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# A $k - \epsilon$ CALCULATION OF TRANSITIONAL BOUNDARY LAYERS

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## ABSTRACT

A recently proposed  $k - \epsilon$  model for low Reynolds number turbulent flows (Yang and Shih, 1991) was modified by introducing a new damping function  $f_\mu$ . The modified model is used to calculate the transitional boundary layer over a flat plate with different freestream turbulence levels. It is found that the model could mimic the transitional flow. However, the predicted transition is found to be sensitive to the initial conditions.

## 1. Introduction

In a quiescent environment, transition is initiated by the amplification of Tollmien-Schlichting waves. These waves eventually breakdown, giving rise to turbulent spots, which are precursors of the turbulent boundary layer. While in an environment with high freestream turbulence, say the flow passing over a turbine blade, turbulent spots are formed due to the transport of turbulence from the freestream to the boundary layer, rather than from the T-S wave amplification.

Priddin (1975) was the first to show that the low Reynolds number two equation models have the potential to predict transitional flows under the influence of the freestream turbulence. A detailed calculation procedure was given by Rodi and Scheuerer (1985), in which the Lam & Bremhorst low Reynolds number  $k - \epsilon$  model was used. More recently, a comparative study of the performance of existing low Reynolds number  $k - \epsilon$  models in predicting laminar-turbulent transition was made by Fujisawa (1990).

Recently, a low Reynolds number  $k - \epsilon$  model was proposed by the authors (Yang and Shih, 1991). The proposed model was found to perform quite well for turbulent channel flows and fully developed turbulent boundary layer flows at different Reynolds numbers. The purpose of this work is to test the capability of the proposed model in predicting the laminar-turbulent transition. The test cases are the transitional boundary layer flows over a flat plate with a freestream turbulence level of 3% (Case T3A) and 6% (Case T3B) respectively. These cases are the benchmark cases in an ongoing project coordinated by

Savill (1991), testing the capability of turbulence models in predicting transitional flows.

The plan of the paper is as follows: in Section 2, we list the equations to be solved, and the corresponding initial and boundary conditions; in Section 3, we brief the numerical aspects of the calculation; the results of the calculation are shown in Section 4 and conclusions are presented in Section 5.

## 2. Mathematical Formulations

The boundary layer approximation is used. Upon using the eddy viscosity assumption, the equations for the mean field over a flat plate boundary layer are:

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \quad (1)$$

$$U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = \frac{\partial}{\partial y} \left[ (\nu + \nu_T) \frac{\partial U}{\partial y} \right] \quad (2)$$

where  $x, y$  are the coordinates along and normal to the plate, and  $U, V$  are the mean velocities in the  $x, y$  directions, respectively.

Recently, Yang and Shih (1991) proposed a low Reynolds number  $k - \epsilon$  model to close the above equations. In this model, the eddy viscosity is given by

$$\nu_T = c_\mu f_\mu k T \quad (3)$$

where the time scale  $T$  has the Kolmogorov time scale as the lower bound:

$$T = \frac{k}{\epsilon} + \left(\frac{\nu}{\epsilon}\right)^{1/2} \quad (4)$$

The transport equations for  $k$  and  $\epsilon$  are

$$U \frac{\partial k}{\partial x} + V \frac{\partial k}{\partial y} = \frac{\partial}{\partial y} \left[ \left(\nu + \frac{\nu_T}{\sigma_k}\right) \frac{\partial k}{\partial y} \right] + \nu_T \left(\frac{\partial U}{\partial y}\right)^2 - \epsilon \quad (5)$$

$$U \frac{\partial \epsilon}{\partial x} + V \frac{\partial \epsilon}{\partial y} = \frac{\partial}{\partial y} \left[ \left(\nu + \frac{\nu_T}{\sigma_\epsilon}\right) \frac{\partial \epsilon}{\partial y} \right] + \left[ C_1 \nu_T \left(\frac{\partial U}{\partial y}\right)^2 - C_2 f_2 \epsilon \right] \frac{1}{T} + E \quad (6)$$

where

$$f_2 = 1.0 - 0.22 \exp\left(-\frac{R_t^2}{36}\right),$$

$$E = \nu \nu_T \left(\frac{\partial U}{\partial y}\right)^2,$$

and the turbulent Reynolds number is defined by  $R_t = \frac{kT}{\nu}$

The model constants used are the same as those in the Standard Model. i.e.

$$c_\mu = 0.09, C_1 = 1.45, C_2 = 1.92, \sigma_k = 1.0, \sigma_\epsilon = 1.3.$$

Thus, the performance of the model in high turbulent Reynolds number flows is ensured.

