forall p : (s(p)).\text{healthy} > 1 \supset f_k((s(p)).\text{proc.state}) = f_k(maj(s))))
\]

\[
\text{cell.recovery: function[\text{RSstate} \rightarrow \text{bool}] =}
\]
\[
(\lambda s : (\forall p, c : 
\quad \text{rec}(c, f_k((s(p)).\text{proc.state}), (s(p)).\text{healthy})
\quad \supset f_t((s(p)).\text{proc.state}, c) = f_t(maj(s), c)))
\]

\[
\text{state.recovery: function[\text{RSstate} \rightarrow \text{bool}] =}
\]
\[
(\lambda s : \text{maj.exists}(s) \wedge \text{control.recovery}(s) \wedge \text{cell.recovery}(s))
\]

The invariant \text{state.recovery} is shown to hold for all reachable states. The control recovery condition of this invariant asserts that if a processor \( p \) has been nonfaulty for at least one frame, then the control state, as extracted by \( f_k \), is equal to the consensus value. Similarly, the cell recovery condition asserts that if cell \( c \) is due to be recovered, as indicated by the predicate \( \text{rec} \), then cell state \( c \), as extracted by \( f_t \), is equal to the consensus value. Proving the invariant requires invoking the axioms presented in section 3.5.

Lemmas showing that a majority among \( \text{RS} \) state values continues to exist after every state transition are also proved in support of the invariant. One such lemma is also central to the proof of \text{frame.commutes}.

\footnote{Note that functions defined with “\( = \)” such as in \text{rs.measure}, are semantically equivalent to those defined with “\( == \)”\; the only difference is automatic expansion of “\( == \)” functions during theorem proving.}
rec_maj_f_c: Lemma
\[ \text{maj(working(s))} \land \text{state_recovery(s)} \land N_{st}(s, t, u) \supset \text{maj(t)} = f_c(u, \text{maj(s)}) \]

With a majority of working processors and state_recovery holding in current state s, this lemma concludes that maj applied to the next state t equals the computation step \( f_c \) applied to maj of s. From this lemma it is clear how RS states and their images under maj will correspond to the desired US states.

With the state_recovery invariant established, most of the work needed to prove the main theorem frame_commutes is in hand. One additional lemma is useful to bridge the gap between the two.

**working_majority: function [RSstate \( \rightarrow \) bool] =
\[
(\lambda s : (\forall p : p \in \text{working_set}(s) \supset (s(p)).\text{proc_state} = \text{maj}(s)))
\]

**consensus_prop: Lemma state_recovery(s) \( \supset \) working_majority(s)**

The lemma consensus_prop allows us to draw a key inference from the state_recovery invariant, which is expressed by the predicate working_majority. This predicate asserts that for all processors \( p \) that belong to the working set, i.e., for all working processors, \( p \)'s value of Pstate is equal to the majority value.

The proof of frame_commutes now follows from rec_maj_f_c and consensus_prop and assorted definitions. The proof of initial_maps follows from definitions and the lemma initial_maj_cond, which states that an initial state satisfies the majority condition.

**initial_maj_cond: Lemma initial_rs(s) \( \supset \) maj_condition(working_set(s))**

This completes the proof that the RS machine implements the US machine.

Note that our proof is in terms of a generic model of fault-tolerant computation and depends on the validity of the axioms of section 3.5. For some choices of definitions for the uninterpreted functions, there will be substantial work required to establish those axioms as theorems. For example, the Minimal Voting scheme presented in our Phase I report [1] requires a nontrivial proof to establish that full recovery is achieved. Such details have been omitted here. Nevertheless, the value of our revised approach is in its generality. The results can now be made to apply to a wide variety of frame-based, fault-tolerant architectures.

## 5 DS Specification

In the Distributed Synchronous layer we focus on two things: expanding the state to include "mailboxes" for interprocessor communication and dividing a frame transition into four sequential subtransitions. The state must also be expanded to include an indicator of which phase of a frame is currently being processed. This is done as follows.

The structure of the mailbox for a four-processor system is shown in figure 6. Each processor contains a mailbox with one slot dedicated to each other processor in the system. Each slot is large enough to contain the largest amount of data to be broadcast during one frame. The \( n \)th slot of processor \( n \) serves as the outgoing mailbox.

The local state for each processor can now be defined:
\textbf{ds\_proc\_state}: Type = Record healthy : nat, 
\hspace{1cm} proc\_state : Pstate, 
\hspace{1cm} mailbox : MBvec 
\hspace{1cm} end record 

The vector of all processors \texttt{ds\_proc\_state} is named \texttt{ds\_proc\_array}:

\texttt{ds\_proc\_array}: Type = array [processors] of ds\_proc\_state

The complete \texttt{DSstate} is:

\texttt{DSstate}: Type = Record phase : phases, 
\hspace{1cm} proc : ds\_proc\_array 
\hspace{1cm} end record

In the DS specification, a frame is decomposed into four phases:

\texttt{phases}: Type = (compute, broadcast, vote, sync)

The first field of \texttt{DSstate} holds the current phase. During each phase a distinct function is performed.

1. \textbf{Computation}. The proc\_state component of the state is updated with the results of computation using the function $f_r$.

2. \textbf{Broadcast}. Interprocessor communication is effected by broadcasting the MB values to all other processors, which are deposited in their respective mailboxes.
3. **Voting.** The received mailbox values are voted and merged with the current \texttt{Pstate} values to arrive at the end-of-frame state.

4. **Synchronization.** The clock synchronization function is performed. (No details of the clocks are introduced until the DA specification layer.)

The transition relation for the frame is defined in terms of a phase-transition relation \( \mathcal{N}_{ds} \).

\[
\text{frame}_N_{ds} \colon \text{function}[\text{DSstate}, \text{DSstate}, \text{inputs} \rightarrow \text{bool}] =
\begin{align*}
( & \lambda s, t, u : ( \exists x, y, z : \\
& \mathcal{N}_{ds}(s, x, u) \land \mathcal{N}_{ds}(x, y, u) \land \mathcal{N}_{ds}(y, z, u) \land \mathcal{N}_{ds}(z, t, u)))
\end{align*}
\]

Note how the intermediate states are defined using existential quantifiers and that the output state of a phase transition becomes the input of the next phase transition. The net result of performing these four phase transitions will be shown to accomplish the same thing as the single transition of the RS specification.

The phase-transition relation is defined as follows:

\[
\mathcal{N}_{ds} \colon \text{function}[\text{DSstate}, \text{DSstate}, \text{inputs} \rightarrow \text{bool}] =
\begin{align*}
( & \lambda s, t, u : \text{maj}_\text{working}(t) \\
& \land t\text{.phase} = \text{next}\_\text{phase}(s\text{.phase}) \\
& \land (\forall i : \\
& \quad \text{if } s\text{.phase} = \text{sync} \\
& \quad \text{then } \mathcal{N}_{ds}^s(s, t, i) \\
& \quad \text{else } t\text{.proc}(i)\text{.healthy} = s\text{.proc}(i)\text{.healthy} \\
& \quad \land (s\text{.phase} = \text{compute} \supset \mathcal{N}_{ds}^c(s, t, i)) \\
& \quad \land (s\text{.phase} = \text{broadcast} \supset \mathcal{N}_{ds}^b(s, t, i)) \\
& \quad \land (s\text{.phase} = \text{vote} \supset \mathcal{N}_{ds}^v(s, t, i)) \\
& \quad \text{end if}))
\end{align*}
\]

Notice that the phase-transition relation only holds when the next state \( t \) has a majority of working processors. This corresponds to the analogous condition in \( \mathcal{N}_{rs} \), presented in section 3.3, where it appears as one conjunct of the allowable_faults relation. Hence, all reachable states in the DS specification must have a majority of working processors.

The phase field of the state is advanced by the function \text{next}\_\text{phase}. The phase-transition relation is defined in terms of four sub-relations: \( \mathcal{N}_{ds}^s, \mathcal{N}_{ds}^c, \mathcal{N}_{ds}^b, \) and \( \mathcal{N}_{ds}^v \), which correspond to the compute, broadcast, vote and sync phases, respectively. The quantifier \( \forall i \) invokes the sub-relations for all of the processors of the system. Note that the statement \( t\text{.proc}(i)\text{.healthy} = s\text{.proc}(i)\text{.healthy} \) after the else requires that the value of healthy remain constant throughout a frame. Thus, if a processor is faulty anywhere in a frame it is considered to be faulty throughout; the value of healthy may only change at the frame boundaries, i.e., at the sync to compute transitions. Similarly, full recovery of state information does not occur until the end of a frame. This is consistent with the previous work [1].

Table 1 provides a summary of the functions that are performed during each phase on nonfaulty processors. In the table \( s_i \) is an abbreviation for \( s\text{.proc}(i) \).

The \( \mathcal{N}_{ds}^c \) sub-relation defines the behavior of a single processor during the compute phase:
<table>
<thead>
<tr>
<th>Phase</th>
<th>Held constant</th>
<th>Modified</th>
</tr>
</thead>
<tbody>
<tr>
<td>compute</td>
<td>healthy</td>
<td>$t_i$.proc_state = $f_s(u, s_i$.proc_state)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$t_i$.mailbox(i) = $f_s(f_s(u, s_i$.proc_state))</td>
</tr>
<tr>
<td>broadcast</td>
<td></td>
<td>$(\forall p : t_i$.mailbox(p) = $s_p$.mailbox(p))</td>
</tr>
<tr>
<td>vote</td>
<td>mailbox</td>
<td>$t_i$.proc_state = $f_v(s_i$.proc_state, $s_i$.mailbox)</td>
</tr>
<tr>
<td>sync</td>
<td>proc_state</td>
<td>$t_i$.healthy = 1 + $s_i$.healthy</td>
</tr>
</tbody>
</table>

Table 1: Summary of activities during various phases

\[ N_{ds}^b : \text{function}[DSstate, DSstate, inputs, processors \rightarrow bool] = \]
\[
(\lambda s, t, i : s_i$.healthy > 0

\[ \lor t_i$.proc(i).proc_state = f_s(u, s_i$.proc(i).proc_state) \]
\[ \land t_i$.proc(i).mailbox(i) = f_s(f_s(u, s_i$.proc(i).proc_state)) \]

During this phase, the proc_state field is updated with the results of the computation:

$f_s(u, s_i$.proc(i).proc_state)

Also, the mailbox is loaded with the subset of the results to be broadcast as defined by the function $f_s$. Recall that a processor's own mailbox slot acts as the place to post outgoing data for broadcast to other processors.

The $N_{ds}^b$ sub-relation defines the behavior of a single processor during the broadcast phase:

\[ N_{ds}^v : \text{function}[DSstate, DSstate, processors \rightarrow bool] = \]
\[
(\lambda s, t, i : s_i$.healthy > 0

\[ \lor t_i$.proc(i).proc_state = s_i$.proc(i).proc_state \]
\[ \land \text{broadcast_received}(s, t, i) \]

During this phase the proc_state field remains unchanged and the broadcast_received relation holds:

broadcast_received: function[DSstate, DSstate, processors \rightarrow bool] =

\[
(\lambda s, t, q : (\forall p : s_i$.proc(p).healthy > 0

\[ \lor t_i$.proc(q).mailbox(p) = s_i$.proc(p).mailbox(p))\]

This states that each nonfaulty processor $q$ receives the values sent by other nonfaulty processors. If the sending processor $p$ is faulty, then the consequent of the relation need not hold and the value found in $p$'s slot of $q$'s mailbox is indeterminate. If the receiving processor $q$ is faulty, the broadcast_received relation is not required to hold in $N_{ds}^v$. In this situation, all of $q$'s mailbox values are unspecified.

The $N_{ds}^v$ sub-relation defines the behavior of a single processor during the vote phase:
During this phase the mailbox field remains unchanged and the local processor state is updated with the result of voting the values broadcast by the other processors. The vote function is named $f_v$.

The $\mathcal{N}_s^t$ sub-relation defines the behavior of a single processor during the sync phase:

$$\mathcal{N}_s^t: \text{function}[\text{DSstate}, \text{DSstate}, \text{processors} \to \text{bool}] =$$

$$( \lambda s, t, i : s.\text{proc}(i).\text{healthy} > 0$$

$$( \Rightarrow t.\text{proc}(i).\text{mailbox} = s.\text{proc}(i).\text{mailbox}$$

$$\land t.\text{proc}(i).\text{proc\_state}$$

$$= f_v(s.\text{proc}(i).\text{proc\_state}, s.\text{proc}(i).\text{mailbox}))$$

During the sync phase, the computation state of a nonfaulty processor remains unchanged. At the end of the sync phase, the current frame ends, so the value of healthy is incremented by one if the processor is to be nonfaulty in the next frame. This is the same condition appearing in the relation allowable faults of section 3.3. Any processor assumed to be faulty in the next frame will have its healthy field set to zero. A limit on how many processors can be faulty simultaneously is imposed by the predicate maj\_working. Therefore, not every possible assignment of values to the healthy fields is admissible; each assignment must satisfy the Maximum Fault Assumption.

The predicate initial\_ds puts forth the conditions for a valid initial state. The initial phase is set to compute and each element of the DS state array has its healthy field equal to recovery\_period and its proc\_state field equal to initial\_proc\_state.

$$\text{initial\_ds: function}[\text{DSstate} \to \text{bool}] =$$

$$( \lambda s : s.\text{phase} = \text{compute}$$

$$\land ( \forall i : s.\text{proc}(i).\text{healthy} = \text{recovery\_period}$$

$$\land s.\text{proc}(i).\text{proc\_state} = \text{initial\_proc\_state}))$$

As before, the constant recovery\_period is the number of frames required to fully recover a processor's state after experiencing a transient fault. By initializing the healthy fields to this value, we are starting the system with all processors working. Note that the mailbox fields are not initialized; any mailbox values can appear in a valid initial DSstate.

## 6 DS to RS Proof

The DS specification performs the functionality of the RS specification in four sequential steps. Thus, we must show that the "frame" transition function, frame\_N\_ds,

$$\text{frame\_N\_ds}(s, t, u) = ( \exists x, y, z : N_{ds}(s, x, u) \land N_{ds}(x, y, u) \land N_{ds}(y, z, u) \land N_{ds}(z, t, u))$$

accomplishes the same function as a single transition of the RS level transition function $N_{rs}(s, t, u)$ under an appropriate mapping function.
6.1 DS to RS Mapping

The DS to RS mapping function, DSmap, is defined as:

\[ DSmap: \text{function}[\text{DSstate} \rightarrow \text{RSstate}] = (\lambda \, ds : \text{ss_update}(ds, nrep)) \]

where ss_update is given by:

\[ \text{ss_update: Recursive function}[\text{DSstate}, \text{nat} \rightarrow \text{RSstate}] = \]
\[ (\lambda \, ds, p : \text{if} (p = 0) \lor (p > nrep) \]
\[ \quad \text{then } rs0 \]
\[ \quad \text{else } \text{ss_update}(ds, p - 1) \]
\[ \quad \text{with } [(p) := \text{rsproc0} \]
\[ \quad \quad \text{with } [(\text{healthy}) := ds.\text{proc}(p).\text{healthy}, \]
\[ \quad \quad \quad (\text{proc.state}) := ds.\text{proc}(p).\text{proc.state}] \]
\[ \quad \text{end if}) \text{ by ssu_measure} \]

This mapping copies the healthy and proc.state fields for each processor as illustrated in figure 7. To establish that DS implements RS, the commutativity diagram of figure 8 must be shown to commute. To establish that the diagram commutes, the following formula must be proved.

\[ \text{frame_commutes: Theorem} \]
\[ s.\text{phase} = \text{compute} \land \text{frame.N.d}(s, t, u) \supset N_{rs}(\text{DSmap}(s), \text{DSmap}(t), u) \]

Note that to make the correct correspondence, we must consider only DS states found at the beginning of each frame, namely those whose phase is compute. Refer to figure 4 on page 12 for a visual interpretation of this theorem.

It is also necessary to show that the initial states are mapped properly:
initial_maps: Theorem initial_ds(s) \supset initial_rs(DSmap(s))

Several basic lemmas follow from the definition of the mapping function:

map.1: Lemma DSmap(s)(i).healthy = s.proc(i).healthy

map.2: Lemma DSmap(s)(i).proc_state = s.proc(i).proc_state

map.3: Lemma allowable_faults(s, t) \supset RS.allowable_faults(DSmap(s), DSmap(t))

map.4: Lemma RS.good_values_sent(DSmap(s), u, w) = good_values_sent(s, u, w)

map.5: Lemma RS.voted_final_state(DSmap(s), DSmap(t), u, h, i) = voted_final_state(s, t, u, h, i)

map.7: Lemma RS.maj_working(DSmap(s)) = DS.maj_working(s)

6.2 The Proof

The proof of the frame_commutes theorem involves the expansion of the frame_N_ds relation and showing that the resulting formula logically implies \( N_{rs}(DSmap(s), DSmap(t), u) \). We begin with the definition of frame_N_ds:

\[
\text{frame_N_ds}(s, t, u) = (\exists x, y, z : N_{ds}(s, x, u) \wedge N_{ds}(x, y, u) \wedge N_{ds}(y, z, u) \wedge N_{ds}(z, t, u))
\]

Since \( s\cdot\text{phase} = \text{compute} \), \( N_{ds}(s, x, u) \) can be rewritten as:

\[
N_{ds}(s, x, u) = \text{maj_working}(x) \wedge x\cdot\text{phase} = \text{broadcast} \\
\wedge (\forall i : x\cdot\text{proc}(i).\text{healthy} = s\cdot\text{proc}(i).\text{healthy} \wedge N_{ds}^c(s, x, u, i))
\]

Substituting for \( N_{ds}(s, x, u) \) we obtain
\[ s\. \text{phase} = \text{compute} \land \text{frame}_N\text{.ds}(s, t, u) \land (\exists x, y, z : \text{maj}\text{.working}(x) \land \text{maj}\text{.working}(y) \land (\forall i : x\. \text{phase} = \text{broadcast} \land x\. \text{proc}(i)\. \text{healthy} = s\. \text{proc}(i)\. \text{healthy} \land s\. \text{proc}(i)\. \text{healthy} > 0) \land (\forall j : x\. \text{proc}(j)\. \text{healthy} > 0) \land (y\. \text{phase} = \text{broadcast} \land y\. \text{proc}(i)\. \text{healthy} = x\. \text{proc}(i)\. \text{healthy} \land (y\. \text{proc}(i)\. \text{healthy} > 0) \land (\forall j : y\. \text{proc}(j)\. \text{healthy} > 0) \land y\. \text{proc}(i)\. \text{mailbox}(j) = f_c((y\. \text{proc}(j)\. \text{proc\_state}))))) \land \text{N}_{ds}(y, z, u) \land \text{N}_{ds}(z, t, u)) \]

Next, expand \( \text{N}_{ds} \), the \( \text{N}_{ds} \) term for the broadcast phase, and combine universal quantifiers:

\[ s\. \text{phase} = \text{compute} \land \text{frame}_N\text{.ds}(s, t, u) \land (\exists x, y, z : \text{maj}\text{.working}(x) \land \text{maj}\text{.working}(y) \land (\forall i : x\. \text{phase} = \text{broadcast} \land x\. \text{proc}(i)\. \text{healthy} = s\. \text{proc}(i)\. \text{healthy} \land (s\. \text{proc}(i)\. \text{healthy} > 0) \land (\forall j : x\. \text{proc}(j)\. \text{healthy} > 0) \land y\. \text{phase} = \text{vote} \land y\. \text{proc}(i)\. \text{healthy} = x\. \text{proc}(i)\. \text{healthy} \land (y\. \text{proc}(i)\. \text{healthy} > 0) \land (\forall j : y\. \text{proc}(j)\. \text{healthy} > 0) \land y\. \text{proc}(i)\. \text{mailbox}(j) = f_s((x\. \text{proc}(j)\. \text{proc\_state))))))) \land \text{N}_{ds}(y, z, u) \land \text{N}_{ds}(z, t, u)) \]

Simplifying to eliminate \( x \) yields:

\[ s\. \text{phase} = \text{compute} \land \text{frame}_N\text{.ds}(s, t, u) \land (\exists y, z : \text{maj}\text{.working}(y) \land (\forall i : y\. \text{phase} = \text{vote} \land y\. \text{proc}(i)\. \text{healthy} = s\. \text{proc}(i)\. \text{healthy} \land (s\. \text{proc}(i)\. \text{healthy} > 0) \land (\forall j : s\. \text{proc}(j)\. \text{healthy} > 0) \land (\forall j : s\. \text{proc}(j)\. \text{healthy} > 0) \land y\. \text{proc}(i)\. \text{mailbox}(j) = f_s((x\. \text{proc}(j)\. \text{proc\_state))))))) \land \text{N}_{ds}(y, z, u) \land \text{N}_{ds}(z, t, u)) \]

Expanding the \( \text{N}_{ds} \) term for the third phase and simplifying produces:

\[ s\. \text{phase} = \text{compute} \land \text{frame}_N\text{.ds}(s, t, u) \land (\exists z : \text{maj}\text{.working}(z) \land (\forall i : z\. \text{phase} = \text{sync} \land z\. \text{proc}(i)\. \text{healthy} = s\. \text{proc}(i)\. \text{healthy} \land (s\. \text{proc}(i)\. \text{healthy} > 0) \land (\forall j : s\. \text{proc}(j)\. \text{healthy} > 0) \land z\. \text{proc}(i)\. \text{mailbox}(j) = f_s((x\. \text{proc}(j)\. \text{proc\_state))))))) \land \text{N}_{ds}(z, t, u)) \]

Expanding the fourth phase \( \text{N}_{ds} \) term and simplifying gives:
\[ s_.phase = \text{compute} \land \text{frame\_N\_ds}(s, t, u) \]
\[ \supset (\exists z \cdot \text{maj\_working}(t)) \land (\forall i : t_.phase = \text{compute} \land (s_.proc(i).healthy > 0 \supset t_.proc(i).proc\_state = f_u(f_c(u, s_.proc(i).proc\_state), z_.proc(i).mailbox)) \land (\forall j : s_.proc(j).healthy > 0 \supset z_.proc(i).mailbox(j) = f_s(f_c(u, (s_.proc(j).proc\_state)))) \land (t_.proc(i).healthy > 0 \supset t_.proc(i).healthy = 1 + s_.proc(i).healthy)) \]

Letting \( h(i) = z_.proc(i).mailbox \),

\[ s_.phase = \text{compute} \land \text{frame\_N\_ds}(s, t, u) \]
\[ \supset \text{maj\_working}(t) \land (\exists h : (\forall i : t_.phase = \text{compute} \land (t_.proc(i).healthy > 0 \supset t_.proc(i).healthy = 1 + s_.proc(i).healthy)) \land (s_.proc(i).healthy > 0 \supset t_.proc(i).proc\_state = f_u(f_c(u, s_.proc(i).proc\_state), h(i))) \land (\forall j : s_.proc(j).healthy > 0 \supset h(i)(j) = f_s(f_c(u, (s_.proc(j).proc\_state)))) \land \text{allowable\_faults}(s, t)) \]

This must be shown to logically imply \( N_s(\text{DSmap}(s), \text{DSmap}(t), u) \), which can be rewritten as:

\[ (\exists h : (\forall i : s_.proc(i).healthy > 0 \supset (\forall j : s_.proc(j).healthy > 0 \supset h(i)(j) = f_s(f_c(u, s_.proc(j).proc\_state)))) \land t_.proc(i).proc\_state = f_u(f_c(u, s_.proc(i).proc\_state), h(i))) \land \text{allowable\_faults}(s, t)) \]

The first conjunct can be seen to follow by inspection. By expanding allowable\_faults,

\[ \text{allowable\_faults}: \text{function}[\text{RSstate}, \text{RSstate} \rightarrow \text{bool}] = \]
\[ (\lambda s, t : \text{maj\_working}(t) \land (\forall i : t(i).healthy > 0 \supset (t(i)).healthy = 1 + s(i).healthy)) \]

the second conjunct can be seen to follow as well. Q.E.D.

7 DA Specification

The DA specification performs the same functions as the DS specification; however, explicit consideration is given to the timing of the system. Every processor of the system has its own clock and consequently task executions on one processor take place at different times than on other processors. Nevertheless, the model at this level explicitly takes advantage of the fact that the clocks of the system are synchronized to within a bounded skew \( \delta \). Therefore, it is necessary to give an overview of clock synchronization theory before elaborating the DA specification.
7.1 Clock Synchronization Theory

In this section we will discuss the synchronization theory upon which the DA specification depends. Although the RCP architecture does not depend upon any particular clock synchronization algorithm, we have used the specification for the interactive consistency algorithm (ICA) [9, 8] since EHDM specifications for ICA already exist.

In this section we show the essential aspects of this theory. The formal definition of a clock is fundamental. A clock can be modeled as a function from real time $t$ to clock time $T$: $C(t) = T$ or as a function from clock time to real time: $c(T) = t$. Since the ICA theory was expressed in terms of the latter, we will also be modeling clocks as functions from clock time to real time. We must be careful to distinguish between an uncorrected clock and a clock which is being resynchronized periodically. We will use the notation $c(T)$ for an uncorrected clock and $r_t(i)(T)$ to represent a synchronized clock during its $i$th frame.\footnote{This differs from the notation, $e^{(i)}(T)$, used in [8].}

Good clocks have different drift rates with respect to perfect time. Nevertheless, this drift rate is bounded. Thus, we can define a good clock as one whose drift rate is strictly bounded by $\frac{p}{2}$. A clock is “good”, (i.e. a predicate $\text{good\_clock}(T_0, T_1)$ is true), between clock times $T_0$ and $T_1$ iff:

$$\forall T_1, T_2: T_0 \leq T_1 \leq T_2 \leq T_1, \quad |c_p(T_1) - c_p(T_2) - (T_1 - T_2)| \leq \frac{p}{2} \times |T_1 - T_2|$$

The synchronization algorithm is executed once every frame of duration $\text{frame\_time}$. The notation $T^{(i)}$ is used to represent the start of the $i$th frame, i.e., $(T^0 + i \times \text{frame\_time})$. The notation $T \in R^{(i)}$ means that $T$ falls in the $i$th frame, i.e.,

$$\exists \Pi: 0 \leq \Pi \leq \text{frame\_time} \land T = T^{(i)}(\Pi)$$

During the $i$th frame the synchronized clock on processor $p$, $r_t_p$, is defined by:

$$r_t_p(i, T) = c_p(T + \text{Corr}_p^{(i)})$$

where $\text{Corr}$ is the cumulative sum of the corrections that have been made to the (logical) clock. It is defined by:

$$\text{Corr}_p^{(i)} = \begin{cases} \text{Corr}_p^{(i-1)} + \Delta_p^{(i-1)} & \text{if } i > 0 \\ \text{initial\_Corr}(p) & \text{else} \end{cases}$$

where initial\_Corr$(p)$ is conveniently equated to zero (i.e. $\text{Corr}_p^{(0)} = 0$). The function $\Delta_p^{(i-1)}$ is the correction factor for the current frame as computed by the clock synchronization algorithm.

We now define what is meant by a clock being nonfaulty in the current frame. The predicate $\text{nonfaulty\_clock}$ is defined as follows:

A1: Lemma $\text{nonfaulty\_clock}(p, i) = \text{good\_clock}(p, T^{(0)} + \text{Corr}_p^{(0)}, T^{(i+1)} + \text{Corr}_p^{(i)})$
Note that in order for a clock to be non-faulty in the current frame it is necessary that it has been working continuously from time 0.\(^9\)

The clock synchronization theory provides two important properties about the clock synchronization algorithm, namely that the skew between good clocks is bounded and that the correction to a good clock is always bounded. The maximum skew is denoted by \(\delta\) and the maximum correction is denoted by \(\Sigma\). More formally,

**Clock Synchronization Conditions:** For all nonfaulty clocks \(p\) and \(q\):

\[
\begin{align*}
S1: & \quad \forall T \in R^{(i)} : |r_{p}^{(i)}(T) - r_{q}^{(i)}(T)| < \delta \\
S2: & \quad |\text{Corr}_{p}^{(i+1)} - \text{Corr}_{p}^{(i)}| < \Sigma
\end{align*}
\]

The value of \(\delta\) is determined by several key parameters of the synchronization system: \(\rho, \epsilon, \delta_0, m, n_{\text{rep}}\) listed in table 2. The formal definition of \(\rho\) has already been given. The parameter \(\epsilon\) is a bound on the error in reading another processor's clock. The synchronization algorithm requires that every processor in the system obtain an estimate of its skew relative to every other clock in the system. The notation \(\Delta_{qp}^{(i)}\) is used to represent the skew between clocks \(q\) and \(p\) during the \(i\)th frame as perceived by \(p\). Thus, the real time at which \(p\)'s clock reads \(T_0 + \Delta_{qp}^{(i)}\) should be very close to the real time that \(q\)'s clock reads \(T_0\). This is constrained by an axiom to be less than \(\epsilon\):

**Axiom** If conditions S1 and S2 hold throughout the \(i\)th frame, then

\[
\begin{align*}
\text{nonfaulty}_\text{clock}(p, i) \land \text{nonfaulty}_\text{clock}(q, i) \\
\supset |\Delta_{qp}^{(i)}| \leq \text{sync}\_\text{time} \\
\land (\exists T_0 : T_0 \in S^{(i)} \land |r_{p}^{(i)}(T_0 + \Delta_{qp}^{(i)}(T_0)) - r_{q}^{(i)}(T_0)| < \epsilon)
\end{align*}
\]

The amount of time reserved for executing the clock synchronization algorithm is denoted by the constant \(\text{sync}\_\text{time}\).

The third parameter, \(\delta_0\), is constrained as follows:

**A0: Axiom** \(|r_{p}^{(0)}(0) - r_{q}^{(0)}(0)| < \delta_0\)

\(^9\)This is a limitation not of the operating system, but of existing, mechanically verified fault-tolerant clock synchronization theory. Future work will concentrate on how to make clock synchronization robust in the presence of transient faults.
Thus, $\delta_0$ bounds the initial clock skew.

The property that the ICA clock synchronization algorithm meets the two synchronization conditions S1 and S2 was proved in [8]. These were named Theorem.1 and Theorem.2: formally as:

**Theorem.1:** Theorem

$S1A(i) \supset (\forall p, q : (\forall T :$

nonfaulty_clock($p, i$) $\land$ nonfaulty_clock($q, i$) $\land$ $T \in R^{(i)}$

$\supset |rt_p^{(i)}(T) - rt_q^{(i)}(T)| \leq \delta$)

**Theorem.2:** Theorem $|Corr_p^{(i+1)} - Corr_p^{(i)}| < \Sigma$

where the premise for Theorem.1, $S1A$, is defined by:

$$(\lambda i : (\forall r : (m + 1 \leq r \text{ and } r \leq n)) \supset \text{nonfaulty_clock}(r, i))$$

and where $m$ is equal to the maximum number of faulty processors.

We have used the following equivalent but more convenient premise: $S1A : \text{function}[\text{period} \rightarrow \text{bool}] = (\lambda i : \text{enough_clocks}(i))$.

where

$\text{enough_clocks : function}[\text{period} \rightarrow \text{bool}] =$

$$(\lambda i : 3 \times \text{num_good_clocks}(i, \text{nrep}) > 2 \times \text{nrep})$$

and

$\text{num_good_clocks : Recursive function}[\text{period}, \text{nat} \rightarrow \text{nat}] =$

$$(\lambda i, k : \text{if } k = 0 \text{ or } k > \text{nrep}$$

$\text{then } 0$

$\text{elsif nonfaulty_clock}(k, i)$

$\text{then } 1 + \text{num_good_clocks}(i, k - 1)$

$\text{else num_good_clocks}(i, k - 1)$

$\text{end if}) \text{ by num_measure}$$

The theorems proved in [8] also depend upon the following axioms not mentioned above.

**A2.aux:** Axiom $\Delta_{pp}^{(i)} = 0$

**C0:** Axiom $m < \text{nrep} \land m \leq \text{nrep} - \text{num_good_clocks}(i, \text{nrep})$

**C1:** Axiom $\text{frame_time} \geq 3 \times \text{sync_time}$

**C2:** Axiom $\text{sync_time} \geq \Sigma$

**C3:** Axiom $\Sigma \geq \Delta$

**C4:** Axiom $\Delta \geq \delta + \epsilon + \frac{\epsilon}{2} \times \text{sync_time}$

**C5:** Axiom $\delta \geq \delta_0 + \rho \times \text{frame_time}$

**C6:** Axiom $\delta \geq 2 \times (\epsilon + \rho \times \text{sync_time}) + 2 \times m \times \Delta / (\text{nrep} - m)$

$+ \text{nrep} \times \rho \times \text{frame_time} / (\text{nrep} - m) + \rho \times \Delta$

$+ \text{nrep} \times \rho \times \Sigma / (\text{nrep} - m)$

Note that this form also subsumes axiom C0 below.
With the S1A premise expanded, the main synchronization theorem becomes:

\[
\text{sync_thm: Theorem} \quad \text{enough_clocks}(i)
\quad \supset (\forall p, q : (\forall T : T \in R(i) \land \text{nonfaulty_clock}(p, i) \land \text{nonfaulty_clock}(q, i))
\quad \supset |r(t_p^i)(T) - r(t_q^i)(T)| \leq \delta)
\]

The proof that DA implements DS depends crucially upon this theorem.

7.2 The DA Formalization

Now that a clock synchronization theory is at our disposal, the DA model can be specified. Two new fields are added to the state vector associated with each processor: lclock and cum_delta:

\[
da\_proc\_state: \text{Type} = \text{Record} \quad \text{healthy : nat,}
\quad \text{proc\_state : Pstate,}
\quad \text{mailbox : MBvec,}
\quad \text{lclock : logical\_clocktime,}
\quad \text{cum\_delta : number}
\quad \text{end record}
\]

The complete DAstate is:

\[
\text{DAstate: Type} = \text{Record} \quad \text{phase : phases,}
\quad \text{sync\_period : nat,}
\quad \text{proc : da\_proc\_array}
\quad \text{end record}
\]

where da\_proc\_state is defined by:

\[
da\_proc\_array: \text{Type} = \text{array [processors] of da\_proc\_state}
\]

The sync\_period field holds the current frame of the system. Note this does not represent the frame counter on any particular processor, but rather the ideal, unbounded frame counter.

The lclock field of a DAstate stores the current value of the processor’s local clock. The real-time corresponding to this clock time can be found through use of the auxiliary function da\_rt.

\[
da\_rt: \text{function[DAstate, processors, logical\_clocktime} \rightarrow \text{realtime}])) =
\quad (\lambda da, p, T : c_p(T + da.p.proc(p).cum\_delta))
\]

This function corresponds to the rt function of the clock synchronization theory. Thus, da\_rt(s,p,T) represents processor p’s synchronized clock. Given a clock time T in the current frame (s.sync\_period), da\_rt returns the real-time that processor p’s clock reads T. The current value of the cumulative correction is stored in the field cum\_delta.

Every frame the clock synchronization algorithm is executed, and \( \Delta_p^i \) is added to cum\_delta. Note that this corresponds to the Corr function of the clock synchronization theory. The relationship between \( c_p \), da\_rt, and cum\_delta is illustrated in figure 9.
Figure 9: Relationship between $c_p$ and $da_{rt}$

Since the original ICA clock theory was not cast into the state-machine framework used in this work, it is necessary to show that the $da_{rt}$ function is equivalent to the $rt$ function of the clock synchronization theory. The first step is to equate the period of the clock synchronization with the length of a frame in the operating system. Since the length of the period in the clock theory is a parameter of the theory, this is accomplished by setting it equal to $frame:length$. Similarly, the execution time of the synchronization algorithm is a parameter of the clock theory which is set equal to $sync:period$. The clock synchronization theory also requires that a constraint be placed on the duration of the sync phase:

**AXIOM:** $duration(sync) >= sync:period$

The next step is to equate the clocks of the state-machine with the clocks in the sync theory. This is done by proving the following lemma:

**da_{rt}: Lemma reachable(da) \land nonfaulty:clock(p, da.sync:period)
\implies da_{rt}(da, p, T) = rt_p^{(da.sync:period)}(T)**

This lemma follows from the fact that in every period (during the sync phase) the $cum:delta$ field is incremented by $\Delta_i$:

$t.proc(i).cum:delta = s.proc(i).cum:delta + \Delta_i^{sync:period}$

The algorithm that is specified in the clock theory uses $\Delta_i$ as its correction factor each frame. The exact same correction factor is used in the DA model. Thus, the RCP system executes

\footnote{These are named $R$ and $S$ in \cite{9, 8}. However, these names conflicted with their use in \cite{1}.}
the same algorithm as specified in the clock theory, and cum.delta will always be equal to Corr. Thus, \( rt_p = da_{rt} \).

The specification of time-critical behavior in the DA model is accomplished using the \( da_{rt} \) function. For example, the broadcast.received function is expressed in terms of \( da_{rt} \):

\[
\text{broadcast.received: function}[DAstate, DAstate, processors \to bool] = \\
(\lambda s, t, q : (\forall p : \\
(s.proc(p).healthy > 0 \\
\land da_{rt}(s, p, s.proc(p).lclock) + max_comm_delay \\
\leq da_{rt}(t, q, t.proc(q).lclock) \\
\lor t.proc(q).mailbox(p) = s.proc(p).mailbox(p)) \\
\text{thus, the data in the incoming bin } p \text{ on processor } q \text{ is only defined to be equal to the value broadcast by } p \text{ (i.e. } s.proc(p).mailbox(p)\text{) when the real time on the receiving end (i.e. } da_{rt}(t, q, t.proc(q).lclock)\text{) is greater than } da_{rt}(s, p, s.proc(p).lclock) \text{ plus max_comm_delay. This specification anticipates the design of a communications system that can deliver a message in a bounded amount of time, in particular within max_comm_delay units of time.}
\]

In the DA level there is no single transition that covers the entire frame. There is only a transition relation for a phase. The \( N_{da} \) relation is:

\[
N_{da}: \text{function}[DAstate, DAstate, inputs \to bool] = \\
(\lambda s, t, u : \text{enough_hardware}(t) \land t.phase = \text{next_phase}(s.phase) \\
\land (\forall i : \text{if } s.phase = \text{sync} \\
\text{then } N_{da}^e(s, t, i) \\
\text{else } t.proc(i).healthy = s.proc(i).healthy \\
\land t.proc(i).cum.delta = s.proc(i).cum.delta \\
\land t.sync.period = s.sync.period \\
\land (\text{nonfaulty_clock}(i, s.sync.period) \\
\lor \text{clock.advanced}(s.proc(i).lclock, t.proc(i).lclock, duration(s.phase))) \\
\land (s.phase = \text{compute } \lor N_{da}^c(s, t, u, i)) \\
\land (s.phase = \text{broadcast } \lor N_{da}^b(s, t, i)) \\
\land (s.phase = \text{vote } \lor N_{da}^v(s, t, i)) \\
\text{end if}))
\]

Note that the transition to a new state is only valid when the enough_hardware function holds in the next state. This function is defined as follows:

\[
\text{enough_hardware: function}[DAstate \to bool] = \\
(\lambda t : \text{maj.working}(t) \land \text{enough_clocks}(t.sync.period))
\]

\text{maj.working} is defined identically in RS, DS, and DA. Its definition is presented in section 3.3. The definition of \text{enough_clocks} appears in section 7.1.

As in the DS level, the state transition relation \( N_{da} \) is defined in terms of four sub-relations, each of which applies to a particular phase type. These are called \( N_{da}^e, N_{da}^b, N_{da}^v \) and \( N_{da}^c \).