The damping function  $f_\mu$  used in Yang and Shih (1991) is a function of  $y^+$ . It has been documented (see Stephens and Crawford, 1990, for example) that the  $y^+$  dependence in the damping function  $f_\mu$  would lead to an early transition in the model prediction, and a lack of minimum  $c_f$  (which signifies the start of transition) when the freestream turbulence is high. We here introduce a new damping function which is a function of  $R_y$ , where  $R_y$  is defined by

$$R_y = \frac{k^{1/2}y}{\nu}. \quad (7)$$

The form of  $f_\mu$  is chosen to be

$$f_\mu = [1 - \exp(-a_1 R_y - a_3 R_y^3 - a_5 R_y^5)]^{1/2} \quad (8)$$

where the constants in the damping function take the following values

$$a_1 = 1.5 \times 10^{-4}, a_3 = 5.0 \times 10^{-7}, a_5 = 1.0 \times 10^{-10}.$$

These values are determined by calibration with the DNS data for channel flows. The damping function chosen gives the correct asymptotic behavior near the wall. Far away from the wall, the damping function approaches one as required. The shape of the damping function is shown in Fig. 1.

The above system of parabolic equations need to be supplemented by the boundary conditions at the wall and at the freestream, and the initial conditions at the starting point of the calculation. At the wall

$$\begin{aligned} U &= V = k = 0, \\ \epsilon_w &= 2\nu \left( \frac{\partial k^{1/2}}{\partial y} \right)^2. \end{aligned} \quad (9)$$

At the edge of the boundary layer, the flow variables are given by the freestream values, i.e.

$$U = U_e, k = k_e, \epsilon = \epsilon_e. \quad (10)$$

The  $k_e, \epsilon_e$  in the above are found from the transport equations for  $k$  and  $\epsilon$ , with the condition that the gradient of the flow variables in the  $y$  direction vanishes as the free stream is approached. Thus,

$$U_e \frac{dk_e}{dz} = -\epsilon_e, \quad (11)$$

$$U_e \frac{d\epsilon_e}{dz} = -\frac{C_2 \epsilon_e}{T}. \quad (12)$$

$k_{e0}$  and  $\epsilon_{e0}$  (i.e., the values of  $k_e$  and  $\epsilon_e$  at the leading edge) are needed.  $k_{e0}$  is obtained from the experiment, and  $\epsilon_{e0}$  is determined in such a way that the resulting  $k_e(x)$  profile agrees with the experiment. In the cases considered here,

$$\frac{k_{e0}}{U_e^2} = 1.18 \times 10^{-3}, \frac{\epsilon_{e0}}{U_e^3/L} = 2.86 \times 10^{-6}$$

for T3A, and

$$\frac{k_{e0}}{U_e^2} = 4.94 \times 10^{-3}, \frac{\epsilon_{e0}}{U_e^3/L} = 1.22 \times 10^{-5}$$

for T3B, where  $L = 1mm$  is the reference length scale.

A major difficulty in calculating the transitional boundary layer through these parabolic equations is the lack of information on the initial conditions, particularly, the initial conditions for  $k$  and  $\epsilon$ . Following Rodi and Scheuerer (1985), we adopt the following initial conditions. At the starting location where the boundary layer is still laminar,

$$\frac{U}{U_e} = U_{Blasius}, k = k_e \left( \frac{U}{U_e} \right)^2, \epsilon = 0.3k \frac{\partial U}{\partial y}, \quad (13)$$

with a further constraint that  $\epsilon \geq \epsilon_e$ .

### 3. Numerical Aspects

A semi-implicit finite difference scheme is used to solve the momentum equation and the transport equations for  $k$  and  $\epsilon$ . The coefficients for the convective terms are lagged one step in the marching direction, and the source terms in the  $k$  and  $\epsilon$  equations are linearized in such a way that numerical stability is ensured.

A variable grid spacing is used to resolve the sharp gradient near the wall. The grid distribution is controlled by  $\delta y_i / \delta y_{i-1} = \alpha$ . Both  $\alpha$  and the total number of grid points,  $N$ , are varied to ensure the grid independence of the numerical results. The marching step size,  $\delta x$ , is also varied to ensure accuracy. It is found that  $N = 150$ ,  $\alpha = 1.03$ , and  $\delta x = 0.02$  are sufficient to give a solution with less than a 1.0% error.

### 4. Results of Calculations

The modification of Yang and Shih (1991) is first tested for fully developed turbulent flow passing a flat plate boundary layer at  $Re_\theta = 1410$ , where the DNS data is available (Spalart, 1988). In the freestream, the turbulence is assumed to be zero. Arbitrary profiles for the initial conditions are used. The results for the mean velocity  $U$  and the turbulent kinetic energy  $k$  are shown in Fig. 2 and Fig. 3, respectively. Also shown in these figures

are the DNS data. The agreement is very good. Same agreement is also obtained for the turbulent boundary layer at  $Re_\theta = 670$  and the turbulent channel flow at  $Re_\tau = 395$ .

Next, calculations are made for the transitional boundary layer passing a flat plate with a freestream turbulence level of 3% (Case T3A) and 6% (case T3B), respectively. The calculations are started at  $Re_\theta = 100$  where the boundary layer is still laminar. The distribution of the shape factor  $H$  as a function of  $Re_x$  is shown in Fig. 4. As  $Re_x$  increases,  $H$  varies from 2.59 (the value for the laminar boundary layer) to 1.41 (the value for the turbulent boundary layer.) The change is more gradual for the high freestream turbulence level (Case T3B) than for the low freestream turbulence level (Case T3A), in agreement with the experiment. Fig. 5 shows the distribution of the skin friction coefficient  $c_f$  as a function of  $Re_x$  for Case T3A. Also shown in this figure is the experimental result reported in Savill (1991). It is found that  $c_f$  varies from a laminar relation to a turbulent relation. Thus, the model prediction does mimic transition. However, the predicted onset of transition (defined as the point of minimum  $c_f$ ) occurs earlier than that found in the experiment. The model prediction gives  $Re_x = 8.0 \times 10^4$  while the experimental result is  $Re_x = 1.3 \times 10^5$ . Similar comparison for Case T3B is presented in Fig. 6. In this case, the predicted transition starts at  $Re_x = 4.2 \times 10^4$  while transition starts at  $Re_x = 5.9 \times 10^4$  in the experiment. The effect of freestream turbulence on transition is captured by the model. It is seen that the higher the freestream turbulence, the earlier the transition. In both cases, the predicted transition from laminar boundary layer to turbulent boundary is sharper than the experimental one.

Since it is unlikely that the initial conditions used are the physical profiles for a transitional boundary layer at  $Re_\theta = 100$  found from an experiment, the effect of the initial conditions has to be studied before the results of the calculations can be assessed. Since we are solving a system of parabolic equations, the change in initial conditions is equivalent to a change in the location of the starting point with the same initial profiles. Calculations were carried out with different starting locations. Fig. 7 and Fig. 8 show the  $c_f$  distribution obtained from starting the calculation at  $Re_\theta = 10$  for Case T3A and Case T3B, respectively. It is found that the predicted transition has moved upstream. Transition starts at  $Re_x = 3.2 \times 10^4$  for Case T3A and  $Re_x = 2.0 \times 10^4$  for Case T3B. Thus, the initial conditions have a significant effect on the results of the calculation. This finding is in agreement with that reported in Schmidt and Patankar (1988).

## 5. Conclusions

A recently proposed low Reynolds number  $k - \epsilon$  model is modified through the introduction of a new damping function which depends on  $R_y$  rather than  $y^+$ . This modification is found to perform well for both fully developed turbulent flat plate boundary layer flows

and channel flows. It is also found that it has the capacity to capture the transition from a laminar boundary layer to a turbulent boundary layer. However, the predicted transition is found to be sensitive to the initial conditions, or equivalently, the starting location of the calculation if the same initial profile is used.