The \( N_{da}^c \) sub-relation is:
\( N_{da} : \text{function}[\text{DAstate}, \text{DAstate}, \text{inputs}, \text{processors} \rightarrow \text{bool}] = \)

\[
( \lambda s, t, u, i:\]

\[
\begin{array}{l}
\quad s.\text{proc}(i).\text{healthy} > 0 \\
\quad \exists t.\text{proc}(i).\text{proc}\_\text{state} = f_s(u, s.\text{proc}(i).\text{proc}\_\text{state}) \\
\quad \land t.\text{proc}(i).\text{mailbox}(i) = f_s(f_u(s.\text{proc}(i).\text{proc}\_\text{state}))
\end{array}
\]

Just as in the corresponding DS relation, the \text{proc\_state} field is updated with the results of the computation, \( f_s(u, s.\text{proc}(i).\text{proc}\_\text{state}) \). Also, the mailbox is loaded with the subset of the results to be broadcast as defined by the function \( f_s \). Unlike the DS model, the local clock time is changed in the new state. This is accomplished by the predicate \text{clock\_advanced}, which is not based on a simple incrementation operation because the number of clock cycles consumed by an instruction stream will exhibit a small amount of variation on real processors. The function \text{clock\_advanced} accounts for this variability, meaning the start of the next phase is not deterministically related to the start time of the current phase.

\( \nu \): number

\text{clock\_advanced}: \text{function}[\text{logical\_clocktime}, \text{logical\_clocktime}, \text{number} \rightarrow \text{bool}] =

\[
( \lambda X, Y, D : X + D * (1 - \nu) \leq Y \land Y \leq X + D * (1 + \nu))
\]

where \( \nu \) represents the maximum rate at which one processor's execution time over a phase can vary from the nominal amount given by the \text{duration} function. \( \nu \) is intended to be a nonnegative fractional value, \( 0 \leq \nu < 1 \). The nominal amount of time spent in each phase is specified by a function named \text{duration}:

\text{duration}: \text{function}[\text{phases} \rightarrow \text{logical\_clocktime}]

However, the actual amount of clock time spent in a phase is not fixed, but can vary within limits. For example, the actual duration of the compute phase can be anything from \((1 - \nu) \times \text{duration(compute)}\) to \((1 + \nu) \times \text{duration(compute)}\). The value of \( \nu \) is a parameter of the specification and can be set to any desired value. However, there are some constraints on the implementation that are expressed in terms of \( \nu \):

\text{broadcast\_duration}: \text{Axiom}

\text{duration(broadcast) \times (1 - } \frac{\nu}{2}) - 2\nu \times \text{duration(compute)} - \nu \times \text{duration(broadcast)} -

\delta \geq \text{max\_comm\_delay}

\text{broadcast\_duration2}: \text{Axiom}

\text{duration(broadcast)} - 2\nu \times \text{duration(compute)} - \nu \times \text{duration(broadcast)} \geq 0

\text{pos\_durations}: \text{Axiom}

0 \leq (1 - \nu) \times \text{duration(compute)} \land 0 \leq (1 - \nu) \times \text{duration(broadcast)}

\land 0 \leq (1 - \nu) \times \text{duration(vote)} \land 0 \leq (1 - \nu) \times \text{duration(sync)}

\text{all\_durations}: \text{Axiom}

(1 + \nu) \times \text{duration(compute)} + (1 + \nu) \times \text{duration(broadcast)}

\leq \text{frame\_time}

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The constants $\rho$ and $\delta$ are drawn from the clock synchronization theory, as explained in section 7.1.

There may be many possible causes of the variation in execution times on different processors. The asynchronous interface between a processor and its memory can lead to different execution times between two processors even when they execute exactly the same instructions on exactly the same data. Another possible cause of different execution times could be the use of different schedules on different processors.

The $A_{\alpha}^\pi_{da}$ sub-relation is:

$$A_{\alpha}^\pi_{da} : \text{function}[\text{Dstate, Dstate, processors } \to \text{bool}] =$$

$$\lambda s, t, i : s.\text{proc}(i).\text{healthy} > 0$$

$$\lor t.\text{proc}(i).\text{proc_state} = s.\text{proc}(i).\text{proc_state}$$

$$\land \text{broadcast.received}(s, t, i))$$

As in the corresponding DS relation, the proc.state field remains unchanged and the broadcast.received relation must hold. When it holds, all the nonfaulty processors receive the values sent by other nonfaulty processors. However, this is now contingent upon certain constraints on the times that things happen.

The $A_{\alpha}^\pi_{da}$ sub-relation is:

$$A_{\alpha}^\pi_{da} : \text{function}[\text{Dstate, Dstate, processors } \to \text{bool}] =$$

$$\lambda s, t, i : s.\text{proc}(i).\text{healthy} > 0$$

$$\lor t.\text{proc}(i).\text{mailbox} = s.\text{proc}(i).\text{mailbox}$$

$$\land t.\text{proc}(i).\text{proc_state} = f_v(s.\text{proc}(i).\text{proc_state}, s.\text{proc}(i).\text{mailbox}))$$

As before, the mailbox field remains unchanged and the local processor state is updated with the result of voting the values broadcast by the other processors.

The $A_{\alpha}^\pi_{da}$ sub-relation is:

$$A_{\alpha}^\pi_{da} : \text{function}[\text{Dstate, Dstate, processors } \to \text{bool}] =$$

$$\lambda s, t, i : (s.\text{proc}(i).\text{healthy} > 0$$

$$\lor t.\text{proc}(i).\text{proc_state} = s.\text{proc}(i).\text{proc_state})$$

$$\land (t.\text{proc}(i).\text{healthy} > 0$$

$$\lor t.\text{proc}(i).\text{healthy} = 1 + s.\text{proc}(i).\text{healthy}$$

$$\land \text{nonfaulty_clock}(i, t.\text{sync_period}))$$

$$\land t.\text{sync_period} = 1 + s.\text{sync_period}$$

$$\land (\text{nonfaulty_clock}(i, s.\text{sync_period})$$

$$\lor t.\text{proc}(i).\text{lclock} = (1 + s.\text{sync_period}) \ast \text{frame.time}$$

$$\land t.\text{proc}(i).\text{cum_delta} = s.\text{proc}(i).\text{cum_delta} + \Delta^4_{\text{sync_period}}))$$

During the sync phase, the processor state remains unchanged. As in the DS specification, the healthy field is incremented by one. Unlike the DS model, the local clock time is changed in the new state. For this sub-relation, the clock is not advanced in accordance with the function clock.advanced, because this phase is terminated by a clock interrupt. At a predetermined local clock time, the clock interrupt fires and the next frame is initiated. The specification requires that the interrupts fire at clock times that are integral multiples of the frame length, frame.time.
In addition to requirements conditioned on having a nonfaulty processor, the DA specifications are concerned with having a nonfaulty clock as well. It is assumed that the clock is an independent piece of hardware whose faults can be isolated from those of the corresponding processor. Although some implementations of a fault-tolerant architecture such as RCP could execute part of the clock synchronization function in software, thereby making clock faults and processor faults mutually dependent, we assume that RCP implementations will have a dedicated hardware clock synchronization function. This means that a clock can continue to function properly during a transient fault period on its adjoining processor. The converse is not true, however. Since the software executing on a processor depends on the clock to properly schedule events, a nonfaulty processor having a faulty clock may produce errors. Therefore, a one-way fault dependency exists.

<table>
<thead>
<tr>
<th>Clock</th>
<th>Function</th>
<th>Processor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Faulty</td>
<td>Voting</td>
<td>Faulty</td>
</tr>
<tr>
<td></td>
<td>Clock sync</td>
<td>N</td>
</tr>
<tr>
<td></td>
<td></td>
<td>N</td>
</tr>
<tr>
<td>Nonfaulty</td>
<td>Voting</td>
<td>Y</td>
</tr>
<tr>
<td></td>
<td>Clock sync</td>
<td>Y</td>
</tr>
</tbody>
</table>

Figure 10: Relationship of clock and processor faults.

Figure 10 summarizes the interaction between clock faults and processor faults. It shows for each combination of fault mode whether a processor can make a sound contribution to voting the state variables and whether a clock can properly contribute to clock synchronization. These conditions have been encoded in the various DA specifications. In particular, the relation $\mathcal{N}_d^a$ shown above requires that for a processor to be nonfaulty in the next frame it must have a nonfaulty clock through the end of that frame. Recall that the definition of nonfaulty clock requires that it be continuously nonfaulty from time zero.\(^\text{12}\)

The predicate initial_da puts forth the conditions for a valid initial state. The initial phase is set to compute and the initial sync period is set to zero. Each element of the DA state array has its healthy field equal to recovery_period and its proc_state field equal to initial_proc.state.

\[
\text{initial_da: function}[\text{DAstate} \rightarrow \text{bool}] = \\
(\lambda s : s.\text{phase} = \text{compute} \land s.\text{sync_period} = 0 \\
\land (\forall i : s.\text{proc}(i).\text{healthy} = \text{recovery_period} \\
\land s.\text{proc}(i).\text{proc_state} = \text{initial}\_\text{proc_state} \\
\land s.\text{proc}(i).\text{cum_delta} = 0 \\
\land s.\text{proc}(i).\text{lclock} = 0 \land \text{nonfaulty_clock}(i, 0)))
\]

As before, the constant recovery_period is the number of frames required to fully recover a processor's state after experiencing a transient fault. By initializing the healthy fields to this

\(^{12}\text{This does not represent a deficiency in the design of the DA model but rather is a limitation imposed by the existing, mechanically verified clock synchronization algorithm. Future work will concentrate on liberating the clock synchronization property from this restriction.\)
value, we are starting the system with all processors working. Note that the mailbox fields are not initialized; any mailbox values can appear in a valid initial DAstate.

8 DA to DS Proof

8.1 DA to DS Mapping

The DA to DS mapping function, DAmap, is defined as:

\[
\text{DAmap: function}[\text{DAstate} \rightarrow \text{DSstate}] = \\
(\lambda \text{da} : \text{ss}_\text{update}(\text{da}, \text{nrep}) \text{ with } [(\text{phase}) := \text{da}.\text{phase}])
\]

where \text{ss}_\text{update} is given by:

\[
\text{ss}_\text{update}: \text{Recursive function}[\text{DAstate}, \text{nat} \rightarrow \text{DSstate}] = \\
(\lambda \text{da}, k : \text{if } (k = 0) \text{ or } (k > \text{nrep}) \text{ then } \text{ds0} \text{ else } \text{ss}_\text{update}(\text{da}, k - 1) \text{ with } [(\text{proc}) (k) := \text{dsproc0} \text{ with } [(\text{healthy}) := \text{da}.\text{proc}(k).\text{healthy}, \text{proc}\_\text{state}) := \text{da}.\text{proc}(k).\text{proc}\_\text{state}, \\
(\text{mailbox}) := \text{da}.\text{proc}(k).\text{mailbox}]] \text{ end if}) \text{ by } \text{da}\_\text{measure}
\]

Thus, the iclock, cum_delta, and sync_period fields are not mapped (i.e., are abstracted away) and all of the other fields are mapped identically. To establish that DA implements DS, the commutativity diagram of figure 11 must be shown to commute. To establish that the

\[
\begin{align*}
\mathcal{N}_{\text{ds}}(s', t', u) \\
\mathcal{N}_{\text{da}}(s, t, u)
\end{align*}
\]

Figure 11: Commutative Diagram for DA to DS Proof
diagram commutes, the following formulas must be proved:

\[
\text{phase}\_\text{commutes: Theorem } \text{reachable}(s) \land \mathcal{N}_{\text{da}}(s, t, u) \supset \mathcal{N}_{\text{ds}}(\text{DAmap}(s), \text{DAmap}(t), u)
\]

\[
\text{initial}\_\text{maps: Theorem } \text{initial}\_\text{da}(s) \supset \text{initial}\_\text{ds}(\text{DAmap}(s))
\]

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The lemmas below directly follow from the definition of the mapping.

map.1: Lemma DAmap(s).proc(i).healthy = s.proc(i).healthy
map.2: Lemma DAmap(s).proc(i).proc_state = s.proc(i).proc_state
map.3: Lemma DAmap(s).phase = s.phase
map.4: Lemma DAmap(s).proc(i).mailbox = s.proc(i).mailbox
map.7: Lemma DS.maj_working(DAmap(s)) = DA.maj_working(s)

8.2 The Proof

The phase_commutes theorem must be shown to hold for all four phases. Thus, the proof is decomposed into four separate cases, each of which is handled by a lemma of the form:

phase_com_\( X \): Lemma

\[ s.\text{phase} = \mathcal{X} \land \mathcal{N}_{sa}(s,t,u) \supset \mathcal{N}_{sd}(\text{DAmap}(s), \text{DAmap}(t), u) \]

where \( \mathcal{X} \) is any one of \{compute, broadcast, vote, sync\}. The proof of this theorem requires the expansion of the \( \mathcal{N}_{sa} \) relation and showing that the resulting formula logically implies \( \mathcal{N}_{sa}(\text{DAmap}(s), \text{DAmap}(t), u) \).

8.2.1 Decomposition Scheme

The proof of each lemma phase_com_\( X \) is facilitated by using a common, general scheme for each phase that further decomposes the proof by means of four subordinate lemmas. The general form of these lemmas is as follows:

Lemma 1: \( s.\text{phase} = \mathcal{X} \land \mathcal{N}_{sa}(s,t,u) \supset (\forall i : \mathcal{N}_{sa}^X(s,t,i)) \)

Lemma 2: \( s.\text{phase} = \mathcal{X} \land \mathcal{N}_{sa}^X(s,t,i) \supset \mathcal{N}_{sa}(\text{DAmap}(s), \text{DAmap}(t), i) \)

Lemma 3: \( s.\text{phase} = \mathcal{X} \land DS.\text{maj_working}(tt) \land (\forall i : \mathcal{N}_{sa}(ss,tt,i)) \supset \mathcal{N}_{sa}(ss,tt,u) \)

Lemma 4: \( s.\text{phase} = \mathcal{X} \land \mathcal{N}_{sa}(s,t,u) \supset DS.\text{maj_working}(\text{DAmap}(t)) \)

A few differences exist among the lemmas for the four phases, but they adhere to this scheme fairly closely. The phase_com_\( X \) lemma follows by chaining the four lemmas together:

\( \mathcal{N}_{sa}(s,t,u) \supset (\forall i : \mathcal{N}_{sa}^X(s,t,i)) \supset (\forall i : \mathcal{N}_{sa}(\text{DAmap}(s), \text{DAmap}(t), i)) \supset \mathcal{N}_{sa}(\text{DAmap}(s), \text{DAmap}(t), u) \)

In three of the four cases above, proofs for the lemmas are elementary. The proof of Lemma 1 follows directly from the definition of \( \mathcal{N}_{sa} \). Lemma 3 follows directly from the definition of \( \mathcal{N}_{sa} \). Lemma 4 follows from the definition of \( \mathcal{N}_{sa} \), enough_hardware and the basic mapping lemmas.

Furthermore, in three of the four phases, the proof of Lemma 2 is straightforward. For all but the broadcast phase, Lemma 2 follows from the definition of \( \mathcal{N}_{sa}^X, \mathcal{N}_{sa}^X \), and the basic mapping lemmas.
However, in the broadcast phase, Lemma 2 from the scheme above, which is named \texttt{com\_broadcast\_2}, is a much deeper theorem. The broadcast phase is where the effects of asynchrony are felt; we must show that interprocessor communications are properly received in the presence of asynchronously operating processors. Without clock synchronization we would be unable to assert that broadcast data is received. Hence the need to invoke clock synchronization theory and its attendant reasoning over inequalities of time.

### 8.2.2 Proof of \texttt{com\_broadcast\_2}

The lemma \texttt{com\_broadcast\_2} is the most difficult of the four lemmas for the broadcast phase. It follows from the definition of $N_{ds}^{b}$, $N_{ds}^{a}$, the basic mapping lemmas and a fairly difficult lemma, \texttt{com\_broadcast\_5}:

\begin{align*}
\texttt{com\_broadcast\_5}: & \text{ Lemma} \\
& \text{reachable}(s) \land N_{ds}^{a}(s,t,u) \land s.\text{phase} = \text{broadcast} \\
& \land s.\text{proc}(i).\text{healthy} > 0 \land \text{broadcast\_received}(s,t,i) \\
& \supset \text{broadcast\_received}(\text{DAmap}(s), \text{DAmap}(t), i)
\end{align*}

This lemma deals with the main difference between the DA level and the DS level—the timing constraint on the function broadcast\_received:

\begin{align*}
\text{broadcast\_received}: & \text{ function}[\text{DAstate, DAstate, processors} \rightarrow \text{bool}] = \\
& (\land s, t, q : (\forall p : \\
& (s.\text{proc}(p).\text{healthy} > 0 \\
& \land \text{da\_rt}(s, p, s.\text{proc}(p).\text{lclock}) + \text{max\_comm\_delay} \leq \text{da\_rt}(t, q, t.\text{proc}(q).\text{lclock}) \\
& \supset t.\text{proc}(q).\text{mailbox}(p) = s.\text{proc}(p).\text{mailbox}(p)
\end{align*}

The timing constraint

$$
\text{da\_rt}(s, p, s.\text{proc}(p).\text{lclock}) + \text{max\_comm\_delay} \leq \text{da\_rt}(t, q, t.\text{proc}(q).\text{lclock})
$$

must be discharged in order to show that the DA level implements the DS level. The following lemma is instrumental to this goal.

\begin{align*}
\text{ELT}: & \text{ Lemma } T_2 \geq T_1 + bb \land (T_1 \geq T^0) \land (bb \geq T^0) \land T_2 \in R^{(sp)} \land T_1 \in R^{(sp)} \\
& \land \text{nonfaulty\_clock}(p, sp) \land \text{nonfaulty\_clock}(q, sp) \land \text{enough\_clocks}(sp) \\
& \supset r_t^{(sp)}(T_2) \geq r_t^{(sp)}(T_1) + (1 - \frac{\delta}{2}) \ast |bb| - \delta
\end{align*}

This lemma establishes an important property of timed events in the presence of a fault-tolerant clock synchronization algorithm and is proved in the next subsection. Suppose that on processor $q$ an event occurs at $T_1$ according to its own clock and another event occurs on processor $p$ at time $T_2$ according to its own clock. Then, assuming that the clock times fall within the current frame and the clocks are working and the system still is safe (i.e. more than two thirds of the clocks are non-faulty), then the following is true about the real times of the events:

$$
rt_{p^{(sp)}}(T_2) \geq rt_{q^{(sp)}}(T_1) + (1 - \frac{\delta}{2}) \ast |bb| - \delta
$$
where $bb = T_2 - T_1$, $T_1 = s.proc(p).lclock$ and $T_2 = t.proc(q).lclock$.

If we apply this lemma to the broadcast phase, letting $T_1$ be the time that the sender loads his outgoing mailbox bin and $T_2$ is the earliest time that the receivers can read their mailboxes (i.e. at the start of the vote phase), we know that these events are separated in time by more than $(1 - \frac{v}{2}) * |bb| - \delta$.

In this case $bb$ is approximately equal to $\text{duration(broadcast)}$. However, since there may be some variations in the time spent in the compute and broadcast phases on different processors (i.e. they can drift from the nominal value at a rate less than $v$), the analysis is a little tricky. First consider the situation where processor $q$ is sending a message to processor $p$ during its broadcast phase. Let $r$ be the state at the start of the compute phase, $s$ be the state at the start of the broadcast phase and $t$ be the state at the start of the vote phase:

$$
\begin{array}{c}
\text{compute} \rightarrow \text{broadcast} \rightarrow \text{t}
\end{array}
$$

Then, let

- $R_q = \text{the clock time at the start of the compute phase on processor } q$
- $S_q = \text{the clock time at the start of the broadcast phase on processor } q$
- $T_q = \text{the clock time at the start of the vote phase on processor } q$
- $R_p = \text{the clock time at the start of the compute phase on processor } p$
- $S_p = \text{the clock time at the start of the broadcast phase on processor } p$
- $T_p = \text{the clock time at the start of the vote phase on processor } p$

This is illustrated in figure 12. By the definition of clock advanced, the following can be established:

$$
( \exists pdurc, pdurb, qdurc, qdurb : \\
\text{near(pdurc, compute) } \land \text{near(pdurb, broadcast) } \\
\land \text{near(qdurc, compute) } \land \text{near(qdurb, broadcast) } \\
\land R_p = R_q)
$$

Figure 12: Relationship between phase times on different processors
\[ \forall p, q : s.\text{phase} = \text{compute} \land \\
\text{nonfaulty_clock}(p, s.\text{sync_period}) \land \text{nonfaulty_clock}(q, s.\text{sync_period}) \\
\Rightarrow s.\text{proc}(p).\text{lclock} = s.\text{proc}(q).\text{lclock} \]

given that the state \( s \) is \text{reachable}(s). This invariant exists in the system because of the use of an interrupt timer to initiate the start of a frame on each of the processors at the pre-determined times \( i \cdot \text{frame_time} \). Using the definition of \( R^{(i)} \) and the axioms \text{pos_durations} and \text{all_durations}, we obtain:

\[ \text{nonfaulty_clock}(p, i) \land \text{nonfaulty_clock}(q, i) \]
\[ \Rightarrow S_q \in R^{(i)} \land T_p \in R^{(i)} \\
\land T_p \geq S_q + \text{duration(broadcast)} \\
- 2 \cdot \nu \cdot \text{duration(compute)} - \nu \cdot \text{duration(broadcast)} \]

where \( i \) is the current synchronization period (i.e. \( i = r.\text{sync_period} = s.\text{sync_period} = t.\text{sync_period} \)). We now have a relationship between the clock time that the message was sent and the clock time that it was received in a form appropriate for application of the ELT theorem. In other words, \( T_2 = T_p, T_1 = S_q \) and \( \text{bb} = \text{pdurc} - \text{qdurc} + \text{pdurb} \). Thus, we can convert the relationship between the events expressed in clock times to a relationship between the real times of these events:

\[ rt_p^{(i)}(T_p) \geq rt_q^{(i)}(S_q) + (1 - \frac{\nu}{2}) \cdot |\text{duration(broadcast)} - \text{Epsi}| - \delta \]

where \( \text{Epsi} = 2 \cdot \nu \cdot \text{duration(compute)} + \nu \cdot \text{duration(broadcast)} \). Using the \text{broadcast_duration} implementation axiom:

\[ \text{broadcast_duration: Axiom} \]
\[ \text{duration(broadcast)} \cdot (1 - \frac{\nu}{2}) - 2 \cdot \nu \cdot \text{duration(compute)} \\
- \nu \cdot \text{duration(broadcast)}) - \delta \geq \text{max_comm_delay} \]

we have:

\[ rt_p^{(i)}(T_p) \geq rt_q^{(i)}(S_q) + \text{max_comm_delay} \]

Using the \text{da_rt_lem} lemma:

\[ \text{da_rt}(t, q, T_q) \Rightarrow \text{da_rt}(s, p, S_q) + \text{max_comm_delay} \]

This will discharge the premise of \text{broadcast_received}. Thus,
Lemma
reach(s) ∧ \( N_{\text{da}}(s, t, u) \) ∧ s.phase = broadcast
∧ s.proc(p).healthy > 0 ∧ broadcast_received(s, t, p)
→ broadcast_received(DAmap(s), DAmap(t), p)

Of course there are several technicalities such as the reachable(da) premise that must be discharged in order to apply the da_rt_lem lemma and the other state invariants and establishing that s.proc(p).healthy > 0 \( \supset \) nonfaulty_clock(p, s.sync_period).

Proof of ELT Lemma: In this section we prove,

Lemma 1 (earliest later time Lemma) \( T_2 = T_1 + BB \)
∧ \( T_1 \geq T^0 \) ∧ \( (BB \geq T^0) \) ∧ nonfaulty_clock(p, i) ∧ nonfaulty_clock(q, i)
∧ enough_clocks(i) ∧ \( T_2 \in R^{(i)} \) ∧ \( T_1 \in R^{(i)} \)
\( \supset rt_p^{(i)}(T_2) \geq rt_q^{(i)}(T_1) + (1 - \frac{\delta}{2}) \cdot |BB| - \delta \)

from which the ELT lemma immediately follows.

Proof. This lemma depends primarily upon the definition of a good clock and the synchronization theorem (i.e. sync_thm). The good clock definition yields:

\[
goodclock(q, T^0, T_1 + BB) ∧ (T_1 \geq T^0) ∧ (BB \geq T^0)
\supset (1 - \frac{\delta}{2}) \cdot |BB| \leq c_q(T_1 + BB) - c_q(T_1)
∧ c_q(T_1 + BB) - c_q(T_1) \leq (1 + \frac{\delta}{2}) \cdot |BB|
\]

Note that the definition of a good clock is defined in terms of the uncorrected clocks, \( c_p(T) \). Using the definition of rt, we can rewrite the first formula as:

\[
Lemma \ goodclock(q, T^0, T_1 + Corr_q^{(i)} + BB)
∧ (T_1 \geq T^0) ∧ (T_1 + Corr_q^{(i)} \geq T^0) ∧ (BB \geq T^0)
\supset (1 - \frac{\delta}{2}) \cdot |BB| \leq rt_q^{(i)}(T_1 + BB) - rt_q^{(i)}(T_1)
∧ rt_q^{(i)}(T_1 + BB) - rt_q^{(i)}(T_1) \leq (1 + \frac{\delta}{2}) \cdot |BB|
\]

and obtain a formula in terms of the function rt.

The sync_thm theorem gives us:

\[
\text{enough_clocks}(i) ∧ \text{nonfaulty_clock}(p, i) ∧ \text{nonfaulty_clock}(q, i) ∧ T \in R^{(i)}
\supset -\delta \leq rt_p^{(i)}(T) - rt_q^{(i)}(T) \leq \delta
\]

Combining the previous two formulas and substituting \( T_2 \) for \( T \) in sync_thm, we obtain:

\[
T_2 = T_1 + BB ∧ (T_1 \geq T^0) ∧ (T_1 + Corr_q^{(i)} \geq T^0) ∧ (BB \geq T^0) ∧ T_2 \in R^{(i)}
∧ \text{enough_clocks}(i) ∧ \text{goodclock}(q, T^0, T_1 + Corr_q^{(i)} + BB) ∧ \text{nonfaulty_clock}(p, i) ∧ \text{nonfaulty_clock}(q, i)
\supset rt_p^{(i)}(T_2) \geq rt_q^{(i)}(T_1) + (1 - \frac{\delta}{2}) \cdot |BB| - \delta
\]

From the definition of nonfaulty and goodclock, we have:

\[
T_1 + BB \leq T^{(i+1)} ∧ \text{nonfaulty_clock}(q, i)
\supset \text{goodclock}(q, T^0, T_1 + Corr_q^{(i)} + BB)
\]
Using these last two results we have:

\[
T_2 = T_1 + BB \land T_2 \leq T^{(i+1)} \land (T_1 \geq T^o) \land (T_1 + Corr^{(i)}_q \geq T^o) \land (BB \geq T^o) \\
\land \text{enough.clocks}(i) \land \text{nonfaulty.clock}(p, i) \land \text{nonfaulty.clock}(q, i) \land T_2 \in R^{(i)} \\
\land r_l^{(i)}(T_2) \geq r_l^{(i)}(T_1) + (1 - \frac{\omega}{2}) \cdot |BB| - \delta
\]

Then from the definition of \( R^{(i)} \), \( T^{(i)} \) and the fact that \( Corr^{(0)}_q = 0 \), we have

\[ \text{ft11: Lemma } T_2 = T_1 + BB \land (T_1 \geq T^o) \land (T_1 + Corr^{(i)}_q \geq T^o) \land (BB \geq T^o) \\
\land \text{enough.clocks}(i) \land \text{nonfaulty.clock}(p, i) \land \text{nonfaulty.clock}(q, i) \land T_2 \in R^{(i)} \\
\land r_l^{(i)}(T_2) \geq r_l^{(i)}(T_1) + (1 - \frac{\omega}{2}) \cdot |BB| - \delta
\]

Using the adj.always_pos theorem from [8], we obtain

\[ \text{ft12: Lemma } T_1 \in R^{(i)} \supset (T_1 + Corr^{(i)}_q \geq T^o)
\]

The key lemma follows immediately from the last two formulas, (ft11 and ft12).

9 Implementation Considerations

Although many RCP design decisions have yet to be made, there are a number of implementation issues that need to be considered early. Some of these have emerged as consequences of the formalization effort completed in Phase 2. Others are the result of preliminary investigations into the needs of implementations that can satisfy the RCP specifications. Following is a discussion of these issues and available options.

9.1 Restrictions Imposed by the DA Model

Recall that the DA extended state machine model described in section 2.4 recognized four different classes of state transition: L, B, R, C. Although each is used for a different phase of the frame, the transition types were introduced because operation restrictions must be imposed on implementations to correctly realize the DA specifications. Failure to satisfy these restrictions can render an implementation at odds with the underlying execution model, where shared data objects are subject to the problems of concurrency. The set of constraints on the DA model's implementation concerns possible concurrent accesses to the mailboxes.

While a broadcast send operation is in progress, the receivers' mailbox values are undefined. If the operation is allowed sufficient time to complete, the mailbox values will match the original values sent. If insufficient time is allowed, or a broadcast operation is begun immediately following the current one, the final mailbox value cannot be assured. Furthermore, we make the additional restriction that all other uses of the mailbox be limited to read-only accesses. This provides a simple sufficient condition for noninterfering use of the mailboxes, thereby avoiding more complex mutual exclusion restrictions.

**Operation Restrictions.** Let \( s \) and \( t \) be successive DA states, \( i \) be the processor with the earliest value of \( c_i(s(i).lclock) \), and \( j \) be the processor with the latest
value of \( c_j(t(j).lclock) \). If \( s \) corresponds to a broadcast (B) operation, all processors must have completed the previous operation of type R by time \( c_i(s(i).lclock) \), and the next operation of type B can begin no earlier than time \( c_j(t(j).lclock) \).

No processor may write to its mailbox during an operation of type B or R.

By introducing a prescribed discipline on the use of mailboxes, we ensure that the axiom describing the net effect of broadcast communication can be legitimately used in the DA proof. Although the restrictions are expressed in terms of real time inequalities over all processors' clocks, it is possible to derive sufficient conditions that satisfy the restrictions and can be established from local processor specifications only, assuming a clock synchronization mechanism is in place.

### 9.2 Processor Scheduling

The DA model of the RCP deals with the timing and coordination of the replicated processors in a fairly complete manner. The model defines in detail the functionality of the system with regard to the activities that are necessary to ensure its fault-masking and transient recovery capability. Nevertheless, the delineation of the task execution process on each local processor has not been elaborated in any more detail than in the US model. This was done deliberately in order to obtain as general a specification as possible. Thus, the 4-level hierarchy presented in this paper could be further refined into a set of entirely different kinds of implementations. They could differ drastically in the types of task scheduling that are utilized as well as the type of hardware or software used.

Nevertheless, one aspect of scheduling needs to be carefully controlled, namely the basic frame structure. The RCP specifications were developed with a very crisp execution model in mind regarding the basic timing of a frame and its major parts. We assume the existence of one or more nonmaskable hardware interrupts, triggered by the clock subsystem, that are used to effect the transition from one frame to the next and one major phase to the next.

As a minimum, the following transitions must be triggered by timer interrupts or an equally strong hardware mechanism.

- **Start of frame.** The last portion of a frame is reserved for clock synchronization activities. This includes not only executing the clock synchronization functions, but also reserving some dead time to be sacrificed when clock adjustments cause local clock time discontinuities. An interrupt is set to fire at the proper value of clock time so that all processors begin the new frame with the same local clock reading.

- **Beginning of vote phase.** After waiting for the completion of broadcast communication from other processors, the vote phase is begun to selectively restore portions of the computation state. Also needing to be recovered are any control state variables used by the operating system. If a transient fault occurs, recovery cannot begin until the control state is first restored through voting. However, a processor operating after a transient fault may be executing with a corrupted memory state. The only way to ensure that corrupted memory does not prevent the eventual recovery of control state information is to force the vote to happen through a nonmaskable interrupt.
The use of timer interrupts are highly desirable in other situations, but those listed above are considered essential.

Scheduling of applications tasks is an area where the implementation retains some flexibility owing to our use of a general fault-tolerant computing model in the US and RS specifications. Often it is considered desirable to achieve some type of schedule diversity across processors as a means of gaining more transient fault immunity. A limited way of accomplishing this is available under the current RCP design. Since the specifications only state what must be true after all tasks have been executed within a frame, it is possible to juggle the order of tasks within each frame to implement diversity. For example, if \( N \) tasks are scheduled in a particular frame, each processor may execute them in a different order up to the limits of data dependency among tasks. It is also possible to introduce different spreads of slack time, dummy tasks, etc. to achieve similar effects.

9.3 Hardware Protection Features

Correct recovery of state information after a transient fault has been formalized in the RS to US proof. Transient recovery of state information occurs gradually, one cell at a time. Consequently, depending on the voting pattern used, some tasks will be executing in the presence of erroneous state information. Implicit in the RS specifications is that computation of task outputs is not subject to interference by other tasks executing with erroneous data inputs. In the specifications, this is due simply to the use of a functional representation of the effects of task execution.

Nonetheless, in a real processor a program in execution can interfere with another unless hardware protection mechanisms are in place. To see why this is so, suppose, for instance, that task \( T_1 \) is followed by task \( T_2 \) in a particular frame and neither’s output is voted during that frame. Suppose further that in the transient fault recovery scheme, \( T_2 \)'s inputs come from recently voted cells while \( T_1 \)'s do not. Thus, we expect \( T_2 \)'s cell to be recovered after this frame. After a transient fault, \( T_1 \) may be executing instructions on erroneous data, possibly overwriting recovered information such as that required by \( T_2 \). This would invalidate our assumption that \( T_2 \)'s state is recovered at the end of the frame.

In a similar manner, interference can be caused in the time domain as well as the data domain. In the example above, if \( T_1 \)'s erroneous input causes it to run longer than its upper execution time bound, \( T_2 \) may not get to execute in this frame. Again, this would result in our assumptions about \( T_2 \)'s output being invalid. Therefore, hardware protection features are required to prevent both kinds of interference in a system that attempts to recover state information selectively.

There are several well-known hardware techniques for providing this type of protection.

- **Memory protection.** Hardware write protection devices are found on many modern computer architectures. What RCP requires is less than a full-blown memory management unit (MMU). All that is necessary is to be able to prevent a task in execution from writing into memory areas for which the operating system has not given explicit write permission. The ability to give a task write access to a small set of physical memory regions is sufficient. Generating hardware exceptions such as traps on illicit write attempts is desirable but not essential.
- **Watch-dog timers.** Timer interrupts or special-purpose timing logic will be required to prevent a task from consuming more than its allotted amount of execution time. When a watch-dog timer is triggered, the operating system need only dispatch the next task on the schedule. The actual hardware used to carry out this timing function needs to have adequate resolution and be distinct from the timer interrupts used to signal the end-of-frame and start-of-voting events.

- **Privileged Operating Modes.** To protect the protection mechanisms, it is usually necessary for a processor to have at least one privileged execution domain. Processors typically provide at least a user domain and a (privileged) supervisor domain to implement conventional operating system designs. In RCP, we need these features so the tasks cannot accidentally change or disable the memory write protection or watch-dog timer functions. There may be other uses for privileged mode as well.

It is important to realize that use of these features may be obviated in special cases. If sufficiently frequent voting is used, for example, it may not be necessary to provide these features as long as a task is always executing with valid data as input.

### 9.4 Voting Mechanisms

Exact-match voting of state information exchanged among processors is usually envisioned as applying the majority function to mailbox values. Note, however, that the voting function $f_v$, described in section 3.3, is unspecified and need not be based on the majority operation. Other types of voting may be used provided that the transient recovery axioms of section 3.5.2 are still true.

A desirable alternative to majority voting is *plurality* voting. If the values subject to voting are $\{a, a, b, c\}$, for example, a majority does not exist, but a plurality does, namely $\{a, a\}$. The reason this can be valuable is that during a massive transient fault that affects more than a majority of processors, the Maximum Fault Assumption no longer holds and transient fault recovery is not assured by the proofs previously described. However, the likelihood is that the affected processors will not exhibit exactly the same errors. If a minority of processors is still working, it is likely that the values produced by the replicated processors will appear something like the example $\{a, a, b, c\}$. Hence, plurality voting has a good chance of recovering the correct state in spite of the absence of a working majority.

This problem has been studied by Miner and Caldwell [26]. They showed that the substitution of plurality voting for majority voting can be used to produce identical results as long as the Maximum Fault Assumption holds:

$$\text{maj_exists}(s) \supset \text{maj}(s) = \text{plur}(s)$$

By using an implementation based on plurality voting, we enjoy the same provable behavior when the Maximum Fault Assumption holds, and we enjoy added transient fault immunity in the rare case that it is violated. All that is necessary to achieve this is to show that the choice of function for $f_v$ meets the requirements of the transient recovery axioms.
10 Future Work

There are four main areas where further work may be profitable.

1. Development of a still more detailed specification and verification that it meets the DA specification.

2. Development of task scheduling/voting strategies that satisfy the axioms of the US model.

3. More detailed specification of the behavior of the actuator outputs.


10.1 Further Refinement

Although the DA specification is a fairly detailed design of the system-wide behavior of the RCP, there is very little implementation detail about what occurs locally on each processor. The next level of the specification hierarchy, the local processor LP specification will define the data structures and algorithms to be implemented on each local processor.

At some point the design must be implemented on hardware. It is anticipated that both standard hardware such as microprocessors and memory management units will be required as well as special hardware to implement the clock synchronization and Byzantine agreement functions. In the same way that this work capitalized on the work done elsewhere in clock synchronization, the LP specification will build on the work being performed under contract to NASA Langley in hardware verification.

NASA Langley has awarded three contracts specifically devoted to formal methods (from the competitive NASA RFP 1-22-9130.0238). The selected contractors were SRI International, Computational Logic Inc., and Odyssey Research Associates. Another task-assignment contract with Boeing Military Aircraft Company (BMAC) is being used to explore formal methods as well. Through this contract BMAC is funding research at the University of California at Davis and California Polytechnic State University to assist them in the use of formal methods in aerospace applications. The efforts are roughly divided as follows:

- SRI: Clock synchronization, operating system
- CLI: Byzantine Agreement Circuits, clock synchronization
- ORA: Byzantine Agreement Circuits, applications
- BMAC: Hardware Verification, formal requirements analysis

The DA specification critically depended upon a clock synchronization property. Previous work by SRI had verified that the ICA algorithm meets this property. Ongoing work at SRI is directed at implementing a synchronization algorithm in hardware verifying it. This will lead to the verification hierarchy shown in figure 13.

Implicit in the RS, DS and DA models is the assumption that it is possible to distribute single source information such as sensor data to the redundant processors in a consistent man-
ner even in the presence of faults. This is the classic Byzantine Generals problem [18]. CLI is investigating the formal verification of such algorithms and their implementation. They have formally verified the original Pease, Shostak, and Lamport version of this algorithm using the Boyer Moore theorem prover [27]. They have also implemented this algorithm down to the register-transfer level and demonstrated that it implements the mathematical algorithm [28]. Future work will concentrate on tying this work together with their verified microprocessor, the FM8502 [29].

ORA has also been investigating the formal verification of Byzantine Generals algorithms. They have focused on the practical implementation of a Byzantine-resilient communications mechanism between Mini-Cayuga micro-processors [30]. The Mini-Cayuga is a small but formally verified microprocessor developed by ORA. It is a research prototype and has not been fabricated. This communications circuitry could serve as a foundation for the RCP architecture. It was designed assuming that the underlying processors were synchronized (say by a clock synchronization circuit). The issues involved with connecting the Byzantine communications circuit with a clock synchronization circuit and verifying the combination have not yet been explored.

Boeing Military Aircraft Company and U. C. Davis have been sponsored by NASA, Langley to apply formal methods to the design of conventional hardware devices. Formal Verification of the following circuits is currently under investigation:

- a floating-point coprocessor similar to the Intel 8087 (but smaller) [31, 32].
- a DMA controller similar to the Intel 8237A (but smaller) [33].
- microprocessors in IICL (small) [34, 35, 36].
- a memory management unit [37, 38].