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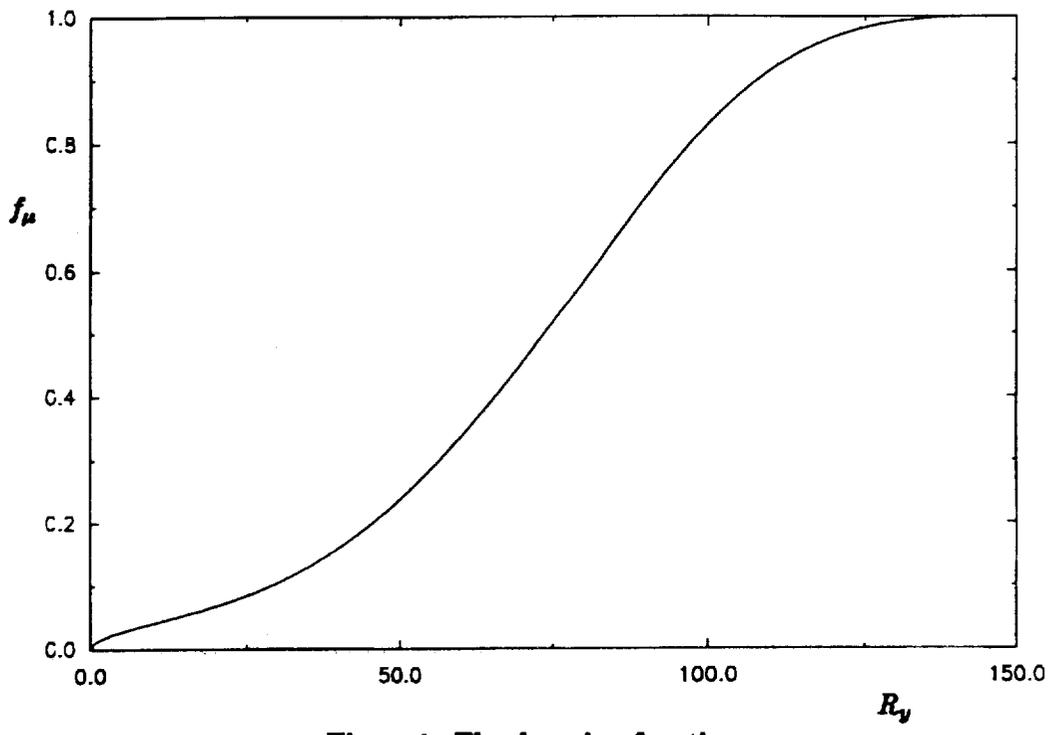


Figure 1. The damping function.

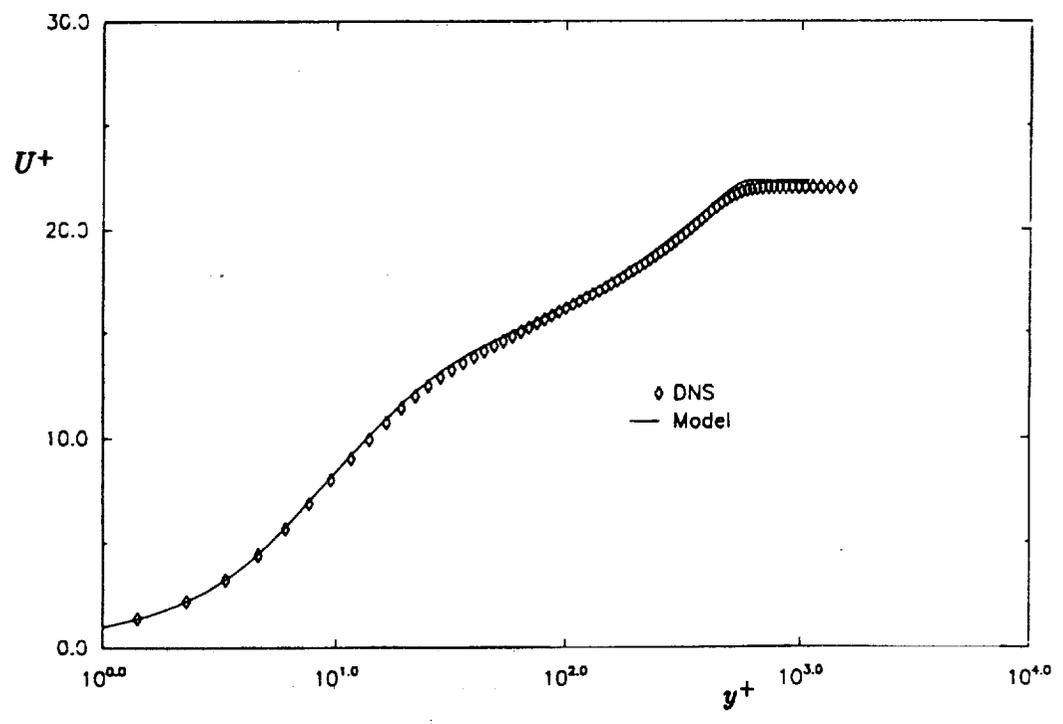


Figure 2. Mean velocity profile for flat plate boundary layer,  $Re_\theta = 1410$ .

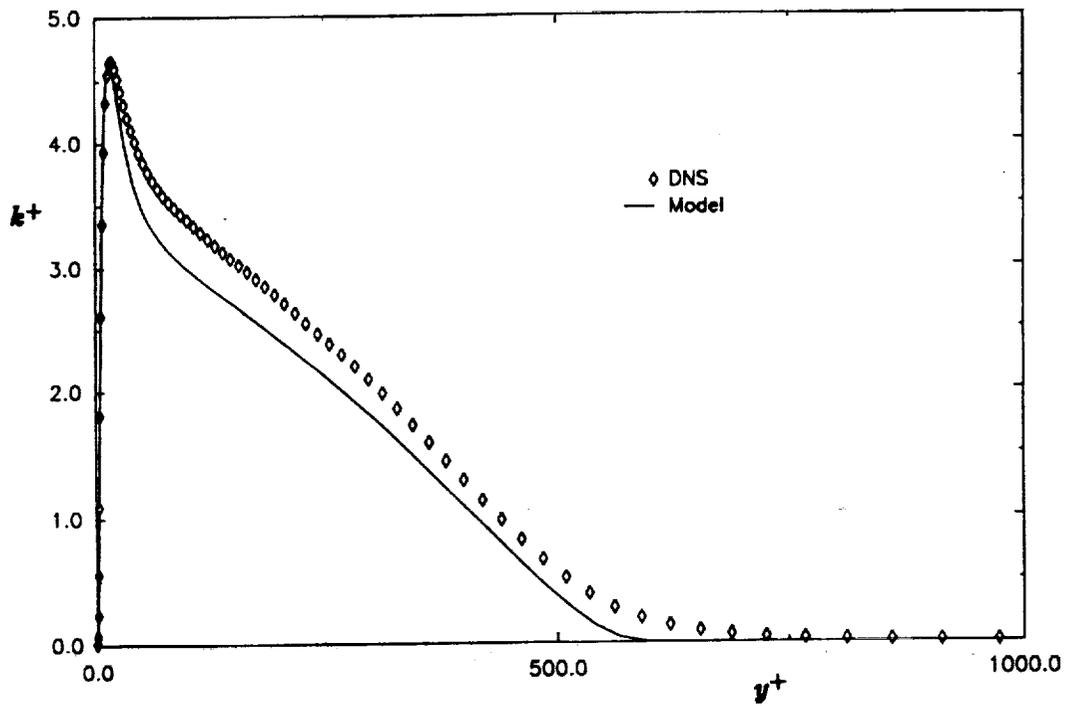


Figure 3. Turbulent kinetic energy for flat plate boundary layer,  $Re_\theta = 1410$ .