Fault-tolerant systems, although internally redundant, must deal with single-source information from the external world. For example, a flight control system is built around the notion of feedback from physical sensors such as accelerometers, position sensors, pressure sensors, etc. Although these can be replicated (and they usually are), the replicates do not produce identical results. In order to use bit-by-bit majority voting all of the computational replicates must operate on identical input data. Thus, the sensor values (the complete redundant suite) must be distributed to each processor in a manner that guarantees all working processors receive exactly the same value even in the presence of some faulty processors.
The team is currently investigating the verification of a composed set of verified hardware devices [39, 40, 41]. Researchers at NASA Langley have begun a new effort on a hardware clock synchronization technique that can serve as a foundation for the RCP architecture. The method, which is based on the Fault-Tolerant Midpoint algorithm [42], is aimed at a fully independent hardware implementation. The primary goals of this work are full mechanical verification, transient fault recovery, and an initialization scheme that provides recovery from large transient upsets.

10.2 Task Scheduling and Voting

The Phase 1 report described a scheduling system that was based upon a deterministic table. In the models presented in this paper, this is no longer strictly required although such an approach clearly fits within the axioms presented in the US model. However, it is conceivable that more sophisticated scheduling strategies could also be shown to conform.

10.3 Actuator Outputs

It is important not only that the replicated outputs sent to the actuators (on separate wires) are identical but that they appear within some bounded time of each other. Although this bound may not be very small, it is still incumbent upon the verification activity that a bound be mathematically established.

10.4 Development of a Detailed Reliability Model

In the Phase 1 paper, a simple reliability model of the RCP system was developed that demonstrated that the speed at which one must remove the effects of a transient fault is not very critical. In other words, flushing the effects of a transient fault over an extended period of time did not significantly decrease the reliability of the system as compared to extremely fast removal. In this model, a fault anywhere in the processor was sufficient to render the entire processor faulty. Clearly, in a fully developed RCP, there will be more than one fault-isolation containment region per processor. The most likely arrangement is to have a separate fault-containment region for the clocking system and one for the Byzantine agreement circuitry.

11 Concluding Remarks

In this paper a hierarchical specification of a reliable computing platform (RCP) has been developed. The top level specification is extremely general and should serve as a model for many fault-tolerant system designs. The successive refinements in the lower levels of abstraction introduce, first, processor replication and voting, second interprocess communication by use of dedicated mailboxes and finally, the asynchrony due to separate clocks in the system.
Although the first phase of this work was accomplished without the use of an automated theorem prover, we found the use of the EHD M system to be beneficial to this second phase of work for several reasons.

- The amount of detail in the lower level models is significantly greater than in the upper level models. It became extremely difficult to keep up with everything using pencil and paper.

- The strictness of the EHD M language (i.e. its requirement to precisely define all variables and functions, etc.) forced us to elaborate the design more carefully.

- Most of the proofs were not very deep but had to deal with large amounts of detail. Without a mechanical proof checker, it would be far too easy to overlook a flaw in the proofs.

- The proof support environment of EHD M, although overly strict in some cases, provided much assistance in assuring us that our proof chains were complete and that we had not overlooked some unproven lemmas.

- The decision procedures of EHD M for linear arithmetic and propositional calculus were valuable in that they relieved us of the need to reduce many formulas to primitive axioms of arithmetic. Especially useful was its ability to reason about inequalities.

Key features of the work completed during Phase 2 and improvements over the results of Phase 1 include the following.

- Specification of redundancy management and the transient fault recovery scheme uses a very general model of fault-tolerant computing similar to one proposed by Rushby [20, 21].

- Specification of the asynchronous layer design uses modeling techniques based on a time-extended state machine approach. This method allows us to build on previous work that formalized clock synchronization mechanisms and their properties.

- Formulation of the RCP specifications is based on a straightforward Maximum Fault Assumption that provides a clean interface to the realm of probabilistic reliability models. It is only necessary to determine the probability of having a majority of working processors and a two-thirds majority of nonfaulty clocks.

- A four-layer tier of specifications has been completely proved to the standards of rigor of the EHD M mechanical proof system. All proofs can be run on a Sun SPARCstation in less than one hour.

- Important constraints on lower level design and implementation constructs have been identified and investigated.

Based on the results obtained thus far, work will continue to a Phase 3 effort, which will concentrate on completing design formalizations and develop the techniques needed to produce verified implementations of RCP architectures.
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Appendix A
LaTeX-printed Specification Listings

The following specifications were formatted with the assistance of the EIDM latex-printer.

us: Module
Using generic_FT
Exporting all
Theory

\begin{verbatim}
  s, t: Var Pstate
  u: Var inputs
  \lambda u: Definition function[Pstate, Pstate, inputs \rightarrow bool] =
    ( \lambda s, t, u : t = f_e(u, s))
  initial_us: function[Pstate \rightarrow bool] = ( \lambda s : s = initial_proc_state)
\end{verbatim}

End

generic_FT: Module
Using rcp_defs, sets[processors], cardinality[processors]
Exporting all with rcp_defs, sets[processors], cardinality[processors]
Theory

\begin{verbatim}
  us, ps, X, Y: Var Pstate
  p, i, j: Var processors
  k, l, q: Var nat
  u: Var inputs
  w: Var MBvec
  h: Var MBmatrix
  A, B: Var set[processors]
  maj_condition: function[set[processors] \rightarrow bool] =
    ( \lambda A : 2 * card(A) > card(fullset[processors]))

  (* The following definitions and axioms are used to model a general class
   of fault-tolerant computation schemes. The elaboration of these
   uninterpreted functions, as well as those in rcp_defs, would be made
   for a particular choice of application-dependent computation style and
   voting pattern. Given suitable choices, the axioms can then be shown
   to be theorems. *)
\end{verbatim}

cell_state: Type
cell: Type
cell_state: Type
c, d, e: Var cell
K: Var cell_state
II: Var nat

suc: function[cell_state \rightarrow cell_state]


\( f_k: \text{function}[\text{Pstate} \rightarrow \text{control\_state}] \)

\( f_i: \text{function}[\text{Pstate}, \text{cell} \rightarrow \text{cell\_state}] \)

\( f_c: \text{function}[\text{inputs}, \text{Pstate} \rightarrow \text{Pstate}] \)

\( f_a: \text{function}[\text{Pstate} \rightarrow \text{outputs} \ (* \ \text{actuator output} *) \)

\( f: \text{function}[\text{Pstate} \rightarrow MB] \)

\( f_m: \text{function}[\text{Pstate}, \text{MBvec} \rightarrow \text{Pstate}] \)

\( \text{(\*) rec}(c, K, H) = \top \text{ i f f cell c's state should have been recovered when in control state K with healthy count H; note that H-1 healthy frames will have occurred previously. \*)} \)

\( \text{rec: function}[\text{cell}, \text{control\_state}, \text{nat} \rightarrow \text{bool}] \)

\( \text{(\*) \ \text{dep}}(c, d, K) = \top \text{ i f f cell c's value in the next state depends on cell d's value in the current state, when in control state K; if cell c is voted during K, or its computation takes only sensor inputs, there is no dependency; if c is not computed during K, c depends only on itself; otherwise, c depends on one or more cells for its new value. \*) \)

\( \text{dep: function}[\text{cell}, \text{cell}, \text{control\_state} \rightarrow \text{bool}] \)

\( \text{dep\_agree: function}[\text{cell}, \text{control\_state}, \text{Pstate}, \text{Pstate} \rightarrow \text{bool}] = \)

\( (\lambda c, K, X, Y : (\forall d : \text{dep}(c, d, K) \supset f_i(X, d) = f_i(Y, d))) \)

\( \text{(\*) Axioms to be satisfied by the generic application \*)} \)

\( \text{succ\_ax: Axiom} \ \ f_k(f_i(u, ps)) = \sup(f_k(ps)) \)

\( \text{full\_recovery: Axiom} \ H \geq \text{recovery\_period} \supset \text{rec}(c, K, H) \)

\( \text{initial\_recovery: Axiom} \ \text{rec}(c, K, H) \supset H > 2 \)

\( \text{dep\_recovery: Axiom} \ \text{rec}(c, \text{succ}(K), H + 1) \wedge \text{dep}(c, d, K) \supset \text{rec}(d, K, H) \)

\( \text{components\_equal: Axiom} \ \ f_k(X) = f_k(Y) \wedge (\forall c : f_i(X, c) = f_i(Y, c)) \supset X = Y \)

\( \text{control\_recovered: Axiom} \)

\( \text{maj\_condition}(A) \wedge (\forall p : p \in A \supset w(p) = f_s(ps)) \supset f_s(f_s(Y, w)) = f_s(ps) \)

\( \text{cell\_recovered: Axiom} \)

\( \text{maj\_condition}(A) \)

\( \wedge (\forall p : p \in A \supset w(p) = f_s(f_i(u, ps))) \)

\( \wedge f_s(X) = K \wedge f_k(ps) = K \wedge \text{dep\_agree}(c, K, X, ps) \)

\( \supset f_i(f_i(f_i(u, X), w), c) = f_i(f_i(u, ps), c) \)

\( \text{vote\_maj: Axiom} \ \text{maj\_condition}(A) \wedge (\forall p : p \in A \supset w(p) = f_s(ps)) \)

\( \supset f_s(ps, w) = ps \)

\( \text{(\*) Lemmas pertaining to sets and cardinalities \*)} \)

\( \text{card\_fullset: Lemma} \ \text{card(fullset[processors])} > 0 \)

\( \text{proc\_extensionality: Lemma} \ (\forall p : p \in A \equiv p \in B) \supset (A = B) \)

**Proof**

\( \text{discharge\_finite: Prove} \)

\( \text{finite[processors]} \ {f \leftarrow (\lambda p \rightarrow \text{nat : p}), \ N \leftarrow \text{nrep}} \)

\( \text{nat\_nit: Sublemma} \ k > 0 \Leftrightarrow k \neq 0 \)

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p_nat_nit: Prove nat_nlt
p_card_fullset: Prove card_fullset from
   empty {a ∈ fullset[processors]}
   card.empty {a ∈ fullset[processors]}
   nat_nit {k ∈ card(fullset[processors])}

p_proc_extensionality: Prove proc_extensionality {p ∈ processors} from
   extensionality {a ∈ A, b ∈ B}

End

RS: Module

Using generic_FT

Exporting all with generic_FT

Theory

rs_proc_state: Type = Record
   healthy : nat,
   proc_state : Pstate
end record

RSstate: Type = array [processors] of rs_proc_state
rs0: RSstate
rsproc0: rs_proc_state
s, t: Var RSstate
u: Var inputs
w: Var MBvec
h: Var MBmatrix
p, q: Var processors
k: Var nat
A: Var set[processors]

working_proc: function[RSstate, processors → bool] =
   (λ s, p : s(p).healthy ≥ recovery_period)
working_set: function[RSstate → set[processors]] =
   (λ s : (λ p : working_proc(s,p)))
maj_working: function[RSstate → bool] =
   (λ t : maj_condition(working_set(t)))

allowable_faults: function[RSstate, RSstate → bool] =
   (λ s, t : maj_working(t)
     ∧ (∀ p : (t(p).healthy > 0 ∨ t(p).healthy = 1 + s(p).healthy))

good_values_sent: function[RSstate, inputs, MBvec → bool] =
   (λ s, u, w : (∀ q :
     s(q).healthy > 0 ∨ w(q) = f_s(f_c(u, s(q).proc_state))))
voted_final_state: function[RSstate, RSstate, inputs, MBmatrix, processors → bool] =
   (λ s, t, u, h, p : t(p).proc_state = f_s(f_c(u, s(p).proc_state), h(p)))

N_r: Definition function[RSstate, RSstate, inputs → bool] =
\( (\lambda s, t, u : (\exists h : s(p).healthy > 0 \\\land \text{good\_values\_sent}(s, u, h(p)) \land \text{voted\_final\_state}(s, t, u, h, p))) \land \text{allowable\_faults}(s, t)) \)

\( \text{initial\_rs: function}[\text{RSstate} \rightarrow \text{bool}] = (\lambda s : (\forall p : s(p).healthy = \text{recovery\_period} \land s(p).proc\_state = \text{initial\_proc\_state})) \)

\( \text{End} \)

**RS\_to\_US: Module**

**Using** RS, US, RS\_majority

**Exporting** all with RS, US, RS\_majority

**Theory**

\( rs, s, t, x, y, z, \text{Var RSstate} \)

\( us, ps, X, Y : \text{Var Pstate} \)

\( p, i, j : \text{Var processors} \)

\( k, l, q : \text{Var nat} \)

\( u : \text{Var inputs} \)

\( w : \text{Var MBvec} \)

\( h : \text{Var MBmatrix} \)

\( \text{MBmatrix0: MBmatrix} \)

\( \text{MBcons\_fn: Type is function[processors \rightarrow MBvec]} \)

\( \text{MBf\_fn: Var MBcons\_fn} \)

\( \text{RSstate\_prop: Type is function[RSstate \rightarrow bool]} \)

\( \text{rs\_prop: Var RSstate\_prop} \)

\( \text{RSmap: function}[\text{RSstate} \rightarrow \text{Pstate}] = (\lambda rs : \text{maj}(rs)) \)

\( \text{rs\_measure: function}[\text{RSstate, nat} \rightarrow \text{nat}] = (\lambda rs, k : k) \)

\( \text{reachable\_in\_n: function}[\text{RSstate, nat} \rightarrow \text{bool}] = (\lambda t, k : \text{if } k = 0 \text{ then initial\_rs}(t) \text{ else } (\exists s, u : \text{reachable\_in\_n}(s, k - 1) \land \text{N}(s, t, u)) \text{ end if}) \text{ by } \text{rs\_measure} \)

\( \text{reachable: function}[\text{RSstate} \rightarrow \text{bool}] = (\lambda t : (\exists k : \text{reachable\_in\_n}(t, k))) \)

\( \text{frame\_commutes: Theorem reachable}(s) \land \text{N}(s, t, u) \cup \text{N}(s, \text{RSmap}(s), \text{RSmap}(t), u) \)

\( \text{initial\_maps: Theorem initial\_rs}(s) \cup \text{initial\_us}(\text{RSmap}(s)) \)

\( \text{End} \)

**RS\_majority: Module**

**Using** US, RS, nat\_inductions

**Exporting** all

**Theory**

\( k : \text{Var nat} \)
p: Var processors
us: Var Pstate
rs: Var RSstate
A: Var set[processors]

maj.exists: function[RSstate → bool] =
( λ rs : (∃ A, us :
maj.condition(A) ∧ (∀ p : p ∈ A ⇒ rs(p).proc_state = us)))
maj: function[RSstate → Pstate]
maj.ax: Axiom (∃ A :
maj.condition(A) ∧ (∀ p : p ∈ A ⇒ rs(p).proc_state = us))
→ maj(rs) = us

End
RS.lemmas: Module
Using RS.to.US
Exporting all with RS.to.US

Theory
rs, s, t, x, y, z: Var RSstate
us: Var Pstate
p, i, j: Var processors
k, l, q: Var nat
w: Var inputs
u: Var inputs
to: Var MBvec
h: Var MBmatrix
MBmatrix0: MBmatrix
MBcons_fn: Type is function[processors → MBvec]
MBfn: Var MBcons_fn
RSstate.prop: Type is function[RSstate → bool]
rstate_prop: Var RSstate.prop
m, n, a, b: Var proc_plus
prop: Var function[proc_plus → bool]
c, d, e: Var cell
K: Var control_state
H: Var nat
A: Var set[processors]

initial.maj: Lemma
initial.rs(s) ⊃ (∀ p : maj.exists(s) ∧ s(p).proc_state = maj(s))

initial.working: Lemma initial.rs(s) ⊃ working.set(s) = fullset[processors]

initial.maj_cond: Lemma initial.rs(s) ⊃ maj.condition(working.set(s))

control.recovery: function[RSstate → bool] =
( λ s : (∀ p, healthy > 1 ⇒ fs(s(p).proc_state) = fs(maj(s)))

cell.recovery: function[RSstate → bool] =
( λ s : (∀ p, c :
rec(c, fs(s(p).proc_state), s(p).healthy)
⇒ fs(s(p).proc_state, c) = fs(maj(s), c)))

state.recovery: function[RSstate → bool] =
( λ s : maj.exists(s) ∧ control.recovery(s) ∧ cell.recovery(s))

working.majority: function[RSstate → bool] =
( λ s : (∀ p : p ∈ working.set(s) ⇒ s(p).proc_state = maj(s)))

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consensus_prop: Lemma state_recovery(s) ⊃ working_majority(s)

working_set_healthy: Lemma working_set(s)(p) ⊃ s(p).healthy > 0

maj_sent: Lemma state_recovery(s) ∧ good_values_sent(s, u, w)
          ⊃ (∀ p : p ∈ working_set(s) ⊃ w(p) = fₜ(u, maj(s)))

rec_maj_exists: Lemma
          maj_working(s) ∧ state_recovery(s) ∧ Nₛ(s, t, u) ⊃ maj_exists(t)

rec_maj_Lc: Lemma
          maj_working(s) ∧ state_recovery(s) ∧ Afrₜ(s, t, u) ⊃ maj(t) = fₜ(u, maj(s))

End

RS_invariants: Module

Using RS_lemmas, nat_inductions

Exporting all with RS_lemmas

Theory

rs, s, t, x, y, z : Var RSstate
us: Var Pstate
p, i, j : Var processors
k, l, q : Var nat
u: Var inputs
w: Var MBvec
h: Var MBmatrix
RSstate_prop: Type is function[RSstate → bool]
rs_prop: Var RSstate_prop
m, n, a, b: Var proc_plus
prop: Var function[proc_plus → bool]
c, d, e: Var cell
K: Var control_state
H: Var nat
A: Var set[processors]
state_invariant: function[RSstate_prop → bool] =
  (λ rs_prop : (∀ t : reachable(t) ⊃ rs_prop(t)))

state.induction: Lemma
  (∀ x : initial.(rs(x) ⊃ rs_prop(x)))
  ∧ (∀ s, t, u : reachable(s) ∧ rs_prop(s) ∧ Nₛ(s, t, u) ⊃ rs_prop(t))
  ⊃ state.invariant(rs_prop)

maj.working.inv: Lemma state.invariant(maj_working)

state_rec.inv: Lemma state.invariant(state_recovery)

Proof

state.invariant.to.n: function[RSstate_prop, nat → bool] =
  (λ rs_prop, k : (∀ t : reachable.in.n(t, k) ⊃ rs_prop(t)))

base.state.ind: Lemma
  (initial.(rs(x) ⊃ rs_prop(x)) ⊃ (reachable.in.n(x, 0) ⊃ rs_prop(x))

ind.state.ind: Lemma
  (∀ s, t, u : reachable(s) ∧ rs_prop(s) ∧ Nₛ(s, t, u) ⊃ rs_prop(t))
  ⊃ (∀ k : state.invariant.to.n(rs_prop, k)
       ⊃ state.invariant.to.n(rs_prop, k + 1))
p_base_state.ind: Prove base_state.ind from reachable_in_n \{t \leftarrow x, \ k \leftarrow 0\}

p_ind.state.ind: Prove
ind_state.ind \{s \leftarrow s@p3, \ t \leftarrow t@p2, \ u \leftarrow u@p3\} from
state_invariant.to_n \{k \leftarrow k, \ t \leftarrow s@p3\},
state_invariant.to_n \{k \leftarrow k + 1, \ t \leftarrow t\},
reachable_in_n \{t \leftarrow t, \ k \leftarrow k + 1\},
reachable \{t \leftarrow s@p3, \ k \leftarrow k\}

p_state.induction: Prove
state_induction \{z \leftarrow t@p3, \ s \leftarrow s@p4,\ t \leftarrow t@p4,\ u \leftarrow u@p4\} from
nat_induction \{p \leftarrow (\lambda k : state_invariant.to_n(rs_prop, k)), \ n_2 \leftarrow k@p7\},
base_state.ind \{x \leftarrow t@p3\},
state_invariant.to_n \{t \leftarrow x, \ k \leftarrow 0\},
ind_state.ind \{k \leftarrow n_1@p1\},
state_invariant_to_n \{t \leftarrow t@p6, \ k \leftarrow k@p7\},
state.invariant,
reachable \{t \leftarrow t@p6\}

maj_working.inv.ll: Lemma initial_rs(s) ⊆ maj_working(s)

maj_working.inv.l2: Lemma \mathcal{N}_x(s, t, u) ⊆ maj_working(t)

p_maj_working.inv.l1: Prove maj_working.inv.l1 from
maj_working \{t \leftarrow s\}, initial.maj_cond

p_maj_working.inv.l2: Prove maj_working.inv.l2 from \mathcal{N}_x, allowable.faults

p_maj_working.inv: Prove maj_working.inv from
state_induction \{rs_prop \leftarrow maj_working\},
maj_working.inv.l1 \{s \leftarrow x@p1\},
maj_working.inv.l2 \{s \leftarrow s@p1, \ t \leftarrow t@p1, \ u \leftarrow u@p1\}

state_rec.inv.l1: Lemma initial_rs(s) ⊆ state_recovery(s)

state_rec.inv.l2: Lemma
maj_working(s) \land state_recovery(s) \land \mathcal{N}_x(s, t, u) \land maj(t) = f_c(u, maj(s))
\supseteq control.recovery(t)

state_rec.inv.l3: Lemma
maj_working(s) \land state_recovery(s)
\land maj(t) = f_c(u, maj(s))
\land t(p).healthy = 1 + s(p).healthy
\land f_k(s(p).proc_state) = f_k(maj(s))
\land f_k(t(p).proc_state) = f_k(maj(t))
\land good.values.sent(s, u, h(p))
\land rec(c, f_k(t(p).proc_state), t(p).healthy)
\supsete f_l(f_c(u, s(p).proc_state), h(p)), c) = f_l(f_c(u, maj(s)), c)

state_rec.inv.l4: Lemma
maj_working(s) \land state_recovery(s)
\land \mathcal{N}_x(s, t, u) \land maj(t) = f_c(u, maj(s)) \land control.recovery(t)
\supsete cell.recovery(t)

state_rec.inv.l5: Lemma
reachable(s) \land state_recovery(s) \land \mathcal{N}_s(s,t,u) \supset state_recovery(t)

p.state_rec_inv_11: Prove state_rec_inv_11 from
control_recovery,
cell_recovery,
state_recovery,
initial_maj \{p \leftarrow p@p1\},
initial_maj \{p \leftarrow p@p2\}

p.state_rec_inv_12: Prove state_rec_inv_12 from
control_recovery \{ s \leftarrow t \},
\mathcal{N}_s \{ p \leftarrow p@p1 \},
control_recovered
\{ ps \leftarrow f_c(u, \text{maj}(s)),
A \leftarrow \text{working_set}(s),
w \leftarrow ((h@p2)p@p1),
Y \leftarrow f_c(u, (s(p@p1)).proc_state),
\text{maj_sent} \{ p \leftarrow p@p3, w \leftarrow (h@p2)p@p1 \},
\text{maj_working} \{ t \leftarrow s \},
state_recovery,
control_recovery \{ p \leftarrow p@p1 \},
voted_final_state \{ h \leftarrow h@p2, p \leftarrow p@p1 \},
allowable_faults \{ p \leftarrow p@p1 \}

p.state_rec_inv_13: Prove state_rec_inv_13 from
dep_agree \{ K \leftarrow f_k(\text{maj}(s)), X \leftarrow s(p).proc_state, Y \leftarrow \text{maj}(s) \},
cell_recovered
\{ ps \leftarrow \text{maj}(s),
w \leftarrow h(p),
X \leftarrow s(p).proc_state,
A \leftarrow \text{working_set}(s),
K \leftarrow f_k(\text{maj}(s)) \},
\text{maj_sent} \{ p \leftarrow p@p2, w \leftarrow h(p) \},
\text{maj_working} \{ t \leftarrow s \},
state_recovery,
cell_recovery \{ p \leftarrow p, c \leftarrow d@p1 \},
dep_recovery \{ d \leftarrow d@p1, K \leftarrow f_k(\text{maj}(s)), ll \leftarrow s(p).\text{healthy} \},
succ_ax \{ ps \leftarrow \text{maj}(s) \}

p.state_rec_inv_14: Prove state_rec_inv_14 from
cell_recovery \{ s \leftarrow t \},
\mathcal{N}_s \{ p \leftarrow p@p1 \},
state_rec_inv_13 \{ p \leftarrow p@p1, h \leftarrow h@p2, c \leftarrow c@p1 \},
state_recovery,
control_recovery \{ p \leftarrow p@p1 \},
control_recovery \{ s \leftarrow t, p \leftarrow p@p1 \},
voted_final_state \{ h \leftarrow h@p2, p \leftarrow p@p1 \},
allowable_faults \{ p \leftarrow p@p1 \},
initial_recovery
\{ c \leftarrow c@p1,\nH \leftarrow \text{health}(p@p1), K \leftarrow f_k((p@p1)).proc_state \},
succ_ax \{ ps \leftarrow \text{maj}(s) \}

p.state_rec_inv_15: Prove state_rec_inv_15 from

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state_rec_inv_12,
rec_maj_exists,
rec_maj_f_c,
state_rec_inv_14,
state_recovery \{s \leftarrow t\},
maj_working.inv,
state_invariant \{rs_prop \leftarrow maj_working, t \leftarrow s\}

p.state_rec_inv: Prove state_rec_inv from
state_induction \{rs_prop \leftarrow state_recovery\},
state_rec_inv_11 \{s \leftarrow z@p1\},
state_rec_inv_15 \{s \leftarrow z@p1, t \leftarrow t@p1, u \leftarrow u@p1\}

End

RS_top_proof: Module

Using RS_invariants

Exporting all

Theory

\begin{align*}
  rs, s, t, x, y, z &: \text{Var RSstate} \\
  us &: \text{Var Pstate} \\
  p, i, j &: \text{Var processors} \\
  k, l, q &: \text{Var nat} \\
  u &: \text{Var inputs} \\
  w &: \text{Var MBvec} \\
  h &: \text{Var MBmatrix} \\
  c, d, e &: \text{Var cell} \\
  K &: \text{Var control_state} \\
  H &: \text{Var nat} \\
  A &: \text{Var set[processors]} \\
  MBmatrix0 &: \text{MBmatrix} \\
  MBcons_fn &: \text{Type is function[processors \rightarrow MBvec]} \\
  MBfn &: \text{Var MBcons_fn} \\
  RSstate_prop &: \text{Type is function[RSstate \rightarrow bool]} \\
  rs_prop &: \text{Var RSstate_prop} \\
  m, n, a, b &: \text{Var proc_plus} \\
  prop &: \text{Var function[proc_plus \rightarrow bool]}
\end{align*}

Proof

p.frame_commutes: Prove frame_commutes from
\begin{align*}
  Ns \{s \leftarrow maj(s), t \leftarrow maj(t)\}, \\
  rec_maj_f_c, \\
  consensus_prop, \\
  maj_working.inv, \\
  state_invariant \{rs_prop \leftarrow maj_working, t \leftarrow s\}, \\
  state_rec_inv, \\
  state_invariant \{rs_prop \leftarrow state_recovery, t \leftarrow s\}, \\
  state_recovery, \\
  RSmap \{rs \leftarrow s\}, \\
  RSmap \{rs \leftarrow t\}
\end{align*}

p.initial_maps: Prove initial_maps from
\begin{align*}
  maj.ax \{A \leftarrow working_set(s), rs \leftarrow s, us \leftarrow initial_proc_state\}, \\
  initial.us \{s \leftarrow RSmap(s)\}, \\
  initial.rs \{p \leftarrow p@p1\}, \\
  RSmap \{rs \leftarrow s\},
\end{align*}
initial.maj.cond

p.initial.working: Prove initial.working from
extensionality \(\{a \rightarrow \text{working.set}(s), b \leftarrow \text{fullset}[\text{processors}]\},\)
initial.rs \(\{p \leftarrow z@p1\},\)
working.set \(\{p \leftarrow z@p1\},\)
working.proc \(\{p \leftarrow z@p1\}\\
\)

p.initial.maj.cond: Prove initial.maj.cond from
maj.condition \(\{A \leftarrow \text{working.set}(s)\},\) initial.working, card.fullset

p.initial.maj: Prove initial.maj from
maj.ax
\(\{rs \leftarrow s,\)\)
\(A \leftarrow \text{fullset}[\text{processors}],\)
\(us \leftarrow \text{initial.proc.state}\},\)
maj.exists
\(\{rs \leftarrow s,\)\)
\(A \leftarrow \text{fullset}[\text{processors}],\)
\(us \leftarrow \text{initial.proc.state}\},\)
maj.condition \(\{A \leftarrow \text{fullset}[\text{processors}]\},\)
initial.rs \(\{p \leftarrow p@p1\},\)
initial.rs \(\{p \leftarrow p@p2\},\)
initial.rs,
card.fullset

p.working.set.healthy: Prove working.set.healthy from
working.set, working.proc, recovery.period.ax

p.consensus.prop: Prove consensus.prop from
working.majority,
components.equal \(\{X \leftarrow (s(p@p1)).proc.state, Y \leftarrow \text{maj}(s)\},\)
control.recovery \(\{p \leftarrow p@p1\},\)
cell.recovery \(\{p \leftarrow p@p1, c \leftarrow c@p2\},\)
full.recovery
\(\{c \leftarrow c@p2,\)\)
\(K \leftarrow f_s((s(p@p1)).proc.state),\)
\(H \leftarrow (s(p@p1)).\text{healthy}\),
state.recovery,
working.set \(\{p \leftarrow p@p1\},\)
working.proc \(\{p \leftarrow p@p1\},\)
recovery.period.ax

p.maj.sent: Prove maj.sent from
good.values.sent \(\{q \leftarrow p\},\)
consensus.prop,
working.majority,
working.set.healthy

p.rec.maj.exists: Prove rec.maj.exists from
\text{maj_exists} \{ rs \rightarrow t, A \rightarrow \textbf{working set}(s), us \rightarrow f_e(u, \textbf{maj}(s)) \}, \\
\mathcal{N}_s \{ p \rightarrow p@p1 \}, \\
\text{vote}_{\text{maj}} \{ ps \rightarrow f_e(u, \textbf{maj}(s)), \\
w \rightarrow ((h@p2)p@p1), \\
A \rightarrow \textbf{working set}(s) \}, \\
\text{maj_sent} \{ p \rightarrow p@p3, w \rightarrow ((h@p2)p@p1) \}, \\
\text{state_recovery}, \\
\text{consensus_prop}, \\
\text{working_majority} \{ p \rightarrow p@p1 \}, \\
\text{voted_final_state} \{ h \rightarrow h@p2, p \rightarrow p@p1 \}, \\
\text{working_set_healthy} \{ p \rightarrow p@p1 \}, \\
\text{maj_working} \{ t \rightarrow s \} \\

\text{p_rec}_{\text{maj}_{\text{f.c}}} : \text{Prove rec}_{\text{maj}_{\text{f.c}}} \text{ from} \\
\text{maj_ax} \{ rs \rightarrow t, A \rightarrow \textbf{working set}(s), us \rightarrow f_e(u, \textbf{maj}(s)) \}, \\
\mathcal{N}_s \{ p \rightarrow p@p1 \}, \\
\text{vote}_{\text{maj}} \{ ps \rightarrow f_e(u, \textbf{maj}(s)), \\
w \rightarrow ((h@p2)p@p1), \\
A \rightarrow \textbf{working set}(s) \}, \\
\text{maj_sent} \{ p \rightarrow p@p3, w \rightarrow ((h@p2)p@p1) \}, \\
\text{state_recovery}, \\
\text{consensus_prop}, \\
\text{working_majority} \{ p \rightarrow p@p1 \}, \\
\text{voted_final_state} \{ h \rightarrow h@p2, p \rightarrow p@p1 \}, \\
\text{working_set_healthy} \{ p \rightarrow p@p1 \}, \\
\text{maj_working} \{ t \rightarrow s \} \\

\text{End} \\
\text{RS_tcc_proof: Module} \\
\text{Using rcp_defs_tcc} \\
\text{Exporting all} \\
\text{Theory} \\
\text{Proof} \\
\text{proc_plus.TCC1_PROOF: Prove proc_plus.TCC1} \{ p \rightarrow 0 \} \\
\text{processors.TCC1_PROOF: Prove processors.TCC1} \{ p \rightarrow nrep \} \text{ from} \\
\text{processors_exist_ax} \\

\text{End} \\
\text{RS_to_US_tcc: Module} \\
\text{Using RS_to_US} \\
\text{Exporting all with RS_to_US} \\
\text{Theory} \\
s: \text{Var RS.RSstate} \\
t: \text{Var RS.RSstate} \\
k: \text{Var naturalnumber} \\

\text{reachable_in_n.TCC1: Formula } \neg(k = 0) \lor (k - 1 \geq 0)
reachable_in_n_TCC2: Formula
\((-((k = 0)) \supset \text{ra.measure}(t, k) > \text{ra.measure}(s, k - 1))\)

Proof
reachable_in_n_TCC1_PROOF: Prove reachable_in_n_TCC1
reachable_in_n_TCC2_PROOF: Prove reachable_in_n_TCC2

End RS_to_US_tcc

DS: Module
Using generic_FT
Exporting all with generic_FT

Theory

ds_proc_state: Type = Record
healthy : nat,
proc_state : Pstate,
mailbox : MBvec
end record
ds_proc_array: Type = array [processors] of ds_proc_state
DSstate: Type = Record
phase : phases,
proc : ds_proc_array
end record
dS0: DSstate
dSproc0: ds_proc_state
s, t, x, y, z: Var DSstate
u: Var inputs
w: Var MBvec
i, j, p, g, qq: Var processors
k: Var nat
ph: Var phases
A: Var set[processors]

working_proc: function[DSstate, processors -> bool] =
( \( \lambda s, p : s.proc(p).healthy \geq \text{recovery-period} \))
working_set: function[DSstate -> set[processors]] =
( \( \lambda s : ( \lambda p : \text{working.proc}(s, p)) \))
maj_working: function[DSstate -> bool] =
( \( \lambda t \text{: maj.condition} (\text{working.set}(t)) \))

allowable_faults: function[DSstate, DSstate -> bool] =
( \( \lambda s, t : \text{maj.working}(t) \)
\wedge (\forall i : t.proc(i).healthy > 0
\supset t.proc(i).healthy = 1 + s.proc(i).healthy))

broadcast.received: function[DSstate, DSstate, processors -> bool] =
( \( \lambda s, t, p : (\forall qq : s.proc(qq).healthy > 0
\supset t.proc(p).mailbox(qq) = s.proc(qq).mailbox(qq))) \))

\( \mathcal{A}_d^e \): function[DSstate, DSstate, inputs, processors -> bool] =
( \( \lambda s, t, u, i : s.proc(i).healthy > 0
\supset t.proc(i).proc_state = f_s(u, s.proc(i).proc_state)
\wedge t.proc(i).mailbox(i) = f_s(f_s(u, s.proc(i).proc_state))) \))
\[ \mathcal{N}_a : \text{function}[	ext{DSstate}, \text{DSstate}, \text{processors} \rightarrow \text{bool}] = \\
( \lambda s, t, i : s.\text{proc}(i).\text{healthy} > 0 \\
\triangleright t.\text{proc}(i).\text{proc\_state} = s.\text{proc}(i).\text{proc\_state} \\
\wedge \text{broadcast\_received}(s, t, i)) \]

\[ \mathcal{N}_b : \text{function}[	ext{DSstate}, \text{DSstate}, \text{processors} \rightarrow \text{bool}] = \\
( \lambda s, t, i : s.\text{proc}(i).\text{healthy} > 0 \\
\triangleright (t.\text{proc}(i).\text{mailbox} = s.\text{proc}(i).\text{mailbox} \\
\wedge t.\text{proc}(i).\text{proc\_state} \\
= f(s.\text{proc}(i).\text{proc\_state}, s.\text{proc}(i).\text{mailbox})) \]

\[ \mathcal{N}_c : \text{function}[	ext{DSstate}, \text{DSstate}, \text{processors} \rightarrow \text{bool}] = \\
( \lambda s, t, i : s.\text{proc}(i).\text{healthy} > 0 \\
\triangleright t.\text{proc}(i).\text{proc\_state} = s.\text{proc}(i).\text{proc\_state} \\
\wedge (t.\text{proc}(i).\text{healthy} > 0 \\
\triangleright t.\text{proc}(i).\text{healthy} = 1 + s.\text{proc}(i).\text{healthy}) \]

\[ \mathcal{N}_d : \text{function}[	ext{DSstate}, \text{DSstate}, \text{inputs} \rightarrow \text{bool}] = \\
( \lambda s, t, u : \text{maj\_working}(t) \\
\wedge t.\text{phase} = \text{next\_phase}(s.\text{phase}) \\
\wedge (\forall i : \\
\quad \text{if } s.\text{phase} = \text{sync} \\
\quad \text{then } \mathcal{N}_d(s, t, i) \\
\quad \text{else } t.\text{proc}(i).\text{healthy} = s.\text{proc}(i).\text{healthy} \\
\quad \wedge (s.\text{phase} = \text{compute} \triangleright \mathcal{N}_d(s, t, u, i)) \\
\quad \wedge (s.\text{phase} = \text{broadcast} \triangleright \mathcal{N}_d(s, t, i)) \\
\quad \wedge (s.\text{phase} = \text{vote} \triangleright \mathcal{N}_d(s, t, i)) \\
\quad \text{end if}) \]

\[ \text{frame\_N\_ds} : \text{function}[	ext{DSstate}, \text{DSstate}, \text{inputs} \rightarrow \text{bool}] = \\
( \lambda s, t, u : (\exists x, y, z : \\
\mathcal{N}_d(s, x, u) \wedge \mathcal{N}_d(s, y, u) \wedge \mathcal{N}_d(s, z, u))) \]

\[ \text{initial\_ds} : \text{function}[	ext{DSstate} \rightarrow \text{bool}] = \\
( \lambda s : s.\text{phase} = \text{compute} \\
\wedge (\forall i : s.\text{proc}(i).\text{healthy} = \text{recovery\_period} \\
\wedge s.\text{proc}(i).\text{proc\_state} = \text{initial\_proc\_state})) \]

End

DS\_to\_RS: Module

Using DS, RS
Exporting all with DS, RS
Theory

\[ ds, s, t, x, y, z \text{ Var DSstate} \]
\textit{rs}: \texttt{Var RSstate}
\textit{i, j}: \texttt{Var processors}
\textit{p}: \texttt{Var nat}
\textit{u}: \texttt{Var inputs}
\textit{w}: \texttt{Var MBvec}
\textit{h}: \texttt{Var MBmatrix}
\textit{MBmatrix0}: \texttt{MBmatrix}
\textit{MBcons_fn}: \texttt{Type is function[processors \to MBvec]}
\textit{MBfn}: \texttt{Var MBcons_fn}
\textit{ssu_measure}: \texttt{function[DSstate, nat \to nat] == (\lambda ds, p : p)}
\textit{ss.update}: \texttt{Recursive function[DSstate, nat \to RSstate] =}
\textit{DSmap: function[DSstate \to RSstate] = (\lambda ds: ss.update(ds, nrep))}
\textit{MBmc_measure: function[MBcons_fn, nat \to nat] == (\lambda MBfn, p : p)}
\textit{MBmatrix_cons: Recursive function[MBcons_fn, nat \to MBmatrix] =}
\textit{frame.commates: Theorem}
\textit{initial_maps: Theorem}
\textit{good_values_sent: function[DSstate, inputs, MBvec \to bool] =}
\textit{voted_final_state: function[DSstate, DSstate, inputs, MBmatrix, processors \to bool] =}
\textit{is_new_proc_state: function[DSstate, DSstate, inputs \to bool] =}
\textit{fr.com.1: Lemma}
\textit{fr.com.2: Lemma}
\textit{fc.A: Lemma}
\textit{fc.B: Lemma}
DS.lemmas: Module