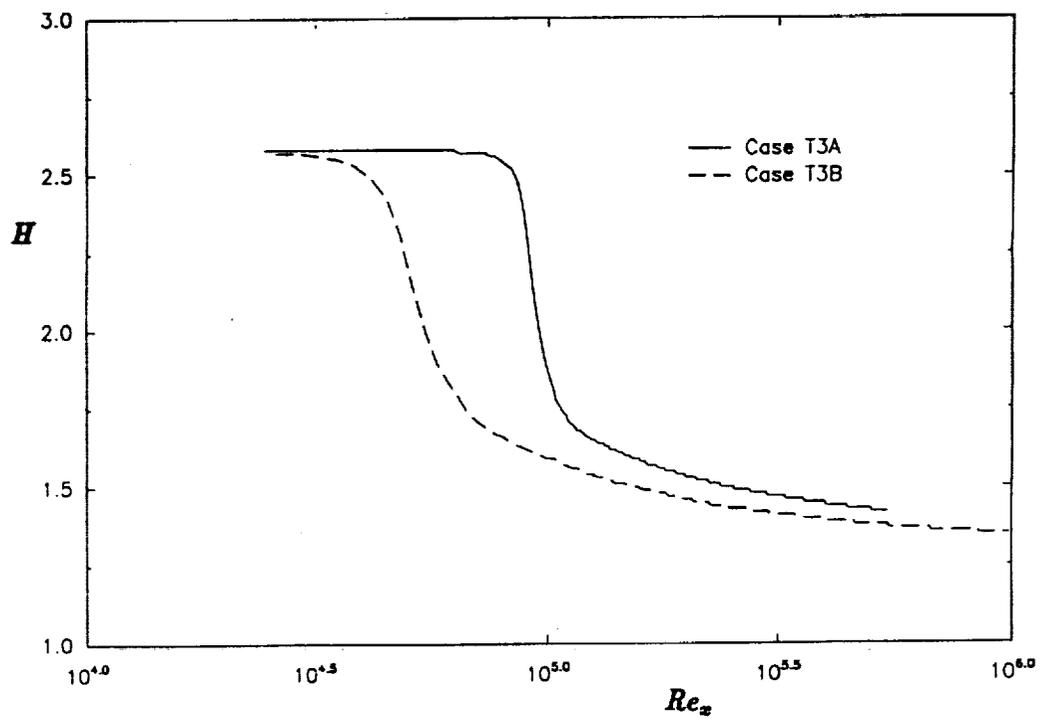


Figure 4. Distribution of the shape factor. Calculation starts at  $Re_\theta = 100$ .

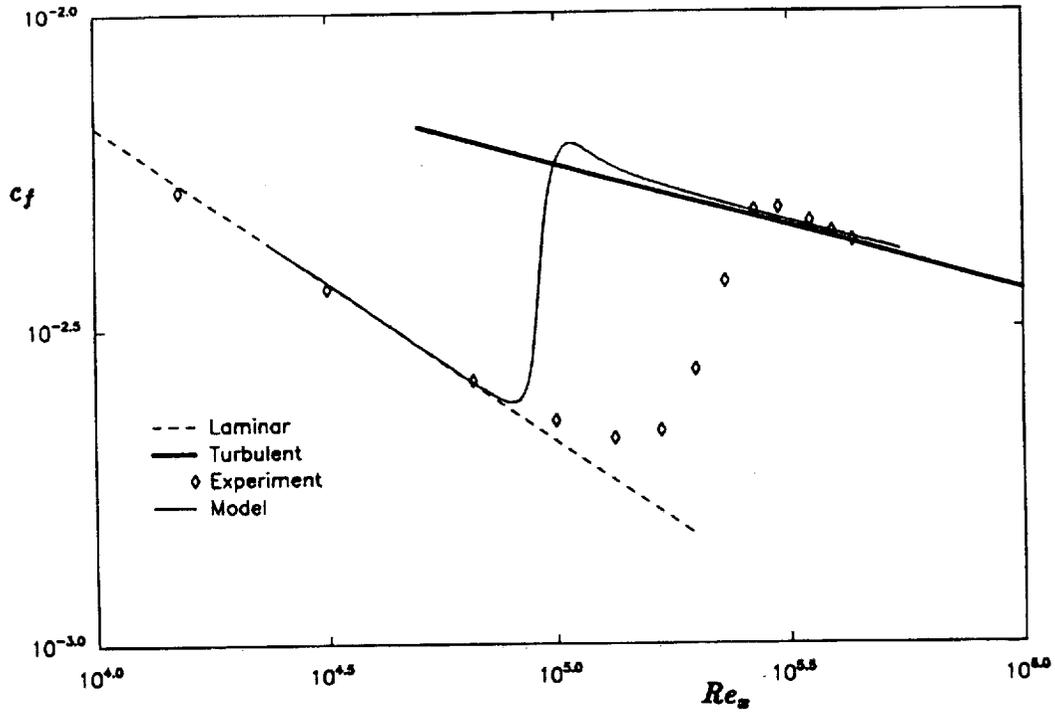


Figure 5. Distribution of the skin friction coefficient for Case T3A.  
Calculation starts at  $Re_\theta = 100$ .