Using DS_to_RS

Exporting all with DS_to_RS

Theory

ds: Var DSstate
rs: Var RState
p, q: Var nat
ph: Var phases
s, t, x, y, z: Var DSstate
i, j, jj: Var processors
u: Var inputs
w: Var MBvec
h: Var MBmatrix
MBfn: Var MBcons_fn
MB: Var MBvec
k, m, n, a, b: Var proc_plus
prop: Var function[proc_plus → bool]

half_frame_N_ds: function[DSstate, DSstate, inputs → bool] =
( λ x, t, u : (∃ y, z : Nds(x, y, u) ∧ Nds(y, z, u) ∧ Nds(z, t, u)))

quarter_frame_N_ds: function[DSstate, DSstate, inputs → bool] =
( λ y, t, u : (∃ z : Nds(y, z, u) ∧ Nds(z, t, u)))

fc_A_1a: Lemma s.phase = compute ∧ frame_N_ds(s, t, u)
  ∃ (x, y, z :
    maj.working(x)
    ∧ (∀ i :
      x.phase = broadcast
      ∧ x.proc(i).healthy = s.proc(i).healthy ∧ Nds(x, i)
      ∧ Nds(x, y, z) ∧ Nds(y, z, u) ∧ Nds(z, t, u)))

fc_A_1b: Lemma s.phase = compute ∧ frame_N_ds(s, t, u)
  ∃ (x, y, z :
    maj.working(x)
    ∧ maj.working(y)
    ∧ (∀ i :
      x.phase = broadcast
      ∧ x.proc(i).healthy = s.proc(i).healthy
      ∧ Nds(x, i)
      ∧ y.phase = next_phase(x.phase)
      ∧ y.proc(i).healthy = x.proc(i).healthy
      ∧ Nds(y, z, u) ∧ Nds(z, t, u)))

fc_A_1c: Lemma s.phase = compute ∧ frame_N_ds(s, t, u)
\( \exists x, y, z:\)
- maj\_working(z)
- maj\_working(y)
- (\( \forall i:\)
  - x\_phase = broadcast
  - x\_proc(i).healthy = s\_proc(i).healthy
  - s\_proc(i).healthy > 0
    \( \implies \)
  - x\_proc(i).proc\_state
    = f_c(u, s\_proc(i).proc\_state))
- y\_phase = vote
- y\_proc(i).healthy = x\_proc(i).healthy
- (x\_proc(i).healthy > 0
  \( \implies \)
  - y\_proc(i).proc\_state
    = x\_proc(i).proc\_state)
- (\( \forall j:\)
  - z\_proc(j).healthy > 0
    \( \implies \)
  - y\_proc(i).mailbox(j)
    = f_c(x\_proc(j).proc\_state)))

\( \land N_{ds}(y, z, u) \land N_{ds}(z, t, u) \)

fc.A.1d: Lemma \( s\_phase = \text{compute} \land \text{frame}_N\_ds(s, t, u) \)
\( \exists x, y, z:\)
- maj\_working(z)
- maj\_working(y)
- (\( \forall jj:\)
  - x\_proc(jj).healthy = s\_proc(jj).healthy
    \( \land \)
  - s\_proc(jj).healthy > 0
    \( \implies \)
  - x\_proc(jj).proc\_state
    = z\_proc(jj).proc\_state))
- (\( \forall i:\)
  - y\_phase = vote
    \( \land \)
  - y\_proc(i).healthy = s\_proc(i).healthy
    \( \land \)
  - s\_proc(i).healthy > 0
    \( \implies \)
  - y\_proc(i).proc\_state
    = f_c(u, s\_proc(i).proc\_state)
    \( \land \)
  - (\( \forall j:\)
    - z\_proc(j).healthy > 0
      \( \implies \)
    - y\_proc(i).mailbox(j)
      = f_c(x\_proc(j).proc\_state)))

\( \land N_{ds}(y, z, u) \land N_{ds}(z, t, u) \)

fc.A.1e: Lemma \( s\_phase = \text{compute} \land \text{frame}_N\_ds(s, t, u) \)
\( \exists x, y, z:\)
- maj\_working(z)
- maj\_working(y)
- (\( \forall i:\)
  - y\_phase = vote
    \( \land \)
  - y\_proc(i).healthy = s\_proc(i).healthy
    \( \land \)
  - s\_proc(i).healthy > 0
    \( \implies \)
  - y\_proc(i).proc\_state
    = f_c(u, s\_proc(i).proc\_state)
    \( \land \)
  - (\( \forall j:\)
    - s\_proc(j).healthy > 0
      \( \implies \)
    - y\_proc(i).mailbox(j)
      = f_c(y\_proc(j).proc\_state)))

\( \land N_{ds}(y, z, u) \land N_{ds}(z, t, u) \)
fc_A_1f: Lemma $s.phase = compute \land frame.N.ds(s, t, u)$
\[ (\exists y, z : \]
\[ \text{maj.working}(y) \land (\forall i : y.phase = \text{vote} \land y.proc(i).healthy = s.proc(i).healthy \land s.proc(i).healthy > 0 \Rightarrow y.proc(i).proc.state = f_s(u, s.proc(i).proc.state) \land (\forall j : s.proc(j).healthy > 0 \Rightarrow y.proc(i).mailbox(j) = f_s(y.proc(j).proc.state)))) \]
\[ \land \mathcal{N}ds(y, z, u) \land \mathcal{N}ds(z, t, u)) \]

fc_A_2a: Lemma $s.phase = compute \land frame.N.ds(s, t, u)$
\[ (\exists y, z : \]
\[ \text{maj.working}(y) \land \text{maj.working}(z) \land (\forall i : y.phase = \text{vote} \land y.proc(i).healthy = s.proc(i).healthy \land s.proc(i).healthy > 0 \Rightarrow y.proc(i).proc.state = f_s(u, s.proc(i).proc.state) \land (\forall j : s.proc(j).healthy > 0 \Rightarrow y.proc(i).mailbox(j) = f_s(y.proc(j).proc.state)))) \]
\[ \land z.phase = \text{next.phase}(y.phase) \land z.proc(i).healthy = y.proc(i).healthy \land \mathcal{N}ds(y, z, i) \]
\[ \land \mathcal{N}ds(z, t, u) \]

fc_A_2b: Lemma $s.phase = compute \land frame.N.ds(s, t, u)$
\[ (\exists y, z : \]
\[ \text{maj.working}(y) \land \text{maj.working}(z) \land (\forall i : z.phase = \text{next.phase}(y.phase) \land z.proc(i).healthy = s.proc(i).healthy \land s.proc(i).healthy > 0 \Rightarrow z.proc(i).proc.state = f_s(u, s.proc(i).proc.state) \land (\forall j : z.proc(j).healthy > 0 \Rightarrow z.proc(i).mailbox(j) = f_s(y.proc(j).proc.state)))) \]
\[ \land s.proc(i).healthy > 0 \Rightarrow (z.proc(i).mailbox = y.proc(i).mailbox \land z.proc(i).proc.state = f_s(y.proc(i).proc.state, y.proc(i).mailbox)))) \]
\[ \land \mathcal{N}ds(z, t, u) \]
Lemma $s$.phase = compute $\land$ frame_N.ds($s$, $t$, $u$)

\[
\begin{align*}
\exists y, z:
& \text{maj}\_\text{working}(y) \\
& \text{maj}\_\text{working}(z) \\
& (\forall i: \\
& \quad z$.phase = \text{next}\_\text{phase}(\text{vote}) \\
& \quad \land z$.proc(i).healthy = s$.proc(i).healthy \\
& \quad \land s$.proc(i).healthy > 0 \\
& \quad \exists y$.proc(i).proc\_state \\
& \quad = f_c(u, s$.proc(i).proc\_state) \\
& \quad \land z$.proc(i).proc\_state \\
& \quad = f_c(y$.proc(i).proc\_state, \\
& \quad z$.proc(i).mailbox) \\
& \quad \land (\forall j: \\
& \quad s$.proc(j).healthy > 0 \\
& \quad \exists z$.proc(i).mailbox(j) \\
& \quad = f_c(y$.proc(j).proc\_state)))) \\
\land \mathcal{N}_d(x, t, u) 
\end{align*}
\]

Lemma $s$.phase = compute $\land$ frame_N.ds($s$, $t$, $u$)

\[
\begin{align*}
\exists z: & \text{maj}\_\text{working}(z) \\
& (\forall i: \\
& \quad z$.phase = \text{sync} \\
& \quad \land z$.proc(i).healthy = s$.proc(i).healthy \\
& \quad \land s$.proc(i).healthy > 0 \\
& \quad \exists z$.proc(i).proc\_state \\
& \quad = f_c(u, s$.proc(i).proc\_state) \\
& \quad \land z$.proc(i).proc\_state \\
& \quad = f_c(f_c(u, s$.proc(i).proc\_state), \\
& \quad z$.proc(i).mailbox) \\
& \quad \land (\forall j: \\
& \quad s$.proc(j).healthy > 0 \\
& \quad \exists z$.proc(i).mailbox(j) \\
& \quad = f_c(f_c(u, s$.proc(j).proc\_state)))) \\
\land \mathcal{N}_d(x, t, u) 
\end{align*}
\]

Lemma $s$.phase = compute $\land$ frame_N.ds($s$, $t$, $u$)

\[
\begin{align*}
\exists z: & \text{maj}\_\text{working}(t) \\
& (\forall i: \\
& \quad z$.phase = \text{sync} \\
& \quad \land z$.proc(i).healthy = s$.proc(i).healthy \\
& \quad \land s$.proc(i).healthy > 0 \\
& \quad \exists z$.proc(i).proc\_state \\
& \quad = f_c(u, s$.proc(i).proc\_state) \\
& \quad \land z$.proc(i).proc\_state \\
& \quad = f_c(f_c(u, s$.proc(i).proc\_state), \\
& \quad z$.proc(i).mailbox) \\
& \quad \land (\forall j: \\
& \quad s$.proc(j).healthy > 0 \\
& \quad \exists z$.proc(i).mailbox(j) \\
& \quad = f_c(f_c(u, s$.proc(j).proc\_state)))) \\
\land t$.phase = \text{next}\_\text{phase}(z$.phase) \land \mathcal{N}_d(x, t, i)) 
\end{align*}
\]
fc_A_3b: Lemma $s.phase = compute \land frame_N.ds(s,t,u)$
\[ (\exists x : maj\.working(t) \land (\forall i : t.phase = next\.phase(sync) \land x.proc(i).healthy = s.proc(i).healthy \land s.proc(i).healthy > 0 \land x.proc(i).proc_state = f_x(f_{x}(u, s.proc(i).proc_state), x.proc(i).mailbox) \land (\forall j : s.proc(j).healthy > 0 \land x.proc(i).mailbox(j) = f_x(f_{x}(u, s.proc(j).proc_state)))) \land (x.proc(i).healthy > 0 \land t.proc(i).proc_state = x.proc(i).proc_state) \land (t.proc(i).healthy > 0 \land t.proc(i).healthy = 1 + x.proc(i).healthy)))

fc_A_3c: Lemma $s.phase = compute \land frame_N.ds(s,t,u)$
\[ (\exists x : maj\.working(t) \land (\forall i : t.phase = compute \land s.proc(i).healthy > 0 \land t.proc(i).proc_state = f_x(f_{x}(u, s.proc(i).proc_state), x.proc(i).mailbox) \land (\forall j : s.proc(j).healthy > 0 \land x.proc(i).mailbox(j) = f_x(f_{x}(u, s.proc(j).proc_state)))) \land (t.proc(i).healthy > 0 \land t.proc(i).healthy = 1 + x.proc(i).healthy)))

fc_A_3d: Lemma $s.phase = compute \land frame_N.ds(s,t,u)$
\[ (\exists x : maj\.working(t) \land (\forall i : t.phase = compute \land (t.proc(i).healthy > 0 \land t.proc(i).healthy = 1 + s.proc(i).healthy) \land s.proc(i).healthy > 0 \land t.proc(i).proc_state = f_x(f_{x}(u, s.proc(i).proc_state), h(i)) \land (\forall j : s.proc(j).healthy > 0 \land h(i)(j) = f_x(f_{x}(u, s.proc(j).proc_state)))))))

map_1: Lemma $(DSmap(s)(i)).healthy = s.proc(i).healthy$

map_2: Lemma $(DSmap(s)(i)).proc.state = s.proc(i).proc.state$

map_3: Lemma allowable_faults(s,t) \supset RS(allowable_faultsDSmap(s),DSmap(t))

map_4: Lemma RS(good_values_sentDSmap(s), u, w) = good_values_sent(s, u, w)

map_5: Lemma RS(voted_final_stateDSmap(s), DSmap(t), u, h, i) = voted_final_state(s, t, u, h, i)

map_7: Lemma RS(maj_workingDSmap(s)) = DS(maj.workings)
support_1: Lemma \( (\forall i: s.proc(i).healthy = x.proc(i).healthy) \)  
\& allowable_faults(x, y)  
\implies allowable_faults(z, y)

support_4: Lemma \( \mathcal{N}_d(s, t, u) \implies t.phase = next_phase(s.phase) \)

support_5: Lemma \( s.phase = ph \land ph \neq sync \land \mathcal{N}_d(s, x, u) \)  
\implies (\forall i: s.proc(i).healthy = x.proc(i).healthy)

support_6: Lemma \( s.phase = ph \land ph \neq sync \land \mathcal{N}_d(s, x, u) \)  
\& allowable_faults(x, y)  
\implies allowable_faults(s, y)

support_7: Lemma \( s.phase = compute \land \text{frame.N_ds}(s, t, u) \)  
\implies (\exists x: \mathcal{N}_d(s, x, u) \land x.phase = broadcast \land \text{half_frame.N_ds}(x, t, u))

support_8: Lemma \( x.phase = broadcast \land \text{frame.N_ds}(x, t, u) \)  
\implies (\exists y: \mathcal{N}_d(x, y, u) \land y.phase = vote \land \text{quarter_frame.N_ds}(y, t, u))

support_9: Lemma \( y.phase = vote \land \text{quarter_frame.N_ds}(y, t, u) \)  
\implies (\exists z: \mathcal{N}_d(y, z, u) \land z.phase = sync \land \mathcal{N}_d(z, t, u))

support_10: Lemma \( s.phase = sync \land \mathcal{N}_d(s, t, u) \)  
\implies allowable_faults(s, t)

support_11: Lemma \( s.phase = compute \land \text{frame.N_ds}(s, t, u) \)  
\implies allowable_faults(s, t)

support_12: Lemma \( s.phase = compute \land \text{frame.N_ds}(s, t, u) \)  
\implies (\exists z: z.phase = sync \land \mathcal{N}_d(z, t, u))

support_13: Lemma MBmatrix_cons(MBfn, nrep)(i) = MBfn(i)

support_14: Lemma initial_ds(s) \implies working_set(s) = fullset[processors]

support_15: Lemma initial_ds(s) \implies maj_condition(working_set(s))

End

DS.top_proof: Module

Using DS.lemmas

Exporting all with DS.lemmas

Theory

ds: Var DSstate  
rs: Var RSstate  
p, q: Var nat  
ph: Var phases  
s, t, x, y, z: Var DSstate  
i, j, ii, jj: Var processors  
u: Var inputs  
w: Var MBvec  
h: Var MBmatrix  
k, m, n, a, b: Var proc_plus  
prop: Var function[proc_plus \rightarrow bool]

Proof

p.initial_maps: Prove initial_maps from
  initial_ds \{i \leftarrow p@p2\},
  initial_rs \{s \leftarrow DSmap(s)\},
  map.1 \{i \leftarrow p@p2\},
  map.2 \{i \leftarrow p@p2\}


  \{s \leftarrow DSmap(s)\},
  is_new_proc_state \{s \leftarrow s, t \leftarrow t, i \leftarrow p@p1\},
  map.3 \{s \leftarrow s, t \leftarrow t\},
  map.4 \{s \leftarrow s, w \leftarrow h@p2(i@p2)\},
  map.5 \{s \leftarrow s, l \leftarrow l, h \leftarrow h@p2, i \leftarrow p@p1\},
  map.1 \{s \leftarrow s, i \leftarrow p@p1\}

p.fc.A: Prove fc.A from
  fc.A.3d \{i \leftarrow i@p2, j \leftarrow j@p3\},
  is_new_proc_state \{h \leftarrow h@p1\},
  DS_to.RS.good_values.sent \{w \leftarrow h@p1(i@p2)\},
  DS_to.RS.voted_final_state \{i \leftarrow i@p2, h \leftarrow h@p1\}

p.fc.B: Prove fc.B from support.11

p.fc.A.1a: Prove fc.A.1a \{x \leftarrow x@p1, y \leftarrow y@p1, z \leftarrow z@p1\} from
  frame.N_ds,
  \{s \leftarrow s@p1, t \leftarrow x@p1\},
  next_phase \{ph \leftarrow s.phase\},
  distinct.phases

p.fc.A.1b: Prove fc.A.1b \{x \leftarrow x@p1, y \leftarrow y@p1, z \leftarrow z@p1\} from
  fc.A.1a, \{s \leftarrow s@p1, t \leftarrow y@p1\}, distinct.phases

p.fc.A.1c: Prove fc.A.1c \{x \leftarrow x@p1, y \leftarrow y@p1, z \leftarrow z@p1\} from
  fc.A.1b, fc.A.1b \{i \leftarrow j\},
  distinct.phases,
  next_phase \{ph \leftarrow s.phase\},
  \{i \leftarrow x\},
  \{t \leftarrow z, i \leftarrow j\},
  \{s \leftarrow x, t \leftarrow y\},
  broadcast_received \{s \leftarrow x, t \leftarrow y, p \leftarrow i, q \leftarrow j\}

p.fc.A.1d: Prove fc.A.1d \{x \leftarrow x@p1, y \leftarrow y@p1, z \leftarrow z@p1\} from
  fc.A.1c, fc.A.1c \{i \leftarrow jj\}

p.fc.A.1e: Prove fc.A.1e \{x \leftarrow x@p1, y \leftarrow y@p1, z \leftarrow z@p1\} from
  fc.A.1d \{jj \leftarrow j\}, fc.A.1d

p.fc.A.1f: Prove fc.A.1f \{y \leftarrow y@p1, z \leftarrow z@p1\} from fc.A.1e

p.fc.A.2a: Prove fc.A.2a \{y \leftarrow y@p1, z \leftarrow z@p1\} from
  fc.A.1f, \{s \leftarrow y@p1, t \leftarrow z@p1\}, distinct.phases

p.fc.A.2b: Prove fc.A.2b \{y \leftarrow y@p1, z \leftarrow z@p1\} from
  fc.A.2a, \{s \leftarrow y, t \leftarrow z, i \leftarrow i@C\}

p.fc.A.2c: Prove fc.A.2c \{y \leftarrow y@p1, z \leftarrow z@p1\} from fc.A.2b

p.fc.A.2d: Prove fc.A.2d \{z \leftarrow z@p1\} from
  fc.A.2c, next_phase \{ph \leftarrow vote\}, distinct.phases, fc.A.2c \{i \leftarrow j\}
p.fc.A.3a: Prove fc.A.3a \( \{ z \leftarrow z@p1 \} \) from fc.A.2d, \( \mathcal{N}_{ds} \) \( \{ s \leftarrow z@p1, \ t \leftarrow t@p1 \} \), distinct_phases

p.fc.A.3b: Prove fc.A.3b \( \{ z \leftarrow z@p1 \} \) from fc.A.3a, \( \mathcal{N}_{ds} \) \( \{ s \leftarrow z, \ i \leftarrow i@C \} \)

p.fc.A.3c: Prove fc.A.3c \( \{ z \leftarrow z@p1 \} \) from fc.A.3b, next_phase \( \{ ph \leftarrow sync \} \), distinct_phases

\( \{ h \leftarrow MBmatrix.cons((\lambda i : z@p1.proc(i).mailbox), nrep) \} \) from fc.A.3c
\( \{ j \leftarrow j@C, \ i \leftarrow i@C, \ u \leftarrow u@C, \ t \leftarrow t@C, \ s \leftarrow s@C \} \), support.13 \( \{ MBfn \leftarrow (\lambda i : z@p1.proc(i).mailbox), i \leftarrow i \} \)

End

DS_map_proof: Module

Using DS.lemmas, nat.inductions

Exporting all with DS.lemmas

Theory

ds: Var DSstate
rs: Var RState
p,qq: Var nat
ph: Var phases
s,t,z,y,z: Var DSstate
i,j: Var processors
u: Var inputs
w: Var MBvec
h: Var MBmatrix
k,m,n,a,b: Var proc_plus
prop: Var function[proc_plus \rightarrow bool]

Proof

mll.prop: function[DSstate, processors \rightarrow function[proc_plus \rightarrow bool]] =
(\lambda ds, i : (\lambda k :
  ss_update(ds,k)(i).healthy
  = if i \leq k then ds.proc(i).healthy else rs0(i).healthy end if))

mll.base: Lemma mll.prop(s,i)(0)

mll.ind: Lemma k < nrep \land mll.prop(s,i)(k) \supset mll.prop(s,i)(k + 1)

p.mll.base: Prove mll.base from
mll.prop \( \{ ds \leftarrow s, \ i \leftarrow i, \ k \leftarrow 0 \} \),
ss.update \( \{ ds \leftarrow s, \ p \leftarrow 0 \} \)

p.mll.ind: Prove mll.ind from
mll.prop \( \{ ds \leftarrow s, \ i \leftarrow i, \ k \leftarrow k \} \),
mll.prop
\( \{ ds \leftarrow s, \ i \leftarrow i, \ k \leftarrow k \}
  ,
  k \leftarrow if k = nrep then nrep else k + 1 end if)
ss.update \( \{ ds \leftarrow s, \ p \leftarrow k + 1 \} \)
p_map_l: Prove map_l from
DSmap (ds ← s),
processors_induction {prop ← ml1_prop(s, i), n ← nrep},
ml1_prop {ds ← s, i ← i, k ← nrep},
ml1_base {s ← s, i ← i},
ml1_ind {s ← s, i ← i, k ← m@P2}

ml2_prop: function[DSstate, processors → function[proc_plus → hool]] =
(λ ds, i: (λ k :
ss_update(ds, k)(i).proc.state
  = if i ≤ k
  then ds.proc(i).proc.state
  else rs0(i).proc.state
  end if))

ml2_base: Lemma ml2_prop(s, i)(0)

ml2_ind: Lemma k < nrep ∧ ml2_prop(s, i)(k) ⊇ ml2_prop(s, i)(k + 1)

p_ml2_base: Prove ml2_base from
ml2_prop {ds ← s, i ← i, k ← 0},
ss_update {ds ← s, p ← 0}

p_ml2_ind: Prove ml2_ind from
ml2_prop {ds ← s, i ← i, k ← k},
ml2_prop
{ds ← s,
  i ← i,
  k ← if k = rep then nrep else k + 1 end if},
ss_update {ds ← s, p ← k + 1}

p_map_2: Prove map_2 from
DSmap (ds ← s),
processors_induction {prop ← ml2_prop(s, i), n ← nrep},
ml2_prop {ds ← s, i ← i, k ← nrep},
ml2_base {s ← s, i ← i},
ml2_ind {s ← s, i ← i, k ← m@P2}

p_map_3: Prove map_3 from
RS allowable_faults {s ← DSmap(s), t ← DSmap(t)},
RS allowable_faults {s ← s, t ← t, i ← p@p1},
map 7 {s ← t},
map 1 {s ← s, i ← p@p1},
map 1 {s ← t, i ← p@p1}

p_map_4: Prove map_4 from
RS good_values_sent {s ← DSmap(s), q ← j@P2},
DS_to_RS good_values_sent {j ← q@P1S},
map 1 {i ← j@p2},
map 2 {i ← j@p2},
map 1 {i ← q@P1},
map 2 {i ← q@P1}

p_map_5: Prove map_5 from
RS voted_final_state {s ← DSmap(s), t ← DSmap(t), p ← i},
DS_to_RS voted_final_state,
map 1 {i ← i},
map 1 {s ← t, i ← i},
map 2 {i ← i},
map 2 {s ← t, i ← i}
p_map.7: Prove map.7 from
proc.extensionality
{A \leftarrow \text{RS}(\text{working.set}DS\text{map}(s)),
B \leftarrow \text{DS}(\text{working.sets})},
\text{RS.maj.working} \{t \leftarrow \text{DS}\text{map}(s)\},
\text{RS.working.set} \{s \leftarrow \text{DS}\text{map}(s), p \leftarrow p@pl\},
\text{RS.working.proc} \{s \leftarrow \text{DS}\text{map}(s), p \leftarrow p@pl\},
\text{DS.maj.working} \{t \leftarrow s\},
\text{DS.working.set} \{s \leftarrow s, p \leftarrow p@pl\},
\text{DS.working.proc} \{s \leftarrow s, p \leftarrow p@pl\},
map.1 \{i \leftarrow p@pl\}

End

DS_support_proof: Module

Using DS.lemmas, nat.inductions

Exporting all with DS.lemmas

Theory

ds: \text{Var DSstate}
rds: \text{Var RState}
p, q: \text{Var nat}
ph: \text{Var phases}
s, t, x, y, z: \text{Var DSstate}
i, j: \text{Var processors}
u: \text{Var inputs}
w: \text{Var MBvec}
h: \text{Var MBmatrix}
MBfn: \text{Var MBcons.fn}
k, m, n, a, b: \text{Var proc.plus}
prop: \text{Var function[proc.plus \rightarrow bool]}

Proof

p_support.1: Prove support.1 \{i \leftarrow i@p2\} from
\text{DS.allowable_faults} \{s \leftarrow x, t \leftarrow y, i \leftarrow i@p2\},
\text{DS.allowable_faults} \{s \leftarrow s, t \leftarrow y\}

p_support.4: Prove support.4 from \mathcal{N}_{ds}

p_support.5: Prove support.5 from
\text{member.phases} \{\text{phases.var} \leftarrow \text{ph}\},
\mathcal{N}_{ds} \{s \leftarrow s, t \leftarrow x, u \leftarrow u, i \leftarrow i\}

p_support.6: Prove support.6 from support.1, support.5 \{i \leftarrow i@p1\}

p_support.7: Prove support.7 \{z \leftarrow z@p1\} from
\text{frame.N.ds},
\text{half.frame.N.ds} \{z \leftarrow z@p1, y \leftarrow y@p1, z \leftarrow z@p1\},
support.4 \{s \leftarrow s, t \leftarrow z@p1, u \leftarrow u\},
next_phase \{\text{ph} \leftarrow \text{compute}\}

p_support.8: Prove support.8 \{y \leftarrow y@p1\} from
\text{half.frame.N.ds},
\text{quarter.frame.N.ds} \{y \leftarrow y@p1, z \leftarrow z@p1\},
support.4 \{s \leftarrow s, t \leftarrow y@p1, u \leftarrow u\},
next_phase \{\text{ph} \leftarrow \text{broadcast}\},
distinct_phases

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p.support_9: Prove support_9 \{ z \leftarrow z@pl \} from 
   quarter.frame.N.ds,
   support.4 \{ s \leftarrow y, t \leftarrow z@p4, u \leftarrow u \},
   next.phase \{ ph \leftarrow vote \},
   distinct.phases

p.support.10: Prove support.10 from 
   DS.allowable_faults, N_ds \{ i \leftarrow i@p1 \}, N'_ds \{ i \leftarrow i@p1 \}

p.support.11: Prove support.11 from 
   support.6 \{ s \leftarrow s, x \leftarrow z@p4, y \leftarrow t, ph \leftarrow compute \},
   support.6 \{ s \leftarrow z@p4, x \leftarrow y@p5, y \leftarrow t, ph \leftarrow broadcast \},
   support.6 \{ s \leftarrow y@p5, x \leftarrow z@p6, y \leftarrow t, ph \leftarrow vote \},
   support.7,
   support.8 \{ x \leftarrow z@p4 \},
   support.9 \{ y \leftarrow y@p4 \},
   support.10 \{ s \leftarrow z@p6 \},
   distinct.phases

p.support.12: Prove support.12 \{ z \leftarrow z@p3 \} from 
   support.7, support.8 \{ x \leftarrow z@p1 \}, support.9 \{ y \leftarrow y@p2 \}

sll3.prop: function[MConc_fn, processors \leftarrow function[proc.plus \leftarrow bool]] =
   ( \lambda MBfn, i: \lambda k:
      MBmatrix.cons(MBfn, k)(i)
   = if i \leq k then MBfn(i) else MBmatrix0(i) end if)

sll3.base: Lemma sll3.prop(MBfn, i)(0)

sll3.ind: Lemma m < nrep \land sll3.prop(MBfn, i)(m) \supset
   sll3.prop(MBfn, i)(m + 1)

p.sll3.base: Prove sll3.base from 
   sll3.prop \{ k \leftarrow 0, i \leftarrow i \}, MBmatrix.cons \{ p \leftarrow 0 \}

p.sll3.ind: Prove sll3.ind from 
   sll3.prop \{ k \leftarrow m, i \leftarrow i \},
   sll3.prop \{ i \leftarrow i, k \leftarrow if m = nrep then nrep else m + 1 end if \},
   MBmatrix.cons \{ p \leftarrow m + 1 \}

p.support.13: Prove support.13 from 
   processors.induction \{ prop \leftarrow sll3.prop(MBfn, i), n \leftarrow nrep \},
   sll3.prop \{ k \leftarrow nrep, i \leftarrow i \},
   sll3.base \{ i \leftarrow i \},
   sll3.ind \{ i \leftarrow i, m \leftarrow m@p1 \}

p.support.14: Prove support.14 from 
   proc.extensionality \{ A \leftarrow working.set(s), B \leftarrow fullset[processors] \},
   initial.ds \{ i \leftarrow p@p1 \},
   DS.working.set \{ p \leftarrow p@p1 \},
   DS.working.proc \{ p \leftarrow p@p1 \}

p.support.15: Prove support.15 from 
   maj.condition \{ A \leftarrow working.set(s) \}, support.14, card.fullset

End

DS.to.RS.tcc: Module

Using DS.to.RS

Exporting all with DS.to.RS
Theory

ds: Var DS.DSstate
p: Var naturalnumber
MBfn: Var function[rcp_defs.processors → rcp_defs.MBvec]
ss_update_TCC1: Formula \(\neg\((p = 0) \lor (p > \text{arep})) \supset (p - 1 \geq 0)\)
ss_update_TCC2: Formula \(\neg((p = 0) \lor (p > \text{arep})) \supset ((p > 0) \land (p \leq \text{arep}))\)
ss_update_TCC3: Formula
\(\neg((p = 0) \lor (p > \text{arep})) \supset \text{ssu_measure}(ds, p) > \text{ssu_measure}(ds, p - 1)\)
MBmatrix_cons.TCC1: Formula
\(\neg((p = 0) \lor (p > \text{arep})) \supset \text{MBmc_measure}(MBfn, p) > \text{MBmc_measure}(MBfn, p - 1)\)

Proof
ss_update_TCC1_PROOF: Prove ss_update_TCC1
ss_update_TCC2_PROOF: Prove ss_update_TCC2
ss_update_TCC3_PROOF: Prove ss_update_TCC3
MBmatrix_cons.TCC1_PROOF: Prove MBmatrix_cons.TCC1

End DS_to_RS_tcc

DS_support_proof_tcc: Module
Using DS_support_proof
Exporting all with DS_support_proof

Theory

p: Var rcp_defs.processors
m: Var rcp_defs.proc_plus
x: Var DS.DSstate
y: Var DS.DSstate
z: Var DS.DSstate
i: Var rcp_defs.processors
p.s113_base.TCC1: Formula \((0 \geq 0) \land (0 \leq \text{arep})\)
p.s113_ind.TCC1: Formula
\((\text{if } m = \text{arep} \text{ then } \text{arep else } m + 1 \text{ end if } \geq 0)\)
\land (\text{if } m = \text{arep} \text{ then } \text{arep else } m + 1 \text{ end if } \leq \text{arep})\)
p.support_13.TCC1: Formula \((\text{arep} \geq 0) \land (\text{arep} \leq \text{arep})\)

Proof
p.s113_base.TCC1_PROOF: Prove p.s113_base.TCC1
p.s113_ind.TCC1_PROOF: Prove p.s113_ind.TCC1
p.support_13.TCC1_PROOF: Prove p.support_13.TCC1

End DS_support_proof_tcc

DS_map_proof_tcc: Module
Using DS_map_proof
Exporting all with DS_map_proof
Theory

\[ k: \text{Var rcpdefs.proc.plus} \]
\[ q: \text{Var rcpdefs.processors} \]
\[ j: \text{Var rcpdefs.processors} \]
\[ p: \text{Var rcpdefs.processors} \]
\[ m: \text{Var rcpdefs.proc.plus} \]

\[ p.mll\text{base.TCCl: Formula }((0 \geq 0) \land (0 \leq \text{nrep})) \]
\[ p.mll\text{ind.TCCl: Formula} \]
\[ ((\text{if } k = \text{nrep} \text{ then } \text{nrep} \text{ else } k + 1 \text{ end if}) \]
\[ \land (\text{if } k = \text{nrep} \text{ then } \text{nrep} \text{ else } k + 1 \text{ end if} \leq \text{nrep})) \]
\[ p.map.l.TCCl: Formula ((\text{nrep} \geq 0) \land (\text{nrep} \leq \text{nrep})) \]

Proof

\[ p.mll\text{base.TCCl.PROOF: Prove } p.mll\text{base.TCCl} \]
\[ p.mll\text{ind.TCCl.PROOF: Prove } p.mll\text{ind.TCCl} \]
\[ p.map.l.TCCl.PROOF: Prove p.map.l.TCCl \]

End DS.map.proof.tcc

DA: Module

Using clkmod, generic.FT

Exporting all with clkmod, generic.FT

Theory

max.comm.delay: realtime (* max broadcast delivery time *)
da.proc.state: Type = Record healthy : nat,
proc.state : Pstate,
mailbox : MBvec,
lclock : logical_clocktime,
cum.delta : number (* = corr; added to logical
to obtain physical *)
end record
da.proc.array: Type = array [processors] of da.proc.state

da.state: Type = Record phase : phases,
sync.period : nat, (* = idealized frame count *)
proc : da.proc.array
end record

s, t, x, y, z, da: Var DAstate
u: Var inputs
w: Var MBvec
i, j, p, q, qq: Var processors
k: Var nat
ph: Var phases
ps: Var da.proc.state
T: Var logical_clocktime
A: Var set[processors]

Corr.implementation: Lemma \( s.proc(p).cum.delta = Corr_p^{(s-sync.period)} \)
working_proc: function[DAstate, processors → bool] =
(λ s, p: s.proc(p).healthy ≥ recovery_period)
working_set: function[DAstate → set[processors]] =
(λ s: (λ p: working_proc(s, p)))
maj_working: function[DAstate → bool] =
(λ t: maj.condition(working_set(t)))

enough_hardware: function[DAstate → bool] =
(λ t: maj.working(t) ∧ enough_clocks(t.sync_period))

da_rt: function[DAstate, processors, logical_clocktime → realtime] =
(λ da, p, T: cp(T + da.proc(p).cum_delta))

unknown: fraction
ν: fraction = unknown (• variability of processor run rates •)
X, Y: Var logical_clocktime
D: Var number
clock_advanced: function[logical_clocktime, logical_clocktime, number → bool] =
(λ X, Y, D: X + D * (1 - ν) ≤ Y ∧ Y ≤ X + D * (1 + ν))
duration: function[phases → logical_clocktime]

broadcast_duration: Axiom
(1 - Rho) * [duration(broadcast) - 2 * ν * duration(compute) - ν * duration(broadcast)] - δ
≥ max_comm_delay

broadcast_duration2: Axiom
duration(broadcast) - 2 * ν * duration(compute) - ν * duration(broadcast) ≥ 0

all_durations: Axiom
(1 + ν) * duration(compute) + (1 + ν) * duration(broadcast) ≤ frame_time

pos_durations: Axiom
0 ≤ (1 - ν) * duration(compute)
∧ 0 ≤ (1 - ν) * duration(broadcast)
∧ 0 ≤ (1 - ν) * duration(vote) ∧ 0 ≤ (1 - ν) * duration(sync)

broadcast_received: function[DAstate, DAstate, processors → bool] =
(λ s, t, p:
∀ qq:
  s.proc(qq).healthy > 0 ∧
  da_rt(s, qq, s.proc(qq).lclock) + max_comm_delay
  ≤ da_rt(t, p, t.proc(p).lclock)
  ⊃ t.proc(p).mailbox(qq) = s.proc(qq).mailbox(qq))

A' de: function[DAstate, DAstate, inputs, processors → bool] =
(λ s, t, u, i:
  s.proc(i).healthy > 0
  ⊃ t.proc(i).proc_state = f_v(u, s.proc(i).proc_state)
  ∧ t.proc(i).mailbox(i) = f_v(f_v(u, s.proc(i).proc_state)))

A' de: function[DAstate, DAstate, processors → bool] =
(λ s, t, i: s.proc(i).healthy > 0
  ⊃ t.proc(i).proc_state = s.proc(i).proc_state
  ∧ broadcast_received(s, t, i))
\( N_{da}^{s}: \text{function}[DAstate, DAstate, processors \rightarrow bool] = \\
(\lambda s, t, i : s.proc(i).healthy > 0 \\
  \land t.proc(i).proc.state = s.proc(i).proc.state \\
  \land t.proc(i).healthy > 0 \\
  \land t.proc(i).healthy = 1 + s.proc(i).healthy \\
  \land \text{nonfaulty.clock}(i, t.sync.period)) \\
  \land t.sync.period = 1 + s.sync.period \\
  \land (\text{nonfaulty.clock}(i, s.sync.period)) \\
  \land t.proc(i).lclock = (1 + s.sync.period) \times \text{frame.time} \\
  \land t.proc(i).cum.delta = s.proc(i).cum.delta + \Delta(s.sync.period)) \\
\)

\( N_{da}^{s}: \text{function}[DAstate, DAstate, inputs \rightarrow bool] = \\
(\lambda s, t, u : \text{enough.hardware}(t) \\
  \land t.phase = \text{next.phase}(s.phase) \\
  \land (\forall i : \\
    \text{if } s.phase = \text{sync} \\
    \text{then } N_{da}^{s}(s, t, i) \\
    \text{else } t.proc(i).healthy = s.proc(i).healthy \\
    \land t.proc(i).cum.delta = s.proc(i).cum.delta \\
    \land t.sync.period = s.sync.period \\
    \land (\text{nonfaulty.clock}(i, s.sync.period)) \\
    \land \text{clock.advanced}(s.proc(i).lclock, \\
    t.proc(i).lclock, \\
    \text{duration}(s.phase))) \\
  \land (s.phase = \text{compute} \lor N_{da}^{s}(s, t, u, i)) \\
  \land (s.phase = \text{broadcast} \lor N_{da}^{s}(s, t, i)) \\
  \land (s.phase = \text{vote} \lor N_{da}^{s}(s, t, i)) \\
\end{if}) \\
\)

initial.da: function[DAstate \rightarrow bool] = \\
(\lambda s : s.phase = \text{compute} \\
  \land s.sync.period = 0 \\
  \land (\forall i : \\
    s.proc(i).healthy = \text{recovery.period} \\
    \land s.proc(i).proc.state = \text{initial.proc.state} \\
    \land s.proc(i).cum.delta = 0 \\
    \land s.proc(i).lclock = 0 \land \text{nonfaulty.clock}(i, 0)))