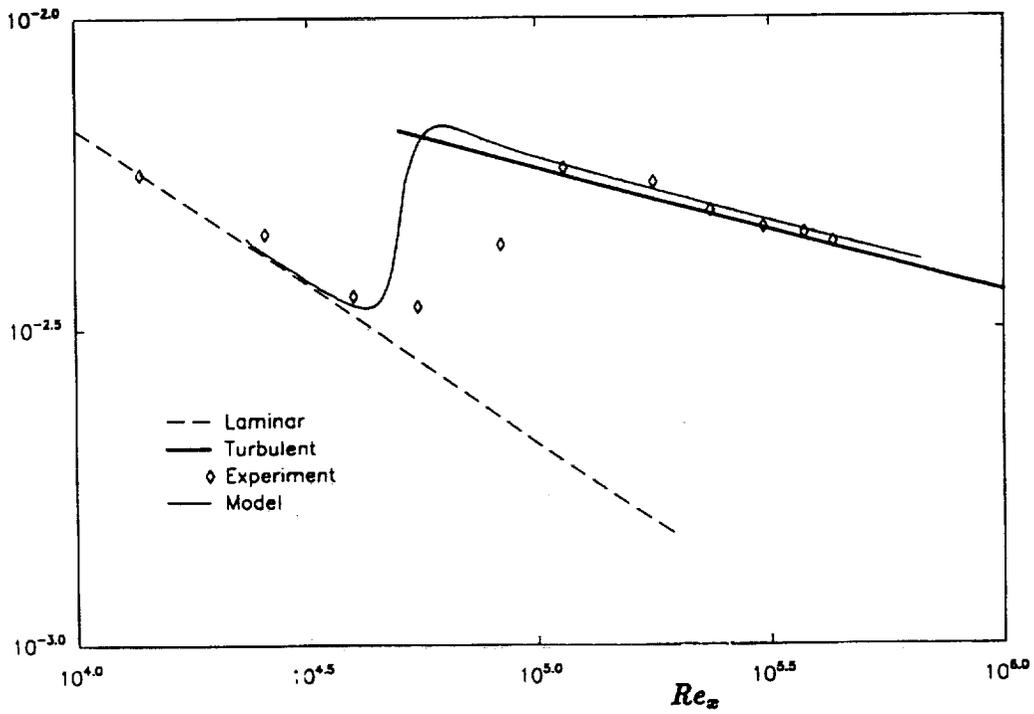


Figure 6. Distribution of the skin friction coefficient for Case T3B.  
Calculation starts at  $Re_\theta = 100$ .