End

DA.to.DS: Module

Using DA, DS

Exporting all with DA, DS

Theory
\[\begin{align*}
d_a, s, t, x, y, z &: \text{Var DAstate} \\
ds &: \text{Var DSstate} \\
p, i, j &: \text{Var processors} \\
k, l &: \text{Var nat} \\
w &: \text{Var inputs} \\
w &: \text{Var MBvec} \\
h &: \text{Var MBmatrix} \\
ph &: \text{Var phases} \\
MBmatrix0 &: \text{MBmatrix} \\
MBcons_fn &: \text{Type is function[processors} \rightarrow \text{MBvec]} \\
MBfn &: \text{Var MBcons_fn} \\
T, T_1, T_2, BB &: \text{Var logical_clocktime} \\
DAstate\_prop &: \text{Type is function[DAstate} \rightarrow \text{bool]} \\
da\_prop &: \text{Var DAstate\_prop} \\
da\_measure &: \text{function[DAstate, nat} \rightarrow \text{nat]} = (\lambda da, k : k) \\
ss\_update &: \text{Recursive function[DAstate, nat} \rightarrow \text{DSstate]} = \\
& \quad (\lambda da, k : \text{if } (k = 0) \lor (k > \text{nrep}) \\
& \quad \quad \text{then } dso \\
& \quad \quad \text{else } ss\_update(da, k - 1) \\
& \quad \quad \text{with } [(\text{proc})(k) := dproc0 \\
& \quad \quad \quad \text{with } [(\text{healthy}) := da\_proc(k).\text{healthy}, \\
& \quad \quad \quad \quad (\text{proc\_state}) := da\_proc(k).\text{proc\_state}, \\
& \quad \quad \quad \quad (\text{mailbox}) := da\_proc(k).\text{mailbox}] \\
& \quad \text{end if}) \text{ by da\_measure} \\
\text{DAmap} &: \text{function[DAstate} \rightarrow \text{DSstate]} = \\
& \quad (\lambda da : \text{ss\_update(da, nrep) with } [(\text{phase}) := da\_phase]) \\
\text{MBmc\_measure} &: \text{function[MBcons\_fn, nat} \rightarrow \text{nat]} = (\lambda MBfn, k : k) \\
\text{MBmatrix\_cons} &: \text{Recursive function[MBcons\_fn, nat} \rightarrow \text{MBmatrix]} = \\
& \quad (\lambda MBfn, k : \text{if } (k = 0) \lor (k > \text{nrep}) \\
& \quad \quad \text{then } MBmatrix0 \\
& \quad \quad \text{else } MBmatrix\_cons(MBfn, k - 1) \\
& \quad \quad \text{with } [(k) := MBfn(k)] \\
& \quad \text{end if}) \text{ by MBmc\_measure} \\
\text{reachable\_in\_n} &: \text{function[DAstate, nat} \rightarrow \text{bool]} = \\
& \quad (\lambda t, k : \text{if } k = 0 \\
& \quad \quad \text{then } initial\_da(t) \\
& \quad \quad \text{else } (\exists s, u : \text{reachable\_in\_n}(s, k - 1) \land \mathcal{N}_{da}(s, t, u)) \\
& \quad \text{end if}) \text{ by da\_measure} \\
\text{reachable} &: \text{function[DAstate} \rightarrow \text{bool]} = (\lambda t : (\exists k : \text{reachable\_in\_n}(t, k))) \\
\text{phase\_commutes} &: \text{Theorem } \text{reachable}(s) \land \mathcal{N}_{da}(s, t, u) \supset \mathcal{N}_{ds}(\text{DAmap}(s), \text{DAmap}(t), u) \\
\text{initial\_maps} &: \text{Theorem } \text{initial\_da}(s) \supset \text{initial\_ds}(\text{DAmap}(s)) \\
\text{End} \\
\text{DA\_invariants}: \text{Module} \\
\text{Using } \text{DA\_to\_DS, nat\_inductions, DA\_lemmas} \\
\text{Exporting all with } \text{DA\_to\_DS} \\
\text{Theory}
\text{da, s, t, z, y, z: Var DAstate}
\text{ds: Var DSstate}
\text{p, i, j: Var processors}
\text{k, l: Var nat}
\text{u: Var inputs}
\text{w: Var MBvec}
\text{h: Var MBmatrix}
\text{ph: Var phases}
\text{cdv: Var number}
\text{ii: Var period}
\text{T, T_1, T_2, BB: Var logical_clocktime}
\text{DAstateProp: Type is function[DAstate \to \text{bool}]}
\text{daProp: Var DAstateProp}
\text{stateInvariant: function[DAstateProp \to \text{bool}] =}
\quad (\lambda \text{daProp} : (\forall t : \text{reachable}(t) \supset \text{daProp}(t)))
\text{stateInduction: Lemma}
\quad (\forall x : \text{initial.da}(x) \supset \text{daProp}(x))
\quad \land (\forall s, t, u : \text{reachable}(s) \land \text{daProp}(s) \land \mathcal{N}_{\text{da}}(s, t, u) \supset \text{daProp}(t))
\quad \supset \text{stateInvariant} (\text{daProp})
\text{enoughInv: Lemma stateInvariant}((\lambda s : \text{enoughHardware}(s)))
\text{nfClks: function[DAstate \to \text{bool}] =}
\quad (\lambda s : (\forall i : 
\quad \text{nonfaultyClock}(i, s.\text{sync_period}))
\quad \land \text{stateInvariant} (\text{nfClks}(s)))
\text{lnClockEq: function[DAstate \to \text{bool}] =}
\quad (\lambda s : (\forall i, j : 
\quad \text{nonfaultyClock}(i, s.\text{sync_period})
\quad \land \text{stateInvariant} (\text{lnClockEq}(s)))
\text{lnClockInv: Lemma stateInvariant}((\lambda s : \text{lnClockEq}(s)))
\text{lnClockVal: function[DAstate \rightarrow \text{bool}] =}
\quad (\lambda s : (\forall i : 
\quad \text{nonfaultyClock}(i, s.\text{sync_period}) \land \text{stateInvariant} (\text{lnClockVal}(s)))
\text{rtl1: Lemma reachable(da) \land \text{nonfaultyClock}(p, da.\text{sync_period})
\quad \supset \text{daProp}(p).\text{cumDelta} = \text{Corr}_{p}(da.\text{sync_period})
\text{daRtlem: Lemma reachable(da) \land \text{nonfaultyClock}(p, da.\text{sync_period})
\quad \supset \text{daRt(da, p, T) = rt}_{p}(da.\text{sync_period})(T)
\text{cumDeltaVal: function[DAstate \rightarrow \text{bool}] =}
\quad (\lambda s : (\forall p : 
\quad \text{nonfaultyClock}(p, s.\text{sync_period}) \land \text{stateInvariant} (\text{cumDeltaVal}(s)))
\text{corrlem: Lemma ii > 0 \supset \text{Corr}_{p}(i) = \text{Corr}_{p}(i-1) + \Delta_{p}(\text{pred}(i))
\text{cdl1: Lemma N_{da}(s, t, u) \land \text{proc}(p).\text{cumDelta} = \text{cdv}
\quad \supset \text{tProp}(p).\text{cumDelta} = \text{cdv} + \Delta_{p}(\text{pred}(i))
\text{cumDeltaInv: Lemma stateInvariant}((\lambda s : \text{cumDeltaVal}(s)))
Proof

state_invariant_to_n: function[DAstate_prop, nat → bool] =
(λ da_prop, k : (∀ t : reachable_in_n(t, k) ⊃ da_prop(t)))

base_state_ind: Lemma
(initial_da(z) ⊃ da_prop(z)) ⊃ (reachable_in_n(z, 0) ⊃ da_prop(z))

ind_state_ind: Lemma
(∀ s, t, u : reachable(s) ∧ da_prop(s) ∧ N_da(s, t, u) ⊃ da_prop(t))
⊃ (∀ k : state_invariant_to_n(da_prop, k)
⊃ state_invariant_to_n(da_prop, k + 1))

p_base_state_ind: Prove base_state.ind from
reachable_in_n {t ← z, k ← 0}

p_ind_state_ind: Prove
ind_state_ind {s ← s@p3, t ← t@p2, u ← u@p3} from
state_invariant_to_n {k ← k, t ← s@p3},
state_invariant_to_n {k ← k + 1, t ← t},
reachable_in_n {t ← t, k ← k + 1},
reachable {t ← s@p3, k ← k}

p_state_induction: Prove
state_induction
{x ← t@p3,
s ← s@p4,
t ← t@p4,
u ← u@p4} from
nat_induction
{p ← (λ k : state_invariant_to_n(da_prop, k)),
n2 ← k@p7},
base_state.ind {x ← t@p3},
state_invariant_to_n {t ← x, k ← 0},
ind_state.ind {k ← n1@p1},
state_invariant_to_n {t ← t@p6, k ← k@p7},
state_invariant,
reachable {t ← t@p6}

enough_inv.ll: Lemma initial_da(s) ⊃ enough_hardware(s)

enough_inv.12: Lemma N_da(s, t, u) ∧ enough_hardware(s) ⊃ enough_hardware(t)

p_enough_inv.ll: Prove enough_inv.ll from
enough_hardware {t ← s},
enough.clocks {i ← s.sync_period},
DA.maj_working {t ← s},
support.14,
support.15,
processors_exist_ax

p_enough_inv.12: Prove enough_inv.12 from N_da

p_enough_inv: Prove enough_inv from
state_induction {da_prop ← (λ s : enough_hardware(s))},
enough_inv.ll {s ← z@p1},
enough_inv.12 {s ← s@p1, t ← t@p1, u ← u@p1}

nfclk_inv.11: Lemma initial_da(s) ⊃ nfclk(s)

nfclk.inv.12: Lemma N_da(s, t, u) ∧ nfclk(s) ⊃ nfclk(t)
p_nfclk_inv_ll: Prove nfclk_inv_ll from nf_clks, initial_da
\{i \leftarrow i@p1\}

p_nfclk_inv_l2: Prove nfclk_inv_l2 from
\mathcal{N}_a \{i \leftarrow i@p3\}, nf_clks \{i \leftarrow i@p3\}, uf_clks \{s \leftarrow t\}, \mathcal{N}'_a \{i \leftarrow i@p3\}

p_nfclk_inv: Prove nfclk_inv from
state_induction \{da_prop \leftarrow (\lambda s : nf_clks(s))\},
nfclk_inv_l1 \{s \leftarrow s@p1\},
nfclk_inv_l2 \{s \leftarrow s@p1, t \leftarrow t@p1, u \leftarrow u@p1\}

clk_inv_l1: Lemma initial_da(s) \supset lclock_eq(s)

clk_inv_l2: Lemma \mathcal{N}_a(s, t, u) \land s.phase = sync \supset lclock_eq(t)

clk_inv_l2b: Lemma
\mathcal{N}_a(s, t, u) \supset (s.phase = sync \supset t.phase = compute)
\land (s.phase = compute \supset t.phase = broadcast)
\land (s.phase = broadcast \supset t.phase = vote)
\land (s.phase = vote \supset t.phase = sync)

p_lclock_inv_l2b: Prove lclock_inv_l2b from
\mathcal{N}_a,
distinct_phases,
ext_phase \{ph \leftarrow compute\},
ext_phase \{ph \leftarrow vote\},
ext_phase \{ph \leftarrow broadcast\},
ext_phase \{ph \leftarrow sync\}

lclock_inv_l2c: Lemma \mathcal{N}_a(s, t, u) \land s.phase \neq sync \supset t.phase \neq compute

p_lclock_inv_l2c: Prove lclock_inv_l2c from
lclock_inv_l2b,
distinct_phases,
member_phases \{phases.var \leftarrow t.phase\},
member_phases \{phases.var \leftarrow s.phase\}

lclock_inv_l3: Lemma
\mathcal{N}_a(s, t, u) \land s.phase \neq sync \supset t.sync_period = s.sync_period

lclock_inv_l4: Lemma
\mathcal{N}_a(s, t, u) \land s.phase \neq sync \land lclock_eq(s) \supset lclock_eq(t)

p_lclock_inv_l1: Prove lclock_inv_l1 from
lclock_eq, initial_da \{i \leftarrow i@p1\}, initial_da \{i \leftarrow j@p1\}

p_lclock_inv_l2: Prove lclock_inv_l2 from
lclock_eq \{s \leftarrow t\},
\mathcal{N}_a \{i \leftarrow i@p1\},
\mathcal{N}_a \{i \leftarrow j@p1\},
\mathcal{N}'_a \{i \leftarrow i@p1\},
\mathcal{N}'_a \{i \leftarrow j@p1\},
afc_lem \{p \leftarrow i@p1, i \leftarrow s.sync_period\},
afc_lem \{p \leftarrow j@p1, i \leftarrow s.sync_period\}

p_lclock_inv_l3: Prove lclock_inv_l3 from \mathcal{N}_a \{i \leftarrow i\}, lclock_inv_l2b

p_lclock_inv_l4: Prove lclock_inv_l4 from
lclock_eq \{s \leftarrow t\},
lclock_eq \{i \leftarrow i@p1, j \leftarrow j@p1\},
lclock_inv_l2c,
distinct_phases,
lclock_inv_l3

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p clk inv: Prove clk inv from state induction \{da prop \rightarrow (\lambda s: lclock eq(s))\},
  lclock inv.1 \{s = s @ p1\},
  lclock inv.2 \{s = s @ p1, t = t @ p1, u = u @ p1\},
  lclock inv.4 \{s = s @ p1, t = t @ p1, u = u @ p1\}

clkval inv.1: Lemma initial da(s) \supset lclock val(s)

clkval inv.2: Lemma reachable(s) \land N_{da}(s, t, u) \supset lclock val(t)

p clkval inv.1: Prove clkval inv.1
  from lclock val, initial da \{i = i @ p1\}

p clkval inv.2: Prove clkval inv.2 from
  lclock val \{s = t\},
  N_{da} \{t = i @ p1\},
  N'_{da} \{t = i @ p1\},
  support 16 \{ph = s phase\},
  prev phase \{ph = t phase\},
  nfc lem \{p = i @ p1, i = s sync period\}

p clkval inv: Prove clkval inv from state induction \{da prop \rightarrow (\lambda s: lclock val(s))\},
  clkval inv.1 \{s = s @ p1\},
  clkval inv.2 \{s = s @ p1, t = t @ p1, u = u @ p1\}

p rtll: Prove rtll from
  cum delta inv,
  state invariant \{da prop \rightarrow (\lambda s: cum delta val(s)), t = da\},
  cum delta val \{s = da\}

d a rt lem: Prove da rt lem from
  da rt \{p = p, t = t@ (s+3) \{i = da sync period, p = p\}, rt l1

cum delta inv.1: Lemma initial da(s) \supset cum delta val(s)

p cum delta inv.1: Prove cum delta inv.1 from
  initial da \{i = p@ p2\}, cum delta val, Corr^{(s2)}_{s1} \{p = p@ p2, i = 0\}

cum delta inv.2: Lemma
  \{N_{da}(s, t, u) \land s phase = sync \land cum delta val(s) \supset cum delta val(t)\}

pt, ps: Var period

cdi 1a: Lemma pt = ps + 1 \supset Corr^{(p t)}_{p} = Corr^{(p s)}_{p} + \Delta^{(p s)}_{p}

p cdi 1a: Prove cdi 1a from Corr^{(s2)}_{s1} \{i = p t, p = p\}

p cum delta inv.2: Prove cum delta inv.2 from
  cum delta val \{s = s, p = p @ p2\},
  cum delta val \{s = t\},
  N_{da} \{i = p @ p2\},
  N'_{da} \{i = p @ p2\},
  cdi 1a \{p = p @ p2, pt = t sync period, ps = s sync period\},
  nfc lem \{p = p @ p2, i = s sync period\}

cum delta inv.4: Lemma
  \{N_{da}(s, t, u) \land s phase \neq s sync \land cum delta val(s) \supset cum delta val(t)\}
p.cum.delta.inv.14: Prove cum.delta.inv.14 from
  cum.delta.val \{s \rightarrow t\},
  cum.delta.val \{p \rightarrow p@p1\},
  \(N_{ds}\) \{i \rightarrow p@p1\},
  distinct_phases

p.cum.delta.inv: Prove cum.delta.inv from
  state.induction \{da.prop \leftarrow (\lambda s: cum.delta.val(s))\},
  cum.delta.inv.11 \{s \rightarrow z@p1\},
  cum.delta.inv.12 \{s \rightarrow s@p1, t \rightarrow t@p1, u \rightarrow u@p1\},
  cum.delta.inv.14 \{s \rightarrow s@p1, t \rightarrow t@p1, u \rightarrow u@p1\}
End

DA.lemmas: Module
Using DA.to_DS, clkprop
Exporting all with DA.to_DS, clkprop

Theory

da: Var DSstate

Theory

ds: Var DSstate
da: Var DAstate
k: Var nat
ph: Var phases
s, t, x, y, z: Var DAstate
ss, tt: Var DSstate
p, q, i, j: Var processors
u: Var inputs
w: Var MBvec
h: Var MBmatrix
MBfn: Var MBcons_fn
m, n, a, b: Var proc_plus
prop: Var function[proc_plus \rightarrow bool]
T, T_1, T_2, BB, bb: Var logical_clocktime
da_prop: Var DAstate.prop

phase.com.compute: Lemma
s.phase = compute \land N_{ds}(s, t, u) \supset N_{ds}(DAmap(s), DAmap(t), u)

hide1: function[DAstate, DAstate, inputs \rightarrow bool] =
( \lambda s, t, u: (enough_hardware(t))
  \land t.phase = next_phase(s.phase)
  \land t.sync_period = s.sync_period
  \land (\forall i:
    t.proc(i).healthy = s.proc(i).healthy
    \land t.proc(i).cum_delta = s.proc(i).cum_delta
    \land t.sync_period = s.sync_period
    \land (nonfaulty_clock(i, s.sync_period)
      \supset clock.advanced(s.proc(i).lclock, t.proc(i).lclock, duration(s.phase)))
  )
  \land N_{ds}(s, t, u, i))

phase.com.lx1: Lemma s.phase = compute \land N_{ds}(s, t, u) \supset hide1(s, t, u)
phase.com.ix2: Lemma
\[ s \text{.phase} = \text{compute} \]
\[ \land (\text{maj\_working}(D\text{Amap}(t)) \]
\[ \land (\forall i:\]
\[ \quad \text{D\text{Amap}(t).phase} = \text{next\_phase}(\text{D\text{Amap}(s).phase}) \]
\[ \land \text{D\text{Amap}(t).proc(i).healthy} = \text{D\text{Amap}(s).proc(i).healthy} \]
\[ \land \mathcal{N}_{da}^s(\text{D\text{Amap}(s)}, \text{D\text{Amap}(t)}, u, i)) \]
\[ \supset \mathcal{N}_{da}(\text{D\text{Amap}(s)}, \text{D\text{Amap}(t)}, u) \]

phase.com.ix4: Lemma
\[ s \text{.phase} = \text{compute} \]
\[ \land (\text{maj\_working}(D\text{Amap}(t)) \]
\[ \land (\forall i:\]
\[ \quad t \text{.phase} = \text{next\_phase}(s \text{.phase}) \]
\[ \land t \text{.proc(i).healthy} = s \text{.proc(i).healthy} \land \mathcal{N}_{da}^s(s, t, u, i)) \]
\[ \supset \mathcal{N}_{da}(D\text{Amap}(s), D\text{Amap}(t), u) \]

phase.com.ix7: Lemma
\[ s \text{.phase} = \text{compute} \land \mathcal{N}_{da}(s, t, u) \]
\[ \supset (\text{maj\_working}(D\text{Amap}(t)) \]
\[ \land (\forall i:\]
\[ \quad t \text{.phase} = \text{next\_phase}(s \text{.phase}) \]
\[ \land t \text{.proc(i).healthy} = s \text{.proc(i).healthy} \land \mathcal{N}_{da}^s(s, t, u, i)) \]
\[ \supset \mathcal{N}_{da}(D\text{Amap}(s), D\text{Amap}(t), u) \]

phase.com.broadcast: Lemma
\[ \text{reachable}(s) \land s \text{.phase} = \text{broadcast} \land \mathcal{N}_{da}(s, t, u) \supset \mathcal{N}_{da}(D\text{Amap}(s), D\text{Amap}(t), u) \]

com.broadcast.1: Lemma
\[ s \text{.phase} = \text{broadcast} \land \mathcal{N}_{da}(s, t, u) \supset (\forall i:\mathcal{N}_{da}^s(s, t, u)) \]

com.broadcast.2: Lemma
\[ s \text{.phase} = \text{broadcast} \]
\[ \land \text{reachable}(s) \]
\[ \land s \text{.proc(i).healthy} = t \text{.proc(i).healthy} \]
\[ \land \mathcal{N}_{da}(s, t, u) \land \mathcal{N}_{da}^s(s, t, i) \]
\[ \supset \mathcal{N}_{da}^s(D\text{Amap}(s), D\text{Amap}(t), i) \]

com.broadcast.3: Lemma
\[ s \text{.phase} = \text{broadcast} \]
\[ \land tt \text{.phase} = \text{next\_phase}(ss \text{.phase}) \]
\[ \land (\forall i:\mathcal{N}_{da}^s(ss, tt, i) \land tt \text{.proc(i).healthy} = ss \text{.proc(i).healthy}) \]
\[ \land DS \text{.maj\_working}(tt) \]
\[ \supset \mathcal{N}_{da}(ss, tt, u) \]

com.broadcast.4: Lemma
\[ s \text{.phase} = \text{broadcast} \land \mathcal{N}_{da}(s, t, u) \]
\[ \supset D\text{Amap}(t).phase = \text{next\_phase}(D\text{Amap}(s).phase) \]
\[ \land D\text{Amap}(t).proc(i).healthy = D\text{Amap}(s).proc(i).healthy \]
\[ \land DS \text{.maj\_working}(D\text{Amap}(t)) \]

com.broadcast.5: Lemma
\[ \text{reachable}(s) \land \mathcal{N}_{da}(s, t, u) \]
\[ \land s \text{.phase} = \text{broadcast} \]
\[ \land s \text{.proc(i).healthy} > 0 \land \text{broadcast\_received}(s, t, i) \]
\[ \supset \text{broadcast\_received}(D\text{Amap}(s), D\text{Amap}(t), i) \]

phase.com.vote: Lemma
\[ s \text{.phase} = \text{vote} \land \mathcal{N}_{da}(s, t, u) \supset \mathcal{N}_{da}(D\text{Amap}(s), D\text{Amap}(t), u) \]

com.vote.1: Lemma
\[ s \text{.phase} = \text{vote} \land \mathcal{N}_{da}(s, t, u) \supset (\forall i:\mathcal{N}_{da}^s(s, t, i)) \]
com.vote.2: Lemma $N^*_d(s, t, i) \supset N^*_d(DAmap(s), DAmap(t), i)$

com.vote.3: Lemma $ss.phase = \text{vote} \land tt.phase = \text{next_phase(ss.phase)}$
\[ \land (\forall i : N^*_d(ss, tt, i) \land tt.proc(i).healthy = ss.proc(i).healthy) \land DS.maj_working(tt) \]
\[ \supset N_d(ss, tt, u) \]

com.vote.4: Lemma $ss.phase = \text{vote} \land N_d(s, t, u)$
\[ \supset DAmap(t).phase = \text{next_phase(DAmap(s).phase)} \land DAmap(t).proc(i).healthy = DAmap(s).proc(i).healthy \land DS.maj_working(DAmap(t)) \]

phase.com.sync: Lemma $s.phase = \text{sync} \land N_d(s, t, u) \supset N_d(DAmap(s), DAmap(t), u)$

com.sync.1: Lemma $s.phase = \text{sync} \land N_d(s, t, u) \supset (\forall i : N^*_d(s, t, i))$

com.sync.2: Lemma $N^*_d(s, t, i) \supset N^*_d(DAmap(s), DAmap(t), i)$

com.sync.3: Lemma $ss.phase = \text{sync} \land tt.phase = \text{next_phase(ss.phase)}$
\[ \land (\forall i : N^*_d(ss, tt, i) \land DS.maj_working(tt)) \]
\[ \supset N_d(ss, tt, u) \]

com.sync.4: Lemma $s.phase = \text{sync} \land N_d(s, t, u)$
\[ \supset DAmap(t).phase = \text{next_phase(DAmap(s).phase)} \land DS.maj_working(DAmap(t)) \]

earliest.later_time: Lemma $T_2 = T_1 + BB \land (T_1 \geq T^o)$
\[ \land (BB \geq T^o) \land \text{nonfaulty_clock}(i, da.sync.period) \land \text{nonfaulty_clock}(j, da.sync.period) \land \text{enough_clocks}(da.sync.period) \land T_2 \in R(da.sync.period) \land T_1 \in R(da.sync.period) \]
\[ \supset r_i^{(da.sync.period)}(T_2) \geq r_j^{(da.sync.period)}(T_1) + (1 - Rho) \ast |BB| - \delta \]

ELT: Lemma $T_2 \geq T_1 + bb$
\[ \land (T_1 \geq T^o) \land (bb \geq T^o) \land \text{nonfaulty_clock}(i, da.sync.period) \land \text{nonfaulty_clock}(j, da.sync.period) \land \text{enough_clocks}(da.sync.period) \land T_2 \in R(da.sync.period) \land T_1 \in R(da.sync.period) \]
\[ \supset r_i^{(da.sync.period)}(T_2) \geq r_j^{(da.sync.period)}(T_1) + (1 - Rho) \ast |bb| - \delta \]

eLT.a: Lemma $(bb \geq T^o) \land BB \geq bb \supset (1 - Rho) \ast |BB| \geq (1 - Rho) \ast |bb|$

map.1: Lemma $DAmap(s).proc(i).healthy = s.proc(i).healthy$

map.2: Lemma $DAmap(s).proc(i).proc_state = s.proc(i).proc_state$

map.3: Lemma $DAmap(s).phase = s.phase$

map.4: Lemma $DAmap(s).proc(i).mailbox = s.proc(i).mailbox$

map.7: Lemma $DS(.maj_working(DAmap(s)) = DA(.maj.workings)$
support.1: Lemma $\text{initial}_\text{da}(s) \supset \text{working}_\text{set}(s) = \text{fullset}[\text{processors}]

support.4: Lemma $s.\text{phase} = ph \land \text{N}da(s, x, u) \supset x.\text{phase} = \text{next}_\text{phase}(ph)

support.5: Lemma $s.\text{phase} = ph \land ph \neq \text{sync} \land \text{N}da(s, x, u)
\supset (\forall i: s.\text{proc}(i).\text{healthy} = x.\text{proc}(i).\text{healthy})$

support.13: Lemma $\text{MBmatrix}\_\text{cons}(\text{MBfn}, \text{nrep})(i) = \text{MBfn}(i)$

support.14: Lemma $\text{initial}_\text{da}(s) \supset \text{maj}_\text{condition}(|\text{working}_\text{set}(s)|)$

support.15: Lemma $\text{initial}_\text{da}(s) \supset \text{num}_\text{good}_\text{clocks}(s.\text{sync}_\text{period}, \text{nrep}) = \text{nrep}$

support.16: Lemma $\text{prev}_\text{phase}(\text{next}_\text{phase}(ph)) = ph$

End

DA_top_proof: Module

Using $\text{DA}\_\text{lemmas}, \text{DA}\_\text{invariants}$

Exporting all with $\text{DA}\_\text{lemmas}$

Theory

d$s$: Var DSstate
da: Var DAstate
k: Var nat
ph: Var phases
s, t, x, y, z: Var DAstate
ss, tt: Var DSstate
p, q, i, j: Var processors
u: Var inputs
w: Var MBvec
h: Var MBmatrix
$MBfn$: Var MBcons_fn
m, n, a, b: Var proc_plus
prop: Var function[proc_plus \to bool]
T, X, Y: Var logical_clocktime

Proof

p_phase_commutes: Prove $\text{phase}_\text{commutes}$ from
\text{phase}_\text{compute},
\text{phase}_\text{broadcast},
\text{phase}_\text{vote},
\text{phase}_\text{sync},
\text{member}\_\text{phases} \{\text{phases}_\text{var} \leftarrow s.\text{phase}\}$

p_initial_maps: Prove $\text{initial}_\text{maps}$ from
\text{initial}_\text{da} \{i \leftarrow \text{i@p2}\},
\text{initial}_\text{ds} \{s \leftarrow \text{DAmap}(s)\},
\text{map}_1 \{i \leftarrow \text{i@p2}\},
\text{map}_2 \{i \leftarrow \text{i@p2}\},
\text{map}_3

p_phase_com_compute: Prove $\text{phase}_\text{com}_\text{compute}$ from
\text{phase}_\text{com}_\text{x4}, \text{phase}_\text{com}_\text{x7} \{i \leftarrow \text{i@p1}\}

p_phase_com_x1: Prove $\text{phase}_\text{com}_\text{x1}$ from
$\text{N}da \{i \leftarrow \text{i@p3}\}, \text{distinct}\_\text{phases}, \text{hide}1$
p.phase_com_lx2: Prove phase_com_lx2 \{ i \mapsto i @ p1 \} from 
\mathcal{N}_{da} \{ s \mapsto DMap(s), t \mapsto DMap(t) \}, distinct_phases, map_3

p.phase_com_lx4: Prove phase_com_lx4 \{ i \mapsto i @ p1 \} from 
phase_com_lx2, 
\mathcal{N}'_{da} \{ s \mapsto DMap(s), t \mapsto DMap(t) \},
\mathcal{N}'_{da} \{ s \mapsto b \& c, t \mapsto b \& c \},
map_1,
map_2,
map_3,
map_4,
map_1 \{ s \mapsto t \},
map_2 \{ s \mapsto t \},
map_3 \{ s \mapsto t \},
map_4 \{ s \mapsto t \}

p.phase_com_lx7: Prove phase_com_lx7 from 
phase_com_lx1, map_7 \{ s \mapsto t \}, hide1, enough_hardware

p.phase_com_broadcast: Prove phase_com_broadcast from 
com_broadcast_1 \{ i \mapsto i @ p3 \},
com_broadcast_2 \{ i \mapsto i @ p3 \},
com_broadcast_3 \{ ss \mapsto DMap(s), tt \mapsto DMap(t) \},
com_broadcast_4 \{ i \mapsto i @ p3 \},
map_1 \{ s \mapsto s, i \mapsto i @ p2 \},
map_1 \{ s \mapsto t, i \mapsto i @ p2 \},
map_3 \{}

p.com_broadcast_1: Prove com_broadcast_1 from 
\mathcal{N}_{da}, next_phase \{ ph \mapsto broadcast \}, distinct_phases

p.com_broadcast_2: Prove com_broadcast_2 from 
com_broadcast_5,
\mathcal{N}'_{da},
\mathcal{N}'_{da} \{ s \mapsto DMap(s), t \mapsto DMap(t) \},
map_1 \{ s \mapsto s \},
map_1 \{ s \mapsto t \},
map_2 \{ s \mapsto s \},
map_2 \{ s \mapsto t \}

p.com_broadcast_3: Prove com_broadcast_3 \{ i \mapsto i @ p1 \} from 
\mathcal{N}_{da} \{ s \mapsto ss, t \mapsto tt \}, distinct_phases

p.com_broadcast_4: Prove com_broadcast_4 from 
\mathcal{N}_{da},
map_1 \{ s \mapsto s \},
map_1 \{ s \mapsto t \},
map_3 \{ s \mapsto s \},
map_3 \{ s \mapsto t \},
map_7 \{ s \mapsto t \},
distinct_phases,
enough_hardware

p.earliest_later_time: Prove earliest_later_time from 
GOAL \{ p \mapsto i, q \mapsto j, i \mapsto da.sync_period \}
p.elt.a: Prove elt.a from
| * 1 | \{ x \leftarrow b \},
| * 1 | \{ x \leftarrow BB \},
* 1 \times 2 \{ y \leftarrow (1 - \text{Rho}), x \leftarrow b \},
* 1 \times 2 \{ y \leftarrow (1 - \text{Rho}), x \leftarrow BB \},
mult.leq \{ x \leftarrow (1 - \text{Rho}), x \leftarrow BB, y \leftarrow b \}

p.ELT: Prove ELT from
earliest\_later\_time \{ BB \leftarrow T_2 - T_1, i \leftarrow p, j \leftarrow q@C \},
etl.a \{ BB \leftarrow T_2 - T_1 \},
* 1 \times 2 \{ x \leftarrow (1 - \text{Rho}), y \leftarrow b \}

p.phase.com.vote: Prove phase\_com\_vote from
com.vote.1 \{ i \leftarrow i@p3 \},
com.vote.2 \{ i \leftarrow i@p3 \},
com.vote.3 \{ ss \leftarrow DAmap(s), tt \leftarrow DAmap(t) \},
com.vote.4 \{ i \leftarrow i@p3 \},
map.3 \{

p.com.vote.1: Prove com.vote.1 from \mathcal{N}_{da}, \text{distinct}\_phases

p.com.vote.2: Prove com.vote.2 from
\mathcal{N}_{da}^{*} \{ s \leftarrow DAmap(s), t \leftarrow DAmap(t) \},
\mathcal{N}_{da}^{*},
map.1 \{ s \leftarrow s \},
map.1 \{ s \leftarrow t \},
map.2 \{ s \leftarrow s \},
map.2 \{ s \leftarrow t \},
map.4 \{ s \leftarrow s \},
map.4 \{ s \leftarrow t \}

p.com.vote.3: Prove com.vote.3 \{ i \leftarrow i@p1 \} from
\mathcal{N}_{da} \{ s \leftarrow ss, t \leftarrow tt \}, \text{distinct}\_phases

p.com.vote.4: Prove com.vote.4 from
\mathcal{N}_{da},
\text{enough}\_hardware,
map.1 \{ s \leftarrow s \},
map.1 \{ s \leftarrow t \},
map.3 \{ s \leftarrow s \},
map.3 \{ s \leftarrow t \},
map.7 \{ s \leftarrow t \},
distinct\_phases

p.phase.com.sync: Prove phase\_com\_sync from
com.sync.1 \{ i \leftarrow i@p3 \},
com.sync.2 \{ i \leftarrow i@p3 \},
com.sync.3 \{ ss \leftarrow DAmap(s), tt \leftarrow DAmap(t) \},
com.sync.4 \{
map.3 \{

p.com.sync.1: Prove com.sync.1 from \mathcal{N}_{da}

p.com.sync.2: Prove com.sync.2 from
\mathcal{N}_{da}^{*} \{ s \leftarrow DAmap(s), t \leftarrow DAmap(t) \},
\mathcal{N}_{da}^{*},
map.1 \{ s \leftarrow s \},
map.1 \{ s \leftarrow t \},
map.2 \{ s \leftarrow s \},
map.2 \{ s \leftarrow t \}
p.com_sync_3: Prove com_sync_3 \{i \leftarrow i@p1\} from \mathcal{N}_s \{s \leftarrow ss, t \leftarrow tt\}

p.com_sync_4: Prove com_sync_4 from 
\mathcal{N}_s, enough_hardware, map_3 \{s \leftarrow s\}, map_3 \{s \leftarrow t\}, map_7 \{s \leftarrow t\}

End

DA_map.proof: Module

Using DA_lemmas, nat_inductions

Exporting all with DA_lemmas

Theory

ds: Var DSstate

da: Var DState

k, q: Var nat

ph: Var phases

s, t, x, y, z: Var DAState

p, i, j: Var processors

u: Var inputs

w: Var MBvec

h: Var MBmatrix

MBfn: Var MBcons_fn

m, n, a, b: Var proc_plus

prop: Var function[proc_plus \rightarrow bool]

Proof

mil_prop: function[DAState, processors \rightarrow function[proc_plus \rightarrow bool]] =
(\lambda da, i : (\lambda a : ss.update(da, a).proc(i).healthy 
   = \text{if } i \leq a 
   \text{then } da.proc(i).healthy 
   \text{else } dso.proc(i).healthy 
   \text{end if})

mil_base: Lemma \text{mil_prop}(s, i)(0)

mil_ind: Lemma \text{a < nrep} \land \text{mil_prop(s, i)(a)} \supset \text{mil_prop(s, i)(a + 1)}

p.mil_base: Prove \text{mil_base} from

mil_prop \{da \leftarrow s, i \leftarrow i, a \leftarrow 0\},

ss.update \{da \leftarrow s, k \leftarrow 0\}

p.mil_ind: Prove \text{mil_ind} from

mil_prop \{da \leftarrow s, i \leftarrow i, a \leftarrow a\},

mil_prop
 \{da \leftarrow s, 
   i \leftarrow i, 
   a \leftarrow \text{if } a = \text{nrep} \text{ then } \text{nrep} \text{ else } a + 1 \text{ end if}, 

ss.update \{da \leftarrow s, k \leftarrow a + 1\}

p.map.1: Prove map_1 from

DAmap \{da \leftarrow s\},

processors.induction \{prop \leftarrow mil_prop(s, i), n \leftarrow \text{nrep}\},

mil_prop \{da \leftarrow s, i \leftarrow i, a \leftarrow \text{nrep}\},

mil_base \{s \leftarrow s, i \leftarrow i\},

mil_ind \{s \leftarrow s, i \leftarrow i, a \leftarrow m@P2\}

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ml2_prop: function[D|Astate, processors → function[proc_plus → bool]] =
( λ da, i : ( λ a :
  ss_update(da, a).proc(i).proc_state
  = if i ≤ a
  then da.proc(i).proc_state
  else ds0.proc(i).proc_state
  end if))

ml2.base: Lemma ml2_prop(s, i)(0)

ml2.ind: Lemma a < nrep ∧ ml2.prop(s, i)(a) ⊃ ml2.prop(s, i)(a + 1)

p_ml2_base: Prove ml2.base from
ml2_prop {da ← s, i ← i, a ← 0},
ss_update {da ← s, k ← 0}

p_ml2.ind: Prove ml2.ind from
ml2_prop {da ← s, i ← i, a ← a},
ml2_prop
{da ← s,
  i ← i,
  a ← if a = nrep then nrep else a + 1 end if},
ss_update {da ← s, k ← a + 1}

p_map_2: Prove map_2 from
DAmap {da ← s},
processors_induction {prop ← ml2_prop(s, i), n ← nrep},
ml2_prop {da ← s, i ← i, a ← nrep},
ml2_base {s ← s, i ← i},
ml2.ind {s ← s, i ← i, a ← m@P2}

p_map_3: Prove map_3 from DAmap {da ← s}

ml4_prop: function[D|Astate, processors → function[proc_plus → bool]] =
( λ da, i : ( λ a :
  ss_update(da, a).proc(i).mailbox
  = if i ≤ a
  then da.proc(i).mailbox
  else ds0.proc(i).mailbox
  end if))

ml4.base: Lemma ml4_prop(s, i)(0)

ml4.ind: Lemma a < nrep ∧ ml4.prop(s, i)(a) ⊃ ml4.prop(s, i)(a + 1)

p_ml4_base: Prove ml4.base from
ml4_prop {da ← s, i ← i, a ← 0},
ss_update {da ← s, k ← 0}

p_ml4.ind: Prove ml4.ind from
ml4_prop {da ← s, i ← i, a ← a},
ml4_prop
{da ← s,
  i ← i,
  a ← if a = nrep then nrep else a + 1 end if},
ss_update {da ← s, k ← a + 1}
p_map_4: Prove map_4 from
DAmap \{da \rightarrow s\},
processors_induction \{prop \leftarrow ml4_prop(s, i), n \leftarrow nrep\},
ml4_prop \{da \rightarrow s, i \leftarrow i, a \leftarrow nrep\},
ml4_base \{s \leftarrow s, i \leftarrow i\},
ml4_ind \{s \leftarrow s, i \leftarrow i, a \leftarrow m\oplus P2\}

p_map_7: Prove map_7 from
proc_extensionality
\{A \leftarrow DS(.working_setDAmap(s)),
B \leftarrow DA(.working_sets)\},
DA_working \{t \leftarrow DAmap(s)\},
DA.working_set \{s \rightarrow DAmap(s), p \leftarrow popl\},
DA.working_proc \{s \rightarrow DAmap(s), p \leftarrow popl\},
DA.maj_working \{t \rightarrow s\},
DA.working_set \{s \rightarrow s, p \leftarrow popl\},
DA.working_proc \{s \rightarrow s, p \leftarrow popl\},
map_1 \{i \leftarrow popl\}

End
DA_support_proof: Module
Using DA_lemmas, nat_inductions, DA_invariants
Exporting all with DA_lemmas

Theory

ds: Var DSstate
da: Var DAstate
k, q: Var nat
ph: Var phases
s, t, x, y, z: Var DAstate
p, i, j: Var processors
u: Var inputs
w: Var MBvec
h: Var MBmatrix
MBfn: Var MBcons_fn
m, n, a, b: Var proc_plus
prop: Var function[proc_plus \rightarrow \text{bool}]