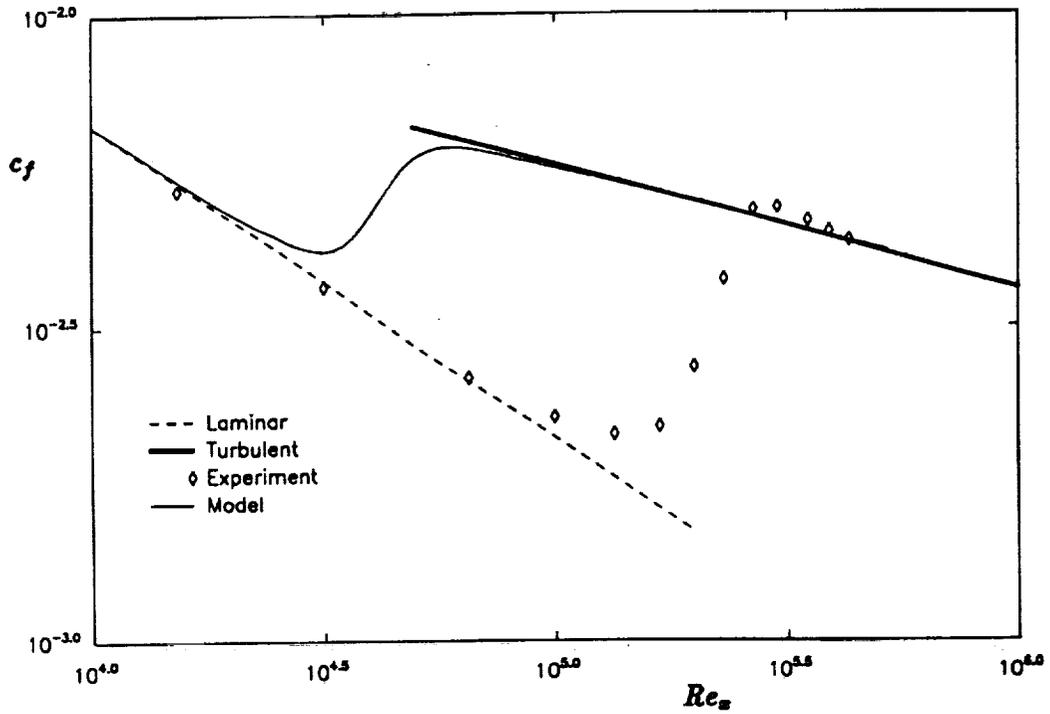


Figure 7. Distribution of the skin friction coefficient for Case T3A.  
Calculation starts at  $Re_\theta = 10$ .

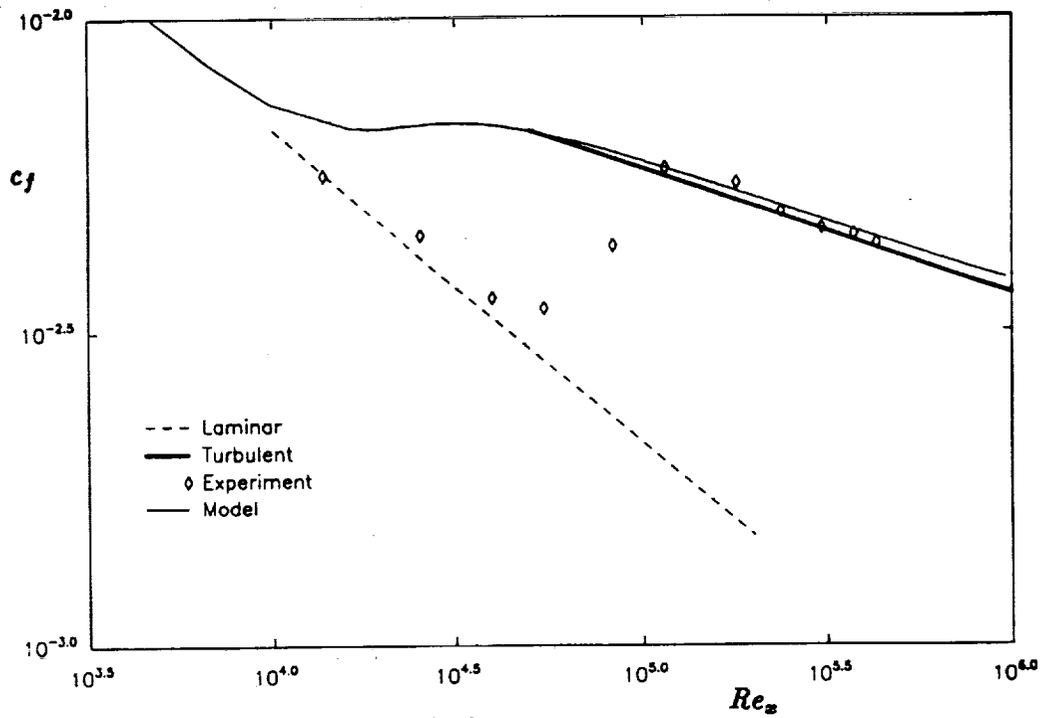


Figure 8. Distribution of the skin friction coefficient for Case T3B.  
Calculation starts at  $Re_\theta = 10$ .



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