Proof

p_support_1: Prove support_1 from
proc_extensionality \{A \leftarrow \text{working_set}(s), B \leftarrow \text{fullset}[processors]\},
initial_da \{i \leftarrow popl\},
DA.working_set \{p \leftarrow popl\},
DA.working_proc \{p \leftarrow popl\}

p_support_4: Prove support_4 from \(N_{da} \{s \leftarrow s, t \leftarrow z\}\)

p_support_5: Prove support_5 from
member_phases \{phases_var \leftarrow ph\},
\(N_{da} \{s \leftarrow s, t \leftarrow x, u \leftarrow u, i \leftarrow i\}\)

sl13.prop: function[MBcons_fn, processors \rightarrow function[proc_plus \rightarrow bool]] =
( \lambda MBfn, i: (\lambda a:
MBmatrix.cons(MBfn, a)(i)
= \text{if } i \leq a \text{ then } MBfn(i) \text{ else } MBmatrix0(i) \text{ end if})

sl13.base: Lemma sl13.prop(MBfn, i)(0)
Lemma \( a < \text{nrep} \land \text{sll3_prop}(MBfn, i)(a) \supset \text{sll3_prop}(MBfn, i)(a + 1) \)

Proof:

- **sll3_base**: Prove \( \text{sll3_base} \) from
  \( \text{sll3_prop}(s \rightarrow 0, i \rightarrow i), MBmatrix.cons \{ k \rightarrow 0 \} \)

- **sll3_ind**: Prove \( \text{sll3_ind} \) from
  \( \text{sll3_prop}(s \rightarrow a, i \rightarrow i), \text{sll3_prop}(i \rightarrow i, a \rightarrow \text{if } a = \text{nrep} \text{ then } \text{nrep} \text{else } a + 1 \text{ end if}), MBmatrix.cons \{ k \rightarrow a + 1 \} \)

- **support_13**: Prove \( \text{support_13} \) from
  \( \text{processors_induction} \{ \text{prop} \rightarrow \text{sll3_prop}(MBfn, i), n \rightarrow \text{nrep} \}, \text{sll3_prop}(s \rightarrow \text{nrep}, i \rightarrow i), \text{sll3_base}(i \rightarrow i), \text{sll3_prop}(i \rightarrow i, a \rightarrow \text{if } a = \text{nrep} \text{ then } \text{nrep} \text{else } a + 1 \text{ end if}), MBmatrix.cons \{ k \rightarrow a + 1 \} \)

- **support_14**: Prove \( \text{support_14} \) from
  \( \text{maj_condition} \{ A \rightarrow \text{working_set}(s) \}, \text{support_1}, \text{card_fullset} \)

- **sll15_prop**: \( \text{function}[\text{DAstate} \rightarrow \text{function}[\text{nat} \rightarrow \text{bool}]] = (\lambda s : (\lambda q : \text{initial_da}(s)
  \supset \text{num_good_clocks}(s.sync_period, q)
  = \text{if } q \leq \text{nrep} \text{ then } q \text{ else } 0 \text{ end if}) \)

- **sll15_base**: Lemma \( \text{sll15_prop}(s)(0) \)

- **sll15_ind**: Lemma \( \text{sll15_prop}(s)(q) \supset \text{sll15_prop}(s)(q + 1) \)

Proof:

- **sll15_base**: Prove \( \text{sll15_base} \) from
  \( \text{sll15_prop}(s \rightarrow s, q \rightarrow 0), \text{num_good_clocks}(i \rightarrow s.sync_period, k \rightarrow 0) \)

- **sll15_ind**: Prove \( \text{sll15_ind} \) from
  \( \text{sll15_prop}(s \rightarrow s, q \rightarrow q), \text{sll15_prop}(s \rightarrow s, q \rightarrow q + 1), \text{num_good_clocks}(i \rightarrow s.sync_period, k \rightarrow q + 1), \text{initial_da}(s \rightarrow s, i \rightarrow \text{if } q < \text{nrep} \text{ then } q + 1 \text{ else } \text{nrep} \text{ end if}) \)

- **support_15**: Prove \( \text{support_15} \) from
  \( \text{nat_induction} \{ p \rightarrow \text{sll15_prop}(s), n2 \rightarrow \text{nrep} \}, \text{sll15_prop}(s \rightarrow s, q \rightarrow \text{nrep}), \text{sll15_base}(s \rightarrow s), \text{sll15_prop}(s \rightarrow s, q \rightarrow n1 @ p1) \)

- **support_16**: Prove \( \text{support_16} \) from
  \( \text{next_phase}, \text{prev_phase}(ph \rightarrow \text{next_phase}(ph)), \text{distinct_phases}, \text{member_phases}(\text{phases_var} \rightarrow ph) \)

End

DA.broadcast.prf: Module

Using \text{DA.lemmas, DA.invariants}

Exporting all with \text{DA.lemmas}

Theory

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ds: Var DSstate
da: Var DAstate
k: Var nat
ph: Var phases
r, s, t, z: Var DAstate
ss, tt: Var DSstate
p, q, pp, qq: Var processors
u, u1, u2: Var inputs
w: Var MBvec
h: Var MBmatrix
MBfu: Var MBcons_fn
m, n, a, b: Var proc_plus
prop: Var function[proc_plus -> bool]
T, X, Y, T1, T2, BB: Var logical.clocktime
bb, xx, yy, zz: Var clocktime
Tp, Sq, Rp, EpSi: Var clocktime

Lemma r.phase = compute
∧ reachable(r)
∧ Nda(r, s, u1) ∧ Nda(s, t, u2) ∧ nonfaulty_clock(q, r.sync_period)
∧ r.proc(q).lclock ∈ R⊂r-sync-period)
∧ s.proc(q).lclock ∈ R⊂s-sync-period)
∧ t.proc(q).lclock ∈ R⊂t-sync-period)

Lemma r.phase = compute ∧ Nda(r, s, u1) ∧ Nda(s, t, u2)
∧ (s.phase = broadcast
∧ t.phase = vote
∧ s.sync_period = r.sync_period
∧ t.sync_period = s.sync_period
∧ (∀ pp :
nonfaulty_clock(pp, r.sync_period)
∨ clock_advanced(r.proc(pp).lclock,
s.proc(pp).lclock,
duration(compute))
∧ clock_advanced(s.proc(pp).lclock,
t.proc(pp).lclock,
duration(broadcast))))

Lemma r.phase = compute ∧ Nda(r, s, u1)
∧ (s.phase = broadcast
∧ s.sync_period = r.sync_period
∧ (∀ pp :
nonfaulty_clock(pp, r.sync_period)
∨ clock_advanced(r.proc(pp).lclock,
s.proc(pp).lclock,
duration(compute)))))
Lemma \( r.\text{phase} = \text{compute} \wedge \mathcal{N}_{\text{da}}(r, s, u_1) \wedge \mathcal{N}_{\text{da}}(s, t, u_2) \)
\( \supset s.\text{phase} = \text{broadcast} \)
\( \wedge t.\text{phase} = \text{vote} \)
\( \wedge s.\text{sync\_period} = r.\text{sync\_period} \)
\( \wedge t.\text{sync\_period} = s.\text{sync\_period} \)
\( \wedge (\text{nonfaulty\_clock}(p, r.\text{sync\_period})) \)
\( \supset (\exists \text{pdurc} : \)
\( \text{near}(\text{pdurc}, \text{compute}) \)
\( \wedge s.\text{proc}(p).\text{lclock} = r.\text{proc}(p).\text{lclock} + \text{pdurc}) \)
\( \wedge (\exists \text{pdurb} : \)
\( \text{near}(\text{pdurb}, \text{broadcast}) \)
\( \wedge t.\text{proc}(p).\text{lclock} = s.\text{proc}(p).\text{lclock} + \text{pdurb}) \)

Lemma \( r.\text{phase} = \text{compute} \wedge \text{reachable}(r) \wedge \mathcal{N}_{\text{da}}(r, s, u_1) \wedge \mathcal{N}_{\text{da}}(s, t, u_2) \)
\( \supset s.\text{phase} = \text{broadcast} \)
\( \wedge t.\text{phase} = \text{vote} \)
\( \wedge s.\text{sync\_period} = r.\text{sync\_period} \)
\( \wedge t.\text{sync\_period} = s.\text{sync\_period} \)
\( \wedge (\text{nonfaulty\_clock}(p, r.\text{sync\_period})) \)
\( \supset r.\text{proc}(p).\text{lclock} = r.\text{proc}(q).\text{lclock} \)
\( \wedge (\exists \text{pdurc} : \)
\( \text{near}(\text{pdurc}, \text{compute}) \)
\( \wedge s.\text{proc}(p).\text{lclock} = r.\text{proc}(q).\text{lclock} + \text{pdurc}) \)
\( \wedge (\exists \text{pdurb} : \)
\( \text{near}(\text{pdurb}, \text{broadcast}) \)
\( \wedge t.\text{proc}(p).\text{lclock} = s.\text{proc}(q).\text{lclock} + \text{pdurb}) \)
\( \wedge (\exists \text{qdurc} : \)
\( \text{near}(\text{qdurc}, \text{compute}) \)
\( \wedge s.\text{proc}(q).\text{lclock} = r.\text{proc}(q).\text{lclock} + \text{qdurc}) \)
\( \wedge (\exists \text{qdurb} : \)
\( \text{near}(\text{qdurb}, \text{broadcast}) \)
\( \wedge t.\text{proc}(q).\text{lclock} = s.\text{proc}(q).\text{lclock} + \text{qdurb}) \)

Lemma \( r.\text{phase} = \text{compute} \wedge \text{reachable}(r) \wedge \text{nonfaulty\_clock}(p, r.\text{sync\_period}) \wedge \text{nonfaulty\_clock}(q, r.\text{sync\_period}) \)
\( \supset r.\text{proc}(p).\text{lclock} = r.\text{proc}(q).\text{lclock} \)

Proof

p.brl: Prove brl from

br1a,
\( \mathcal{N}_{\text{da}} \{ s \leftarrow s, t \leftarrow t, u \leftarrow u_2, i \leftarrow pp \}, \)
\( \text{next\_phase} \{ \text{ph} \leftarrow \text{broadcast} \}, \)
\( \text{distinct\_phases} \)

p.brla: Prove brla from

\( \mathcal{N}_{\text{da}} \{ s \leftarrow r, t \leftarrow s, u \leftarrow u_1, i \leftarrow pp \}, \)
\( \text{next\_phase} \{ \text{ph} \leftarrow \text{compute} \}, \)
\( \text{distinct\_phases} \)
p.br2: Prove br2
{pdurc ← s.proc(p).lclock − r.proc(p).lclock,
pdurb ← t.proc(p).lclock − s.proc(p).lclock} from
br1 {pp ← p},
clock.advanced
{X ← r.proc(p).lclock,
Y ← s.proc(p).lclock,
D ← duration(compute)},
clock.advanced
{X ← s.proc(p).lclock,
Y ← t.proc(p).lclock,
D ← duration(broadcast)}

p.br3.aa: Prove br3.aa from
state.invariant {t ← r, da_prop ← (λ s : lclock_eq(s))},
lclock.inv,
lclock.eq {s ← r, i ← p, j ← q}

p.br3: Prove br3
{pdurc ← pdurc@p1,
pdurb ← pdurc@p1,
qdurc ← pdurc@p2,
qdurb ← pdurc@p2} from br2, br2 {p ← q}, br3.aa

br4: Lemma r.phase = compute ∧ reachable(r) ∧ N_da(r, s, w1) ∧ N_da(s, t, w2)
⇒ s.phase = broadcast
∧ t.phase = vote
∧ s.sync_period = r.sync_period
∧ t.sync_period = s.sync_period
∧ (nonfaulty_clock(p, r.sync_period)
∧ nonfaulty_clock(q, r.sync_period)
∧ Rq = r.proc(q).lclock
∧ Rp = r.proc(p).lclock
∧ Sq = s.proc(q).lclock ∧ Tp = t.proc(p).lclock
⇒ (∃ pdurc, pdurb, qdurc, qdurb:
near(pdurc, compute)
∧ near(pdurc, broadcast)
∧ near(qdurc, compute)
∧ near(qdurc, broadcast)
∧ Rp = Rq
∧ Sq = Rq + pdurc
∧ Tp = Sq − pdurc + pdurc + pdurb)}

p.br4: Prove br4
{pdurc ← pdurc@p1,
pdurb ← pdurc@p1,
qdurc ← pdurc@p1,
qdurb ← pdurc@p1} from br3
br5: Lemma \( r \text{.phase} = \text{compute} \land \text{reachable}(r) \land \mathcal{N}_d(r, s, u_1) \land \mathcal{N}_d(s, t, u_2) \)
\(\supset s \text{.phase} = \text{broadcast} \)
\land t \text{.phase} = \text{vote}
\land s \text{.sync\_period} = r \text{.sync\_period}
\land t \text{.sync\_period} = s \text{.sync\_period}
\land (\text{nonfaulty\_clock}(p, r \text{.sync\_period})
\land \text{nonfaulty\_clock}(q, r \text{.sync\_period})
\supset s \text{.proc}(q) \text{.lclock} \in R^{r \text{.sync\_period}})
\land t \text{.proc}(p) \text{.lclock} \in R^{t \text{.sync\_period}}
\land s \text{.proc}(q) \text{.lclock} \geq s \text{.proc}(q) \text{.lclock} + \text{duration(broadcast)}
- 2 \ast \nu \ast \text{duration(compute)}
- \nu \ast \text{duration(broadcast)})
\(p \text{.br5: Prove br5 from br4}\)

br4
\{ Rq \leftarrow r \text{.proc}(q) \text{.lclock},
Rp \leftarrow r \text{.proc}(p) \text{.lclock},
Sq \leftarrow s \text{.proc}(q) \text{.lclock},
Tp \leftarrow t \text{.proc}(p) \text{.lclock} \},
nint\{ p \leftarrow q \}

br6: Lemma \( \exists r: r \text{.phase} = \text{compute} \)
\land \text{reachable}(r) \land \mathcal{N}_d(r, s, u_1) \land \text{sync\_period} = r \text{.sync\_period})
\land \mathcal{N}_d(s, t, u_2)
\supset s \text{.phase} = \text{broadcast}
\land t \text{.phase} = \text{vote}
\land t \text{.sync\_period} = s \text{.sync\_period}
\land (\text{nonfaulty\_clock}(p, s \text{.sync\_period})
\land \text{nonfaulty\_clock}(q, s \text{.sync\_period})
\supset s \text{.proc}(q) \text{.lclock} \in R^{t \text{.sync\_period}})
\land t \text{.proc}(p) \text{.lclock} \in R^{t \text{.sync\_period}}
\land s \text{.proc}(q) \text{.lclock} \geq s \text{.proc}(q) \text{.lclock} + \text{duration(broadcast)}
- 2 \ast \nu \ast \text{duration(compute)}
- \nu \ast \text{duration(broadcast)})
\(p \text{.br6: Prove br6 from br5}\)

br7: Lemma \( \mathcal{N}_d(x, s, u) \)
\(\supset x \text{.phase} = \text{prev\_phase}(s \text{.phase}) \land (x \text{.phase} \neq \text{sync} \supset x \text{.sync\_period} = s \text{.sync\_period})\)
\(p \text{.br7: Prove br7 from}\)
support\_16 \{ ph \leftarrow x \text{.phase} \},
\mathcal{N}_d \{ s \leftarrow x, t \leftarrow s, u \leftarrow u \},
distinct\_phases

br8: Lemma \( \text{reachable}(s) \land s \text{.phase} = \text{broadcast} \)
\(\supset ( \exists z, u: \mathcal{N}_d(x, s, u) \land \text{reachable}(z) \land x \text{.phase} = \text{compute} \land x \text{.sync\_period} = s \text{.sync\_period})\)
p.br8: Prove br8 \{x → s@p2, u → w@p2\} from
reachable \{t → s\},
reachable.in.n \{t → s, k → k@p1\},
reachable \{t → s@p2, k → if k@p1 = 0 then 0 else k@p1 − 1 end if\},
initial.da \{s → s\},
br7 \{x → s@p2, s → s, u → u@p2\},
prev.phase \{ph → s.phase\},
distinct_phases

br9: Lemma reachable(s) \land N_{da}(s, t, u2) \land s.phase = broadcast
\lor t.sync.period = s.sync.period
\land (nonfaulty_clock(p, s.sync.period) \land nonfaulty_clock(q, s.sync.period)
\lor s.proc(q).lclock \in R^{s.sync.period})
\land t.proc(p).lclock \in R^{t.sync.period})
\land t.proc(p).lclock ≥ s.proc(q).lclock + duration(broadcast)
− 2 \ast \nu \ast duration(compute)
− \nu \ast duration(broadcast))

p.br9: Prove br9 from br8 \{x → x@p2, u1 → u@p2\}, br8

rtp0: Lemma Sq \in R^{s.sync.period} \supset Sq \geq 0

rtp0a: Lemma T \geq 0 \lor frame.time \ast k + T \geq 0

p.rtp0a: Prove rtp0a from
mult.non_neg \{x → frame.time, y → k\},
\ast 1 \ast \ast 2 \{x → frame.time, y → k\}

p.rtp0: Prove rtp0 from
\ast 1 \in R^{s^2} \{T → Sq, i → s.sync.period, II → 0\},
T^{s^1} \{i → s.sync.period\},
rt0a \{T → \Pi@p1, k → s.sync.period\}

rtp1: Lemma reachable(s) \land N_{da}(s, t, u2) \land s.phase = broadcast
\lor t.sync.period = s.sync.period
\land (nonfaulty_clock(p, t.sync.period)
\land nonfaulty_clock(q, s.sync.period)
\land enough_clocks(s.sync.period)
\land T_p = t.proc(p).lclock
\land Sq = s.proc(q).lclock
\land E_{psi} = 2 \ast \nu \ast duration(compute) + \nu \ast duration(broadcast)
\land duration(broadcast) − E_{psi} \geq 0
\lor rt_1^{s.sync.period}(T_p)
\geq rt_1^{s.sync.period}(Sq)
− (1 − \rho) \ast [duration(broadcast) − E_{psi}]
− \delta\}

p.rtp1: Prove rtp1 from
rtp0,
br9,
ELT
\{da → s,
T_3 → T_p,
T_1 → Sq,
q → q,
bb → duration(broadcast) − E_{psi}\}

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rtp2: Lemma reachable(s)
\[ N_{da}(s, t, u_2) \]
\[ \land s.\text{phase} = \text{broadcast} \]
\[ \land \text{nonfaulty\_clock}(p, s.\text{sync\_period}) \]
\[ \land \text{nonfaulty\_clock}(q, s.\text{sync\_period}) \]
\[ \land \text{enough\_clocks}(s.\text{sync\_period}) \]
\[ \land T_p = t.\text{proc}(p).\text{lclock} \]
\[ \land S_q = s.\text{proc}(q).\text{lclock} \]
\[ \land \text{Epsi} = 2 * \nu * \text{duration(compute)} + \nu * \text{duration(broadcast)} \]
\[ \land \text{duration(broadcast)} - \text{Epsi} \geq 0 \]
\[ \implies rt_{t \text{\_sync\_period}}(T_p) \]
\[ \geq rt_{t \text{\_sync\_period}}(S_q) + (1 - \text{Rho}) * [\text{duration(broadcast)} - \text{Epsi}] - \delta \]

p.rtp2: Prove rtp2 from rtp1

rtp3: Lemma reachable(s)
\[ N_{da}(s, t, u_2) \]
\[ \land s.\text{phase} = \text{broadcast} \]
\[ \land \text{nonfaulty\_clock}(p, s.\text{sync\_period}) \]
\[ \land \text{nonfaulty\_clock}(q, s.\text{sync\_period}) \]
\[ \land \text{enough\_clocks}(s.\text{sync\_period}) \]
\[ \land t.\text{sync\_period} = s.\text{sync\_period} \]
\[ \implies rt_{t \text{\_sync\_period}}(t.\text{proc}(p).\text{lclock}) \]
\[ \geq rt_{t \text{\_sync\_period}}(s.\text{proc}(q).\text{lclock}) + \text{max\_comm\_delay} \]

p.rtp3: Prove rtp3 from rtp2
\{ Epsi = 2 * \nu * \text{duration(compute)} + \nu * \text{duration(broadcast)}, \}
\{ S_q = s.\text{proc}(q).\text{lclock}, \}
\{ T_p = t.\text{proc}(p).\text{lclock} \},
broadcast\_duration,
broadcast\_duration2

rtp4: Lemma reachable(s)
\[ N_{da}(s, t, u_2) \]
\[ \land s.\text{phase} = \text{broadcast} \]
\[ \land \text{nonfaulty\_clock}(p, s.\text{sync\_period}) \]
\[ \land \text{nonfaulty\_clock}(q, s.\text{sync\_period}) \]
\[ \land \text{enough\_clocks}(s.\text{sync\_period}) \]
\[ \implies da._rt(t, p, t.\text{proc}(p).\text{lclock}) \]
\[ \geq da._rt(s, q, s.\text{proc}(q).\text{lclock}) + \text{max\_comm\_delay} \]

rtp4a: Lemma reachable(s) \[ N_{da}(s, t, u_2) \]
\[ \land s.\text{phase} = \text{broadcast} \]
\[ \implies t.\text{sync\_period} = s.\text{sync\_period} \]

p.rtp4a: Prove rtp4a from \[ N_{da} \{ s \leftarrow s, t \leftarrow t, u \leftarrow u_2 \}, \text{distinct\_phases} \]

rtp4b: Lemma reachable(s) \[ N_{da}(s, t, u) \]
\[ \land s.\text{phase} = \text{broadcast} \]
\[ \implies \text{reachable}(t) \]

p.rtp4b: Prove rtp4b from reachable \[ k \leftarrow k@p3, \]
reachable \[ t \leftarrow s, \]
reachable_in_n \[ k \leftarrow k@p2 + 1, s \leftarrow s, u \leftarrow u \]
p.rtp4: Prove rtp4 from
rtp3,
rtp4b \{ u \leftarrow u_2 \},
rtp4a,
da rt.lem \{ da \leftarrow t, p \leftarrow p, T \leftarrow t.proc(p).lclock \},
da rt.lem \{ da \leftarrow s, p \leftarrow q, T \leftarrow s.proc(q).lclock \}

rtp5: Lemma reachable(s)

\land \mathcal{N}_{da}(s, t, u)
\land s.\text{phase} = \text{broadcast}
\land s.proc(p).\text{healthy} > 0
\land \text{broadcast}\_\text{received}(s, t, p)
\land (\forall q:
\land s.proc(q).\text{healthy} > 0
\Rightarrow da rt(s, q, s.proc(q).lclock) + \text{max}\_\text{comm}\_\text{delay}
\leq da rt(t, p, t.proc(p).lclock))
\cup \text{broadcast}\_\text{received}(\text{DA}\_\text{map}(s), \text{DA}\_\text{map}(t), p)

p.rtp5: Prove rtp5 \{ q \leftarrow \text{qq@p2} \} from
\text{distinct}\_\text{phases},
\text{DS}\_\text{broadcast}\_\text{received} \{ s \leftarrow \text{DA}\_\text{map}(s), t \leftarrow \text{DA}\_\text{map}(t), qq \leftarrow q \},
\text{DA}\_\text{broadcast}\_\text{received} \{ qq \leftarrow \text{qq@p2} \},
\text{map}\_1 \{ s \leftarrow s, i \leftarrow \text{qq@p2} \},
\text{map}\_4 \{ s \leftarrow s, i \leftarrow \text{qq@p2} \},
\text{map}\_4 \{ s \leftarrow t, i \leftarrow p \},
\mathcal{N}_{da}

rtp6: Lemma reachable(s) \land s.proc(p).\text{healthy} > 0
\cup \text{nonfaulty}\_\text{clock}(p, s.\text{sync}\_\text{period})

p.rtp6: Prove rtp6 from
\text{nfc}\_\text{clk}\_\text{inv},
\text{state}\_\text{invariant} \{ t \leftarrow s, \text{da}\_\text{prop} \leftarrow (\lambda s : \text{nf}\_\text{clks}(s)) \},
\text{nf}\_\text{clks} \{ i \leftarrow p \}

rtp7: Lemma reachable(s) \land \mathcal{N}_{da}(s, t, u) \land s.\text{phase} = \text{broadcast}
\cup \text{enough}\_\text{circuits}(s, s.\text{sync}\_\text{period}) \land t.\text{phase} = \text{vote}

p.rtp7: Prove rtp7 from
\mathcal{N}_{da},
\text{state}\_\text{invariant} \{ \text{da}\_\text{prop} \leftarrow (\lambda s : \text{enough}\_\text{hardware}(s)), t \leftarrow s \},
enough.inv,
enough.\text{hardware} \{ t \leftarrow s \},
\text{next}\_\text{phase} \{ ph \leftarrow s.\text{phase} \},
distinct.\text{phases}

p.\text{com}\_\text{broadcast}\_5: Prove \text{com}\_\text{broadcast}\_5 from
rtp4 \{ u_2 \leftarrow u, q \leftarrow \text{qq@p2}, p \leftarrow i \},
rtp5 \{ p \leftarrow i \},
rtp6 \{ p \leftarrow i \},
rtp6 \{ p \leftarrow \text{qq@p2} \},
rtp7

End

DA.intervals: Module
Using DA.broadcast.prf
Exporting all with DA.lemmas
Theory
ds: Var DSstate
da: Var DAstate
k: Var nat
ph: Var phases
r, s, t, x: Var DAstate
ss, tt: Var DSstate
p, q, pp, qq: Var processors
u, u1, u2: Var inputs
w: Var MBvec
h: Var MBmatrix
MBfn: Var MBcons_fn
m, n, a, b: Var proc_plus
prop: Var function[proc_plus : bool]
T, Y, Y1, T1, T2, BB: Var logical_clocktime
bb, bb, yy, zz, x2, y2: Var clocktime
Tp, Sq, Eps: Var logical_clocktime
p_br: Var clocktime
qdurc: Var clocktime
qturb: Var clocktime
qdurb: Var clocktime
dur: Var clocktime

Proof

br_int: Lemma r.phase = compute \& reachable(r) \& N_{da}(r, s, u_1) \& N_{da}(s, t, u_2)
   \& s.phase = broadcast
   \& t.phase = vote
   \& s.sync_period = r.sync_period
   \& t.sync_period = s.sync_period
   \& nonfaulty_clock(q, r.sync_period)
   \& (\exists qdurc, qdurc : near(qdurc, compute)
     \& near(qdurc, broadcast)
     \& s.proc(q).l_clock = r.proc(q).l_clock + qdurc
     \& t.proc(q).l_clock = s.proc(q).l_clock + qdurc))

p_br_int: Prove br_int {qdurc ← qdurc@p1, qdurc ← qdurc@p1} from
   br3 (p ← q)

int0: Lemma r.phase = compute
   \& reachable(r)
   \& N_{da}(r, s, u_1) \& N_{da}(s, t, u_2) \& nonfaulty_clock(q, r.sync_period)
   \& r.proc(q).l_clock = r.sync_period + frame.time
   \& r.proc(q).l_clock \in R^{+ sync.period})

p_int0: Prove int0 from
   clkval_inv,
   state_invariant {da_prop ← (λ r : l_clock.val(r)), t ← r},
   l_clock.val {t ← q, s ← r},
   \&1 \in R^{(α2)} \{T ← r.proc(q).l_clock, i ← r.sync_period, II ← 0},
   T^{(*1)} \{i ← r.sync_period}
int1: Lemma \( r.\text{phase} = \text{compute} \land \text{reachable}(r) \land N_{da}(r, s, u_1) \land N_{da}(s, t, u_2) \)
\( \supset s.\text{phase} = \text{broadcast} \)
\( \land t.\text{phase} = \text{vote} \)
\( \land s.\text{sync-period} = r.\text{sync-period} \)
\( \land t.\text{sync-period} = s.\text{sync-period} \)
\( \land (\text{nonfaulty.clock}(q, r.\text{sync-period})) \)
\( \supset r.\text{proc}(q).l\text{clock} = r.\text{sync-period} \ast \text{frame.time} \)
\( \land r.\text{proc}(q).l\text{clock} \in R^{r.\text{sync-period}} \)
\( \land s.\text{proc}(q).l\text{clock} \in R^{s.\text{sync-period}} \)

int1a: Lemma \( xx \leq yy \land yy \leq zz \supset xx \leq zz \)

p.int1: Prove int1a

p.int1: Prove int1 from

int0,
br.int,
\(+ 1 \in R'_{(\ast)}\)
\{ \( T \leftarrow s.\text{proc}(q).l\text{clock}, \)
\( i \leftarrow s.\text{sync-period}, \)
\( \Pi \leftarrow qdurl_{p2}\}, \)
\( T'_{(\ast)}\{ i \leftarrow s.\text{sync-period}\}, \)
all_durations,
int1a
\{ \( xx \leftarrow qdurl_{p2}, \)
\( yy \leftarrow (1 - \nu) \ast \text{duration(compute)}, \)
\( zz \leftarrow 0\}, \)
int1a
\{ \( xx \leftarrow qdurl_{p2}, \)
\( yy \leftarrow (1 + \nu) \ast \text{duration(compute)}, \)
\( zz \leftarrow \text{frame.time}\) \)

int2: Lemma \( r.\text{phase} = \text{compute} \land \text{reachable}(r) \land N_{da}(r, s, u_1) \land N_{da}(s, t, u_2) \)
\( \supset s.\text{phase} = \text{broadcast} \)
\( \land t.\text{phase} = \text{vote} \)
\( \land s.\text{sync-period} = r.\text{sync-period} \)
\( \land t.\text{sync-period} = s.\text{sync-period} \)
\( \land (\text{nonfaulty.clock}(q, r.\text{sync-period})) \)
\( \supset r.\text{proc}(q).l\text{clock} = r.\text{sync-period} \ast \text{frame.time} \)
\( \land r.\text{proc}(q).l\text{clock} \in R^{r.\text{sync-period}} \)

int2a: Lemma \( \text{near(qdurl, compute)} \land \text{near(qdurl, broadcast)} \)
\( \supset 0 \leq \text{qdurl + qdurb} \land \text{qdurl + qdurb} \leq \text{frame.time} \)

p.int2a: Prove int2a from

pos_durations,
all_durations,
\(+ 1 \times \ast 2\{ x \leftarrow (1 - \nu), y \leftarrow \text{duration(compute)}\}, \)
\(+ 1 \times \ast 2\{ x \leftarrow (1 - \nu), y \leftarrow \text{duration(broadcast)}\} \)

p.int2: Prove int2 from

\( T'_{(\ast)}\{ i \leftarrow t.\text{sync-period}\}, \)
br.int,
\(+ 1 \in R'_{(\ast)}\)
\{ \( T \leftarrow t.\text{proc}(q).l\text{clock}, \)
\( i \leftarrow t.\text{sync-period}, \)
\( \Pi \leftarrow qdurl_{p2} + qdurb_{p2}\}, \)
int2a \( \{ qdurl \leftarrow qdurl_{p2}, qdurb \leftarrow qdurb_{p2}\} \)

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int3: Lemma \( r.\text{phase} = \text{compute} \land \text{reachable}(r) \land N_{da}(r, s, u_1) \land N_{da}(s, t, u_2) \)
\( \lor \) (nonfaulty_\text{clock}(q, r.\text{sync.\ period})
\( \lor \) \( r.\text{proc}(q)\text{.lclock} = r.\text{sync.\ period} \times \text{frame\_time} \land r.\text{proc}(q)\text{.lclock} \in R^{r.\text{sync.\ period}} \)
\( \land \) \( s.\text{proc}(q)\text{.lclock} \in R^{s.\text{sync.\ period}} \)
\( \land t.\text{proc}(q)\text{.lclock} \in R^{t.\text{sync.\ period}} \))

p.int3: Prove int3 from int1, int2

int4: Lemma \( (r.\text{phase} = \text{compute} \land \text{reachable}(r) \land N_{da}(r, s, u_1) \land N_{da}(s, t, u_2) \land \text{nonfaulty\_\text{clock}}(q, r.\text{sync.\ period}) \)
\( \lor \) \( (r.\text{proc}(q)\text{.lclock} = r.\text{sync.\ period} \times \text{frame\_time} \land r.\text{proc}(q)\text{.lclock} \in R^{r.\text{sync.\ period}} \)
\( \land \) \( s.\text{proc}(q)\text{.lclock} \in R^{s.\text{sync.\ period}} \)
\( \land t.\text{proc}(q)\text{.lclock} \in R^{t.\text{sync.\ period}} \))

p.int4: Prove int4 from int3

p.int5: Prove int5 from int4

End
clk.types: Module
Exporting all

Theory

realtime: Type is number
logical_clocktime: Type is number
physical_clocktime: Type is number
clocktime: Type is number
z: Var number
posnum: Type from number with \( (\lambda x : x > 0) \)
pos_logical_clocktime: Type is posnum
posrealtime: Type is posnum
fraction: Type from number with \( (\lambda x : 1 \geq x \land x \geq 0 \land x \neq 1) \)
period: Type is nat

End
clkmod: Module
Using rcp.defs, absmod, clk.types

Exporting all with rcp.defs, clk.types, absmod Theory

\( e, \delta, \delta: \text{posrealtime} \)
\( \Sigma, \Delta: \text{pos\_logical\_clocktime} \)
frame_time, sync_time: pos\_logical\_clocktime (* Changed from R, S *)
i: Var period
k: Var nat
\( T^0: \text{logical\_clocktime} = 0 \)
\( T^{(i+1)}: \text{function[period} \rightarrow \text{logical\_clocktime}] = (\lambda i: T^0 + i \times \text{frame\_time}) \)
T.next: Lemma \( T^{(i+1)} = T^{(i)} + \text{frame\_time} \)
T, \Pi: \textit{Var} logical\_clocktime
T_1, T_2, T_0, T_N: \textit{Var} physical\_clocktime
s_1 \in R^{(2)}: \text{function}[logical\_clocktime, period \rightarrow boolean] =
( \lambda T, i: (3 \Pi : 0 \leq \Pi \land \Pi \leq \text{frame\_time} \land T = T(i) + \Pi))
s_1 \in S^{(2)}: \text{function}[logical\_clocktime, period \rightarrow boolean] =
( \lambda T, i: (3 \Pi :
0 \leq \Pi \land \Pi \leq \text{sync\_time} \land T = T(i) + \text{frame\_time} - \text{sync\_time} + \Pi))
p, q, r: \textit{Var} processors
c_{sr}(\ast): \text{function}[processors, physical\_clocktime \rightarrow real\_time]
\text{log\_to\_phys: function}[logical\_clocktime \rightarrow physical\_clocktime] =
( \lambda T \rightarrow physical\_clocktime: T)
z: \textit{Var} number
\frac{z}{2}: \text{function}[number \rightarrow number] = (\lambda z: z/2)
p: \textit{fraction}
Rho: \textit{fraction} = \frac{\Pi}{2}
goodclock: \text{function}[processors, physical\_clocktime, physical\_clocktime \rightarrow bool] =
( \lambda p, T_0, T_N :
(\forall T_1, T_2 :
T_0 \leq T_1 \land T_0 \leq T_2 \land T_1 \leq T_N \land T_2 \leq T_N
\Rightarrow |c_p(T_1) - c_p(T_2) - (T_1 - T_2)| \leq \text{Rho} \cdot |T_1 - T_2|)
\text{monotonicity: Theorem}
(\exists T_0, T_N : \text{goodclock}(p, T_0, T_N) \land T_0 \leq T_1 \land T_0 \leq T_2 \land T_1 \leq T_N \land T_2 \leq T_N
\Rightarrow (T_1 > T_2 \Rightarrow c_p(T_1) \geq c_p(T_2)))
\Delta^{(2)}: \text{function}[processors, period \rightarrow clocktime]
\text{(* mean of the skew within tolerance *)}
Delta_{12}: \text{function}[processors, processors, period \rightarrow clocktime]
\text{(* measured skew *)}
\text{initial\_corr: function}[processors \rightarrow clocktime] = (\lambda p \rightarrow number: 0)
\text{second\_arg: function}[processors, period \rightarrow nat] = (\lambda p, i: i)
\text{corr}_{s1}(\ast): \text{Recursive function}[processors, period \rightarrow clocktime] =
(\lambda p, i : if i > 0
then \text{corr}_{p}(pred(i)) + \Delta_{p}(pred(i))
else \text{initial\_corr}(p)
end if) \text{by second\_arg}
A_{r1}(\ast): \text{function}[processors, period, logical\_clocktime \rightarrow physical\_clocktime] = (\lambda p, i, T: T + \text{corr}_{p}(i))
r_{r1}(\ast): \text{function}[processors, period, logical\_clocktime \rightarrow real\_time] =
(\lambda p, i, T: c_p(A_{r1}(T)))
skew: \text{function}[processors, processors, clocktime, period \rightarrow clocktime] =
(\lambda p, q, T, i \rightarrow clocktime: |r_{r1}(T) - r_{r1}(T(i))|
\text{nonfault\_clock: function}[processors, period \rightarrow boolean] =
(\lambda p, i: \text{goodclock}(p, A_{r1}(T(i)), A_{r1}(T(i+1))))
num_measure: function[period, nat \rightarrow nat] \equiv (\lambda i, k : k)
num_good_clocks: Recursive function[period, nat \rightarrow nat] =
(\lambda i, k : 
  if k = 0 \lor k > nrep
  then 0
  else if nonfaulty_clock(k, i)
    then 1 + num_good_clocks(i, k - 1)
    else num_good_clocks(i, k - 1)
end if) by num_measure

enough_clocks: function[period \rightarrow bool] =
(\lambda i : 3 \ast num_good_clocks(i, nrep) > 2 \ast nrep)

SIA: function[period \rightarrow bool] \equiv (\lambda i : enough_clocks(i))
(* in current clock sync theory =
 (\lambda i : enough_clocks(i))
*)

S1C: function[processors, processors, period \rightarrow bool] =
(\lambda p, q, i : (\forall T :: nonfaulty_clock(p, i) \land nonfaulty_clock(q, i) \land T \in R(i) \land skew(p, q, T, i) \leq \delta))
S1C.lemma: Lemma S1C(p, q, i) \supset S1C(q, p, i)
S1: function[period \rightarrow bool] = (\lambda i : SIA(i) \supset (\forall p, q : S1C(p, q, i)))

S2: function[processors, period \rightarrow bool] =
(\lambda p, i : ((Corr_p^{i+1} - Corr_p^i) \leq \Sigma))

(* The following three theorems were proved in the clock sync theory. They are taken as axioms here. *)
adj.always.pos: Axiom A_p^{(b)}(T) \geq T^0
Theorem_1: Axiom S1(i)
(* THEOREM *)
Theorem_2: Axiom S2(p, i)
(* THEOREM *)
A0: Axiom skew(p, q, T^0, 0) < \delta_0
A1: Lemma nonfaulty_clock(p, i) = goodclock(p, A_p^{(0)}(T^0), A_p^{(i)}(T^{i+1}))
A2: Axiom nonfaulty_clock(p, i) 
\land S1C(p, q, i) \land S2(p, i)
\supset (\exists T_0 : T_0 \in S(i) \land [r_{f_p}^i(T_0 + \Delta_p^{(i)}) - r_{f_q}^i(T_0)] < \epsilon)
A2.aux: Axiom \Delta_p^{(i)} = 0
m: processors (* maximum number of faulty clocks *)
C0: Axiom m < nrep \land m \leq nrep - num_good_clocks(i, nrep)
C1: Axiom frame_time \geq 3 \ast sync_time
C2: Axiom \( \text{sync.time} \geq \Sigma \)

C3: Axiom \( \Sigma \geq \Delta \)

C4: Axiom \( \Delta \geq \delta + \epsilon + \frac{\delta}{\text{frame.time}} \)

C5: Axiom \( \delta \geq \delta_0 + \rho \cdot \text{frame.time} \)

C6: Axiom \( \delta \geq 2 \cdot (\epsilon + \rho \cdot \text{sync.time}) + 2 \cdot m \cdot \Delta/(\text{nrep} - m) \)
\[ + \text{nrep} \cdot \rho \cdot \text{frame.time}/(\text{nrep} - m) \]
\[ + \rho \cdot \Delta \]
\[ + \text{nrep} \cdot \rho \cdot \Sigma/(\text{nrep} - m) \]

\( \text{sync.thm: Theorem} \)

\( \text{enough.clocks(i)} \)
\[ \exists (\forall p, q : \] (\text{nonfaulty.clock}(p, i) \land \text{nonfaulty.clock}(q, i) \land T \in R^{(i)} \]
\[ \exists |r_{p}^{(i)}(T) - r_{q}^{(i)}(T)| \leq \delta)) \]

\( \text{Proof} \)

\( \text{p.sync.thm: Prove sync.thm from} \)
\[ \text{Theorem.1 [i \rightarrow i], S1 [i \rightarrow i], S1C [i \rightarrow i]} \]

\( \text{End} \)

\( \text{clkprop: Module} \)

\( \text{Using clkmod, DA} \)

\( \text{Exporting all} \)

\( \text{Theory} \)

\( T, T_1, T_2, T_3, T_4, BB, T_0, T_N, TX, TY: \text{Var logical.clock.time} \)

\( p, q: \text{Var processors} \)

\( da: \text{Var Dstate} \)

\( i: \text{Var period} \)

\( \text{ft2: Lemma goodclock(q, T_0, T_1 + BB) \land (T_1 \geq T_0) \land (BB \geq T_0)} \)
\[ \exists |c_q(T_1 + BB) - c_q(T_1)| \leq \rho \cdot |BB| \]

\( \text{ft3: Lemma goodclock(q, T_0, T_1 + BB) \land (T_1 \geq T_0) \land (BB \geq T_0)} \)
\[ \exists (1 - \rho) \cdot |BB| \leq c_q(T_1 + BB) - c_q(T_1) \]
\[ \land c_q(T_1 + BB) - c_q(T_1) \leq (1 + \rho) \cdot |BB| \]

\( \text{ft4: Lemma enough.clocks(i)} \)
\[ \land \text{nonfaulty.clock}(p, i) \land \text{nonfaulty.clock}(q, i) \land T \in R^{(i)} \]
\[ \exists -\delta \leq r_{p}^{(i)}(T) - r_{q}^{(i)}(T) \]
\[ \land r_{p}^{(i)}(T) - r_{q}^{(i)}(T) \leq \delta \]

\( \text{ft5: Lemma goodclock(q, T_0, T_1 + \text{Corr}^{(i)} + BB)} \)
\[ \land (T_1 \geq T_0) \land (T_1 + \text{Corr}^{(i)} + BB) \land (BB \geq T_0) \]
\[ \exists (1 - \rho) \cdot |BB| \leq r_{p}^{(i)}(T_1 + BB) - r_{q}^{(i)}(T_1) \]
\[ \land r_{q}^{(i)}(T_1 + BB) - r_{q}^{(i)}(T_1) \leq (1 + \rho) \cdot |BB| \]

\( \text{ft6: Lemma T}_2 = T_1 + BB \)
\[ \land \text{goodclock(q, T_0, T_1 + \text{Corr}^{(i)} + BB)} \]
\[ \land (T_1 \geq T_0) \land (T_1 + \text{Corr}^{(i)} \geq T_0) \land (BB \geq T_0) \]
\[ \land \text{enough.clocks(i)} \]
\[ \land \text{nonfaulty.clock}(p, i) \land \text{nonfaulty.clock}(q, i) \land T_2 \in R^{(i)} \]
\[ \exists r_{p}^{(i)}(T_2) \geq r_{q}^{(i)}(T_1) + (1 - \rho) \cdot |BB| - \delta \]

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Lemma $T_3 \leq T_4 \land \text{goodclock}(q, T^0, T_4) \supset \text{goodclock}(q, T^0, T_3)$

Lemma $T_1 + BB \leq T^{(i+1)} \land \text{nonfaulty_clock}(q, i)$
$\supset \text{goodclock}(q, T^0, T_1 + \text{Corr}^{(i)}_q + BB)$

Lemma $T_2 = T_1 + BB$
$\land T_2 \leq T^{(i+1)}$
$\land (T_1 \geq T^0)$
$\land (T_1 + \text{Corr}^{(i)}_q \geq T^0)$
$\land (BB \geq T^0)$
$\land \text{enough_clocks}(i)$
$\land \text{nonfaulty_clock}(p, i) \land \text{nonfaulty_clock}(q, i) \land T_2 \in R^{(i)}$
$\supset rt^{(i)}_p (T_2) \geq rt^{(i)}_q (T_1) + (1 - \text{Rho}) \cdot |BB| - \delta$

Lemma $T_2 \in R^{(i)} \supset T_2 \leq T^{(i+1)}$

Lemma $T_2 = T_1 + BB$
$\land (T_1 \geq T^0)$
$\land (T_1 + \text{Corr}^{(i)}_q \geq T^0)$
$\land (BB \geq T^0)$
$\land \text{enough_clocks}(i)$
$\land \text{nonfaulty_clock}(p, i) \land \text{enough_clocks}(i) \land T_2 \in R^{(i)}$
$\land T_1 \in R^{(i)}$\land T_2 \in R^{(i)}
$\supset rt^{(i)}_p (T_2) \geq rt^{(i)}_q (T_1) + (1 - \text{Rho}) \cdot |BB| - \delta$

Lemma $T_1 \in R^{(i)} \supset (T_1 + \text{Corr}^{(i)}_q \geq T^0)$

GOAL: Lemma $T_2 = T_1 + BB$
$\land (T_1 \geq T^0)$
$\land (BB \geq T^0)$
$\land \text{nonfaulty_clock}(p, i)$
$\land \text{enough_clocks}(i) \land T_2 \in R^{(i)} \land T_1 \in R^{(i)}$
$\supset rt^{(i)}_p (T_2) \geq rt^{(i)}_q (T_1) + (1 - \text{Rho}) \cdot |BB| - \delta$

Lemma nonfaulty_clock(p, i + 1) \supset nonfaulty_clock(p, i)

Proof

nfc.a: Lemma $T^{(i+1)} + \text{Corr}^{(i)}_p \leq T^{(i+2)} + \text{Corr}^{(i+1)}_p$

p.nfc.a: Prove nfc.a from
$T^{(i+1)} \{ i \leftarrow i + 1 \}$
$T^{(i+1)} \{ i \leftarrow i + 2 \}$
Theorem.2 $\{ i \leftarrow i \}$
$S2 \{ i \leftarrow i \}$
abs.main $\{ x \leftarrow \text{Corr}^{(i+1)}_p - \text{Corr}^{(i)}_p, z \leftarrow \Sigma \}$
C1
C2

p.nfc.a: Prove nfc.a from
nonfaulty_clock,
nonfaulty_clock $\{ i \leftarrow i + 1 \}$,
goodclock $\{ T_0 \leftarrow T^{(i+2)} + \text{Corr}^{(i+1)}_p, T_0 \leftarrow T^{(i+2)} + \text{Corr}^{(i+1)}_p \}$,
$T_0 \leftarrow T_0 \oplus p4$
$T_2 \leftarrow T_2 \oplus p4$
goodclock $\{ T_0 \leftarrow T^{(i+1)} + \text{Corr}^{(i)}_p, T_0 \leftarrow T^{(i+1)} + \text{Corr}^{(i)}_p \}$
nfc.a
p_ft2: Prove ft2 from
goodclock
\{ p \leftarrow q, \\
T_0 \leftarrow T^0, \\
T_N \leftarrow T_1 + BB, \\
T_2 \leftarrow T_1, \\
T_1 \leftarrow (T_1 + BB) \}

p_ft3: Prove ft3 from
ft2,
abs_leq \{ x \leftarrow c_q(T_1 + BB) - c_q(T_1) - BB, \quad z \leftarrow \text{Rho} * |BB|, \\
abs_geq \{ x \leftarrow BB \}

p_ft4: Prove ft4 from
sync_thm \{ i \leftarrow i \},
abs_leq \{ x \leftarrow rt_p^{(i)}(T) - rt_q^{(i)}(T), \quad z \leftarrow \delta \}

p_ft5: Prove ft5 from
ft3 \{ T_1 \leftarrow T_1 + \text{Corr}_q \},
rt^{(2)}_p(i+3) \{ p \leftarrow q, \quad T \leftarrow T_1, \quad i \leftarrow i \},
rt^{(2)}_q(i+3) \{ p \leftarrow q, \quad T \leftarrow T_1 + BB, \quad i \leftarrow i \}

p_ft6: Prove ft6 from ft4 \{ T \leftarrow T_2 \}, ft5

p_ft7: Prove ft7 from
goodclock
\{ p \leftarrow q, \\
T_0 \leftarrow T^0, \\
T_1 \leftarrow T_1 \Theta p2, \\
T_2 \leftarrow T_2 \Theta p2, \\
T_N \leftarrow T_1, \}
goodclock \{ p \leftarrow q, \quad T_0 \leftarrow T^0, \quad T_N \leftarrow T_1 \}

ft8a: Lemma \text{Corr}_q \{ i \leftarrow i \} = 0

p_ft8: Prove ft8 from
nonfaulty_clock \{ p \leftarrow q, \quad i \leftarrow i \},
ft7 \{ T_3 \leftarrow T_1 + \text{Corr}_q^{(i)} + BB, \quad T_4 \leftarrow T^{(i+1)} + \text{Corr}_q^{(i)} \},
T^{(i)} \{ i \leftarrow 0 \},
ft8a

p_ft8a: Prove ft8a from \text{Corr}_q^{(2)} \{ i \leftarrow 0, \quad p \leftarrow q \}

p_ft9: Prove ft9 from ft6, ft8

p_ft10: Prove ft10 from
*1 \in \mathcal{R}^{(2)} \{ T \leftarrow T_2, \quad \Pi \leftarrow \text{frame_time} \},
T^{(x)} \{ i \leftarrow i \},
T^{(x)} \{ i \leftarrow i + 1 \}

p_ft11: Prove ft11 from ft10 \{ i \leftarrow i \}, ft9

p_ft12: Prove ft12 from
adj_always_pos \{ k \leftarrow i, \quad p \leftarrow q \},
*1 \in \mathcal{R}^{(2)} \{ T \leftarrow T_1, \quad i \leftarrow i \}

p_GOAL: Prove GOAL from ft11, ft12

End
DA_invariants_tcc: Module

Using DA_invariants

Exporting all with DA_invariants

Theory
\( ii: \text{Var natural number} \)
\( p: \text{Var rcp_defs.processors} \)
\( j: \text{Var rcp_defs.processors} \)
\( i: \text{Var rcp_defs.processors} \)
\( z: \text{Var DA.DAstate} \)
\( n_1: \text{Var natural number} \)
\( k: \text{Var natural number} \)
\( u: \text{Var rcp_defs.inputs} \)
\( t: \text{Var DA.DAstate} \)
\( s: \text{Var DA.DAstate} \)

**Corr_lem_TCC1:** Formula \((ii > 0) \lor (ii - 1 \geq 0)\)

**Proof**

Corr_lem_TCC1.PROOF: Prove Corr_lem.TCC1

End DA.invariants.tcc

**DA_map_proof.tcc:** Module

Using DA.map.proof

Exporting all with DA.map.proof

**Theory**

\( a: \text{Var rcp_defs.proc_plus} \)
\( p: \text{Var rcp_defs.processors} \)
\( m: \text{Var rcp_defs.proc_plus} \)
\( \text{p_mll_base_TCC1: Formula } ((0 \geq 0) \land (0 \leq \text{nrep})) \)

\( \text{p_mll_ind_TCC1: Formula } \)

\( (( \text{if } a = \text{nrep then } \text{nrep else } a + 1 \text{ end } \text{if } \geq 0)) \)
\( \land ( \text{if } a = \text{nrep then } \text{nrep else } a + 1 \text{ end } \text{if } \leq \text{nrep})) \)

\( \text{p_map_l_TCC1: Formula } ((\text{nrep} \geq 0) \land (\text{nrep} \leq \text{nrep})) \)

**Proof**

p_mll_base_TCC1.PROOF: Prove p_mll_base_TCC1

p_mll_ind_TCC1.PROOF: Prove p_mll_ind_TCC1

p_map_l_TCC1.PROOF: Prove p_map_l_TCC1

End DA.map.proof.tcc

**DA_support_proof.tcc:** Module

Using DA.support.proof

Exporting all with DA.support.proof

**Theory**

\( q: \text{Var natural number} \)
\( a: \text{Var rcp_defs.proc_plus} \)
\( n_1: \text{Var natural number} \)
\( m: \text{Var rcp_defs.proc_plus} \)
\( p: \text{Var rcp_defs.processors} \)
\( \text{p_sll3_base_TCC1: Formula } ((0 \geq 0) \land (0 \leq \text{nrep})) \)

\( \text{p_sll3_ind_TCC1: Formula } \)

\( (( \text{if } a = \text{nrep then } \text{nrep else } a + 1 \text{ end } \text{if } \geq 0)) \)
\( \land ( \text{if } a = \text{nrep then } \text{nrep else } a + 1 \text{ end } \text{if } \leq \text{nrep})) \)
\( \text{p..support.13..TCC1: Formula } (\text{arep} \geq 0) \land (\text{arep} \leq \text{nrep}) \)

\( \text{p..sl15..ind_.TCC1: Formula } \)
\((\text{if } q < \text{nrep} \text{ then } q + 1 \text{ else } \text{nrep} \text{ end if } > 0) \land \text{ (if } q < \text{nrep} \text{ then } q + 1 \text{ else } \text{nrep} \text{ end if } \leq \text{nrep}) \)

**Proof**

\( \text{p..sl13..base_.TCC1.PROOF: Prove p..sl13..base_.TCC1} \)
\( \text{p..sl13..ind_.TCC1.PROOF: Prove p..sl13..ind_.TCC1} \)
\( \text{p..support.13..TCC1.PROOF: Prove p..support.13..TCC1} \)
\( \text{p..sl15..ind_.TCC1.PROOF: Prove p..sl15..ind_.TCC1} \)

End \text{DA_support..proof..tcc}

\text{DA..to..DS..tcc: Module}

Using \text{DA..to..DS}

Exporting all with \text{DA..to..DS}

**Theory**

\( \text{da: Var DA.DAstate} \)
\( \text{s: Var DA.DAstate} \)
\( \text{t: Var DA.DAstate} \)
\( \text{k: Var naturalnumber} \)
\( MBf: \text{Var function[rcp..defs..processors} \rightarrow \text{rcp..defs..MBvec]} \)

\( \text{ss_update.TCC1: Formula } (\neg((k = 0) \lor (k > \text{nrep}))) \lor (k - 1 \geq 0) \)

\( \text{ss_update.TCC2: Formula } (\neg((k = 0) \lor (k > \text{nrep}))) \lor ((k > 0) \land (k \leq \text{nrep})) \)

\( \text{ss_update.TCC3: Formula } \\
(\neg((k = 0) \lor (k > \text{nrep}))) \lor \text{da_measure(da, } k > \text{da_measure(da, } k - 1) \)

\( \text{MBmatrix..cons.TCC1: Formula } \\
(\neg((k = 0) \lor (k > \text{nrep}))) \lor \text{MBmc_measure(MBf, } k > \text{MBmc_measure(MBf, } k - 1) \)

\( \text{reachable_in.n.TCC1: Formula } (\neg(k = 0)) \lor (k - 1 \geq 0) \)

\( \text{reachable_in.n.TCC2: Formula } \\
(\neg(k = 0)) \lor \text{da_measure(t, } k > \text{da_measure(s, } k - 1) \)

**Proof**

\( \text{ss_update.TCC1.PROOF: Prove ss_update.TCC1} \)
\( \text{ss_update.TCC2.PROOF: Prove ss_update.TCC2} \)
\( \text{ss_update.TCC3.PROOF: Prove ss_update.TCC3} \)
\( \text{MBmatrix..cons.TCC1.PROOF: Prove MBmatrix..cons.TCC1} \)
\( \text{reachable_in.n.TCC1.PROOF: Prove reachable_in.n.TCC1} \)
\( \text{reachable_in.n.TCC2.PROOF: Prove reachable_in.n.TCC2} \)

End \text{DA..to..DS..tcc}

\text{DA..tcc..proof: Module}

Using \text{clk..types..tcc, clkmod..tcc, DA..map..proof..tcc, DA..support..proof..tcc}
Exporting all

Theory

Proof

posnum_TxCC1.PROOF: Prove posnum_TxCC\{z \rightarrow 1\}

fraction_TxCC1.PROOF: Prove fraction_TxCC\{x \rightarrow 0\}

C6_TxCC1.PROOF: Prove C6_TxCC1 from C0

p.sll15.ind.TxCC1.PROOF: Prove p.sll15.ind.TxCC1 from processors.exist_ax

Rho_TxCC1.PROOF: Prove Rho.TxCC1 (* needs printerdivide = yes *)

End

DA_broadcast_prf_Tcc: Module

Using DA.broadcast_prf

Exporting all with DA.broadcast_prf

Theory

i: Var rcp_defs.processors
q: Var rcp_defs.processors
qq: Var rcp_defs.processors
ll: Var number
x: Var DA.DAstate
k: Var naturalnumber
u: Var rcp_defs.inputs
s: Var DA.DAstate
qdurb: Var number
qdurc: Var number
pdurb: Var number
pdurc: Var number
p_br8_TxCC1: Formula ( if k = 0 then 0 else k - 1 end if \geq 0)

Proof

p.br8_TxCC1.PROOF: Prove p.br8_TxCC1

End DA.broadcast_prf_Tcc

clk_types_Tcc: Module

Using clk_types

Exporting all with clk_types

Theory

x: Var number
posnum_TxCC1: Formula (\exists x : x > 0)

fraction_TxCC1: Formula (\exists x : 1 \geq x \land x \geq 0 \land x \neq 1)

Proof

posnum_TxCC1.PROOF: Prove posnum_TxCC1

fraction_TxCC1.PROOF: Prove fraction_TxCC1
End clk.types.tcc
clkmod.tcc: Module
Using clkmod
Exporting all with clkmod

Theory

i: Var naturalnumber
k: Var naturalnumber
p: Var rcp.defs.processors
half.TCCI: Formula (2 \neq 0)

Rho.TCCI: Formula (1 \geq i \land j \geq 0 \land k \neq 1)

Corr.TCCI: Formula (i > 0) \supset \text{second.arg}(p, i) > \text{second.arg}(p, \text{pred}(i))

num.good_clocks.TCCI: Formula
\neg(k = 0 \lor k > \text{nrp}) \supset ((k > 0) \land (k \leq \text{nrp}))

num.good_clocks.TCCI2: Formula
\neg(\text{nonfaulty_clock}(k, i)) \land (\neg(k = 0 \lor k > \text{nrp})) \supset (k - 1 \geq 0)

num.good_clocks.TCCI3: Formula
\neg(\text{nonfaulty_clock}(k, i)) \land (\neg(k = 0 \lor k > \text{nrp})) \supset (k - 1 \geq 0)

num.good_clocks.TCCI4: Formula
\neg(\text{nonfaulty_clock}(k, i)) \land (\neg(k = 0 \lor k > \text{nrp}))
\supset \text{num.measure}(i, k) > \text{num.measure}(i, k - 1)

num.good_clocks.TCCI5: Formula
\neg(\text{nonfaulty_clock}(k, i)) \land (\neg(k = 0 \lor k > \text{nrp}))
\supset \text{num.measure}(i, k) > \text{num.measure}(i, k - 1)

C6.TCCI: Formula (\text{nrp} - m \neq 0)

Proof

half.TCCI.PROOF: Prove half.TCCI
Rho.TCCI.PROOF: Prove Rho.TCCI

num.good_clocks.TCCI1.PROOF: Prove num.good_clocks.TCCI1
num.good_clocks.TCCI2.PROOF: Prove num.good_clocks.TCCI2
num.good_clocks.TCCI3.PROOF: Prove num.good_clocks.TCCI3
num.good_clocks.TCCI4.PROOF: Prove num.good_clocks.TCCI4
num.good_clocks.TCCI5.PROOF: Prove num.good_clocks.TCCI5
C6.TCCI.PROOF: Prove C6.TCCI

End clkmod.tcc

top: Module
Using rcp_defs, generic_FT, sets[processors], cardinality[processors], 
nat_inductions, noetherian[proc_plus, lesep], US, RS, RS_majority, RS_to_US, 
RS.lemmas, RS.invariants, RS.top_proof, RS.tcc_proof, rcp_defs.tcc, 
RS_to_US.tcc, DS, DS_to_RS, DS.lemmas, DS.top_proof, DS.map_proof, 
DS.support_proof, multiplication, absmod, clk_types, clkmod, DA, DA_to_DS, 
DA.invariants, clkprop, DA.lemmas, DA.top_proof, DA.map_proof, 
DA.support_proof, DA.broadcast_prf, DA.intervals, rcp_defs.tcc, 
DS_to_RS.tcc, DS.support_proof.tcc, DS.map_proof.tcc, DA.invariants_tcc, 
DA.map_proof.tcc, DA_support_proof.tcc, DA_to_DS.tcc, clk_types_tcc, 
clkmod_tcc, DA.tcc_proof, DA.broadcast_prf_tcc

Theory

u: Var inputs
us1, us2: Var Pstate
ts1, ts2: Var RSstate
ds1, ds2: Var DSstate
da1, da2: Var DAstate

RS.frame.commutes: Theorem
  reachable(rsl) \land N_u (rsl, rs2, u) \supset N_u (RSmap(rsl), RSmap(rs2), u)

RS.initial.maps: Theorem
  initial.rs (rsl) \supset initial.us (RSmap(rsl))

DS.frame.commutes: Theorem
  ds1.phase = compute \land frame_N_ds (ds1, ds2, u) 
  \supset N_u (DSmap(ds1), DSmap(ds2), u)

DS.initial.maps: Theorem
  initial.ds (ds1) \supset initial.rs (DSmap(ds1))

DA.phase.commutes: Theorem
  reachable(da1) \land N_u (da1, da2, u) \supset N_u (DAmap(da1), DAmap(da2), u)

DA.initial.maps: Theorem
  initial.da (da1) \supset initial.ds (DAmap(da1))

Proof

p.RS.frame.commutes: Prove RS.frame.commutes from
  RS_to_US.frame_commutes \{s \leftarrow rsl, t \leftarrow rs2\}

p.RS.initial.maps: Prove RS.initial.maps from
  RS_to_US.initial.maps \{s \leftarrow rsl\}

p.DS.frame.commutes: Prove DS.frame.commutes from
  DS_to_RS.frame_commutes \{s \leftarrow ds1, t \leftarrow ds2\}

p.DS.initial.maps: Prove DS.initial.maps from
  DS_to_RS.initial.maps \{s \leftarrow ds1\}

p.DA.phase.commutes: Prove DA.phase.commutes from
  DA_to_DS.phase_commutes \{s \leftarrow da1, t \leftarrow da2\}

p.DA.initial.maps: Prove DA.initial.maps from
  DA_to_DS.initial.maps \{s \leftarrow da1\}

End
rcp_defs: Module
Exporting all
Theory
p: Var nat
Pstate: Type (* computation state of a single processor *)
inputs: Type (* type of external sensor input *)
outputs: Type (* actuator output type *)
MB: Type (* mailbox exchange type *)
nrep: nat (* number of replicated processors *)
initial.proc.state: Pstate (* assumes each processor begins identically *)
recovery.period: nat (* number of healthy frames required to recover from transient fault plus one *)

recovery.period_ax: Axiom recovery_period > 2

processors_exist_ax: Axiom nrep > 0

processors: Type from nat with (λ p : (p > 0) ∧ (p ≤ nrep))
MBvec = array [processors] of MB
MBmatrix = array [processors] of MBvec
ph: Var phases
next_phase: function[phases → phases] =
  (λ ph : if ph = compute then broadcast
   elseif ph = broadcast then vote
   elseif ph = vote then sync
   else compute
   end if)

prev_phase: function[phases → phases] =
  (λ ph : if ph = compute
   then sync
   elseif ph = broadcast
   then compute
   elseif ph = vote
   then broadcast
   else vote
   end if)

proc_plus: Type from nat with (λ p : (p ≥ 0) ∧ (p ≤ nrep))
k, m, a, n, b: Var proc_plus
prop: Var function[proc_plus → bool]
lessp: function[proc_plus, proc_plus → bool] == (λ m, n : m < n)

processors_induction: Lemma
  (∀ prop : prop(0) ∧ (∀ m : m < nrep ∧ prop(m) ⊃ prop(m + 1)))
  ⊃ (∀ n : prop(n)))

Proof
Using noetherian[proc_plus, lessp]

reachability: Lemma a ≠ 0 ⇔ (∃ b : a = b + 1)
p_processors_induction: Prove processors_induction {m ← b@P2} from
general_induction {p ← prop, d ← n, d2 ← m},
reachability {a ← d1@P1}

p_well_founded: Prove well_founded {measure ← (λ k → nat : k)}
p_reachability: Prove reachability {b ← if a = 0 then 0 else a - 1 end if}

End
sets: Module [T: Type]
Exporting all
Theory
set: Type is function[T → bool]

z,y,z: Var T

*1 U *2: function[set, set → set] == (λ a, b: (λ x: a(x) ∨ b(x)))

*1 ∩ *2: function[set, set → set] == (λ a, b: (λ x: a(x) ∧ b(x)))

add: function[T, set → set] == (λ x, a: (λ y: x = y ∨ a(y)))

singleton: function[set, set → bool] == (λ a, b: (λ z: a(z) ⊆ b(z)))

empty: function[set → bool] == (λ a: (λ x: ¬a(x)))

extensionality: Axiom (V x : T, (λ a: a(x) = a(x)) → (a = a))

End sets

cardinality: Module [T: Type]

Using sets[T]

Exporting all

Assuming

z, y, z: Var T
N: Var nat
f: Var function[T → nat]

finite: Formula (∃ N, f: (∀ z, y: f(z) ≤ N ∧ (f(x) = f(y) ∨ x = y)))

Theory

a, b, c: Var set

card: function[set → nat]

card_ax: Axiom card(a U b) + card(a ∩ b) = card(a) + card(b)

card_subset: Axiom a ⊆ b ⊆ card(a) ≤ card(b)

card_empty: Axiom card(a) = 0 ⇔ empty(a)

card_subset: Lemma card(a) > 0 ⇔ (∃ x: x ∈ a)

card_prop: Lemma a ⊆ c ∧ b ⊆ c ∧ 2 * card(a) > card(c) ∧ 2 * card(b) > card(c)

Proof

card_subset: Sublemma a ⊆ c ∧ b ⊆ c ⊆ a U b ⊆ c

card_subset_union: Sublemma a ⊆ c ∧ b ⊆ c ⊆ a U b ⊆ c

Proof card_subset_union from

1 ⊆ 2 {x ← z@p3, b ← c},
1 ⊆ 2 {x ← z@p3, a ← b, b ← c},
1 ⊆ 2 {a ← a U b, b ← c}

m, n, p: Var nat

twice_prop: Sublemma 2 * m > p ∧ 2 * n > p ⊆ m + n > p

twice_prop: Prove twice_prop
card_proof: Prove card_prop from
  twice_prop {m \rightarrow \text{card}(a), n \rightarrow \text{card}(b), p \rightarrow \text{card}(c)},
card_ax,
subset_union,
card_subset \{a \rightarrow a \cup b, b \rightarrow c\}

End cardinality

nat_inductions: Module

Theory

i, j: Var nat
n_1, n_2, n_3: Var nat
p: Var function[nat \rightarrow bool]

nat_complete: Axiom

(\forall n_1 : (\forall n_3 : (n_3 \neq n_1) \supset p(n_3)) \supset (\forall n_2 : p(n_2)))

nat_induction: Axiom

(p(0) \land (\forall n_1 : p(n_1) \supset p(n_1 + 1))) \supset (\forall n_2 : p(n_2))

nat_induct_by_2: Axiom

(p(0) \land p(1) \land (\forall n_1 : p(n_1) \supset p(n_1 + 2))) \supset (\forall n_2 : p(n_2))

End nat_inductions

noetherian: Module [dom: Type, <: function[dom, dom \rightarrow bool]]

Assuming

measure: Var function[dom \rightarrow nat]
a, b: Var dom

well_founded: Formula (\exists measure : a < b \supset measure(a) < measure(b))

Theory

p, A, B: Var function[dom \rightarrow bool]
d, d_1, d_2: Var dom

general_induction: Axiom

(\forall d : (\forall d_2 : d_2 < d \supset p(d_2)) \supset p(d)) \supset (\forall d : p(d))

End noetherian

multiplication: Module

Exporting all

Theory

x, y, z, x_1, y_1, z_1, x_2, y_2, z_2: Var number

*1 \times 2: function[number, number \rightarrow number] = (\lambda x, y : (x \times y))

mult_distrib: Lemma x \times (y + z) = x \times y + x \times z

mult_distrib_minus: Lemma x \times (y - z) = x \times y - x \times z

mult_rident: Lemma x \times 1 = x

mult_lident: Lemma 1 \times x = x

distrib: Lemma (x + y) \times z = x \times z + y \times z

distrib_minus: Lemma (x - y) \times z = x \times z - y \times z
mult_non_neg: Axiom \((x \geq 0 \land y \geq 0) \lor (x \leq 0 \land y \leq 0) \leftrightarrow x \times y \geq 0\)
mult_pos: Axiom \((x > 0 \land y > 0) \lor (x < 0 \land y < 0) \leftrightarrow x \times y > 0\)
mult_com: Lemma \(x \times y = y \times x\)

pos_product: Lemma \(x \geq 0 \land y \geq 0 \supset x \times y \geq 0\)
mult_leq: Lemma \(x \geq 0 \land y \geq 0 \supset x \times z \geq y \times z\)
mult_leq_2: Lemma \(x \geq 0 \land z \geq y \supset x \times z \geq y \times z\)
mult_leq_2: Lemma \(x \geq 0 \land z \geq y \supset x \times z \geq y \times z\)
mult_l0: Axiom \(0 \times x = 0\)
mult_gt: Lemma \(x > 0 \land y > 0 \supset x \times z > y \times z\)

Proof

mult_gt_pr: Prove mult_gt from
mult_pos \(\{x \leftarrow x - y, y \leftarrow z\}\), distrib_minus

distrib_minus_pr: Prove distrib_minus from
mult_l_distrib_minus \(\{x \leftarrow x - y, y \leftarrow z\},\)
mult_com \(\{x \leftarrow x - y, z \leftarrow y\},\)
mult_com \(\{y \leftarrow z\},\)
mult_com \(\{x \leftarrow x, y \leftarrow z\}\)

mult_leq_2_pr: Prove mult_leq_2 from
mult_l_distrib_minus \(\{z \leftarrow x \times y, y \leftarrow z\},\)
mult_non_neg \(\{z \leftarrow y, y \leftarrow z\}\)

mult_leq_pr: Prove mult_leq from
distrib_minus, mult_non_neg \(\{x \leftarrow x - y, y \leftarrow z\}\)

mult_com_pr: Prove mult_com from
*1 \times \ast 2 , *1 \times \ast 2 \(\{x \leftarrow x - y, y \leftarrow z\}\)

pos_product_pr: Prove pos_product from mult_non_neg
mult_rident: Prove mult_rident from *1 \times \ast 2 \(\{y \leftarrow 1\}\)

mult_lident: Prove mult_lident from *1 \times \ast 2 \(\{x \leftarrow 1, y \leftarrow z\}\)

distrib_pr: Prove distrib from
*1 \times \ast 2 \(\{x \leftarrow z + y, y \leftarrow z\},\)
*1 \times \ast 2 \(\{y \leftarrow z\},\)
*1 \times \ast 2 \(\{z \leftarrow y, y \leftarrow z\}\)

mult_l_distrib_pr: Prove mult_l_distrib from
*1 \times \ast 2 \(\{y \leftarrow y + z, x \leftarrow z\},\)
*1 \times \ast 2 , *1 \times \ast 2 \(\{y \leftarrow z\}\)

mult_l_distrib_minus_pr: Prove mult_l_distrib minus from
*1 \times \ast 2 \(\{y \leftarrow y - z, x \leftarrow z\},\)
*1 \times \ast 2 , *1 \times \ast 2 \(\{y \leftarrow z\}\)

End

absmod: Module
Using multiplication
Exporting all
Theory
\[ z, y, z_1, y_1, z_2, y_2, z_2 : \text{Var number} \]
\[ | \ast | : \text{Definition function [number \to number]} = \]
\[ (\lambda x : (\text{if } x < 0 \text{ then } -x \text{ else } x \text{ end if}) \]

abs.main: Lemma \[ | x | < z \supset (x < z \wedge -x < z) \]

abs.\text{leq.0}: Lemma \[ | x - y | \leq z \supset (x - y) \leq z \]

abs.\text{diff}: Lemma \[ | x - y | < z \supset ((x - y) < z \wedge (y - x) < z) \]

abs.\text{leq}: Lemma \[ | x | \leq z \supset (x \leq z \wedge -x \leq z) \]

abs.\text{bnd}: Lemma \[ 0 \leq x \wedge 0 \leq x \wedge x \leq z \wedge 0 \leq y \wedge y \leq z \supset |x - y| \leq z \]

abs.\text{1.bnd}: Lemma \[ | x - y | \leq z \supset x \leq y + z \]

abs.\text{2.bnd}: Lemma \[ | x - y | \leq z \supset x \geq y - z \]

abs.\text{3.bnd}: Lemma \[ x \leq y + z \wedge x \geq y - z \supset |x - y| \leq z \]

abs.\text{drift}: Lemma \[ | x - y | \leq z \supset |x_1 - x| \leq z_1 \supset |x_1 - y_1| \leq z + z_1 \]

abs.\text{com}: Lemma \[ | x - y | = |y - z| \]

abs.\text{drift.2}: Lemma
\[ |x - y| \leq z \wedge |x_1 - x| \leq z_1 \wedge |y_1 - y| \leq z_2 \supset |x_1 - y_1| \leq z + z_1 + z_2 \]

abs.\text{geq}: Lemma \[ x \geq y \wedge y \geq 0 \supset |x| \geq |y| \]

abs.\text{ge0}: Lemma \[ x \geq 0 \supset |x| = x \]

abs.\text{plus}: Lemma \[ |x + y| \leq |x| + |y| \]

abs.\text{diff.3}: Lemma \[ x - y \leq z \wedge y - x \leq z \supset |x - y| \leq z \]

abs.\text{eq}: Lemma \[ |x - y| = |y - z| \]

Proof

abs.\text{plus.pr}: Prove abs.\text{plus} from \[ \ast | \{ x \leftarrow x + y \}, \ast | \}, \ast | \{ z \leftarrow y \} \]

abs.\text{diff.3.pr}: Prove abs.\text{diff.3} from \[ \ast | \{ x \leftarrow x - y \} \]

abs.\text{ge0}.\text{proof}: Prove abs.\text{ge0} from \[ \ast | \]

abs.\text{geq}.\text{proof}: Prove abs.\text{geq} from \[ \ast | \}, \ast | \{ x \leftarrow y \} \]

abs.\text{drift.2}.\text{proof}: Prove abs.\text{drift.2} from

abs.\text{drift},
abs.\text{drift} \{ x \leftarrow y, y \leftarrow y_1, z \leftarrow z_1, z_1 \leftarrow z + z_1 \},
abs.\text{com} \{ z \leftarrow y_1 \}

abs.\text{com}.\text{proof}: Prove abs.\text{com} from \[ \ast | \{ x \leftarrow (x - y) \}, \ast | \{ x \leftarrow (y - z) \} \]

abs.\text{drift}.\text{proof}: Prove abs.\text{drift} from

abs.\text{1.bnd},
abs.\text{1.bnd} \{ x \leftarrow z_1, y \leftarrow y_1, z \leftarrow z_1 \},
abs.\text{2.bnd},
abs.\text{2.bnd} \{ x \leftarrow z_1, y \leftarrow y_1, z \leftarrow z_1 \},
abs.\text{3.bnd},
abs.\text{3.bnd} \{ x \leftarrow z_1, z \leftarrow z + z_1 \}

abs.\text{3.bnd}.\text{proof}: Prove abs.\text{3.bnd} from \[ \ast | \{ x \leftarrow (x - y) \} \]

abs.\text{main}.\text{proof}: Prove abs.\text{main} from \[ \ast | \]

abs.\text{leq.0}.\text{proof}: Prove abs.\text{leq.0} from \[ \ast | \{ x \leftarrow x - y \} \]

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abs_diff_proof: Prove abs_diff from \( \ast 1 \mid \{ z \leftarrow (x - y) \} \)

abs_leq_proof: Prove abs_leq from \( \ast 1 \mid \{ z \leftarrow (x - y) \} \)

abs_bnd_proof: Prove abs_bnd from \( \ast 1 \mid \{ z \leftarrow (x - y) \} \)

abs_1_bnd_proof: Prove abs_1_bnd from \( \ast 1 \mid \{ z \leftarrow (x - y) \} \)

abs_2_bnd_proof: Prove abs_2_bnd from \( \ast 1 \mid \{ x \leftarrow (x - y) \} \)

End absmod

rcp_defs.tcc: Module

Using rcp_defs

Exporting all with rcp_defs

Theory

\( p \): Var naturalnumber

\( m \): Var proc_plus

\( a \): Var proc_plus

\( prop \): Var function[proc_plus \rightarrow boolean]

\( dj \): Var proc_plus

\( b \): Var proc_plus (* Existence TCC generated for processors *)

processors_TCC1: Formula (\( \exists p : (p > 0) \land (p \leq \text{ nrep}) \))

processors_TCC1: Formula (\( \exists p : (p \geq 0) \land (p \leq \text{ nrep}) \))

processors_induction_TCC1: Formula ((0 \geq 0) \land (0 \leq \text{ nrep}))

processors_induction_TCC2: Formula

(\( m < \text{ nrep} \land \text{ prop}(m) \land (\text{ prop}(0)) \lor ((m + 1 \geq 0) \land (m + 1 \leq \text{ nrep})) \))

p_reachability_TCC1: Formula

(\( \text{ if } a = 0 \text{ then } 0 \text{ else } a - 1 \text{ end if } \geq 0 \))

\( \land (\text{ if } a = 0 \text{ then } 0 \text{ else } a - 1 \text{ end if } \geq 0) \)

\( \land (\text{ if } a = 0 \text{ then } 0 \text{ else } a - 1 \text{ end if } \leq \text{ nrep}) \))

Proof

processors_TCC1_PROOF: Prove processors_TCC1

proc_plus_TCC1_PROOF: Prove proc_plus_TCC1

processors_induction_TCC1_PROOF: Prove processors_induction_TCC1

processors_induction_TCC2_PROOF: Prove processors_induction_TCC2

p_reachability_TCC1_PROOF: Prove p_reachability_TCC1

End rcp_defs.tcc
Appendix B
LaTeX-printed Supplementary Specification Listings

rcp_defs: Module
(* This rcp_defs module differs slightly from the original. Several
  definitions have been moved to new modules; the originals have
  been commented out. *)

Exporting all

Theory

p: Var nat
inputs: Type  (* type of external sensor input *)
outputs: Type  (* actuator output type *)
nrep: nat  (* number of replicated processors *)
recovery_period: nat  (* number of healthy frames required to recover
  from transient fault plus one *)
recovery_period_ax: Axiom recovery_period > 2

processors_exist_ax: Axiom nrep > 0

processors: Type from nat with (λ p : (p > 0) ∧ (p ≤ nrep))
phases: Type = (compute, broadcast, vote, sync)
ph: Var phases
next_phase: function[phases → phases] =
  (λ ph: if ph = compute
   then broadcast
   elseif ph = broadcast then vote elseif ph = vote then sync else compute
   end if)
prev_phase: function[phases → phases] =
  (λ ph: if ph = compute
   then sync
   elseif ph = broadcast
   then compute
   elseif ph = vote then broadcast else vote
   end if)
proc_plus: Type from nat with (λ p : (p ≥ 0) ∧ (p ≤ nrep))
k, m, a, n, b: Var proc_plus
prop: Var function[proc_plus → bool]
lessp: function[proc_plus, proc_plus → bool] = (λ m, n : m < n)

processors_induction: Lemma
  (∀ prop : prop(0) ∧ (∀ m : m < nrep ∧ prop(m) ⊃ prop(m + 1))
  ⊃ (∀ n : prop(n)))

Proof

Using noetherian[proc_plus, lessp]

reachability: Lemma a ≠ 0 ⇒ (∃ b : a = b + 1)

p_processors_induction: Prove processors_induction {m ↔ k@P2} from
genral_induction {p ↔ prop, d ↔ n, d2 ↔ m},
reachability {a ↔ d1@P1}

p_well_founded: Prove well_founded {measure ↔ (λ k → nat : k)}

p_reachability: Prove reachability {b ↔ if a = 0 then 0 else a - 1 end if}
End

(* This module introduces an interpretation for a basic task-oriented style of computation state. It is common to both the continuous voting and cyclic voting interpretations. *)

Using rcp_defs, sets[processors], cardinality[processors], nat_inductions

Exporting all with rcp_defs, sets[processors], cardinality[processors]

Theory

p, i, j: Var processors
k, l, q: Var nat
u: Var inputs
A: Var set[processors] (* Basic definitions for schedules *)
maj_condition: function[set[processors] → bool] =
( λ A : 2 * card(A) > card(fullset[processors]))
schedule_length: nat (* Number of frames in schedule cycle *)
schedule_length_ax: Axiom schedule_length > 0

control_state: Type from nat with ( λ k : k < schedule_length )
K, L: Var control_state
mod_plus: function[control_state, control_state → control_state] =
( λ K, L → control_state : 
  if K + L ≥ schedule_length 
  then K + L - schedule_length 
  else K + L 
end if)

mod_minus: function[control_state, control_state → control_state] =
( λ K, L → control_state : 
  if L ≥ K then K - L else schedule_length - L + K end if)

num_cells: nat
num_cells_ax: Axiom num_cells > 0

cell: Type from nat with ( λ k : k < num_cells )
cell_state: Type
cell_array: Type = array [cell] of cell_state
c, d, e: Var cell
H: Var nat
C, D: Var cell_array

(* Task schedule concepts. Each cell occupies a unique place in the schedule, being computed only once per schedule cycle. *)
cell_frame: function[cell → control_state] (* scheduled frame of cell *)
cell_subframe: function[cell → nat] (* scheduled subframe of cell *)
sched_cell: function[control_state, nat → cell] (* cell of frame, subframe *)
um_subframes: function[control_state → nat] (* subframes for this frame *)

(* Well-formedness axioms constraining these functions *)
cell_frame_ax: Axiom c = sched_cell(K', k) ⊃ cell_frame(c) = K

cell_subframe_ax: Axiom c = sched_cell(K', k) ⊃ cell_subframe(c) = k
sched_cell_ax: Axiom
\[ K = \text{cell.frame}(c) \land k = \text{cell.subframe}(c) \supset \text{sched.cell}(K, k) = c \]

num_subframes.ax: Axiom
\[ K = \text{cell.frame}(c) \supset \text{cell.subframe}(c) < \text{num.subframes}(k) \]

(* Processor state definition *)

Pstate: Type = Record
  control : control.state,
  cells : cell_array
end record

null_cell_array: cell_array (* default value *)

initial.proc.state: Pstate (* assumes each processor begins identically *)

MB: Type is Pstate

MBvec: Type = array [processors] of MB

MBmatrix: Type = array [processors] of MBvec

w: Var MBvec

h: Var MBmatrix

us, ps, X, Y: Var Pstate

cell_array.equal: Axiom
\[ \forall c: C(c) = D(c) \supset C = D \]

Pstate.equal: Axiom
\[ X.\text{control} = Y.\text{control} \land X.\text{cells} = Y.\text{cells} \supset X = Y \]

(* Interpretations for task-related functions *)

succ: function[control.state -> control.state] =
  \( (\lambda K. \text{control.state} : \text{if } K + 1 < \text{schedule.length} \text{ then } K + 1 \text{ else } 0 \text{ end if}) \)

fs: function[Pstate -> control.state] == \( (\lambda ps : ps.\text{control}) \)

ff: function[Pstate, cell -> cell.state] == \( (\lambda ps, c : ps.\text{cells}(c)) \)

(* Functions modeling task execution *)

eexec_task: function[inputs, control.state, cell_array, nat -> cell.state]

exec.measure: function[inputs, control.state, cell_array, nat -> nat] ==
  \( (\lambda u, K, C, k : k) \)

eexec: Recursive function[inputs, control.state, cell_array, nat
  -> cell.array] =
  \( (\lambda u, K, C, k :
    \text{if } k = 0
    \text{ then } C
    \text{ else exec}(u, K, C, k - 1)
    \text{ with } [(\text{sched.cell}(K, k - 1)) :=
      \text{exec.task}(u, K, \text{exec}(u, K, C, k - 1), k - 1)]
    \text{ end if})
  \)
  by exec.measure

f.: function[inputs, Pstate -> Pstate] =
  \( (\lambda u, ps : ps \text{ with } [(\text{control}) := \text{succ}(ps.\text{control}),
    (\text{cells}) := \text{exec}(u, ps.\text{control},
      ps.\text{cells}, \text{num.subframes}(ps.\text{control}))]) \)

f.: function[Pstate -> outputs] (* actuator output *)

(* Axioms to be satisfied by the generic application *)

succ_ax: Formula \( f_x(f_x(u, ps)) = \text{succ}(f_x(ps)) \)
components_equal: Formula \( f_k(X) = f_k(Y) \land (\forall c : f_k(X,c) = f_k(Y,c)) \supset X = Y \)

(* Support lemmas *)

succ_le_plus: Lemma \( \text{succ}(K) \leq K + 1 \)

mod_minus_zero: Lemma \( \text{mod\_minus}(K, L) = 0 \iff K = L \)

mod_minus_succ: Lemma \( \text{mod\_minus}(\text{succ}(K), L) = \text{succ}(\text{mod\_minus}(K, L)) \)

mod_minus_plus: Lemma \( \text{succ}(K) \neq L \supset \text{mod\_minus}(\text{succ}(K), L) = \text{mod\_minus}(K, L) + 1 \)

exec_element: Lemma
\[
\text{exec}(u, K, C, \text{num\_subframes}(K))(c) = \begin{cases} 
\text{if cell\_frame}(c) = K \\
\text{then exec\_task}(u, K, \text{exec}(u, K, C, \text{cell\_subframe}(c)), \text{cell\_subframe}(c)) \\
\text{else } C(c) 
\end{cases}
\]

Proof

p_succ_ax: Prove succ_ax from \( f_k \)

p_components_equal: Prove components_equal \( \{ c \rightarrow \text{false} \} \) from cell_array_equal \( \{ C \rightarrow X\text{\_cells}, D \rightarrow Y\text{\_cells} \} \), Pstate_equal

p_succ_le_plus: Prove succ_le_plus from succ

p_mod_minus_zero: Prove mod_minus_zero from mod_minus

p_mod_minus_succ: Prove mod_minus_succ from \( \text{mod\_minus}\{ K \rightarrow \text{succ}(K) \}, \text{mod\_minus}\{ K \rightarrow \text{mod\_minus}(K, L) \}, \text{succ} \)

p_mod_minus_plus: Prove mod_minus_plus from \( \text{mod\_minus}\{ K \rightarrow \text{succ}(K) \}, \text{mod\_minus}, \text{succ} \)

exec_prop: function\[\text{inputs, control\_state, cell\_array, cell, nat} \\
\rightarrow \text{function}[\text{nat} \rightarrow \text{bool}] = \]
\[
(\lambda u, K, C, c, k : \\
(\lambda q : \text{cell\_subframe}(c) = k \land q \leq \text{num\_subframes}(K)) \\
\supset \text{exec}(u, K, C, q)(c) = \begin{cases} 
\text{if } k < q \land \text{cell\_frame}(c) = K \\
\text{then exec\_task}(u, K, \text{exec}(u, K, C, k), k) \\
\text{else } C(c) 
\end{cases}
\]

exe_base: Lemma exe_prop(u, K, C, c, k)(0)

exe_ind_1: Lemma exe_prop(u, K, C, c, k)(q) \land \text{cell\_subframe}(c) = q \\
\supset \text{exec}(u, K, C, q)(q + 1)

exe_ind_2: Lemma exe_prop(u, K, C, c, k)(q) \land \text{cell\_subframe}(c) \neq q \\
\supset \text{exec}(u, K, C, c, k)(q + 1)

p_exe_base: Prove exe_base from exe_prop \( \{ q \rightarrow 0 \}, \text{exec} \{ k \rightarrow 0 \} \)

p_exe_ind_1: Prove exe_ind_1 from 
exe_prop \( \{ q \rightarrow q \} \), 
exe_prop \( \{ q \rightarrow q + 1 \} \), 
\text{exec} \{ k \rightarrow q + 1 \}, 
\text{sched\_cell\_ax} \{ k \rightarrow q \}, 
\text{cell\_frame\_ax} \{ k \rightarrow q \}, 
\text{num\_subframes\_ax} \]
p.exe.ind.2: Prove exe.ind.2 from
exe.prop \{q \to q\},
exe.prop \{q \to q+1\},
exec \{k \leftarrow q+1\},
cell.frame.ax \{k \leftarrow q\},
cell.subframe.ax \{k \leftarrow q\},
num.subframes.ax

p.exec.element: Prove exec.element from
nat.induction
\{p \leftarrow \text{exe.prop}(u, K, C, c, \text{cell.subframe}(c)),
n_2 \leftarrow \text{num.subframes}(K)\},
exe.prop \{q \leftarrow \text{num.subframes}(K), k \leftarrow \text{cell.subframe}(c)\},
exe.base \{k \leftarrow \text{cell.subframe}(c)\},
exe.ind.1 \{q \leftarrow u_1 \otimes p_1, k \leftarrow \text{cell.subframe}(c)\},
exe.ind.2 \{q \leftarrow u_1 \otimes p_1, k \leftarrow \text{cell.subframe}(c)\},
num.subframes.ax

End

cont.voting: Module
(* Following is the interpretation for the continuous voting scheme. *)

Using task_model, nat.inductions

Exporting all with task_model

Theory

us, ps, X, Y: Var Pstate
p, i, j: Var processors
k, l, q: Var nat
u: Var inputs
w: Var MBvec
h: Var MBmatrix
A: Var set[processors]
c, d, e: Var cell
cs: Var cell.state
K: Var control.state
H: Var nat (* Majority functions *)
k.maj: function[MBvec \to control.state]

k.maj.ax: Axiom (\exists A : maj.condition(A) \land (\forall p : p \in A \supset w(p).control = K))
\supset k.maj(w) = K

t.maj: function[MBvec, cell \to cell.state]
t.maj.ax: Axiom (\exists A :
  maj.condition(A) \land (\forall p : p \in A \supset ((w(p)).cellc) = cs))
\supset t.maj(w, c) = cs

cell.measure: function[MBvec, nat \to nat] == (\lambda w, k : k)
cell_maj: Recursive function[Mvec, nat → cell.array] =
( λ w, k : if k = 0 ∨ k > num.cells
then null.cell.array
else cell_maj[w, k - 1]
with [(k - 1) := t_maj(w, k - 1)]
end if) by cell.measure

(* Interpretations for voting-related functions *)

f_k: function[Pstate → MB] == ( λ ps : ps)
f_w: function[Pstate, MBvec → Pstate] =
( λ ps, w : ps with [(control) := k_maj(w), (cells) :=
cell_maj[w, num.cells]])

rec: function[cell, control.state, nat → bool] == ( λ c, K, H : H > 2)
dep: function[cell, cell, control.state → bool] == ( λ c, d, K : false)

recovery_period_value: Axiom recovery_period = 3

(* Definitions derived from uninterpreted functions *)

dep_agree: function[cell, control.state, Pstate, Pstate → bool] =
( λ c, K, X, Y : ( ∀ d : dep(c, d, K) ⊃ f_k(X, d) = f_k(Y, d)))
w_condition: function[set[processors], MBvec, Pstate → bool] =
( λ A, w, ps : ( ∀ p : p ∈ A ⊃ w(p) = f_k(ps)))

(* Axioms to be satisfied by the generic application *)

full_recovery: Formula H ⊳ recovery_period ⊃ rec(c, K, H)
initial_recovery: Formula rec(c, K, H) ⊃ H > 2
dep_recovery: Formula rec(c, succ(K), H + 1) ∧ dep(c, d, K) ⊃ rec(d, K, H)
control_recovered: Formula
maj_condition(A) ∧ ( ∀ p : p ∈ A ⊃ w(p) = f_k(ps)) ⊃ f_k(f_k(Y, w)) = f_k(ps)
cell_recovered: Formula
maj_condition(A) ∧ ( ∀ p : p ∈ A ⊃ w(p) = f_k(u, ps)))
∧ f_k(f_k(u, X), w), c) = f_k(f_k(u, ps), c)
vote_maj: Formula
maj_condition(A) ∧ ( ∀ p : p ∈ A ⊃ w(p) = f_k(ps)) ⊃ f_k(ps, w) = ps

(* Support lemmas *)

cell_maj_element: Lemma cell_maj[w, num.cells](c) = t_maj[w, c]
f_v_components: Lemma f_k(f_k(ps, w)) = k_maj[w] ∧ f_k(f_k(ps, w), c) = t_maj[w, c]
Proof

\( p_{\text{full.recovery}}: \text{Prove full.recovery from recovery.period.value} \)

\( p_{\text{initial.recovery}}: \text{Prove initial.recovery} \)

\( p_{\text{dep.recovery}}: \text{Prove dep.recovery} \)

\( p_{\text{control.recovered}}: \text{Prove control.recovered} \{p \leftarrow p@p1\} \text{ from} \)
\( \text{k.maj.ax} \{K \leftarrow \text{ps.control}\}, f_c \{\text{ps} \leftarrow Y, w \leftarrow w\} \)

\( p_{\text{cell.recovered}}: \text{Prove cell.recovered} \{p \leftarrow p@p1\} \text{ from} \)
\( \text{t.maj.ax} \{\text{cs} \leftarrow ((f_c(u, \text{ps})).\text{cellsc})\}, \)
\( f_c \{\text{ps} \leftarrow X\}, \)
\( f_c, \)
\( f_c \{\text{ps} \leftarrow f_c(u, X), w \leftarrow w\}, \)
\( \text{cell.maj.element} \)

\( p_{\text{vote.maj}}: \text{Prove vote.maj} \{p \leftarrow p@p4\} \text{ from} \)
\( \text{components.equal} \{X \leftarrow f_c(\text{ps}, w), Y \leftarrow \text{ps}\}, \)
\( \text{k.maj.ax} \{K \leftarrow \text{ps.control}\}, \)
\( \text{t.maj.ax} \{\text{cs} \leftarrow \text{ps}.\text{cellsc}@p1\}, c \leftarrow c@p1\}, \)
\( \text{w.condition}, \)
\( \text{w.condition} \{p \leftarrow p@p2\}, \)
\( \text{w.condition} \{p \leftarrow p@p3\}, \)
\( \text{f.v.components} \{c \leftarrow c@p1\} \)

\( \text{cme.prop: function}[\text{MBvec, cell} \to \text{function}[\text{nat} \to \text{bool}]] = \)
\( (\lambda w, c: (\lambda q : \text{cell.maj}(w, q)(c) \) \)
\( \quad \text{if } c < q \land q \leq \text{num.cells} \)
\( \quad \text{then } t\text{.maj}(w, c) \)
\( \quad \text{else } \text{null.cell.array}(c) \)
\( \text{end if}) \)

\( \text{cme.base: Lemma cme.prop}(w, c)(0) \)

\( \text{cme.ind.1: Lemma cme.prop}(w, c)(q) \land c = q \supset \text{cme.prop}(w, c)(q + 1) \)

\( \text{cme.ind.2: Lemma cme.prop}(w, c)(q) \land c \neq q \supset \text{cme.prop}(w, c)(q + 1) \)

\( p_{\text{cme.base}}: \text{Prove cme.base from cme.prop} \{q \leftarrow 0\}, \text{cell.maj} \{k \leftarrow 0\} \)

\( p_{\text{cme.ind.1}}: \text{Prove cme.ind.1 from} \)
\( \text{cme.prop} \{q \leftarrow q\}, \text{cme.prop} \{q \leftarrow q + 1\}, \text{cell.maj} \{k \leftarrow q + 1\} \)

\( p_{\text{cme.ind.2}}: \text{Prove cme.ind.2 from} \)
\( \text{cme.prop} \{q \leftarrow q\}, \text{cme.prop} \{q \leftarrow q + 1\}, \text{cell.maj} \{k \leftarrow q + 1\} \)

\( p_{\text{cell.maj.element}}: \text{Prove cell.maj.element from} \)
\( \text{nat.induction} \{p \leftarrow \text{cme.prop}(w, c), n_2 \leftarrow \text{num.cells}\}, \)
\( \text{cme.prop} \{q \leftarrow \text{num.cells}\}, \)
\( \text{cme.base}, \)
\( \text{cme.ind.1} \{q \leftarrow n_1@p1\}, \)
\( \text{cme.ind.2} \{q \leftarrow n_1@p1\} \)

\( p_{\text{f.v.components}}: \text{Prove f.v.components from } f_c, \text{cell.maj.element} \)

End
cyclic_voting: Module
(* Following is the interpretation for the cyclic voting scheme. *)

Using task_model, nat_inductions

Exporting all with task_model

Theory

us, ps, X, Y: Var Pstate
p, i, j: Var processors
k, l, q: Var nat
w: Var inputs
u: Var MBvec
k: Var MBmatrix
A: Var set[processors]
c, d, e: Var cell
cs: Var cell.state
K, L: Var control.state
H: Var nat
C, D: Var cell.array
cell_fn: Type is function[cell → cell.state]
cfn: Var cell_fn (* Majority functions *)
k_maj: function[MBvec → control.state]

k_maj.ax: Axiom (∃A:
    maj_condition(A) ∧ (∀p ∈ A ∋ w(p).control = K))
    ⇒ k_maj(w) = K

t_maj: function[MBvec, cell → cell.state]

t_maj.ax: Axiom (∃A:
    maj_condition(A) ∧ (∀p ∈ A ∋ ((w(p)).cells) = cs))
    ⇒ t_maj(w, c) = cs

cell_measure: function[cell_fn, control.state, cell.array, nat → nat] ==
(λcnf, K, C, k → cell.array) =
(λcnf, K, C, k:
    if k = 0 ∨ k > num_cells
    then C
    elseif K = succ(cell.frame(k - 1))
    then cell_apply(cnf, K, C, k - 1)
    with [(k - 1) := cnf(k - 1)]
    else cell_apply(cnf, K, C, k - 1)
    end if)
by cell_measure

(* Interpretations for voting-related functions *)

f_s: function[Pstate → MB] =
(λps: ps with [(control) := ps.control, (cells) :=
cell_apply((λc: ps.cells(c)),
ps.control,
null.cell.array,
um.cells)])
f_\,: \text{function}[\text{Pstate, MBvec} \rightarrow \text{Pstate}] =
( \lambda ps, w : ps \text{ with } [(\text{control}) := k_{\text{maj}}(w),
(cells) := \text{cell_apply}((\lambda c : t_{\text{maj}}(w, c)),
ps.control,
ps.cells,
um.cells)])

rec: \text{function}[\text{cell, control_state, nat} \rightarrow \text{bool}] =
( \lambda c, K, H : H > 1 + ( \text{if } K = \text{cell_frame}(c)
\text{then } \text{schedule_length}
\text{else } \text{mod_minus}(K, \text{cell_frame}(c))
\text{end if}))

dep: \text{function}[\text{cell, cell, control_state} \rightarrow \text{bool}] =
( \lambda c, d, K : \text{cell_frame}(c) \neq K \land c = d)

\text{recovery_period.value: Axiom } \text{recovery_period} = \text{schedule_length} + 2

(* Definitions derived from uninterpreted functions *)
\text{dep_agree: function}[\text{cell, control_state, Pstate, Pstate} \rightarrow \text{bool}] =
( \lambda c, K, X, Y : (\forall d : \text{dep}(c, d, K) \supset f_{\lambda}(X, d) = f_{\lambda}(Y, d)))

w_condition: \text{function}[\text{set[processors], MBvec, Pstate} \rightarrow \text{bool}] =
( \lambda A, w, ps : (\forall p : p \in A \supset w(p) = f_{\lambda}(ps)))

(* Axioms to be satisfied by the generic application *)

\text{full_recovery: Formula } H \geq \text{recovery_period} \supset \text{rec}(c, K, H)

\text{initial_recovery: Formula } \text{rec}(c, K, H) \supset H > 2

\text{dep.recovery: Formula } \text{rec}(c, \text{succ}(K), H + 1) \land \text{dep}(c, d, K) \supset \text{rec}(d, K, H)

\text{control.recovered: Formula}
\text{maj.condition}(A) \land (\forall p : p \in A \supset w(p) = f_{\lambda}(ps)) \supset f_{\lambda}(f_{\lambda}(Y, w)) = f_{\lambda}(ps)

\text{cell.recovered: Formula}
\text{maj.condition}(A) \land (\forall p : p \in A \supset w(p) = f_{\lambda}(f_{\lambda}(u, ps)))
\land f_{\lambda}(X) = K \land f_{\lambda}(ps) = K \land \text{dep_agree}(c, K, X, ps)
\supset f_{\lambda}(f_{\lambda}(f_{\lambda}(u, X), w), c) = f_{\lambda}(f_{\lambda}(u, ps), c)

\text{vote_maj: Formula}
\text{maj.condition}(A) \land (\forall p : p \in A \supset w(p) = f_{\lambda}(ps)) \supset f_{\lambda}(ps, w) = ps

(* Support lemmas *)

\text{cell.apply_element: Lemma}
\text{cell.apply(cfn, K, C, num.cells)}(c) = \text{if } K = \text{succ(cell.frame}(c)) \text{ then cfn}(c) \text{ else } C(c) \text{ end if}
\textbf{f.s. components: Lemma}
\[ K = \text{ps.control} \supset f_s(f_u(ps)) = K \]
\[ \land f_s(f_u(ps), c) \]
\[ = \begin{cases} \text{if succ(cell.frame}(c)) = K & \text{then ps.cells}(c) \\ \text{else null.cell.array}(c) \end{cases} \]
\textbf{f.v. components: Lemma}
\[ f_u(f_s(ps, w)) = k.maj(w) \]
\[ \land f_u(f_s(ps, w), c) \]
\[ = \begin{cases} \text{if succ(cell.frame}(c)) = \text{ps.control} & \text{then t.maj}(w, c) \\ \text{else ps.cells}(c) \end{cases} \]
\textbf{f.c. uncomputed_cells: Lemma}
\[ \text{cell.frame}(c) \neq X.\text{control} \supset f_c(u, X).\text{cells}(c) = X.\text{cells}(c) \]

\textbf{Proof}
\textbf{p.full.recovery: Prove full.recovery from}
\[ \text{rec,} \]
\[ \text{recovery.period.value,} \]
\[ \text{control.state.invariant} \]
\[ \{ \text{control.state.var} \leftarrow \text{mod.minus}(K, \text{cell.frame}(c@pl)) \} \]
\textbf{p.initial.recovery: Prove initial.recovery from}
\[ \text{rec,} \]
\[ \text{schedule.length.ax,} \]
\[ \text{mod.minus.zero} \{ L \leftarrow \text{cell.frame}(c@pl) \}, \]
\[ \text{nat.invariant} \{ \text{nat.var} \leftarrow \text{mod.minus}(K, \text{cell.frame}(c@pl)) \} \]
\textbf{p.dep.recovery: Prove dep.recovery from}
\[ \text{rec} \{ K \leftarrow \text{succ}(K), H \leftarrow H + 1 \}, \]
\[ \text{dep,} \]
\[ \text{rec} \{ c \leftarrow d \}, \]
\[ \text{control.state.invariant} \{ \text{control.state.var} \leftarrow \text{mod.minus}(K, \text{cell.frame}(c)) \}, \]
\[ \text{mod.minus.plus} \{ L \leftarrow \text{cell.frame}(c) \} \]
\textbf{p.control.recovered: Prove control.recovered \{ p \leftarrow p@p1 \} from}
\[ k.maj.ax \{ K \leftarrow \text{ps.control} \}, f_s \{ \text{ps} \leftarrow Y, w \leftarrow w, f_s \}, \]
\textbf{p.cell.recovered: Prove cell.recovered \{ p \leftarrow p@p1 \} from}
\[ t.maj.ax \{ cs \leftarrow ((f_s(f_u(ps))).\text{cells}) \}, \]
\[ \text{dep.agree} \{ Y \leftarrow \text{ps}, d \leftarrow c \}, \]
\[ \text{dep} \{ d \leftarrow c \}, \]
\[ f_s.\text{components} \{ \text{ps} \leftarrow f_s(u, ps), K \leftarrow (f_u(u, X)).\text{control} \}, \]
\[ f_c.\text{components} \{ X \leftarrow \text{ps} \}, \]
\[ f_c.\text{components}, \]
\[ f_u \{ \text{ps} \leftarrow X \}, \]
\[ f_u \text{,} \]
\[ f_v.\text{components} \{ \text{ps} \leftarrow f_u(u, X) \} \]
p_vote.maj: Prove vote_maj {p p p4} from components.equal {X f1(ps w), Y ps}, k_maj.ax {K ps.control}, t_maj.ax {cs ps.cellac p1}, c c p1}, w.condition, w.condition {p p p2}, w.condition {p p p3}, f , cell.apply_element
{cfn (λ c : ps.cells(c)), c c p1, K ps.control, C null.cell.array}, f_v.components {c c p1}
cae.prop: function[cel_fn, control.state, cell_array, cell
→ function[nat → bool] =
(λ cf, K, C, c :
(λ q : cell.apply(cf, K, C, q)(c)
  = if c q ∧ q ≤ num.cells ∧ K = succ.cell.frame(c)
    then cf(c)
    else C(c)
  end if))
cae_base: Lemma cae_prop(cf, K, C, c)(0)
cae.ind.1: Lemma cae_prop(cf, K, C, c)(q) ∧ c q
  ⊂ cae.prop(cf, K, C, c)(q + 1)
cae.ind.2: Lemma cae_prop(cf, K, C, c)(q) ∧ c q
  ⊂ cae.prop(cf, K, C, c)(q + 1)
p.ca.base: Prove cae_base from cae.prop {q 0}, cell.apply {k 0}
p.ca.ind.1: Prove cae.ind.1 from
cae.prop {q q}, cae.prop {q q + 1}, cell.apply {k q + 1}
p.ca.ind.2: Prove cae.ind.2 from
cae.prop {q q}, cae.prop {q q + 1}, cell.apply {k q + 1}
p.cell.apply.element: Prove cell.apply.element from
nat.induction {p cae.prop(cf, K, C, c), n2 num.cells},
cae.prop {q num.cells},
cae.base,
cae.ind.1 {q n1 p1},
cae.ind.2 {q n1 p1}
p.f.s.components: Prove f_s.components from
f ,
cell.apply_element
{cfn (λ c : ps.cells(c)), K ps.control, C null.cell.array}
p.f.v.components: Prove f_v.components from
f ,
cell.apply_element
{cfn (λ c : t_maj(u, c)), K ps.control, C ps.cells}
p.f.c.uncomputed.cells: Prove f.c.uncomputed.cells from
f: {ps → X}, exec.element {C → X.cells, K → X.control}

End
Appendix C

Results of Proof Chain Analysis

The following pages were obtained from Elndm using the proof-chain analyzer command (M-x apcs) applied to the module top.

T terse proof chains for module top

Use of the formula

\[ RS\_to\_US\_frame\_commutes \]
requires the following TCCs to be proven

\[ RS\_to\_US\_tcc\_reachable\_in\_n\_TCC1 \]
\[ RS\_to\_US\_tcc\_reachable\_in\_n\_TCC2 \]

Formula \( RS\_to\_US\_tcc\_reachable\_in\_n\_TCC2 \) is a termination TCC for \( DA\_to\_DS\_reachable\_in\_n \)
Proof of

\[ RS\_to\_US\_tcc\_reachable\_in\_n\_TCC2 \]
must not use

\[ DA\_to\_DS\_reachable\_in\_n \]

Use of the formula

\[ rcp\_defs\_recovery\_period\_ax \]
requires the following TCCs to be proven

\[ rcp\_defs\_tcc\_processors\_TCC1 \]
\[ rcp\_defs\_tcc\_proc\_plus\_TCC1 \]
\[ rcp\_defs\_tcc\_processors\_induction\_TCC1 \]
\[ rcp\_defs\_tcc\_processors\_induction\_TCC2 \]
\[ rcp\_defs\_tcc\_p\_reachability\_TCC1 \]

Use of the formula

\[ \text{cardinality}[rcp\_defs\_processors]\_card\_empty \]
requires the following assumptions to be discharged

\[ \text{cardinality}[rcp\_defs\_processors]\_finite \]

Use of the formula

\[ DS\_to\_RS\_frame\_commutes \]
requires the following TCCs to be proven

\[ DS\_to\_RS\_tcc\_ss\_update\_TCC1 \]
\[ DS\_to\_RS\_tcc\_ss\_update\_TCC2 \]
\[ DS\_to\_RS\_tcc\_ss\_update\_TCC3 \]
\[ DS\_to\_RS\_tcc\_MBmatrix\_cons\_TCC1 \]

Formula \( DS\_to\_RS\_tcc\_ss\_update\_TCC3 \) is a termination TCC for \( DA\_to\_DS\_ss\_update \)
Proof of

\[ DS\_to\_RS\_tcc\_ss\_update\_TCC3 \]
must not use

\[ DA\_to\_DS\_ss\_update \]

Formula \( DS\_to\_RS\_tcc\_MBmatrix\_cons\_TCC1 \) is a termination TCC for \( DA\_to\_DS\_MBmatrix\_cons \)
Proof of

\[ DS\_to\_RS\_tcc\_MBmatrix\_cons\_TCC1 \]
must not use

\[ DA\_to\_DS\_MBmatrix\_cons \]
Use of the formula
\texttt{noetherian[rcp\_defs.proc\_plus, rcp\_defs.lessp].general\_induction}
requires the following assumptions to be discharged
\texttt{noetherian[rcp\_defs.proc\_plus, rcp\_defs.lessp].well\_founded}

Use of the formula
\texttt{DS\_support\_proof.sll3\_prop}
requires the following TCCs to be proven
\texttt{DS\_support\_proof\_tcc.p_sll3\_base\_TCC1}
\texttt{DS\_support\_proof\_tcc.p_sll3\_ind\_TCC1}
\texttt{DS\_support\_proof\_tcc.p_support\_13\_TCC1}

Use of the formula
\texttt{DS\_map\_proof.mll\_prop}
requires the following TCCs to be proven
\texttt{DS\_map\_proof\_tcc.p_mll\_base\_TCC1}
\texttt{DS\_map\_proof\_tcc.p_mll\_ind\_TCC1}
\texttt{DS\_map\_proof\_tcc.p_map\_l\_TCC1}

Use of the formula
\texttt{DA\_to\_DS.phase\_commutes}
requires the following TCCs to be proven
\texttt{DA\_to\_DS\_tcc.ss\_update\_TCC1}
\texttt{DA\_to\_DS\_tcc.ss\_update\_TCC2}
\texttt{DA\_to\_DS\_tcc.ss\_update\_TCC3}
\texttt{DA\_to\_DS\_tcc.MBmatrix\_cons\_TCC1}
\texttt{DA\_to\_DS\_tcc.reachable\_in\_n\_TCC1}
\texttt{DA\_to\_DS\_tcc.reachable\_in\_n\_TCC2}

Formula \texttt{DA\_to\_DS\_tcc.ss\_update\_TCC3} is a termination TCC
for \texttt{DA\_to\_DS\_ss\_update}
Proof of
\texttt{DA\_to\_DS\_tcc.ss\_update\_TCC3}
must not use
\texttt{DA\_to\_DS\_ss\_update}

Formula \texttt{DA\_to\_DS\_tcc.MBmatrix\_cons\_TCC1} is a termination TCC for
\texttt{DA\_to\_DS\_MBmatrix\_cons}
Proof of
\texttt{DA\_to\_DS\_tcc.MBmatrix\_cons\_TCC1}
must not use
\texttt{DA\_to\_DS\_MBmatrix\_cons}

Formula \texttt{DA\_to\_DS\_tcc.reachable\_in\_n\_TCC2} is a termination TCC for
\texttt{DA\_to\_DS\_reachable\_in\_n}
Proof of
\texttt{DA\_to\_DS\_tcc.reachable\_in\_n\_TCC2}
must not use
\texttt{DA\_to\_DS\_reachable\_in\_n}

Use of the formula
\texttt{DA\_map\_proof.mll\_prop}
requires the following TCCs to be proven
\texttt{DA\_map\_proof\_tcc.p_mll\_base\_TCC1}
\texttt{DA\_map\_proof\_tcc.p_mll\_ind\_TCC1}
\texttt{DA\_map\_proof\_tcc.p_map\_l\_TCC1}
Use of the formula
   DA_broadcast_prf.rtp4
requires the following TCCs to be proven
   DA_broadcast_prf_tcc.p_br8_TCC1

Use of the formula
   clkmod.in_R_interval
requires the following TCCs to be proven
   clkmod_tcc.half_TCC1
   clkmod_tcc.Corr_TCC1
   clkmod_tcc.Corr_TCC2
   clkmod_tcc.Corr_TCC3
   clkmod_tcc.Corr_TCC4
   clkmod_tcc.Corr_TCC5
   clkmod_tcc.Corr_TCC6_TCC1

Formula clkmod_tcc.Corr_TCC1 is a termination TCC for clkmod.Corr
Proof of
   clkmod_tcc.Corr_TCC1
must not use
   clkmod.Corr

Formula clkmod_tcc.num_good_clocks_TCC4 is a termination TCC for
clkmod.num_good_clocks
Proof of
   clkmod_tcc.num_good_clocks_TCC4
must not use
   clkmod.num_good_clocks

Formula clkmod_tcc.num_good_clocks_TCC5 is a termination TCC for
clkmod.num_good_clocks
Proof of
   clkmod_tcc.num_good_clocks_TCC5
must not use
   clkmod.num_good_clocks

Use of the formula
   DA_invariants.state_invariant
requires the following TCCs to be proven
   DA_invariants_tcc.Corr_lem_TCC1

Use of the formula
   DA_support_proof.sll5_prop
requires the following TCCs to be proven
   DA_support_proof_tcc.p_sll3_base_TCC1
   DA_support_proof_tcc.p_sll3_ind_TCC1
   DA_support_proof_tcc.p_support_13_TCC1
   DA_support_proof_tcc.p_sll3_ind_TCC1

SUMMARY

The proof chain is complete

The axioms and assumptions at the base are:
   DA.all_durations
   DA.broadcast_duration
The definitions and type-constraints are:

DA.N_da
DA.N_da_broadcast
DA.N_da_compute
DA.N_da_sync
DA.N_da_vote
DA.broadcast_received
DA.clock_advanced
DA.da_rt
DA.enough_hardware
DA.initial_da
DA.maj_working
DA.working_proc
DA.working_set
DA.invariants.cum_delta_val
DA.invariants.lclock_eq
DA.invariants.lclock_val
DA.invariants.nf_clks
DA.invariants.state_invariant
DA.invariants.state_invariant_to_n
DA.lemmas.hide1
DA.map_proof.ml1_prop
DA.map_proof.ml2_prop
DA.map_proof.ml4_prop
DA.support_proof.s115_prop
DA.to_DS.DAmap
DA.to_DS.reachable
DA.to_DS.reachable_in_n
DA.to_DS.ss_update
DS.N_ds
DS.N_ds_broadcast
DS.N_ds_compute

Total: 26
The formulae used are:
DA\_broadcast\_prf\_br1
DA\_broadcast\_prf\_br1a
DA\_broadcast\_prf\_br2
DA\_broadcast\_prf\_br3
DA\_broadcast\_prf\_br3\_aa
DA\_broadcast\_prf\_br4
DA\_broadcast\_prf\_br5
DA\_broadcast\_prf\_br6
DA\_broadcast\_prf\_br7
DA\_broadcast\_prf\_br8
DA\_broadcast\_prf\_br9
DA\_broadcast\_prf\_int5
DA\_broadcast\_prf\_rtp0
DA\_broadcast\_prf\_rtp0a
DA\_broadcast\_prf\_rtp1
DA\_broadcast\_prf\_rtp2
DA\_broadcast\_prf\_rtp3
DA\_broadcast\_prf\_rtp4
DA\_broadcast\_prf\_rtp4a
DA\_broadcast\_prf\_rtp4b
DA\_broadcast\_prf\_rtp5
DA\_broadcast\_prf\_rtp6
DA\_broadcast\_prf\_rtp7
DA\_broadcast\_prf\_tcc\_p\_br8\_TCC1
DA\_intervals\_br\_int
DA\_intervals\_int0
DA\_intervals\_int1
DA\_intervals\_int1a
DA\_intervals\_int2
DA\_intervals\_int2a
DA\_intervals\_int3
DA\_intervals\_int4
DA\_invariants\_base\_state\_ind
DA\_invariants\_cdi\_12a
DA\_invariants\_clkval\_inv
DA\_invariants\_clkval\_inv\_11
DA\_invariants\_clkval\_inv\_12
DA\_invariants\_cum\_delta\_inv
DA\_invariants\_cum\_delta\_inv\_11
DA\_invariants\_cum\_delta\_inv\_12
DA\_invariants\_cum\_delta\_inv\_14
DA\_invariants\_da\_rt\_lem
DA\_invariants\_enough\_inv
DA\_invariants\_enough\_inv\_11
DA\_invariants\_enough\_inv\_12
DA\_invariants\_ind\_state\_ind
DA\_invariants\_lclock\_inv
DA\_invariants\_lclock\_inv\_11
DA\_invariants\_lclock\_inv\_12
DA\_invariants\_lclock\_inv\_12b
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DA_invariants.nfclk_inv_11
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DA_support_proof_tcc.p_s113_base_TCC1
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DA_support_proof_tcc.p_support_13_TCC1
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DS_lemmas.fc_A_1b
DS_lemmas.fc_A_1c
DS_lemmas.fc_A_1d
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DS_lemmas.fc_A_2d
DS_lemmas.fc_A_3a
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DA_intervals.p_int0
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DA_map_proof_tcc.p_map_11_ind_TCC1_PROOF
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DA_support_proof.p_s115_ind
DA_support_proof_p_support_1
DA_support_proof_p_support_14
DA_support_proof_p_support_15
DA_support_proof_p_support_16
DA_support_proof_tcc.p_s113_base_TCC1_PROOF
DA_support_proof_tcc.p_s113_ind_TCC1_PROOF
DA_support_proof_tcc.p_support_13_TCC1_PROOF
DA_tcc_proof.C6_TCC1_PROOF
DA_tcc_proof.Rho_TCC1_PROOF
DA_tcc_proof.p_s115_ind_TCC1_PROOF
DA_to_DS_tcc.MNmatrix_cons_TCC1_PROOF
DA_to_DS_tcc.reachable_in_n_TCC1_PROOF
DA_to_DS_tcc.reachable_in_n_TCC2_PROOF
DA_to_DS_tcc.ss_update_TCC1_PROOF
DA_to_DS_tcc.ss_update_TCC2_PROOF
DA_to_DS_tcc.ss_update_TCC3_PROOF
DA_top_proof.p_ELT
DA_top_proof.p_com_broadcast_1
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DA_top_proof.p_com_vote_2
DA_top_proof.p_com_vote_3
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In this paper the design and formal verification of the Reliable Computing Platform (RCP), a fault-tolerant computing system for digital flight control applications, is presented. The RCP utilizes N-Multiply Redundant (NMR) style redundancy to mask faults and internal majority voting to flush the effects of transient faults. The system is formally specified and verified using the Ehdm verification system. A major goal of this work is to provide the system with significant capability to withstand the effects of High Intensity Radiated Fields (HIRF).